QCD Corrections for B meson Weak Radiative Decays

The $B_{s(d)}$-meson (weak) rare decays have been the focus point of theorists and experimentalists for some time, which is due to the potential they provide for the tests of the Standard Model (SM) at scales of several hundreds GeV. Getting experimental information about rare decays puts strong constraints on the extensions of the SM, or can lead to disagreements with the SM predictions, providing an evidence of new physics. To make a rigorous comparison between experiment and theory, one has to get refined theoretical predictions for the rare decay at hand. For the *inclusive* rare B-decays, the perturbative strong interaction effects result in sizable contributions.

Intensive experimental studies led to observation of the rare decay $\bar{B} \to X_s \gamma$ and later of the decay $\bar{B} \to X_s l^+ l^-$. In view of the expected increase in the precision of the experimental measurements, full next-to-next-to-leading logarithmic order (NNLL) calculation was necessary to reduce the theoretical uncertainties and to enable us to perform a rigorous comparison with future experimental data. Some parts of the program of the full NNLL calculation for $\bar{B} \to X_s \gamma$

have already been accomplished (including our papers). The combination of all these individual contributions culminated in the first estimate of the $\bar{B} \to X_s\gamma$ branching ratio at $O\left(\alpha_s^2\right)$: $B\left(\bar{B} \to X_s\gamma\right) = (3.15 \pm 0.23) \times 10^{-4}$, which is consistent with the experimental averages at the 1.2 sigma level. An update of predictions of the branching ratio of $\bar{B} \to X_s\gamma$ decay, incorporating all (including our results) results for the $O\left(\alpha_s^2\right)$ and lower-order perturbative corrections that have been calculated after 2006, was published in our paper. For the CP- and isospin-averaged branching ratio we find $B_{s\gamma} = (3.36 \pm 0.23) \times 10^{-4}$ which is in agreement with the current experimental averages. This gives some bounds on charge Higgs boson mass:

$$
\begin{aligned}
M_{H^\pm} &> 480\,\mathrm{GeV} \quad \text{at } 95\%\ \mathrm{C.L.}, \\
M_{H^\pm} &> 358\,\mathrm{GeV} \quad \text{at } 99\%\ \mathrm{C.L.}
\end{aligned}
$$

Another interesting process is the $\bar{B} \to X_s\gamma\gamma$ decay, which is rather sensitive to new physics contributions and is expected to be much smaller in its branching ratio relative to $\bar{B} \to X_s\gamma$, has attracted new attention in view of the planned experiments at Super B factories, which may test branching ratios as low as 10^{-8}. The main problem of the theoretical description of $\bar{B} \to X_s\gamma\gamma$ is due to the long-distance contributions from $\bar{c}c$ resonant states. However, when the invariant mass of the photon pair $\sqrt{s}$ is restricted to regions far from the resonance, the long-distance effects are under control. All available studies indicate that for the regions 0.0 ≤ s/m_b^2 ≤ 0.3 and for s/m_b^2 ≥ 0.5 the effect of the η_c resonance is practically negligible. Consequently, the decay rate for $\bar{B} \to X_s\gamma\gamma$ can be precisely predicted in this region, using renormalization group improved perturbation theory. At the leading logarithmic (LL) precision the main uncertainty in $B(b \to s\gamma\gamma)$ comes from the dependence on the renormalization scale μ_b, which is 25-30%. The NNL QCD corrections associated with the operator $(O_7 - O_7)$ and $(O_8 - O_8)$ were calculated by the members of our team.

The next process we will consider is the mixing in $B_s - \bar{B}_s$ system. In the SM the mixing of neutral B-mesons is described by box diagrams. The absorptive part of the box diagrams is sensitive to the light internal particles and the dispersive part is sensitive to the heavy internal particles. These two complex quantities can be related to three physical measurable: the mass difference between heavy and light mass eigenstates of neutral B mesons, the decay rate difference between heavy and light mass eigenstates and the CP asymmetry in the semileptonic B-decays. Currently the SM prediction for decay widths difference includes the LO contribution in α_s for LO, NLO and NNLO terms in the heavy quark expansion (HQE) and NLO contribution in α_s for LO terms in HQE. Recently the calculation of $\alpha_s^2 N_f$ (f=b,c,u,d,s) corrections to $\Delta\Gamma_s$ has been completed by our group, with the value of charm quark mass m_c taken equal to zero in the week loop, and in we considered the non-zero value for m_c but only for the penguin sector.

Our main purpose was to calculate QCD corrections for $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s\gamma\gamma$ decays, mixing in $B_s - \bar{B}_s$ system. We will get improved predictions for the physical quantities to perform a comparison with experimental data. Expected results can be used to put constraints on new physics scenarios

The starting point of our calculation is effective Hamiltonian obtained by integrated out heavy particles in the SM

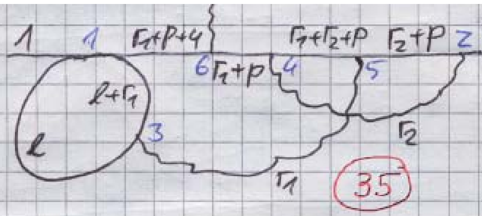
$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^{\star} V_{tb} \sum_{i=1}^{8} C_i(\mu) \mathcal{O}_i(\mu)$$

$$
\begin{aligned}
\mathcal{O}_1 &= (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T_a b_L)\,, & \mathcal{O}_2 &= (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)\,,\\
\mathcal{O}_3 &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_q(\bar{q}\gamma^\mu q)\,, & \mathcal{O}_4 &= (\bar{s}_L\gamma_\mu T^a b_L)\textstyle\sum_q(\bar{q}\gamma^\mu T_a q)\,,\\
\mathcal{O}_5 &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho q)\,, & \mathcal{O}_6 &= (\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\rho T^a b_L)\textstyle\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho T_a q)\,,\\
\mathcal{O}_7 &= \tfrac{e}{16\pi^2}\,\bar{m}_b(\mu)\,(\bar{s}_L\sigma^{\mu\nu}b_R)\,F_{\mu\nu}\,, & \mathcal{O}_8 &= \tfrac{g_s}{16\pi^2}\,\bar{m}_b(\mu)\,(\bar{s}_L\sigma^{\mu\nu}T^a b_R)\,G^a_{\mu\nu}\,.
\end{aligned}
$$

$\bar{B}\to X_s\gamma$: α_s^2 **corrections for physical (non-zero) c quark mass.**

It is well known that a part of the α_s^2 contribution was obtained via interpolation using the results for charm quark mass for the large m_c asymptotic expression and the $m_c = 0$. Now we will try to perform the calculations for physical value of m_c. To be more precise, we will calculate the $(O_7 - O_{1,2})$ operator's interference contribution at α_s^2 order for the physical value of m_c. These calculations are very complicated and we divide them into two tasks.

We first consider diagrams all gluons are not touching b-quark line.



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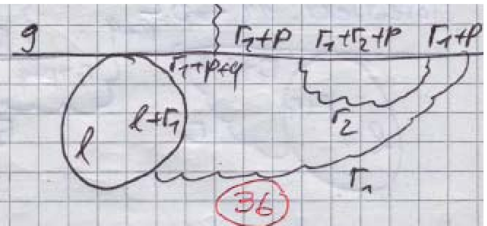

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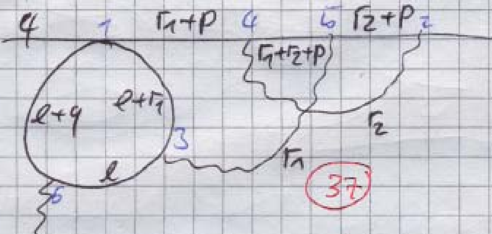

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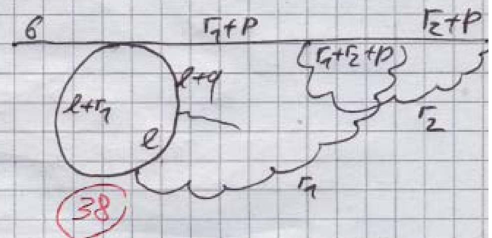

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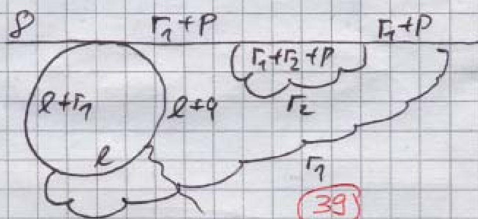

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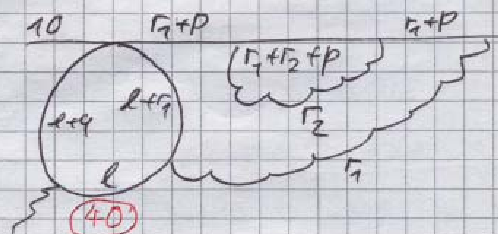

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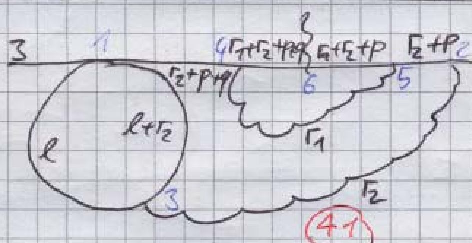

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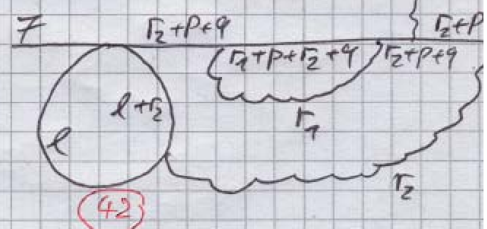

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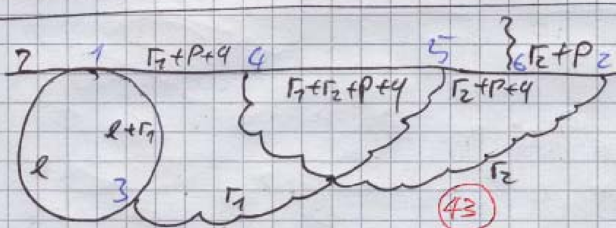

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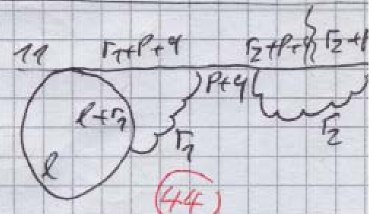

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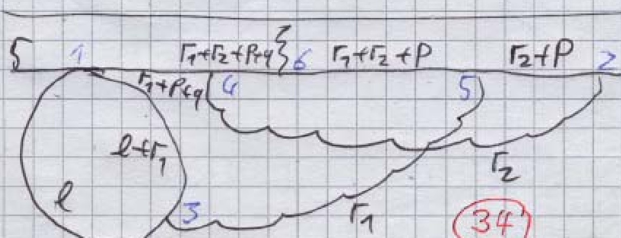

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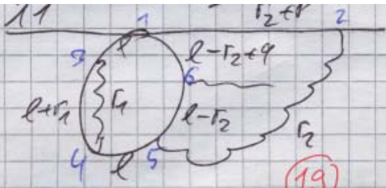

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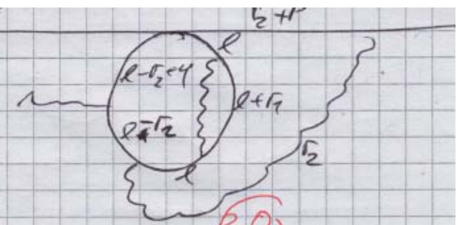

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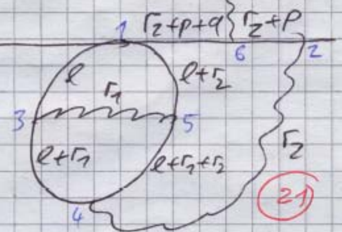

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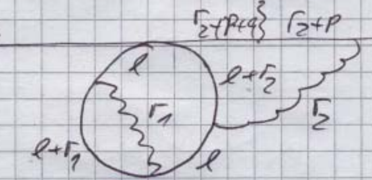

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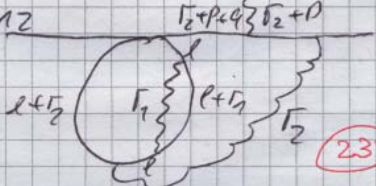

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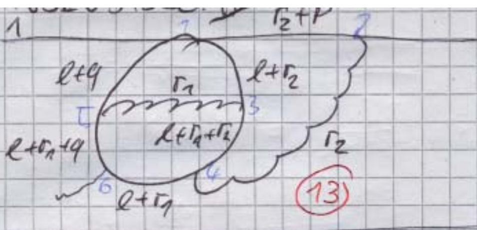

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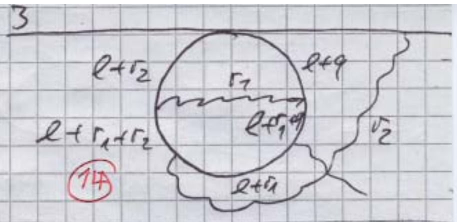

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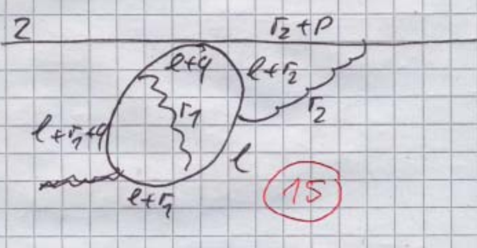

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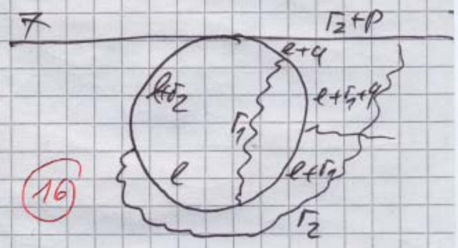

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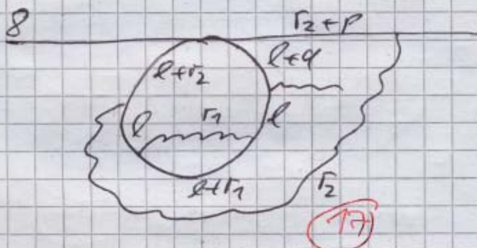

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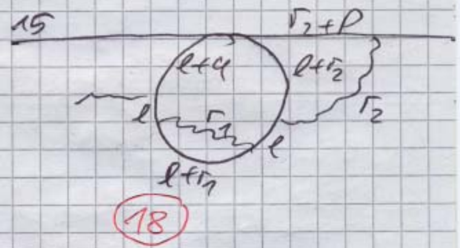

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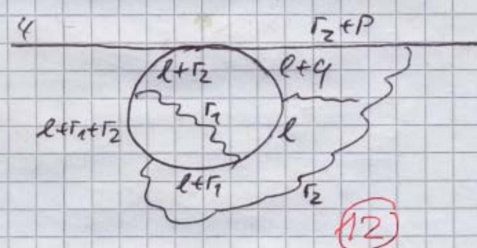

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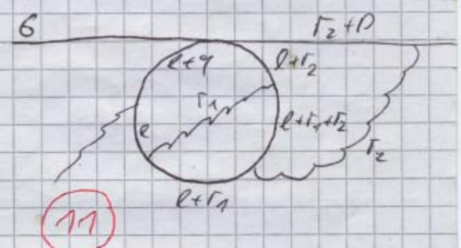

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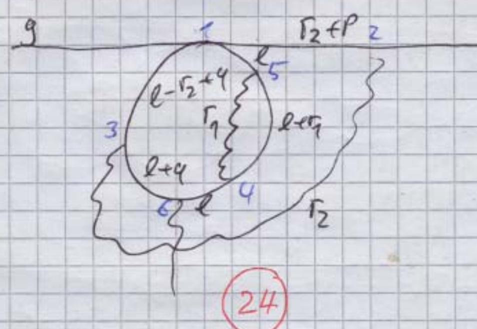

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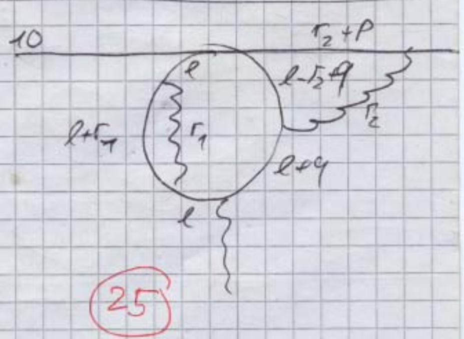

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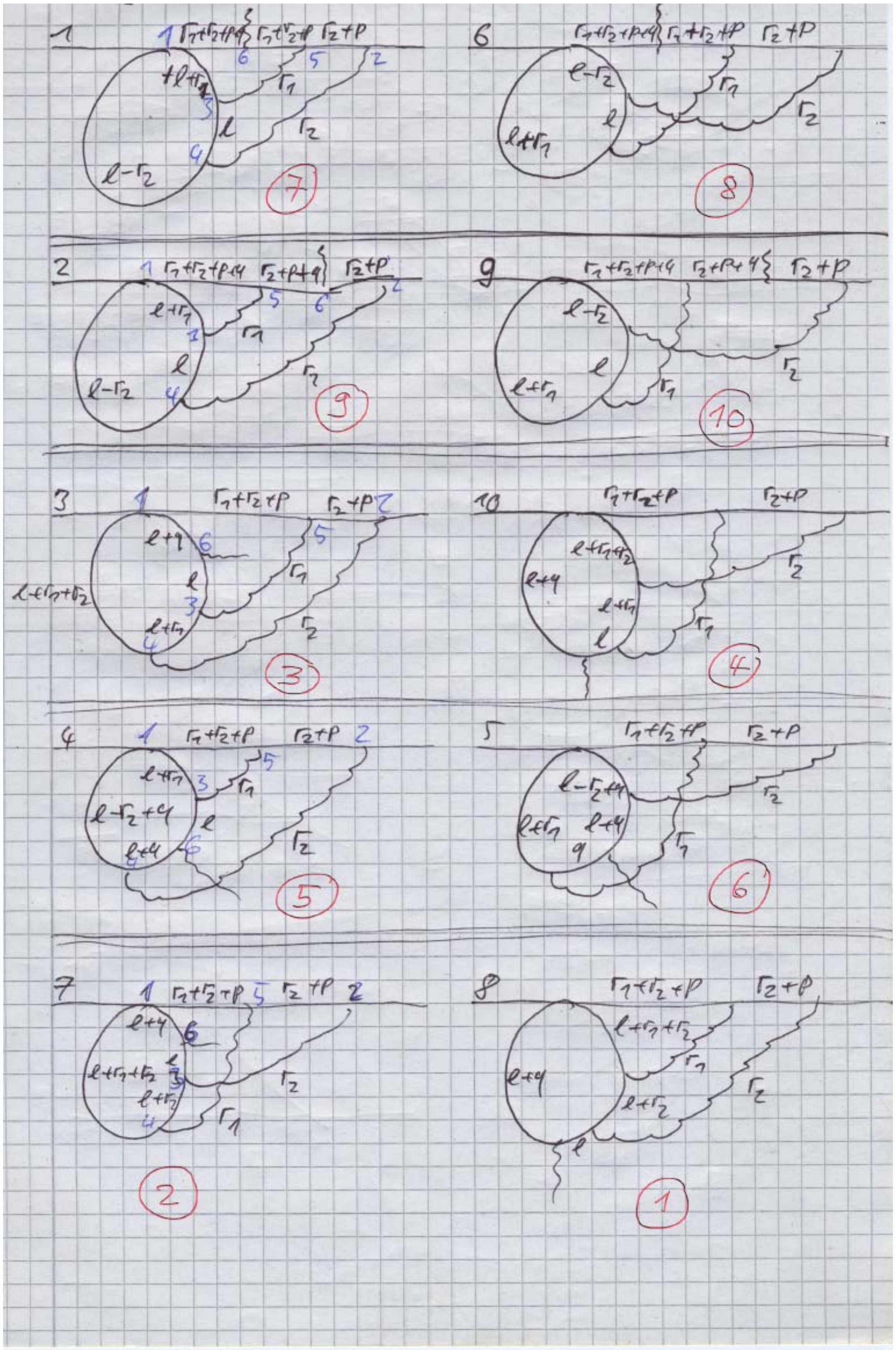

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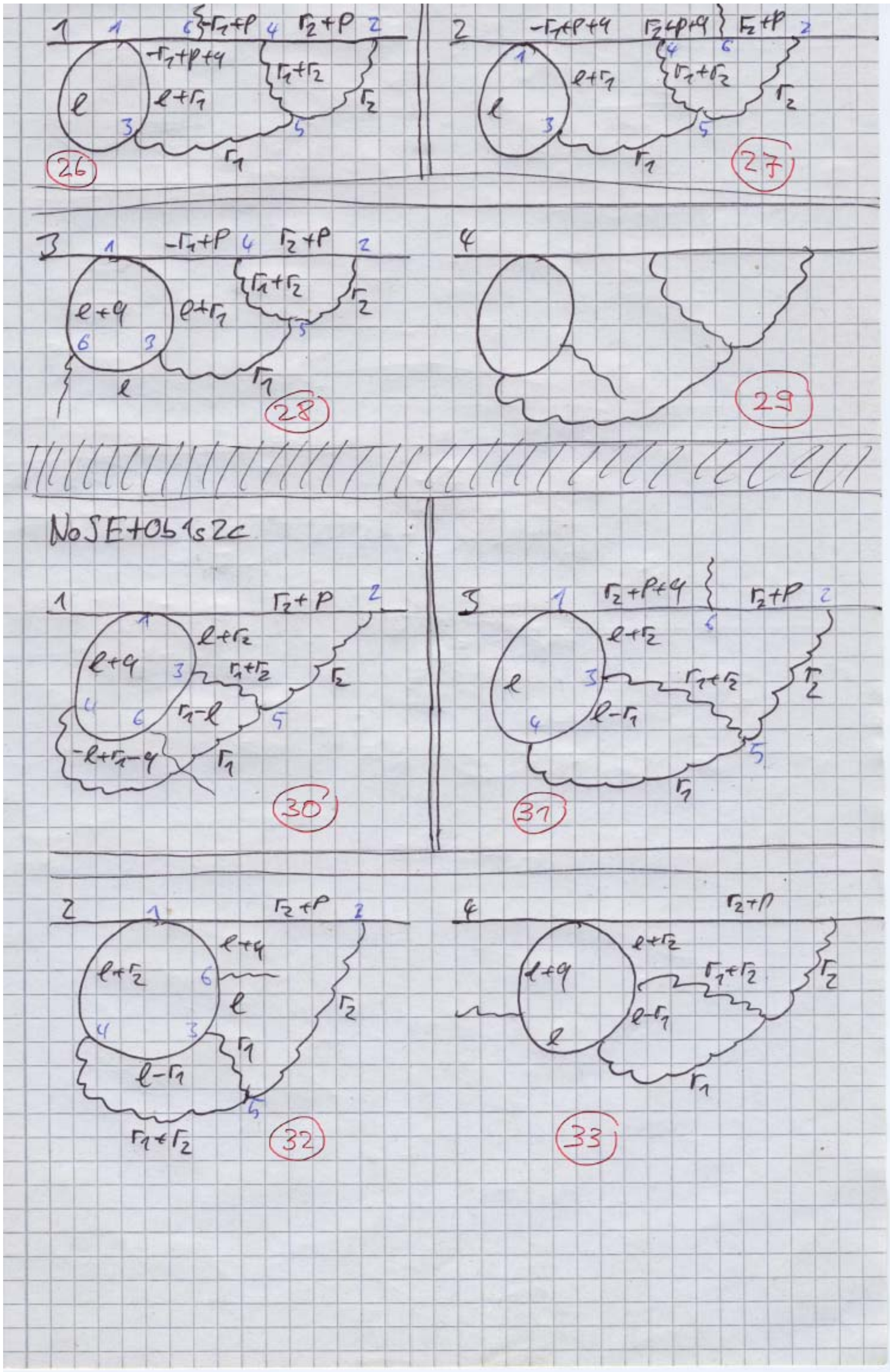


We performed analytic calculations of α_s^2 QCD corrections for the $\bar{B} \to X_s \gamma$ decay for the physical values of m_c. We carry out the reduction to the master integrals by means of package Kira. We use the differential equations method to calculate master integrals as we did it in our recent paper. We get system of differential equations for $z = m_c^2/m_b^2$ for each set of master integrals. We solve differential equations in the so called canonical basis, which considerably simplify the calculations. To get differential equations in canonical basis, we use program Canonica or Libra. Usually it is necessary to also do variable (z) change. For the most of diagrams result is presented in the terms of Generalized Polylogarithms (GPLs) functions. For some diagrams, for which we cannot get canonical basis we solve differential equations expanding that equations on z. For that case we use Fiesta5 program or pysecdec-1.5.5 program to get initial conditions for differential equations.

Canonical basis for n differential equatios

df(x,eps)/dx=eps A(x) f(x,eps),

where eps is parameter of dimensional regularization d=4-2 eps A is n-dimentional matrix and f is n dimentional vector.

The solution of that equations can be presented in term of Generalized Polylogarithms (GPL).

$$G(w_1,\dots,w_n;y) = \int_0^y \frac{dt}{t-w_1} G(w_2,\dots,w_n;t)\ ; \quad G(;y)=1\ ; \quad G(\vec{0}_n;x) = \frac{\log^n x}{n!}\ ,$$

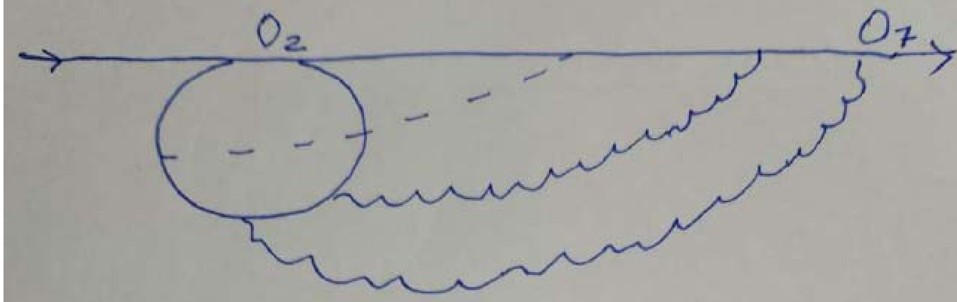
where x is some function of variable z=mc^2/mb^2, for instance: x=1/Sqrt[1-4z]. We fix integration constants using limit z->Infinity.

For all above 44 diagrams we have final results. The paper is published.

The double differential decay rate of the $\bar{B} \to X_s\gamma\gamma$ process at α_s order.

As the $\bar{B} \to X_s\gamma\gamma$ decay, which is rather sensitive to new physics contributions, is expected to be measured at Super B factories, we plan to carry out further investigation of the $B \to X_s\gamma\gamma$ decay. We are planning to perform the calculations of the double differential decay rate of $\bar{B} \to X_s\gamma\gamma$ process at the NLL order in α_s and calculate the contribution of the $(O_7 - O_{1,2})$ operators interference.

We first consider most complicated diagrams when gluon goes from c-quark loop to b-quark line or s quark line. Diagrams where gluon is not touching c quark line or is going from c-quark line to c-quark line are much simpler.



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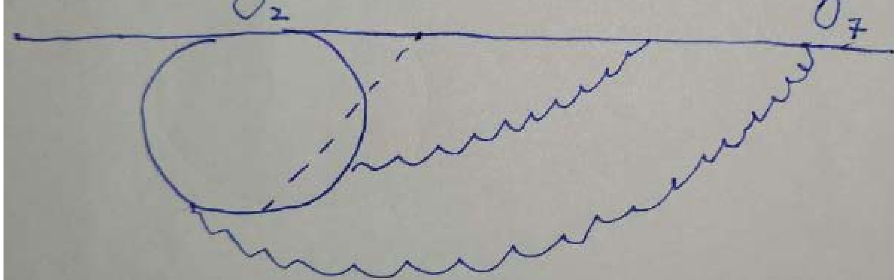

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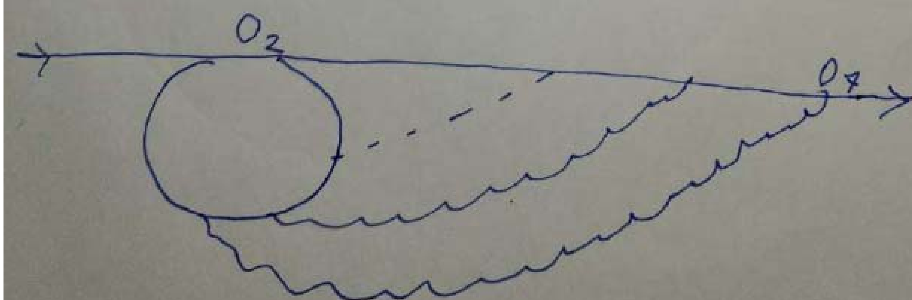

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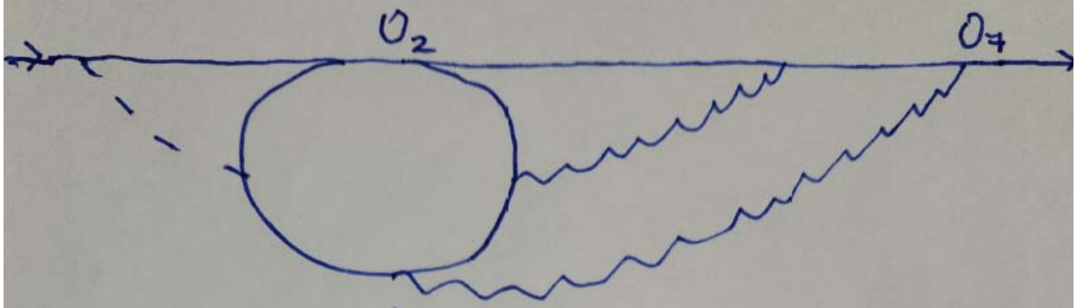

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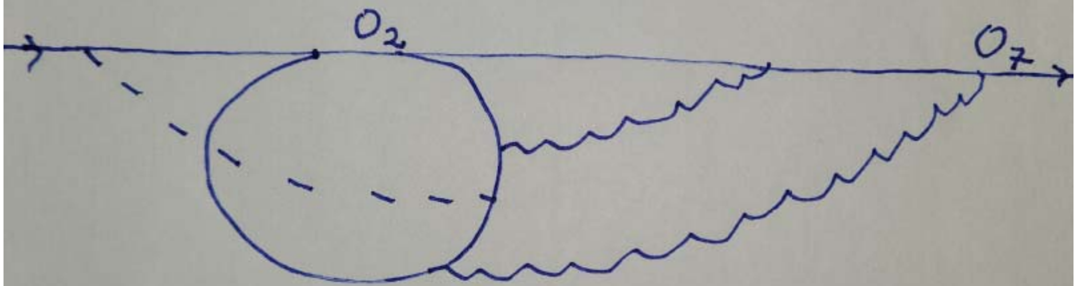

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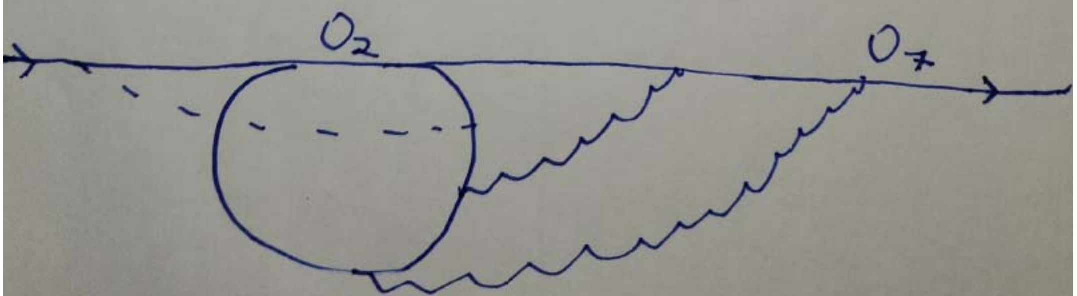

6.

$\overline{B} \to X_s \gamma\gamma$

pb=ps+q1+q2, s1 = (pb - q1)^2, s2 = (pb – q2)^2, ms^2 = (pb - q1 - q2)^2

Integration by part equations.

$$\int [d^D k_1] \ldots [d^D k_{N_k}] \frac{\partial}{\partial (k_j)_\mu} \left((p_l)_\mu V'_{ni\alpha\beta} \right) = 0 \,, \quad j = 1, \ldots, N_k, \; l = 1, \ldots, N_p,$$
$$\int [d^D k_1] \ldots [d^D k_{N_k}] \frac{\partial}{\partial (k_j)_\mu} \left((k_l)_\mu V'_{ni\alpha\beta} \right) = 0 \,, \quad j, l = 1, \ldots, N_k \;,$$

Using Kira program for diag 3 we reduce 311 scalar integrals to 37 master integrals (MI). It took about 3 months on server, data 136GB. Now we are at the stage of calculations of MI-s.

We have now MI-s for all 6 diagrams (37, 47, 37, 37, 47 37).

Propagators for diag. 3.

P1=k^2 - mc^2, P2=(k-q1)^2-mc^2, P3=(k-q1-q2)^2-mc^2
P4 =(k-q1-q2-r)^2-mc^2, P5=r^2, P6 =(pb - r - q1 - q2)^2- ms^2

MI-s for diag 3.

MI={{0,0,1,1,0,1},{0,0,1,1,0,2},{0,1,0,1,0,1},{0,1,0,1,0,2},{0,1,0,1,1,1},{0,1,0,2,0,1},{0,1,1,1,0,1},{0,1,1,1,0,2},{1,0,0,0,0,1},{1,0,0,1,0,0},{1,0,0,1,0,1},{1,0,0,1,0,2},{1,0,0,1,1,0},{1,0,0,1,1,1},{1,0,0,1,1,2},{1,0,0,1,2,0},{1,0,0,2,0,1},{1,0,1,0,0,1},{1,0,1,1,0,0},{1,0,1,1,0,1},{1,0,1,1,0,2},{1,0,1,2,0,1},{1,0,2,1,0,1},{1,1,0,1,0,1},{1,1,0,1,0,2},{1,1,0,1,1,0},{1,1,0,1,1,1},{1,1,0,1,1,2},{1,1,0,2,1,1},{1,1,1,0, 0,1},{1,1,1,1,0,0},{1,1,1,1,0,1},{1,1,1,1,0,2},{1,1,1,2,0,1},{1,1,2,1,0,1},{1,2,0,1,1,1},{2,1,0,1,1,1}}

In our paper which was devoted to O7-O7 order alphas contribution we have a Figure which is illustrating that contribution.

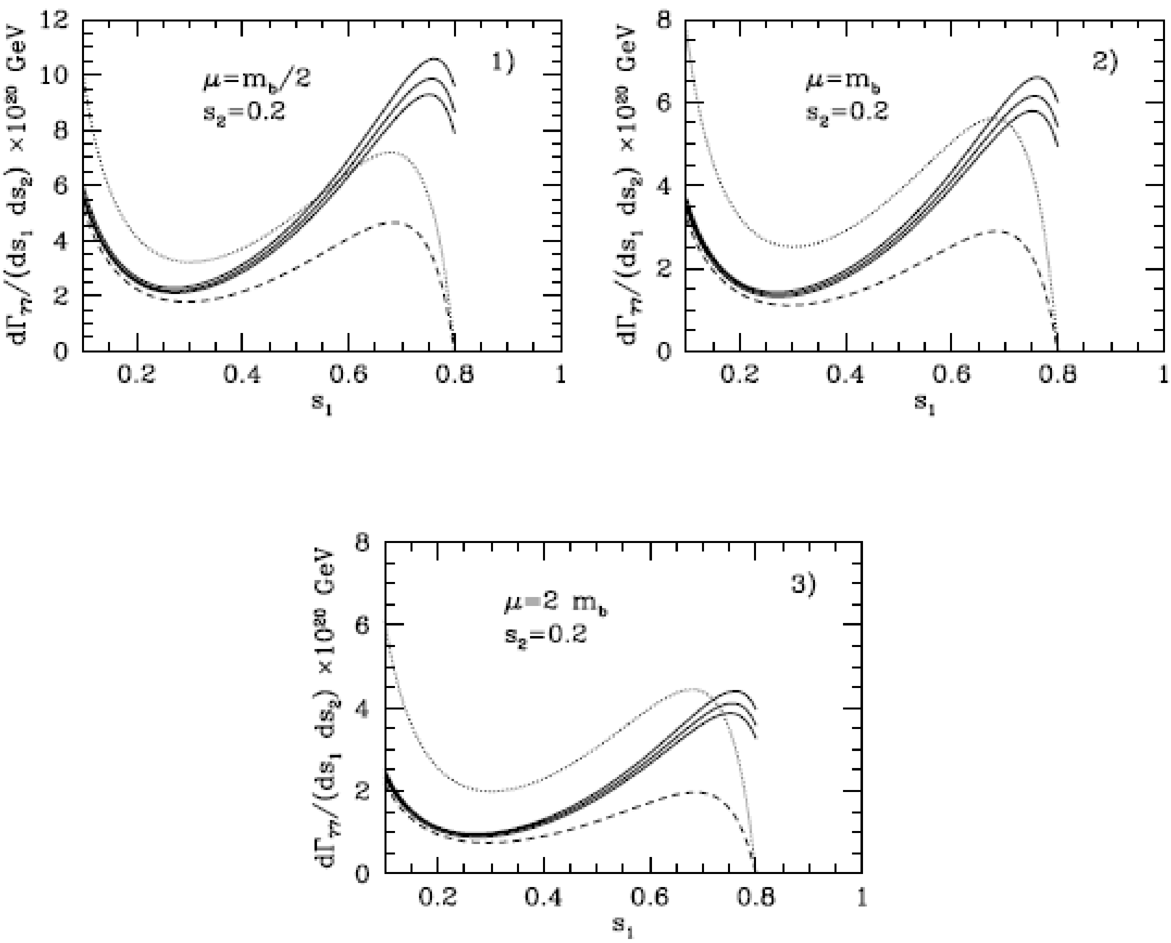

Figure 7: Double differential decay width $d\Gamma_{77}/(ds_1 ds_2)$, based on the operator $\mathcal{O}_7$ only, as a function of s_1 for s_2 fixed at $s_2 = 0.2$. The dotted, the dashed and the solid lines show the LL result, the NLL when only retaining leading power terms as in ref. [8] and the full NLL result of the present paper, respectively. Among the three solid lines, the highest, middle and lowest curve correspond to $m_s = 400$ MeV, $m_s = 500$ MeV and $m_s = 600$ MeV, respectively. In the frames 1), 2) and 3) the renormalization scale is chosen to be $\mu = m_b/2$, $\mu = m_b$ and $\mu = 2\,m_b$, respectively. See text for details.

To get such a Figure for our case and taking into account that we have also dependence from c-quark mass, we are calculating Mi-s for mc/mb=0.27,0.29,0.31

ms/mb=0.4/4.8, 0.5/4.8,0.6/4.8,

s1=1/1000,
s1=1/1000+1*399/25000…1/1000+50*399/25000*399/25000;s2=1/5
(s1+s2<1).
Calculations are in progress.