Lattice study of dense two-color QCD

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Outline:

- Introduction
  - Lattice simulation of QCD
  - Dense matter and sign problem
  - Simulation of QCD-like theories
- Two-color QCD at low density
- Moderate and large densities
- String breaking and charmonium dissociation
- Conclusion

Quantum chromodynamics (QCD)

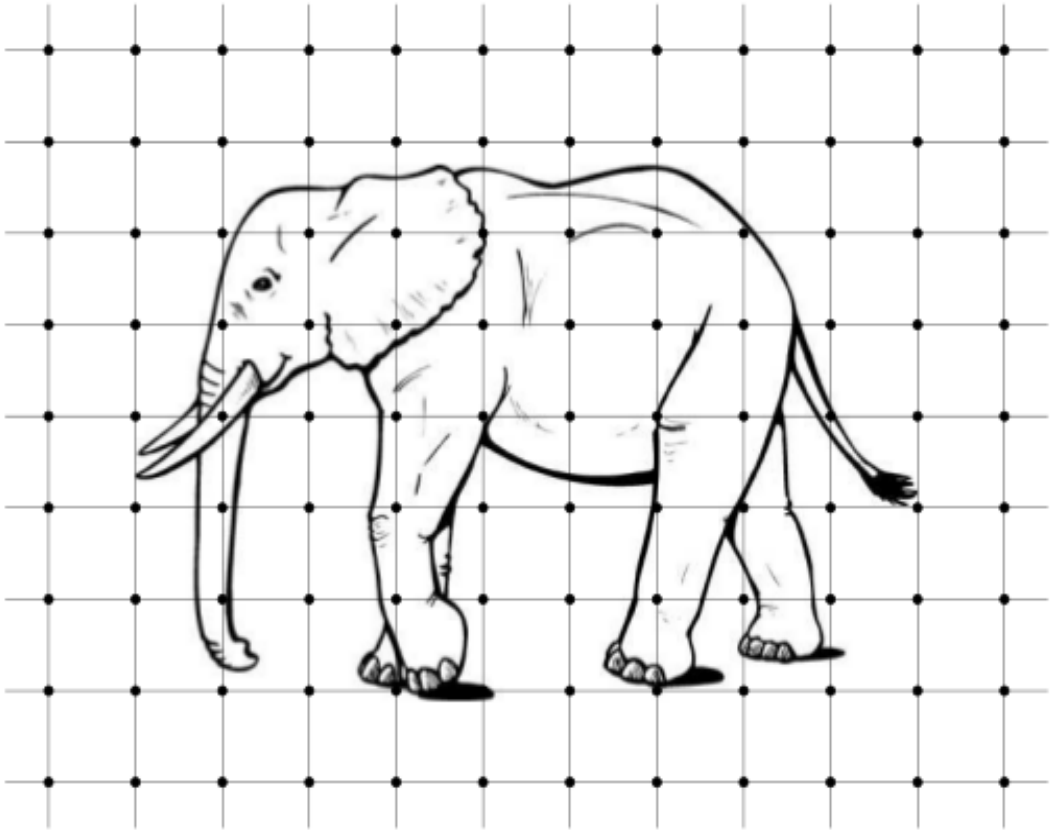
QCD properties

- Degrees of freedom
  - Quarks q
  - Gluons A
- QCD Lagrangian

$$L = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_{f=u,d,s,\ldots} \bar{q}_f (i\gamma^\mu \partial_\mu - m) q_f + g \sum_{f=1}^{N_f} \bar{q}_f \gamma^\mu \hat{A}_\mu q_f$$

- Nonlinear equations of motion with $g \sim 1$
- QCD Lagrangian is known, but observables cannot be calculated analytically
  - In particular: **Cofinement from the QCD Lagrangian – Millenium problem**

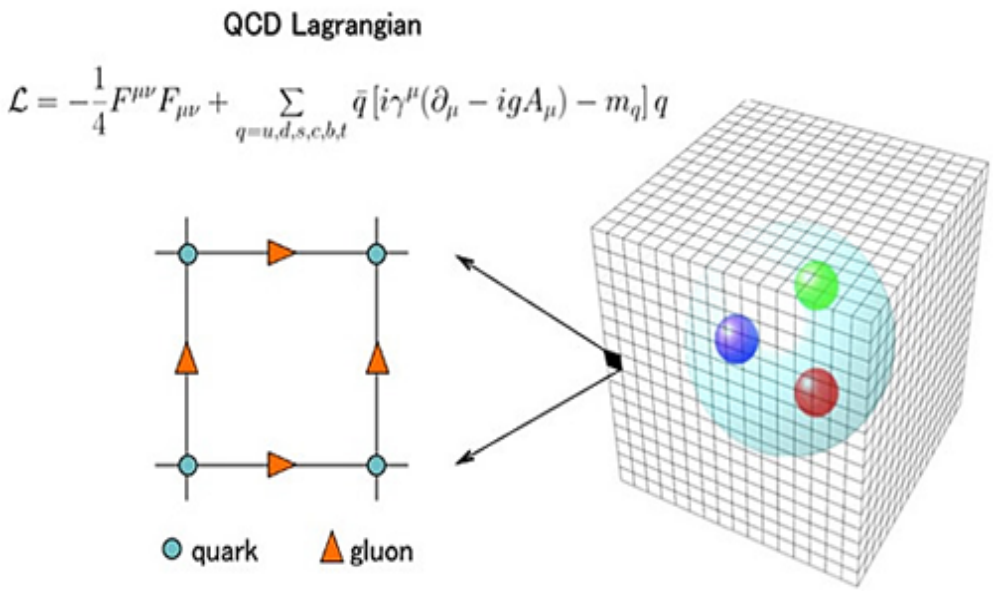
Lattice simulation of QCD



Lattice simulation

- Allows to study strongly interacting nonlinear systems
- Based on the first principles of quantum field theory
- The most powerful and perspective approach due to development of supercomputers and algorithms

Building Lattice QCD



- Introduce 4-dimensional lattice
  $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- Lattice spacing $-a$
- **Degrees of freedom**
  - **Gluon fields:** 3x3 matrix $U \in SU(3)$, links
  - **Quark fields:** $q, \bar{q}$, nodes

Building Lattice QCD

- One calculates QCD partition function (thermodynamic equilibrium!)
  $Z_l = \int DU D\bar{q}Dq \ \exp\left(-S_G(U) - \int d^4x \bar{q}(\hat{D}(U) + m)q\right)$
- The partition function is calculated in 3+1 Euclidean space
- In the continuum limit Z_l exactly reproduces $Z_{\text{КХД}}$
  - The gluon contribution: $S_G(U)\Big|_{a\to 0} = -\frac{1}{4}\sum_{a=1}^{8} F_a^{\mu\nu} F_{\mu\nu}^a$
  - The quark contribution:
    $\bar{q}(\hat{D}(U) + m)q\Big|_{a\to 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$
- The quark fields can be integrated out exactly
  $Z = \int DU \exp\left(-S_G(U)\right) \times \prod_{i=u,d,s\ldots} \det\left(\hat{D}_i(U) + m_i\right)$

Lattice simulation of QCD

- One calculates the partition function
  $Z_l \sim \int DU e^{-S_G(U)} \prod_i \det\left(\hat{D}_i(U) + m_i\right) = \int DU e^{-S_{eff}(U)}$
- Hybrid Monte Carlo is used
  (generation of the gluon configurations with the weight $e^{-S_{eff}(U)}$)
- Carry out the extrapolation $a \to 0, \quad V \to \infty$
- The method is based on the first principles
- Exact numerical evaluation - no expansion in a small parameter! No assumptions!
- Parameters: g и quark masses
- Statistical and systematic (discretization and finite volume effects) uncertainties can be systematically reduced

Modern capabilities of the approach

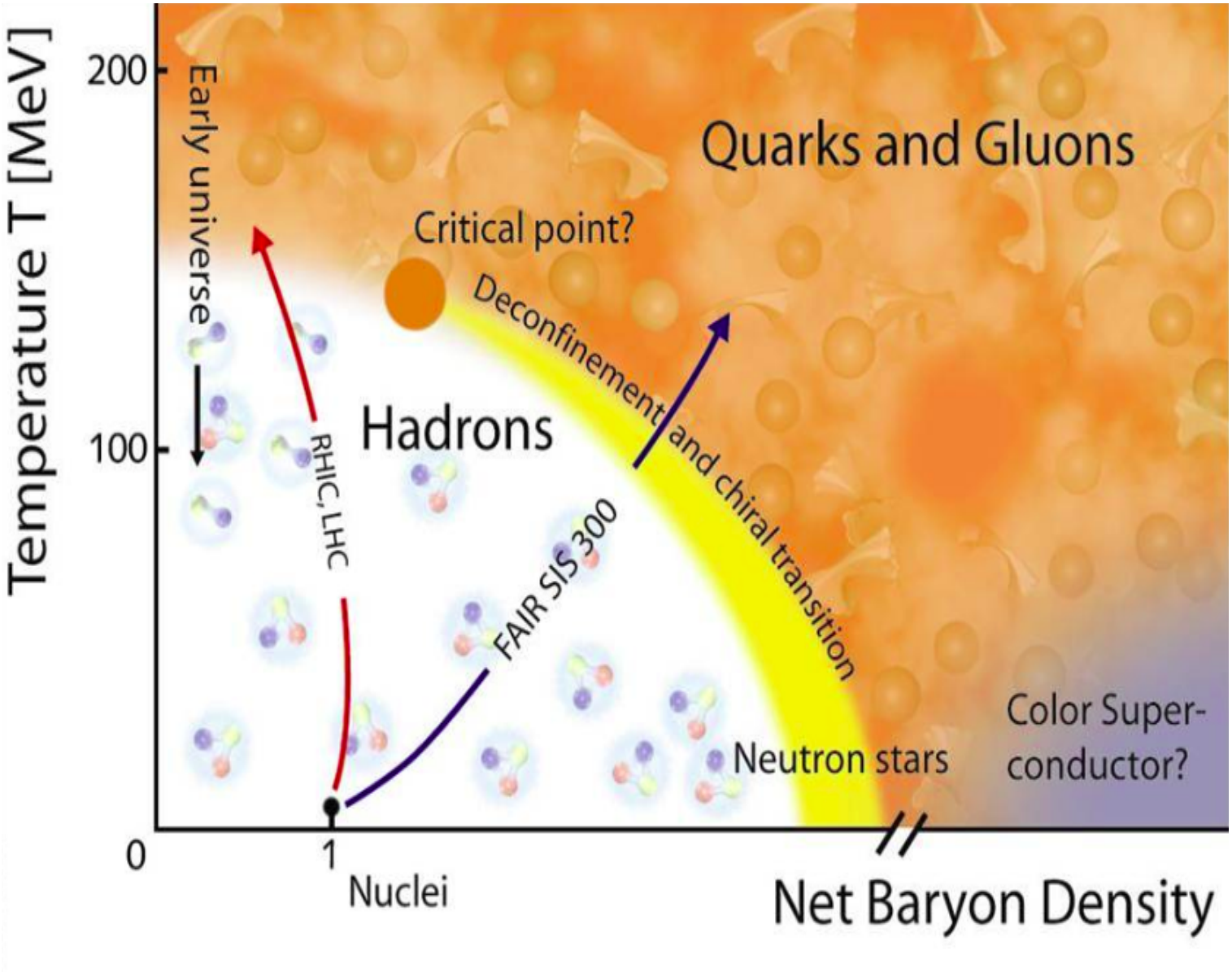
$$Z_l \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s\ldots} \det\left(\hat{D}_i(U) + m_i\right)$$

- Lattices
  - 96×48^3
  - Integration variables: $96 \cdot 48^3 \cdot 4 \cdot 8 \sim 300 \cdot 10^6$
  - Matrices manipulation: $100 \cdot 10^6 \times 100 \cdot 10^6$
- Calculation with dynamical u, d, s, c–quarks
- Physical masses of the u, d, s, c quarks
- Lattice spacing up to $a \sim 0.05\,\mathrm{fm}$

Applications

- Spectroscopy
- Matrix elements, correlation functions
- Thermodynamic properties
- Transport properties
- QCD phase transitions
- Nuclear physics
- QCD properties in extreme conditions (strong magnetic field, baryon density, isospin density, rotation,...)
- Topological properties and objects in QCD
- Beyond the SM at strong coupling
- ...

QCD phase diagram



QCD at finite baryon density

- Poor knowledge of dense QCD
- Phenomenological models (unknown systematic uncertainties)
- Lattice QCD (small densities $\frac{\mu}{T} < few$)

  Taylor expansion, Imaginary chemical potential,...
- **QCD-like theories**
  - SU(2) QCD (two-color) with chemical potential
  - SU(3) QCD isospin chemical potential

The sign problem in QCD

SU(3) QCD

- $Z = \int DU \exp\left(-S_G\right) \times \det\left(\hat{D} + m\right)$
- Eigenvalues go in pairs $\hat{D}: \ \pm i\lambda \Rightarrow$
  $\det\left(\hat{D} + m\right) = \prod_\lambda (\lambda^2 + m^2) > 0$ i.e. simulations can be used
- Introduce chemical potential:
  $\det\left(\hat{D} + m\right) \to \det\left(\hat{D} - \mu\gamma_4 + m\right) \Rightarrow$
  the determinant becomes complex (**sign problem**)

SU(2) QCD

- $(\gamma_5 C \tau_2) \cdot D^* = D \cdot (\gamma_5 C \tau_2)$
- Eigenvalues go in pairs $\hat{D} - \mu\gamma_4$: λ, λ^*
- For even N_f $\ \det\left(\hat{D} - \mu\gamma_4 + m\right) > 0 \ \Rightarrow$ **free from sign problem**

Two-color QCD at finite baryon density

Why SU(2) and SU(3) at finite μ are different?

- No phase of the fermion determinant
- The Lagrangian of the SU(2) QCD has the symmetry: $SU(2N_f)$ as compared to $SU_R(N_f) \times SU_L(N_f)$ for $SU(3)$ QCD
- Goldstone bosons ($N_f = 2$) $\pi^+, \pi^-, \pi^0, d, \bar{d}$
- The baryon d is composed of two quarks, i.e. it is a boson

However, in dense medium:

- **Relevant degrees of freedom are quarks and gluons** rather than goldstone bosons

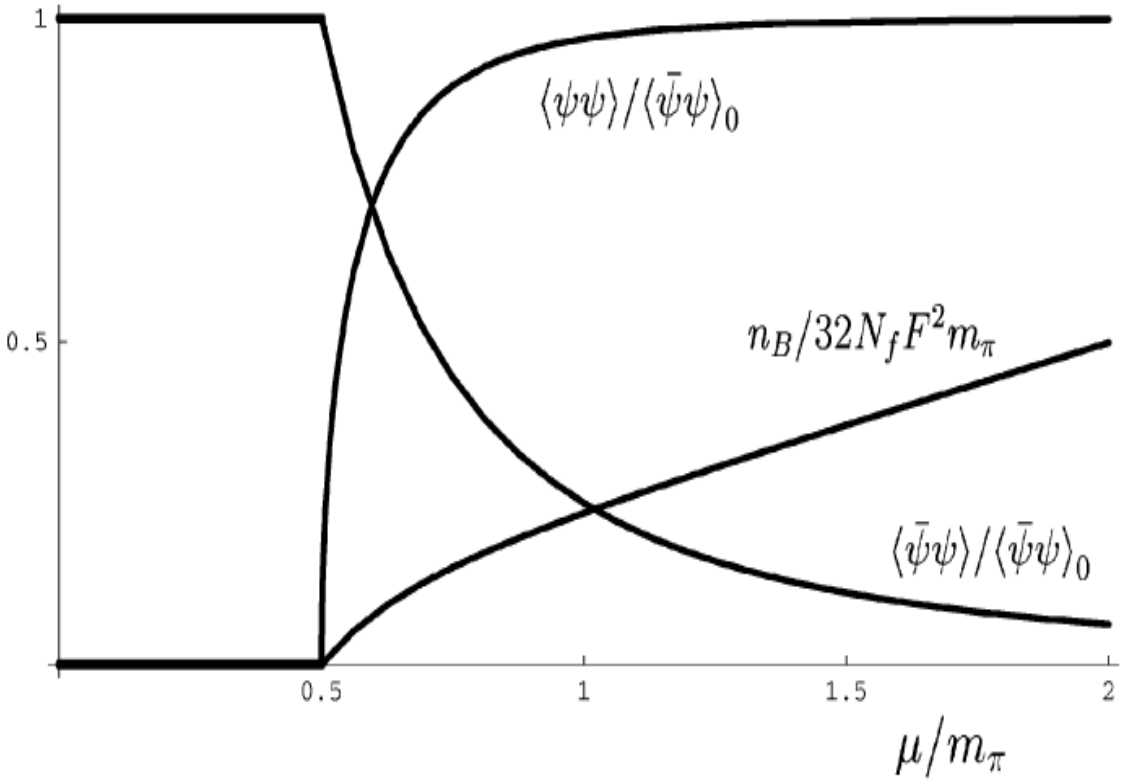
Two-color QCD at finite baryon density

How QCD-like theories can be used to study dense QCD

- Lattice study of QCD-like theories contains full dynamics of real system (contrary to phenomenological models)
- Adjust phenomenological models, check different approximations (Phys.Rev. D99 (2019) no.1, 014518)
- Study of different physical phenomena in dense medium

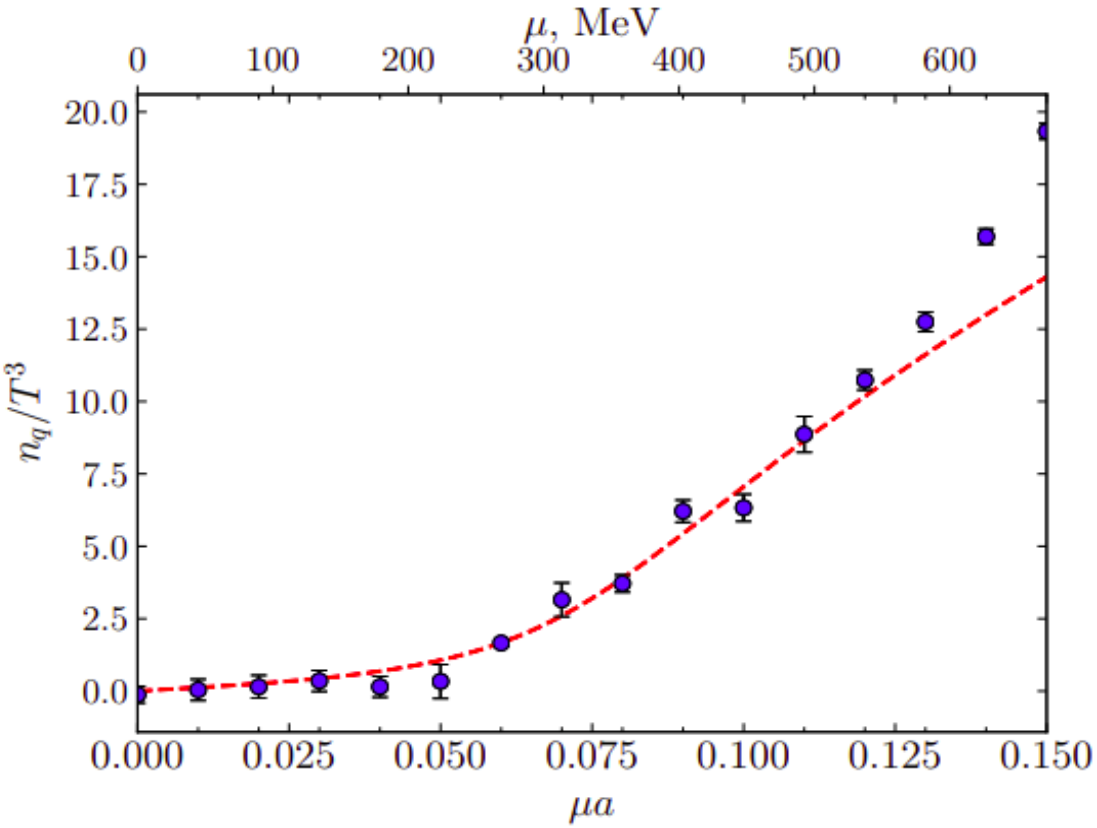
Small and moderate densities

Predictions of CHPT



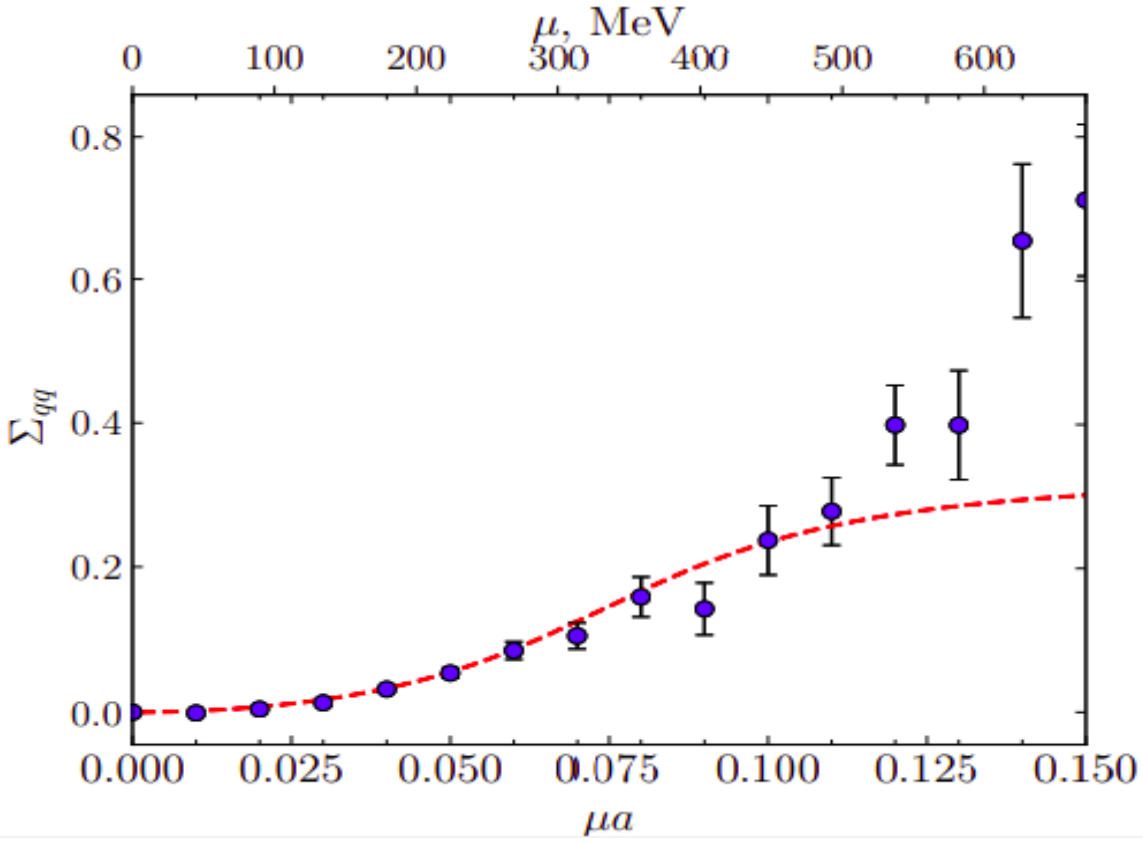
- Due to the chiral symmetry $SU(2N_f)$ one can build CHPT for small densities
- CHPT gives reliable results
- Phase transitions can be studied for sufficiently small μ

The baryon density

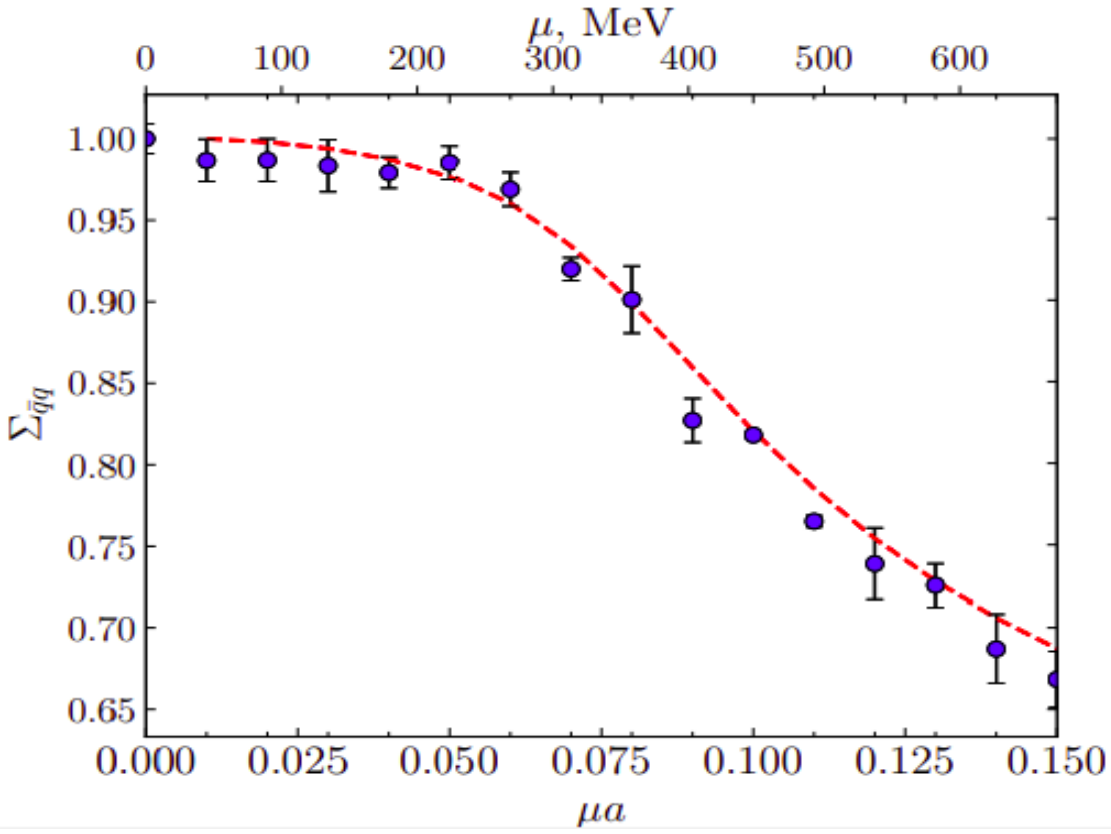


- Good agreement with CHPT
- We observe BEC of diquarks (BEC phase)
- Dilute baryon gas in the region $\mu > m_\pi/2$
- At moderate $\mu_0 \sim 500$ MeV the deviation from CHPT is seen
- The region $\mu < \mu_0$ – **delute baryon gas, degrees of freedom: hadrons**
- The region $\mu > \mu_0$ – **dense quark matter, degrees of freedom: quarks**

The diquark condensate



The chiral condensate

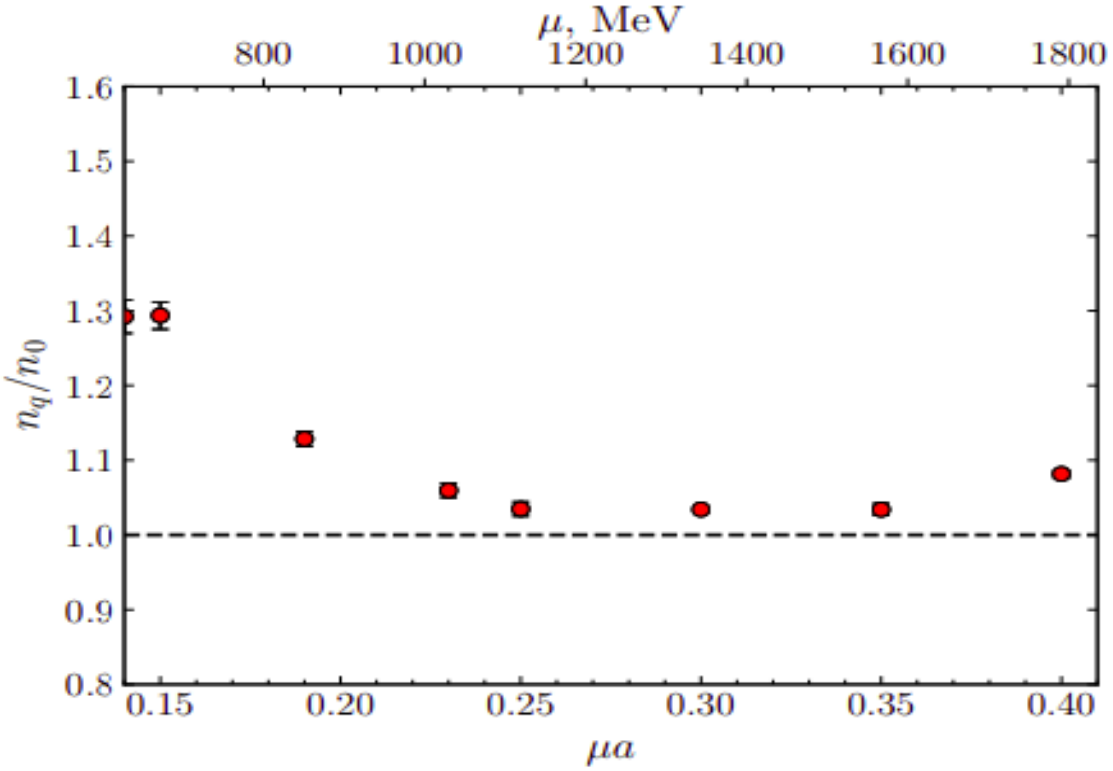


Phase diagram for $N_c \to \infty$

- Hadron phase $\mu < M_N/N_c$ ($p \sim O(1)$)
- Dilute baryon gas $\mu > M_N/N_c$ (width $\delta\mu \sim \frac{\Lambda_{QCD}}{N_c^2}$)
- Quarkyonic phase $\mu > \Lambda_{QCD}$ ($p \sim N_c$)
  - Degrees of freedom:
    - Baryons (on the surface)
    - Quarks (inside the Fermi sphere $|p| < \mu$)
  - No chiral symmetry breaking
  - The system is in confinement phase
- Deconfinement ($p \sim N_c^2$)

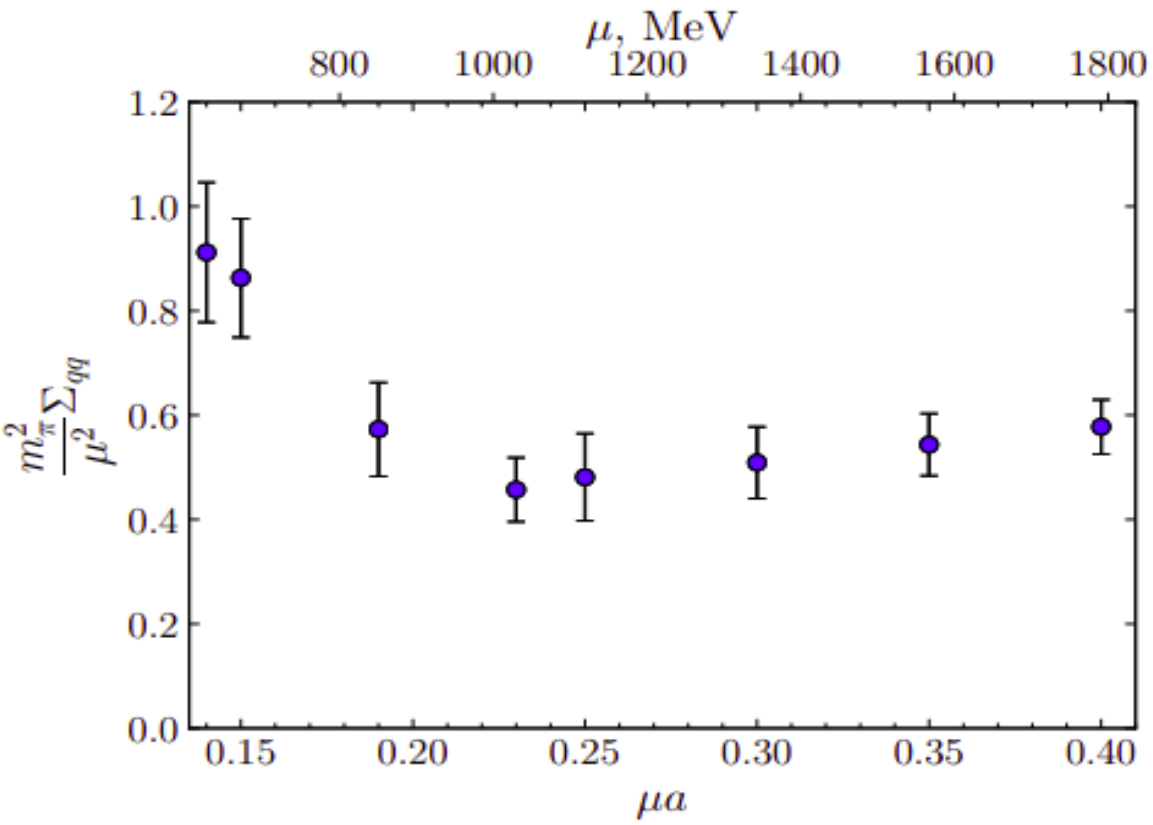
L. McLerran, R.D. Pisarski, Nucl.Phys. A796 (2007) 83-100

Baryon density



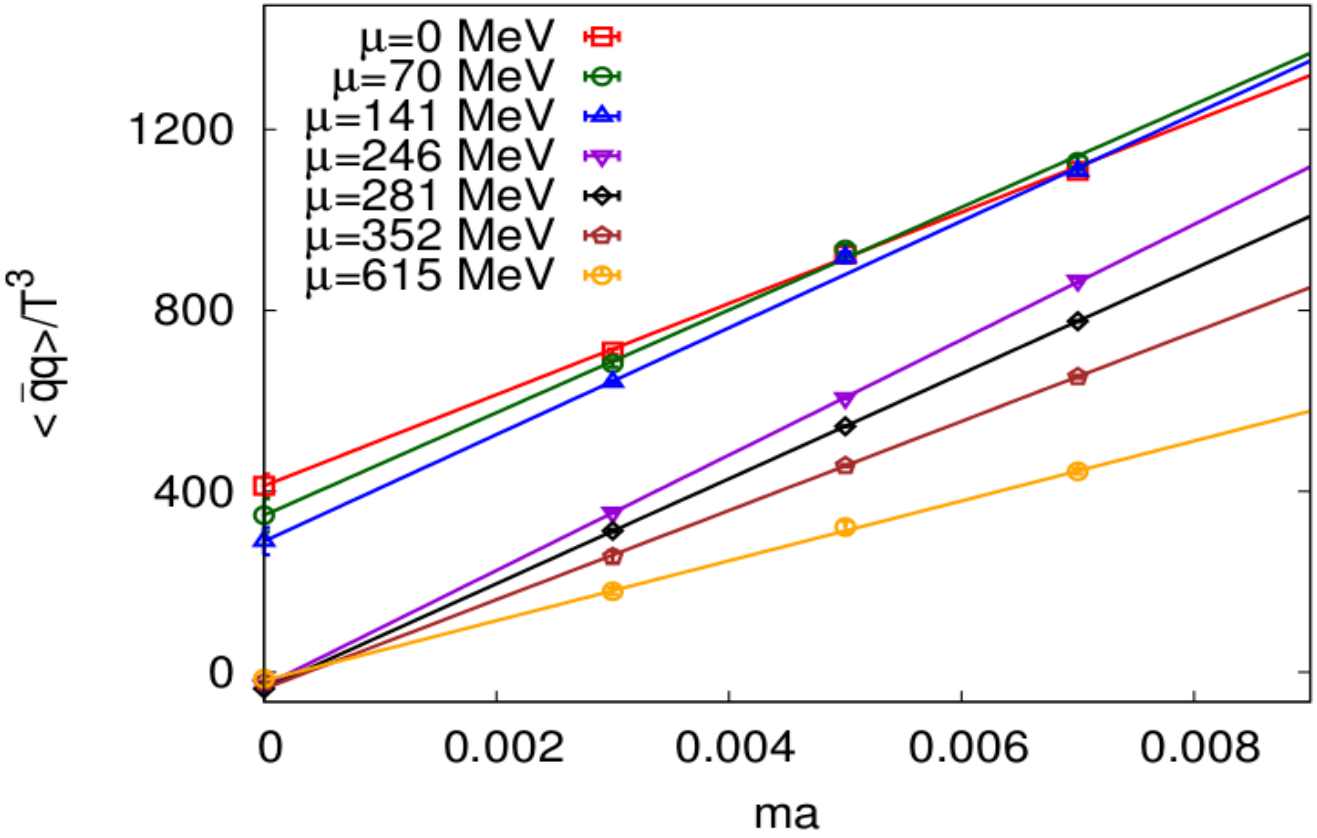
- Free quarks $n_0 = N_f \times N_c \times (2s+1) \times \int \frac{d^3p}{(2\pi)^3}\theta(|p|-\mu) = \frac{4}{3\pi^2}\mu^3$
- **Quarks inside Fermi sphere**
- Quarks inside Fermi sphere dominate over the surface: $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$

Diquark condensate



- Bardeen–Cooper–Schrieffer (BCS) phase $\mu > 800$ MeV, $\Sigma_{qq} \sim \mu^2$
- Degrees of freedom: Cooper pairs (color-singlet diquarks qq - baryons)
- **Baryons (on the surface)**

Chiral condensate (chiral limit $m \to 0$)



No chiral symmetry breaking

Properties of Quarkyonic phase:

- Baryons (on the surface)

Properties of Quarkyonic phase:

- Baryons (on the surface) ✓
- Quarks (inside the Fermi sphere $|p| < \mu$)

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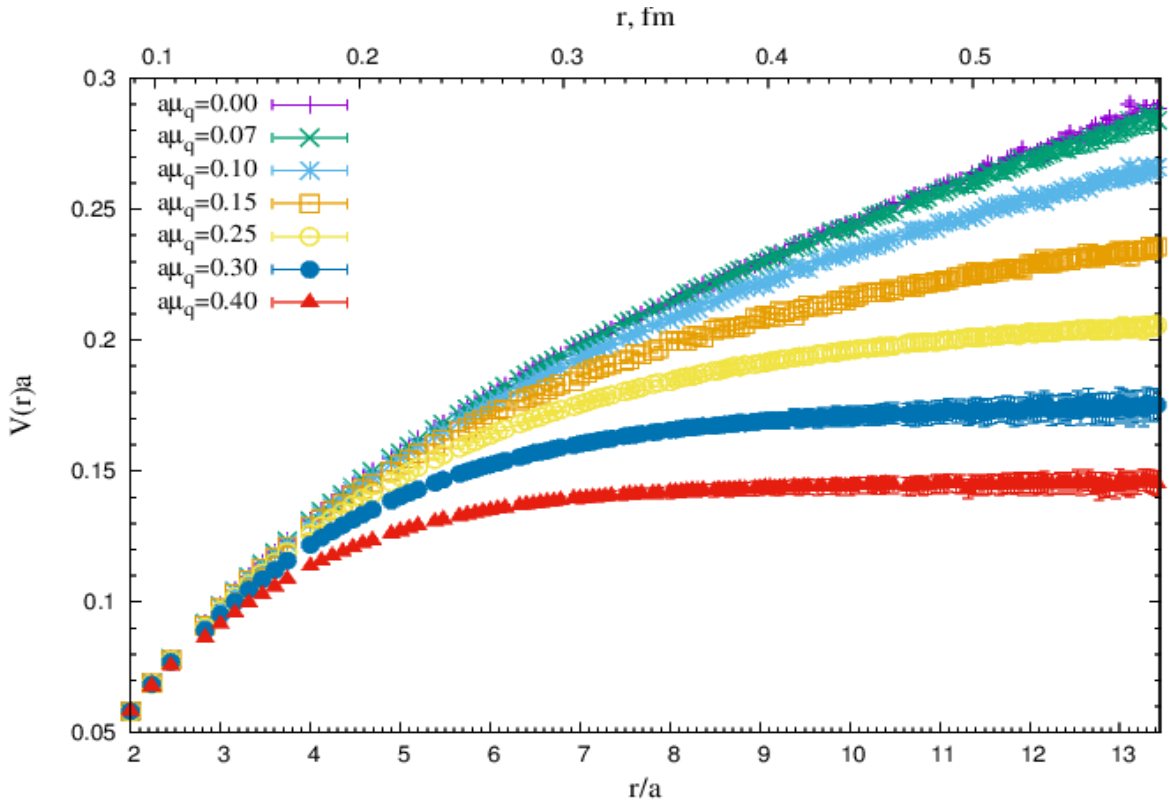
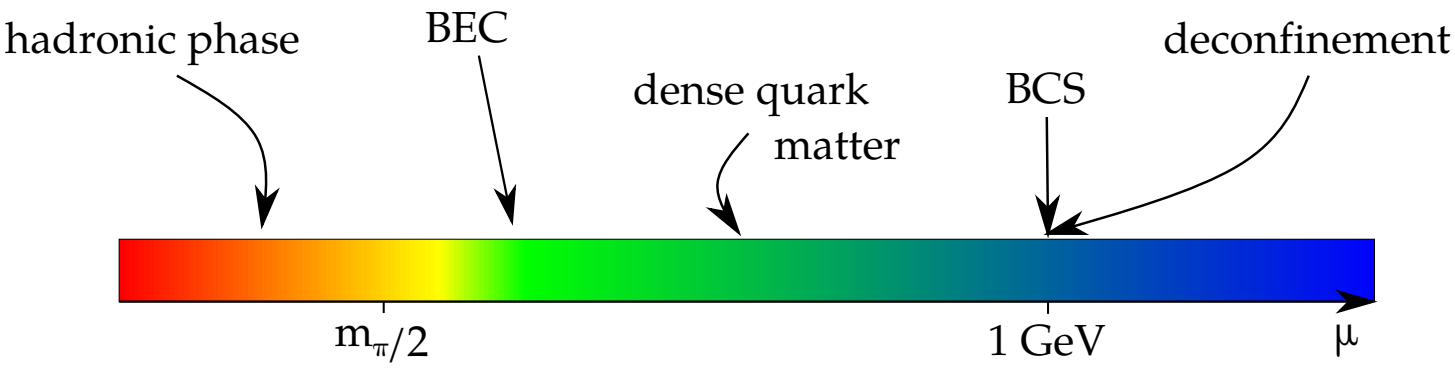
What about confinement in cold dense quark matter?

Potential between static quark-antiquark pair



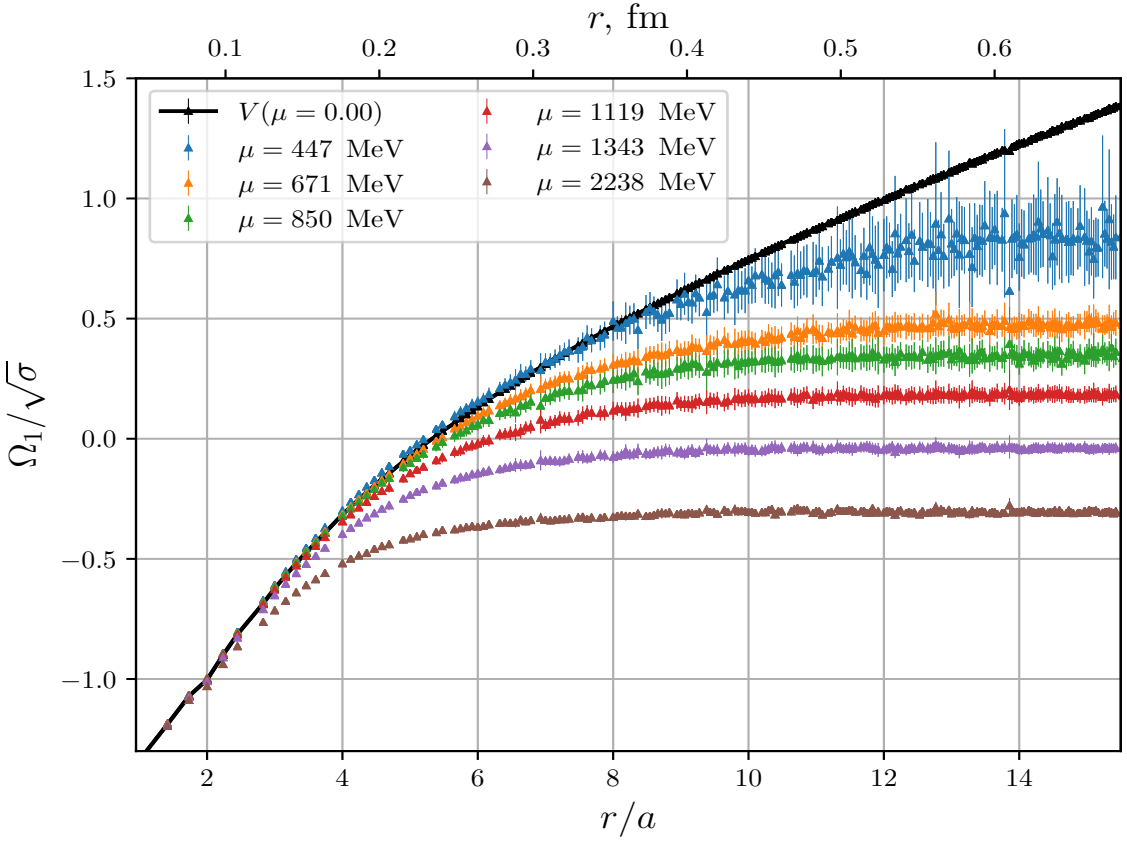
- We observe deconfinement in dense medium!
- It might be due to the finite temperature effects
- Still under debate

Tentative phase diagram of dense SU(2) QCD



- Low densities can be well described by CHPT (BEC phase)
- Dense matter in the region $\mu > 500$ MeV
- BCS phase take place at $\mu \sim 1$ GeV
- Relevant degrees of freedom in the region $\mu > 1$ GeV: color-singlet Cooper pairs(baryons) and quarks with $\Delta(\mu)$
- No chiral symmetry breaking in dense matter
- Deconfinement at sufficiently large density?

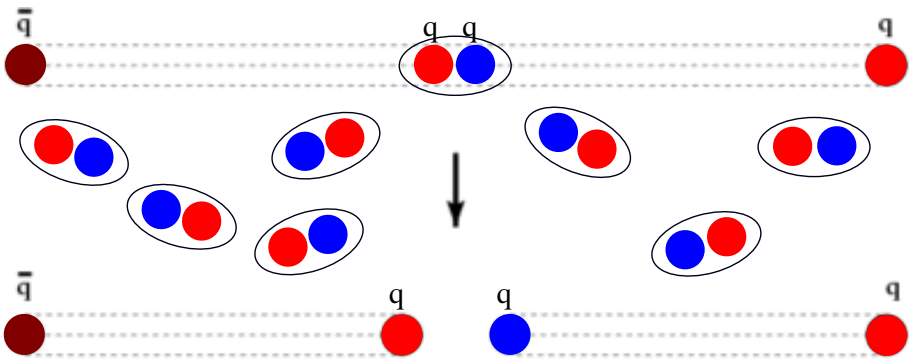
String breaking in dense matter



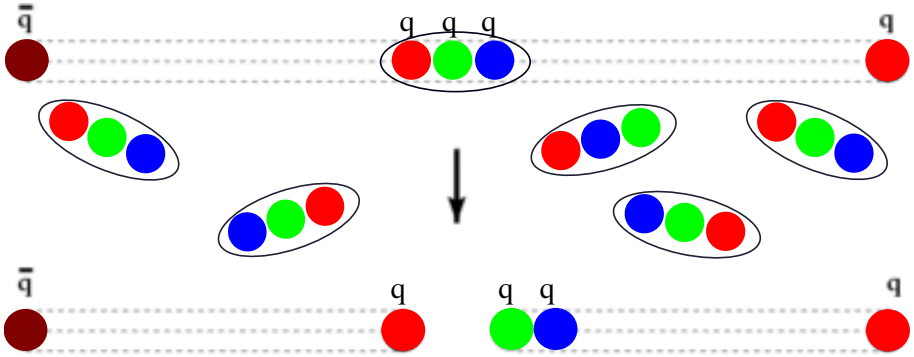
- Free energy of quark-antiquark pair $\Omega_1 \sim F_{q\bar{q}} \sim \langle P^+(r)P(0)\rangle$
- The confinement implies linear rise of Ω_1
- Because of the string breaking Ω_1 goes to plateau
- The larger the baryon density the smaller string breaking distance

Mechanism of string breaking in dense medium

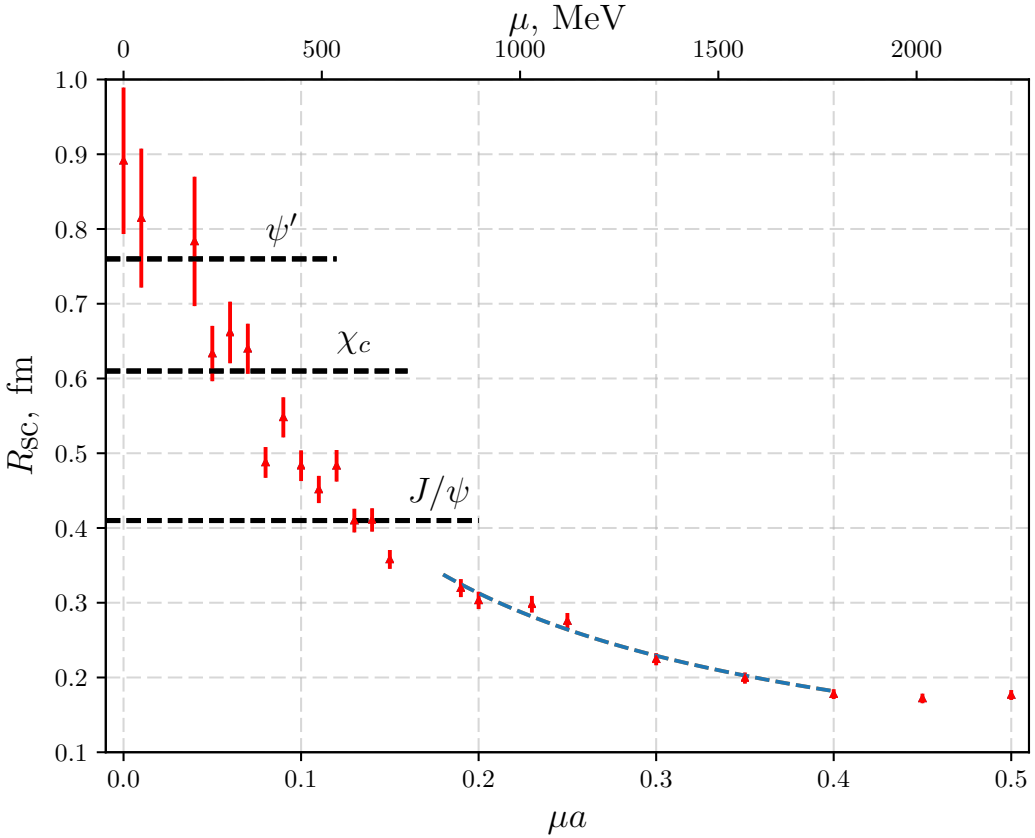
In SU(2) QCD:



Analogous mechanism may be proposed in SU(3) QCD:



Charmonium dissociation in dense medium



- String breaking at distance R_{SC}
- Charmonium dissociation takes place before the deconfinement
- **Charmonium dissociation can be observed at modern experiments**

Conclusion

- One can carry out lattice study of dense QCD-like theories
- QCD-like theories allow us to understand the properties of SU(3) QCD dense matter
- We have studied dense two-color QCD and found the following phases
  - QCD vacuum
  - BEC phase
  - Dense matter
  - BCS phase

THANK YOU FOR ATTENTION!