

Electromagnetic and gravitational effects for spin dynamics in accelerators

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OUTLINE

- Preliminary comments
- Electromagnetic interactions of a Dirac particle in Earth's frame and effective fields acting on the particle and the spin
- Manifestations of Earth's rotation
- Manifestations of Earth's gravity
- Summary
- Extra slides

Preliminary comments

Chapter in monograph:

Yu.N. Obukhov, A.J. Silenko, O. V. Teryaev, Manifestations of the rotation and gravity of the Earth in spin physics experiments. Gribov-85 Memorial Volume: Exploring Quantum Field Theory, edited by Y.L Dokshitzer, P. Levai, J. Nyiri, World Scientific, 2016, pp. 297-308.

Articles:

Yu.N. Obukhov, A.J. Silenko, O. V. Teryaev, Manifestations of the rotation and gravity of the Earth in high-energy physics experiments, Phys. Rev. D 94, 044019 (2016). Yu.N. Obukhov, A.J. Silenko, O. V. Teryaev, Manifestations of the rotation and gravity of the Earth in spin physics experiments, Int. J. Mod. Phys. A 31, 1645030 (2016). Yu.N. Obukhov, A.J. Silenko, O. V. Teryaev, Non-Maxwellian electrodynamics in Earth's reference system: applications in high-energy physics, Nonl. Phen. Compl. Sys. 19, 303 (2016).



However, a small g-2 factor can condition an importance of gravity

Homopolar generator – Faraday disc

A homopolar generator is a DC electrical generator comprising an electrically conductive disc or cylinder rotating in a plane perpendicular to a uniform static magnetic field. A potential difference is created between the center of the disc and the rim (or ends of the cylinder) with an electrical polarity that depends on the direction of rotation and the orientation of the field. It is also known as a unipolar generator, a cyclic generator, disk dynamo, or Faraday disc.



Electromagnetic interactions of a Dirac particle in a rotating frame and effective fields acting on the particle and the spin

The metric of the rotating frame is given by $ds^{2} = dx^{0} dx^{0} - \left(dx^{a} - K^{a} dx^{0} \right) \left(dx^{a} - K^{a} dx^{0} \right), \quad \mathbf{K} = -\frac{\mathbf{\omega} \times \mathbf{r}}{c}.$

where ω is an angular velocity of a frame rotation.

We need the tetrad in the Schwinger gauge:

$$e_i^{\hat{0}} = \delta_i^{\hat{0}}, \qquad e_i^{\hat{a}} = \delta_i^{\hat{a}} - cK^{\hat{a}}e_i^{\hat{0}},$$

 $ds^2 = \eta_{ij}dx^{\hat{i}}dx^{\hat{j}}.$

We generalize the initial Dirac equation to introduce anomalous magnetic and electric dipole moments

electromagnetic field tensor ,

$$\begin{pmatrix} i\hbar\gamma^{a}D_{a} + \frac{\mu'}{2}\sigma^{ab}F_{ab} + \frac{d}{2}\sigma^{ab}G_{ab} - mc \end{pmatrix} \psi = 0, \quad F_{ab} = e_{a}^{\mu}e_{b}^{\nu}F_{\mu\nu}. \\ G_{\mu\nu} = \frac{1}{2}\eta_{\mu\nu\rho\lambda}F^{\rho\lambda}, \quad F_{\mu\nu} = (\mathbf{E},\mathbf{B}), \quad G_{\mu\nu} = (-\mathbf{B},\mathbf{E}).$$

Effective fields in the rotating frame

$$G_{ab} = \frac{1}{2} \eta_{abcd} F^{cd}, \quad F_{ab} = (\mathcal{E}, \mathcal{B}), \quad G_{ab} = (-\mathcal{B}, \mathcal{E}).$$
$$\mathcal{E} = \mathbf{E} - (\mathbf{\omega} \times \mathbf{r}) \times \mathbf{B}, \quad \mathcal{B} = \mathbf{B},$$
$$\mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}.$$

Generalized Dirac Hamiltonian

$$\mathcal{H} = \beta mc^{2} + e\Phi + c\mathbf{a} \cdot \boldsymbol{\pi} - \boldsymbol{\omega} \cdot (\mathbf{r} \times \boldsymbol{\pi}) - \frac{\hbar}{2} \boldsymbol{\omega} \cdot \boldsymbol{\Sigma} -\beta \Big[\boldsymbol{\Sigma} \cdot (\mu' \boldsymbol{\mathcal{B}} + d\boldsymbol{\mathcal{E}}) - i\boldsymbol{\alpha} \cdot (\mu' \boldsymbol{\mathcal{E}} - d\boldsymbol{\mathcal{B}}) \Big].$$

Relativistic Foldy-Wouthuysen transformation in an arbitrarily strong external field

see A.J. Silenko, Phys. Rev. A 77, 012116 (2008).

$$\mathcal{H}_{FW} = \mathcal{H}_{FW}^{(D)} + \mathcal{H}_{FW}^{(add)}, \qquad \varepsilon' = \sqrt{m^2 c^4 + c^2 \pi^2},$$
$$\mathcal{H}_{FW}^{(D)} = \beta \varepsilon' + e \Phi - \mathbf{\omega} \cdot (\mathbf{r} \times \boldsymbol{\pi}) - \frac{\hbar}{2} \mathbf{\omega} \cdot \boldsymbol{\Sigma} - \frac{e \hbar c^2}{4} \left\{ \frac{1}{\varepsilon'}, \boldsymbol{\Pi} \cdot \boldsymbol{\mathcal{B}} \right\}$$
$$+ \frac{e \hbar c^2}{8} \left\{ \frac{1}{\varepsilon' (\varepsilon' + mc^2)}, \left[\boldsymbol{\Sigma} \cdot (\boldsymbol{\pi} \times \boldsymbol{\mathcal{E}} - \boldsymbol{\mathcal{E}} \times \boldsymbol{\pi}) - \hbar \nabla \cdot \boldsymbol{\mathcal{E}} \right] \right\},$$

$$\mathcal{H}_{FW}^{(add)} = \frac{1}{4} \left\{ \frac{1}{\varepsilon'}, \left[\boldsymbol{\Sigma} \cdot \left(\boldsymbol{\pi} \times \boldsymbol{\mathcal{P}} - \boldsymbol{\mathcal{P}} \times \boldsymbol{\pi} \right) - \hbar \nabla \cdot \boldsymbol{\mathcal{P}} \right] \right\} - \boldsymbol{\Pi} \cdot \boldsymbol{\mathcal{M}} \\ + \frac{1}{8} \left\{ \frac{1}{\varepsilon' \left(\varepsilon' + mc^2 \right)}, \left[\left\{ \boldsymbol{\Pi} \cdot \boldsymbol{\pi}, \left(\boldsymbol{\pi} \cdot \boldsymbol{\mathcal{M}} + \boldsymbol{\mathcal{M}} \cdot \boldsymbol{\pi} \right) \right\} \\ + \beta \left(\boldsymbol{\pi} \cdot \boldsymbol{\mathcal{J}} + \boldsymbol{\mathcal{J}} \cdot \boldsymbol{\pi} \right) - \beta \left\{ \pi_a, \left(\boldsymbol{\omega} \times \mathbf{r} \right)^b \partial_b \boldsymbol{\mathcal{P}}^a \right\} \right] \right\}, \\ \mathcal{P} = \mu' \mathcal{E} - d\mathcal{B}, \quad \mathcal{M} = \mu' \mathcal{B} + d\mathcal{E}, \quad \mathcal{J} = \nabla \times \mathcal{M} - \frac{\partial \mathcal{P}}{\partial t}.$$

When the condition of the Wentzel–Kramers–Brillouin approximation is satisfied, the transition to the classical limit in the relativistic case can be done by replacing the operators in the Foldy-Wouthuysen Hamiltonian and equations of motion by the respective classical quantities

Equation of spin motion



Manifestations of Earth's rotation

The angular velocity of the spin precession in the Cartesian coordinates

$$\Omega_{res} = \Omega + \Omega_g,$$

The angular velocity of the spin precession in the Frenet-Serret coordinates

$$\Omega_{res}^{(FS)} = \Omega - O + \Omega_g.$$

where O is the angular velocity of rotation of the direction of the particle motion v/|v|.

The angular velocity of the spin precession in the cylindrical coordinates

$$\Omega_{res}^{(cyl)} = \Omega - O_z e_z + \Omega_g,$$

where the z axis is orthogonal to the plane of the ring.

Equation of spin motion in the Frenet-Serret coordinates



Effects conditioned by global and local fields significantly differ



In local fields, corrections to the motion of the particle and the spin conditioned by Earth's rotation are rather small.



On other points of the Earth surface, corrections to the motion of the particle and the spin conditioned by Earth's rotation are of the same order and are rather small.

The Sagnac effect takes also place. In storage rings, it manifests in a nonequivalence of different parts of a particle orbit. The time spent by the particle to pass a certain part of the orbit depends on a direction of its velocity relative to the velocity of the Earth rotation. Thus, the particle passes two halves of the orbit during different intervals of time. While the *coordinate* velocity of the particle depends on the Earth rotation, the *measurable* velocity of the particle cannot exceed c.

Other corrections conditioned by the Earth rotation can usually be neglected for particles and nuclei in storage rings.

Manifestations of Earth's gravity

The Earth attraction manifests in additional forces acting on particles/nuclei and in additional torques acting on the spin. The additional forces are the Newton force and the reaction force provided by a focusing system. The additional torques are caused by the corresponding focusing field and by the geodetic effect (the spin precession in a gravitational field). In storage ring EDM experiments, the latter torque leads to the spin rotation about the radial axis with the angular velocity

$$\mathbf{\Omega}_{g} = \frac{2\gamma + 1}{c(\gamma + 1)} \mathbf{\beta} \times \mathbf{g}, \quad \mathbf{\beta} = \frac{\mathbf{v}}{c}$$

where **g** is the Newtonan acceleration and γ is the Lorentz factor. When magnetic focusing is used, this gravitational force defines the nonzero radial magnetic field

$$B_r = \frac{\left(2\gamma^2 - 1\right)mg}{\sqrt{\gamma^2 - 1}\left|e\right|}.$$

20

This field causes the spin rotation with the average angular velocity

$$\mathbf{\Omega}_{m} = -\frac{\left(1+G\gamma\right)\left(2\gamma^{2}-1\right)}{c\left(\gamma^{2}-1\right)}\mathbf{\beta} \times \mathbf{g}.$$

When electric focusing is used, the gravitational force gives rise to the vertical electric field

$$E_z = \frac{\left(2\gamma^2 - 1\right)mg}{\gamma e}$$

The corresponding average angular velocity of the spin rotation is given by

$$\mathbf{\Omega}_{e} = -\frac{2\gamma^{2} - 1}{c\gamma} \left(G + \frac{1}{\gamma + 1} \right) \mathbf{\beta} \times \mathbf{g}.$$

The total effect of the Earth gravity on the spin motion reads

$$\boldsymbol{\Omega}_{g} + \boldsymbol{\Omega}_{m} = -\frac{\gamma \left[1 + G\left(2\gamma^{2} - 1\right)\right]}{c\left(\gamma^{2} - 1\right)} \boldsymbol{\beta} \times \boldsymbol{g},$$
$$\boldsymbol{\Omega}_{g} + \boldsymbol{\Omega}_{e} = -\frac{1 - G\left(2\gamma^{2} - 1\right)}{c\gamma} \boldsymbol{\beta} \times \boldsymbol{g}.$$

In the planned dEDM experiment with magnetic focusing [D. Anastassopoulos *et al* (EDM Collaboration), ``AGS Proposal: Search for a Permanent Electric Dipole Moment of the Deuteron Nucleus at the 10^{-29} e·cm Level], the Earth gravity would bring the effect identical to that given by the deuteron EDM of d= 1.5×10^{-29} e·cm. The effect of the Earth gravity can be important, because the expected sensitivity of the dEDM experiment is of the same order.

For the Schwarzschild metric in the isotropic coordinates,



There is not a significant gravitational correction to $\Omega - \dot{\Omega}$.

Summary

- The initial covariant Dirac equation can be generalized to introduce anomalous magnetic and electric dipole moments
- Relativistic Foldy-Wouthuysen transformation allows to derive the Foldy-Wouthuysen Hamiltonians and the equations of spin motion
- Classical and quantum-mechanical descriptions fully agree
- Earth's rotation does not bring any significant effects for particles and nuclei in storage rings
- Earth's gravity may be important for EDM experiments in storage rings but is usually negligible in other cases

Extra slides

JEDI (Jülich Electric Dipole moment Investigations) collaboration

Some new achievements of JEDI Collaboration (Julich, COSY)

PRL 115, 094801 (2015)

PHYSICAL REVIEW LETTERS

week ending 28 AUGUST 2015

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New Method for a Continuous Determination of the Spin Tune in Storage Rings and Implications for Precision Experiments

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(JEDI collaboration)

A new method to determine the spin tune is described and tested. In an ideal planar magnetic ring, the spin tune—defined as the number of spin precessions per turn—is given by $\nu_s = \gamma G (\gamma \text{ is the Lorentz factor}, G \text{ the gyromagnetic anomaly})$. At 970 MeV/c, the deuteron spins coherently precess at a frequency of $\approx 120 \text{ kHz}$ in the Cooler Synchrotron COSY. The spin tune is deduced from the up-down asymmetry of deuteron-carbon scattering. In a time interval of 2.6 s, the spin tune was determined with a precision of the order 10^{-8} , and to 1×10^{-10} for a continuous 100 s accelerator cycle. This renders the presented method a new precision tool for accelerator physics; controlling the spin motion of particles to high precision is mandatory, in particular, for the measurement of electric dipole moments of charged particles in a storage ring.

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How to Reach a Thousand-Second in-Plane Polarization Lifetime with 0.97-GeV/c Deuterons in a Storage Ring

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(JEDI Collaboration)

We observe a deuteron beam polarization lifetime near 1000 s in the horizontal plane of a magnetic storage ring (COSY). This long spin coherence time is maintained through a combination of beam bunching, electron cooling, sextupole field corrections, and the suppression of collective effects through beam current limits. This record lifetime is required for a storage ring search for an intrinsic electric dipole moment on the deuteron at a statistical sensitivity level approaching $10^{-29} e$ cm.

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rf Wien filter in an electric dipole moment storage ring: The "partially frozen spin" effect

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An rf Wien filter (WF) can be used in a storage ring to measure a particle's electric dipole moment (EDM). If the WF frequency equals the spin precession frequency without WF, and the oscillating WF fields are chosen so that the corresponding transverse Lorentz force equals zero, then a large source of systematic errors is canceled but the EDM signal is not. This effect, discovered by simulation, can be called the "partially frozen spin" effect.

A rf vertical-magnetic-field resonator does not influence the EDM effect. This effect is defined by a rf radial-electric-field resonator.

The angular velocity of the spin rotation in the cylindrical coordinates is given by

$$\Omega^{(cyl)} = \omega_0 \left[1 + b_z \cos\left(\omega t + \chi\right) \right] e_z - \frac{e\eta}{2m} \beta B_0 \left[1 + b_r \cos\left(\omega t + \chi\right) \right] e_r, \qquad \omega_0 = -\frac{eG}{m} B_0.$$

For the rf vertical-magnetic-field resonator,

$$b_r = b_r^{(m)} = b_z = b_z^{(m)} = \frac{a_n B_0^{(osc)}}{B_0}.$$

(A. J. Silenko, arXiv:1508.00742)

Thank you for your attention