Grassmannians and Form factors in $\mathcal{N} = 4$ SYM

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Contents

Preliminaries

Introduction

- Form factor $\Leftrightarrow$ $\langle 0|\mathcal{O}(x)|1, ..., n\rangle$
- On-shell diagram $\Leftrightarrow$ integral over Grassmannian
- Generalization
- Gluing operation

$$(p_i^\mu)^{a\dot{a}} = \begin{pmatrix} p_i^0 + p_i^3 & p_i^1 - ip_i^2 \\ p_i^1 + ip_i^2 & p_i^0 - p_i^3 \end{pmatrix} \Leftrightarrow p_i^{\dot{a}a} = \lambda_i^a \tilde{\lambda}_i^{\dot{a}}$$

$$\langle ij \rangle := \epsilon_{ab} \lambda_i^a \lambda_j^b, [ij] := \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}}$$

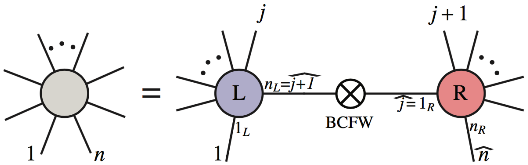
$$\Omega = g^+ + \tilde{\eta}_A \lambda^A - \frac{1}{2!} \tilde{\eta}_A \tilde{\eta}_B S^{AB} - \frac{1}{3!} \tilde{\eta}_A \tilde{\eta}_B \tilde{\eta}_C \lambda^{ABC} + \tilde{\eta}_1 \tilde{\eta}_2 \tilde{\eta}_3 \tilde{\eta}_4 g^-$$

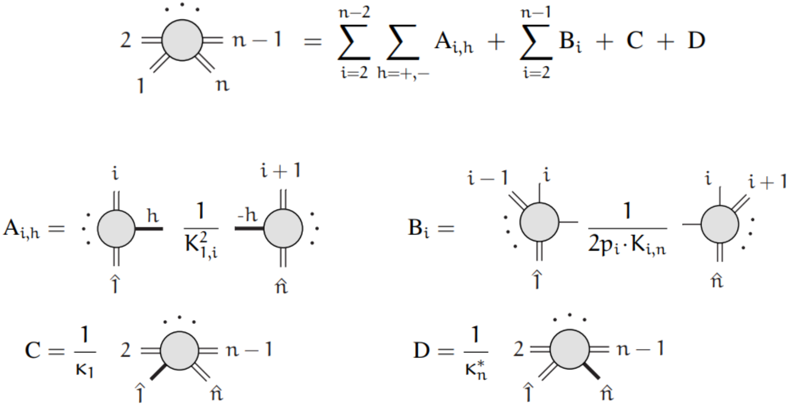
$$\mathcal{A}_n^{\mathrm{MHV}} = \frac{\delta^4(\sum_{i=1}^n p_i)\delta^8(\sum_{i=1}^n \lambda_i \tilde{\eta}_i)}{\langle 12 \rangle \langle 23 \rangle ... \langle n1 \rangle}$$

On-shell recursion

- momentum deformation: $\hat{p}_i = p_i - zq$, $\hat{p}_j = p_j + zq$
- Cauchy theorem: $0 = \oint\limits_{\mathcal{C}} dz \frac{\hat{A}(z)}{z} = A + \sum_{z\neq 0} \mathrm{res}\left(\frac{\hat{A}(z)}{z}\right) \Rightarrow$

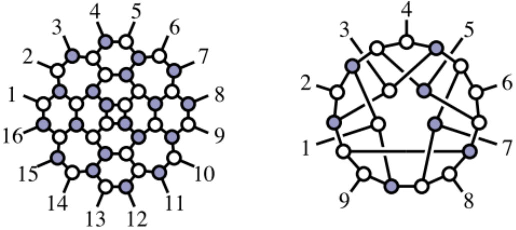
  $A = \sum_k \hat{A}_L(z_k) \frac{1}{P_k^2} \hat{A}_R(z_k)$





2 n − 1 1 n = $\sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D$
$A_{i,h} =$ i h $\frac{1}{K^2_{1,i}}$ -h i + 1 $\hat{1}$ $\hat{n}$
$B_i =$ i − 1 i $\frac{1}{2p_i \cdot K_{i,n}}$ i i + 1 $\hat{1}$ $\hat{n}$
$C = \frac{1}{\kappa_1}$ 2 n − 1 $\hat{1}$ $\hat{n}$
$D = \frac{1}{\kappa^*_n}$ 2 n − 1 $\hat{1}$ $\hat{n}$

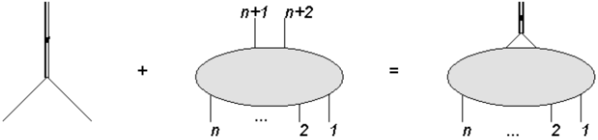
$$\sum_I \int d^4\tilde{\eta}_I \int \frac{d^2\lambda_I d^2\tilde{\lambda}_I}{vol[GL(1)]}$$



- Non-local operator
- WLO form factors ⇔ reggeon amplitudes

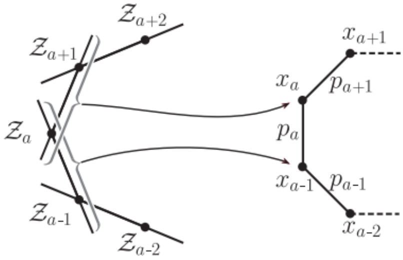
$$\mathcal{W}_p^c(k) = \int d^4x e^{ik\cdot x} \mathrm{Tr}\left\{\frac{1}{\pi g} t^c \mathcal{P}\exp\left[\frac{ig}{\sqrt{2}}\int_{-\infty}^{+\infty} dsp \cdot A_b(x+sp)t^b\right]\right\}$$

$$A^*_{m+n}(\Omega_1, ..., \Omega_m, g^*_{m+1}, ..., g^*_{m+n}) = \langle\Omega_1...\Omega_m|\prod_{i=1}^{n}\mathcal{W}_{p_{m+i}}^{c_{m+i}}(k_{m+i})|0\rangle$$



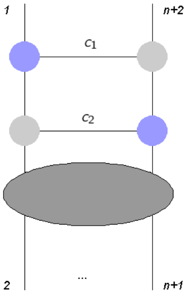
$$A^*_{k,n+1} = \frac{\langle \xi p \rangle}{\kappa^*} \int \frac{d^{k\times(n+2)}C}{Vol[GL(k)]} \frac{\hat{\delta}^{2\times k}(C\cdot\underline{\tilde{\lambda}})\hat{\delta}^{4\times k}(C\cdot\underline{\tilde{\eta}})\hat{\delta}^{2\times(n+2-k)}(C^{\perp}\cdot\underline{\lambda})}{\prod_{i=1}^{n} M_i}$$

- automatic momentum conservation and mass-shell condition
- null-rays in space-time ⇔ points in twistor space



Incidence relations:

$$\mu_{\dot\alpha} = x_{\alpha\dot\alpha}\lambda^\alpha,\; x_{\alpha\dot\alpha} = (p-q)_\mu \sigma^\mu_{\alpha\dot\alpha} \Rightarrow Z = (\lambda^\alpha, \mu_{\dot\alpha})$$



Corollary

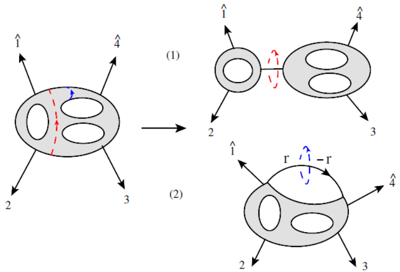
$$\tilde{\mathcal{G}}^{m.tw.}_{i-1,i}[\mathcal{P}^{4(k-2)}_{n+2}] = \mathcal{P}^{4(k-2)}_{n+2}(\ldots, \mathcal{Z}_i - \frac{\langle i-1 i\rangle}{\langle i-1 i+1\rangle}\mathcal{Z}_{i+1}, \mathcal{Z}_{i+1}, \ldots)$$

Integrand recursion

on-shell BCFW:

$$\mathcal{A}_n^L = \mathcal{A}^0_{n,\mathrm{MHV}}\Big(Y^L_{n-1} + \sum_{j=3}^{n-2}[j-1,j,n-1,n,1]\,Y^{L_1}_{left}\,Y^{L_2}_{right} +$$

$$+\int_{AB}[A,B,n-1,n,1]\,Y^{L-1}_{n+2}(\ldots,\hat{\mathcal{Z}}_{n_{AB}},\mathcal{Z}_A,\mathcal{Z}_B)\Big)$$



Off-shell recursion from gling operation

Observation:

- Gluing operator maps terms of the on-shell recursion to terms of the off-shell recursion

The Conjecture:

$$\mathcal{I}^{*,L}_{(n-2)+1} = \mathcal{I}^{*,L}_{n-1} + \sum_{j=3}^{n-2} [j-1, j, n-1, n^*, 1] \mathcal{I}^{*,L_1}_{left} \mathcal{I}^{*,L_2}_{right} +$$

$$+ \int_{AB} [A, B, n-1, n*, 1] \mathcal{I}^{*,L-1}_{n+2}(..., \hat{\mathcal{Z}}^*_{n_{AB}}, \mathcal{Z}_A, \mathcal{Z}_B)$$

Conclusion

- A method for deriving Grassmannian integral representation for various S-matrix elements is obtained and successfully tested on different representations of external data
- Recursion relation for all-loop off-shell integrand is derived