Z_N symmetry and confinement-deconfinement transition in SU(N) Higgs theory

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22 August 2018

Outline

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- Z_N symmetry (with fundamental Higgs field).
- Simulation study of Z_N symmetry in presence of Higgs.
- N_τ (number of lattice sites in temporal direction) dependence.
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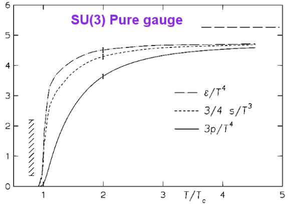
Introduction.



- Pure SU(N) gauge theories undergo confinement deconfinement (CD) phase transition. The nature of the phase transition depends on N. [1]
- The CD transition is present in all SU(N) $[N \geq 2]$ gauge theories like QCD and Electroweak theory.
- This transition is described using the order parameter Polyakov loop and the Z_N symmetry.

[1] *F. Karsch*, Lect.Notes Phys. 583 (2002) 209-249 (arXiv:hep-lat/0106019).

Z_N symmetry

- Partition function of a pure SU(N) gauge theory at high temperature ($T = \frac{1}{\beta}$) is

$$\mathcal{Z} = \mathrm{Tr} e^{-\beta \hat{H}} = \int dA \langle A | e^{-\beta \hat{H}} | A \rangle = \int_{b.c.} DA e^{-S(A)} \quad (1)$$

$$S(A) = \int_0^{\beta} d\tau \int_V d^3x \left\{ \frac{1}{2} \mathrm{Tr} \left(F^{\mu\nu} F_{\mu\nu} \right) \right\} \quad (2)$$

- Where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

- The allowed A's in the path integral are periodic in β,

$$A_\mu(\vec{x}, 0) = A_\mu(\vec{x}, \beta) \quad (3)$$

Contd...

- $S(A)$ and $\mathcal{Z}$ are invariant under the gauge transformation $V(\vec{x}, \tau)$, A_μ transforms

$$A_\mu \longrightarrow V A_\mu V^{-1} - \frac{i}{g}(\partial_\mu V) V^{-1} \quad (4)$$

- $V(\vec{x}, \tau)$ need not be periodic, as long as it satisfies the following eqn.

$$V(\vec{x}, \tau = 0) = z V(\vec{x}, \tau = \beta) \quad (5)$$

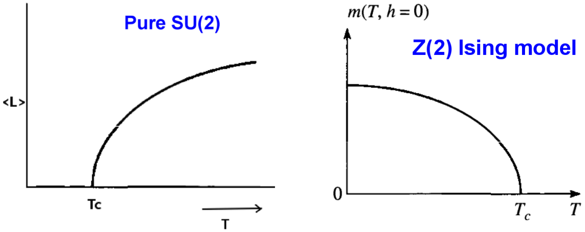
  Where $z \in Z_N$,with $z = \mathbb{1} exp(\frac{2\pi i n}{N}),\ n = 0, 1, 2...N - 1,$
- Therefore, all the allowed gauge transformations at finite temperature are classified by Z_N group.
- Z_N is a symmetry of $\mathcal{Z}$.

Contd...

- The Polyakov loop transforms nontrivially under Z_N.

$$L(\vec{x}) = \frac{1}{N}\mathrm{Tr}\left\{ P e^{\left(-ig\int_0^{\beta} A_0(\vec{x},\tau)d\tau\right)} \right\} \qquad (6)$$

Under $Z_N, L \longrightarrow zL$.



- $\langle L \rangle$ is an order parameter for CD transition and it is analogous to the magnetization in a $Z(N)$ spin system.

Z_N symmetry (with fundamental Higgs field)

- The action in presence of fundamental Higgs field is given by,

$$S_E = \int_0^\beta d\tau \int_V d^3x \left[\frac{1}{2} \mathrm{Tr}\left(F^{\mu\nu} F_{\mu\nu}\right) + \frac{1}{2} |D_\mu \Phi|^2 + \frac{m^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4!} (\Phi^\dagger \Phi)^2 \right. \tag{7}$$

- Where $D_\mu \Phi = \partial_\mu \Phi + igA_\mu \Phi$. Under a gauge transformation $V(\vec{\mathbf{x}}, \tau)$, the Φ field transforms as, $\Phi' = V\Phi$.
- But being a bosonic field, $\Phi(\vec{x}, 0) = \Phi(\vec{x}, \beta)$. Under above non-periodic gauge transformations, $\Phi'(0) \neq \Phi'(\beta)$ (when $z \neq \mathbb{1}$).
- It is not clear how this Z_N explicit breaking will affect the CD transition. Fluctuations of the gauge and Higgs fields need to be considered.
- We plan to do this by numerically simulating the partition function.

Monte Carlo simulations of the CD transition

- For simulations, we discretise the action on a 4-D euclidean space,

$$S = \beta_g \sum_p \frac{1}{2}\mathrm{Tr}(2 - U_p - U_p^\dagger) - \kappa \sum_{\mu,n} \mathrm{Re}\left[\mathrm{Tr}(\Phi^\dagger_{n+\mu} U_{n,\mu} \Phi_n)\right] + \sum_n \left[\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger \Phi_n\right) + \lambda\left(\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger \Phi_n\right) - 1\right)^2\right]. \tag{8}$$

- Here $\beta_g = \frac{2N}{g^2}$, Φ_n is Higgs field at site n, link $U_{n,\mu} = e^{iagA_{n,\mu}}$.
- Plaquette U_p is the product of links around an elementary square 'p' ($U_p = U_{n,\mu} U_{n+\mu,\nu} U^\dagger_{n+\nu,\mu} U^\dagger_{n,\nu}$).
- To study the Z_N symmetry, we compute the distribution of the Polyakov loop and other properties using Monte Carlo simulations [2],

[2] *M. Biswal, S. Digal and P. S. Saumia,* **Nucl. Phys. B 910, 30 (2016)**.

Our plan of study

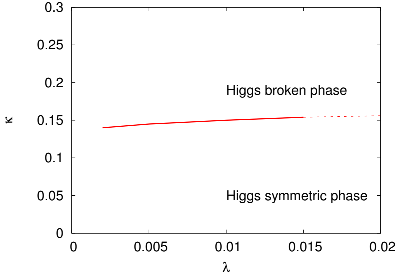


Figure: Higgs phase diagram

- In this Higgs phase diagram the Higgs symmetric ($\langle\Phi\rangle = 0$) and broken phase ($\langle\Phi\rangle \neq 0$) are separated by the Higgs transition line.
- We compute the Polyakov loop distribution at various points on this phase diagram to study the Z_N symmetry.

Polyakov loop distribution (close to Higgs transition line)

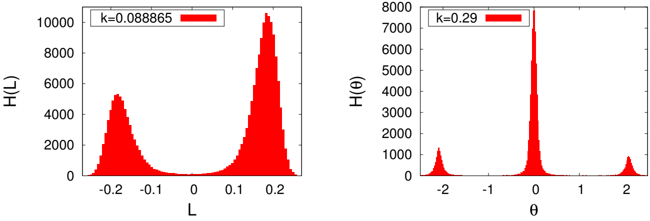


Figure: SU(2) and SU(3)

- There is no Z_2 symmetry in the distribution H(L) of the Polyakov loop for SU(2).
- Similarly for SU(3) there is no Z_3 symmetry of the Polyakov loop distribution.
- Here Z_N symmetry is explicitly broken.
- Largest peak corresponds to the stable state and others correspond to meta-stable states.

Polyakov loop distribution (Away from Higgs transition line)

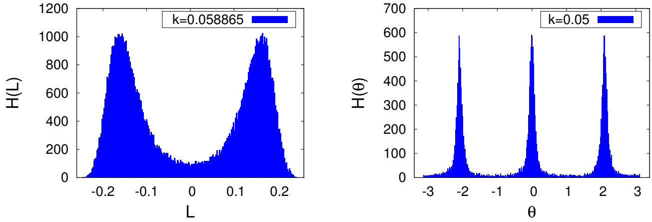
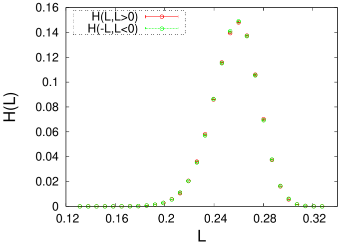
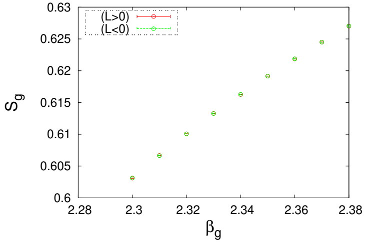


Figure: SU(2) and SU(3)

- The two peaks in case of SU(2) are related by Z_2 symmetry($L \to -L$).
- Similarly the distribution of the Polyakov loop for SU(3) has the Z_3 symmetry.
- So away from the Higgs transition line the Z_N symmetry is restored.

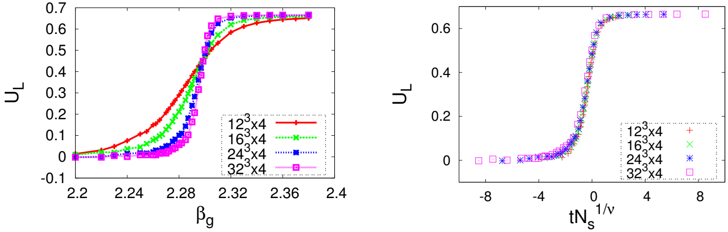
$H(L)$ and Gauge action showing Z_2 symmetry





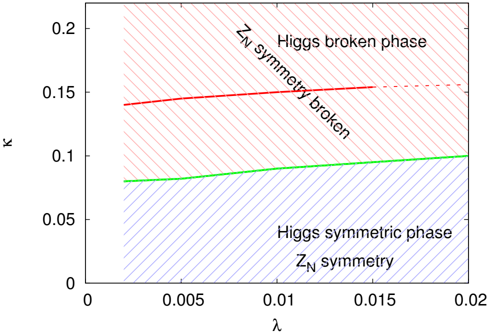
- Within errors $H(L) = H(-L)$ and $S_g(L) = S_g(-L)$.
- This is clear evidence that there is Z_2 symmetry.

This symmetry restoration leads to critical behavior



- The value of the Binder cumulant$(U_L = 1 - \frac{\langle L\rangle^4}{3\langle L^2\rangle^2})$ at the crossing point for different volumes is consistent with the 3D-Ising Universality class.
- It is clearly seen that, by scaling β_g by $(\frac{\beta_g - \beta_{gc}}{\beta_{gc}})N_s^{\frac{1}{\nu}}$ all different volume curves collapse on one line.
- This corresponds to a second order phase transition.

Results for Z_2 symmetry ($N_\tau = 4$)



- The Z_N symmetry is explicitly broken in the Higgs broken phase and close to the Higgs transition line in the Higgs symmetric phase.
- Restoration of Z_N symmety happens in the part of Higgs symmetric phase away from Higgs transition line.

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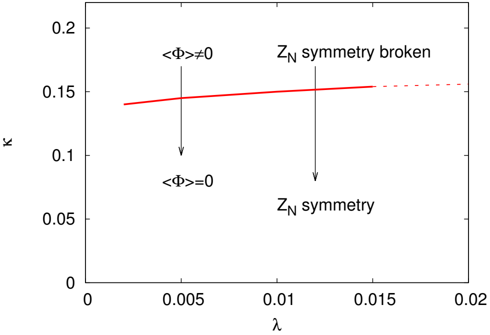
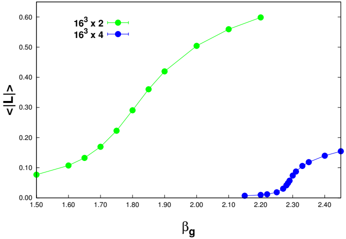


- Z_N symmetry explicit breaking decreases with decrease in κ.
- On the other hand, Higgs condensate decreases with decrease in κ.
- Therefore, we believe that the Higgs condensate plays the role of symmetry breaking field like external field in the Ising model.

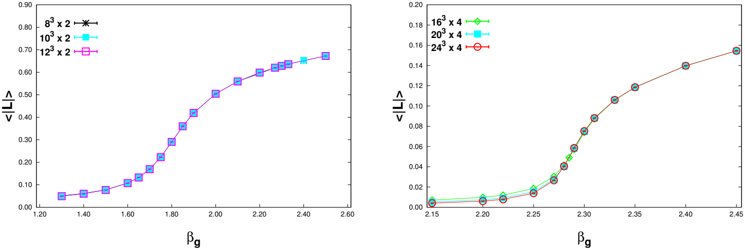
N_τ dependence of the Z_N symmetry and CD transition.

- For pure gauge theory, the nature of the CD transition doesn't depend on N_τ. However the Polyakov loop does.



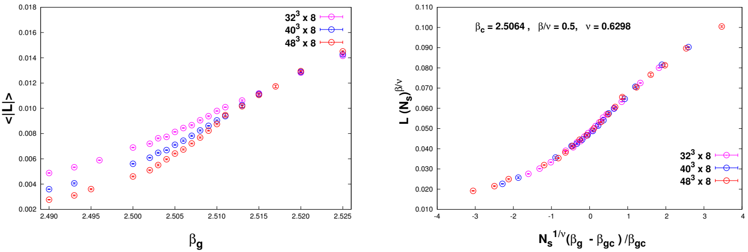
- It is importrant to check, if the explicit symmetry breaking depends on N_τ.
- Therefore we consider larger N_τ. To start with, we take SU(2) Higgs with $\lambda = 0$ and $ma = 0$ [3].
- With the choice of the parameters, we can compare our results with the perturbative calculations [4]

CD transition for smaller N_τ ($N_\tau = 2, 4$).



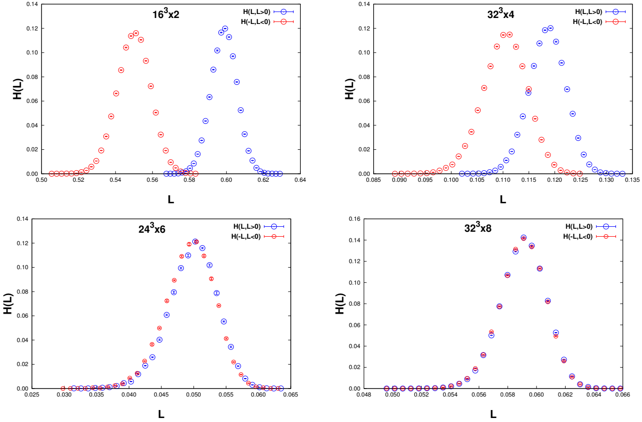
- The figure clearly show that the Polyakov loop does not show any volume dependence.
- Nevertheless the Polyakov loop varies sharply in a small range of β.
- Therefore,the CD transition is a cross over for $N_\tau = 2, 4$.

CD transition for larger N_τ ($N_\tau = 8$)



- In the left figure, the Polyakov loop avg. shows volume dependence.
- By scaling X-axis and Y-axis, the different volume curves collapse into one line.
- This is a signature of second order phase transition.

Z_2 symmetry for different N_τ



- $H(L, L > 0)$ and $H(-L, L < 0)$ are converging with increase in N_τ.
- For $N_\tau = 8$ they are same.

Why the $Z_2(Z_N)$ restoration happening for larger N_τ??

- Its possible that the Z_2 restoration can happen because of decoupling between gauge and Higgs fields.

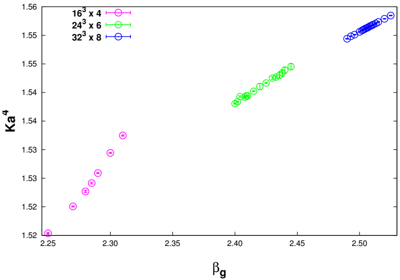


Figure: The avg. of $Ka^4 = \frac{1}{8}\sum_{\mu,\nu} \mathrm{Re}(\phi^\dagger_{\mathrm{n}+\mu} \mathrm{U}_{\mathrm{n},\mu} \phi_{\mathrm{n}})$ for different N_τ near transition point.

- However, the interaction increases form $N_\tau = 4$ to $N_\tau = 8$. Although the rate of increase is decreasing.

SU(N) Higgs theory N_τ dependence

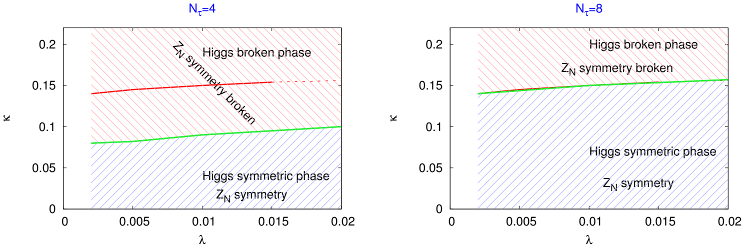


Figure: Higgs phase diagram

- The Z_N symmetry breaking line will approach Higgs transition line for higher N_τ.

Summary.

- Our results suggest that the Higgs condensate plays a role of symmetry breaking field like external field in the Ising model.
- We believe increase in phase space of the Higgs field is responsible for Z_2 restoration.
- Perturbative calculation for massless bosons show that the explicit symmetry breaking is large, But surprisingly our simulation results show that the symmetry is restored.

References

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- *N. Weiss,* **Phys. Rev. D 2667, 25 (1982)**.

Thank You

$$\left[\frac{f}{T^4}\right]_{\beta_0}^{\beta} = {N_\tau}^4 \int_{\beta_0}^{\beta} d\beta' (S_T - S_0) \tag{9}$$

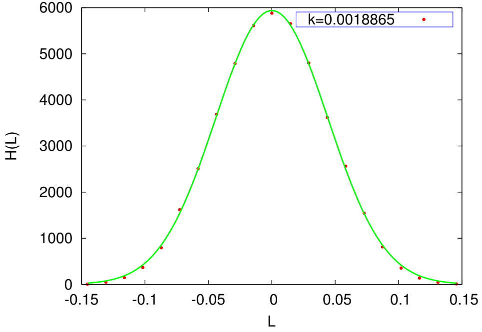
$$D_\mu \Phi = \partial_\mu \Phi + \frac{(1 - U_\mu)}{a}\Phi \tag{10}$$

$$(D_\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{a^2}[\Phi^\dagger_{x+\mu}\Phi_{x+\mu} + \Phi^\dagger_x \Phi_x - \Phi^\dagger_x U^\dagger_{x,\mu}\Phi_{x+\mu} - \Phi^\dagger_{x+\mu} U_{x,\mu}\Phi_x] \tag{11}$$

$$\begin{aligned} S_H = \sum_x \Big[& \frac{8a^2}{2} Tr(\Phi^\dagger_x \Phi_x) - a^2 ReTr(\Phi^\dagger_{x+\mu} U_{x,\mu}\Phi_x) \\ & - \frac{m^2 a^4}{2} Tr(\Phi^\dagger_x \Phi_x) + \frac{\lambda a^4}{2} Tr(\Phi^\dagger_x \Phi_x)^2] \end{aligned} \tag{12}$$

$$\Phi(x) \to \frac{\sqrt{\kappa}\Phi_n}{a}, \lambda \to \frac{\lambda}{\kappa^2}, m^2 \to \frac{(1 - 2\lambda - 8\kappa)}{\kappa a^2} \tag{13}$$

$$S_H = \sum_n [-\kappa Tr(\Phi^\dagger_{n+\mu} U_{n,\mu}\Phi_n) + \frac{1}{2} Tr(\Phi^\dagger_n \Phi_n) + \lambda(\frac{1}{2} Tr(\Phi^\dagger_n \Phi_n) - 1)^2] \tag{14}$$



6000
5000
4000
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2000
1000
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H(L)
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-0.1
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