

# Invariants of angular distributions in hadronic dilepton production

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The dependence of the angular distribution of lepton pairs in hadronic collisions on the choice of the frame of reference is discussed. The rotational invariants including all coefficients, as well as their restrictions, are considered. The data of the E615 experiment have been processed and the verification of rotational invariants based on them has been presented.

## 1 Introduction

We analyze the dependencies of the rotational invariants on the transverse momentum  $q_T$  in the Collins-Soper [5], Gottfried-Jackson [6], u-channel [4] reference frames. To estimate the errors of the rotational invariants it is necessary to calculate their standard deviation  $\sigma$ , for this purpose we used the characteristics of the coefficients  $\lambda, \mu, \nu$  whose functions are the rotational invariants. The calculations were performed on the basis of data from the E615 experiment on the scattering of  $\pi^-$ -mesons with energy 252 GeV on a fixed tungsten target. The result of checking the rotational invariants is their good agreement with the theory: the invariants are qualitatively closer than the coefficients of the angular distributions in different reference frames and lie within one standard deviation. We also analyze the rotational invariants with a zero coefficient  $\mu$ , the result of which is the appearance of a scatter of values of the rotational invariants, i.e., we can conclude about the non-zero value of this coefficient.

## 2 Angular distributions in hadronic processes

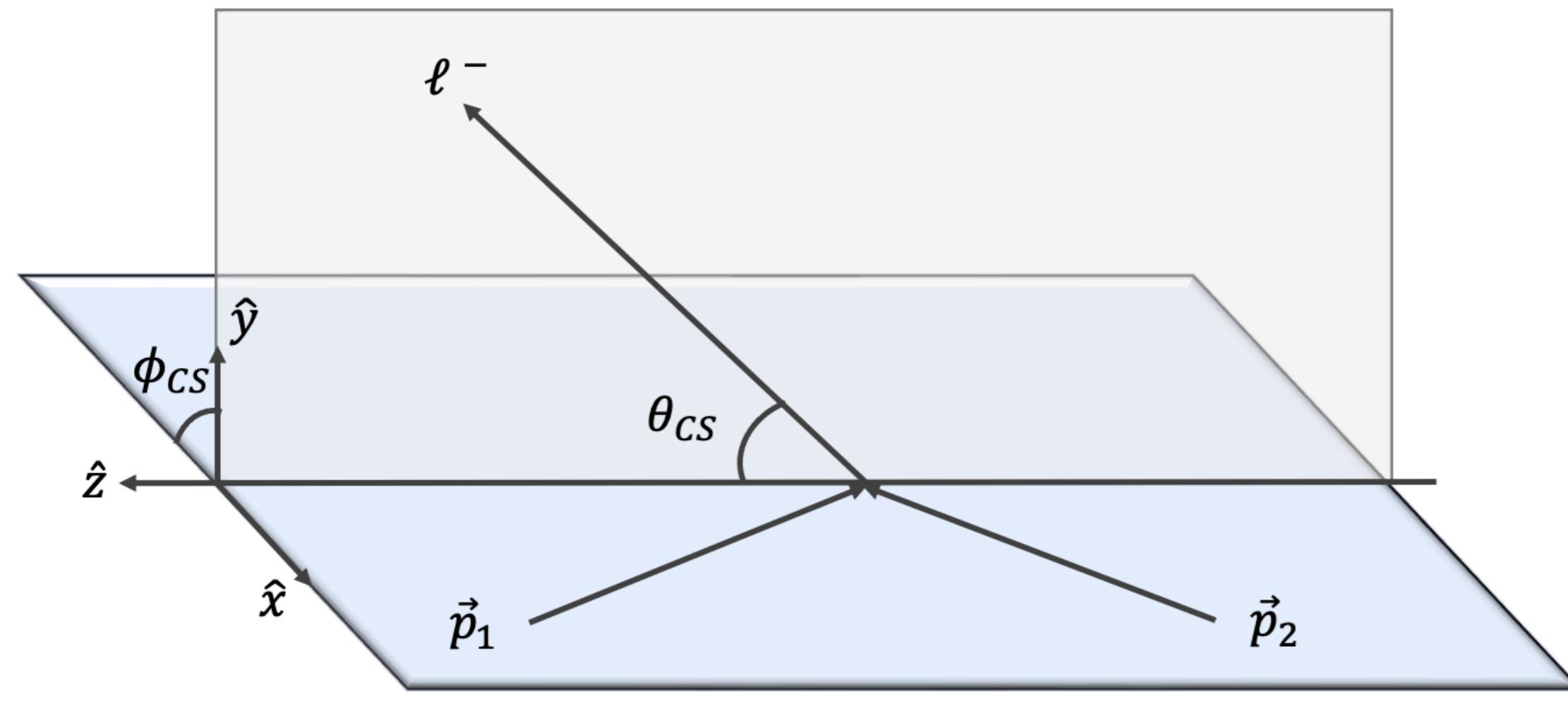


Figure 1: The Collins-Soper reference frame: the angles  $\theta_{CS}$  and  $\phi_{CS}$  are defined for a negatively charged lepton To produce a lepton pair (Z-boson) in the Drell-Jan process in hadronic collisions, the well-known Collins-Soper reference frame (FIG.1) is used. Dileptons are formed in decays of gauge bosons  $Z \rightarrow \ell^-\ell^+$  born in proton-proton collisions in the Drell-Yan process.

The general expression of the angular distribution has the form:

$$\frac{1}{\sigma d\Omega} = \frac{3}{4\pi} \frac{1}{3+\lambda} \cdot (1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi + \rho \sin 2\theta \sin \phi + \sigma \sin^2 \theta \sin 2\phi + 2A_\theta \cos \theta + 2A_\phi \sin \theta \cos \phi + 2A_{\perp\phi} \sin \theta \sin \phi), \quad (1)$$

where  $\theta$  and  $\phi$  – are the polar and azimuthal angles of decay in the rest frame of the dilepton. The coefficients  $\lambda, \mu, \nu, \rho, \sigma, A_\theta, A_\phi, A_{\perp\phi}$  carry information about the tensor polarization of the virtual photon or boson.

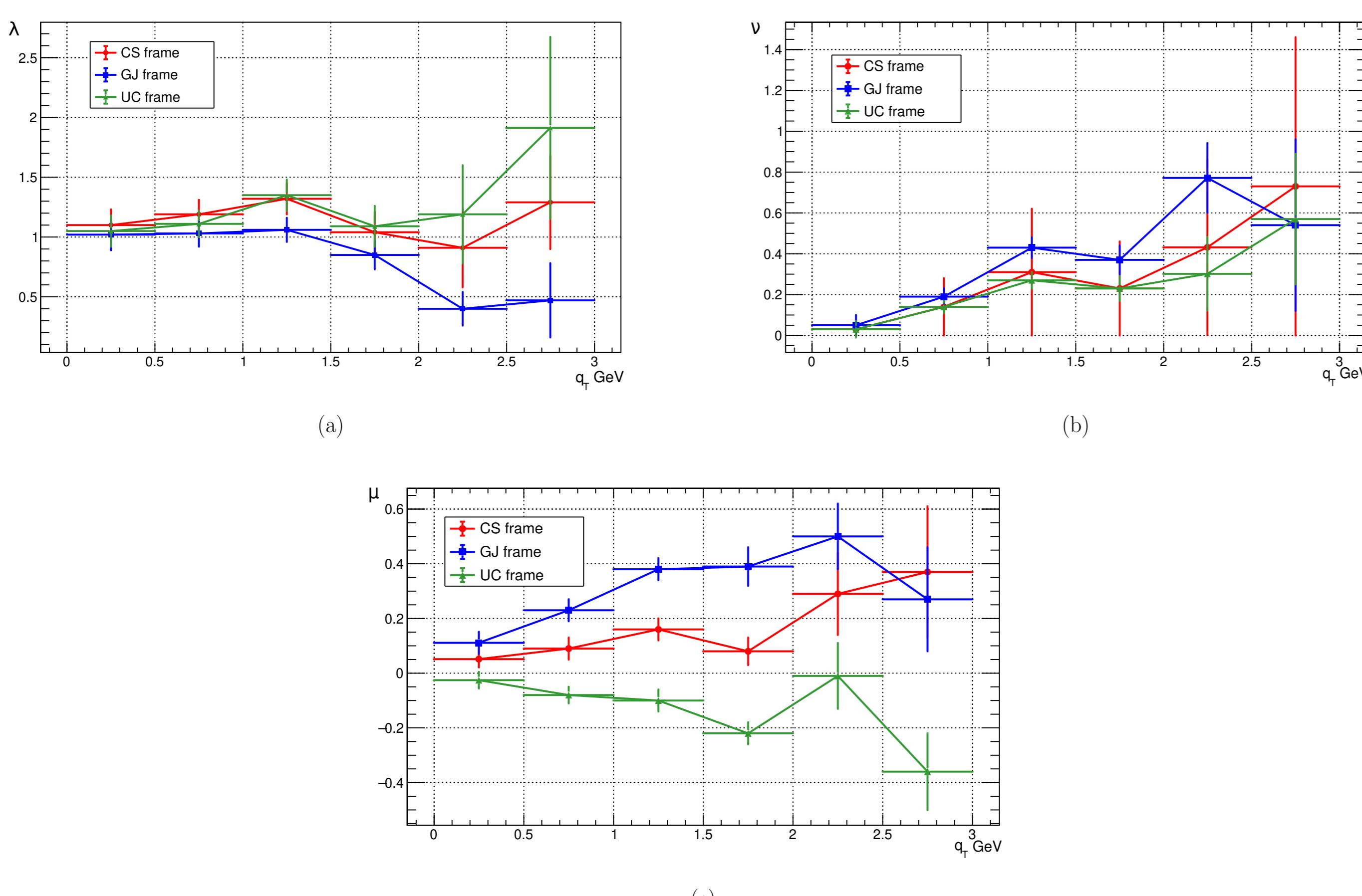


Figure 2: Dependencies of the parameters of angular distributions a)  $\lambda$ , b)  $\nu$ , c)  $\mu$  on the transverse momentum  $q_T$  in different reference frames.

## 3 Invariants

Cross section is proportional to the contraction of a hadronic  $W^{\mu\nu}$  and lepton  $L^{\mu\nu}$  tensors:

$$\frac{1}{\sigma d\Omega} \propto W^{\mu\nu} L_{\mu\nu}. \quad (2)$$

Omitting the detailed derivation, let us use the result from [?]:

$$W^{ij} = \begin{pmatrix} \frac{1-\lambda}{2} & -\mu - iA_{\perp\phi} & -\sigma + iA_\phi \\ -\mu + iA_{\perp\phi} & \frac{1+\lambda-2\nu}{2} & -\rho - iA_\theta \\ -\sigma - iA_\phi & -\rho + iA_\theta & \frac{1+\lambda+2\nu}{2} \end{pmatrix}. \quad (3)$$

Normalization condition:  $Tr \mathbf{W} = 1$ . The density matrix (3) is associated with several rotation-invariant quantities proposed in [3]

$$U_1 = \frac{A_\theta^2 + A_\phi^2 + A_{\perp\theta\phi}^2}{(3+\lambda)^2}, \quad (4)$$

$$U_2 = \frac{\lambda^2 + 3(\frac{\nu^2}{4} + \mu^2 + \rho^2\sigma^2)}{(3+\lambda)^2}, \quad (5)$$

$$T = \frac{(\lambda + \frac{3}{2}\nu)(2\lambda^2 - 3\lambda\nu + 9\mu^2) + 9(\lambda\sigma^2 - 2\lambda\rho^2 + 6\mu\sigma\rho - \frac{3}{2}\nu\sigma^2)}{(3+\lambda)^3}. \quad (6)$$

## 4 Results

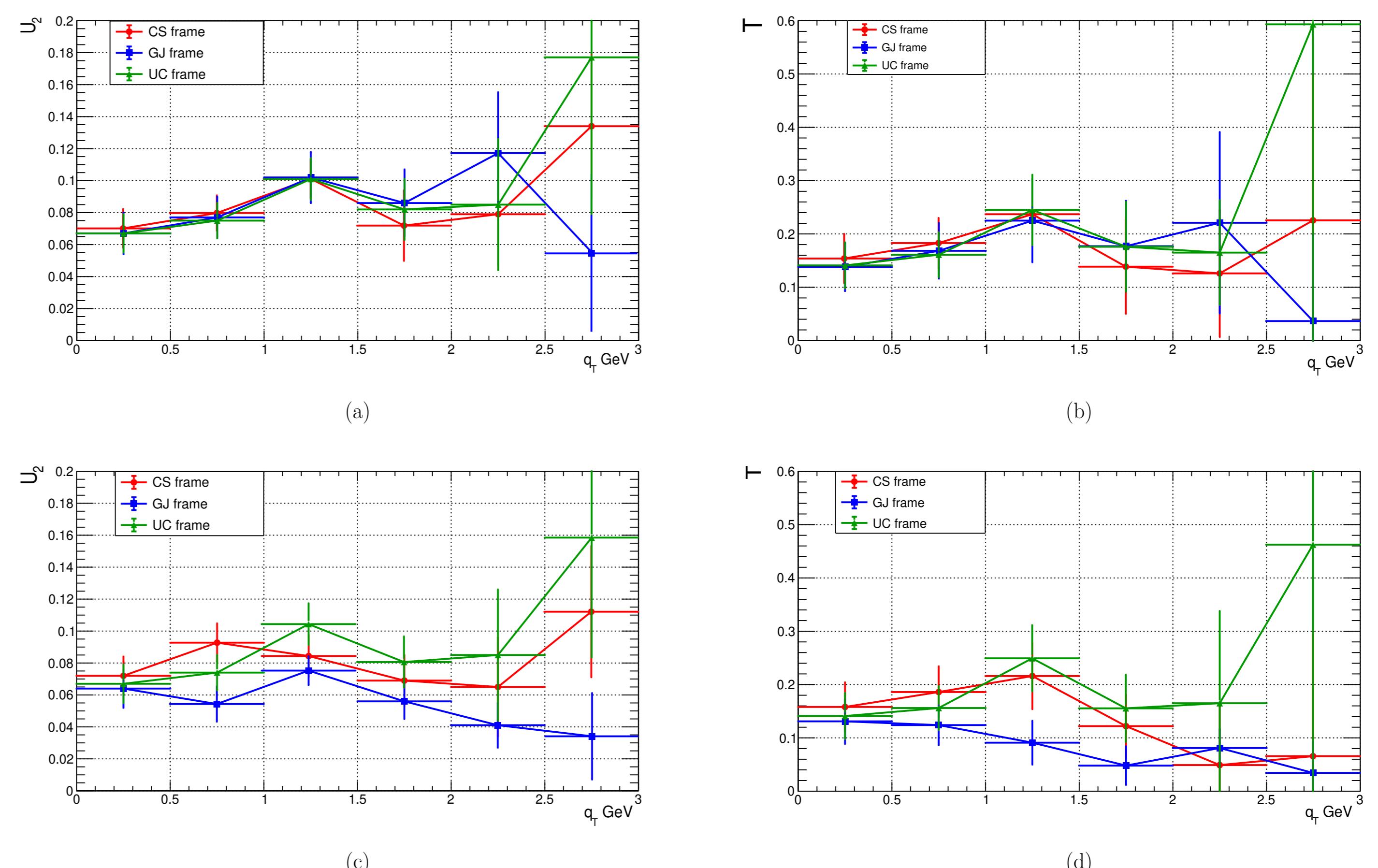


Figure 3: Dependencies of rotational invariants  $U_2$  and  $T$  on transverse momentum  $q_T$  in different reference frames, (a) - (b)  $\mu \neq 0$ , (c) - (d)  $\mu = 0$

Table 1: Average values of coefficients of angular distributions and rotational invariants with standard deviation in different reference frames,  $\mu \neq 0$ .

$q_T$ [GeV]	$\langle \lambda \rangle$	$\langle \nu \rangle$	$\langle \mu \rangle$	$\langle U_2 \rangle$	$\langle T \rangle$
0.25	$1.061 \pm 0.046$	$0.043 \pm 0.011$	$0.061 \pm 0.053$	$0.077 \pm 0.003$	$0.158 \pm 0.011$
0.75	$1.111 \pm 0.085$	$0.165 \pm 0.032$	$0.133 \pm 0.094$	$0.088 \pm 0.004$	$0.182 \pm 0.016$
1.25	$1.247 \pm 0.164$	$0.343 \pm 0.081$	$0.213 \pm 0.151$	$0.119 \pm 0.001$	$0.233 \pm 0.014$
1.75	$0.993 \pm 0.197$	$0.284 \pm 0.082$	$0.237 \pm 0.162$	$0.085 \pm 0.008$	$0.172 \pm 0.019$
2.25	$0.831 \pm 0.401$	$0.515 \pm 0.246$	$0.271 \pm 0.251$	$0.093 \pm 0.019$	$0.175 \pm 0.047$
2.75	$1.226 \pm 0.723$	$0.618 \pm 0.113$	$0.339 \pm 0.063$	$0.125 \pm 0.061$	$0.291 \pm 0.281$

Table 2: Average values of coefficients of angular distributions and rotational invariants with standard deviation in different reference frames,  $\mu = 0$ .

$q_T$ [GeV]	$\langle \lambda \rangle$	$\langle \nu \rangle$	$\langle U_2 \rangle$	$\langle T \rangle$
0.25	$1.061 \pm 0.046$	$0.043 \pm 0.011$	$0.027 \pm 0.004$	$0.143 \pm 0.014$
0.75	$1.111 \pm 0.085$	$0.165 \pm 0.032$	$0.029 \pm 0.019$	$0.155 \pm 0.031$
1.25	$1.247 \pm 0.164$	$0.343 \pm 0.081$	$0.091 \pm 0.013$	$0.181 \pm 0.089$
1.75	$0.993 \pm 0.197$	$0.284 \pm 0.082$	$0.066 \pm 0.009$	$0.103 \pm 0.048$
2.25	$0.831 \pm 0.401$	$0.515 \pm 0.246$	$0.067 \pm 0.017$	$0.098 \pm 0.059$
2.75	$1.226 \pm 0.723$	$0.618 \pm 0.113$	$0.103 \pm 0.068$	$0.187 \pm 0.372$

## 5 Summary

- Rotational invariant do not change in different reference frames and lie within one standard deviation.
- The result with a zero  $\mu$  coefficient is worse, consequently we can conclude that it must be non-zero.

## References

- [1] D. Volkova, N. Gramotkov, O. Teryaev, // Phys. Part. Nuclei Lett., 2023 doi:10.1134/S154747712401014X
- [2] M. Gavrilova and O. Teryaev, // Phys. Rev. D **99** 2019 no.7, 076013 doi:10.1103/PhysRevD.99.076013 [arXiv:1901.04018 [hep-ph]].
- [3] P. Faccioli, C. Lourenco and J. Seixas, // Phys. Rev. Lett. **105** 2010, 061601 doi:10.1103/PhysRevLett.105.061601 [arXiv:1005.2601 [hep-ph]].
- [4] J. S. Conway, C. E. Adolphsen, J. P. Alexander, K. J. Anderson, J. G. Heinrich, J. E. Pilcher, A. Posoz, E. I. Rosenberg, C. Biino and J. F. Greenhalgh, et al. // Phys. Rev. D **39** 1989, 92-122 doi:10.1103/PhysRevD.39.92
- [5] J. C. Collins and D. E. Soper, // Phys. Rev. D **16** 1977, 2219 doi:10.1103/PhysRevD.16.2219
- [6] K. Gottfried and J. D. Jackson, // Nuovo Cim. **33** 1964, 309-330 doi:10.1007/BF02750195