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Debye mass of photon and graviton in de Sitter space

Damir Sadekov

MIPT

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Plan

- Set up. Definitions of 'Debye masses'
- Polarization operator in curved space-time
- General properties of masses in the curved background
- Expanding Poincare patch of dS
- What's about graviton?

Setup for the problem. Definitions of 'Debye masses'

We start from a scalar QED on a some gravitational background (which is diagonal and depends only on time):

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + ieA_\mu)\,\phi|^2 - m^2|\phi|^2\right], \quad g_{\mu\nu} = a^2(t)\eta_{\mu\nu} \tag{1}$$

Integrating out the scalar fields the effective equation is as follows:

$$\Box_x A_\mu(x,t) + \int_{-\infty}^{t} dt' d^3y\sqrt{-g\,(t')}\Pi_{\mu\nu'}\left(x,t \mid y,t'\right) g^{\nu'\nu}\left(t'\right) A_\nu\left(y,t'\right) = 0 \tag{2}$$

with the polarization operator:

$$\Pi_{\mu\nu}\left(x,t \mid y,t'\right) = i\theta\left(t-t'\right)\left\langle\Psi\,\middle|\left[J_\mu(x,t), J_\nu\left(y,t'\right)\right]\middle|\,\Psi\right\rangle. \tag{3}$$

Or, supposing approximately that $A_\nu(x,t) \simeq A_\nu(y,t')$:

$$\Box_x A_\mu(x,t) + A_\nu(x,t)\int_{-\infty}^{t} dt' d^{d-1}y\sqrt{-g\,(t')}\Pi_{\mu\nu'}\left(x,t \mid y,t'\right) g^{\nu'\nu}\left(t'\right) = 0. \tag{4}$$

The notion of mass depends on polarization. Use $SO(3)$ symmetry and Ward identities o write:

$$\Pi_{00}\left(q,t,t'\right)=C_E\left(q,t,t'\right)=M_E\left(q,t,t'\right),\quad \Pi_{0\alpha}=\nabla_t M_E\left(q,t,t'\right)\frac{q_\alpha}{q^2},$$
$$\Pi_{\alpha\beta}=-C_M\left(q,t,t'\right)\left(\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{q^2}\right)+\nabla_t^2 M_E\left(q,t,t'\right)\frac{q_\alpha q_\beta}{q^4}, \tag{5}$$

where $\nabla_t f(t)=\partial_t\left(\frac{1}{a^2(t)}f(t)\right)$. Then in the Coloumb gauge:

$$\Box_p A_0(q,t)+A_0(q,t)\int_{-\infty}^{t}dt'\sqrt{-g}g^{00}C_E\left(q,t,t'\right)=0,$$
$$\Box_p A_\alpha(q,t)+A_\alpha(q,t)\int_{-\infty}^{t}dt'\sqrt{-g}g^{00}C_M\left(q,t,t'\right)=0.$$

Hence we can introduce two notions of mass:

$$m^2_{Deb}(t)=\lim_{q\to 0}\int_{-\infty}^{t}dt'\sqrt{-g}g^{00}C_E\left(q,t,t'\right),\quad m^2_{mag}(t)=\lim_{q\to 0}\int_{-\infty}^{t}dt'\sqrt{-g}g^{00}C_M\left(q,t,t'\right) \tag{6}$$

and, if there exists the limit $t\to+\infty$:

$$m^2_{Deb,mag}=\lim_{t\to\infty}m^2_{Deb,mag}(t). \tag{7}$$

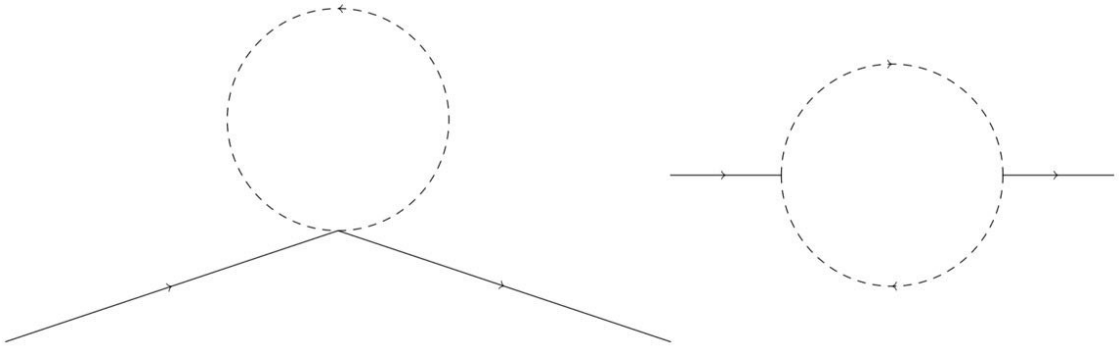

Figure: The one-loop contributions to the polarization operator. The dashed lines correspond to the scalar field. The solid ones are photon propagators.

General properties of magnetic and Debye masses in the curved background

Make the decomposition of the field ϕ through creation and annihilation operators:

$$\phi = \int \frac{d^3p}{(2\pi)^3} \left[a_p e^{ipx} f_p(t) + b_p^\dagger f_p^*(t) e^{-ipx} \right], \quad \left[a_p, a_q^\dagger \right] = \left[b_p, b_q^\dagger \right] = (2\pi)^3 \delta(p-q). \tag{8}$$

$$\left[\partial_t^2 + 2\partial_t \log a(t) \partial_t + p^2 + m^2 a^2(t) \right] f_p(t) = 0. \tag{9}$$

Then the direct computation gives:

$$m_{\text{Deb}}^2 = \lim_{q\to 0} 2e^2 \int_{-\infty}^{t} dt \sqrt{-g} g^{00} \int \frac{d^3k}{(2\pi)^3} \operatorname{Im} \left[\left(f_{p+q}(t) \overleftrightarrow{\partial}_t f_p(t) \right) \left(f_{p+q}^*(t') \overleftrightarrow{\partial}_t f_p^*(t') \right) \right]; \tag{10}$$

$$m_{\text{mag}}^2 \delta_{\alpha\beta} = 8e^2 \int \frac{d^3p}{(2\pi)^3} p_\alpha p_\beta \int_{-\infty}^{t} dt' \sqrt{-g} g^{00} \operatorname{Im} \left[f_p^2(t) f_p^{*2}(t') \right] + 2e^2 \delta_{\alpha\beta} \int \frac{d^3p}{(2\pi)^3} |f_p(t)|^2 . \tag{11}$$

Interesting, that for any choosing modes and background:

$$m_{\text{mag}}^2 \delta_{\alpha\beta} \equiv 0. \tag{12}$$

For Debye mass we have representation with the commutator:

$$m_{Deb}^2 \propto \lim_{q\to 0} \int_{-\infty}^{t} dt' \sqrt{-g} g^{00} \left\langle \Psi \left| \left[J_0(t,q), J_0\left(t', -q\right) \right] \right| \Psi \right\rangle . \tag{13}$$

Because of the relation with total charge $\lim_{q\to 0} J_0(t,q) \propto Q$, we conclude that m_{Deb}^2 is non-zero if the integral has divergences. For example, for alpha-vacuums in Minkowski space-time:

$$\begin{gathered} f_p(t) = \frac{1}{\sqrt{2\omega_p}} \left[\alpha_p e^{-i\omega_p t} + \beta_p e^{i\omega_p t} \right], \quad |\alpha_p|^2 - |\beta_p|^2 = 1, \\ m_{Deb}^2 = 2e^2 \lim_{q\to 0} \int \frac{d^3 p}{(2\pi)^3} \frac{|\alpha_p|^2 |\beta_{p+q}|^2 - |\alpha_{p+q}|^2 |\beta_p|^2}{\omega_{p+q} - \omega_p} = -2e^2 \int \frac{d^3 p}{(2\pi)^3} \frac{\partial |\beta_p|^2}{\partial \omega_p}. \end{gathered} \tag{14}$$

Expanding Poincare Patch

$$ds^2 = \frac{d\eta^2 - d\vec{x}^2}{\eta^2}, \quad \eta \in (0, +\infty) \tag{15}$$

We will consider the case $m > \frac{D-1}{2}$:

$$f_p(\eta) = \eta^{\frac{d-1}{2}} h_{i\mu}(p\eta) = \eta^{\frac{d-1}{2}} \left[\alpha H^{(1)}_{i\mu}(p\eta) + \beta H^{(2)}_{i\mu}(p\eta)\right], \quad |\alpha|^2 - |\beta|^2 = 1. \tag{16}$$

Then

$$m^2_{\text{Deb}} =$$

$$= \lim_{q\to 0} 2e^2 \int_\eta^\infty d\eta' \eta^3 \eta' \int \frac{d^3p}{(2\pi)^3} \operatorname{Im}\left[\left(h_{i\mu}(p\eta) \overleftrightarrow{\partial_\eta} h_{i\mu}(|p+q|\eta)\right)\left(h^*_{i\mu}(p\eta') \overleftrightarrow{\partial_{\eta'}} h^*_{i\mu}(|p+q|\eta')\right)\right]. \tag{17}$$

The result is as follows:

$$m^2_{\text{Deb}} = -\frac{32e^2}{\pi} \int \frac{d^3p}{(2\pi)^3} \operatorname{Im}\left[\alpha^*\beta^*\left(h_{i\mu}(p)h'_{i\mu}(p) - p\left(h'_{i\mu}(p)\right)^2 + p h_{i\mu}(p) h''_{i\mu}(p)\right)\right]. \tag{18}$$

Note, that for Bunch Davies vacuum $m^2_{\text{Deb, BD}} = 0$. However, it is divergent and may be negative or positive for specific chosen alpha-vacuums.

Breaking dS invariance

Consider the situation when the scale parameter $a(t)$ is switched off in a while:

$$a(t) = e^{T \tanh\left(\frac{t}{T}\right) - T} \tag{19}$$

After the expansion phase the system will be in an excited state with level population (in the limit $T \to \infty$)

$$n_p = \begin{cases} 0, |p| > \mu, |p| < \mu e^{-2T} \\ \frac{1}{e^{2\pi\mu}-1}, \mu e^{-2T} < |p| < \mu \end{cases}, \tag{20}$$

where $\mu = \sqrt{m^2 - \left(\frac{D-1}{2}\right)^2}$. In such a case

$$m^2_{\text{Deb}} = \frac{e^2\mu^2}{2\pi^2}\frac{1}{e^{2\pi\mu}-1}. \tag{21}$$

Compare this to the situation with the strong electric field instead of dS:

$$m^2_{\text{Deb}} = \frac{e^2}{4\pi^2} e^{-\frac{\pi m^2}{eE}} (eET)^2. \tag{22}$$

Hence, any electric field is screened, including the initial impulse, which indicates unphysical situation – we have to take into account the backreaction of the scalar field.

What's about graviton?

$$S[g_{\mu\nu},\phi] = -\frac{1}{16\pi G}\int d^D x\sqrt{|g|}\left[R + 2\Lambda_{dS}\right] + \frac{1}{2}\int d^D x\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right], \tag{23}$$

Consider the small perturbation of the metric around dS:

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + h_{\mu\nu}, \;\; \hat{g}_{\mu\nu} = \frac{1}{H^2\eta^2}\mathrm{diag}\left(1,-1,\ldots,-1\right). \tag{24}$$

Then interaction terms of the scalar field with this perturbation have the form:

$$\Delta S = -\int d^D x\sqrt{|\hat{g}|}h^{\mu\nu}T_{\mu\nu} + \int d^D x\sqrt{|\hat{g}|}h^{\mu\nu}_{cl}\Gamma_{\mu\nu|\alpha\beta}h^{\alpha\beta}_{q}, \tag{25}$$

where

$$T_{\mu\nu} = -\frac{1}{2}\hat{g}_{\mu\nu}\left(\hat{g}^{\lambda\omega}\partial_\lambda\phi\partial_\omega\phi - m^2\phi^2\right) + \partial_\mu\phi\partial_\nu\phi, \tag{26}$$

$$\Gamma_{\mu\nu|\alpha\beta} =$$
$$= \frac{1}{8}\left(\hat{g}_{\alpha\beta}\hat{g}_{\mu\nu} - 2\hat{g}_{\mu\alpha}\hat{g}_{\nu\beta}\right)\left[\hat{g}^{\lambda\omega}\partial_\lambda\phi^{cl}\partial_\omega\phi^{cl} - m^2\phi^{cl\,2}\right] - \frac{1}{2}\hat{g}_{\alpha\beta}\partial_\mu\phi^{cl}\partial\nu\phi^{cl} + \frac{1}{2}\hat{g}_{\mu\alpha}\partial_\beta\phi^{cl}\partial_\nu\phi^{cl}. \tag{27}$$

We will use Schwinger-Keldysh diagrammatic technique with the propagators

$$G(x,x') = \langle \mathcal{T}_{\mathcal{C}} \varphi(x)\varphi(x')\rangle = F(x,x') - \frac{i}{2}\mathrm{sign}_{\mathcal{C}}(\eta-\eta')\rho(x,x'). \tag{28}$$

Then the effective equation of motion in the momentum space over the d space-coordinates has the form:

$$\frac{1}{16\pi G}\hat{D}_{\mu\nu|\alpha\beta}h^{\mu\nu}(\vec{k},\eta)+$$
$$+\frac{1}{2}\int_{\eta}^{\infty}\frac{d\eta'}{H^D\eta'^D}\Pi^{\mathrm{bub}}_{\mu\nu|\alpha\beta}\left(\vec{k}|\eta',\eta\right)h^{\mu\nu}(\vec{k},\eta')+\Pi^{\mathrm{tad}}_{\mu\nu|\alpha\beta}(\eta)\,h^{\mu\nu}(\vec{k},\eta)=0, \tag{29}$$

where the operator $\hat{D}_{\mu\nu|\alpha\beta}$ appears due to the Einstein-Gilbert part of the action.

Tensor structure

General SO(D-1)-invariant tensor structure in this situation:

$$\Pi^{\text{bub}}_{00|00} = a, \quad \Pi^{\text{bub}}_{00|0k} = i\frac{k_k}{k^2}b,$$

$$\Pi^{\text{bub}}_{00|kl} = f_1'\delta^{\perp}_{kl} + f_2'\frac{k_k k_l}{k^4} = f_1\delta^{\perp}_{kl} + f_2\frac{k_k k_l}{k^4} + f\left(\delta_{kl} - (D-1)\frac{k_k k_l}{k^2}\right),$$

$$\Pi^{\text{bub}}_{0i|0k} = c_1\delta^{\perp}_{ik} + c_2\frac{k_i k_k}{k^4},$$

$$\Pi^{\text{bub}}_{0i|kl} = i\frac{k_i}{k^2}d_1'\delta^{\perp}_{kl} + id_2'\frac{k_i k_k k_l}{k^6} + id_3\left[\delta^{\perp}_{il}\frac{k_k}{k^2} + \delta^{\perp}_{ik}\frac{k_l}{k^2}\right] =$$

$$= i\frac{k_i}{k^2}d_1\delta^{\perp}_{kl} + id_2\frac{k_i k_k k_l}{k^6} + id\left(\delta_{kl} - (D-1)\frac{k_k k_l}{k^2}\right) + id_3\left[\delta^{\perp}_{il}\frac{k_k}{k^2} + \delta^{\perp}_{ik}\frac{k_l}{k^2}\right],$$

$$\Pi^{\text{bub}}_{ij|kl} = e_1\delta^{\perp}_{ij}\delta^{\perp}_{kl} + \left[-\overline{e}_2\frac{k_i k_j}{k^4}\delta^{\perp}_{kl} + e_2\frac{k_k k_l}{k^4}\delta^{\perp}_{ij}\right] + e_3\frac{k_i k_j k_k k_l}{k^8} +$$

$$+ e_4\left[\delta^{\perp}_{ik}\frac{k_j k_l}{k^4} + \delta^{\perp}_{jk}\frac{k_i k_l}{k^4} + \delta^{\perp}_{il}\frac{k_j k_k}{k^4} + \delta^{\perp}_{jl}\frac{k_i k_k}{k^4}\right] +$$

$$+ e_5\left[\delta^{\perp}_{ik}\delta^{\perp}_{jl} + \delta^{\perp}_{il}\delta^{\perp}_{jk}\right], \tag{30}$$

where we've also denoted e.g. $\overline{a}(\vec{k}|\eta,\eta') = a(\vec{k}|\eta',\eta)$ and $\delta^{\perp}_{kl} = \delta_{kl} - \frac{k_k k_l}{k^2}$.

The Ward identities

$$\begin{cases} \nabla_{\eta'} \Pi^{\text{bub}}_{\mu\nu|00} - (-ik_k)\Pi^{\text{bub}}_{\mu\nu|0k} + \frac{1}{\eta'}\left[\Pi^{\text{bub}}_{\mu\nu|00} - \Pi^{\text{bub}}_{\mu\nu|kk}\right] = 0, \\ \nabla_{\eta'} \Pi^{\text{bub}}_{\mu\nu|0k} - (-ik_l)\Pi^{\text{bub}}_{\mu\nu|kl} = 0, \end{cases} \tag{31}$$

reduces the number of coefficients to 7 independent ones $A_1, A_2, A_3, B_1, B_2, c_1, e_5$. Solution:

$$a = A_1 - A_2 + \frac{m^2}{\eta'^2} A_3, \; b = \nabla_{\eta'} a - \frac{D-2}{\eta'}\left(A_1 + A_2 - \frac{m^2}{\eta'^2} A_3\right) + \frac{2}{\eta'}\frac{m^2}{\eta'^2} A_3,$$

$$c_2 = -\nabla_{\eta'} \overline{b} + \frac{D-2}{\eta'}\overline{b} + \frac{D-1}{\eta'} B_1 + \frac{D-2}{\eta'} B_2,$$

$$f_1 = A_1 + A_2 - \frac{m^2}{\eta'^2} A_3, \; f_2 = k^2\left[A_1 - A_2 - \frac{m^2}{\eta'^2} A_3\right],$$

$$f = \frac{1}{k^2}\frac{1}{D-2}\left(f_2 + \nabla_{\eta'} b\right), \; \left(f_1' = f_1 + f, \; f_2' = -\nabla_{\eta'} b\right),$$

$$d_1 = \overline{b} + B_1 + B_2, \; d_2 = k^2\left(\overline{b} + B_1\right), \; d_3 = \nabla_{\eta'} c_1,$$

$$d = \frac{1}{k^2}\frac{1}{D-2}\left(d_2 - \nabla_{\eta'} c_2\right), \; \left(d_1' = d_1 + d, \; d_2' = \nabla_{\eta'} c_2\right),$$

$$e_2 = -\nabla_{\eta'} \overline{d}_1, \; e_3 = -\nabla_{\eta'} \overline{d}_2, \; e_4 = -\nabla_{\eta'} \overline{d}_3,$$

$$e_1 = -\frac{2}{D-2} e_5 - \frac{1}{k^2}\frac{1}{D-2} e_2 - \frac{1}{D-2}\overline{f_1'} - \eta'\left(\nabla_{\eta'} \overline{f_1'} + \overline{d_1'}\right), \tag{32}$$

And for the tadpole diagram:

$$\Pi^{\text{tad}}_{00|00}(\eta) = -\frac{1}{8}\int_p \left[4\partial_\eta \partial_{\eta'} F\Big|_{\eta'=\eta} - \left(p^2 + \frac{m^2}{\eta^2}\right) F\right] =: \pi_1(\eta),$$

$$\Pi^{\text{tad}}_{00|0k} = \Pi^{\text{tad}}_{0i|kl} = 0,$$

$$\Pi^{\text{tad}}_{0i|0k} = \frac{1}{8}\int_p \left[\partial_\eta \partial_{\eta'} F\Big|_{\eta'=\eta} + \left(\frac{D-3}{D-1}p^2 + \frac{m^2}{\eta^2}\right) F\right] \delta_{ik} =: \pi_2(\eta)\delta_{ik},$$

$$\Pi^{\text{tad}}_{00|kl} = \pi_2(\eta)\delta_{kl},$$

$$\Pi^{\text{tad}}_{ij|kl} = \frac{1}{8}\int_p \left[\partial_\eta \partial_{\eta'} F\Big|_{\eta'=\eta} - \left(\frac{D-5}{D-1}p^2 + \frac{m^2}{\eta^2}\right) F\right] \times \left(\delta_{ij}\delta_{kl} - \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right) =$$

$$=: \pi_3(\eta)\left(\delta_{ij}\delta_{kl} - \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right). \tag{33}$$

After this, we can write an effective EOM in scalar, vector and tensor sectors of graviton's modes separately:

$$h_{00} = 2\Phi, \quad h_{0k} = ik_k Z + Z_k^T,$$
$$h_{kl} = -2\Psi\delta_{kl} - 2k_k k_l E + i(k_k W_j^T + k_l W_i^T) + h_{kl}^{TT}, \tag{34}$$

where $k_k Z_k^T = k_k W_k^T = k_k h_{kl}^{TT} = 0$ and $h_{kk}^{TT} = 0$. For example (which is the most simple), in tensor sector we have an appropriate definition for the mass:

$$\nabla_\eta \partial_\eta h_{ij}^{TT} + \left(k^2 + m_{TT}^2(\eta)\right) h_{ij}^{TT} \simeq 0, \tag{35}$$

$$m_{TT}^2 = -32\pi G \lim_{\eta\to 0} \lim_{k\to 0} \left[\int_\eta^\infty \frac{d\eta'}{H^{D-2}\eta'^{D-2}} \overline{e}_5\left(\vec{k}|\eta,\eta'\right) - 2\pi_3(\eta)\right]. \tag{36}$$

Discussion

- We considered Debye and magnetic photon masses in the framework of particle creation by external electromagnetic and gravitational fields
- We didn't calculate the higher corrections to the photon mass, but the propagators may receive large IR contributions
- In the similar framework we calculate an appropriate masses for graviton in dS
- It would be interesting to investigate different patches of de Sitter space and other space-time geometries

Thank you for your attention!