All-loop quantum corrections to effective potentials and its applications

R.M. Iakhibbaev[1], D.I. Kazakov[1,2] , D.M. Tolkachev[1,3]

[1] JINR BLTP, Dubna, Russia
[2] MIPT, Doloprudny, Russia
[3] Stepanov Institute of Physics, Minsk, Belarus

INTRODUCTION

We study the construction of effective potentials for scalar theories with an arbitrary kind of interaction. We derive a generalised renormalisation-group equation for the leading logarithms in such a theory and show on some examples how all-loop corrections affect the form of the potential. We apply our formalism to the study of potentials of different kinds, which are often used in inflationary cosmology.

GENERALISED RG-EQUATION AND ITS APPLICATION

The effective potential is defined as part of the effective action without derivatives. The standard formalism to calculate it is to consider the generating functional for the Green functions in the path integral formalism [1].

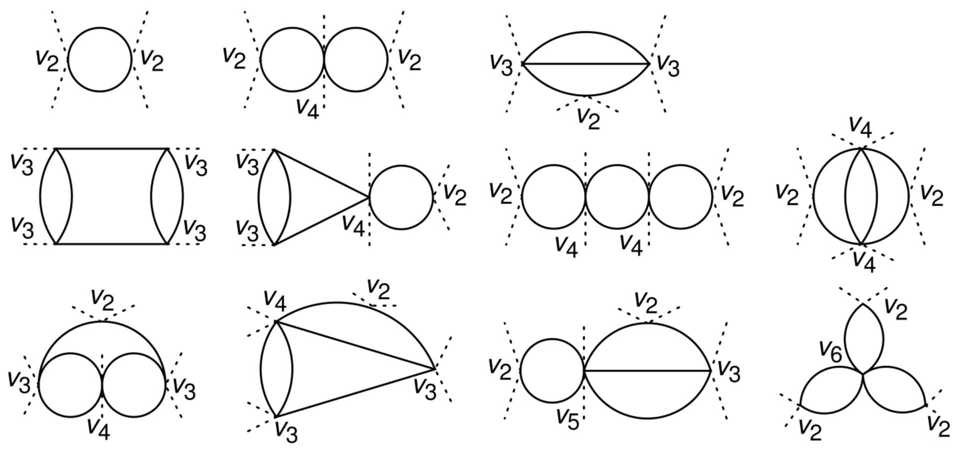
A direct way to calculate the effective action perturbatively is to sum over all 1PI vacuum diagrams acquired using the Feynman rules derived from the shifted action S[ϕ + φ], where ϕ is the classical field obeying the equation of motion and φ(x) is the quantum field. Converting this prescription into the form of Feynman diagrams for a scalar field theory with the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - gV_0(\phi)$$

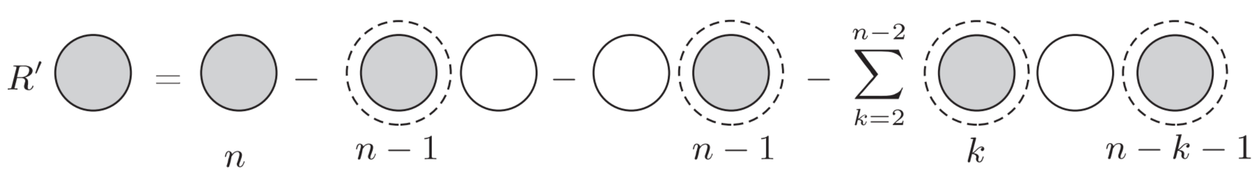
one has to take the 1PI vacuum diagrams with the propagator containing an infinite number of insertions of which acts like a mass term which, however, depends on the field

$$m^2(\phi) = gv_2(\phi) \qquad\qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$

The vertices are also generated by expansion of V(ϕ + φ) over φ. So that we have the next Feynman diagrams up to three-loop order:



R-operation acting on a *n*-loop diagram subtracts first of all the UV divergences in subgraphs starting from one loop and up to *(n-1)*-loops and then finally subtracts the remaining *n*-loop divergence which is always local due to the Bogoliubov-Parasyuk theorem [2]. This *n*-loop divergence left after the incomplete *R'* -operation is precisely what we are looking for. Graphically it can be represented as:



After calculations and taking into account combinatoric factor one can get the generalized RG-equation which sum up all leading divergences in the theory (with *z = g/ε*) :

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2, \quad \Sigma(0,\phi) = V_0(\phi) \quad \Sigma(z,\phi) = \sum_{n=0}^{\infty}(-z)^n\Delta V_n(\phi)$$

This equation is a PDE, which is difficult to solve. To get the effective potential one has just make the substitution

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log gv_2/\mu^2}$$

This formalism and the obtained equation are valid for any type of potential whether it is renormalisable or non-renormalisable. Coleman-Weinberg potential [3] can be obtained from the RG-equation after inserting corresponding quartic scalar interaction:

$$f'(y) = -\frac{3}{2}f(y)^2$$

and after solution and the substitution gives the desired result

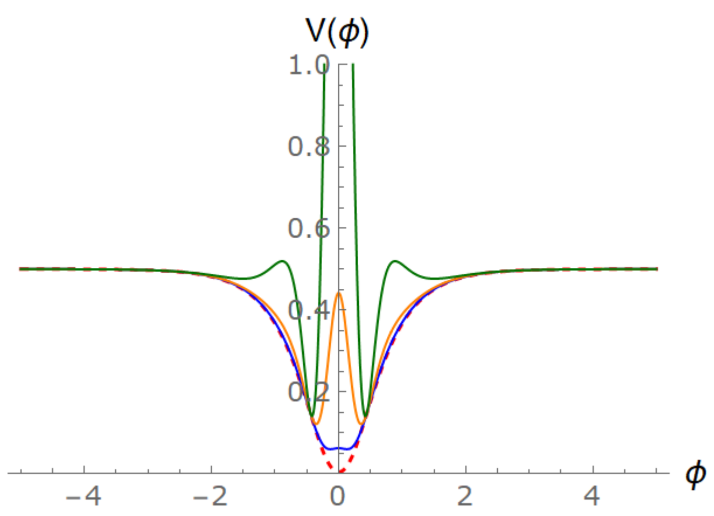
$$V_{eff}(\phi) = \frac{g\phi^4/4!}{1 - \frac{3}{2}\frac{g}{16\pi^2}\log\left(\frac{g\phi^2}{2\mu^2}\right)}$$

The most interesting application of this formalism for obtaining effective potentials is inflationary cosmology, in which all potentials are non-renormalisable.

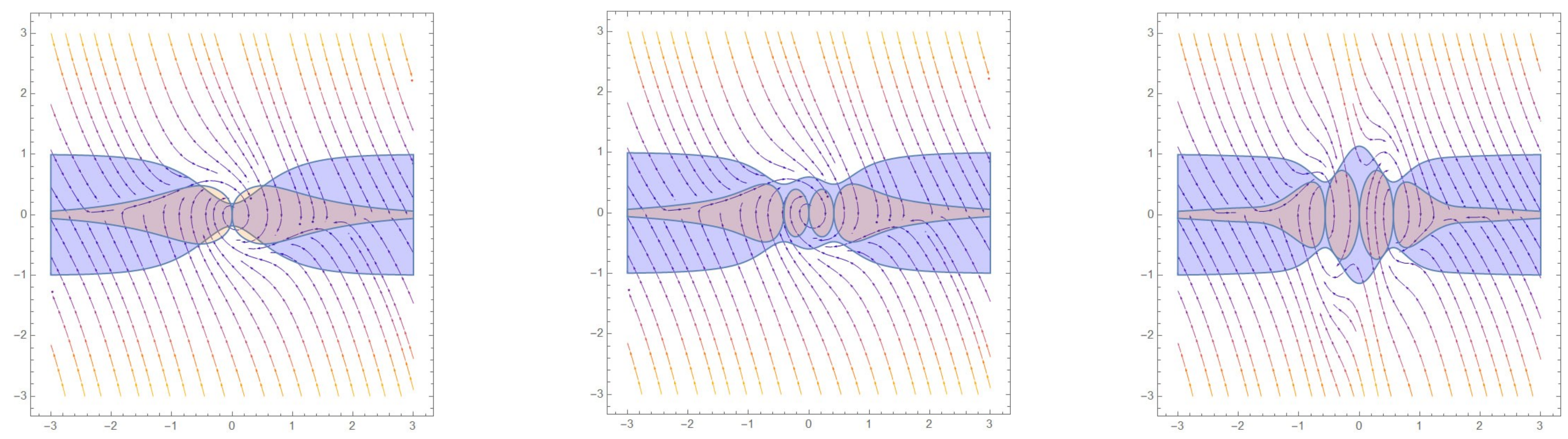
The most interesting potentials in inflationary cosmology is an alpha-attractor potentials first presented in [4]. Cosmological α-attractors are a compelling class of inflationary models. They lead to universal predictions for large-scale observables, broadly independent from the functional form of the inflaton potential. α-attractors arise from supergravity and have a good agreement with observational data, in the simplest form they are potentials of the following type (they are called T-model and E-model respectively):

$$V = g\tanh(\frac{\phi}{\sqrt{6\alpha}M_P})^{2n}, \quad V = g(1-e^{\sqrt{\frac{2}{3\alpha}}\phi/M_P})^{2n}$$

A feature of the alpha-attractor models is the flatness of the potential at infinity, and the slope of the potential satisfies the values of the Hubble flow parameter ratios. We can analyse behaviour of the first case with n=1 and α=1 after summing up all loop corrections with the help of the RG-equation



Comparison of classical and effective potentials at different values of the dimensional transmutation parameter. It is important that in this comparison for all potentials the flat asymptotics is preserved, also one can notice a change of the potential near zero. There occurs a spontaneous symmetry breaking due to quantum corrections. While the one-loop correction strongly affects the potential breaking its original symmetry, the contribution of higher loops smoothes the behaviour of the potential



Phase-space of Friedmann equations: one can notice attractor-like behaviour: purple region corresponds to inflation realization, blue contour correspond to slow-rolling regime

Occurrence of additional maxima can serve as a good tool for modeling of potentials necessary for obtaining primordial black holes. The value of the potential at small values of the transmutation parameter in the global minimum also leads to eternal inflation.

CONCLUSION

We conclude that quantum corrections for the effective potential can be calculated for an arbitrary initial classical potential without renormalizability restriction. Of course, this can be done assuming that the UV divergences are removed by some subtraction prescription. Then the effective potential in the LL approximation becomes fully defined and obeys the RG master equation which is a partial non-linear second order differential equation. In some cases, this equation can be reduced to the ordinary differential one.

We applied our RG-equation to evaluate the potentials of the T-model, which are often used in modern cosmology. We found that while the one-loop correction strongly affects the potential breaking its original symmetry, the contribution of higher loops smoothes the behaviour of the potential. This may lead to effects related to the decay of the metastable vacuum. In the case of T-models of α-attractors, we have found that the asymptotics of the potential remains unchanged when all-loop corrections are taken into account

REFERENCES

[1] R. Jackiw. Functional evaluation of the effective potential. Phys. Rev. D, 9:1686, 1974

[2] N. Bogoliubow, O. Parasiuk. Uber die multiplikation der kausalfunktionen ¨ in der quantentheorie der felder. Acta Mathematica, 97:227–266, 1957

[3] S. Coleman, E. Weinberg. Radiative Corrections as the Origin of Spontaneous Symmetry Breaking. Phys. Rev. D, 7:1888–1910, 1973.

[4] A. Linde, R. Kallosh, Universality Class in Conformal Inflation, JCAP 07 (002), 2013

ACKNOWLEDGEMENTS

Authors are grateful to RSF grant 21-12-00129