$\mathcal{N} = 2$ Supersymmetric Higher Spins from Harmonic Superspace

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Summary and outlook

Supersymmetry and higher spins

- Supersymmetric higher-spin theories are under intensive development for last decades. One of the basic origins of interest in them is that they could provide a bridge between superstring theory and low-energy (super)gauge theories.

- Free massless bosonic and fermionic higher spin field theories have been pioneered in Fronsdal, 1978; Fang, Fronsdal, 1978.

- The natural tools to deal with supersymmetric theories are off-shell superfield methods. In the superfield approach the supersymmetry is closed on the off-shell supermultiplets with the correct sets of the auxiliary fields and so is manifest. Unconstrained superfield formulations are most preferable.

- The component approach to the description of $4D$, $\mathcal{N} = 1$ supersymmetric free massless higher spin models was initiated in Courtright, 1979; Vasiliev, 1980.

- The complete off-shell Lagrangian formulation of $4D$ free higher spin $\mathcal{N} = 1$ models (including those on the AdS background) has been given in terms of $\mathcal{N} = 1$ superfields in a series of works by S. Kuzenko with collaborators (Kuzenko et al, 1993, 1994).

- Until present, an off-shell superfield Lagrangian formulation for higher-spin **extended** supersymmetric theories, with all supersymmetries manifest, was unknown even for free theories.

- This gap was recently filled in I. Buchbinder, E. Ivanov, N. Zaigraev, JHEP 12 (2021) 016, arXiv: 2109.07639 [hep-th]. An off-shell manifestly $\mathcal{N} = 2$ supersymmetric unconstrained formulation of $4D, \mathcal{N} = 2$ superextension of the Fronsdal theory for integer spins was constructed for the first time, based on the harmonic superspace approach.

- Manifestly $\mathcal{N} = 2$ supersymmetric off-shell cubic couplings of $4D, \mathcal{N} = 2$ to the hypermultiplets were given in I. Buchbinder, E. Ivanov, N. Zaigraev, JHEP 05 (2022) 104, arXiv: 2202.08196 [hep-th].
- Our papers open a new area of applications of the harmonic superspace formalism, that time in $\mathcal{N} = 2$ higher-spin theories.

Harmonic superspace

- At present, in four-dimensions, the self-consistent off-shell superfield formalism for $\mathcal{N}=2$ and $\mathcal{N}=3$ theories is the harmonic superspace approach (Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, CQG 1984, 1985).

- Harmonic $\mathcal{N}=2$ superspace:

$$Z=(x^m\,,\ \theta^\alpha_i\,,\ \bar\theta^{\dot\alpha\, j}, u^{\pm i}),\quad u^{\pm i}\in SU(2)/U(1),\ u^{+i}u^-_i=1.$$

- Analytic harmonic $\mathcal{N}=2$ superspace:

$$\zeta_A=(x^m_A,\theta^{+\alpha},\bar\theta^{+\dot\alpha},u^{\pm i}),\ \theta^{+\alpha,\dot\alpha}:=\theta^{\alpha,\dot\alpha i}u^+_i,\ x^m_A:=x^m-2i\theta^{(i}\sigma^m\bar\theta^{j)}u^+_iu^+_j$$

- All basic $\mathcal{N}=2$ superfields are analytic:

$$\underline{\text{SYM}}:\qquad V^{++}(\zeta_A)\,,\ \ \underline{\text{matter hypermultiplets}}:\ q^+(\zeta_A)\,,\ \bar q^+(\zeta_A)$$

$$\underline{\text{supergravity}}:\qquad H^{++m}(\zeta_A)\,,\ H^{++\alpha+}(\zeta_A)\,,\ H^{++5}(\zeta_A)\,.$$

$\mathcal{N} = 2$ spin 1 multiplet

- An instructive example is supplied by Abelian $\mathcal{N} = 2$ gauge theory,

$$V^{++}(\zeta_A)\,, \quad \delta V^{++} = D^{++}\Lambda(\zeta_A)\,,\; D^{++} = \partial^{++} - 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\,.$$

- Wess-Zumino gauge:

$$V^{++}(\zeta_A) = (\theta^+)^2\phi + (\bar{\theta}^+)^2\bar{\phi} + 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}}$$
$$+ (\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i + (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 D^{(ik)}u^-_i\,u^-_k\,.$$

- $4D$ fields ϕ, $\bar{\phi}$, $A_{\alpha\dot{\alpha}}$, ψ^i_α, $\bar{\psi}^i_{\dot{\alpha}}$, $D^{(ik)}$ constitute an Abelian gauge $\mathcal{N} = 2$ off-shell multiplet (8 + 8 off-shell degrees of freedom).

- Invariant action:

$$S \sim \int d^{12}Z\,\left(V^{++}V^{--}\right),\; D^{++}V^{--} - D^{--}V^{++} = 0\,,\; \delta V^{--} = D^{--}\Lambda\,,$$
$$[D^{++}, D^{--}] = D^0, \quad D^0\,V^{\pm\pm} = \pm 2\,V^{\pm\pm}\,.$$

$\mathcal{N}=2$ spin 2: linearized $\mathcal{N}=2$ supergravity

- Analogs of $V^{++}(\zeta_A)$ are the following set of analytic gauge potentials:

$$\Big(h^{++m}(\zeta_A)\,,\ h^{++5}(\zeta_A)\,,\ h^{++\hat{\mu}+}(\zeta_A)\Big),\quad \hat{\mu}=(\mu\,,\dot{\mu})\,,$$
$$\delta_\lambda h^{++m}=D^{++}\lambda^m+2i\big(\lambda^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}}+\theta^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big)\,,$$
$$\delta_\lambda h^{++5}=D^{++}\lambda^5-2i\big(\lambda^{+\alpha}\theta^+_\alpha-\bar{\theta}^+_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big),\delta_\lambda h^{++\hat{\mu}+}=D^{++}\lambda^{+\hat{\mu}}\,.$$

- Wess-Zumino gauge:

$$h^{++m}=-2i\theta^+\sigma^a\bar{\theta}^+\Phi^m_a+\big[(\bar{\theta}^+)^2\theta^+\psi^{m\,i}u^-_i+c.c.\big]+\dots$$
$$h^{++5}=-2i\theta^+\sigma^a\bar{\theta}^+C_a+\dots\,,\quad h^{++\mu+}=\dots$$

- The residual gauge freedom:

$$\lambda^m\ \Rightarrow\ a^m(x)\,,\ \lambda^5\ \Rightarrow\ b(x)\,,\ \lambda^{\mu+}\ \Rightarrow\ \epsilon^{\mu i}(x)u^+_i+\theta^{+\nu}l_{(\nu}{}^{\mu)}(x)\,.$$

- The physical fields are $\Phi^m_a,\psi^{m\,i}_\mu,C_a$ ($(\mathbf{2},\mathbf{3}/\mathbf{2},\mathbf{3}/\mathbf{2},\mathbf{1})$ on shell). The spin 1 part of Φ^m_a is gauged away by the local "Lorentz" parameters $l_{(\nu}{}^{\mu)}(x),l_{(\dot{\nu}}{}^{\dot{\mu})}(x)$:

$$\Phi^m_a\sim\Phi_{\beta\dot{\beta}\alpha\dot{\alpha}}\Rightarrow\Phi_{(\beta\alpha)(\dot{\beta}\dot{\alpha})}+\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\Phi\,.$$

- The invariant action:

$$S \sim \int d^4x d^8\theta du \Big(G^{++\alpha\dot\alpha} G^{--}_{\alpha\dot\alpha} + G^{++5} G^{--5} \Big),$$
$$G^{++\mu\dot\mu} := h^{++\mu\dot\mu} + 2i\big(h^{++\mu+}\bar\theta^{-\dot\mu} + \theta^{-\mu} h^{++\dot\mu+}\big)\,,$$
$$G^{++5} := h^{++5} - 2i\big(h^{++\mu+}\theta^-_\mu - \bar\theta^-_{\dot\mu} h^{++\dot\mu+}\big)\,,$$
$$D^{++} G^{--\mu\dot\mu} = D^{--} G^{++\mu\dot\mu}\,, \quad D^{++} G^{--5} = D^{--} G^{++5}\,.$$

- After passing to components, the spin 2 part of the Lagrangian reads:

$$G^{++\alpha\dot\alpha}_{(\Phi)} G^{--}_{(\Phi)\alpha\dot\alpha} + G^{++5}_{(\Phi)} G^{--5}_{(\Phi)} \quad \Rightarrow$$
$$\mathcal{L}_{(\Phi)} = -\frac{1}{4}\Big[\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\Box\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - \Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\partial_{\alpha\dot\alpha}\partial^{\rho\dot\rho}\Phi_{(\rho\beta)(\dot\rho\dot\beta)}$$
$$+2\,\Phi\partial^{\alpha\dot\alpha}\partial^{\beta\dot\beta}\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - 6\Phi\Box\Phi\Big].$$

- It is invariant under $\delta\Phi_{\beta\dot\beta\alpha\dot\alpha} = \frac{1}{2}\left(\partial_{\alpha\dot\alpha} a_{\beta\dot\beta} + \partial_{\beta\dot\beta} a_{\alpha\dot\alpha}\right)$, $\delta\Phi = \frac{1}{4}\partial_{\alpha\dot\alpha} a^{\alpha\dot\alpha}$.

$\mathcal{N} = 2$ spin 3 and higher spins

A generalization to higher integer spin $\mathcal{N} = 2$ supermultiplets goes rather straightforwardly. The spin 3 example is significative.

- The spin 3 triad of analytic gauge superfields is introduced as :

$$h^{++(\alpha\beta)(\dot\alpha\dot\beta)}(\zeta)\,,\ h^{++\alpha\dot\alpha}(\zeta),\ h^{++(\alpha\beta)\dot\alpha+}(\zeta),\ h^{++(\dot\alpha\dot\beta)\alpha+}(\zeta)\,,$$

  and has the following transformation laws, with the analytic gauge parameters:

$$\delta h^{++(\alpha\beta)(\dot\alpha\dot\beta)} = D^{++}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 2i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{+\dot\beta)} + \theta^{+(\alpha}\bar\lambda^{+\beta)(\dot\alpha\dot\beta)}\big],$$

$$\delta h^{++\alpha\dot\alpha} = D^{++}\lambda^{\alpha\dot\alpha} - 2i\big[\lambda^{+(\alpha\beta)\dot\alpha}\theta^{+}_{\beta} + \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\bar\theta^{+}_{\dot\beta}\big],$$

$$\delta h^{++(\alpha\beta)\dot\alpha+} = D^{++}\lambda^{+(\alpha\beta)\dot\alpha}\,,\ \delta h^{++(\dot\alpha\dot\beta)\alpha+} = D^{++}\lambda^{+(\dot\alpha\dot\beta)\alpha}\,.$$

- The bosonic physical fields in the WZ gauge are collected in

$$\begin{aligned} h^{++(\alpha\beta)(\dot\alpha\dot\beta)} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho} + \dots \\ h^{++\alpha\dot\alpha} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + \dots \end{aligned}$$

- The physical gauge fields are $\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho}$ (spin 3 gauge field), $C^{\alpha\dot\alpha}_{\rho\dot\rho}$ (spin 2 gauge field) and $\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}_{\gamma}$ (spin 5/2 gauge field). The rest of fields are auxiliary. On shell, we end up with the multiplet $(\mathbf{3}, \mathbf{5/2}, \mathbf{5/2}, \mathbf{2})$.

- Some residual gauge freedom can be used to put the physical bosonic gauge fields into the irreducible form

$$\Phi_{\gamma\dot\gamma(\alpha\beta)(\dot\alpha\dot\beta)} = \Phi_{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)} + \varepsilon_{\dot\gamma(\dot\alpha}\varepsilon_{\gamma(\beta}\Phi_{\alpha)\dot\beta)}\,,$$
$$C_{\gamma\dot\gamma\alpha\dot\alpha} = C_{(\gamma\alpha)(\dot\gamma\dot\alpha)} + \varepsilon_{\gamma\alpha}\varepsilon_{\dot\gamma\dot\alpha}C\,,$$

with the following gauge transformations

$$\delta\Phi_{(\alpha\gamma\beta)(\dot\alpha\dot\gamma\dot\beta)} = \partial_{(\beta(\dot\beta}a_{\alpha\gamma)\dot\alpha\dot\gamma)}\,, \quad \delta\Phi_{\alpha\dot\beta} = \frac{4}{9}\partial^{\gamma\dot\gamma}a_{(\alpha\gamma)(\dot\beta\dot\gamma)}\,,$$
$$\delta C_{(\alpha\beta)(\dot\alpha\dot\beta)} = \partial_{(\beta(\dot\beta}b_{\alpha)\dot\alpha)}, \quad \delta C = \frac{1}{4}\partial_{\alpha\dot\alpha}b^{\alpha\dot\alpha}.$$

- These are just the correct gauge transformations for the Fronsdal spin 3 fields ($\Phi_{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)}, \Phi_{\alpha\dot\beta}$) and spin 2 fields ($C_{(\alpha\beta)(\dot\alpha\dot\beta)}, C$).

- The invariant superfield action is constructed literally on the pattern of the spin 2 case

$$S_{s=3} = \int d^4x d^8\theta du \left\{ G^{++(\alpha\beta)(\dot\alpha\dot\beta)} G^{--}_{(\alpha\beta)(\dot\alpha\dot\beta)} + G^{++\alpha\dot\beta} G^{--}_{\alpha\dot\beta} \right\},$$

with

$$G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = h^{++(\alpha\beta)(\dot\alpha\dot\beta)} + 2i\left[h^{++(\alpha\beta)(\dot\alpha+}\bar\theta^{-\dot\beta)} - h^{++(\dot\alpha\dot\beta)(\alpha+}\theta^{-\beta)}\right],$$

$$G^{++\alpha\dot\beta} = h^{++\alpha\dot\beta} - 2i\left[h^{++(\alpha\beta)\dot\beta+}\theta^-_\beta - \bar\theta^-_{\dot\alpha} h^{++(\dot\alpha\dot\beta)\alpha+}\right],$$

$$D^{++} G^{--(\alpha\beta)(\dot\alpha\dot\beta)} - D^{--} G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,,\ D^{++} G^{--\alpha\dot\beta} - D^{--} G^{++\alpha\dot\beta} = 0.$$

- The component spin 3 bosonic action reads

$$\begin{aligned} S_{(s=3)} = \int d^4x \Big\{ & \Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \Box \Phi_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \\ & - \frac{3}{2} \Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \partial_{\alpha_1\dot\alpha_1} \partial^{\rho\dot\rho} \Phi_{(\rho\alpha_2\alpha_3)(\dot\rho\dot\alpha_2\dot\alpha_3)} \\ & + 3\Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \partial_{\alpha_1\dot\alpha_1} \partial_{\alpha_2\dot\alpha_2} \Phi_{\alpha_3\dot\alpha_3} - \frac{15}{4} \Phi^{\alpha\dot\alpha} \Box \Phi_{\alpha\dot\alpha} \\ & + \frac{3}{8} \partial_{\alpha_1\dot\alpha_1} \Phi^{\alpha_1\dot\alpha_1} \partial_{\alpha_2\dot\alpha_2} \Phi^{\alpha_2\dot\alpha_2} \Big\}. \end{aligned}$$

- The general case with the maximal spin **s** is spanned by the following analytic gauge potentials

$$h^{++\alpha(s-1)\dot\alpha(s-1)}(\zeta), h^{++\alpha(s-2)\dot\alpha(s-2)}(\zeta), h^{++\alpha(s-1)\dot\alpha(s-2)+}(\zeta), h^{++\dot\alpha(s-1)\alpha(s-2)+}(\zeta),$$

  where $\alpha(s) := (\alpha_1 \dots \alpha_s), \dot\alpha(s) := (\dot\alpha_1 \dots \dot\alpha_s)$.
- The relevant gauge transformations can also be defined and shown to leave, in the WZ-like gauge, the physical field multiplet $(\mathbf{s}, \mathbf{s}-\mathbf{1}/\mathbf{2}, \mathbf{s}-\mathbf{1}/\mathbf{2}, \mathbf{s}-\mathbf{1})$.
- The invariant action has the universal form for any **s**

$$S_{(s)} = (-1)^{s+1} \int d^4x d^8\theta du \Big\{ G^{++\alpha(s-1)\dot\alpha(s-1)} G^{--}_{\alpha(s-1)\dot\alpha(s-1)} + G^{++\alpha(s-2)\dot\alpha(s-2)} G^{--}_{\alpha(s-2)\dot\alpha(s-2)} \Big\},$$

  where

$$\begin{aligned} G^{\pm\pm\alpha(s-1)\dot\alpha(s-1)} &= h^{\pm\pm\alpha(s-1)\dot\alpha(s-1)} + 2i\big[h^{\pm\pm\alpha(s-1)(\dot\alpha(s-2)+}\bar\theta^{-\dot\alpha_{s-1})} \\ &\quad - h^{\pm\pm\dot\alpha(s-1)(\alpha(s-2)+}\theta^{-\alpha_{s-1})}\big], \\ G^{\pm\pm\alpha(s-2)\dot\alpha(s-2)} &= h^{\pm\pm\alpha(s-2)\dot\alpha(s-2)} - 2i\big[h^{\pm\pm(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)+}\theta^-_{\alpha_{s-1}} \\ &\quad + h^{\pm\pm\alpha(s-2)(\dot\alpha(s-2)\dot\alpha_{(s-1)})+}\bar\theta^-_{\alpha_{s-1}}\big], \end{aligned}$$

  and the negatively charged potentials are related to the basic ones by the appropriate harmonic zero-curvature conditions.

Hypermultiplet couplings

- The construction of interactions in the theory of higher spins is a very important (albeit difficult) task. The simplest higher spin interaction is described by a cubic vertex, e.g., bilinear in the matter fields and of the first order in gauge fields. At present, there is an extensive literature related to the construction of cubic higher spin interactions (e.g., Bengtsson et al, 1983; Fradkin, Metsaev, 1991; Metsaev, 1993; Manvelyan, Mkrtchyan, Ruehl, 2010, 2011, and many others).

- Supersymmetric $\mathcal{N} = 1$ generalizations of the purely bosonic cubic vertices with matter and the corresponding supercurrents were explored in terms of $\mathcal{N} = 1$ superfields in the papers by Gates, Koutrolikos, Kuzenko, I. Buchbinder, E. Buchbinder and others.

- In JHEP 05 (2022) 104, arXiv: 2202.08196 [hep-th] we have constructed, for the first time, the off-shell manifestly $\mathcal{N} = 2$ supersymmetric cubic couplings $\frac{1}{2} - \frac{1}{2} - \mathbf{s}$ of an arbitrary higher integer superspin $\mathbf{s}$ gauge $\mathcal{N} = 2$ multiplet to the hypermultiplet matter in $4D, \mathcal{N} = 2$ harmonic superspace.

- Our starting point is the $\mathcal{N}=2$ hypermultiplet off-shell free action:

$$S=\int d\zeta^{(-4)}\,\mathcal{L}^{+4}_{free}=-\int d\zeta^{(-4)}\,\frac{1}{2}q^{+a}\mathcal{D}^{++}q^{+}_{a}\,,\,a=1,2.$$

- We reproduce the gauge higher-spin $\mathcal{N}=2$ superfields from gauging the appropriate higher-derivative rigid (super)symmetries of this free hypermultiplet action. The simplest symmetry is of zero order in derivatives, it is $U(1)$ transformation of q^{+a},

$$\delta q^{+a}=-\lambda_0 Jq^{+a},\quad Jq^{+a}=i(\tau_3)^{a}{}_{b}q^{+b}$$

Gauging of this symmetry is just replacing λ_0 by analytic superparameter, $\lambda_0\to\lambda(\zeta)$, and in order to make the q^{+a} action gauge-invariant, we perform the change

$$\mathcal{D}^{++}\;\Rightarrow\;\mathfrak{D}^{++}_{(1)}=\mathcal{D}^{++}+\hat{\mathcal{H}}^{++}\,,\quad \hat{\mathcal{H}}^{++}=h^{++}J\,,$$
$$\delta_\lambda\hat{\mathcal{H}}^{++}=[\mathcal{D}^{++},\hat{\Lambda}]\,,\quad \hat{\Lambda}=\lambda J\;\Rightarrow\;\delta_\lambda h^{++}=\mathcal{D}^{++}\lambda\,.$$

There is no direct relation between J and ∂_5: one can choose $\partial_5 q^{+a}=0$ which corresponds to massless q^{+a} or $\partial_5 q^{+a}\sim mJq^{+a}$, which corresponds to massive q^{+a}.

- The global symmetry to be gauged in the spin 2 case looks somewhat more complicated, it is of first order in derivatives

$$\delta_{rig} q^{+a} = -\hat{\Lambda}_{rig} q^{+a},$$

$$\begin{aligned}\hat{\Lambda}_{rig} &= \left(\lambda^{\alpha\dot{\alpha}} - 2i\lambda^{-\alpha}\bar{\theta}^{+\dot{\alpha}} - 2i\theta^{+\alpha}\bar{\lambda}^{-\dot{\alpha}}\right)\partial_{\alpha\dot{\alpha}} + \lambda^{+\alpha}\partial^{-}_{\alpha} + \bar{\lambda}^{+\dot{\alpha}}\partial^{-}_{\dot{\alpha}} \\ &+ \left(\lambda^{5} + 2i\lambda^{\hat{\alpha}-}\theta^{+}_{\hat{\alpha}}\right)\partial_{5} := \Lambda^{M}\partial_{M}\,, \quad [\mathcal{D}^{++}, \hat{\Lambda}_{rig}] = 0\end{aligned}$$

  It involves five constant bosonic parameters $\lambda^{\alpha\dot{\alpha}}$, λ^{5}, four constant spinor parameters $\lambda^{\pm\hat{\alpha}} = \lambda^{\hat{\alpha}i}u^{\pm}_{i}$ and can be treated as a copy of the rigid $\mathcal{N} = 2$ supersymmetry transformations in their active form. We gauge just these transformations, leaving $\mathcal{N} = 2$ supersymmetry realized on the coordinates rigid.

- In this case there are two possibilities for gauge transformations of the hypermultiplet:

$$\delta_1 q^{+a} = -\hat{\Lambda}_{(2)} q^{+a}, \qquad \hat{\Lambda}_{(2)} := \lambda^{M}\partial_{M} = \lambda^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \lambda^{+\alpha}\partial^{-}_{\alpha} + \bar{\lambda}^{+\dot{\alpha}}\partial^{-}_{\dot{\alpha}} + \lambda^{5}\partial_{5}\,,$$

$$\delta_2 q^{+a} = -\frac{1}{2}\Omega_{(2)} q^{+a}, \qquad \Omega_{(2)} := (-1)^{P(M)}\partial_{M}\lambda^{M} = \partial_{\alpha\dot{\alpha}}\lambda^{\alpha\dot{\alpha}} - \partial^{-}_{\alpha}\lambda^{+\alpha} - \partial^{-}_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}.$$

$$(\delta_1 + \delta_2)\,\mathcal{L}^{+4}_{free} = \frac{1}{2} q^{+a}[\mathcal{D}^{++}, \hat{\Lambda}_{(2)}] q^{+}_{a}.$$

- Next, we introduce the differential operator

$$\hat{\mathcal{H}}^{++}_{(2)} = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\hat{\mu}+}\partial^{-}_{\hat{\mu}} + h^{++5}\partial_5 \,,$$

with the transformation law

$$\delta\hat{\mathcal{H}}^{++}_{(2)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(2)}].$$

Then the linear in gauge superfields part of the gauge-invariant action reads

$$\mathcal{L}^{+4}_{free} \quad \rightarrow \quad \mathcal{L}^{+4(s=2)}_{gauge} = \mathcal{L}^{+4}_{free} - \frac{1}{2} q^{+a}\hat{\mathcal{H}}^{++}_{(2)} q^{+}_{a} \,.$$

- In fact $\mathcal{L}^{+4(s=2)}_{gauge}$ can be made fully gauge invariant (and not at the linearized level) by deforming the gauge transformation law of $\hat{\mathcal{H}}^{++}_{(2)}$ as

$$\delta_{full}\hat{\mathcal{H}}^{++}_{(2)} = [\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}_{(2)}, \hat{\Lambda}_{(2)}] \,.$$

- A complete nonlinear harmonic superfield action of $\mathcal{N} = 2$ supergravity including the pure supergravity part was constructed in A.Galperin, Nguen A. Ky and E. Sokatchev, 1987.

- A surprising peculiarity of the superspin 3 case is that the relevant rigid two-derivative transformations to be gauged and the resulting couplings of the relevant gauge superfields to the hypermultiplet can be consistently defined only at cost of breaking rigid $SU(2)_{PG}$ symmetry down to $U(1)$ generator which is manifestly present in all formulas. This peculiarity extends to all odd $\mathcal{N} = 2$ spins.
- Though in this case from the beginning one can define 4 independent transformations of q^{+a}, only two their fixed combinations can be compensated by the appropriate transformations of gauge superfields:

$$\delta_\lambda q^{+a} = -\left[\lambda^{\alpha\dot\alpha M}\partial_M\partial_{\alpha\dot\alpha} + \frac{1}{2}(\partial_{\alpha\dot\alpha}\lambda^{\alpha\dot\alpha M})\partial_M + \frac{1}{2}\Omega_{(3)}\right](\tau_3)^a{}_b q^{+b}\,,$$

$$\delta_\xi q^{+a} = -\xi\,\Omega_{(3)}\,Jq^{+a} = -i\xi\,\Omega_{(3)}\,(\tau_3)^a{}_b q^{+b}\,,$$

where

$$\lambda^{\alpha\dot\alpha M}\partial_M = \lambda^{(\alpha\beta)(\dot\alpha\dot\beta)}\partial_{\beta\dot\beta} + \lambda^{(\alpha\beta)\dot\alpha+}\partial^-_\beta + \bar\lambda^{(\dot\alpha\dot\beta)\alpha+}\partial^-_{\dot\beta} + \lambda^{\alpha\dot\alpha}\partial_5\,,$$

$$\Omega_{(3)} = (\partial_{\alpha\dot\alpha}\Omega^{\alpha\dot\alpha}_{(3)})\,,\quad \Omega^{\alpha\dot\alpha} = (-1)^{P(M)}(\partial_M\lambda^{\alpha\dot\alpha M})\,.$$

- Defining

$$\hat{\mathcal{H}}^{++\alpha\dot{\alpha}} = h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})}\partial_{\beta\dot{\beta}} + h^{++(\alpha\beta)\dot{\alpha}+}\partial^-_{\beta} + \bar{h}^{++(\dot{\alpha}\dot{\beta})\alpha+}\partial^-_{\dot{\beta}} + h^{++\alpha\dot{\alpha}}\partial_5\,,$$

$$\Gamma^{++\alpha\dot{\alpha}} = \partial_{\beta\dot{\beta}}h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} - \partial^-_{\beta}h^{++(\alpha\beta)\dot{\alpha}+} - \partial^-_{\dot{\beta}}h^{++\alpha(\dot{\alpha}\dot{\beta})+},$$

$$\hat{\mathcal{H}}^{++}_{(3)} = \hat{\mathcal{H}}^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}},\ \Gamma^{++}_{(3)} = \partial_{\alpha\dot{\alpha}}\Gamma^{++\alpha\dot{\alpha}}$$

$$\delta\hat{\mathcal{H}}^{++} = [\mathcal{D}^{++}, \hat{\Lambda}_{(3)}]\,,\ \hat{\Lambda}_{(3)} = \lambda^{\alpha\dot{\alpha}M}\partial_M\partial_{\alpha\dot{\alpha}}\,,\ \delta\Gamma^{++}_{(3)} = \mathcal{D}^{++}\Omega_{(3)}\,,$$

we obtain a gauge invariant extension of the q^+ action as

$$\mathcal{L}^{+4(s=3)}_{gauge} = \mathcal{L}^{+4}_{free} - \frac{1}{2}q^{+a}\left(\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}_{(3)}J + \xi\Gamma^{++}_{(3)}J\right)q^+_a$$

- The presence of constant ξ in the gauged Lagrangian shows that off shell there are 2 types of possible interactions of the $\mathcal{N}=2$ spin **3** with the hypermultiplet. The coefficient ξ is a dimensionless coupling constant that measures the relative strength of these interactions. Actually, recently we have checked that on shell the ξ term does not contribute to cubic vertex, it survives only off-shell and perhaps can play some role in the quantum theory (I.B., E.I., N.Z., under way).

- In the superspin 4 case the transformations of q^{+a} do not require internal symmetry generators, so they preserve $SU(2)_{PG}$ invariance. The rigid symmetry transformations are of third order in derivatives and well defined. Their localization in the analytic subspace admit 6 independent variations, but only 3 of them turn out to finally matter

$$\delta_1 q^{+a} = -\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\hat\Lambda^{\alpha\beta\dot\alpha\dot\beta} q^{+a}\,,\ \delta_2 q^{+a} = -\hat\Lambda^{\alpha\beta\dot\alpha\dot\beta}\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta} q^{+a}\,,$$
$$\delta_3 q^{+a} = -\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\Omega^{\alpha\beta\dot\alpha\dot\beta} q^{+a}\,,$$

where

$$\hat\Lambda^{\alpha\beta\dot\alpha\dot\beta} = \lambda^{(\alpha\beta)(\dot\alpha\dot\beta)M}\partial_M\,,\ \Omega^{\alpha\beta\dot\alpha\dot\beta} = (-1)^{P(M)}(\partial_M\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)M})$$

and derivatives freely act to the right.

- One can calculate

$$(\delta_1+\delta_2+\delta_3)\,\mathcal{L}^{+4}_{free} = \frac{1}{2} q^{+a}[\mathcal{D}^{++},\hat\Lambda_{(4)}]q^+_a\,.$$

- Then the modified q^{+a} action reads

$$\mathcal{L}^{+4(s=4)}_{gauge} = -\frac{1}{2}\,q^{+a}\left(\mathcal{D}^{++}+\hat{\mathcal{H}}^{++}_{(4)}\right)q^+_a\,,$$

with

$$\hat{\mathcal{H}}^{++}_{(4)} = h^{++(\alpha\beta)(\dot\alpha\dot\beta)M}\partial_M\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\,,$$
$$\delta\hat{\mathcal{H}}^{++}_{(4)} = [\mathcal{D}^{++},\hat\Lambda_{(4)}] \quad \hat\Lambda_{(4)} = \lambda^{(\alpha\beta)(\dot\alpha\dot\beta)M}\partial_M\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta}$$

- The hypermultiplet couplings of the $\mathcal{N}=2$ gauge multiplets of higher superspins **s** can be constructed quite analogously and have the uniform structure: gauge superfields and gauge parameters are the appropriate differential operators of the rank $\mathbf{s}-\mathbf{1}$.
- Everything can be easily extended to a system of several hypermultiplets. The free Lagrangian of n hypermultiplets can be written in the manifestly $\mathrm{USp}(2n)$ invariant form as

$$\mathcal{L}^{+4}_{free,n} = \frac{1}{2} q^{+A} \mathcal{D}^{++} q^{+}_{A} \,, \quad \widetilde{q^{+}_{A}} = \Omega^{AB} q^{+}_{B} \,, \quad A = 1, 2, \ldots, 2n \,,$$

where $\Omega^{AB} = -\Omega^{BA}$ is $\mathrm{USp}(2n)$ invariant constant $2n \times 2n$ symplectic metric. It can be rewritten in an equivalent complex form as

$$\mathcal{L}^{+4}_{free,n} \sim \tilde{q}^{+a} \mathcal{D}^{++} q^{+}_{a} - \mathcal{D}^{++} \tilde{q}^{+a} q^{+}_{a} \,, \quad a = 1, 2, \ldots, n \,, \quad q^{+}_{A} = (q^{+}_{a}, -\tilde{q}^{+a})$$

so that the manifest symmetry is $\mathrm{U}(n) = \mathrm{SU}(n) \times \mathrm{U}(1) \subset \mathrm{USp}(2n)$, with respect to which q^{+}_{a} and $\tilde{q}^{+a}$ transform in the fundamental and co-fundamental representations. Then one can identify the $U(1)$ generator needed for description of odd spins as

$$J q^{+}_{a} = i q^{+}_{a} \,, \quad J \tilde{q}^{+a} = -i \tilde{q}^{+a} \,,$$

so that $\mathrm{USp}(2n)$ gets broken to $\mathrm{U}(n)$. Some other options with $\mathrm{SU}(n)$ also broken, are as well admissible.

Summary and outlook

The theory of $\mathcal{N}=2$ higher spins $s \geq 3$ opens a new promising direction of applications of the harmonic superspace approach which earlier proved to be indispensable for description of more conventional $\mathcal{N}=2$ theories with the maximal spins $s \leq 2$. Once again, the basic property underlying these new higher-spin theories is the harmonic Grassmann analyticity (all basic gauge potentials are unconstrained analytic superfields involving an infinite number of degrees of freedom off shell before fixing WZ-type gauges).

Under way:

- The natural next steps are the construction and investigation of $4D, \mathcal{N}=2$ higher-spin supercurrents of the hypermultiplet and more detailed study of the component structure. Almost finished.
- $\mathcal{N}=2$ supersymmetric half-integer spins?
- An extension to AdS background?
- From the linearized theory to its full nonlinear version? At present, the latter is known only for $s \leq 2$ ($\mathcal{N}=2$ super Yang - Mills and $\mathcal{N}=2$ supergravities). This problem will seemingly require accounting for ALL higher $\mathcal{N}=2$ superspins simultaneously. New supergeometries?

THANK YOU FOR YOUR ATTENTION!