Factorization Theorem and Duality: from Low-energy to High-energy Regimes

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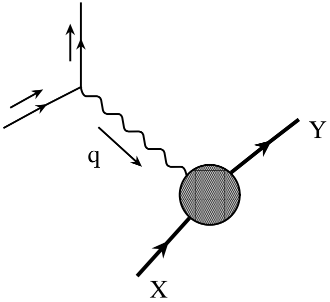
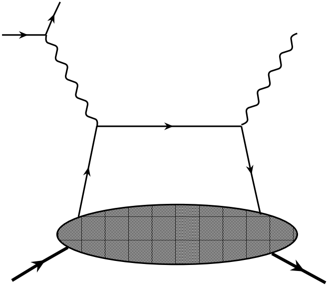
Ultimate Goal: Study of 3-dimensional hadron structure inspired by SPD (talk of R. Tsenov). Theoretical support by Theory of fundamental interaction dept, sector N4 in Bogoliubov Lab.

We study one-photon and two-photon processes

$$\gamma^*(q) + X(P_1) \to Y(P_2), \quad \gamma^*(q) + X(P_1) \to \gamma(q') + Y(P_2)$$

Hard processes imply the processes with large momentum transfers: $-q^2 = Q^2 \to \infty$.

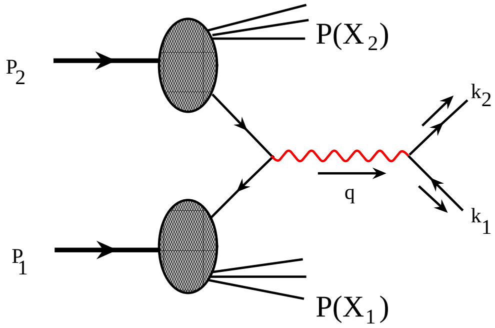




We study

$$A^{(\uparrow\downarrow)}(p_1) + B(p_2) \to \gamma^*(q) + X(P_X) \to \ell(l_1) + \bar{\ell}(l_2) + X(P_X),$$

where $l_1 + l_2 = q$ has a large mass squared ($q^2 = Q^2$).



Independently proposed by Muradyan, Matveev, Tavkhelidze '1969

$$\boxed{\text{Gravitation Form Factors} \Longleftrightarrow \text{Spin Structure}}$$

Since the quark/gluon confinement in QCD, instead of explicit calculation we estimate the amplitude/hadron tensor at the asymptotical regime.

This leads to the Factorization theorem which states that the short (hard) and long (soft) distance dynamics can be separated out provided Q^2 is large, *i.e.*, schematically,

$$\boxed{\text{Ampl./Had. Tensor} = \{\text{Hard part (pQCD)}\} \otimes \{\text{Soft part (npQCD)}\}}$$

where both hard and soft parts are independent of each other, UV- and IR-renormalizable and, finally, parton distributions must possess the universality property.

- 1978: Efremov, Radyushkin, based on α-representation for DIS- and DY-like processes.
- 1982: Efremov, Teryaev, based on EFP-scheme in p-repres for DIS-like processes with S_T, higher twist.
- 2000: Anikin, Teryaev, based on ET-EFP-scheme in p-repres for DVCS-like processes with Δ_T, higher twist.

- The geometrical twist defined for local quark-gluon operators as

$$\tau(\text{twist}) = d(\text{dimension}) - j(\text{spin})$$

- The collinear twist defined for non-local quark-gluon operators as

$$t(\text{coll.twist}) = d(\text{dimension}) - j_a(\text{spin projection})$$

In DIS-case, one has

$$\boxed{\text{Loc.}\mathcal{O}^{\,\text{twist}=\tau}(\bar{\psi}, \psi, A) \Longrightarrow \left(\frac{1}{Q^2}\right)^{\tau/2-1}}$$

The twist is a very useful tool for classification of $1/Q^2$-corrections in the hard processes.

Brodsky-Farrar-Matveev-Muradyan-Tavkhelidze's quark counting rules

Matching:

leading twist-t $\Longleftrightarrow$ leading twist-τ

next-to-leading twist-t $\Longleftrightarrow$ $\tau \leq$ next-to-leading twist-t

That is, any “amplitude" can be presented as

(L-twist-t operator) $\oplus$ (NL-twist-t operator) $\oplus$ ....

where

(NL-twist-t operator) $\ni$ (L-twist-τ operator) , To apply the factorized amplitude/hadron tensor for the low energy, we use the dispersion relations (based on the analytical properties) and the quark-gluon duality that states that the hadron process is equivalent to the parton process provided the asymptotical regime.

Main Subjects

Important information on the Hadron Structure have being extracted from both the low-energy and high-energy regimes (FermiLab, JPAC, RHIC; NICA, COMPASS etc).

- We first deal with the high-energy regime, that is we are interested in the Hard Processes with, at least, one large kinematical parameter: $Q^2 \to \infty$ (the other possible large parameters related to the heavy mass etc.) The main method based on the Factorization Theorem
- Then, owing on the dispersion relations and duality conception, we can use the issues of factorization to study the processes with rather moderate Q^2.

Main results I

- Higher twist nucleon distribution amplitudes in Wandzura-Wilczek approximation, Conformal group and representations for coefficient functions;
- Nucleon Resonance Form Factors, the Axial Form Factors, Threshold pion electroproduction and DAs in QCD;
- Nucleon DAs and OPE for three-quark operators.
- Gravitation FFs and Hadron Spin (in perspective).

- I.V.A., V.M. Braun and N. Offen // PRD88, 114021 (2013); PRD92, 014018 (2015); PRD94, 034011 (2016); PRD93, 034024 (2016)
- I.V.A. and A. N. Manashov // PRD89, 014011 (2014); PRD93, 034024 (2016)
- I.V.A. and V.M. Braun// Work in progress (2018)

Main results II

- Based on the use of Contour Gauge and Collinear Factorization, we propose a new set of SSA which can be measured in Polarized DY process by SPD@NICA.
- All of discussed SSA exists owing to the Gluon Poles manifesting in the $twist-3$ or $(twist-2 \otimes twist-3)$ parton distributions related to the transverse-polarized DY process.

- I.V.A. and O.V. Teryaev // PLB690, 519 (2010); EPJC75, 184 (2015); PLB751, 495 (2015)
- I.V.A., L. Szymanowski, O.V. Teryaev and N. Volchanskiy // PRD95, 111501 (2017)
- I.V.A., I.O. Cherednikov and O.V. Teryaev // PRD95, 034032 (2017)

Nucleon Form Factors within LCSR

I.V.A., V.M. Braun and N. Offen '2013, '2015, '2016

The work consist of three parts:

- Calculations within LCSR;
- Factorized amplitude at LO up to twist-6 and at NLO up to twist-4. We calculated 22 coefficient functions at NLO and 20 of them are new ones. To avoid the mixture with the so-called evanescent operators, we use the renormalization procedure for operators with open Dirac indices;
- Distribution amplitudes: the light-cone expansion up to the twist-4 accuracy of the three-quark matrix elements with generic quark positions

The LCSR approach allows one to calculate the form factors in terms of the nucleon (proton) DAs. To this end we consider the correlation function

$$T_\nu(P,q) = i\int d^4x\, e^{iqx}\langle 0|\,T\left[\eta(0) j_\nu^{\rm em}(x)\right]|P\rangle$$

where T denotes time-ordering and $\eta(0)$ is the Ioffe interpolating current

$$\eta(x) = \epsilon^{ijk}\left[u^i(x) C\gamma_\mu u^j(x)\right]\,\gamma_5\gamma^\mu d^k(x)\,,$$
$$\langle 0|\eta(0)|P\rangle = \lambda_1 m_N N(P)\,.$$

The matrix element of the electromagnetic current

$$j_\mu^{\rm em}(x) = e_u \bar{u}(x)\gamma_\mu u(x) + e_d \bar{d}(x)\gamma_\mu d(x)$$

taken between nucleon states is conventionally written in terms of the Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$:

$$\langle P'|j_\mu^{\rm em}(0)|P\rangle = \bar{N}(P')\left[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N}F_2(Q^2)\right]N(P).$$

In terms of the electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ Sachs form factors, we have

$$\begin{aligned} G_M(Q^2) &= F_1(Q^2) + F_2(Q^2), \\ G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2). \end{aligned}$$

We consider the "plus" spinor projection of the correlation function involving the "plus" component of the electromagnetic current, which can be parametrized in terms of two invariant functions

$$\Lambda_+ T_+ = p_+ \left\{ m_N \mathcal{A}(Q^2, P'^2) + \hat{q}_\perp \mathcal{B}(Q^2, P'^2) \right\} N^+(P)\,,$$

where $Q^2 = -q^2$ and $P'^2 = (P-q)^2$ and

$$N^\pm(P) = \Lambda^\pm N(P),$$
$$\Lambda^+ = \frac{\hat{p}\,\hat{n}}{2pn}\,, \quad \Lambda^- = \frac{\hat{n}\,\hat{p}}{2pn}$$

Making the Borel transformation

$$\frac{1}{s - P'^2} \longrightarrow e^{-s/M^2}$$

one obtains the sum rules

$$2\lambda_1 F_1(Q^2) = \frac{1}{\pi} \int_0^{s_0} ds\, e^{(m_N^2 - s)/M^2} \mathrm{Im}\, \mathcal{A}^{\mathrm{QCD}}(Q^2, s)\,,$$
$$\lambda_1 F_2(Q^2) = \frac{1}{\pi} \int_0^{s_0} ds\, e^{(m_N^2 - s)/M^2} \mathrm{Im}\, \mathcal{B}^{\mathrm{QCD}}(Q^2, s)\,.$$

The correlation functions $\mathcal{A}(Q^2, P'^2)$ and $\mathcal{B}(Q^2, P'^2)$ can be written as a sum:

$$\mathcal{A} = e_d\,\mathcal{A}_d + e_u\mathcal{A}_u\,, \qquad \mathcal{B} = e_d\,\mathcal{B}_d + e_u\mathcal{B}_u\,.$$

Each of the functions has a perturbative expansion which we write as

$$\mathcal{A} = \mathcal{A}^{\rm LO} + \frac{\alpha_s(\mu)}{3\pi}\mathcal{A}^{\rm NLO} + \dots$$

and similar for $\mathcal{B}$; μ is the renormalization scale.

The following Feynman diagrams contribute to the NLO amplitude.

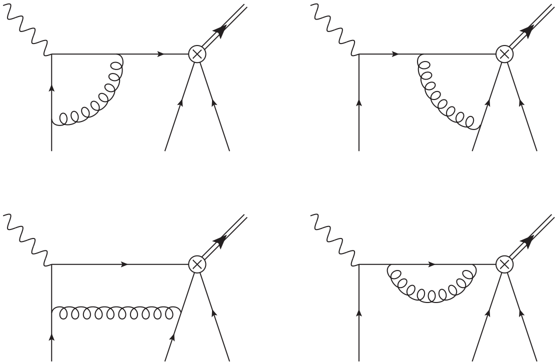


Figure: NLO corrections to the light-cone sum rule for baryon form factors.

Discussion of parameters Schematically, the general structure of form factors has the following form:

$$\mathcal{F} = \mathcal{F}_0^{\text{tw-4}} + \frac{f_N}{\lambda_1}\mathcal{F}_{f_N}^{\text{tw-3}} + \sum_{i=0,1} \eta_{1i}\mathcal{F}_{\eta_{1i}}^{\text{tw-4}} + \frac{f_N}{\lambda_1}\sum_{i=1}^{2}\sum_{j=0;j\leq i}^{2} \varphi_{ij}\mathcal{F}_{\varphi_{ij}}^{\text{tw-3}}\,.$$

Or, in other words, we have

- tw-3: $\left\{\varphi_{10},\varphi_{11},\varphi_{20},\varphi_{21},\varphi_{22}\right\}, f_N$;
- tw-4: $\left\{\eta_{10},\eta_{11}\right\},\lambda_1$;

The other parameters that enter LCSRs are

- the interval of duality (continuum threshold) s_0 ($s_0 = 2.25$ GeV^2);
- Borel parameter M^2 ($M^2 = 1.5$ GeV^2 and $M^2 = 2$ GeV^2 and $M^2 \simeq s_0$) ;
- factorization/renormalization scale μ^2 ($\mu^2 = 2$ GeV^2 and $\mu^2 \sim (1-x)Q^2 - xP'^2$ or $\mu^2 \leq (1-x_0)Q^2 + x_0 M^2 \leq \frac{2s_0 Q^2}{s_0+Q^2} < 2s_0$).
- We use a two-loop expression for the QCD coupling with $\Lambda^{(4)}_{QCD} = 326$ MeV resulting in the value $\alpha_s(2\text{ GeV}^2) = 0.374$.

Model	Method	f_N/λ_1	φ_{10}	φ_{11}	φ_{20}	φ_{21}	φ_{22}
ABO1	LCSR (NLO)	-0.17	0.05	0.05	0.075	-0.027	0.17
ABO2	LCSR (NLO)	-0.17	0.05	0.05	0.038	-0.018	-0.13
BLW	LCSR (LO)	-0.17	0.0534	0.0664	-	-	-
BK	pQCD	-	0.0357	0.0357	-	-	-
COZ	QCDSR (LO)	-	0.163	0.194	0.41	0.06	-0.163
KS	QCDSR (LO)	-	0.144	0.169	0.56	-0.01	-0.163
	QCDSR (NLO)	-0.15	-	-	-	-	-
BS(HET)	QCDSR(LO)	-	0.152	0.205	0.65	-0.27	0.020
LAT09	LATTICE	-0.083	0.043	0.041	0.038	-0.14	-0.47
LAT13	LATTICE	-0.075	0.038	0.039	-0.050	-0.19	-0.19

Model	Method	η_{10}	η_{11}
ABO1	LCSR (NLO)	-0.039	0.140
ABO2	LCSR (NLO)	-0.027	0.092
BLW	LCSR (LO)	0.05	0.0325
BK	pQCD	-	-
COZ	QCDSR (LO)	-	-
KS	QCDSR (LO)	-	-
	QCDSR (NLO)	-	-
LAT09	LATTICE	-	-
LAT13	LATTICE	-	-

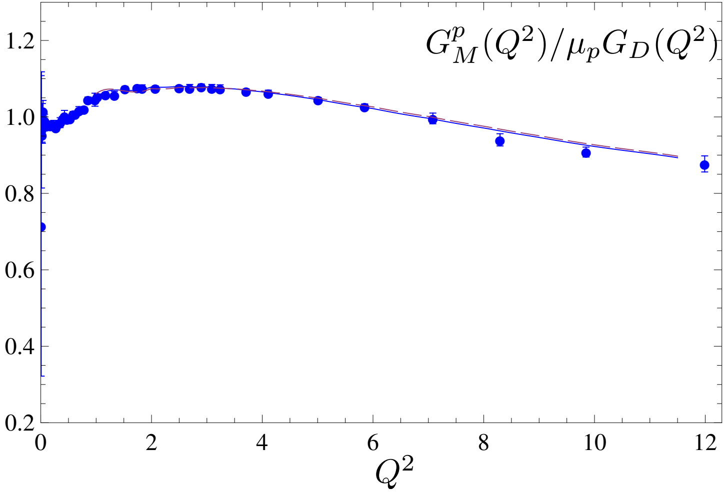
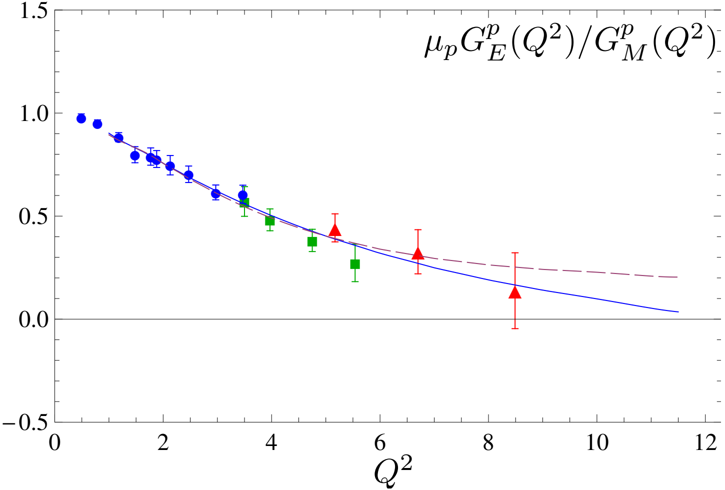


Figure: Nucleon electromagnetic form factors from LCSRs compared to the experimental data [CLAS Coll., Jeff.Lab. Hall A Coll.]. Parameters of the nucleon DAs correspond to the sets ABO1 and ABO2 in Table for the solid and dashed curves, respectively. Borel parameter $M^2 = 1.5$ GeV^2 for ABO1 and $M^2 = 2$ GeV^2 for ABO2.



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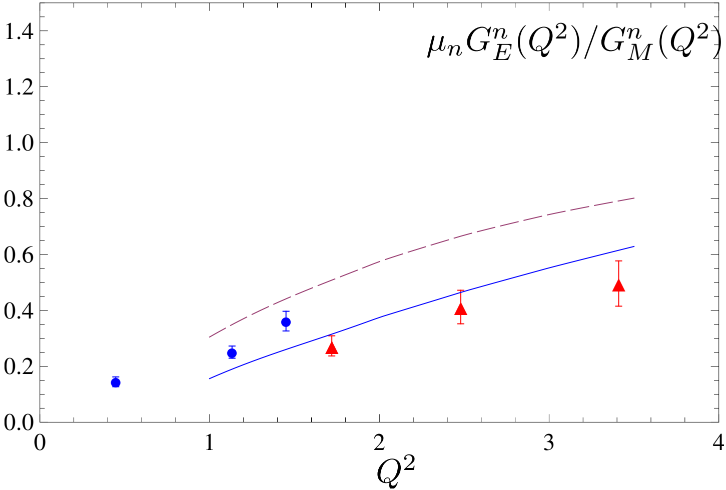

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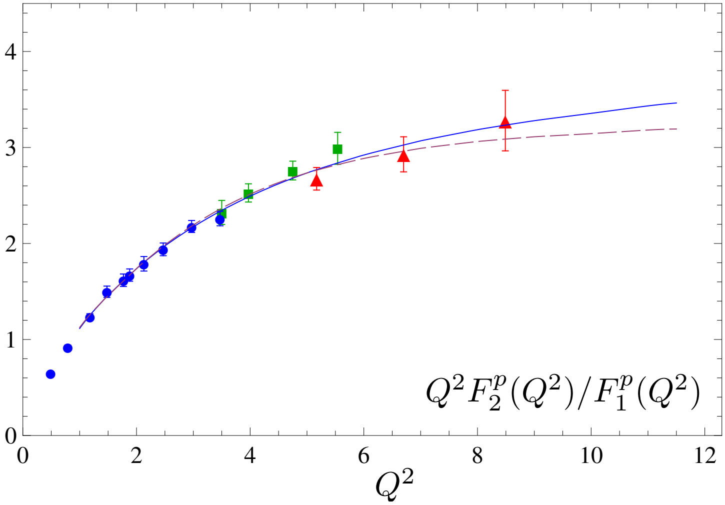


Figure: The ratio of Pauli and Dirac electromagnetic proton form factors from LCSRs compared to the experimental data [Jeff.Lab. Hall A Coll.]. Parameters of the nucleon DAs correspond to the sets ABO1 and ABO2 in Table for the solid and dashed curves, respectively. Borel parameter $M^2 = 1.5$ GeV^2 for ABO1 and $M^2 = 2$ GeV^2 for ABO2.

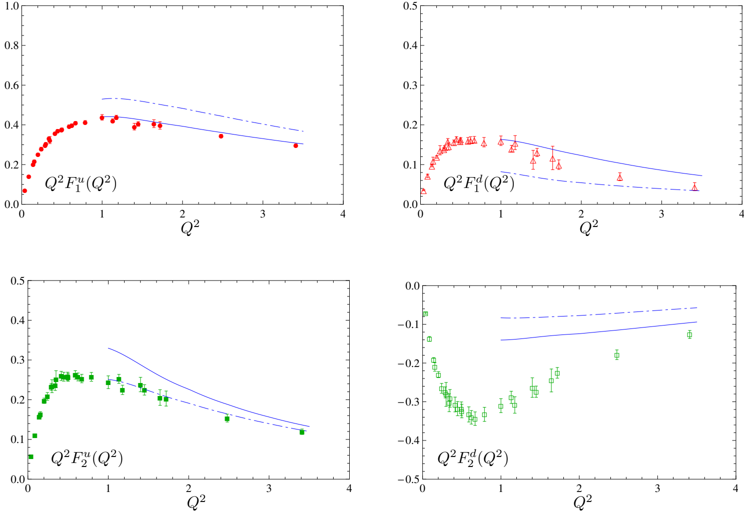


Figure: The corresponding leading-order results are shown by the dash-dotted curves for comparison. Parameters of the nucleon DAs correspond to the set ABO1 in the table.

Gravitation Form Factors

Based on the previous developed approach and results derived within the Light Cone Sum Rules, we study the Gravitation Form Factors and the corresponding relations to the Hadron Spin Structures.

Work in progress....

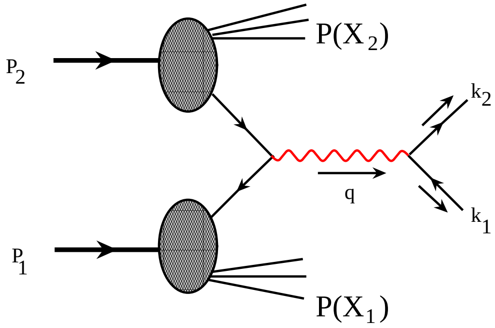
I.V.A., O.V. Teryaev in collaboration with V.M. Braun (Regensburg), M. Vanderhaegen (Mainz) and S. Kumano (Tsukuba)

Drell-Yan-like processes

We study

$$A^{(\uparrow\downarrow)}(p_1) + B(p_2) \to \gamma^*(q) + X(P_X) \to \ell(l_1) + \bar{\ell}(l_2) + X(P_X),$$

where $l_1 + l_2 = q$ has a large mass squared ($q^2 = Q^2$).

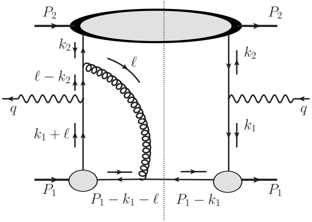


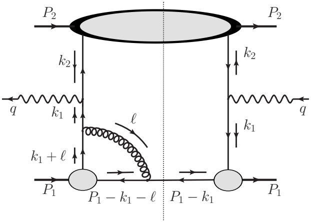
Pion-Nucleon Drell-Yan process

SSA under our consideration is given by

$$\mathcal{A} = \frac{d\sigma^{(\uparrow)} - d\sigma^{(\downarrow)}}{d\sigma^{(\uparrow)} + d\sigma^{(\downarrow)}}, \qquad \frac{d\sigma^{(\uparrow\downarrow)}}{d^4 q d\Omega} = \frac{\alpha_{em}^2}{2jq^4} \mathcal{L}_{\mu\nu} H_{\mu\nu} \,,$$

where $\mathcal{L}_{\mu\nu}$ is a lepton tensor, and $H_{\mu\nu}$ – the QED gauge invariant hadron tensor (direct channel minus mirror channel; $x_F \to 1$).





- The "standard" diagram (*a*) and the "non-standard" diagram (b) differ by the hard parts. (Factorization links: IVA, O.V.Teryaev '09.)

1) The upper blob (with “$-$” dominant direction):

$$\langle P_2, S_\perp | \big[\bar\psi \, \sigma^{-\perp} \, \psi\big]^{\text{tw-2}} | S_\perp, P_2\rangle \stackrel{\mathcal{F}}{\sim} \varepsilon^{-\perp S^\perp P_2} \, \bar h_1(y)$$

2) The lower blob (with “$+$” dominant direction):

$$\langle P_1, S | \big[\bar\psi \, \gamma^+ \, A^\alpha_\perp \, \psi\big]^{\text{tw-3}} | S, P_1\rangle \Big|_{\text{DY}} \Longrightarrow$$

$$\mathcal{D}^{\alpha\beta} \, \langle 0 | \big[\bar\psi \gamma^+ (\gamma_5) \psi\big]^{\text{tw-2}} | S, P_1\rangle \, \gamma_\beta \, \langle P_1, S | \big[\bar\psi \sigma^{-+} (\gamma_5) \psi\big]^{\text{tw-3}} | 0 \rangle \Big|_{\pi N\text{-DY}}$$

which finally leads to

$$\boxed{\widetilde{B}(x_1, x_2) = \frac{1}{2} \frac{\Phi^{\text{tw-3}}_{(1)}(x_1) \overset{\vec{\mathbf{k}}_1^\perp}{\circledast} \Phi^{\text{tw-2}}_{(2)}(x_2)}{x_2 - x_1 - i\epsilon} \, .}$$

where $-i\epsilon$ stems from the contour gauge: $[z^-, -\infty^-]_{A^+} = 1$.

For c-gauge details, see I.V.A. and O.V. Teryaev PLB690, 519 (2010); EPJC75, 184 (2015); PLB751, 495 (2015)

For the chiral-odd contributions, we predict a new SSA which reads

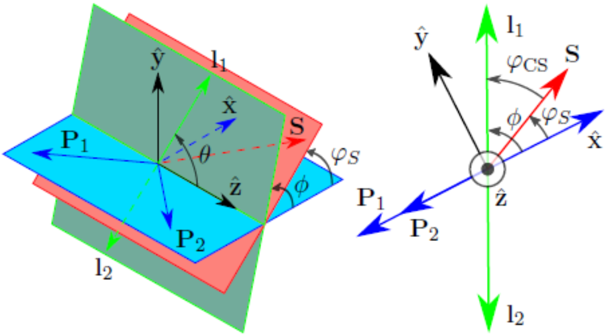
$$\boxed{\mathcal{A}_T = \frac{S_\perp}{Q}\,\frac{D_{[\sin 2\theta]}\;\sin\phi_S B_{UT}^{\sin\phi_S}}{\bar{f}_1(y_B)\,H_1(x_B)},\; D_{[\sin 2\theta]} = \frac{\sin 2\theta}{1+\cos^2\theta}}$$

where

$$B_{UT}^{\sin\phi_S} = 2\bar{h}_1(y_B)\,\Phi_{(1)}^{\text{tw-3}}(x_B) \overset{\vec{\mathbf{k}}_1^\perp}{\circledast} \Phi_{(2)}^{\text{tw-2}}(x_B) \quad \Longleftarrow \quad \Im\mathrm{m}\widetilde{B}(x_1, x_2)$$

and $\bar{f}_1(y_B),\ H_1(x_B)$ emanate from the unpolarized cross-section and they parameterize the following matrix elements.

The result is presented in terms of M. Aghasyan *et al.* [COMPASS Collaboration], arXiv:1704.00488 [hep-ex]



Definition of Collins-Soper frame: cyan, olive and red planes are scattering, lepton and nucleon-spin planes, respectively. The colour of a vector corresponds to the colour of a plane the vector lies in.

$\boxed{\text{Our } \mathcal{A}_T \text{ is a new SSA compared to the recent results from COMPASS.}}$

Indeed, the leading twist Sivers asymmetry $A_{UT}^{\sin\phi_S}$ which formally stands at the similar tensor combination $\varepsilon^{l_1^\perp S^\perp P_1 P_2}$, appears only together with the depolarization factor $D_{[1+\cos^2\theta]}$.
In its turn, the higher twist asymmetries $A_{UT}^{\sin(\phi_S\pm\phi)}$ at $D_{[\sin 2\theta]}$ correspond to the different tensor structures, $\varepsilon^{qS^\perp P_1^\perp P_2} \sim \sin(\phi_S \pm \phi)$.

I.V.A. and O.V. Teryaev, '2009-2010

- Single Transverse SA from Polarized DY:

$$A_T = \frac{d\sigma^{(\uparrow)} - d\sigma^{(\downarrow)}}{d\sigma^{(\uparrow)} + d\sigma^{(\downarrow)}}, \quad d\sigma^{(\uparrow\downarrow)} = (dP.S.)\mathcal{L}_{\mu\nu}\,\overline{\mathcal{W}}^{\text{GI}}_{\mu\nu},$$

$$\mathcal{L}_{\mu\nu} = \ell_{1\,\mu}\ell_{2\,\nu} + \ell_{1\,\nu}\ell_{2\,\mu} - g_{\mu\nu}\frac{q^2}{2}, \quad q = \ell_1 + \ell_2$$

and

$$\boxed{\mathcal{L}_{\mu\nu}\,\overline{\mathcal{W}}^{\text{GI}}_{\mu\nu} = -2\cos\theta\,\varepsilon_{\ell_1 S^T p_1 p_2}\,\bar{q}(y_B)\,T(x_B, x_B)}$$

where $T(x,x) = x\tilde{f}_T(x)$ with T-odd function $\tilde{f}_T(x)$.

Factor of 2 is because of the non-standard diagram contributes as well as the standard one.

Summary

Nucleon Resonance FFs:

- Next-to-leading order QCD corrections to the contributions of twist-three and twist-four DAs;
- Exact account of “kinematic” contributions to the nucleon DAs of twist-four and twist-five induced by lower geometric twist operators (Wandzura-Wilczek terms);
- Light-cone expansion to the twist-four accuracy of the three-quark matrix elements with generic quark positions;
- A new calculation of twist-five off-light cone contributions;
- A more general model for the leading-twist DA, including contributions of second-order polynomials.

Drell-Yan process:

- It is mandatory to include a contribution of the extra diagram which naively does not have an imaginary part;
- This additional contribution emanates from the complex gluon pole prescription in the representation of the twist 3 correlator $B^V(x_1, x_2)$ owing to the corresponding contour gauge;
- For SPD@NICA, we predict new single transverse spin asymmetries to be measured experimentally which are associated with the spin transversity and with the nontrivial φ-angular dependence.

Direct Photon Production (as a Bridge to pA collision, SPD):

- In contrast to DY, this process includes both **ISI** and **FSI** that leads to the different gluon pole prescriptions in the diagrams under our consideration; In turn, the different gluon pole prescriptions ensure the QCD gauge invariance.
- We find that the non-standard new terms, which exist in the case of the complex twist-3 B^V-function with the corresponding prescriptions, do contribute to the hadron tensor in the same way as the standard term known previously. This is another important result of our work. We also observe that this is exactly similar to the case of Drell-Yan process.
- We observed the universality breaking, which spoils the standard factorization. However, the factorization procedure we proposed can still be applied for calculations.

Sector N4 "Standard Model" of SD TFI Bogoliubov Lab

Efremov	Anatoly	Vasilevich	Principal researcher
Anikin	Igor	Valerevich	Head of Sector
Goloskokov	Sergey	Vitalevich	Leading researcher
Mikhailov	Sergey	Vladimirovich	Leading researcher
Radyushkin	Anatoly	Vladimirovich	Leading researcher
Selyugin	Oleg	Victorovich	Leading researcher
Silenko	Alexander	Yakovlevich	Leading researcher
Kotlaz	Dorota	Barbara	Leading researcher
Bytiev	Vladimir	Vyacheslavovich	Senior researcher
Klopot	Yaroslav		Senior researcher
Gavrilova	Margarita	Vyacheslavovna	PhD student
Deka	Mridupavan		Senior researcher
Pivovarov	Alexey	Alexandrovich	Junior researcher
Prokhorov	Gregory	Yurevich	Junior researcher
Seylkhanova	Gulnazym		Junior researcher
Sklyarov	Igor	Konstantinovich	PhD student
Krasnikov	Nikolay	Velerevich	Principal researcher *(collabor.)*
Oganissian	Armen	Gurgenovich	Senior researcher *(collabor.)*
Volchanskiy	Nikolay	Igorevich	Senior researcher *(collabor)*
Teryaev	Oleg	Valerianovich	Principal researcher *(collabor.)*