Equation of state of the QCD matter in the PNJL model

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CNRS IN2P3 Les deux infinis
Subatech
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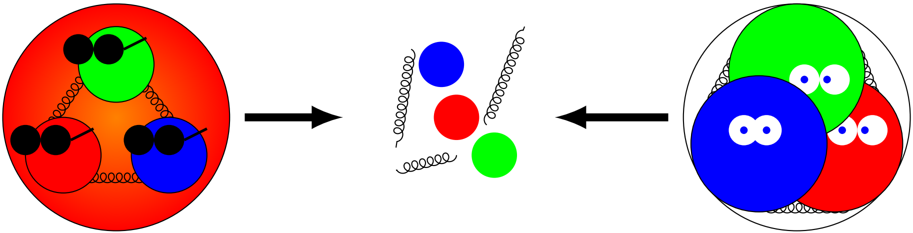
- Better understanding of the strong interaction.
- Thermodynamical (phase diagram, equation of state) and dynamical (cross sections) studies, to build a transport theory describing the hadronisation process.
- Study of the phase diagram of QCD.
- To go to finite μ, effective models are necessary
- PNJL provides such an approach including a first order phase transition for the chiral condensate.

Two phases predicted for QCD matter :

- Hadronic phase :
  Quarks and gluons are bound into hadrons
  This is nuclear matter, we can observe it experimentally
- QGP phase :
  Quarks and gluons are free in the medium
  We don't observe this phase experimentally



QCD lagrangian : life is tough

$$\mathscr{L}_{QCD} = i\delta_{ij}\bar{\psi}^i_k\gamma^\mu\partial_\mu\psi^j_k + g_s\bar{\psi}^i_k\gamma^\mu\lambda^a_{ij}A^a_\mu\psi^j_k - m_k\bar{\psi}^i_k\psi^j_k - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Perturbative approach pQCD

Need of a small coupling constant = large momentum transfer.

- Not usable around hadronization process yet.

Lattice approach lQCD

Static study which does not work at finite chemical potential yet.

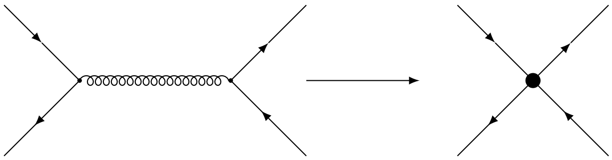
PNJL Model

Effective model

Works only in a special domain of energy but allows finite chemical potential studies.

Contact interaction

Static approximation : no gluons propagating the interaction



Frozen gluons

$$\frac{1}{p^2 - \epsilon_g^2} = -\frac{1}{\epsilon_g^2}$$

if $p << \epsilon_g^2$

NJL Lagrangian

$$\mathscr{L}_{PNJL} = \delta_{ij}\overline{\psi}^i_k(i\gamma^\mu\partial_\mu - m)\psi^j_k + G(\overline{\psi}^i_k\lambda_{ij}\psi^j_k)^2 + \text{'t Hooft term}$$

Symmetries

- Chiral symmetries $SU_L(3) \otimes SU_R(3)$
- Color symmetry $SU_c(3)$
- Flavour symmetry $SU_f(3)$

Problem

Center symmetry is missing

Confinement is not described

Free parameters

$m^0_q = 0.0055\, GeV$

$m^0_s = 0.134\, GeV$

$\Lambda = 0.569\, GeV$

$G = \frac{2.3}{\Lambda^2}\, GeV^{-2}$

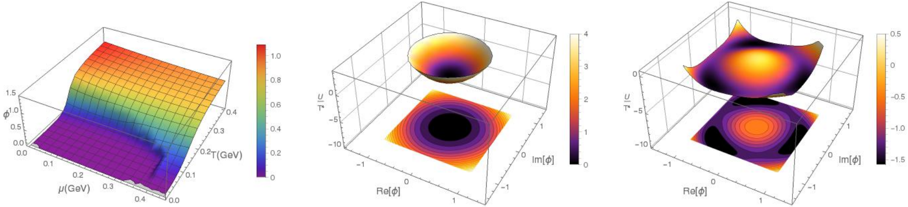
$K = \frac{11}{\Lambda^5}\, GeV^{-5}$

Polyakov loop

Confinement is taken into consideration using an effective potential $U(\phi, \bar{\phi}, T)$, function of the Polyakov loop ϕ.

PNJL Lagrangian

$$\mathscr{L}_{PNJL} = \overline{\psi}_k(i\partial\!\!\!/_\mu - m)\psi_k + G(\overline{\psi}_k\lambda_i\psi_k)^2 + \text{'t Hooft} - U(\phi, \bar{\phi}, T)$$



Still no gluons in the interaction

U : static gluon field corresponding to $\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$, fit P_{YM} of lQCD.

Traditional PNJL - Before

One of the parameter is $T_0 = 270 MeV$, the critical temperature for confinement.
This is the pure Yang-Mills critical temperature.

Quarks are here too ! - Better

Slight change in the critical temperature. We use the reduced temperature to quantify it.

$T^{eff} = \frac{T - T_c}{T_c} \rightarrow T^{eff}_{YM} \simeq 0.57 T^{eff}_{rs}$ *https ://arxiv.org/abs/1302.1993, Haas and al.*

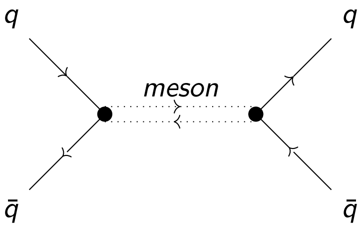
This rescale the critical temperature to $T_0 = 190 MeV$

Not enough ! - Our personnal touch

In addition to this rescaling we consider that the parameter T_0 depends on the temperature : $T_0(T)$.

Quark-antiquark bound states

In NJL, degrees of freedom are quarks. Mesons need to be build from quark-antiquarks bound states



Propagator of the mesons

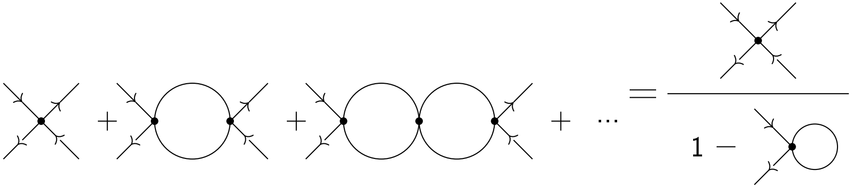
$$iU(k^2) = \Gamma \frac{-ig_m^2}{k^2 - m^2} \Gamma$$

Mesons masses

The mass is given by the poles : $m = k$

Equation of Bethe-Salpeter

$$iU(k^2) = \Gamma \frac{2ig_m^2}{1-2g_m^2\Pi(k^2)}\Gamma$$



Mesons masses

By analogy, the mass is given by the poles :

$$1 - 2G^2\Pi(k^2 = m^2) = 0$$

Equation of State

Partition function

As always in statistical physics, we need the partition function :
$Z = Tr[\exp -\beta(H - \mu N)] = \exp(-\beta\Omega)$

Grand potential

The grand potential is calculated from the partition function.

$$\Omega = -2\int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_p$$

$$+2T\int_0^\infty (\ln[1+\exp(-\beta(E_p-\mu))] + \ln[1+\exp(-\beta(E_p+\mu))]$$

$$+2G\sum_k < \bar{\psi}_k\psi_k >^2 -4K\Pi_i < \bar{\psi}_k\psi_j > +U_{PNJL})$$

Non interactive fermion gas + PNJL mean field

Don't forget mesons !

Below T_{Mott}, mesons are lighter than their constituants. They are stable, present in the medium and contribute to the pressure.

Mesonic fluctuations

To add mesons, we need to go beyond the mean field approximation, to the next to leading order in the $\frac{1}{N_c}$ expansion and consider ring diagrams.

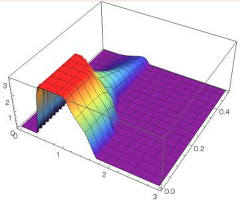
Mesonic grand potential

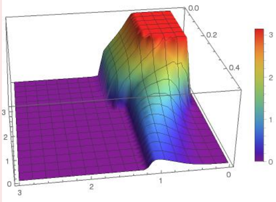
$$\Omega_M = -\frac{g_M}{8\pi^3}\int dp p^2 \int \frac{ds}{\sqrt{s+p^2}}\left[\frac{1}{\exp(\beta(\sqrt{s+p^2}-\mu)-1)}+\frac{1}{\exp(\beta(\sqrt{s+p^2}+\mu)-1)}\right]\delta_M$$

Phase shift : the physics

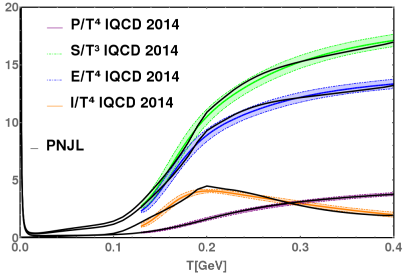
The phase shift depends on the mesons masses

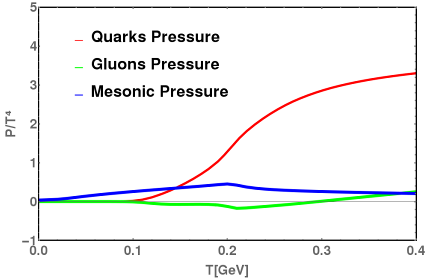
$$\delta_M = -Arg[1-2K_M\Pi_M]$$





Equation of state at zero μ

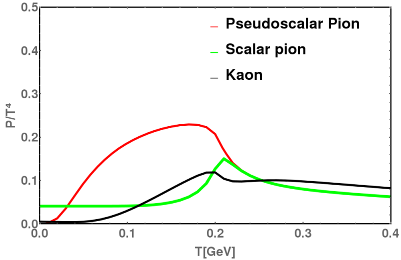


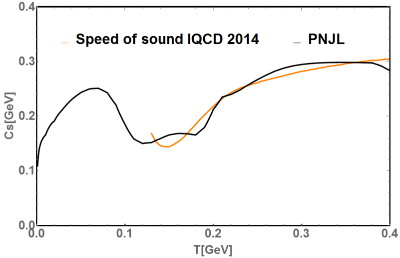


We reproduce lattice results ! ! !

We have an effective model based on a lagrangian that shares QCD symmetry and match lattice results.
This is an effective theory, no sign problem, we can expand to finite chemical potential.

Equation of state at zero μ





https ://arxiv.org/abs/1407.6387v2, HotQCD Collaboration

Mesonic contributions to the pressure

As expected, Mesons contribute only at low temperature.

Critical temperature

Minimum of speed of sound : localisation of the cross over region.

Lattice at finite μ

Lattice can handle Taylor expansion around zero chemical potential.

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \ldots$$

The κ coefficient is the second order derivative of our function :

$$\kappa = \left. \frac{\partial^2 \frac{T_c(\mu_B)}{T_c(0)}}{\partial \mu_B^2} \right|_{\mu_B=0}$$

"On the critical line of 2+1 flavor QCD" Cea, Cosmai,Papa

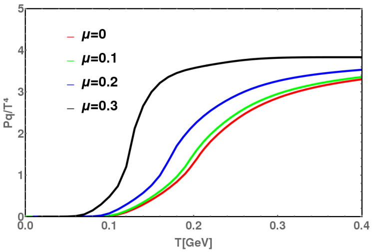
Our critical temperature

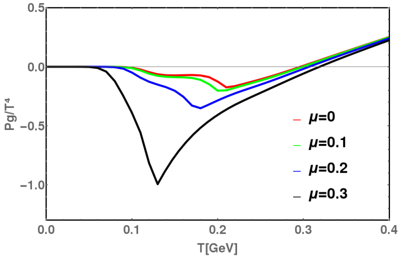
At $\mu_B = 0$, we get the critical temperature : $T_c = 198 MeV$

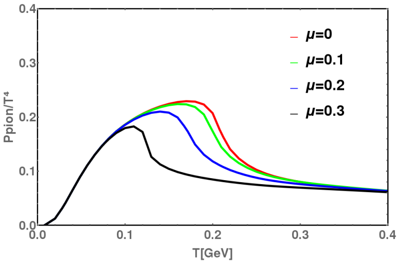
Our coefficient

The corresponding κ coefficient is : $\kappa = 0.018$

At finite μ







Finite μ

- Results before the first order transition.
- The transition gets sharper though.
- Mesons are vanishing.

Conclusion :

PNJL + T0(T) + Pressure beyond mean field (mesons)

=

- Lattice equation of state at 0 μ.
- Lattice equation of state at almost zero μ.
- 1st order phase transition not available yet (but soon !).
- But the transition gets sharper when μ increases.

Thank you for attention ! !

Sign problem

- Partition function : $Z = \int \mathscr{D}_U \mathscr{D}_{\bar{\psi}} \mathscr{D}_\psi \exp(-S)$
- With the action :
  $S = \int d^4x \bar{\psi}(\gamma_\nu(\partial_\nu + iA_\nu) + \mu\gamma_4 + m)\psi = \int d^4x \bar{\psi} M \psi$
- μ appears as an A_4 imaginary quadrivector and :
  $M = \gamma_\nu\partial_\nu + i\gamma_\nu A_\nu + \mu\gamma_4 + m$
- We then have :
  $M^\dagger(\mu) = M(-\mu^*)$
- The action is now complex. It can be seen using the hermiticity of the γ_5 matrix. M hermiticity valide at $\mu = 0$ and but not for finite μ.

$U_A(1)$ anomaly

- Classical action invariant $\rightarrow$ symmetry.
- Quantum action not invariant $\rightarrow$ symmetry broken.
- Symmetry broken by quantum fluctuation : Anomalies !