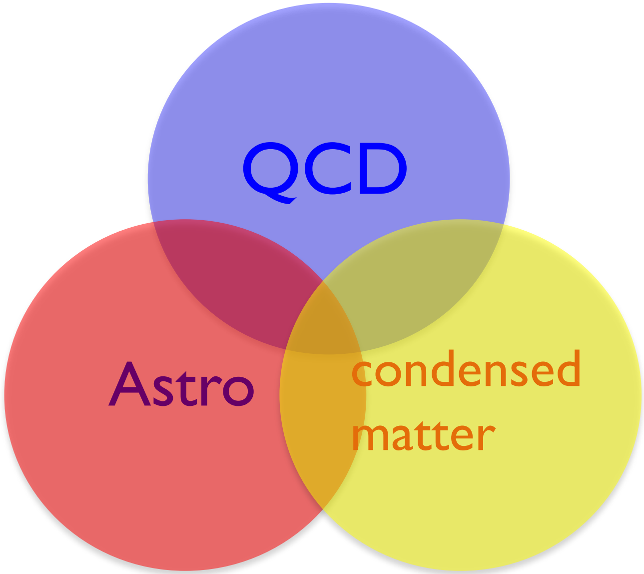
Three-window approach to (cold) dense matter

Toru Kojo (CCNU, Wuhan)



References (3-window modeling)

- Masuda-Hatsuda-Takatsuka (2012, 2013) : the 1st version
- Kojo-Powell-Song-Baym (2014), Fukushima-Kojo (2015) : extension
- Kojo (2015) : concise **review** of 3-window modeling
- Baym-Hatsuda-Kojo-Powell-Song-Takatsuka (2017) : comprehensive **review**

  Rept. Prog. Phys. 81 (2018) no.5, 056902 (arXiv: 1707.04966 [astro-ph])

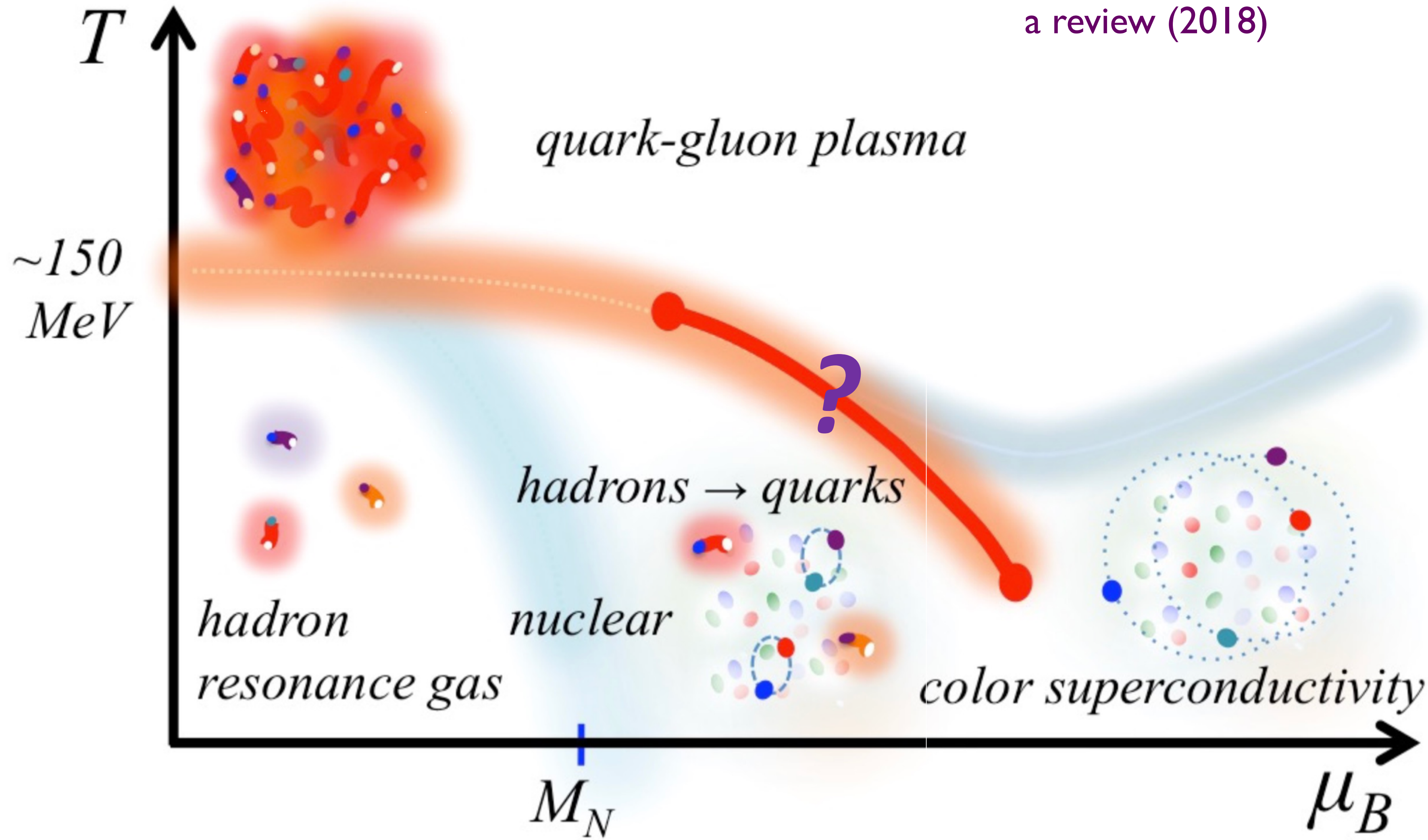
  including EoS: Quark-Hadron-Crossover (**QHC18**)

- Annala-Gorda-Kurkela-Vuorinen (2017) & refs. therein

  More conservative use of 3-window modeling

Lattice + HIC + HIC + Astro

- Baym-Hatsuda-TK-Powell-Song-Takatsuka: a review (2018)



Lattice + HIC + HIC + Astro

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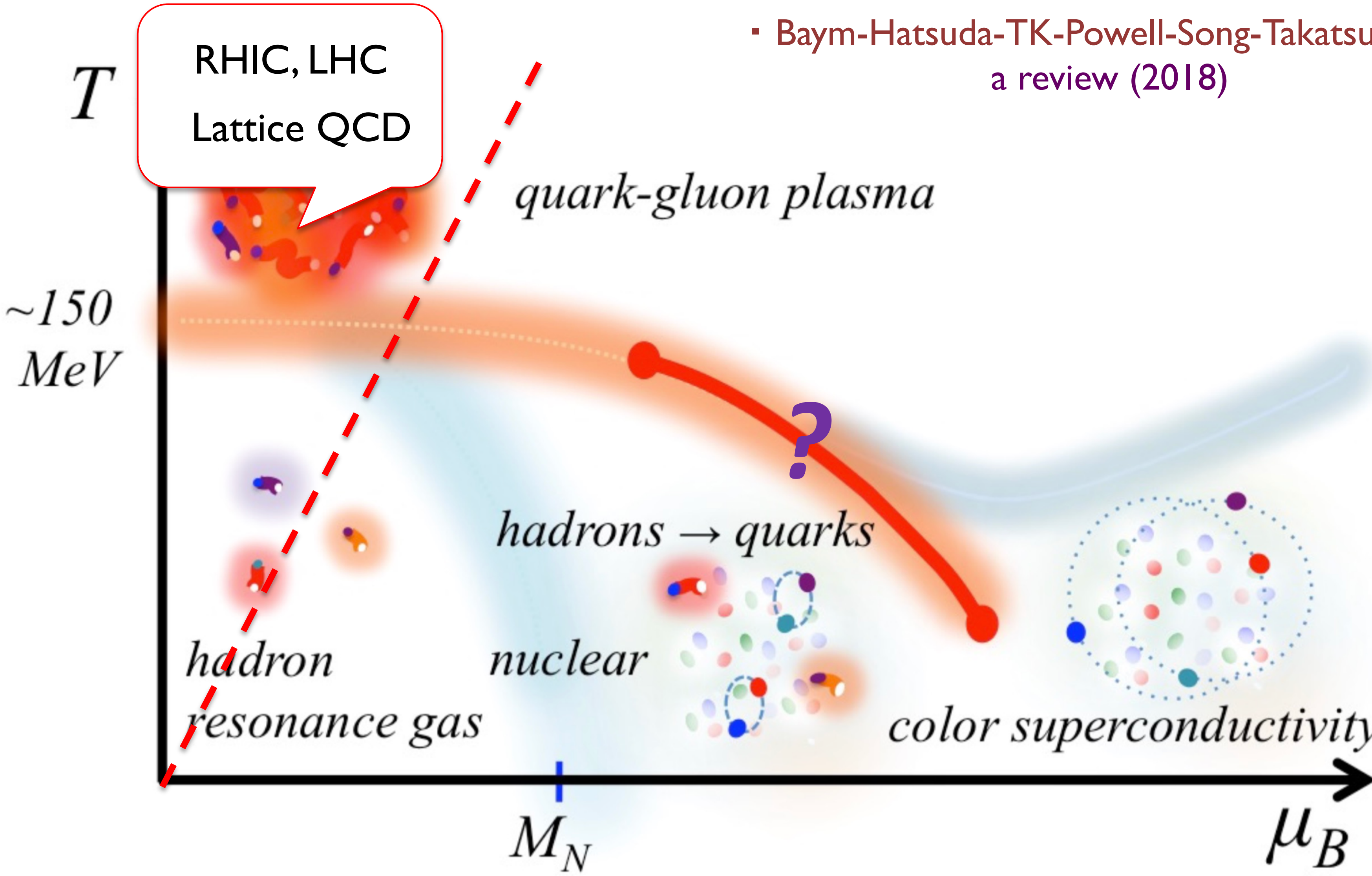

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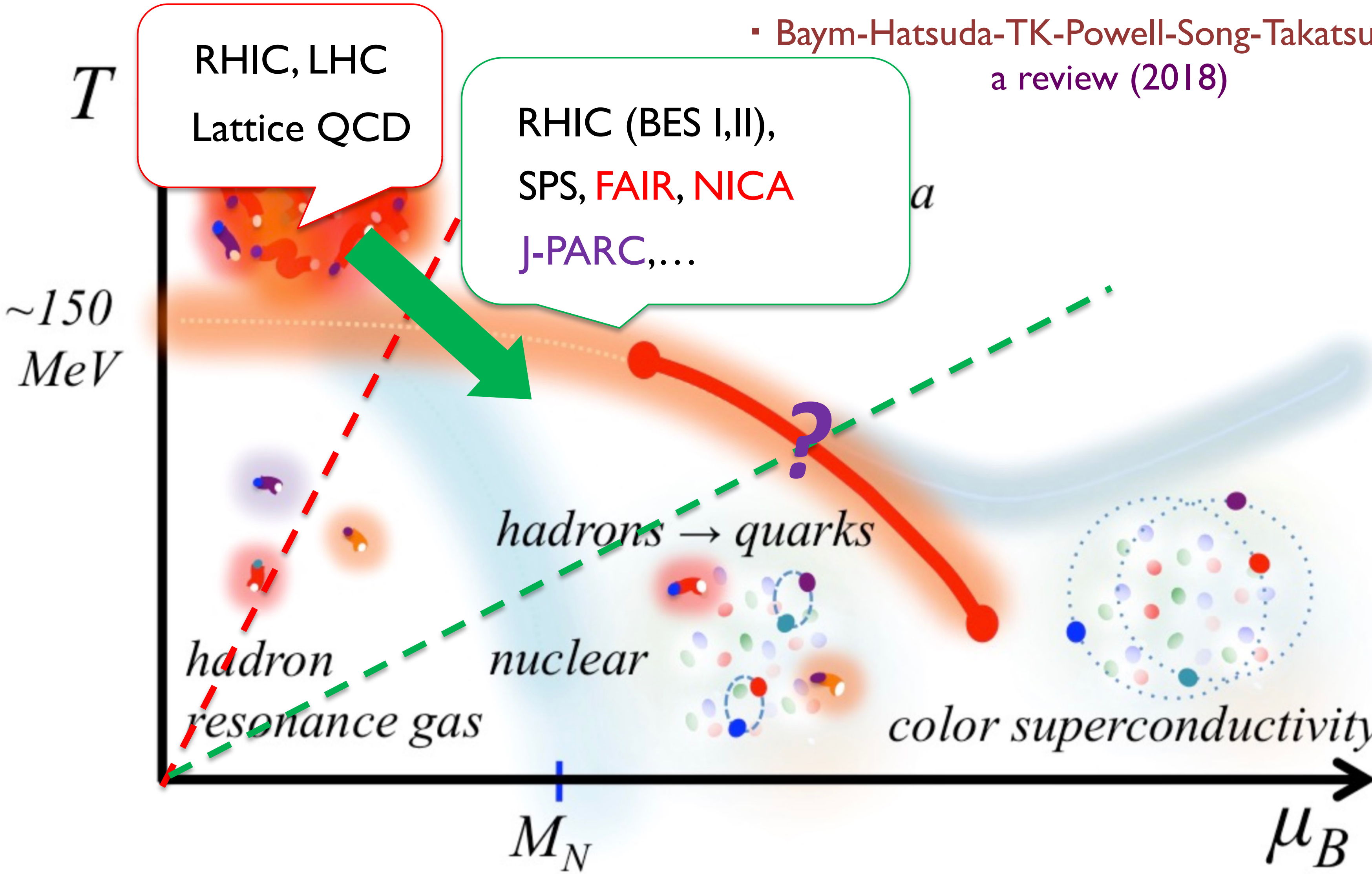
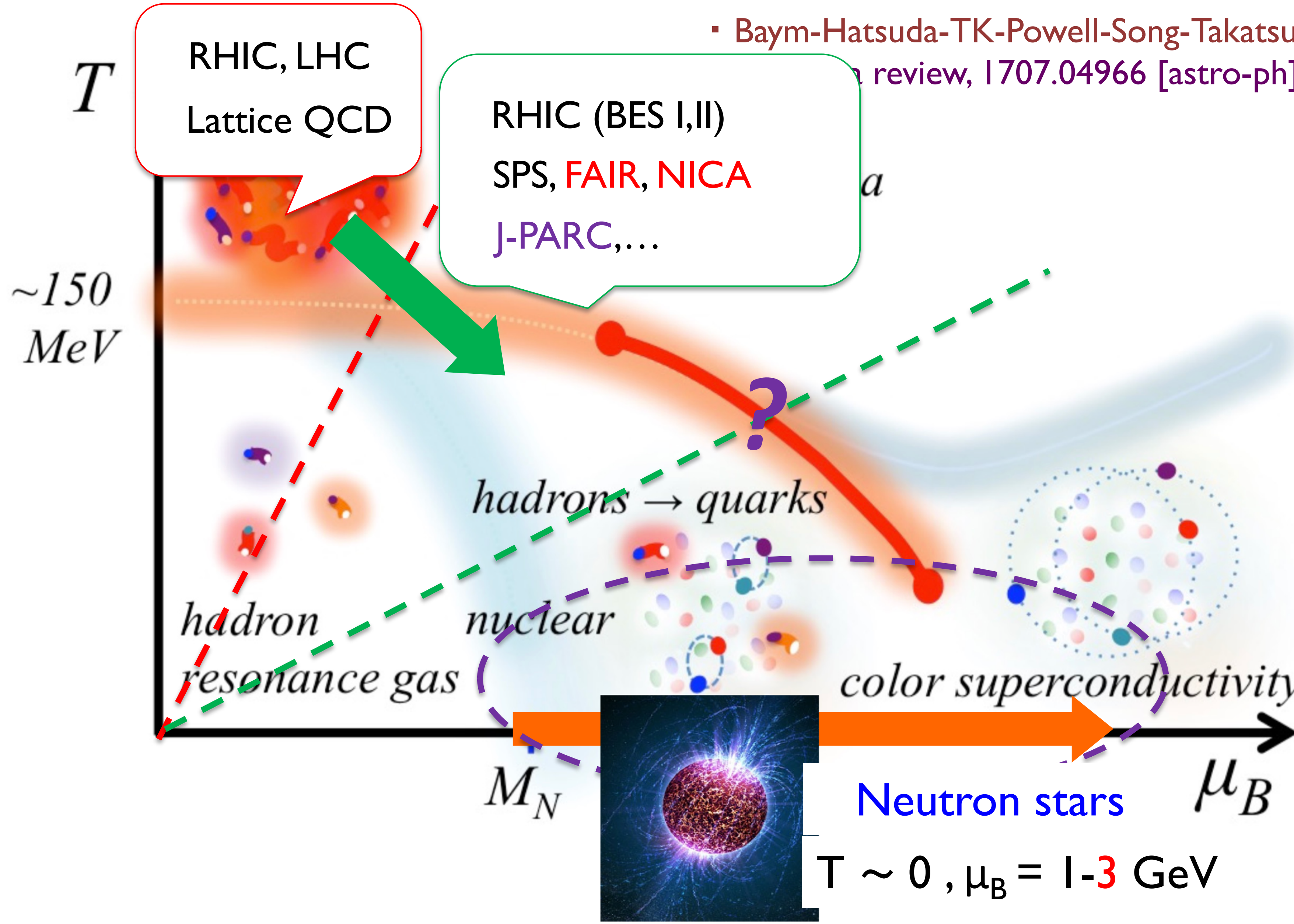


Lattice + HIC + HIC + Astro

- Baym-Hatsuda-TK-Powell-Song-Takatsuka: review, 1707.04966 [astro-ph]



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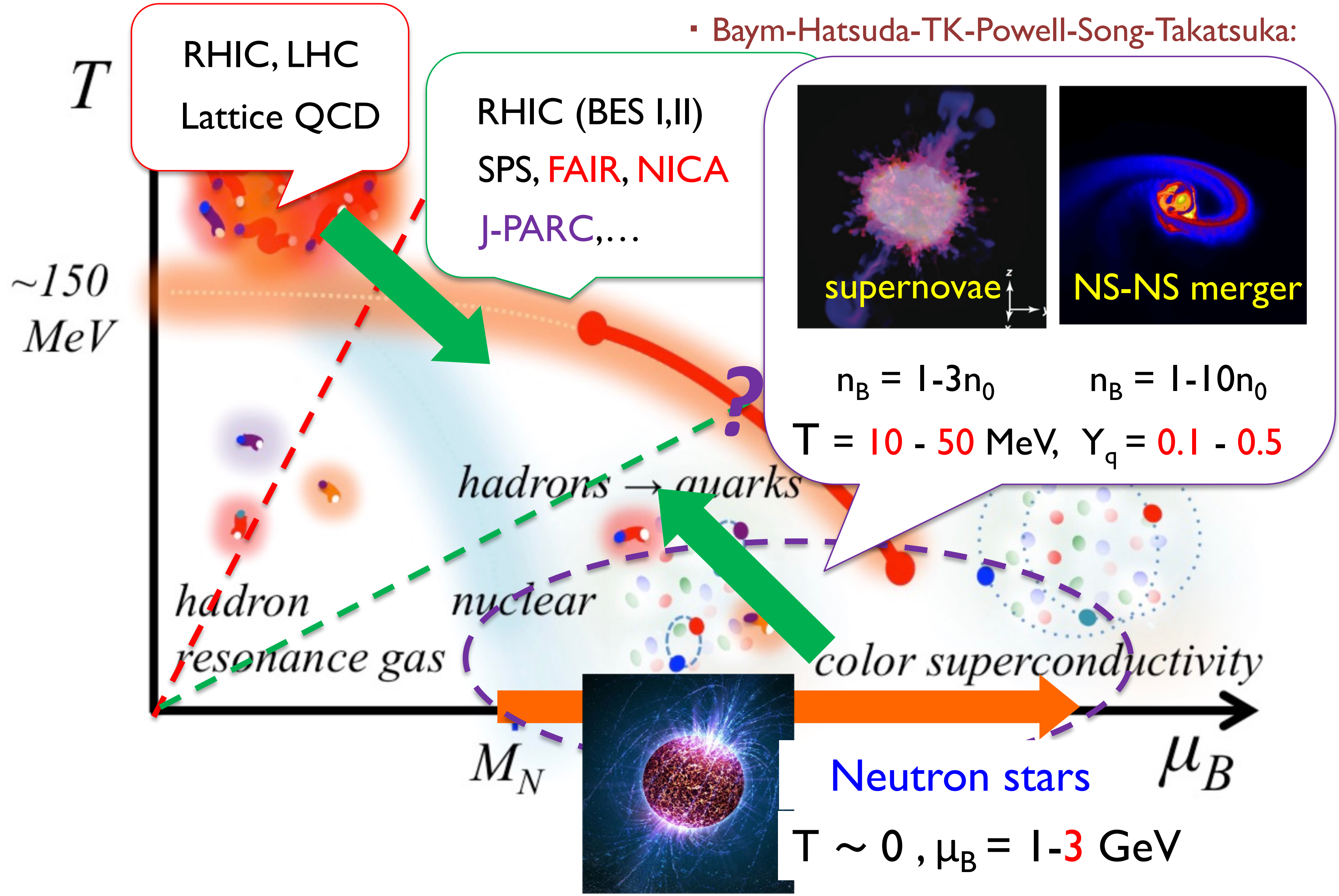


3-characteristic regimes in QCD matter

1, **Low** density regime (hadrons **dilute**)

Theory reliable : EFT with exp. inputs (hadron spectroscopy)

Effective d.o.f : hadrons

2, **High** density regime (hadrons **overlapped**)

Theory reliable : weak coupling computations

Effective d.o.f : quarks & gluons

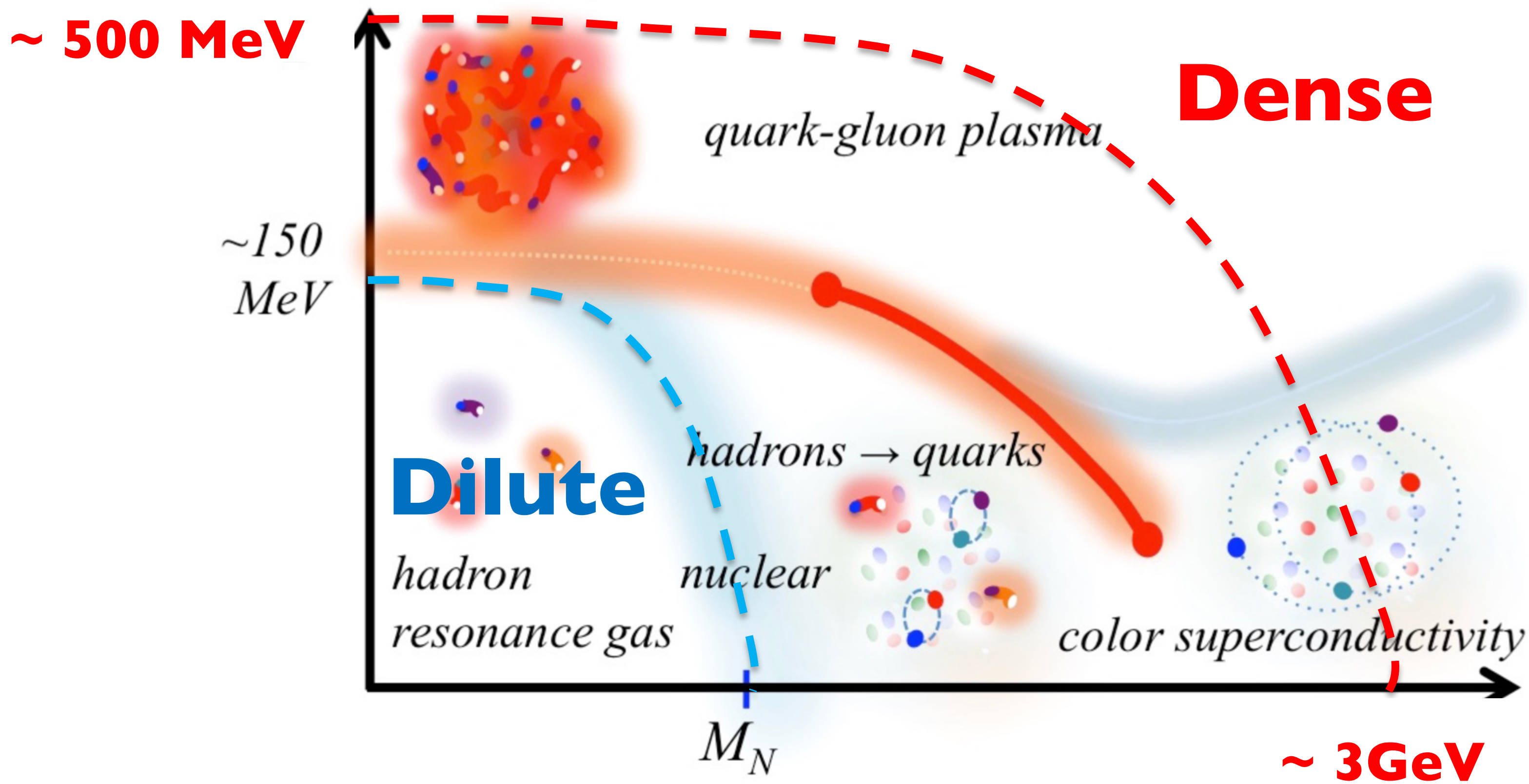
3, **Intermediate** regime

Theoretically most difficult, most important in phenomenology

Effective d.o.f : **NOT clear-cut**, collective something?

Difficulties in the QCD case

The domains of (theoretically reliable)
low & high density regimes DO NOT overlap !



Difficulties in predicting physics around phase transitions

What we will discuss

- the **gross pictures** on the QCD phase diagram
- **effective** d.o.f. & interactions
- how to use the thermodynamic relations **in practice**
- how to use the astrophysical data **in practice**
- **orientations** toward the **future** QCD computations

Many details remain to be worked out

Plan of lectures

1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

3, NS constraints on EoS : hints for **soft**-**stiff** EoS

4, Crossover scenario: chiral restorations, etc.

5, A quark model : delineating the properties of matter

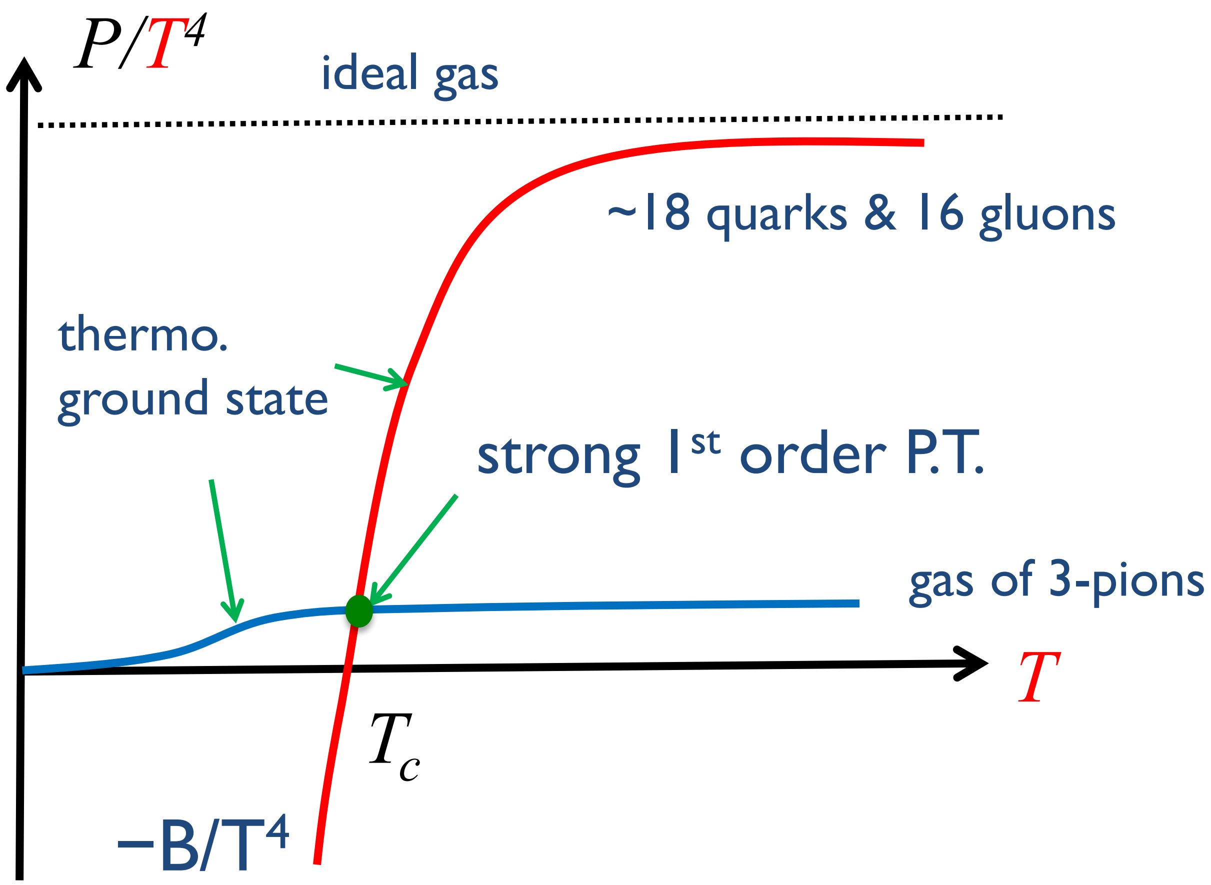
6, The astrophysical results from EoS QHC18

7-, Other topics: warm EoS, beyond-MF, etc.

Hot QCD case I : textbook example

Textbook example: **pion gas** vs **bag model QGP**

($P_{bag} = P_{ideal} - B$)



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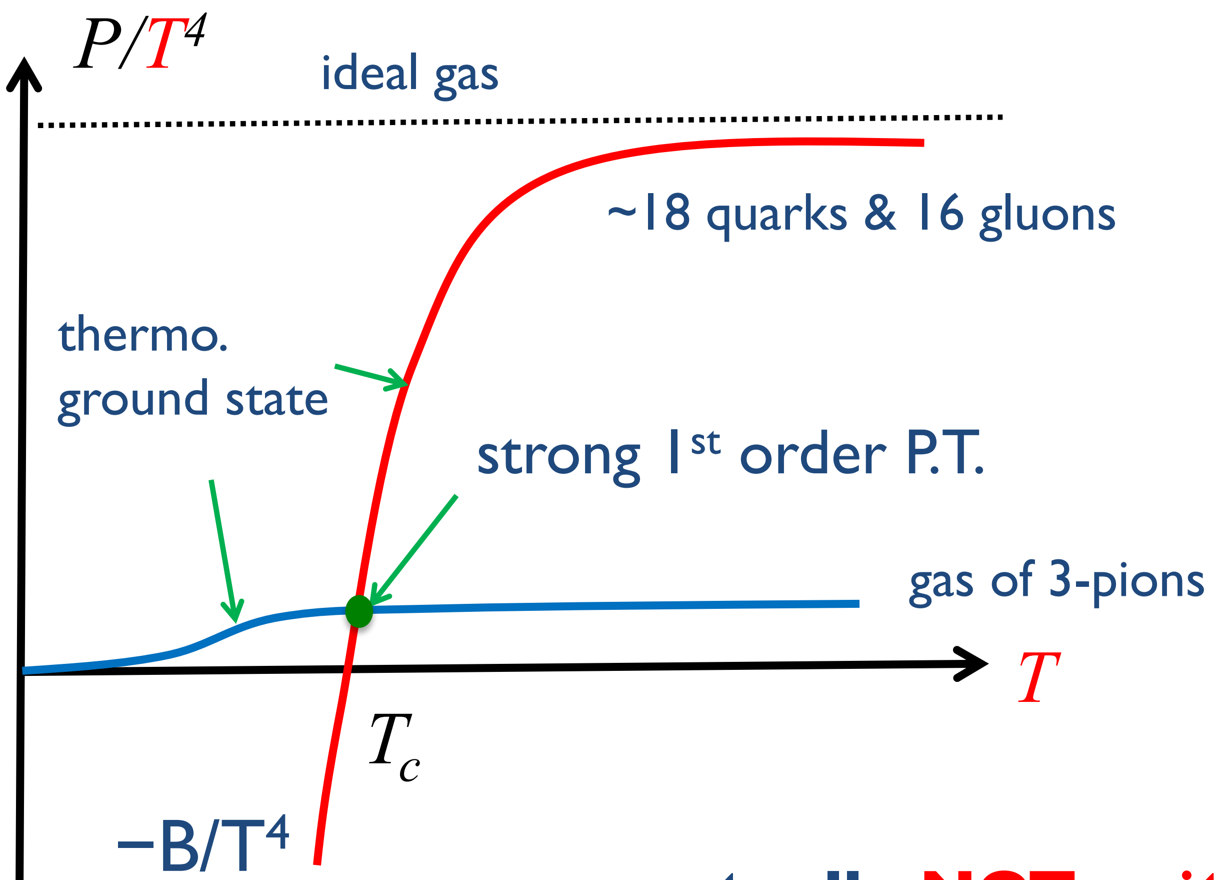
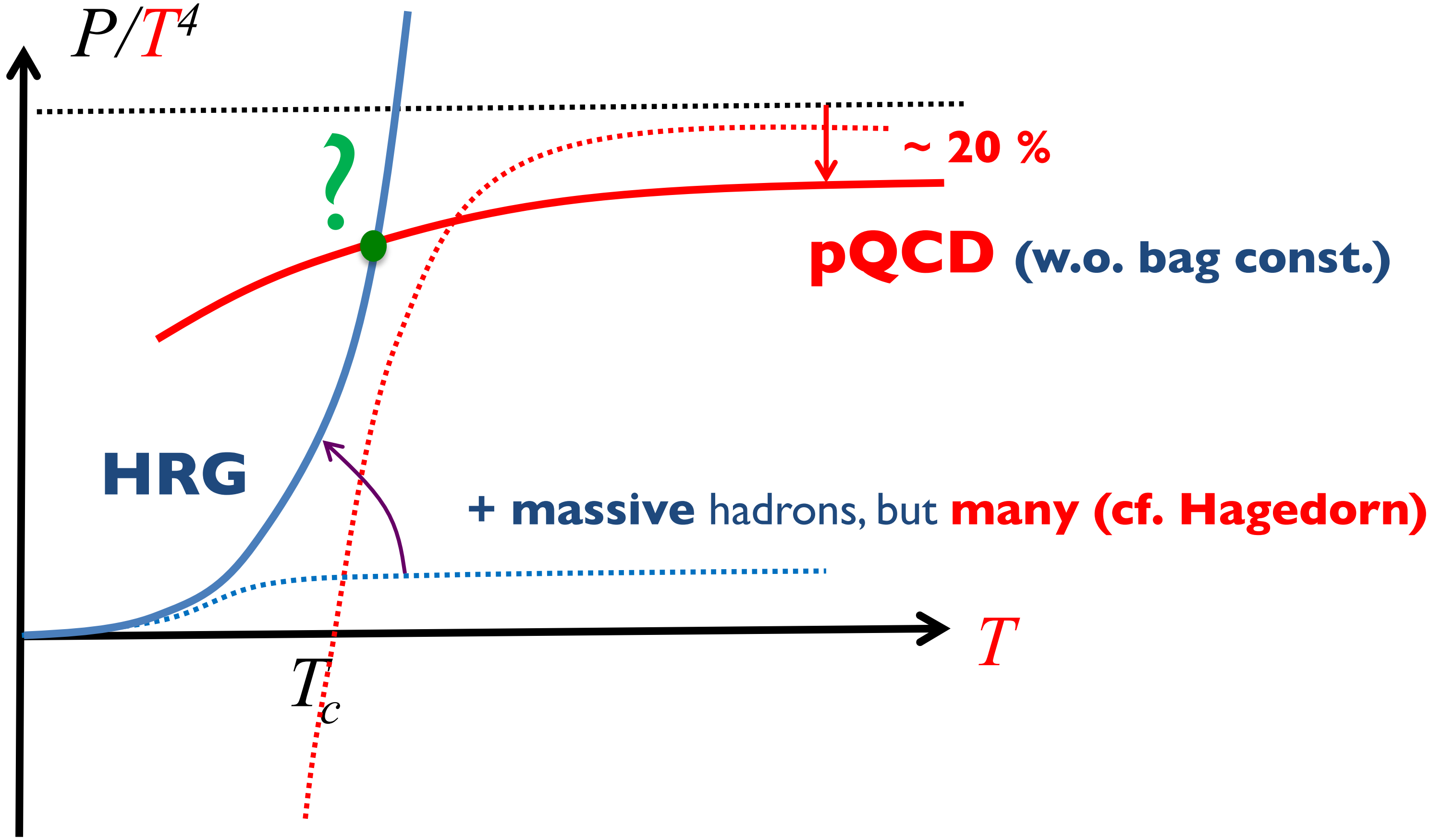


conceptually NOT quite correct

Hot QCD case 2 : improved low & high T EoS

modern version: **HRG** (hadron resonance gas) vs **pQCD** **(resummed)**



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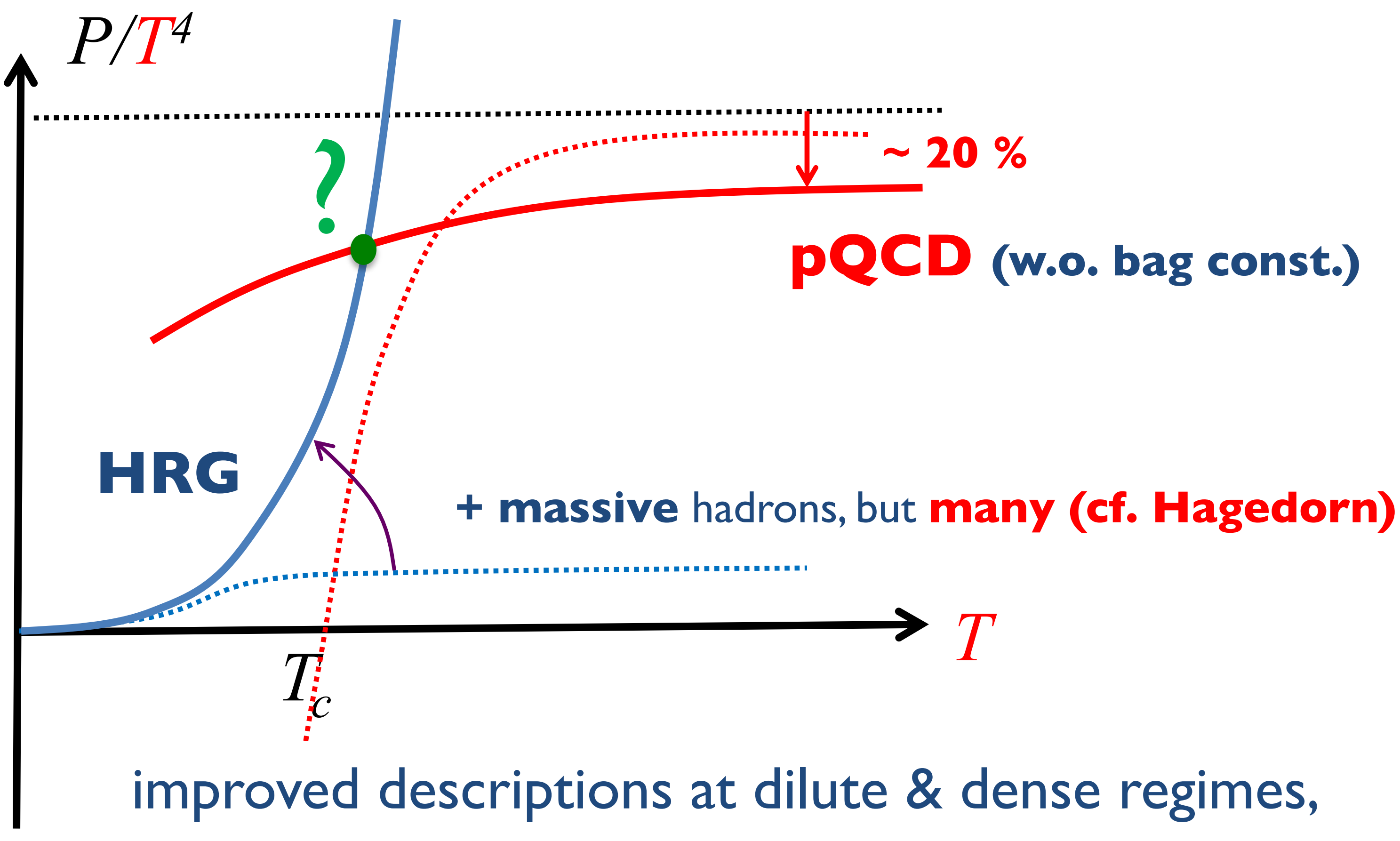
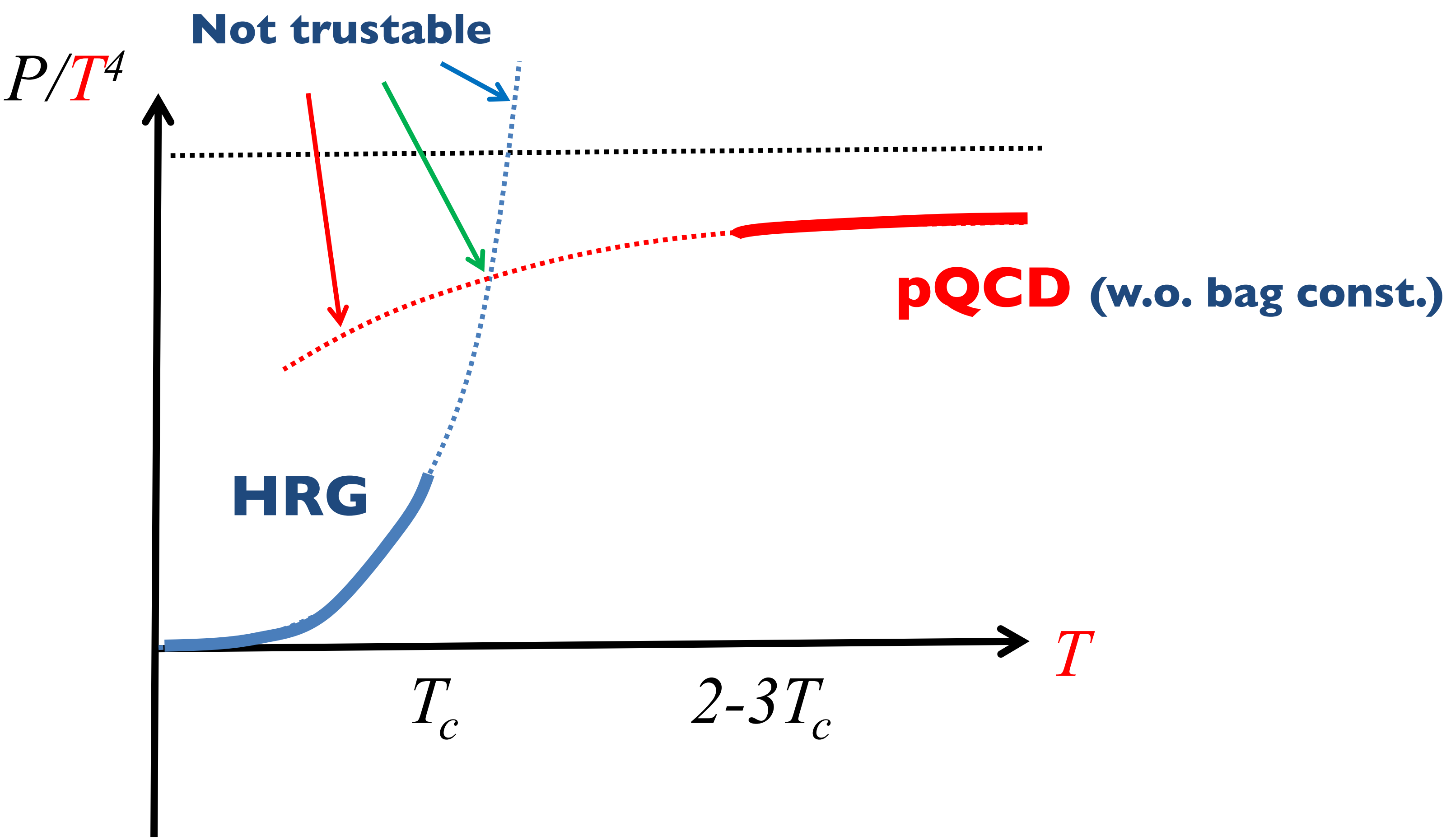


improved descriptions at dilute & dense regimes,

but wrong picture in the **intermediate** region

Hot QCD case 3 : 3-window modeling

use **only** trustable parts of **HRG** & **pQCD** **(resummed)**



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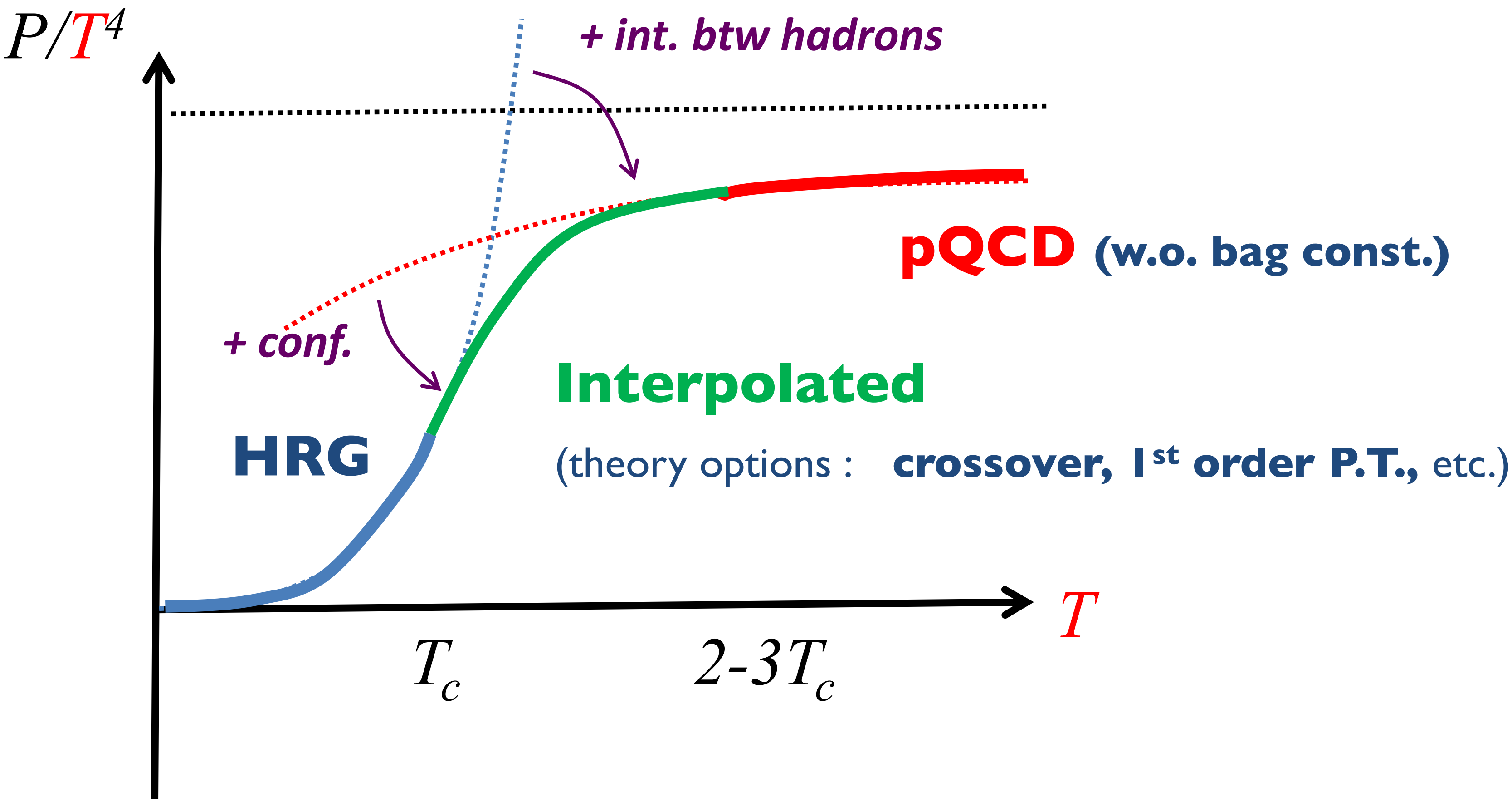
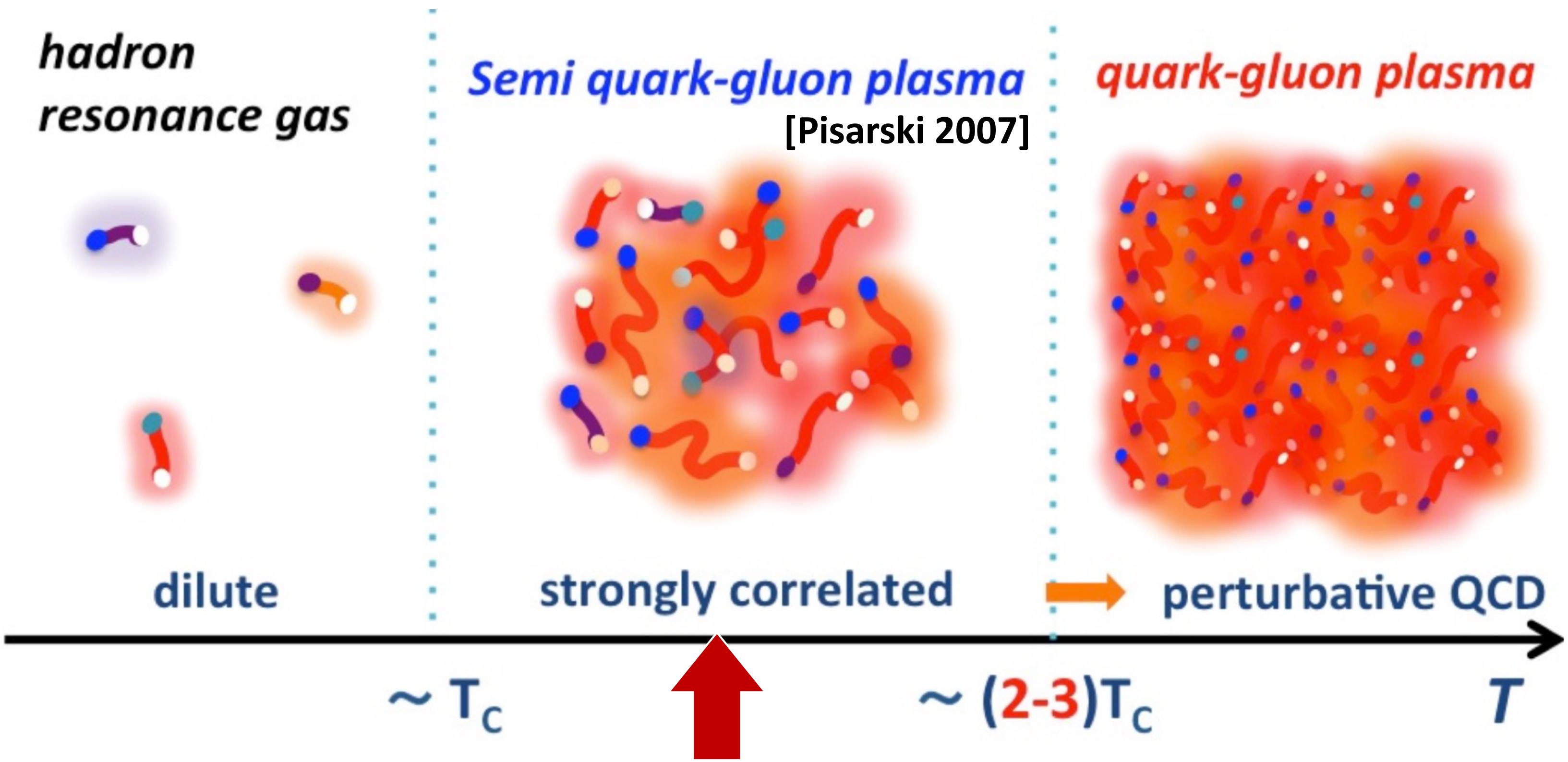


picture reasonably consistent with **lattice** & **exp. data**

Plausible picture for HOT QCD

Fig. from Baym et al. 2018



new state of matter

rather than a mixed state of HRG & pQCD gas

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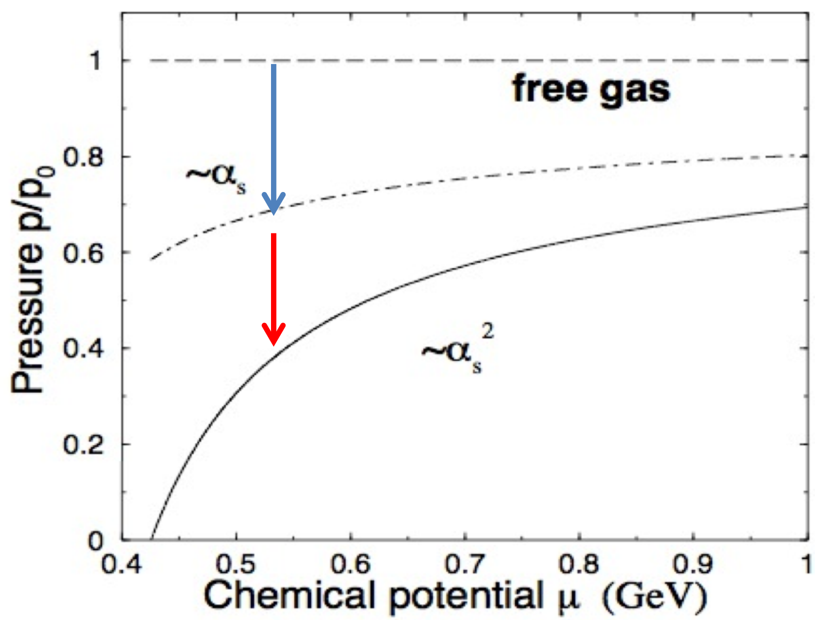
Cold, dense EoS : High density

3-loop pQCD : Freedman-McLerran 78; Baluni 78; Kurkela-Romatschke-Vuorinen 09

[some **4-loop** contributions: E. Sappi et al.]

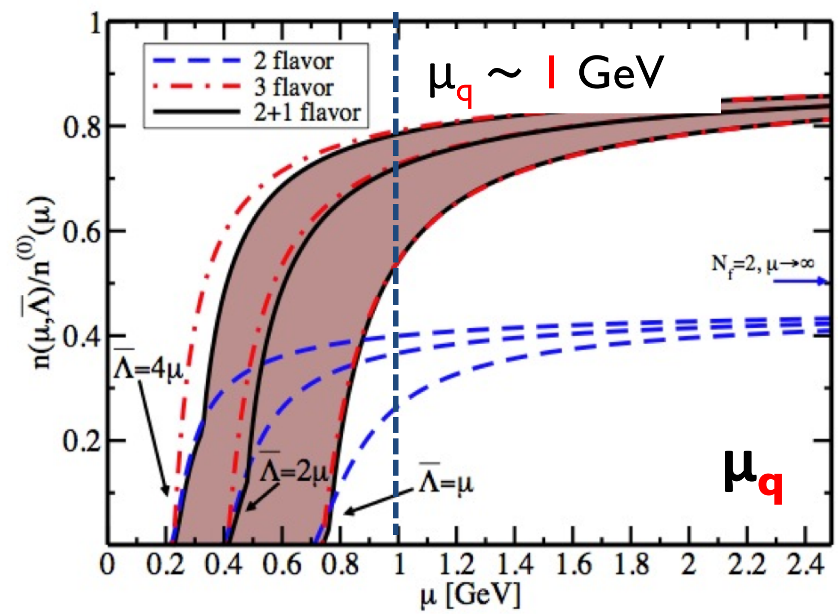
check of convergence

(Fraga-Pisarski-Schaffner-Bielich 01)



check of renorm. scale dep.

(Kurkela-Romatschke-Vuorinen 09)



- Interactions crucial for μ_q < ~ **1** GeV or n_B < ~ **50**n_0
- Hints for effective **repulsion** (more μ needed to reach n_{ideal})

Cold, dense EoS : Low density

calculations based on microscopic interactions

NN + 3N forces + ...

a) Fit to data
- to E ~ 350 MeV for NN (well constrained)
- fit to nuclei for NNN (uncertain)

Illinois, Argonne, Bonn, ...

b) ChEFT (N^3LO)
- systematics
- symmetry of QCD

Epelbaum, Heberer, Kaiser, Schwenk, ...

c) Lattice QCD
- NN & YN, YY pot.

HAL collaboration, ...

Many-body calculations (non-perturbative for soft nucleons)

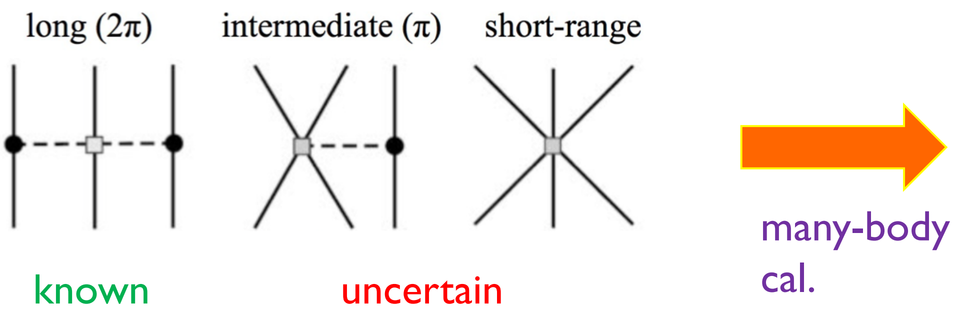
- Hartree-Fock, BHF, ...
- Quantum Monte-Carlo — Carlson. Gandolfi, ...
- Variational — Pandharipande, Takano, Togashi, ...

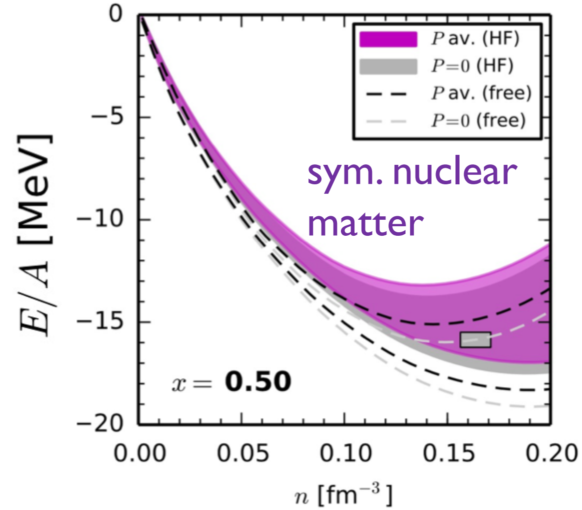
→ **EoS**

Cold, dense EoS : Low density

- short range part of 3N forces is uncertain

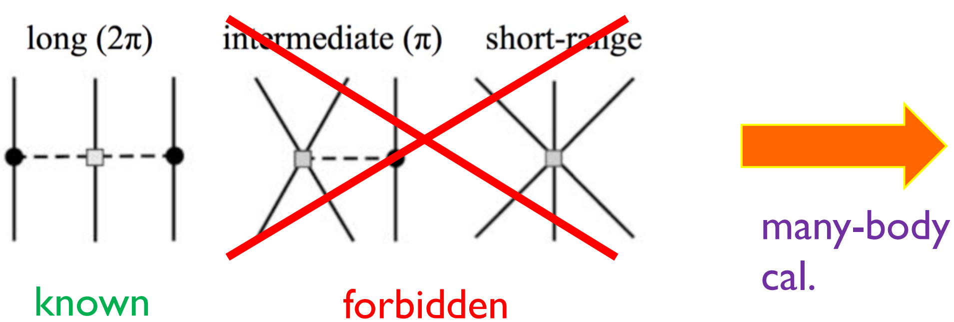
Drischler-Hebeler-Schwenk, 2016

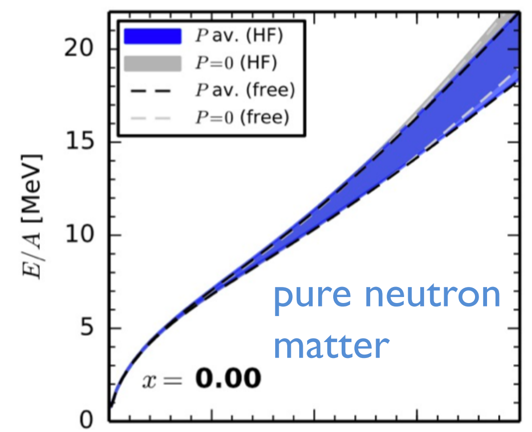




- pure neutron matter is less uncertain:

(Good for NS community)





microscopic calculations at n_B = 1-2 n_0 : consistent with empirical facts

Cold, dense EoS : Low density

For NS applications (n_B=1-10n_0), the fundamental question is:

convergence of **many-body forces**

e.g.1) parameterized **pure neutron** matter EoS [Gandolfi+, 2009]

~kin. + 2-body ~3-body

$$\varepsilon = n_0 \left[(12 \pm 1\,\mathrm{MeV}) \left(\frac{n_B}{n_0} \right)^{1.45 \pm 0.05} + (4 \pm 2\,\mathrm{MeV}) \left(\frac{n_B}{n_0} \right)^{3.3 \pm 0.3} \right]$$

e.g.2) Akmal-Pandharipande-Ravenhall EoS (**APR 98**) [Table V of APR paper]

pure neutron matter

n_B	2 –body int. $\langle v_{ij}^{\pi} \rangle$	$\langle v_{ij}^{R} \rangle$	3 –body int. $\langle V_{ijk}^{2\pi} \rangle$	$\langle V_{ijk}^{R} \rangle$
n_0	−4.1	−29.9	1.2	4.5
2 n_0	−25.1	−36.4	−17.4	30.6
3 n_0	−35.7	−44.7	−34.1	78.0
4 n_0	−52.2	−41.1	−76.9	160.3

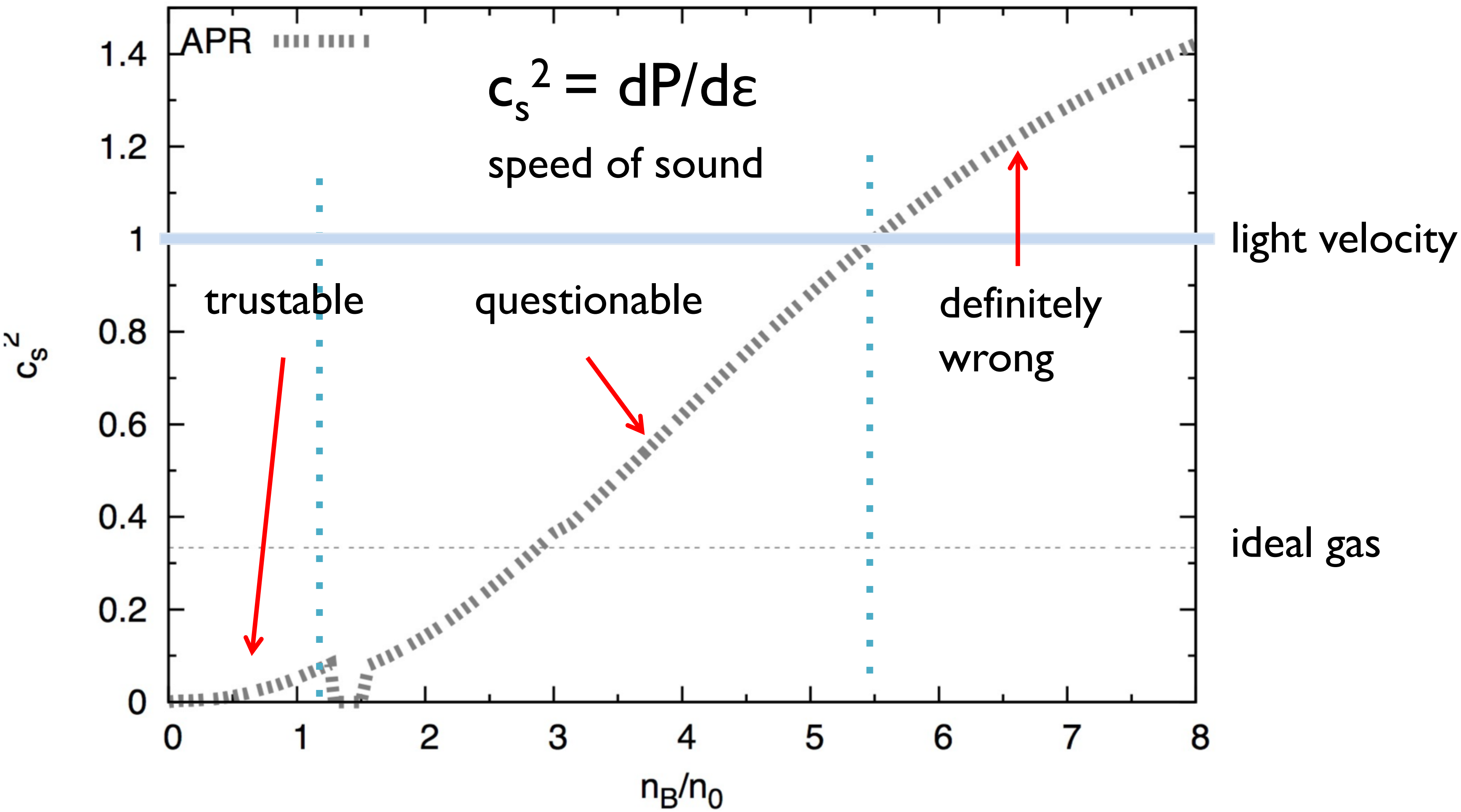
grow rapidly!

4-, 5- or more-body forces should be important as well beyond ~ **2n_0**

$$\langle V_{N\text{-body}} \rangle \sim (n_B/n_0)^N$$

Cold, dense EoS : Low density

Akmal-Pandharipande-Ravenhall EoS (**APR 98**)



Picture to be developed

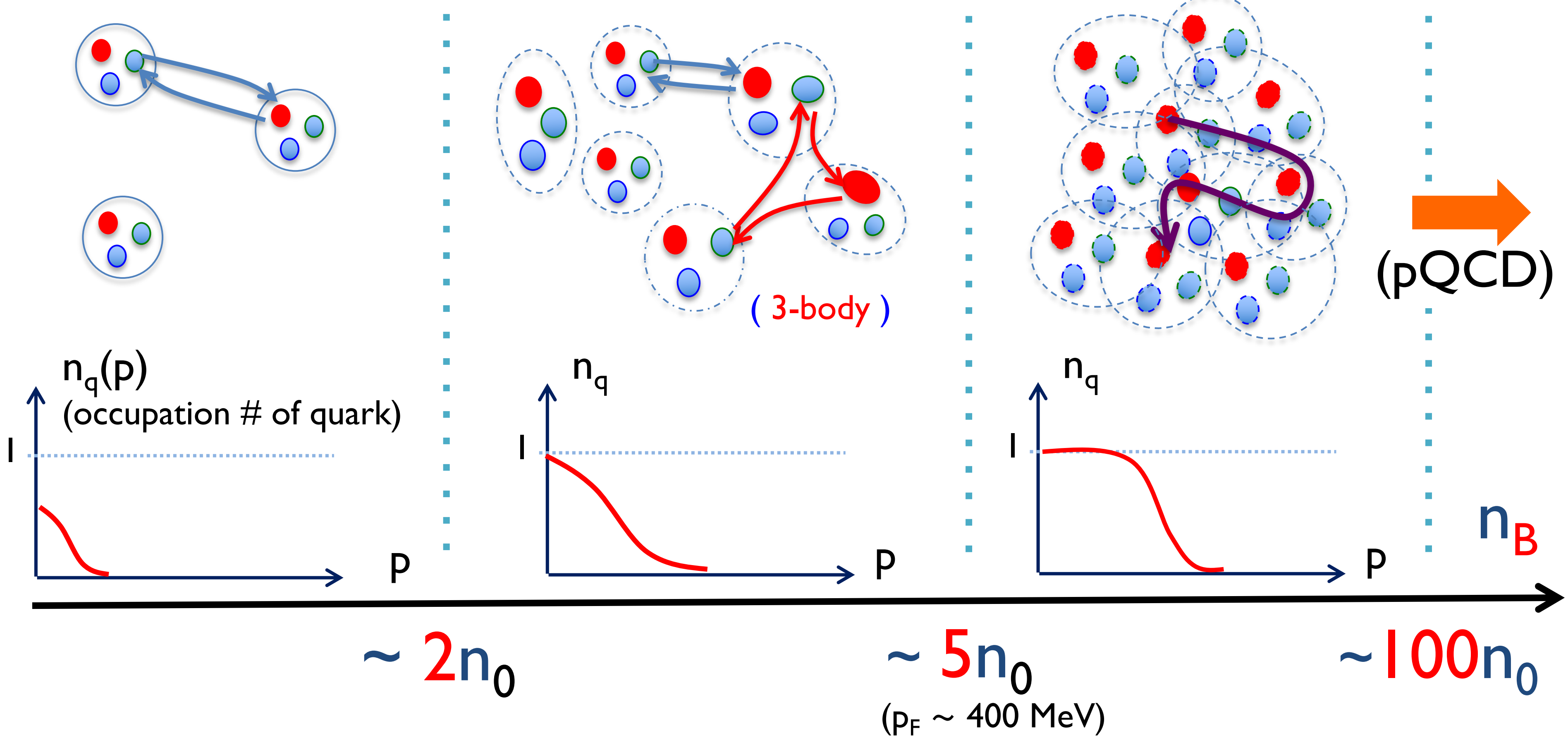
Masuda-Hatsuda-Takatsuka 2012
TK-Powell-Song-Baym 2014

Hints from neutron stars

- few meson ex.
- nucleons only

- many-quark exchange
- structural change

- Baryons overlap
- Quark Fermi sea



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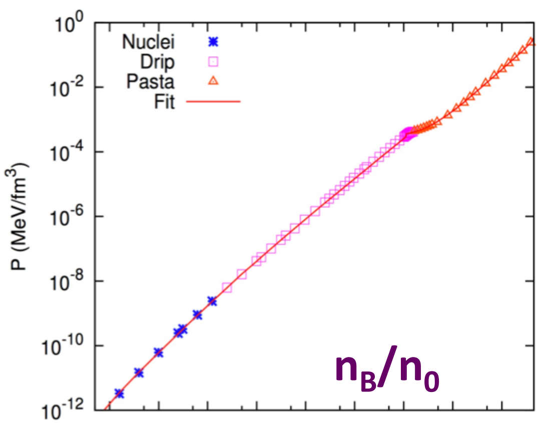
EoS & M-R relation

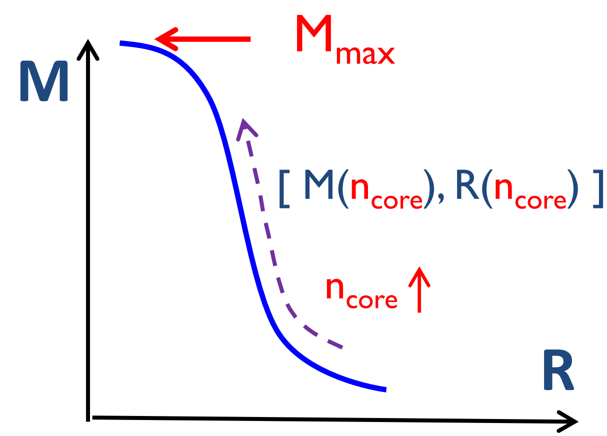
Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ .... QCD EoS



[for spherical NS → TOV eq.]

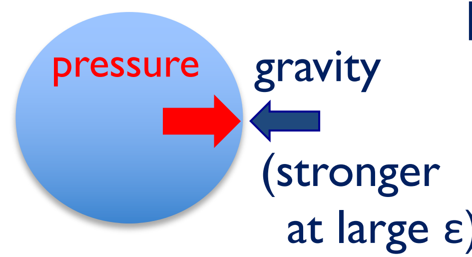
1-to-1 correspondence

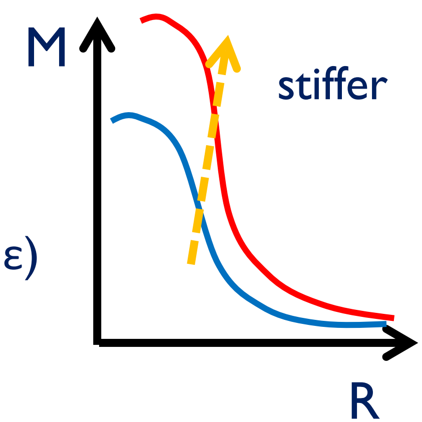
Lindblom (1992)



Terminology (my convention)

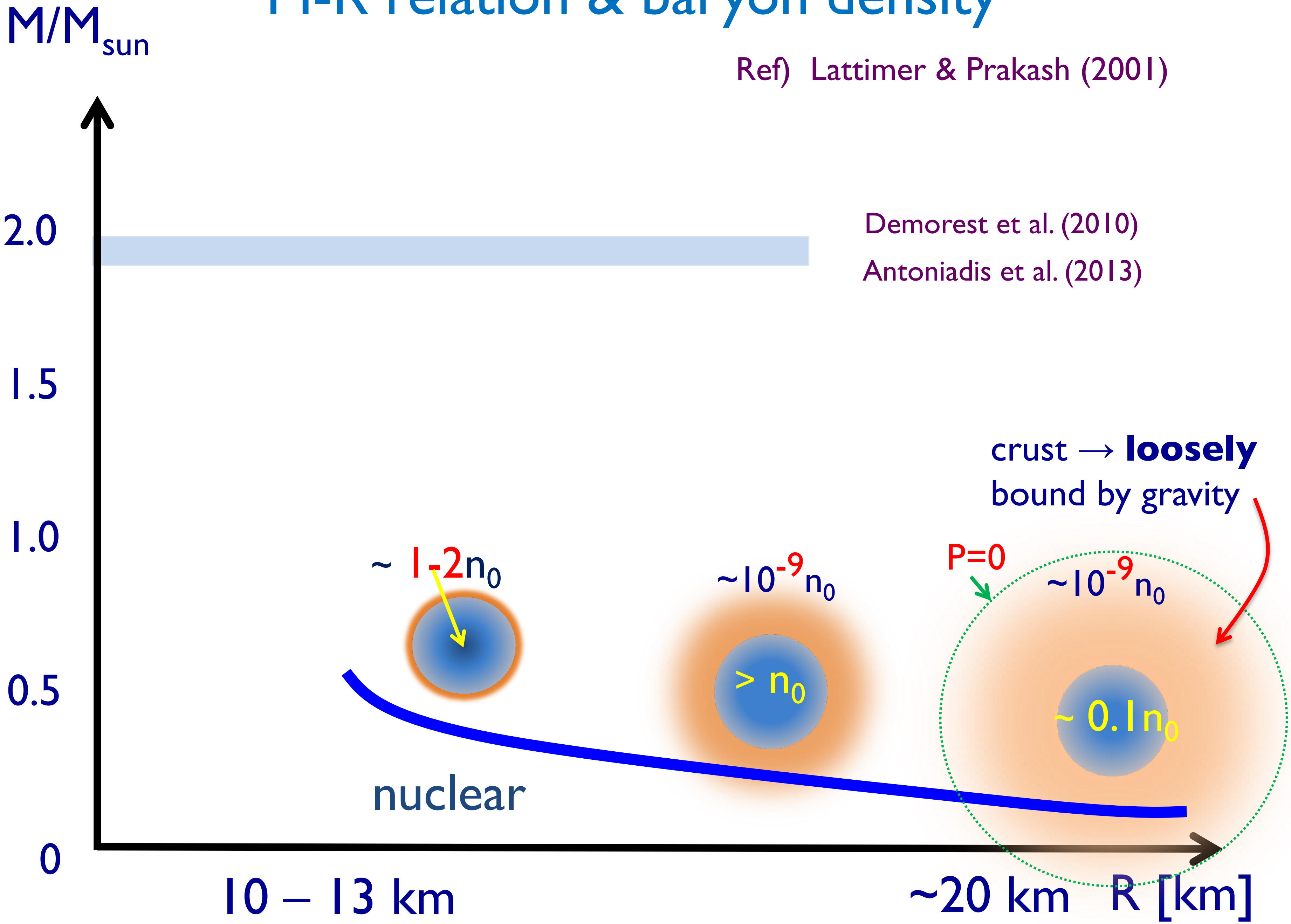
1) **Stiff** EoS : P is large at given ε





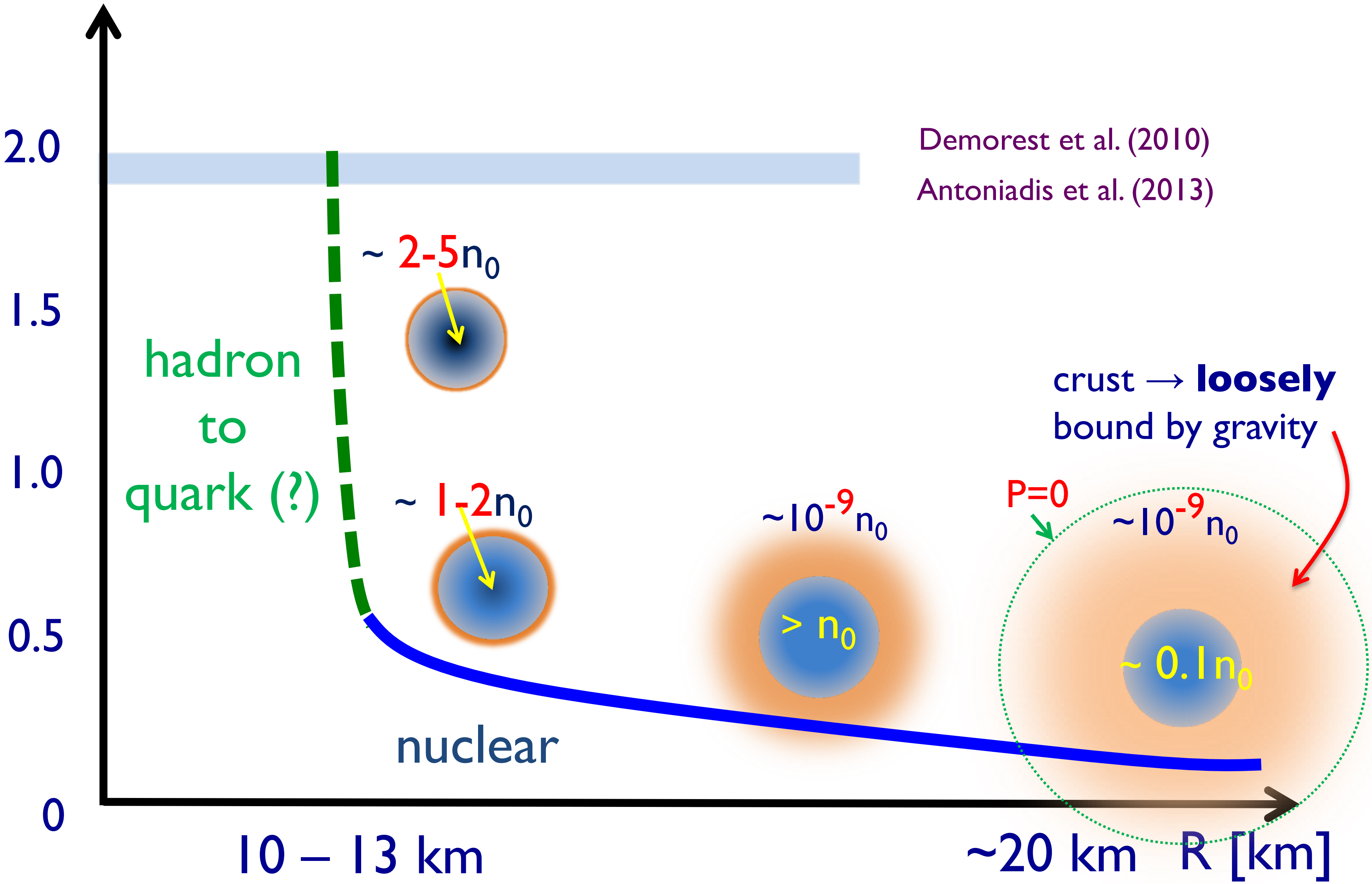
2) **Soft**-**Stiff** EoS : Soft at $n_B < 2n_0$ & Stiff at $n_B > 5n_0$

M-R relation & baryon density



M-R relation & baryon density

Ref) Lattimer & Prakash (2001)



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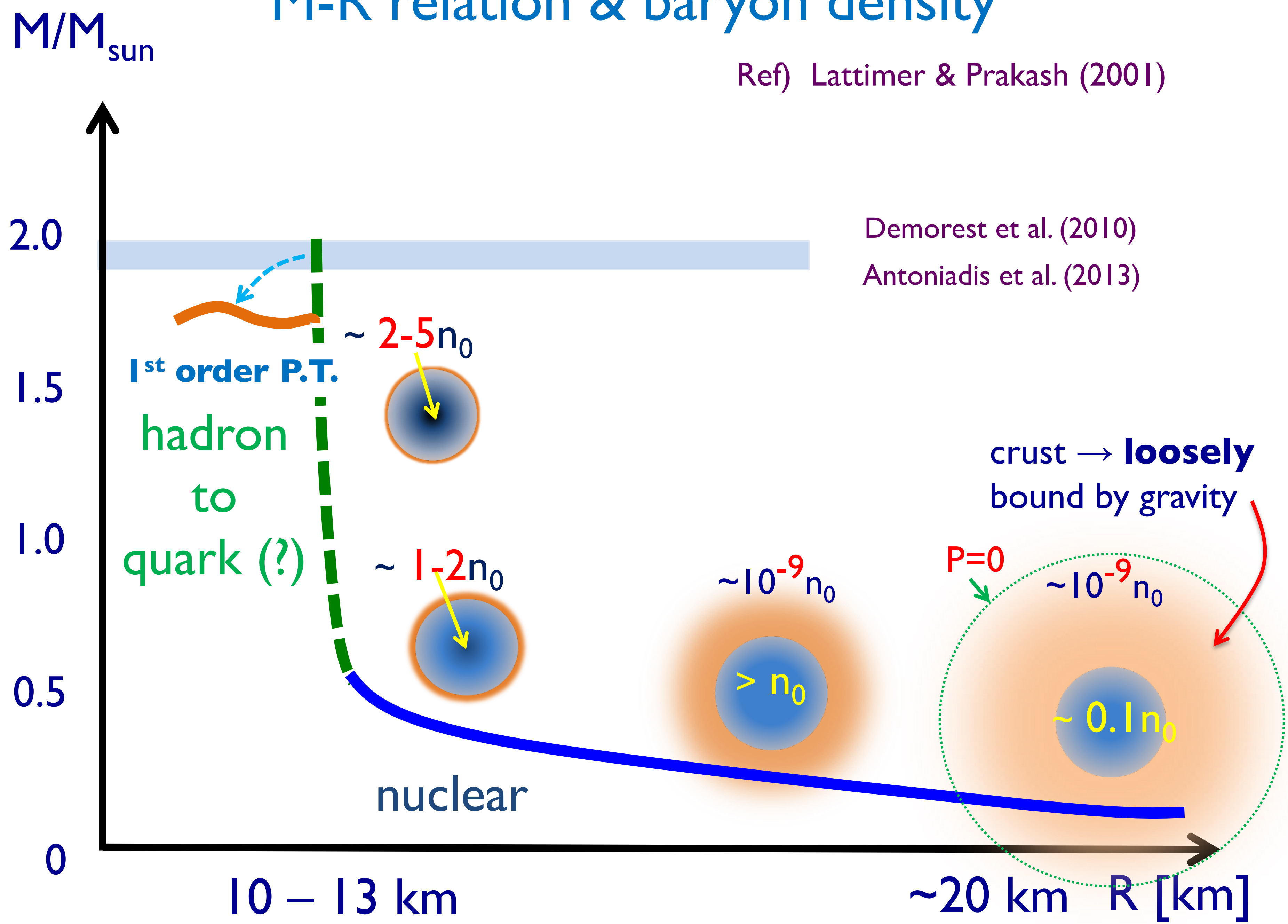

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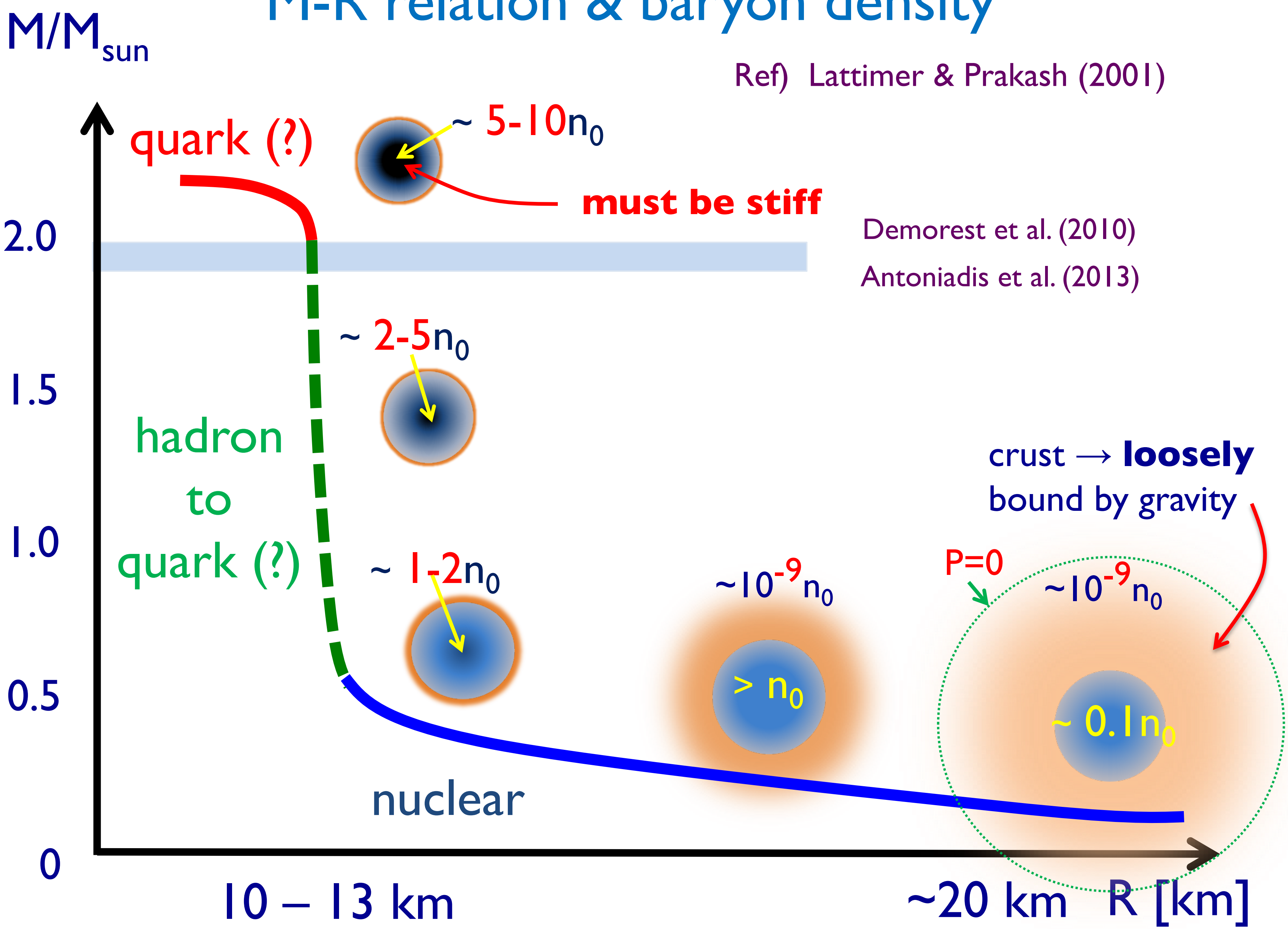
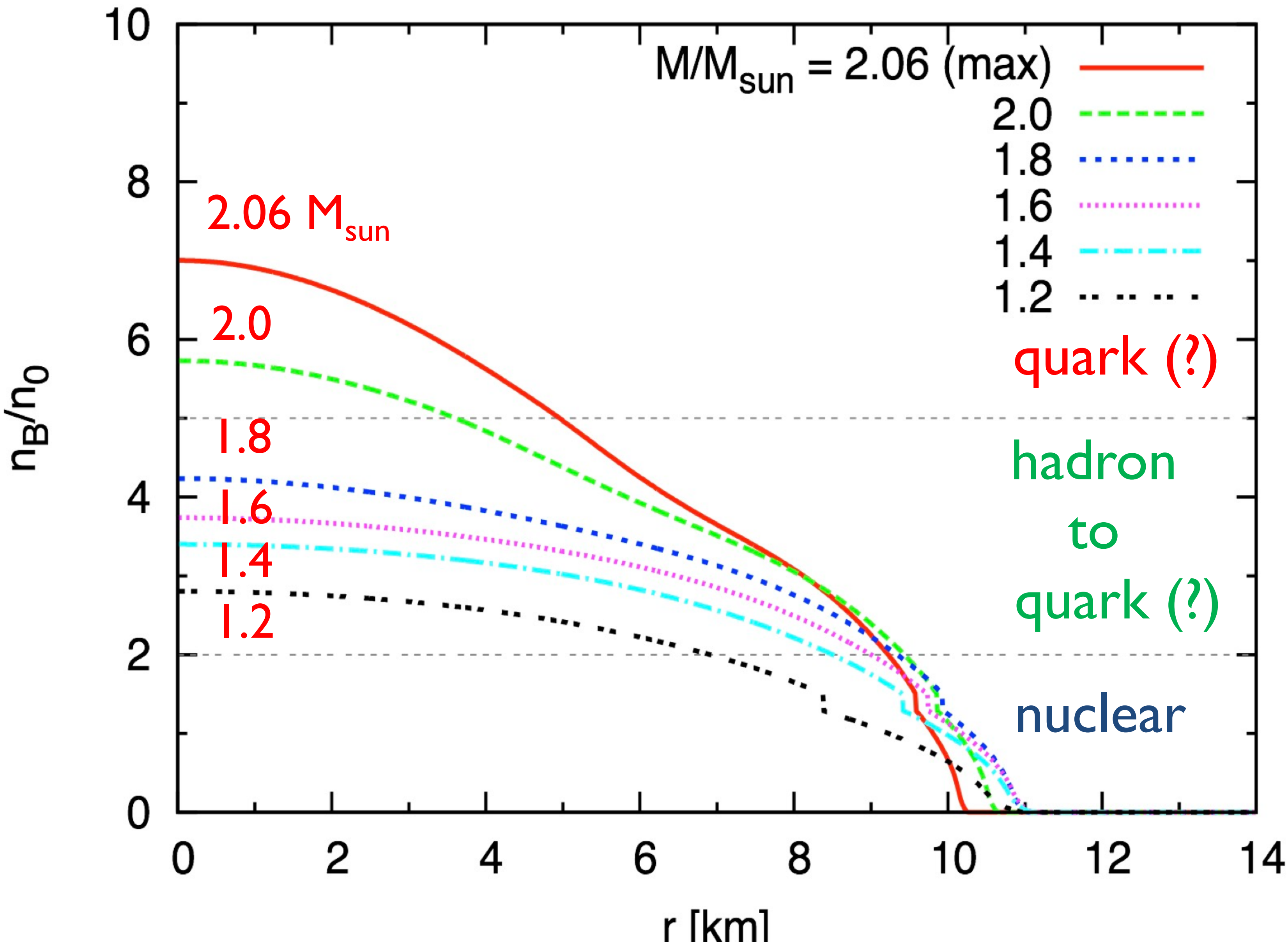
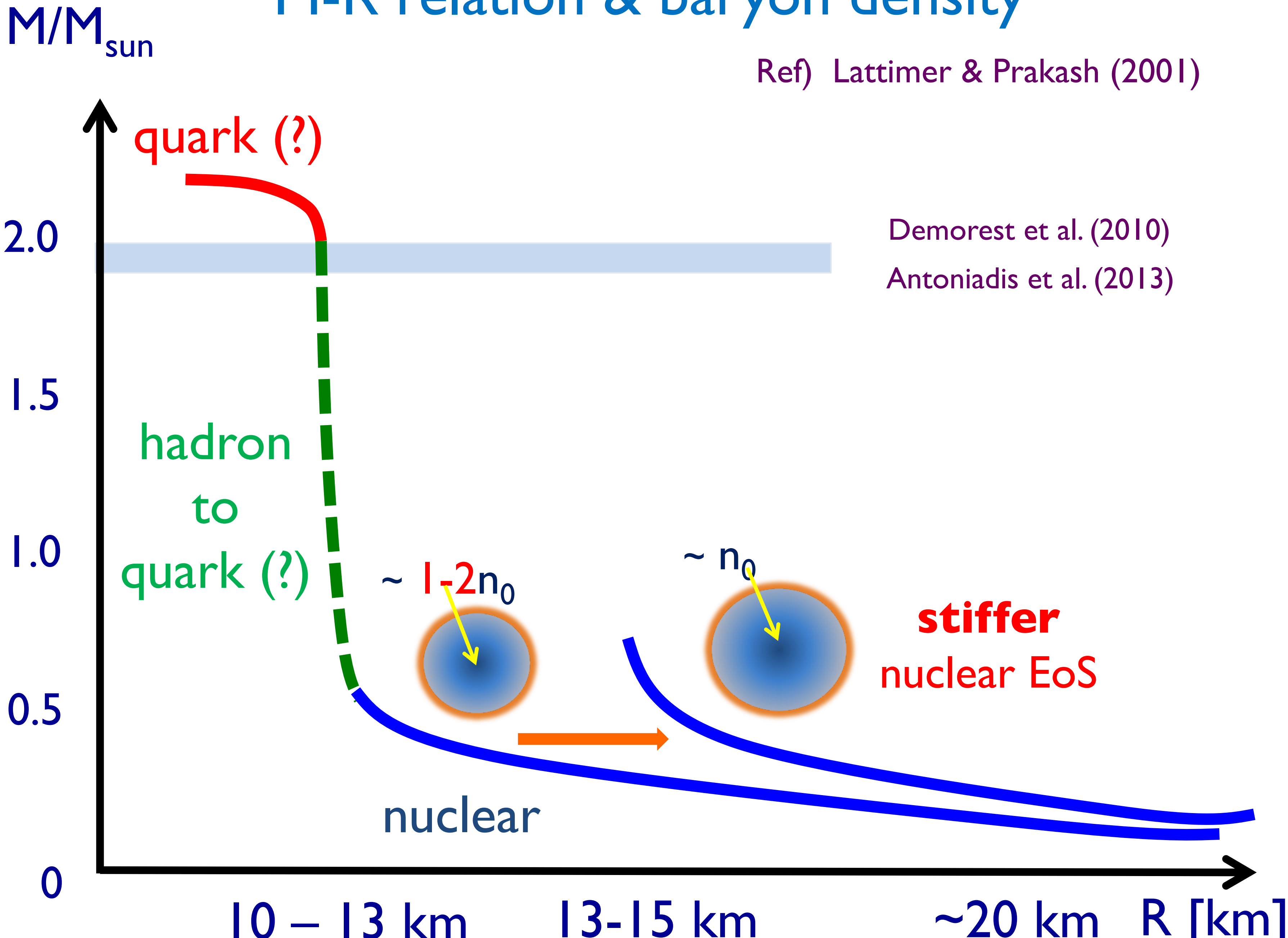


Baryon density in a neutron star (QHC18)



M-R relation & baryon density



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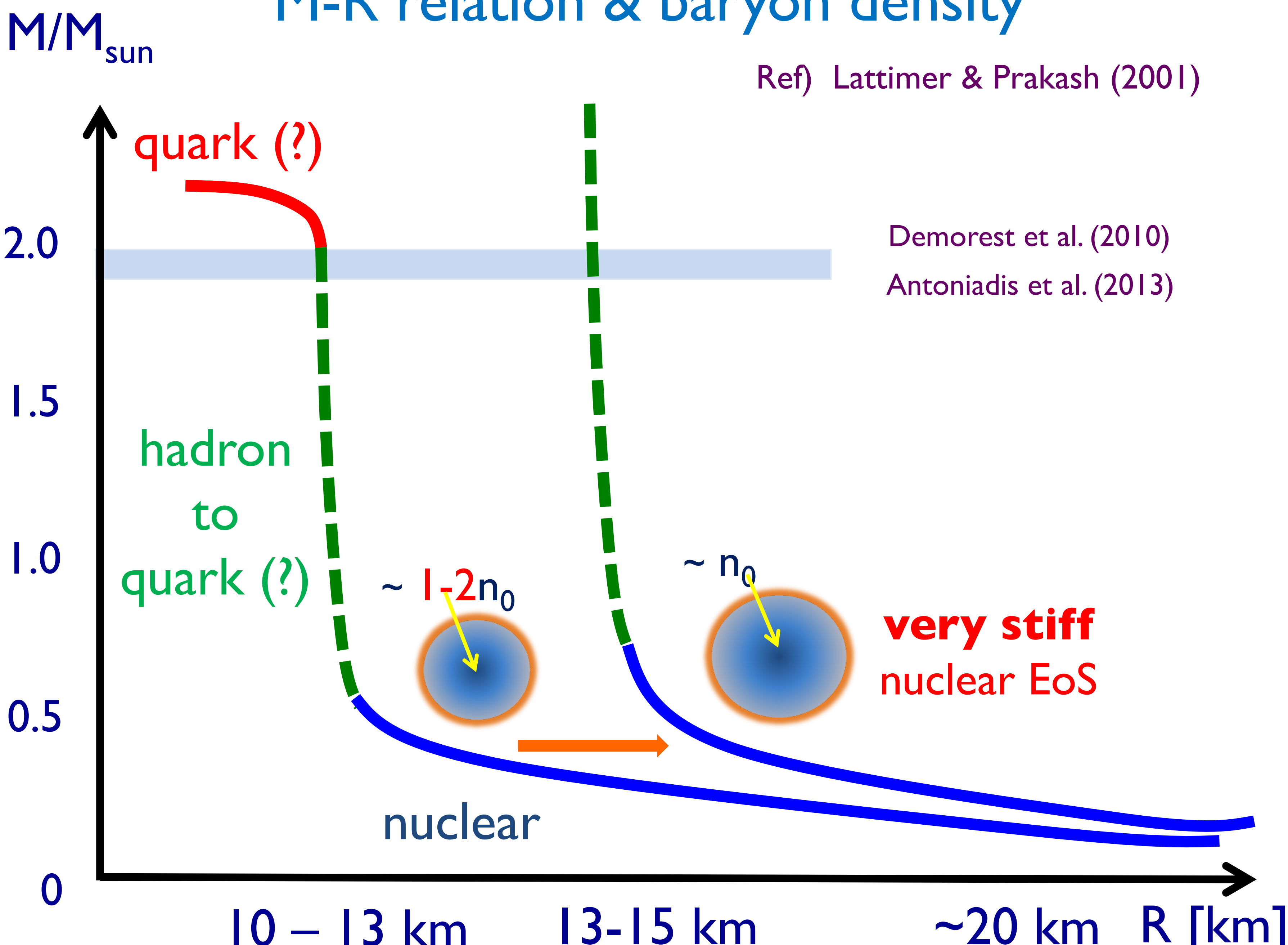

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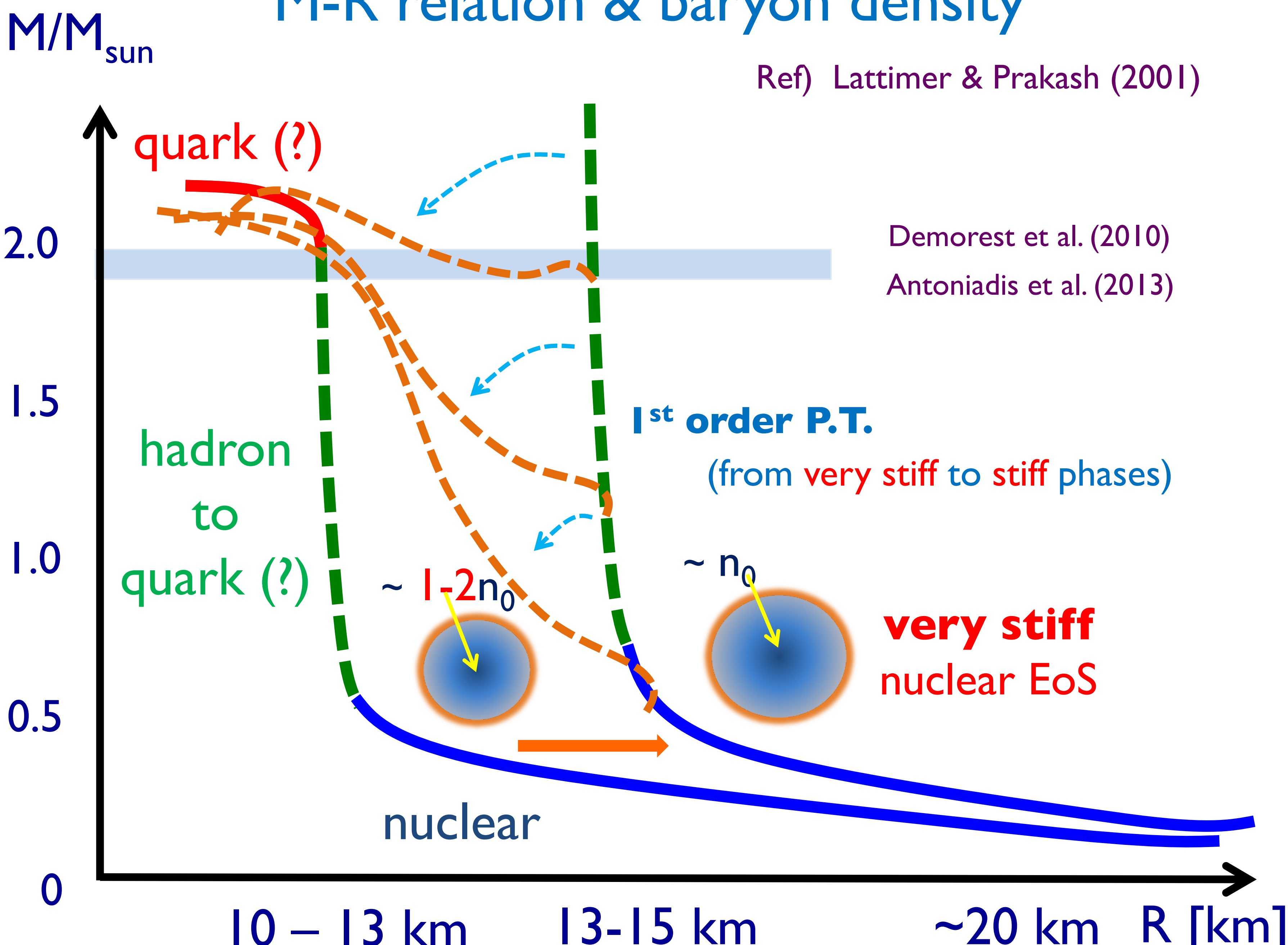


M-R re

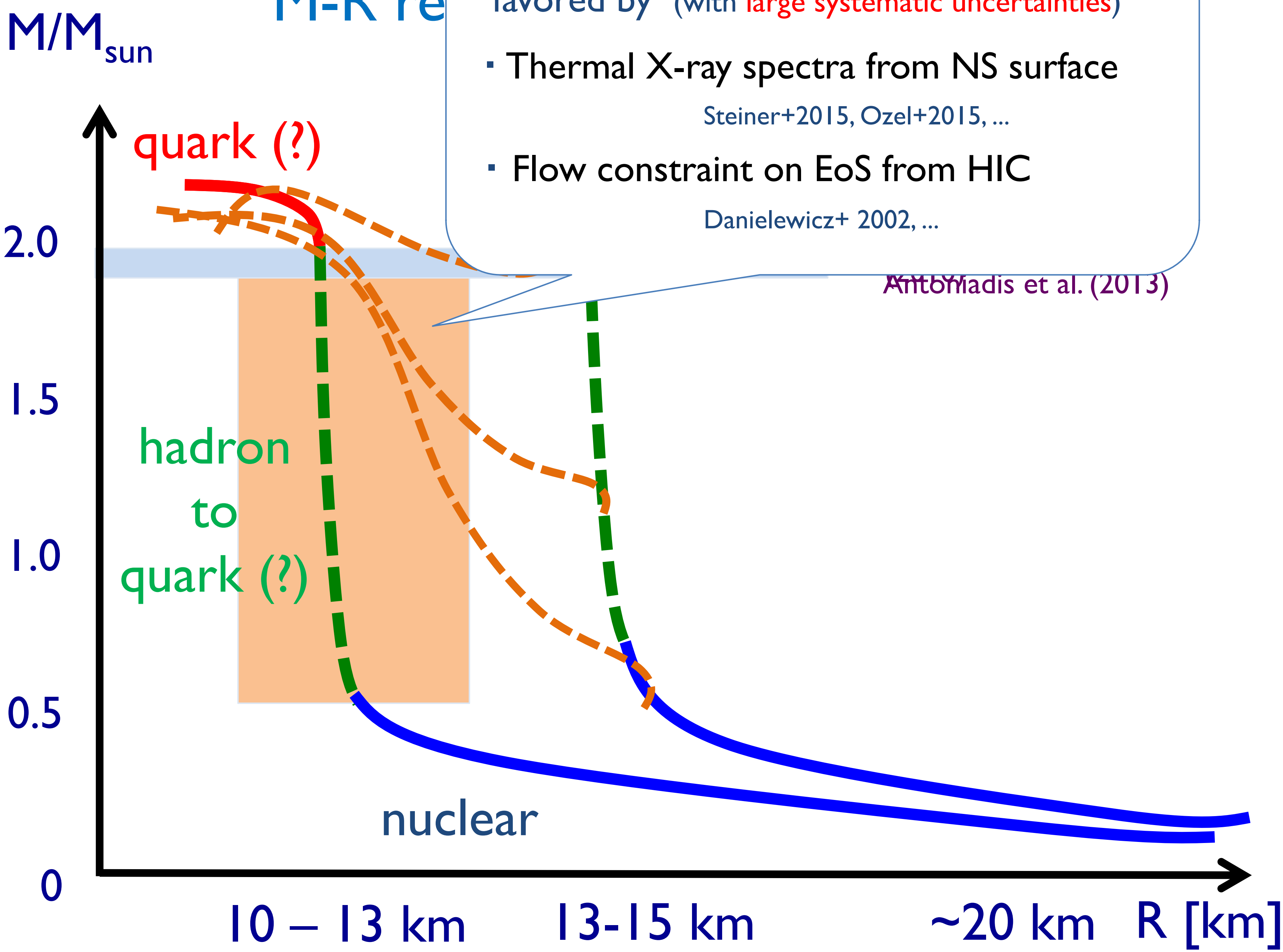
favored by (with large systematic uncertainties)

- Thermal X-ray spectra from NS surface

  Steiner+2015, Ozel+2015, …

- Flow constraint on EoS from HIC

  Danielewicz+ 2002, …



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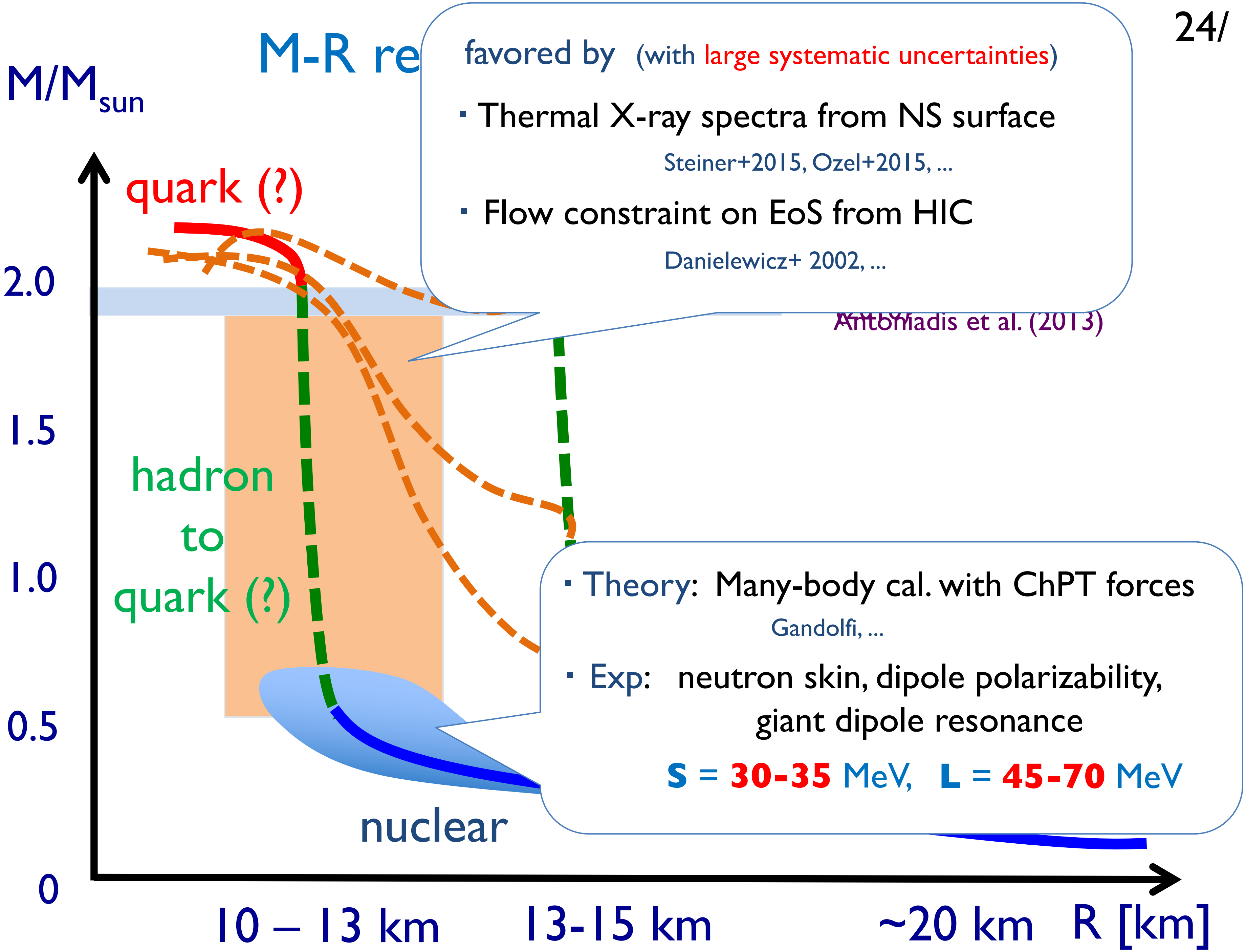
M/M$_{sun}$
2.0
1.5
1.0
0.5
0
quark (?)
hadron
to
quark (?)
nuclear
Antoniadis et al. (2013)
10 – 13 km
13-15 km
~20 km
R [km]
favored by (with large systematic uncertainties)
· Thermal X-ray spectra from NS surface
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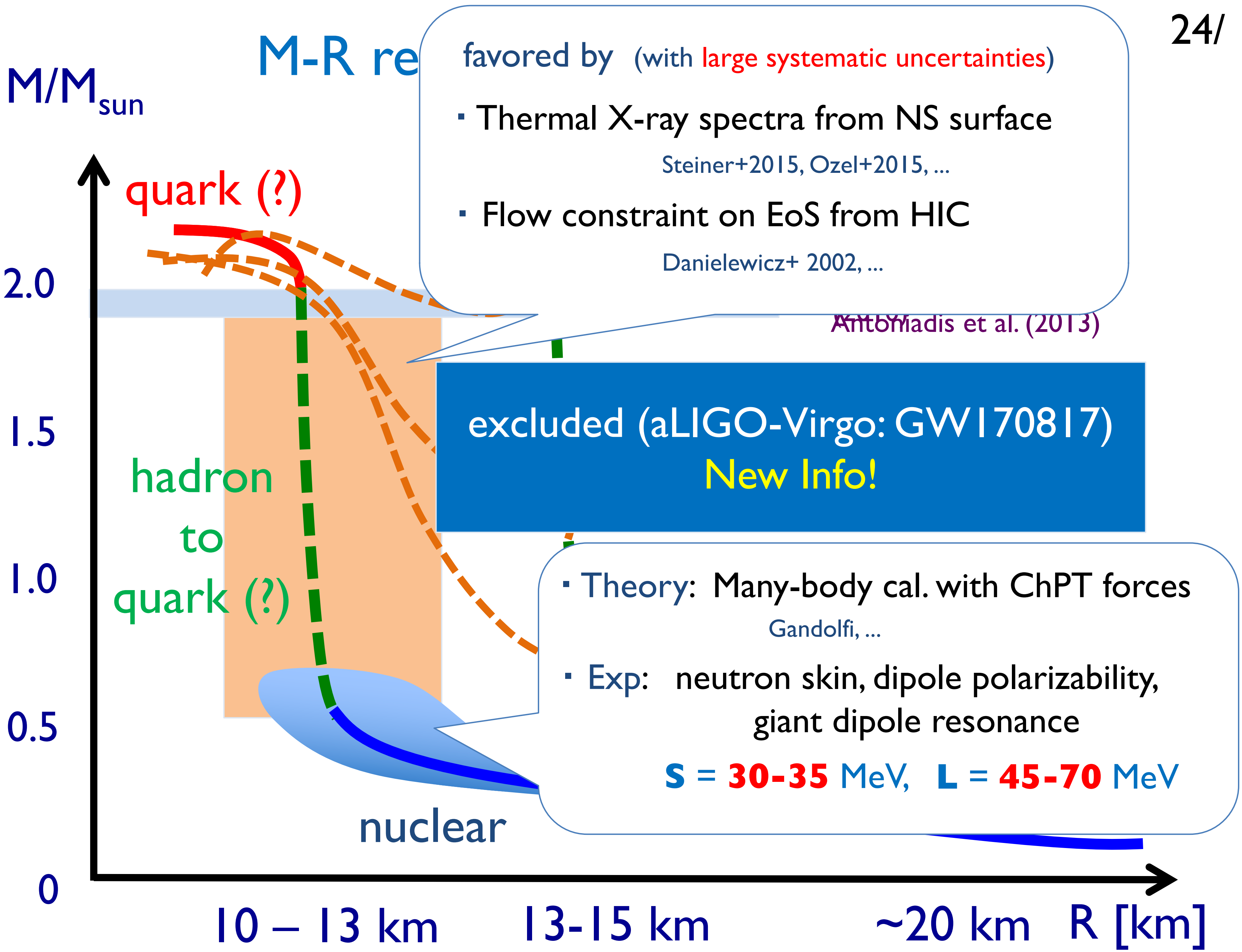
Danielewicz+ 2002, ...
· Theory: Many-body cal. with ChPT forces
Gandolfi, ...
· Exp: neutron skin, dipole polarizability,
giant dipole resonance
S = **30-35** MeV, **L** = **45-70** MeV


M-R re
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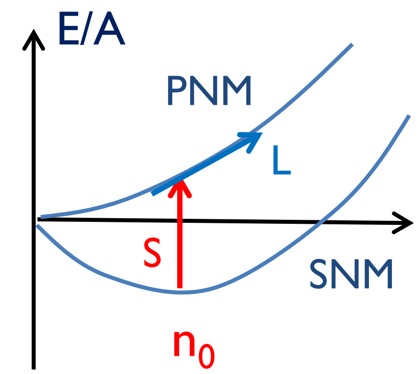
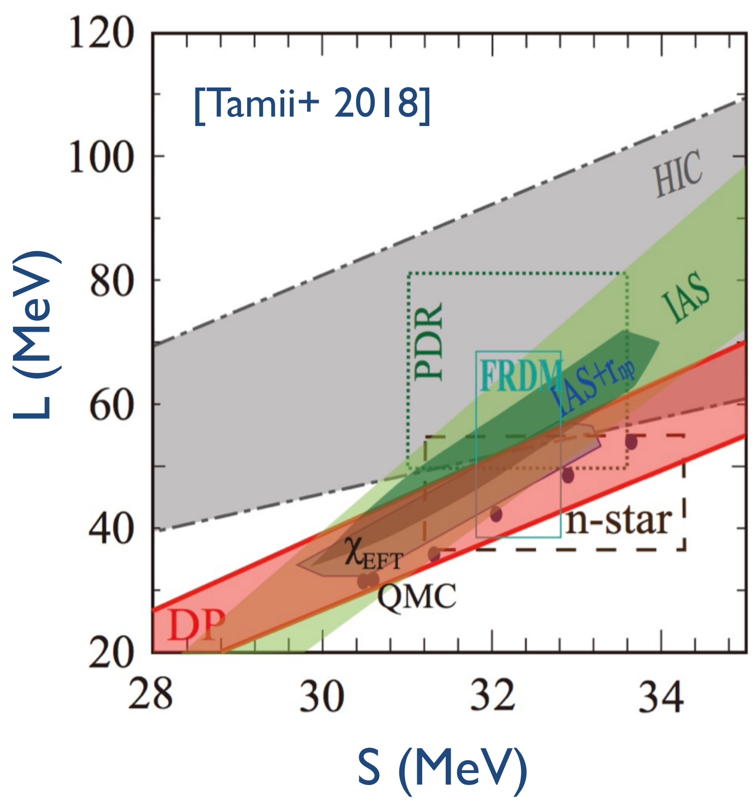
excluded (aLIGO-Virgo: GW170817)
New Info!
▪ Theory: Many-body cal. with ChPT forces
Gandolfi, ...
▪ Exp: neutron skin, dipole polarizability, giant dipole resonance
S = 30-35 MeV, L = 45-70 MeV

Hints for **soft** EoS at $n_B < 2n_0$

$$\frac{E}{A} = -16\,\mathrm{MeV} + S + \frac{L}{3}\frac{n-n_0}{n_0} + \cdots$$

pure neutron matter EoS — sym. energy — density dep.





Theory
- Many-body cal. with ChPT forces

Exp.
- Neutron skin
- Dipole polarizability
- Giant dipole resonance

} **T ~ 0 MeV**

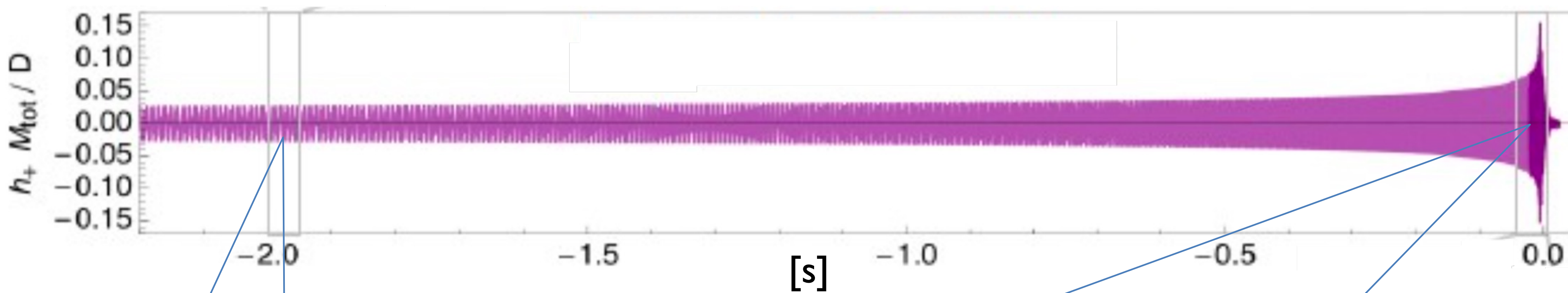
- Heavy ion (E_{lab}/A ~ 200 MeV)

S = 30-35 MeV, L = 45-70 MeV

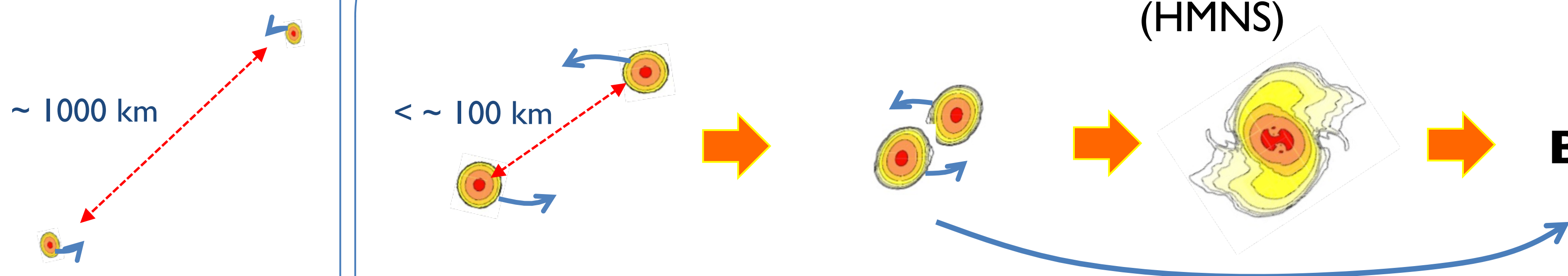
Then, EoS extrapolated to $2n_0$ leads to

$R_{1.4}$ = 11-13 km

GWs from NS-NS mergers

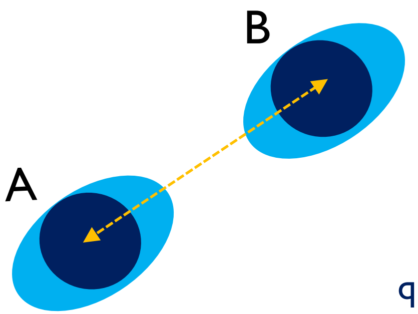


Early inspiral	Tidally deformed	Merger	Hyper Massive NS (HMNS)	BH



Post Newtonian (point particle)	Finite size effect	strong GR + MHD + neutrino transport
M_1 & M_2	**R_1 & R_2**	**M_{max} & hot EoS & ...**
spins		
cleaner		**Messy**

Tidal deformation → accelerated phase evolution

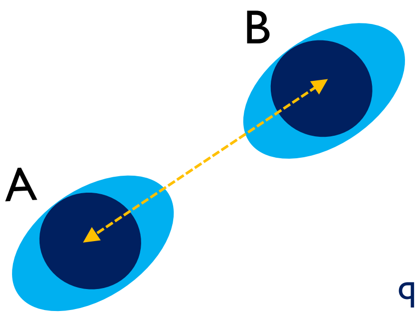


1) grav. fields from star B → the deformation of star A
2) deformed energy density → quadrupole grav. fields

polarizability

quadrupole moment $$Q_{ij} = -\lambda(M)E_{ij}$$ external field $$E_{ij} = -\frac{\partial^2 V}{\partial x_i \partial x_j}$$

Tidal deformation → accelerated phase evolution



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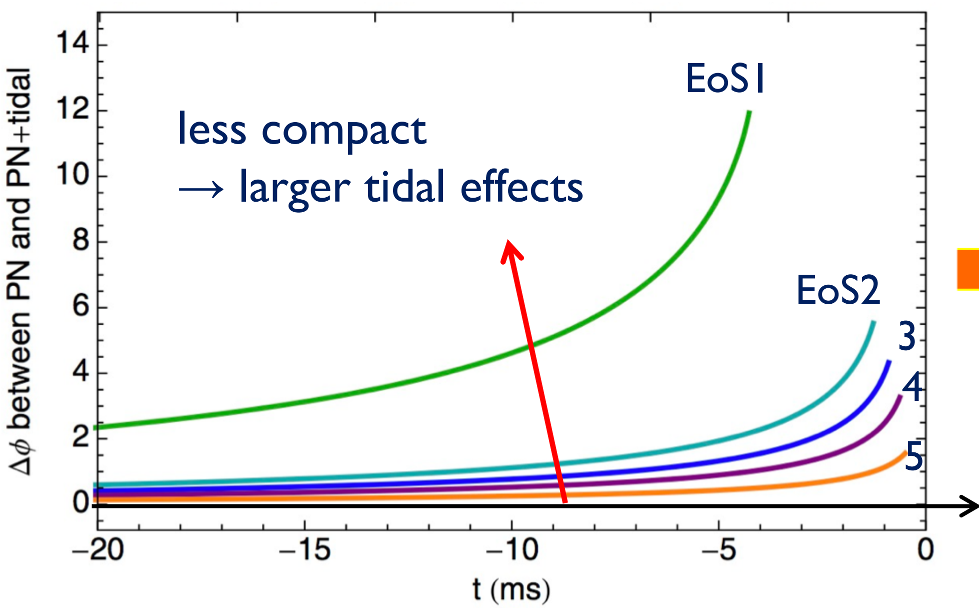
gravitational pot. from the star A
$$V_A(r) \simeq -\frac{GM_A}{r} - \underline{\frac{GQ_{AB}}{r^3}} \simeq -\frac{GM_A}{r} - \underline{\frac{G}{r^3}\left(\frac{\lambda G M_B}{r^3}\right)}$$

attractive
→ acceleration

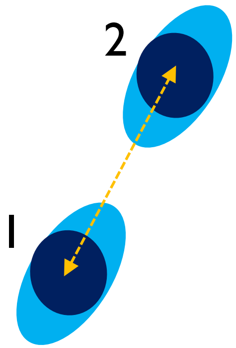


Read+ 2012

point particle

→ **upperbound** on **λ** & **R**

Dimensionless tidal deformability → R_{NS}



more common to use $$\Lambda(M) = 32\frac{\lambda G}{R^5}$$

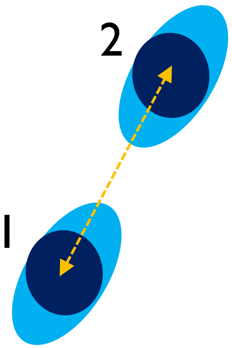
What GW analyses measure: combination of Λ for star 1 & 2 :

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$

(measured)

2-parameters: M_1 & M_2

Dimensionless tidal deformability → R_{NS}



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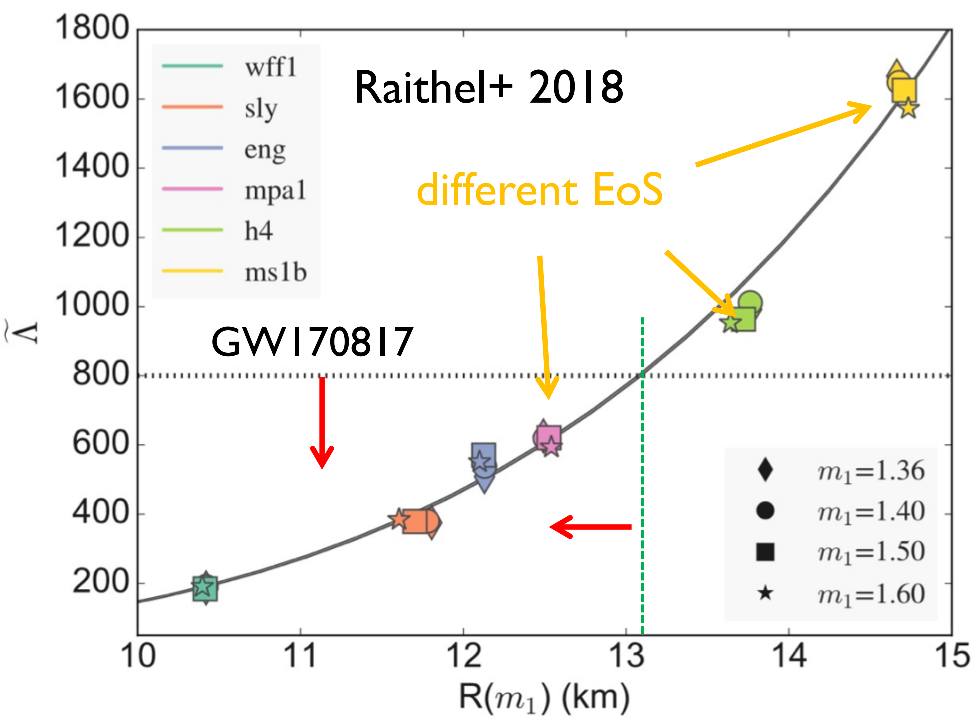
(measured)

2-parameters: M_1 & M_2



For **GW170817** :

chirp mass (1.188 M_{sun}) (determined)

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = m_1\frac{q^{3/5}}{(1+q)^{1/5}}$$

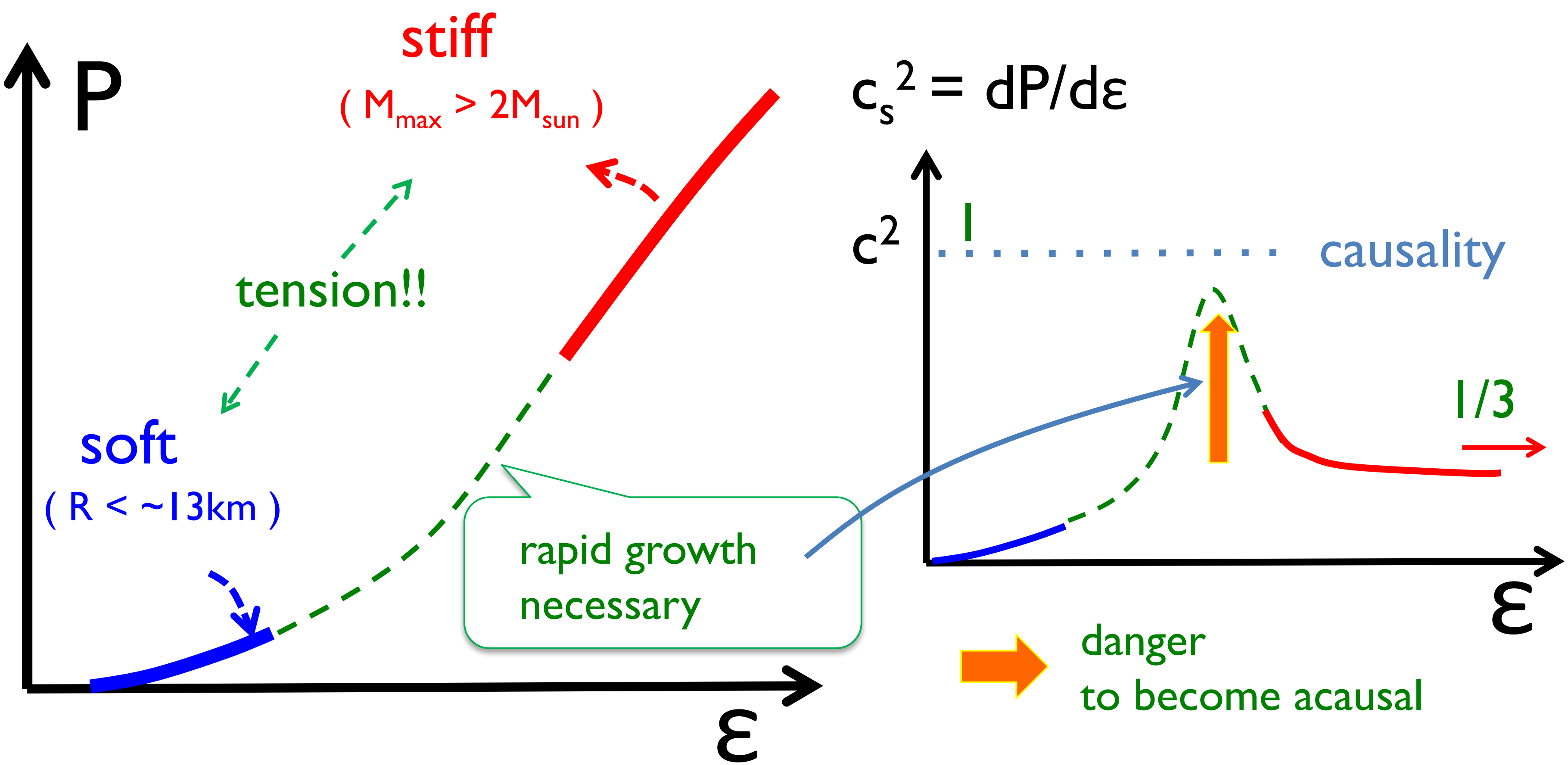
mass ratio

$q = M_2/M_1$ (undetermined)

- different q degenerate !
- R < ~ 13 km

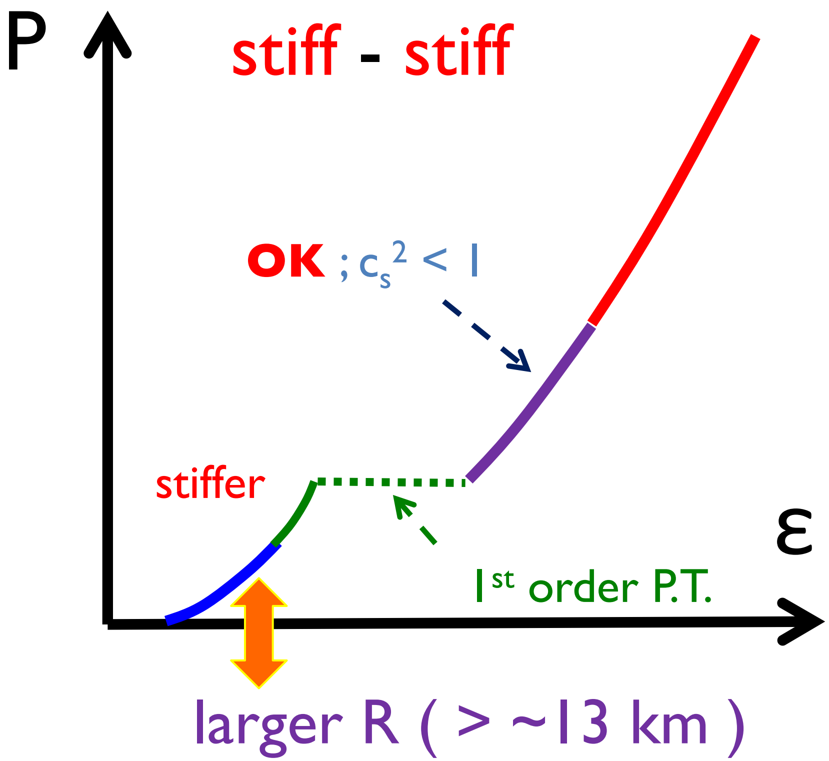
Causality constraint on $2n_0$-$5n_0$ region

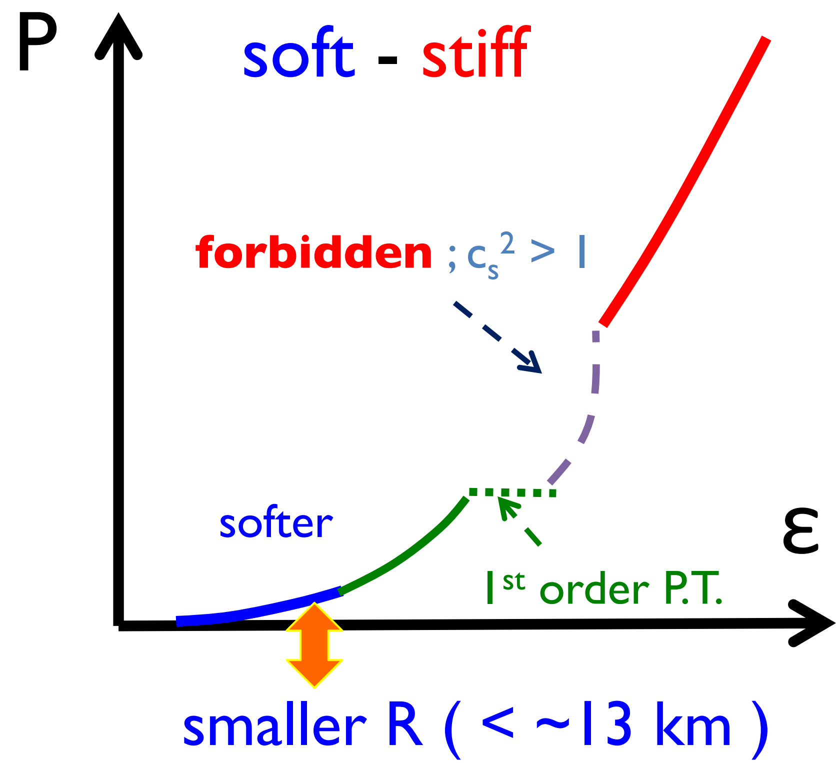
assume: $R < 13\text{km}$ & $M_{max} > 2M_{sun}$



Stiff-Stiff v.s. Soft-Stiff EoS

[more quantitative analyses → Han-Alford-Prakash 13]



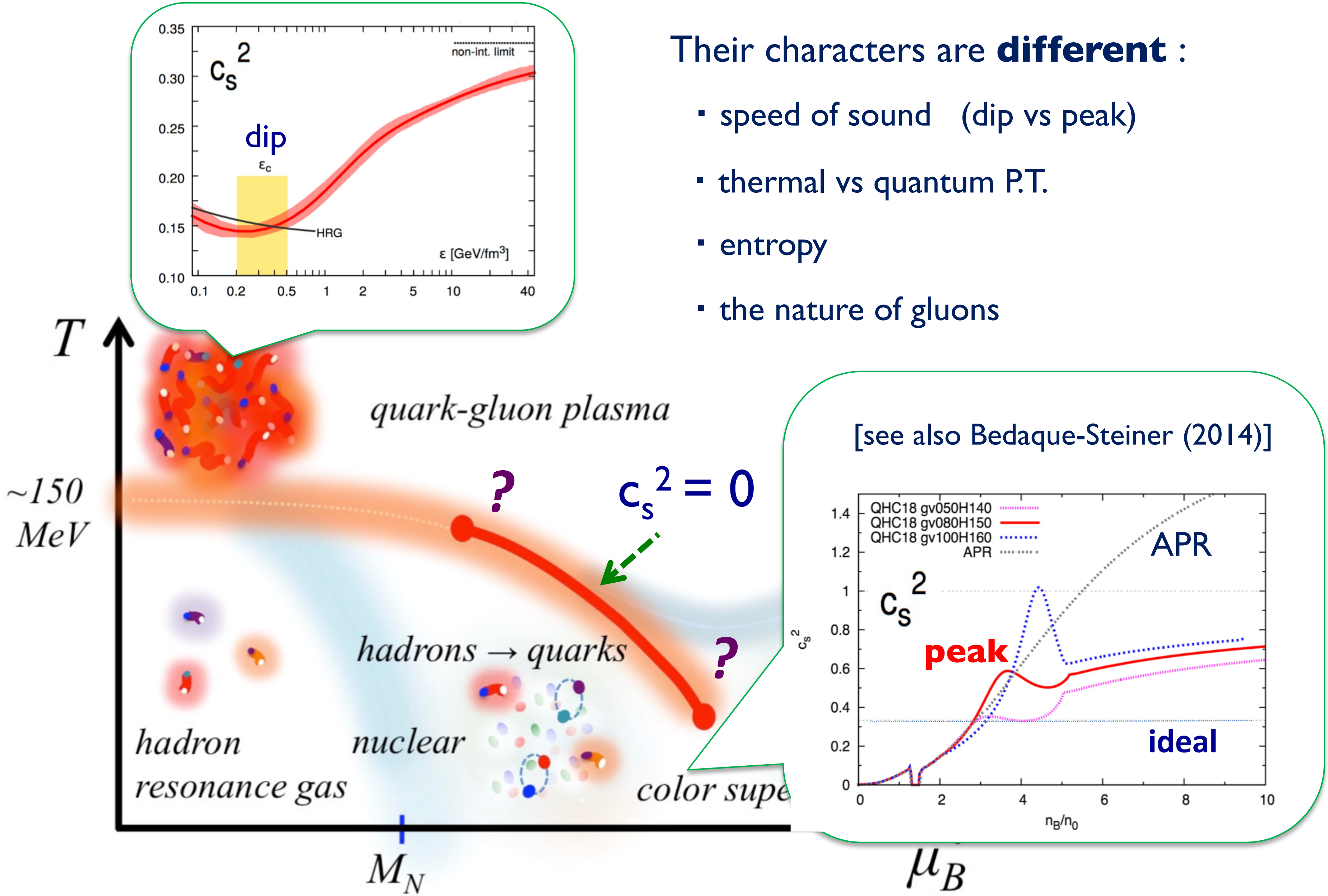


→ we consider a **soft-stiff** EoS with **crossover** (or weak 1st order)

Finite T vs low T crossover

Their characters are **different** :

- speed of sound (dip vs peak)
- thermal vs quantum P.T.
- entropy
- the nature of gluons



Summary of lecture 1

1, QCD has reliable high & low density limits,
but be careful in extrapolating these results:

interpolation of these limits are **much safer** procedure

2, 1st principle methods
→ the validity range of quasi-particle pictures

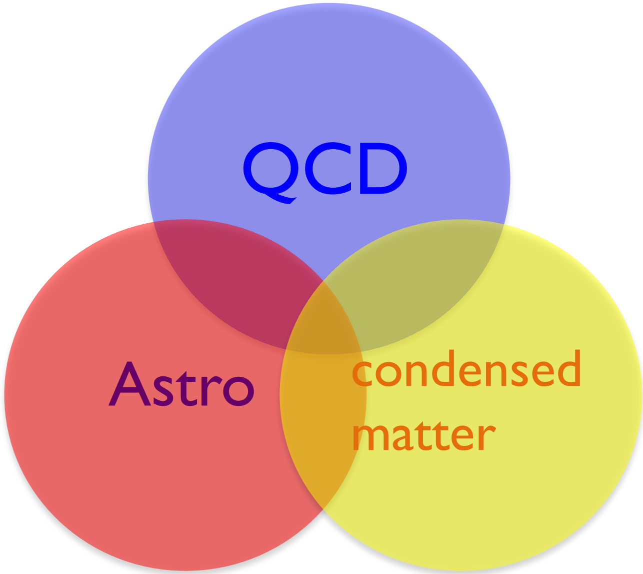
3, NS observations → Hints for **soft-stiff** EoS
+ **causality**

→ Hadron-quark P.T. : **crossover** or **weak 1st order**

Lecture 2 : **Discussions from microscopic point of view**

Three-window approach to (cold) dense matter: Lect. 2

Toru Kojo (CCNU, Wuhan)



Picture to be developed

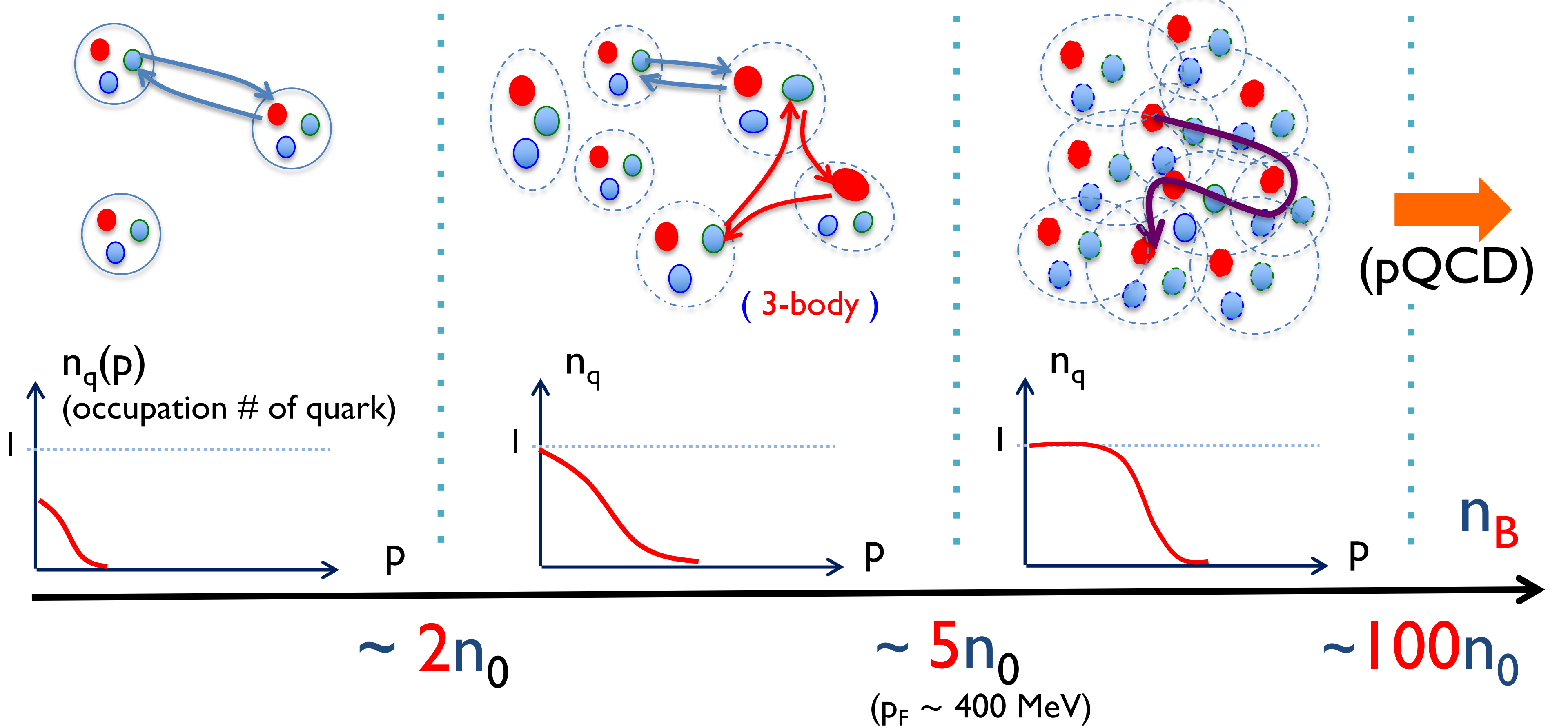
Masuda-Hatsuda-Takatsuka 2012
TK-Powell-Song-Baym 2014

Hints from neutron stars

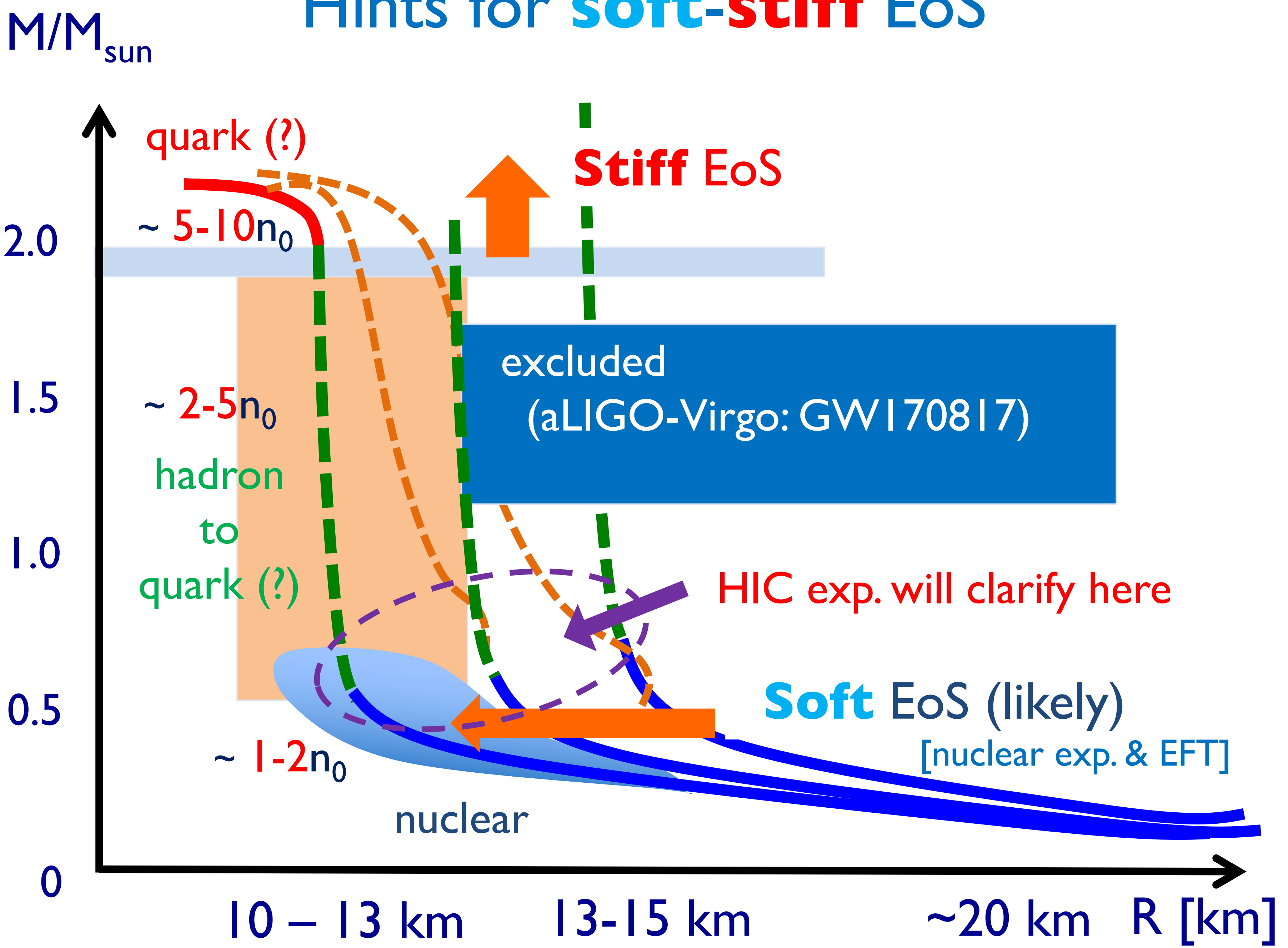
- few meson ex.
- nucleons only

- many-quark exchange
- structural change

- Baryons overlap
- Quark Fermi sea

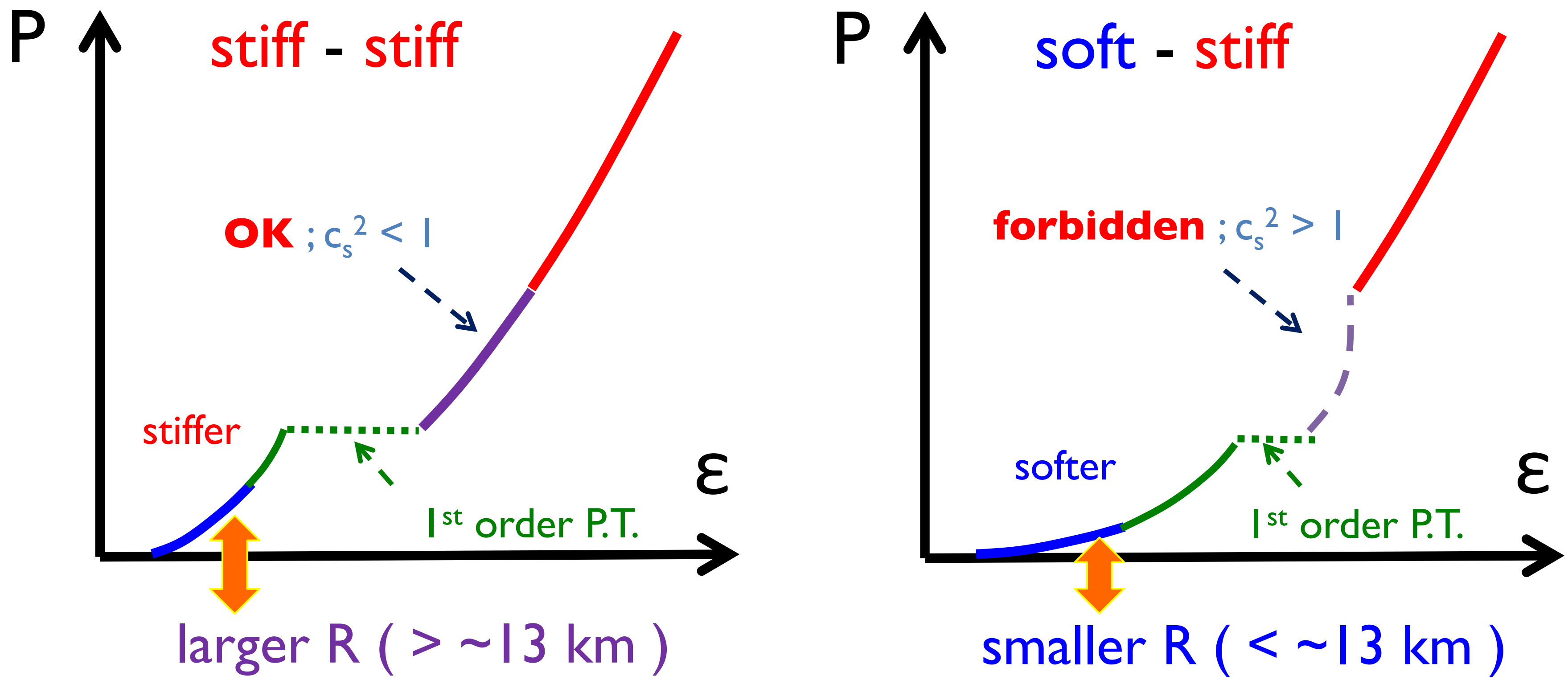


Hints for **soft-stiff** EoS



Stiff-Stiff v.s. Soft-Stiff EoS

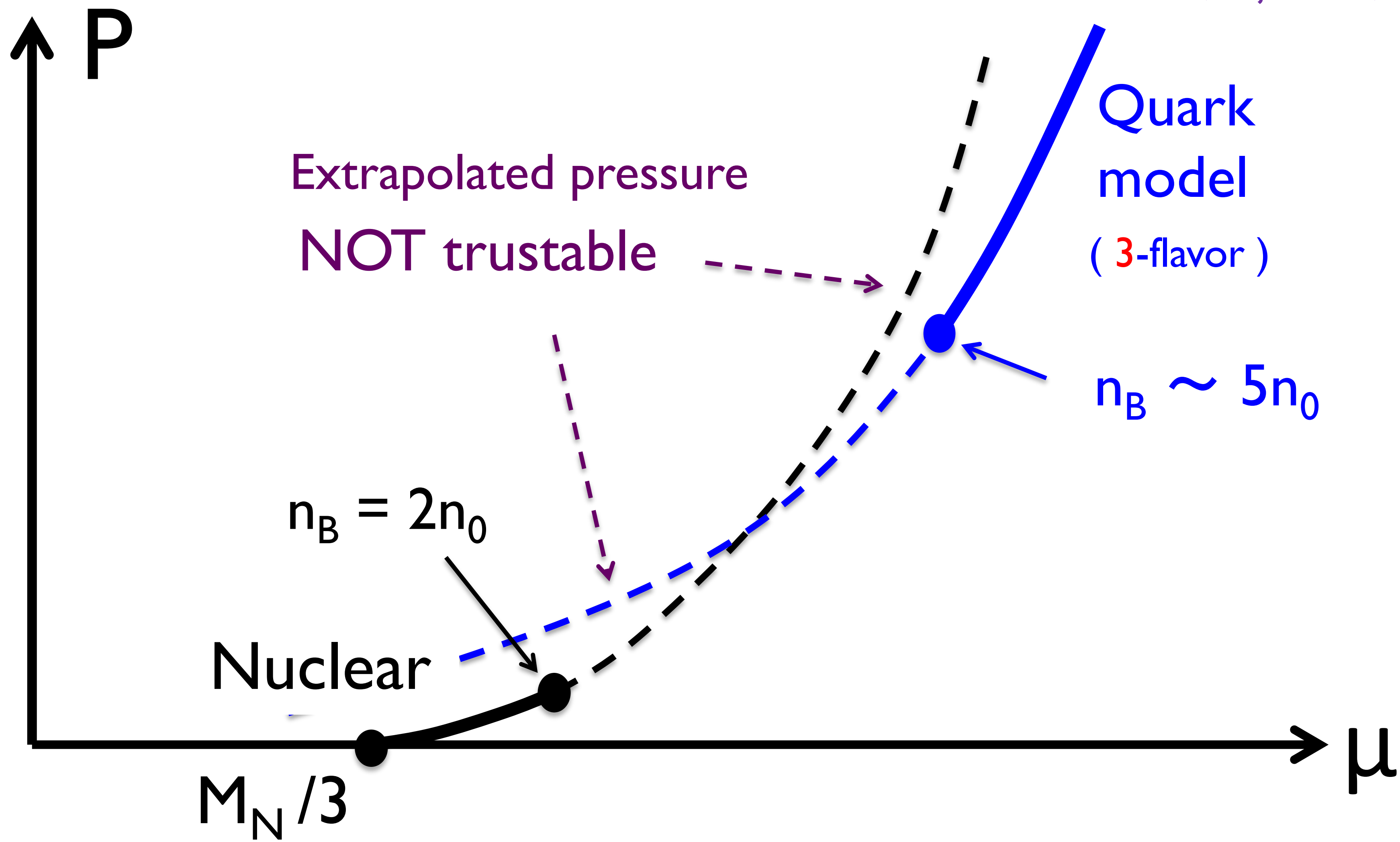
$c_s^2 = dP/d\varepsilon < c^2$ (**causality**)



→ we consider a **soft-stiff** EoS with **crossover** (or weak 1st order)

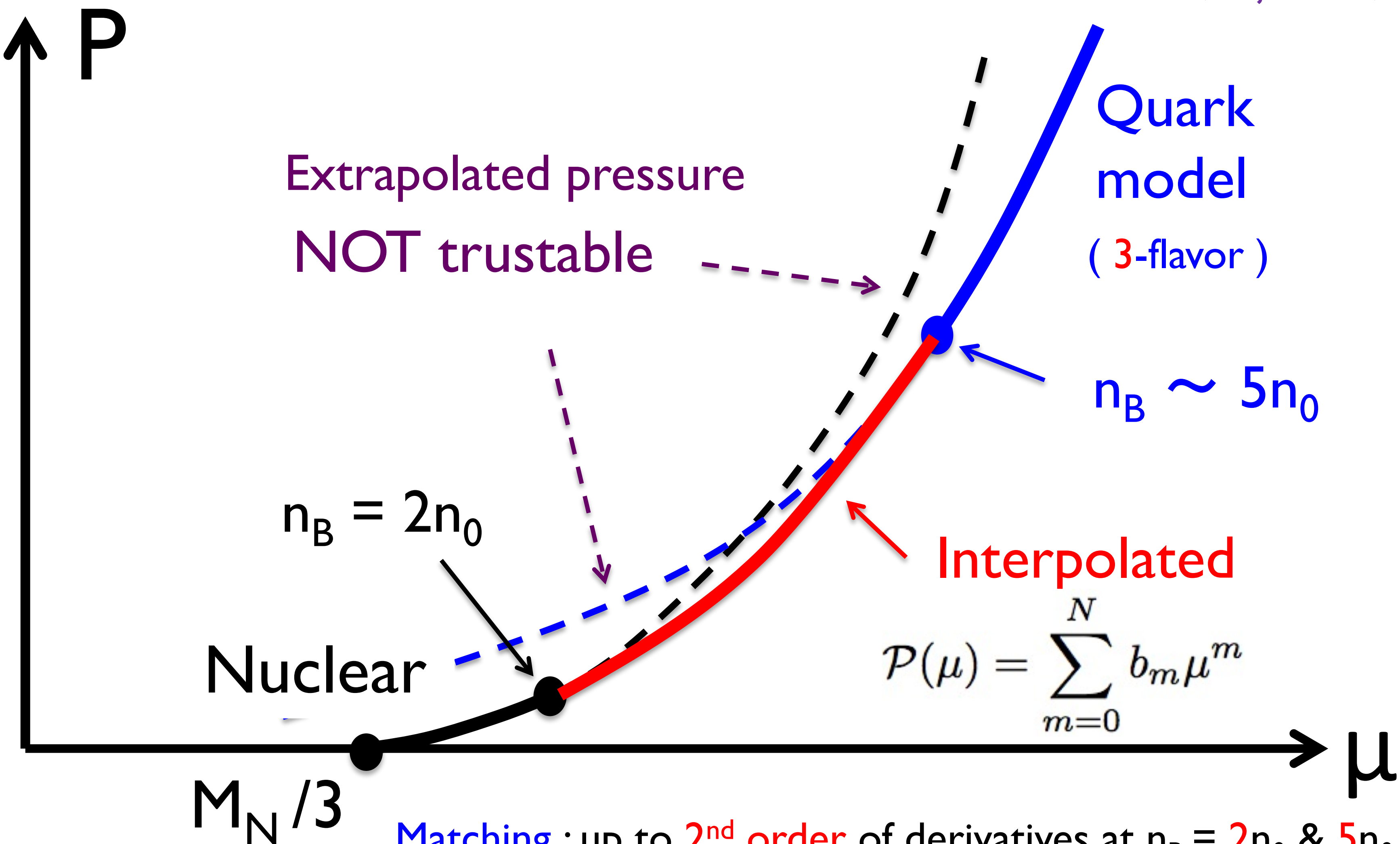
3-window modeling : P vs μ

Masuda+2012, Kojo+2014, ...



3-window modeling : P vs μ

Masuda+2012, Kojo+2014, ...



Matching : up to 2nd order of derivatives at $n_B = 2n_0$ & $5n_0$

(if you wish, put a small kink for weak 1st order P.T.)

Caveats

Matching or interpolation of 2-EoSs look innocent,
but actually it is NOT a trivial task at all.

(especially when underlying microphysics are different)

EoS must be
- thermodynamically consistent
- causal ($dP/d\varepsilon|_s = c_s^2 < c^2$)

Otherwise numerical simulations easily stop by instability.

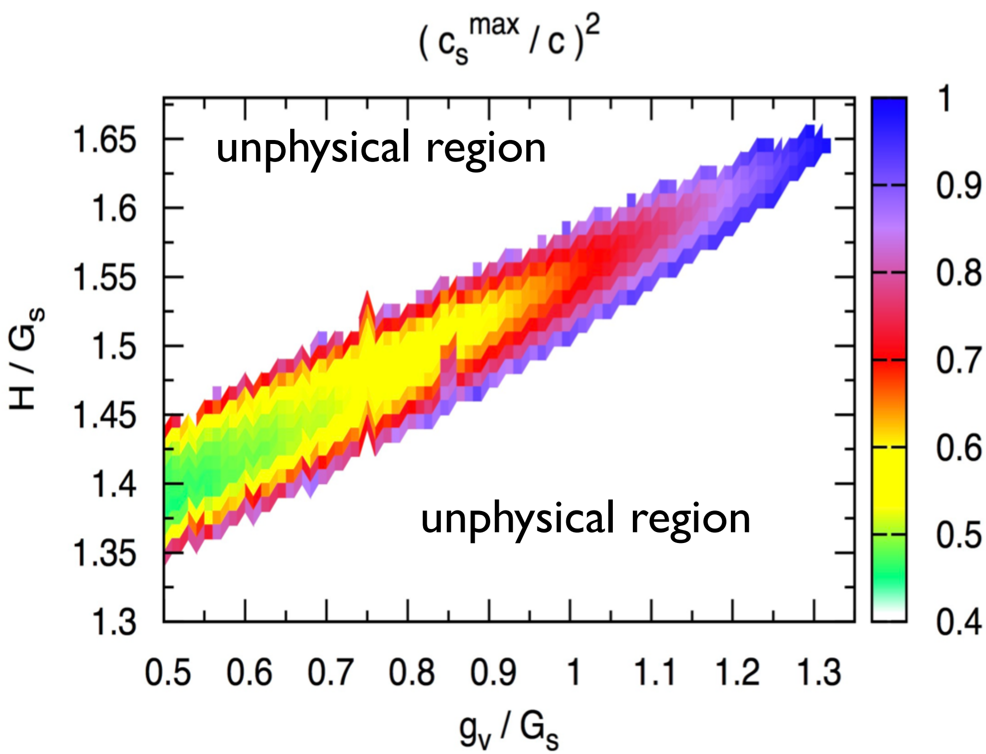
(In fact some EoS tables in the website are not usable...)

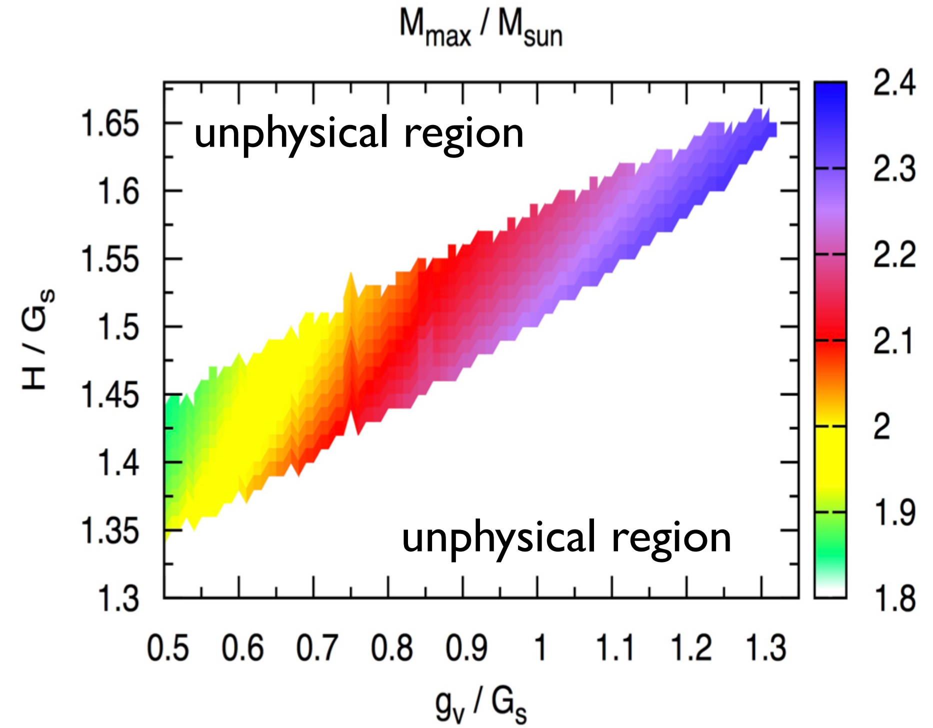
In addition, we have nuclear & astrophysical constraints :

Stronger constraints for **softer**-**stiffer** EoS.

($\rightarrow$ more chances to select out the correct EoS)

Constraints -> quark model parameters





The allowed range of (gv, H) is constrained;

-> predictions for other domains, e.g. (Ye, T, ...)

(explanations for plots -> Lect.3)

Plan of lectures

1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

3, NS constraints on EoS : hints for **soft**-**stiff** EoS

4, The constraints on P(μ) curves

5, Order parameter & symmetry

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9, Other topics: warm EoS, beyond-MF, etc.

From $\varepsilon(n)$ to $\Omega(\mu)$ $(= -P(\mu))$ (at T=0)

astro. people prefer ↗ n QCD people prefer ↖ μ

energy density at a given number density

$$\varepsilon(n) : \quad d\varepsilon(n) = \mu \, dn \qquad \qquad \mu(n) = d\varepsilon(n)/dn$$

For QCD calculations, more common to work at fixed μ

change of variables : **Legendre transf.**

$$-P = \Omega(\mu) = \varepsilon - \mu n$$

$$\rightarrow d\Omega(\mu) = d\varepsilon - \mu \, dn - n \, d\mu = - n \, d\mu$$

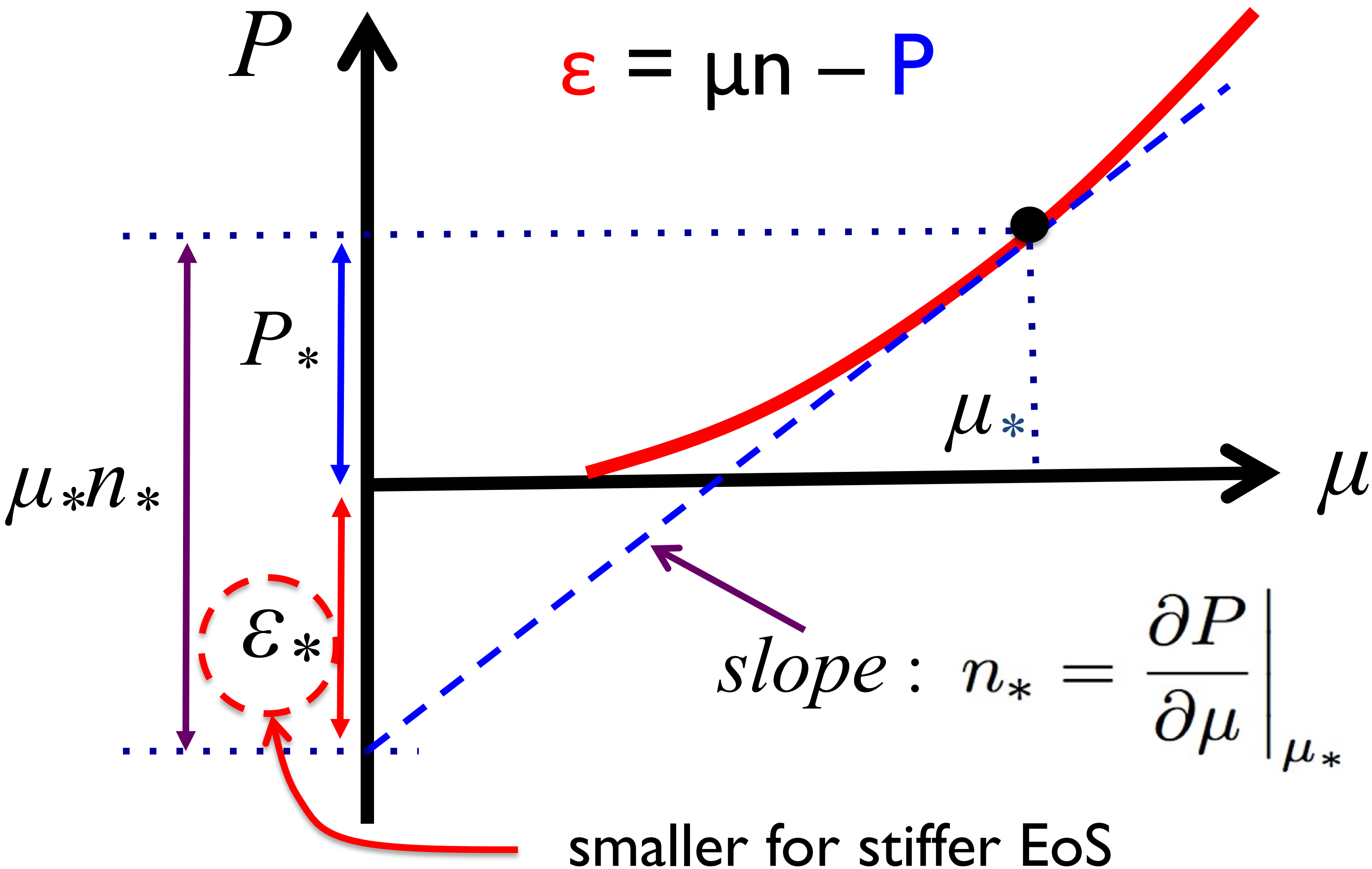
With the expression of $P(\mu)$ given ⇒

$$n(\mu) = dP/d\mu$$

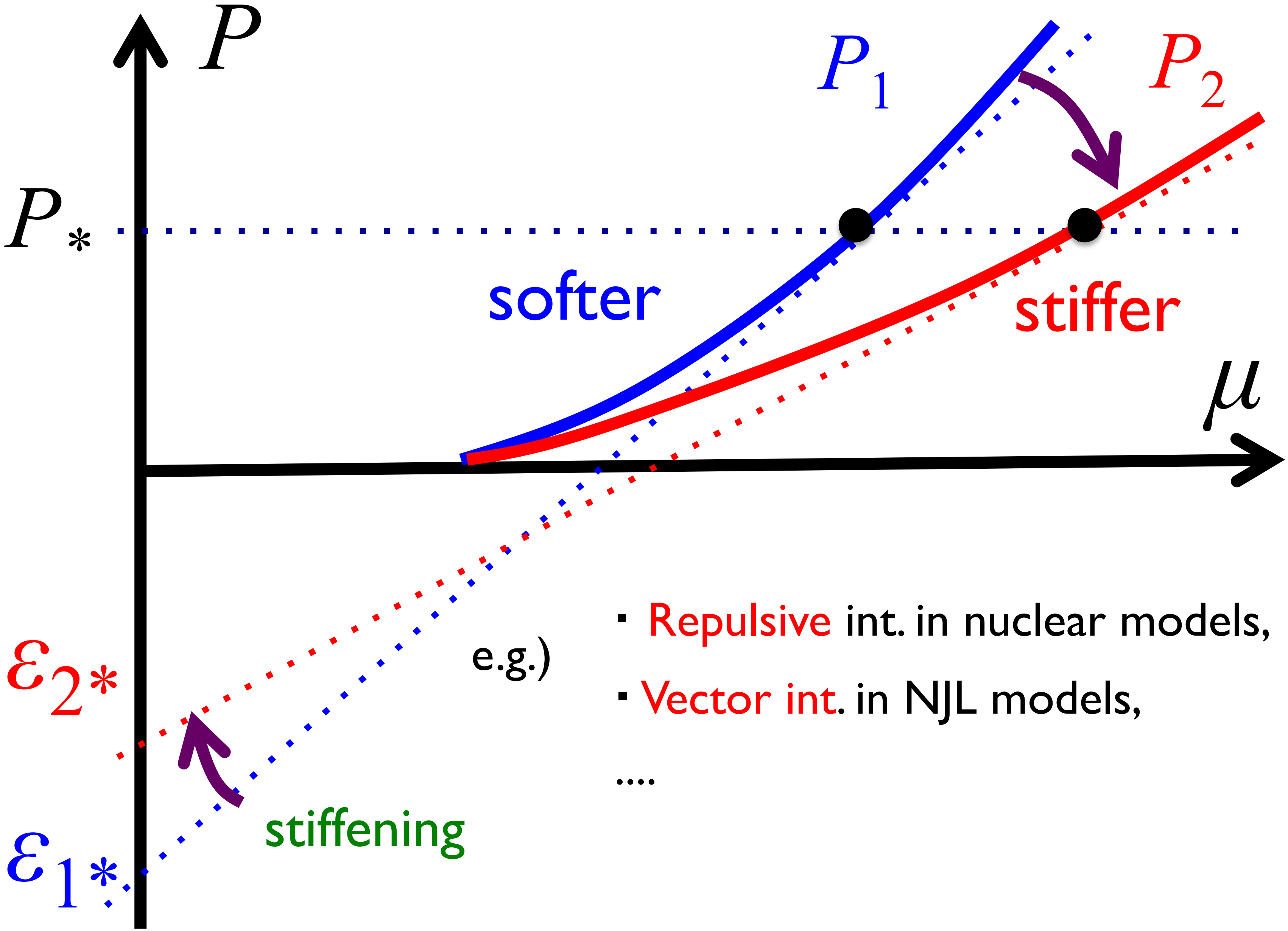
$$\varepsilon(\mu) = \mu \, n(\mu) - P(\mu)$$

↗ **all info** about EoS included

How stiff EoS looks like in P(μ) curves



Stiffening I : Rotation

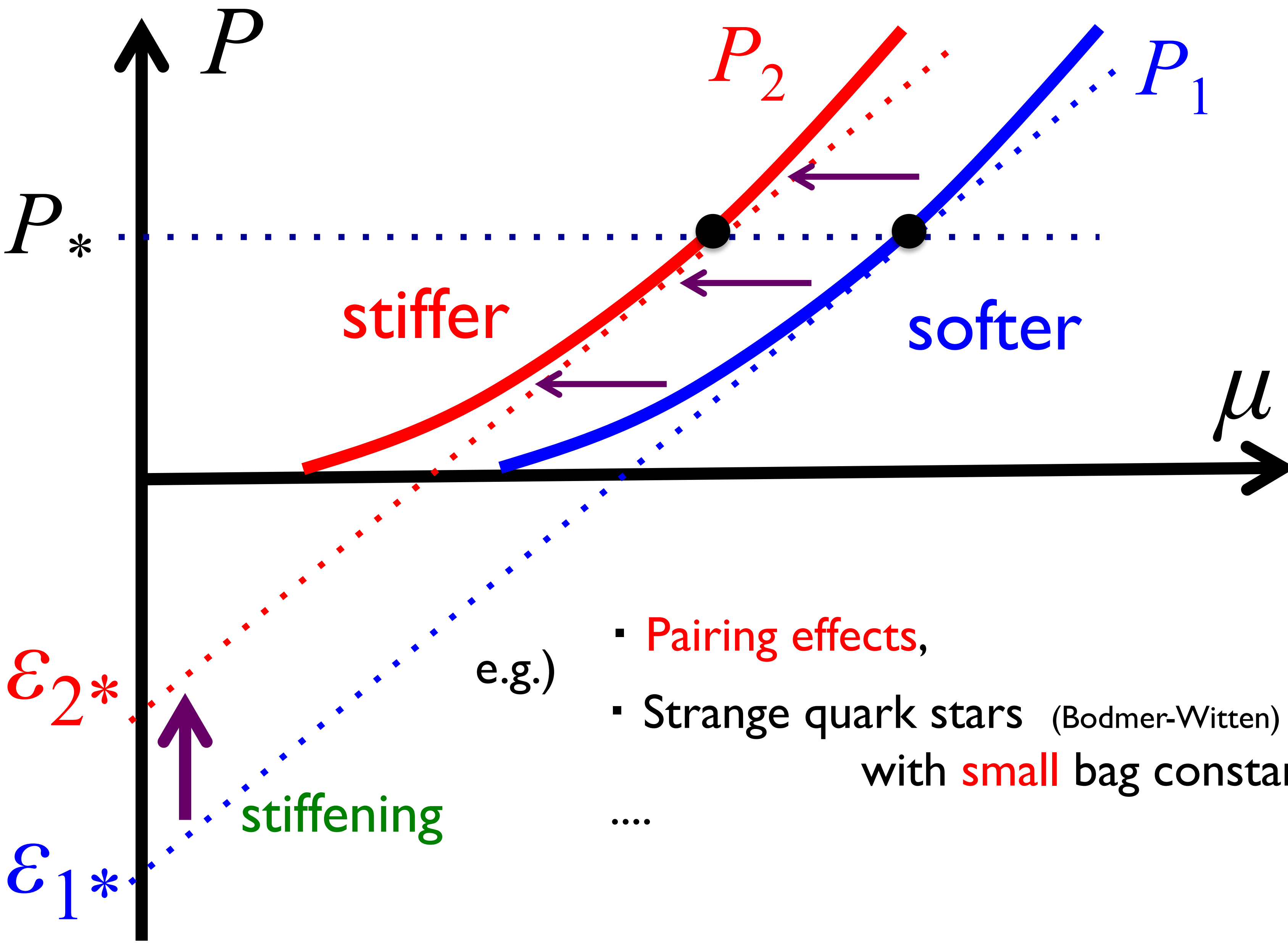


e.g.)

- Repulsive int. in nuclear models,
- Vector int. in NJL models,

....

Stiffening 2 : Parallel shift



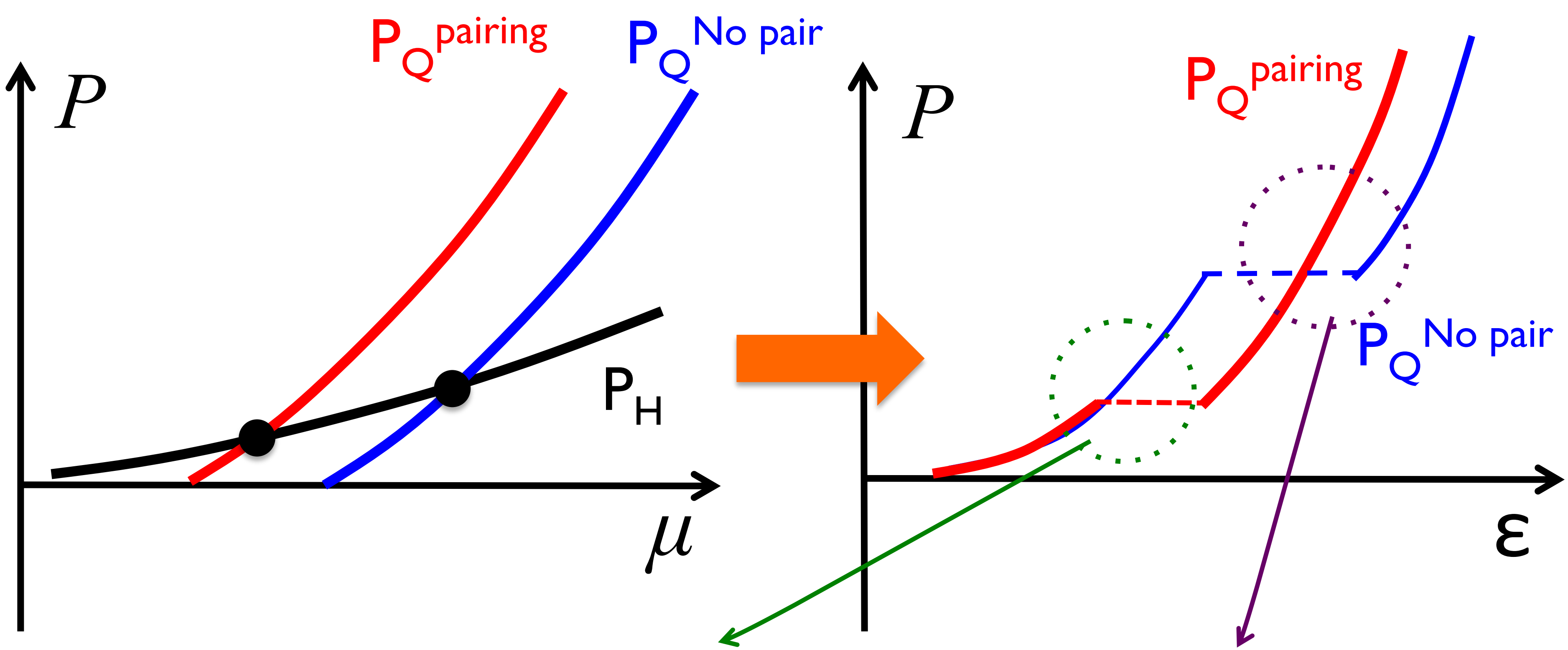
e.g.)

- Pairing effects,
- Strange quark stars (Bodmer-Witten) with small bag constant,

....

App.1 :"Pairing" can stiffen EoS

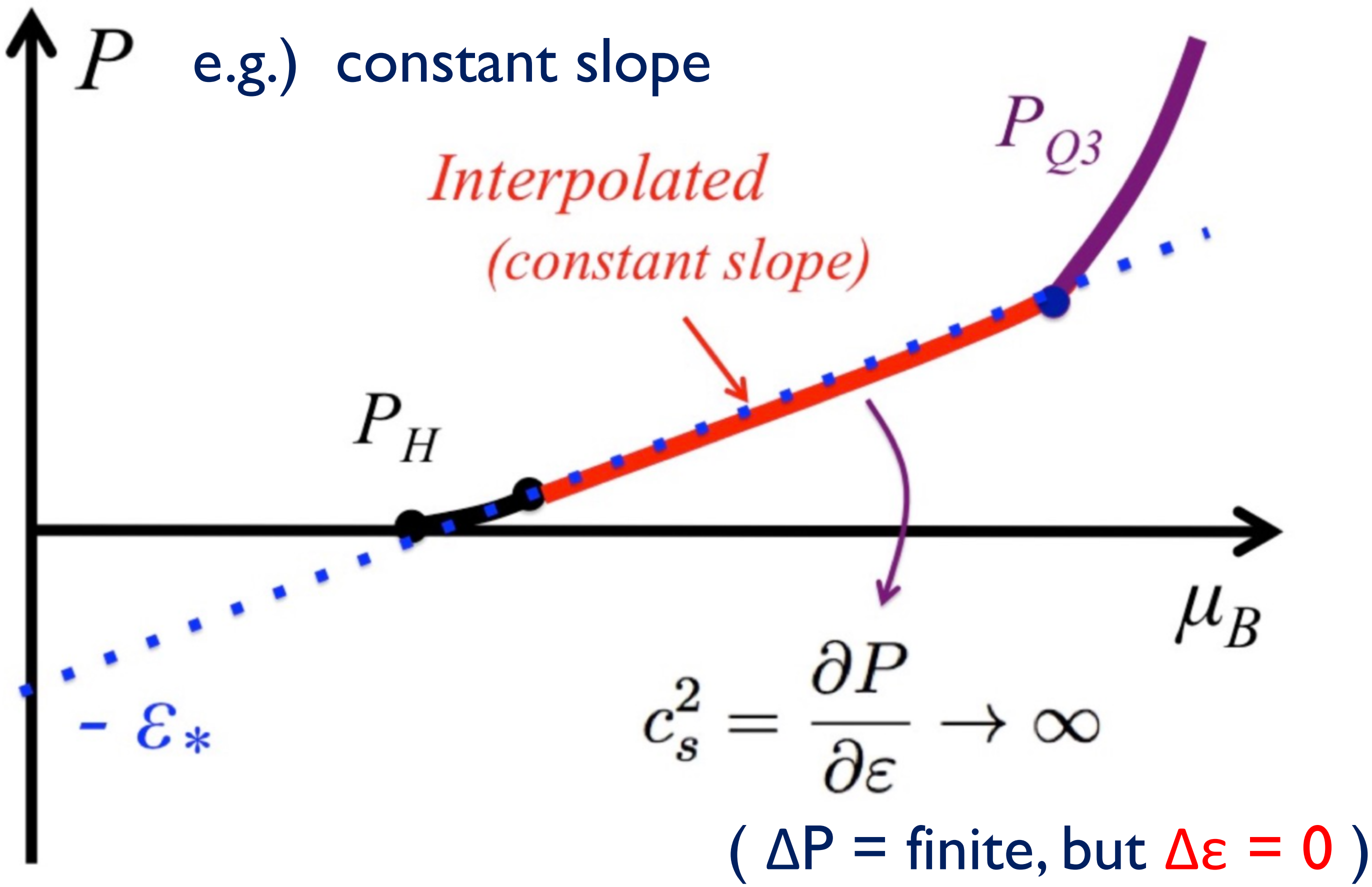
Do exotic phases always give softening? → Not necessarily



→ Softening at low n_B & stiffening at high n_B

App.2 : P(μ) must grow sufficiently fast

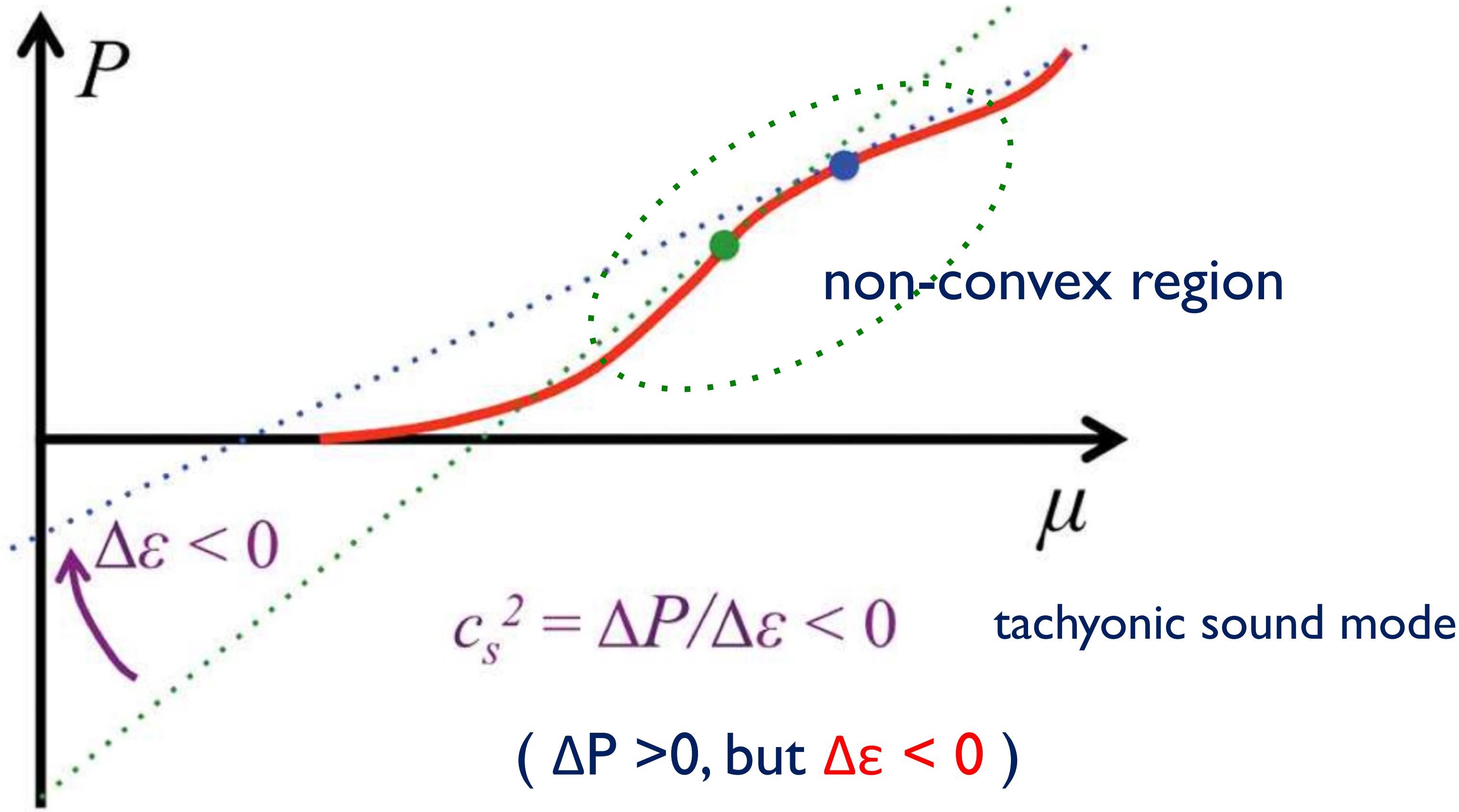
(otherwise the speed of sound becomes superluminal)



(more analyses → P(μ) should grow faster than μ^2)

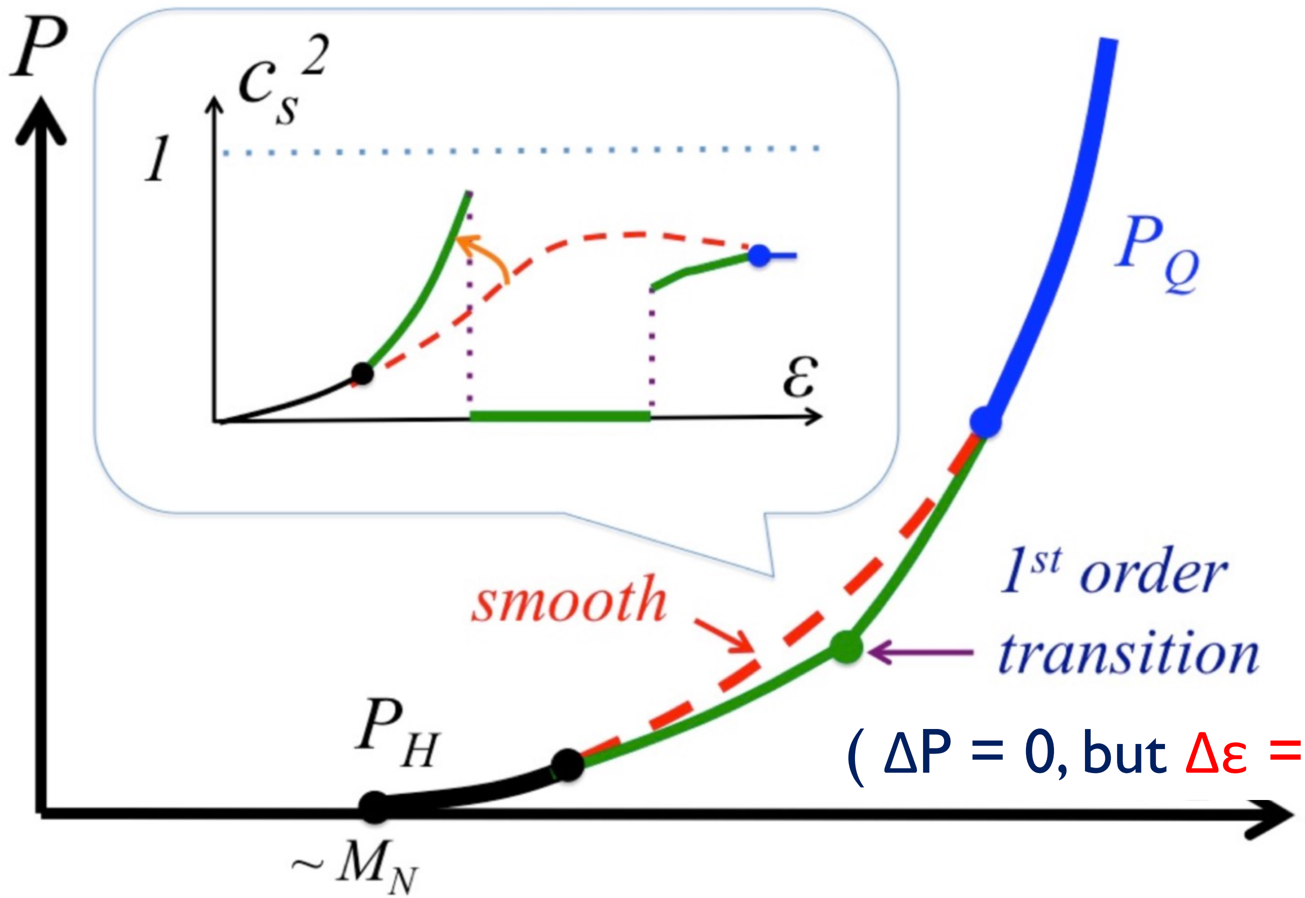
App.3 : P(μ) must NOT have inflection points

(or P(μ) must be convex)



(ΔP >0, but Δε < 0)

App.4 : 1st order P.T. & speed of sound



(ΔP = 0, but Δε = finite)

$c_s{}^2$ grows rapidly before P.T., then suddenly reduces to zero.

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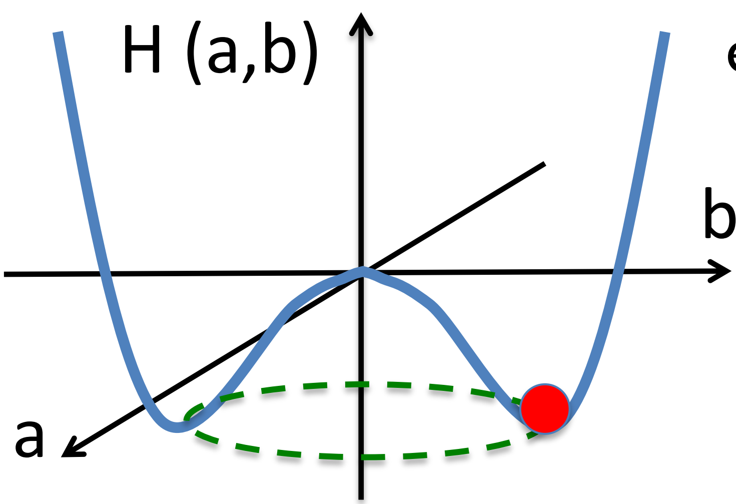
Symmetry & Order parameters I

Spontaneous Symmetry Breaking (**SSB**)

(Heisenberg, Landau, Nambu 60, Goldstone 61)

Sym. of **Hamiltonian** ≠ Sym. of **States**



e.g.) A ball in a wine bottle (**classical**)

- Hamiltonian:
  rotational symmetric
- Ground state:
  NOT rotational symmetric

Symmetry & Order parameters 2 (**Quantum**)

Symmetry of H → symmetry generator Q

$$e^{i\theta\hat{Q}}\,\hat{H}\,e^{-i\theta\hat{Q}} = \hat{H}$$ "rotation" invariant

Suppose the G.S. is $|0\rangle$. "Rotate" the G.S. as

$$|\theta\rangle = e^{-i\theta\hat{Q}}|0\rangle$$

The "rotated" state has the **same energy** as $|0\rangle$.

$$|\theta\rangle = e^{i\delta}|0\rangle$$ **trivial** ($|0\rangle$ & $|\theta\rangle$ are the **same** state)

$$|\theta\rangle \neq e^{i\delta}|0\rangle$$ **SSB** ($|0\rangle$ & $|\theta\rangle$ can be G.S., but $|0\rangle$ was **chosen**)

How to check? We look for **order parameter** :

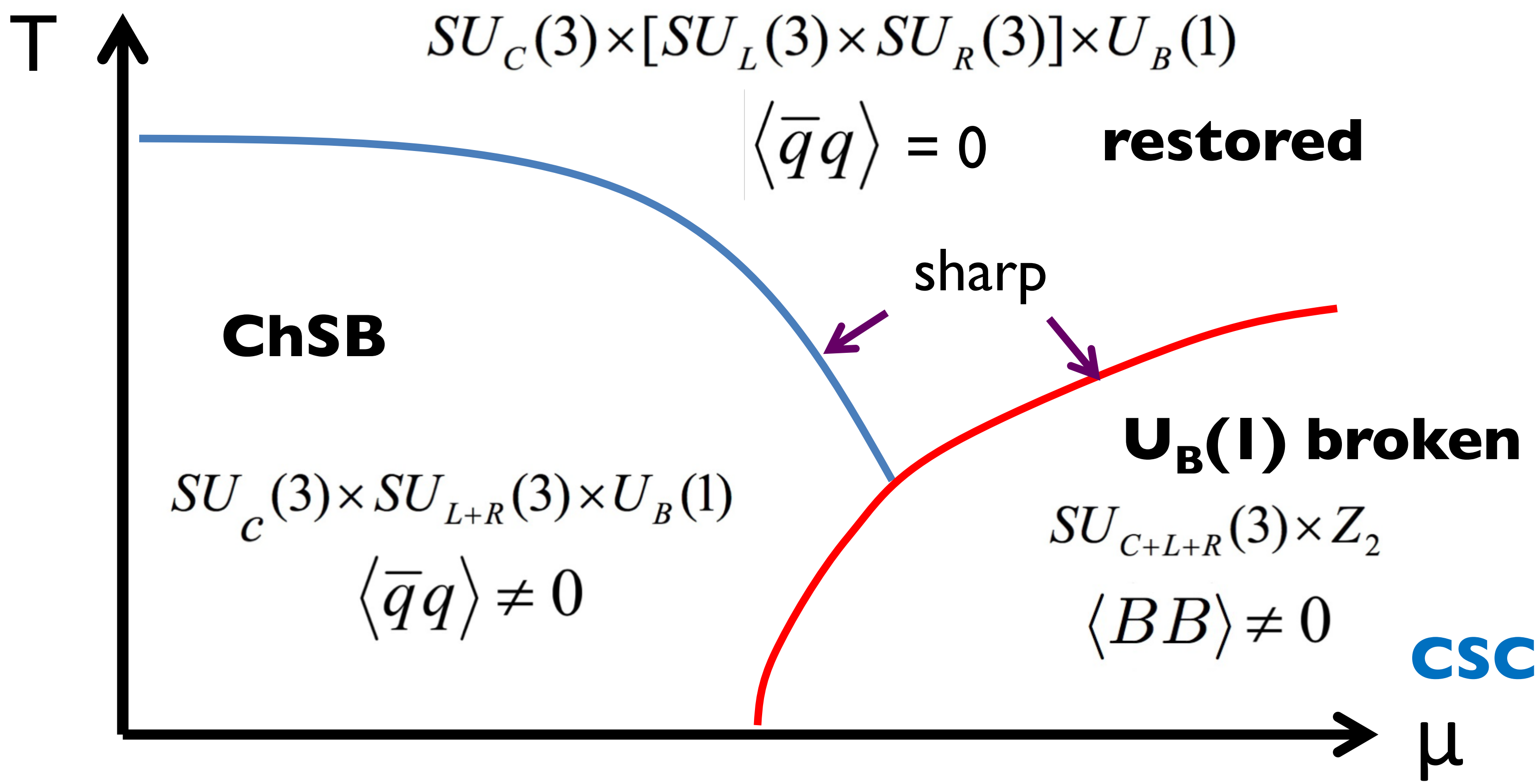
$$\langle\theta|\hat{O}|\theta\rangle - \langle 0|\hat{O}|0\rangle = i\theta\underline{\langle 0|\delta_Q\hat{O}|0\rangle} + \cdots$$

If $\langle\delta O\rangle$ is nonzero, we can say $|0\rangle$ & $|\theta\rangle$ are different.

Symmetry & Order parameters 3

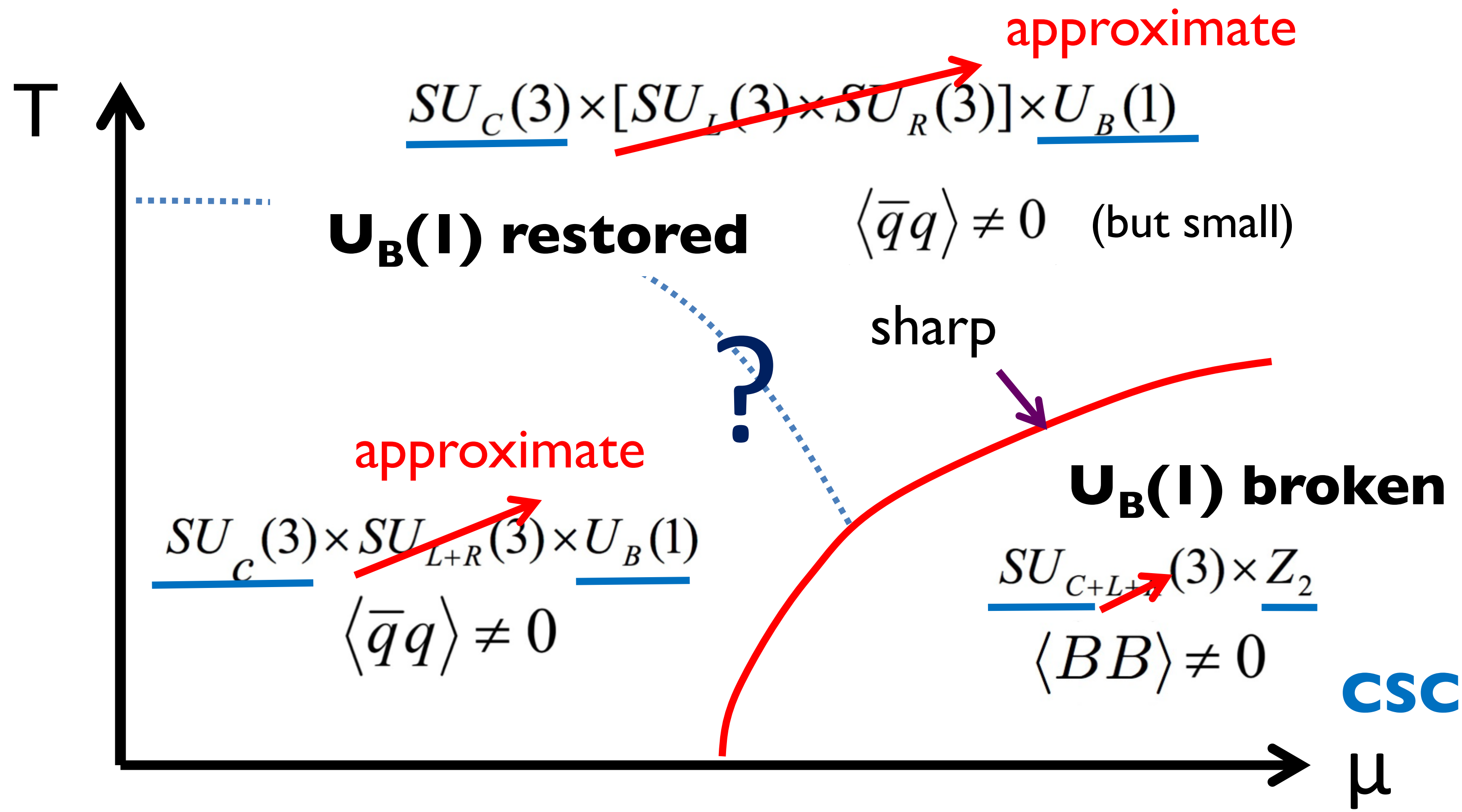
Sym. **unambiguously** distinguishes the phases

If chiral sym. of QCD **were exact**...



Symmetry & Order parameters 4

In reality: $m_u \neq m_d \neq m_s$



Symmetry & Order parameters 5

Because of explicit sym. breaking, the possible
chiral phase transitions can be **any**. (crossover, 1st, 2nd, ...)

Also, rigorous order parameters for confinement are **NOT** known.
(except for pure YM)

Hadron-quark P.T. is difficult to define in a formal way.
(even Confinement-Higgs is difficult to distinguish; Fradkin-Shenkar 79)

Then what can we do?

1, If we are lucky, we can find **abrupt changes**
(e.g. 1st order P.T. or radical crossover)

2, If not, need to examine the validity of **effective d.o.f.**
(if proper d.o.f are used, calculations converge quickly)

Dynamics must be discussed

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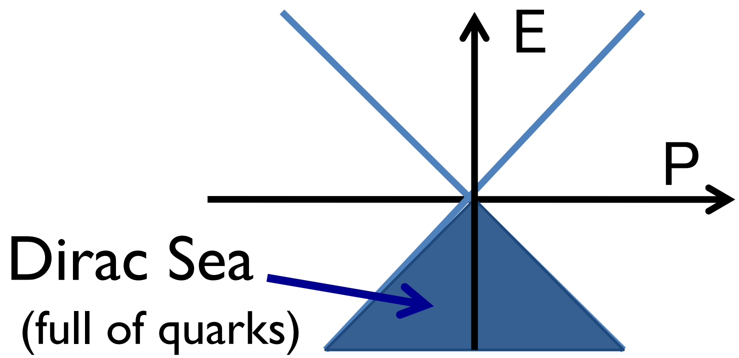
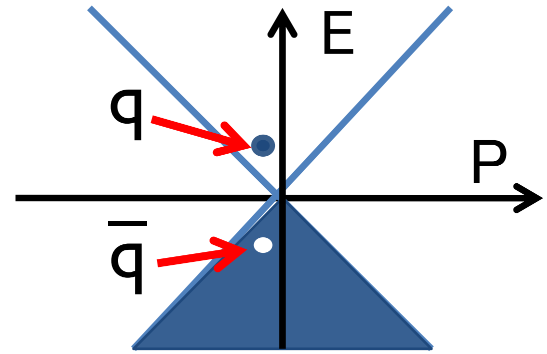
Generation of the chiral condensate





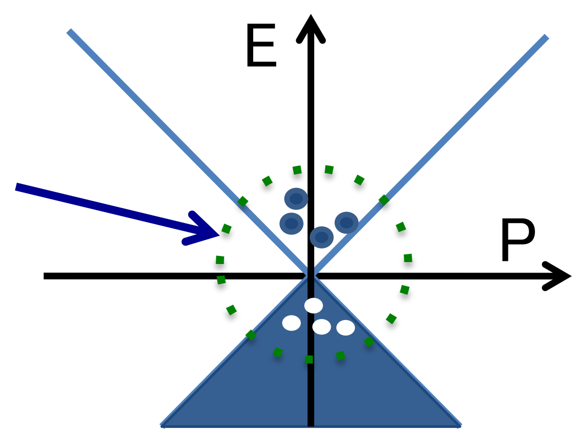
Usually : $E_0 < E_0 + E_q + E_{\bar{q}}$

But with (strong) attractive interaction :

$E_0 > E_0 + E_{int} + E_q + E_{\bar{q}}$

Then **macroscopic** number of $q\bar{q}$ are kept excited :

" Condensation "

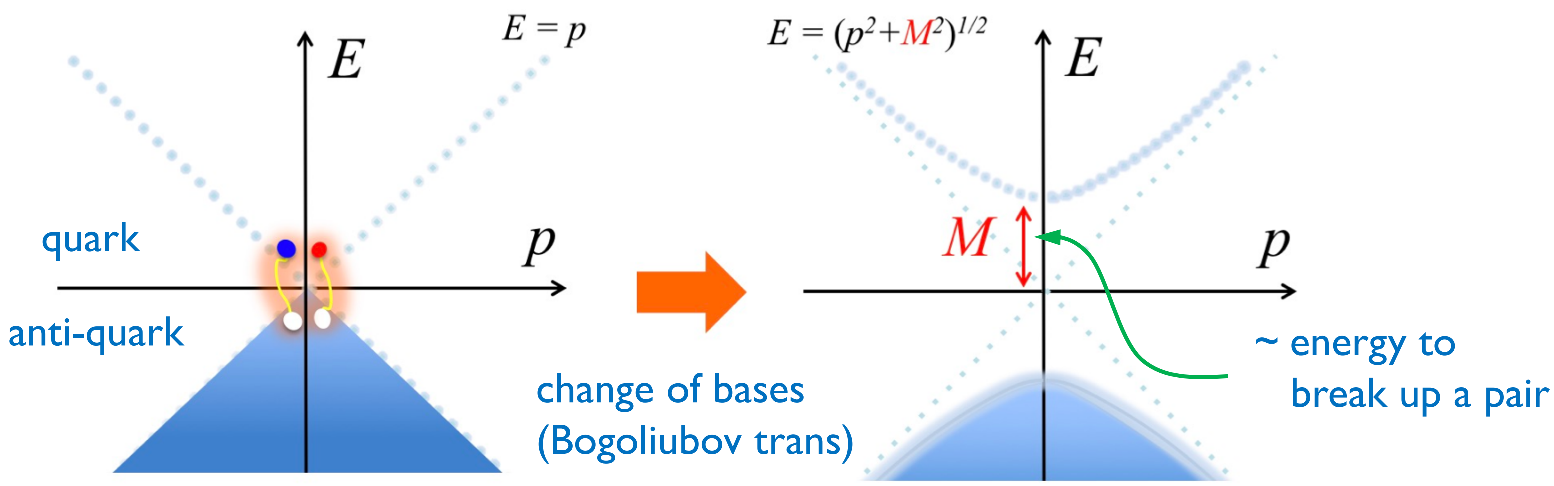


quantified by

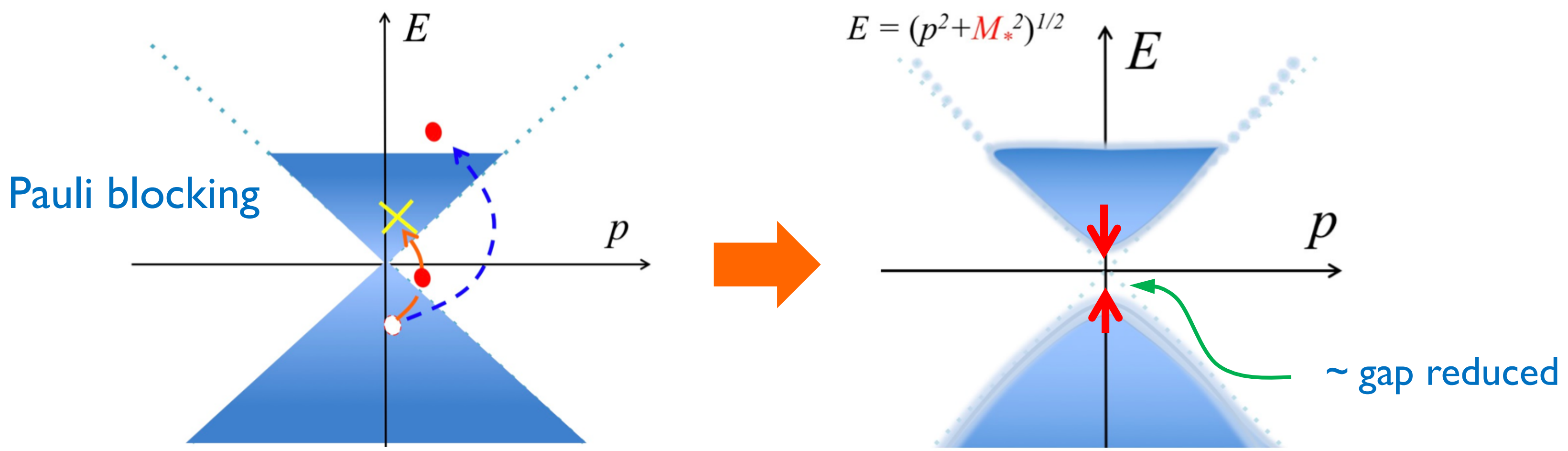
Chiral Condensate : $\langle 0|\bar{q}q|0\rangle$

Chiral sym. breaking & restoration

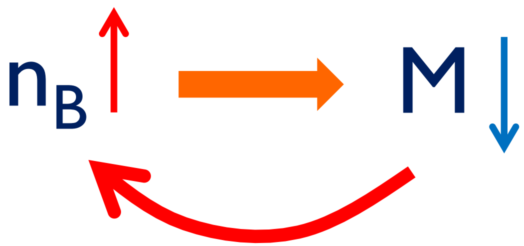
vac



finite density



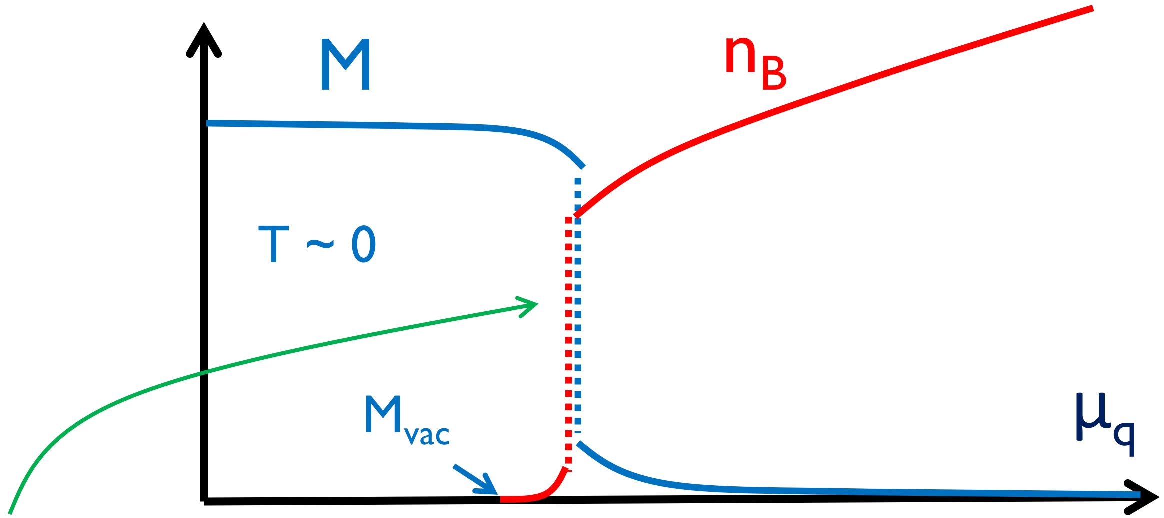
1^{st} order chiral transition (typical quark **models**)

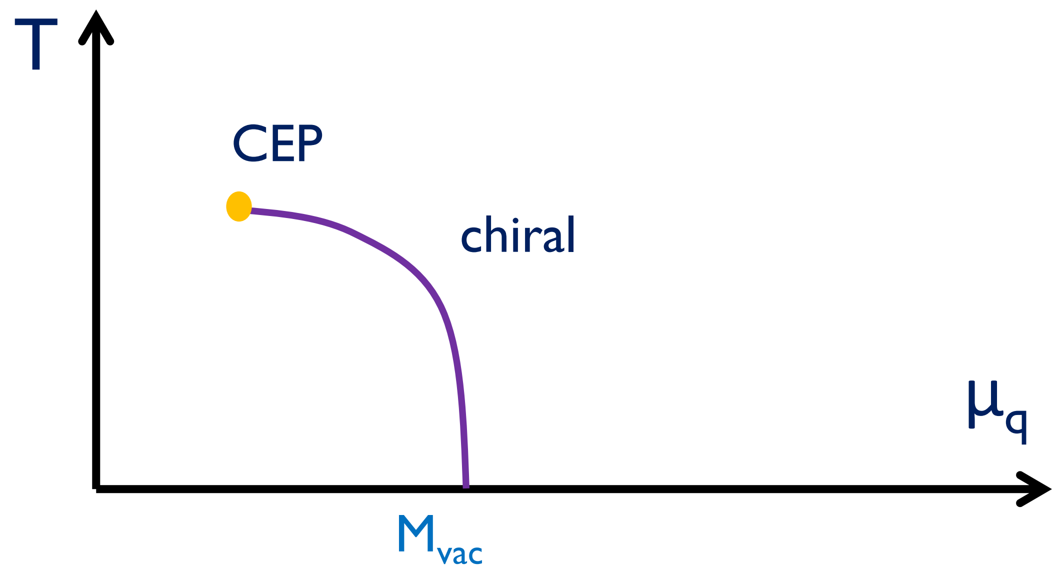


"feedback"

more phase space

→ radical changes in n_B & M

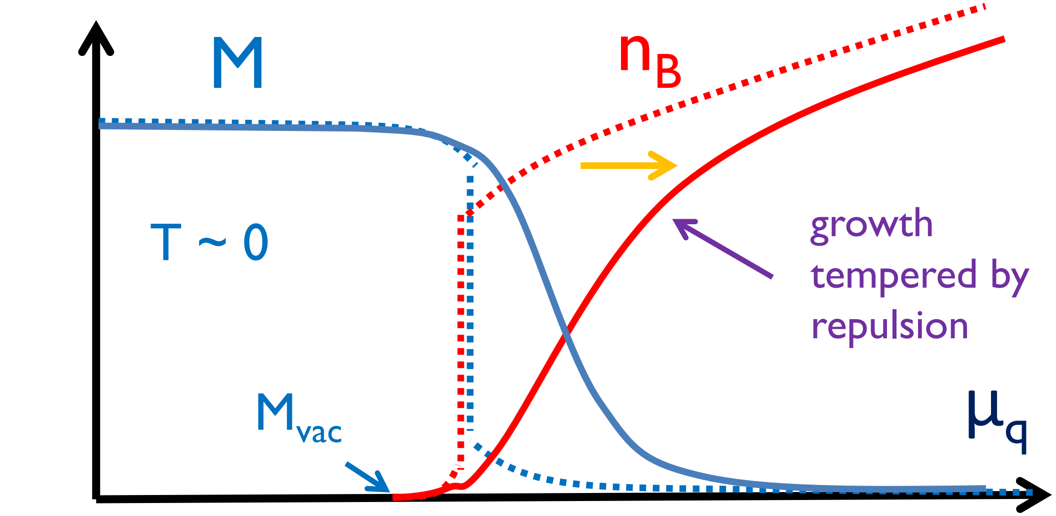




Braking density evolution: I st → crossover

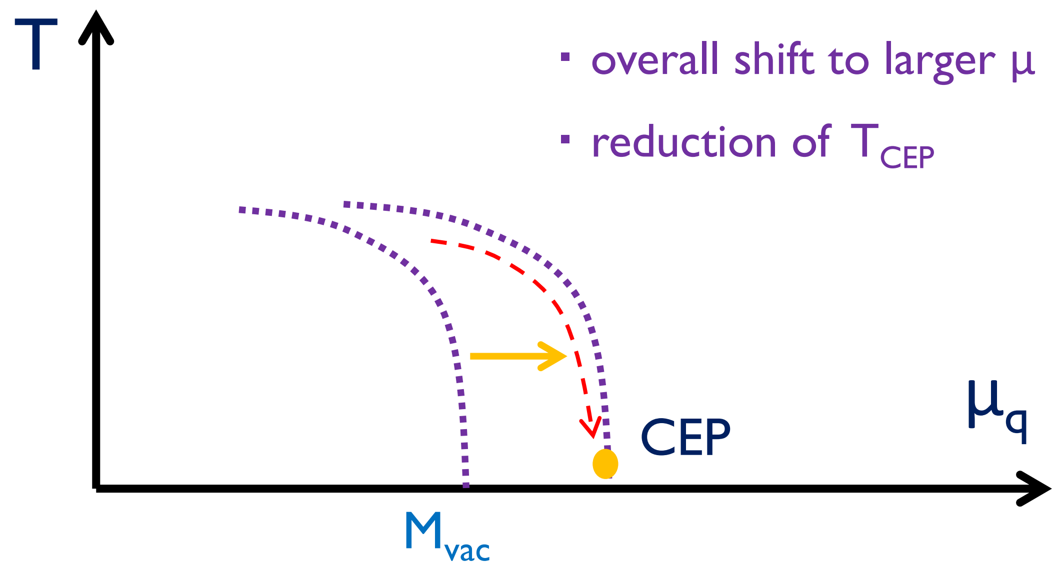
Now add
density-density repulsion

$$\Delta H \sim g_V\,(n_B)^2$$



braking the evolution of n_B

→ milder changes in M



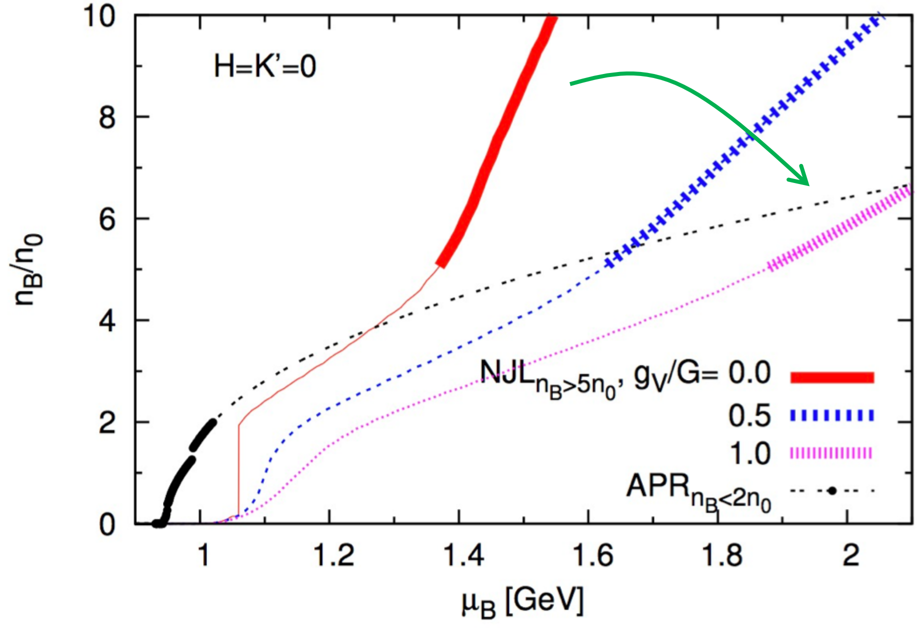
- overall shift to larger μ
- reduction of T_{CEP}

Details of int. are crucial

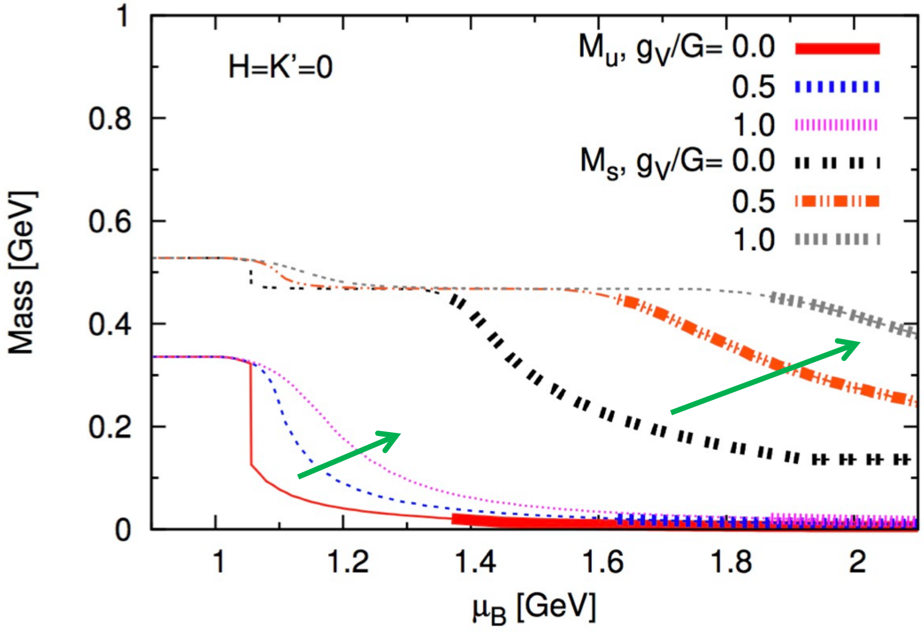
Some quark model results

$$\Delta H \sim g_V (n_B)^2$$

number density



Dirac mass



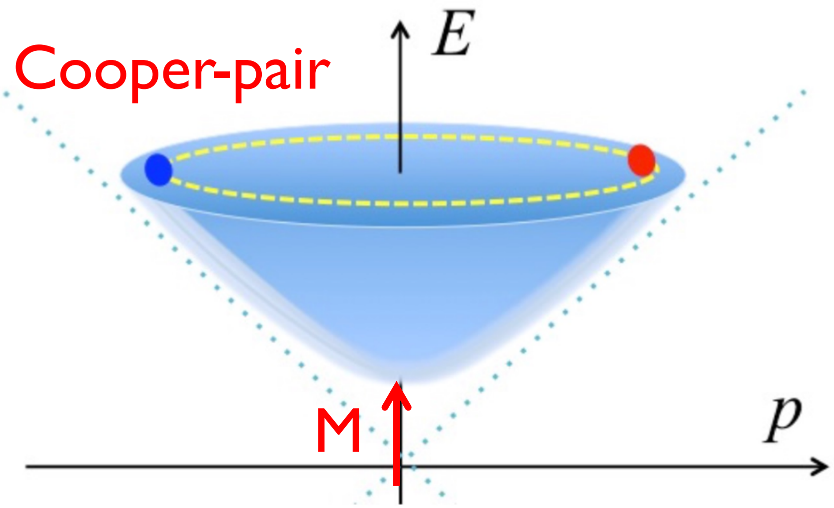
repulsion tempers the growth of n_B

→ milder chiral phase transition (1st -> crossover)

Di-fermion pairing

As density increases, another kind of condensation takes place:

(particle-particle & hole-hole pairing)



di-baryon or di-quark

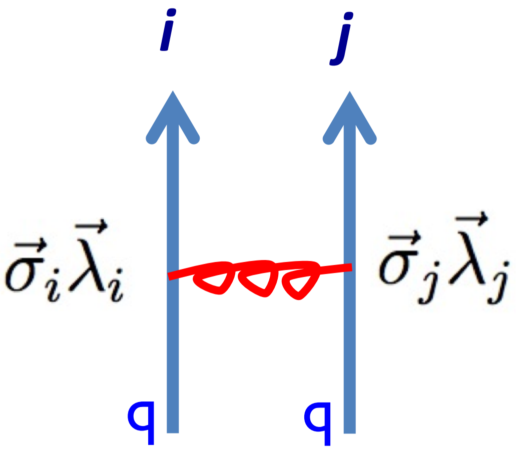
Key elements for condensations

- Fermi surface : large phase space for **gapless** excitations (many pairs can be formed)
- attractive interactions (small int. is already enough)

Diquark pairing : quantum number

[Bailin-Love, Alford, Rajagopal, Wilzcek, Schafer, ...]

qq-pairing



less (more) color charges

color: $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$

So we consider **color anti-symmetric** channel.

Next consider a color-magnetic interaction.

$$\sim \alpha_s \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{M_i M_j} \delta(\vec{r}_{ij})$$ (at short distance)

Most attractive for **spin-singlet** & **S-wave**

& fermion statistics -> **flavor anti-symmetric**

qq-"condensate" color-antisym

$$\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle = \epsilon^{\alpha\beta A} \epsilon_{ijB} \Delta_B^A$$

scalar 0+ flavor-antisym

CFL & 2SC pairing

$$\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle = \epsilon^{\alpha\beta A} \epsilon_{ijB} \Delta_B^A$$

many strange quarks

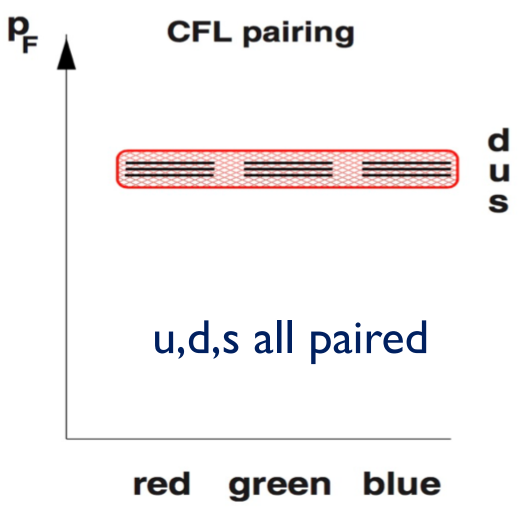
little strange quarks

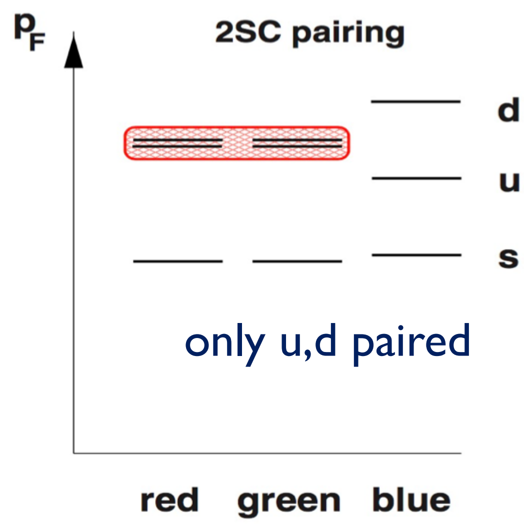
Color-**F**lavor-**L**ocked

$$\Delta_B^A = \Delta_{\rm CFL} \underline{\delta_B^A}$$

2SC

$$\Delta_B^A = \Delta_{\rm 2SC} \underline{\delta_3^A \delta_B^3}$$



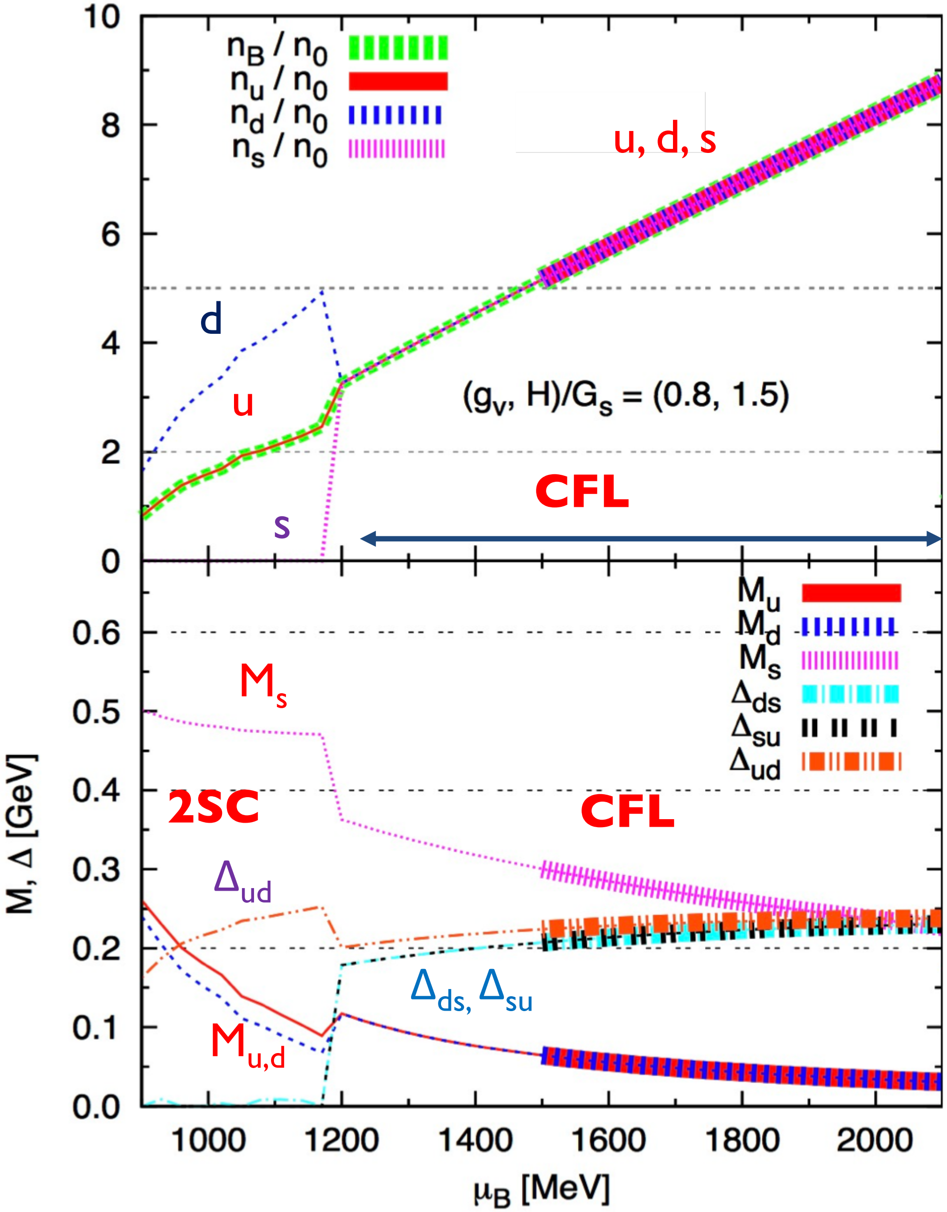


All quarks & gluons are gapped

gapped and gapless quarks & gluons

Some quark model results

[β-equilibrium]



($n_B < 5n_0$ is not trustable)

Remark:

- repulsive forces included
- chiral & diquark condensates coexist
- pairing favors $n_u = n_d = n_s$

many strange quarks!

[more will be explained in Lect. 3]

Summary of lecture 2

1, Interpolation procedure looks innocent,
but many constraints must be taken care.
(thermo., causality, astro & nuclear)

2, How to graphically extract EoS info from P(μ)

3, The nature of chiral restoration strongly depends on
the presence of repulsive interactions;
should be examined when we build models

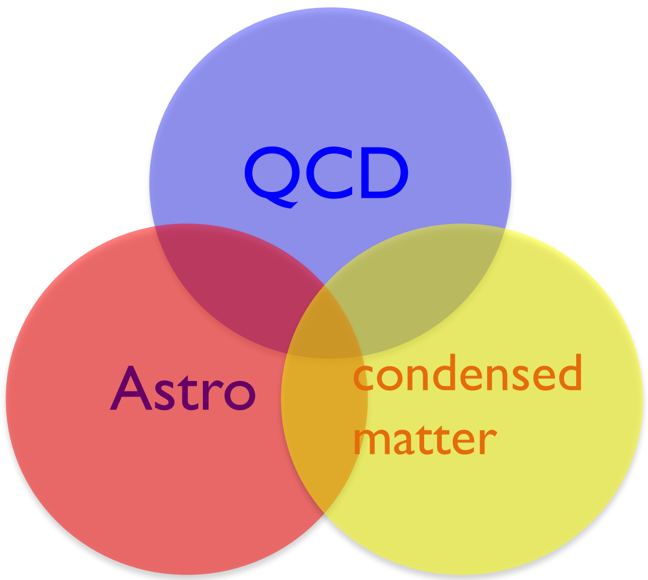
4, Color-superconductivity; theoretically well-motivated

we should include, or should explain why it is not generated...

Lecture 3 : **A quark model & impacts on observables**

Three-window approach to (cold) dense matter: Lect. 3

Toru Kojo (CCNU, Wuhan)



Quark-Hadron continuity (some history)

- 1, Percolation picture   Baym-Chin 1978; Satz-Karsch 1979,...

- 2, In the context of color-superconductivity (CSC)   Schafer-Wilczek 1998

symmetry: hadron super fluidity ~ color-flavor-locked (CFL) phases

same order parameters : $\langle BB \rangle \sim \langle (qqq)^2 \rangle$

color singlet, but break $U(1)_B$; chiral sym. is also broken

confinement-Higgs complementarity   Fradkin-Shenkar 1979

dynamics: the interplay between chiral & diquark

proposal of double CEP   Kitazawa+ 2002; Hatsuda+2006; Zhang+ 2009, ...

- 3, Inferred from the NS constraints (for $2n_0 - 5n_0$)   Masuda+2012, Kojo+2014, ...

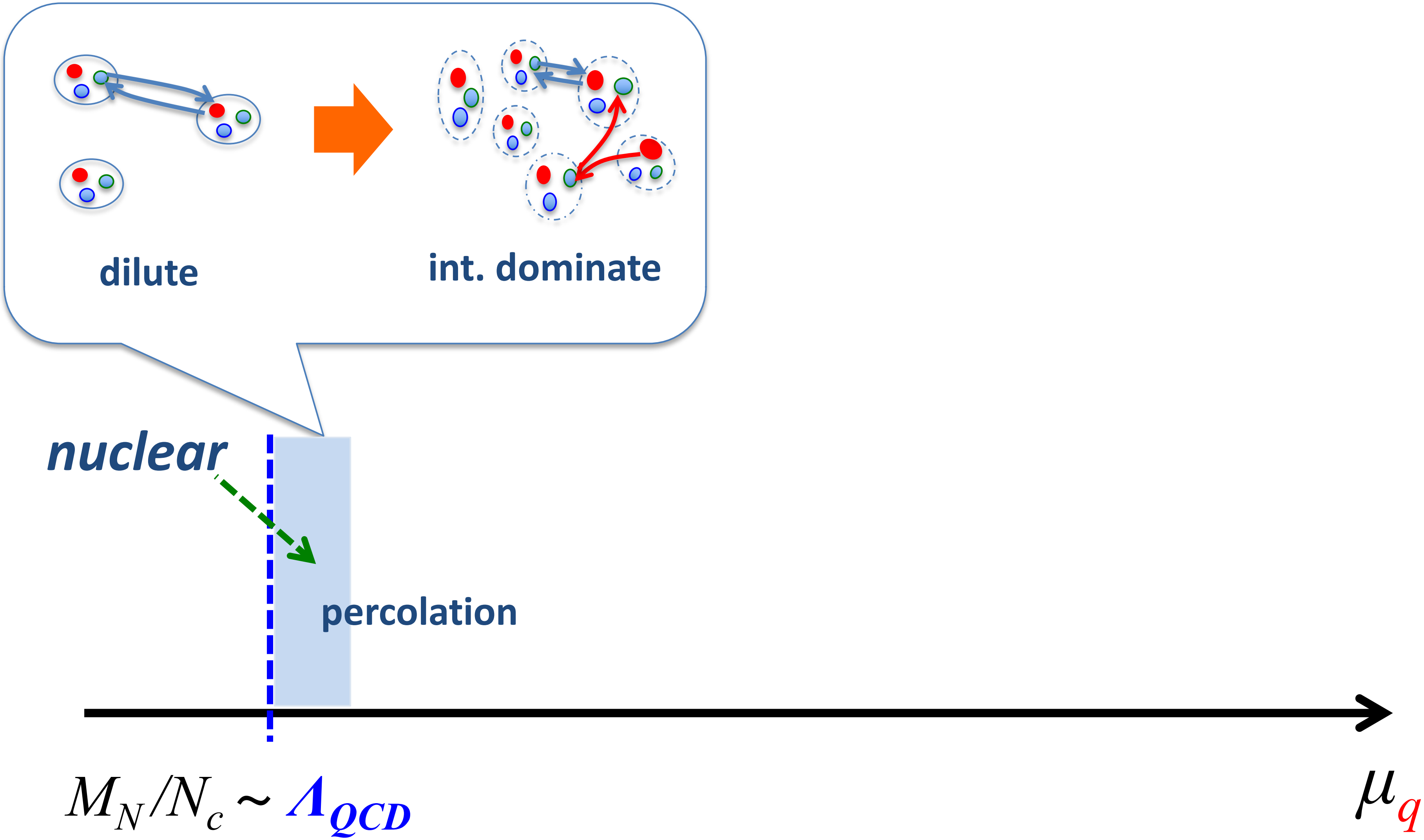
soft-stiff EoS & causality $\longrightarrow$ **crossover** or **weak** 1st order

McLerran-Pisarski's picture

[McLerran-Pisarski '07]

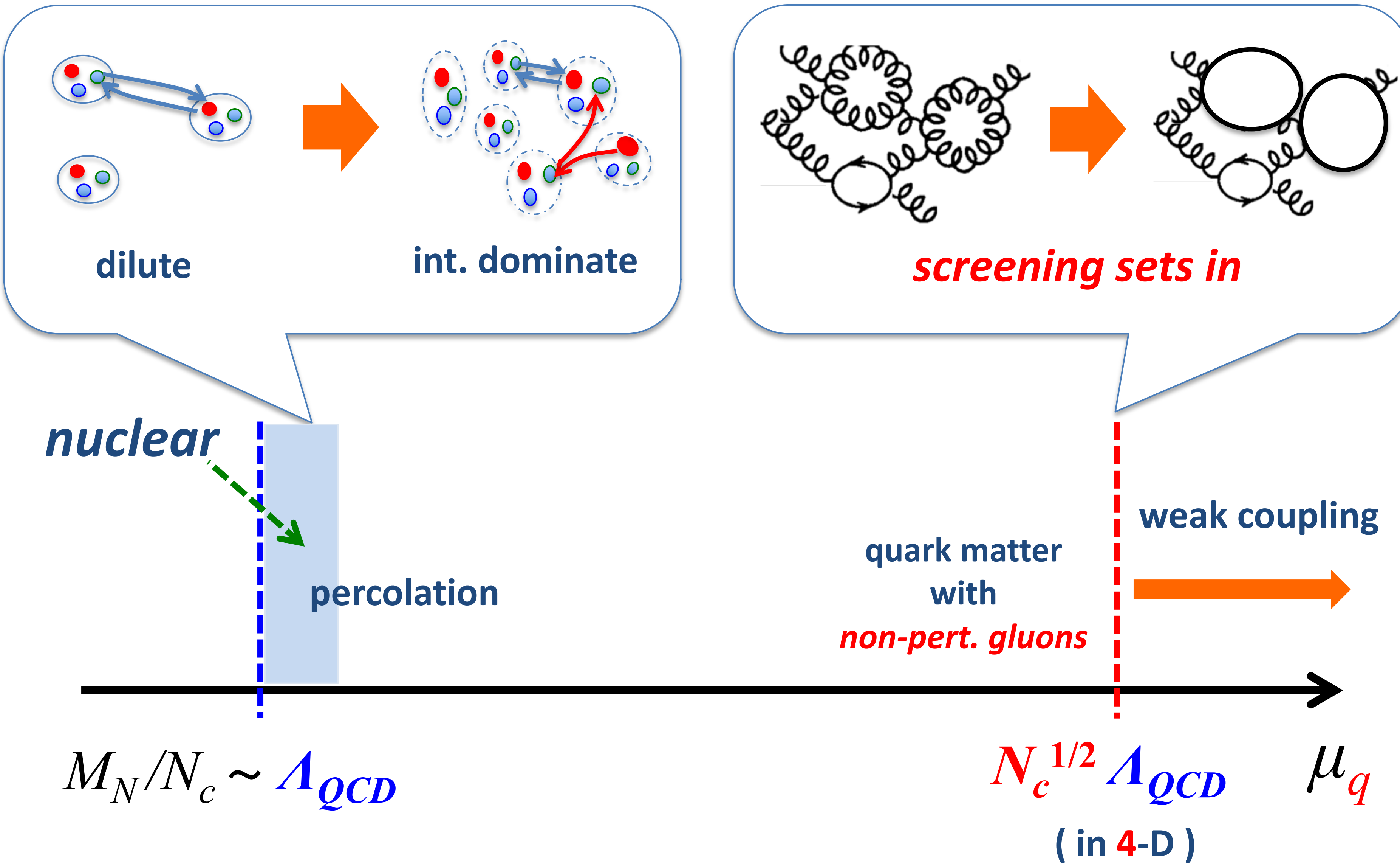
McLerran-Pisarski's picture

[McLerran-Pisarski '07]



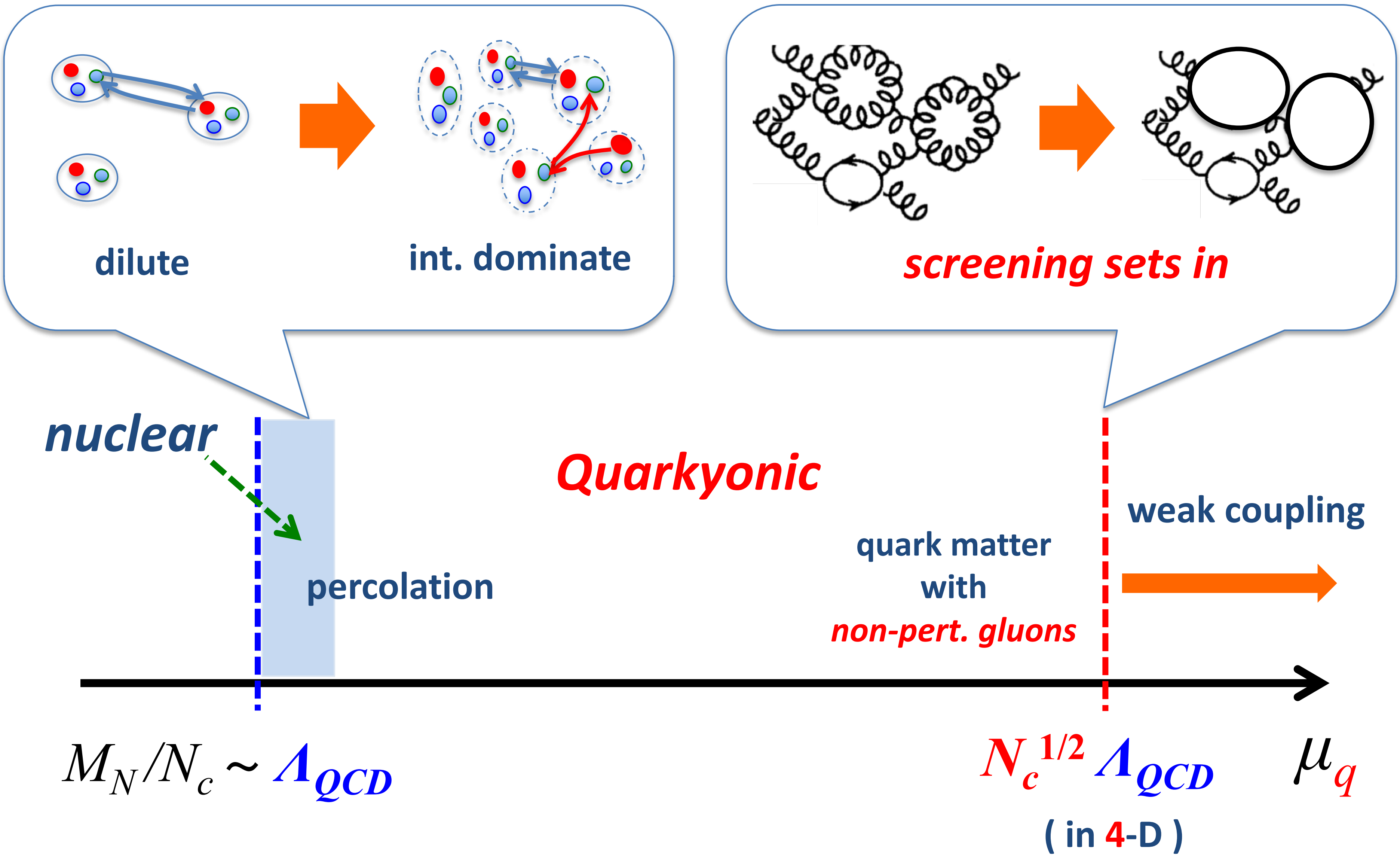
McLerran-Pisarski's picture

[McLerran-Pisarski '07]



McLerran-Pisarski's picture

[McLerran-Pisarski '07]



Several branches

- **Confined, *but chiral symmetric* matter** (many papers ...)
  - **have been *challenged* by many model calculations** [Glozman et al. 2007, ...]

- **Confined, *inhomogeneous* chiral *SSB*** (still ongoing ...)

  (*chiral sym. broken only locally*)
  - **Skyrme crystals, ...**
  - **Chiral density wave (1-D periodic structure)** [Carignano-Nickel-Bubbala]
  - **Quarkyonic Chiral Spirals**
  - **Interweaving Chiral Spirals**

  } [TK-Hidaka-Fukushima -McLerran-Pisarski-Tsvelik 09-11]

- **Reinterpretation of *Hadron-Quark Continuity***
  - **Original proposal : Schafer-Wilczek**
  - **CSC in quarkyonic matter & NS context** [Fukushima-TK '15]

Plan of lectures

1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

3, NS constraints on EoS : hints for **soft-stiff** EoS

4, Crossover scenario: chiral restorations, etc.

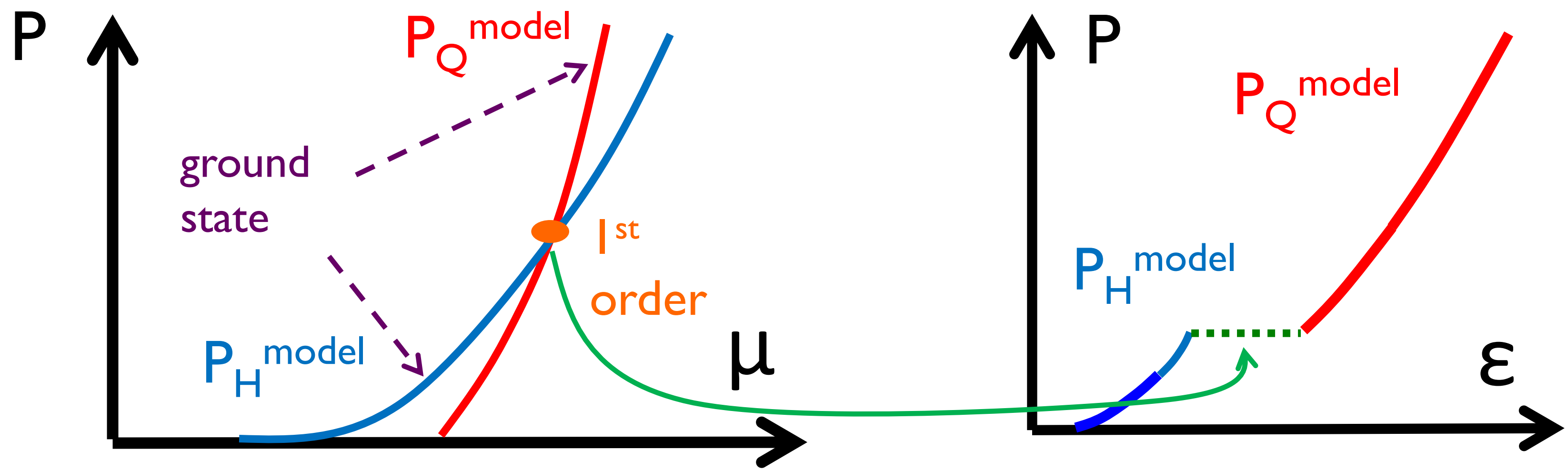
5, A quark model : delineating the properties of matter

6, The astrophysical results from EoS QHC18

7-, Other topics: warm EoS, beyond-MF, etc.

0, quark matter can be stiff
1, chiral restoration, color-super
2, expand quark-hadron continuity picture,
  percolation model, quarkyonic matter,
  Wilczek-Schafer, interplay b.t.w chiral & diquark,
  phases separated by symmetry
3, P vs mu : graphical rep.
4, NN, NY interaction: universal repulsion & strangeness?
5, a schematic quark model
6, astrophysical outputs

Traditional hybrid construction



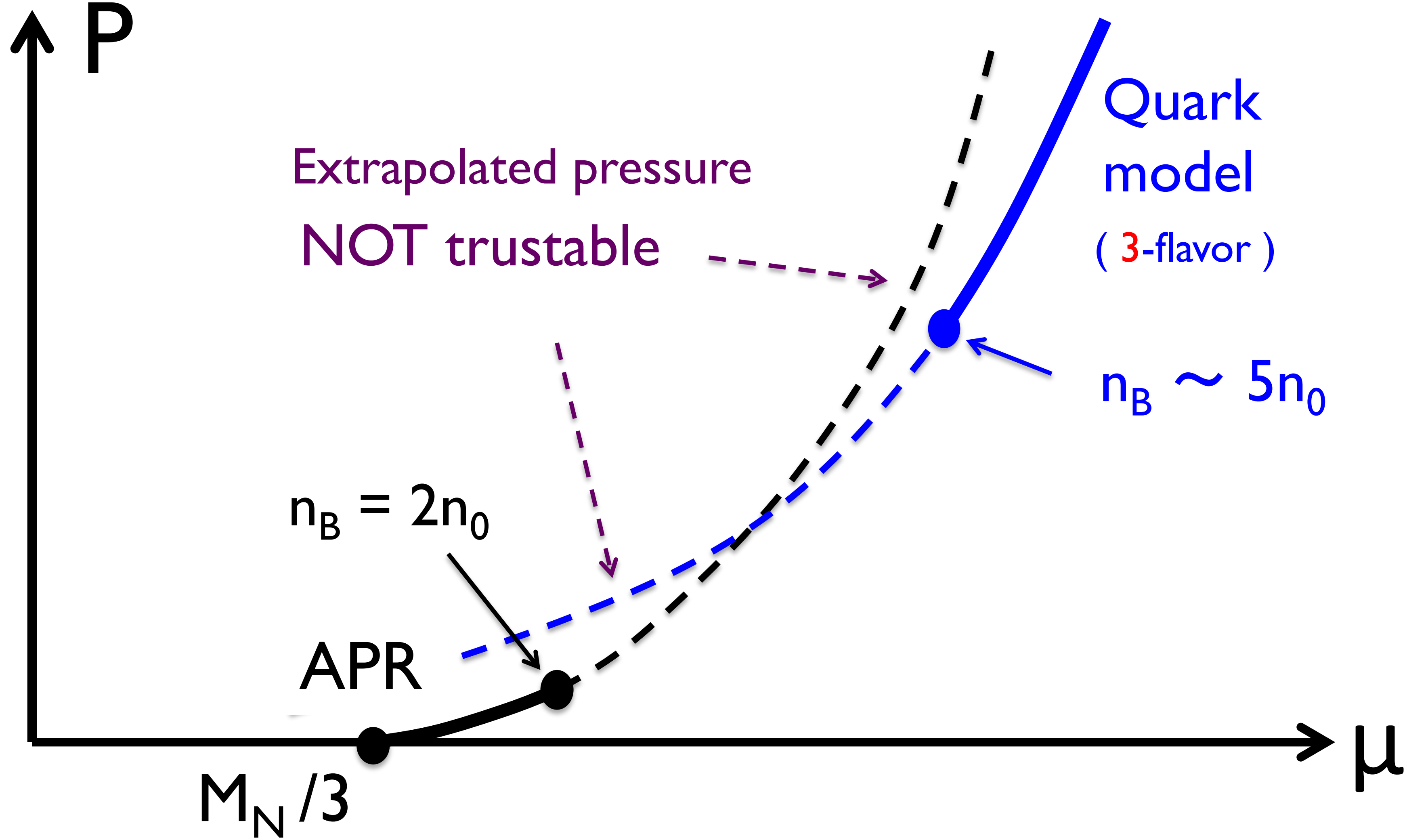
- Key (implicit) **assumptions** :
  - 1) Hadronic & quark phases are distinct (e.g. by order parameters)
  - 2) Both P_H and P_Q are reliable in the overlap region

→ by construction, Q-EoS must be much softer than H-EoS

(unless fine tuning worked out)

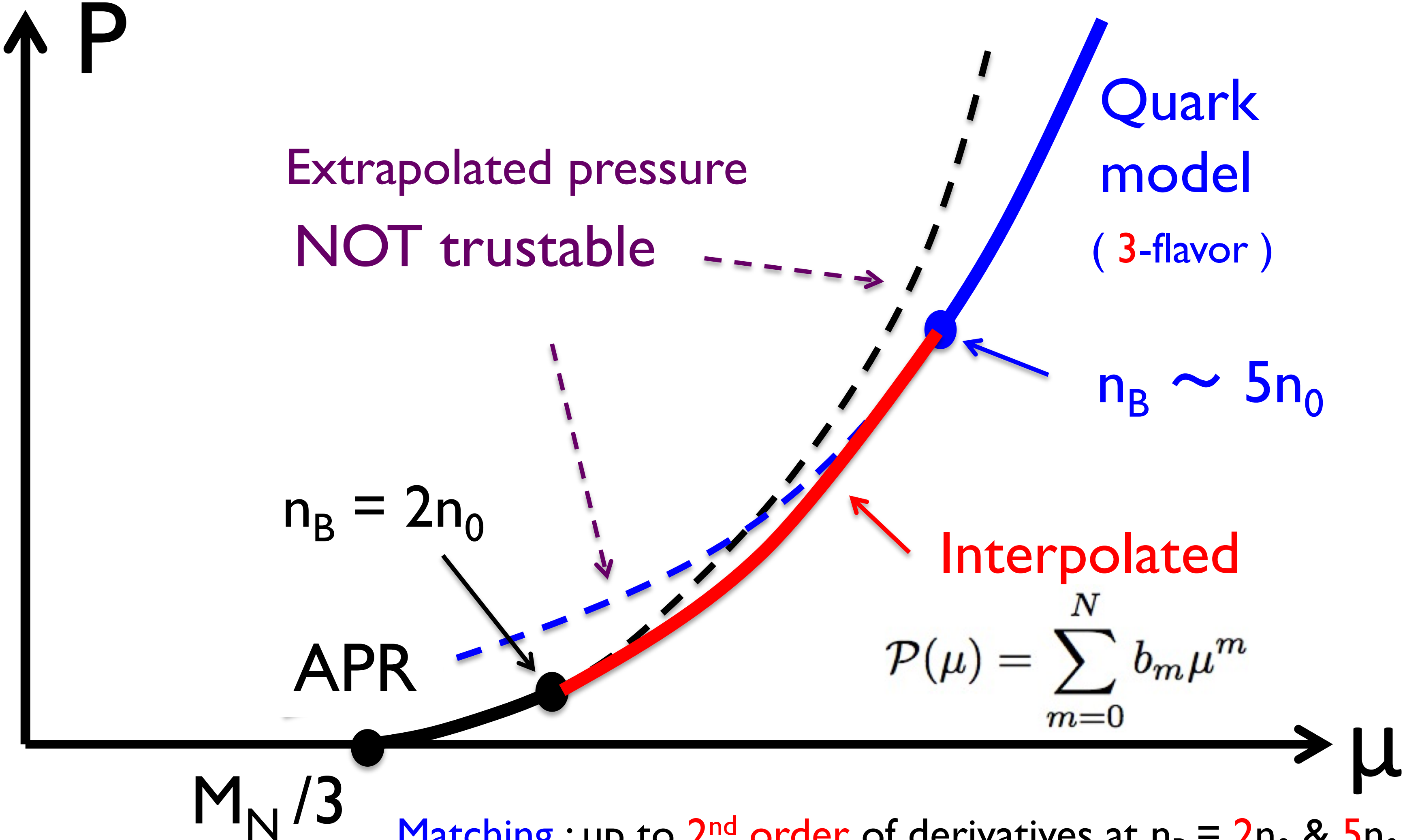
3-window modeling : P vs μ

Masuda+2012, Kojo+2014, ...



3-window modeling : P vs μ

Masuda+2012, Kojo+2014, ...



Matching : up to 2nd order of derivatives at $n_B = 2n_0$ & $5n_0$

(if you wish, put a small kink for weak 1st order P.T.)

3-flavor quark MF model : template

Kojo+2014

$$\mathcal{H}_{\rm eff} \sim \bar{\psi}\left[-i\vec{\alpha}\cdot\vec{\partial}+m\right]\psi \;+\; \mathcal{H}_{\rm NJL}^{\rm 4Fermi+KMT}$$

→ change in Dirac sea, beyond no-sea approximation

$$+\;\; \mathcal{H}_{\rm conf}^{3q\to B}$$ ➡ will be ignored at $n_B >\sim 5n_0$

mag. part

$$+\;\; \mathcal{H}_{\rm OGE}$$ ➡ $$- H \sum_{A,A'=2,5,7}\left(\bar{\psi} i\gamma_5 \lambda_A \tau_{A'} \psi_c\right)^2$$

(cf: N-Δ splitting)

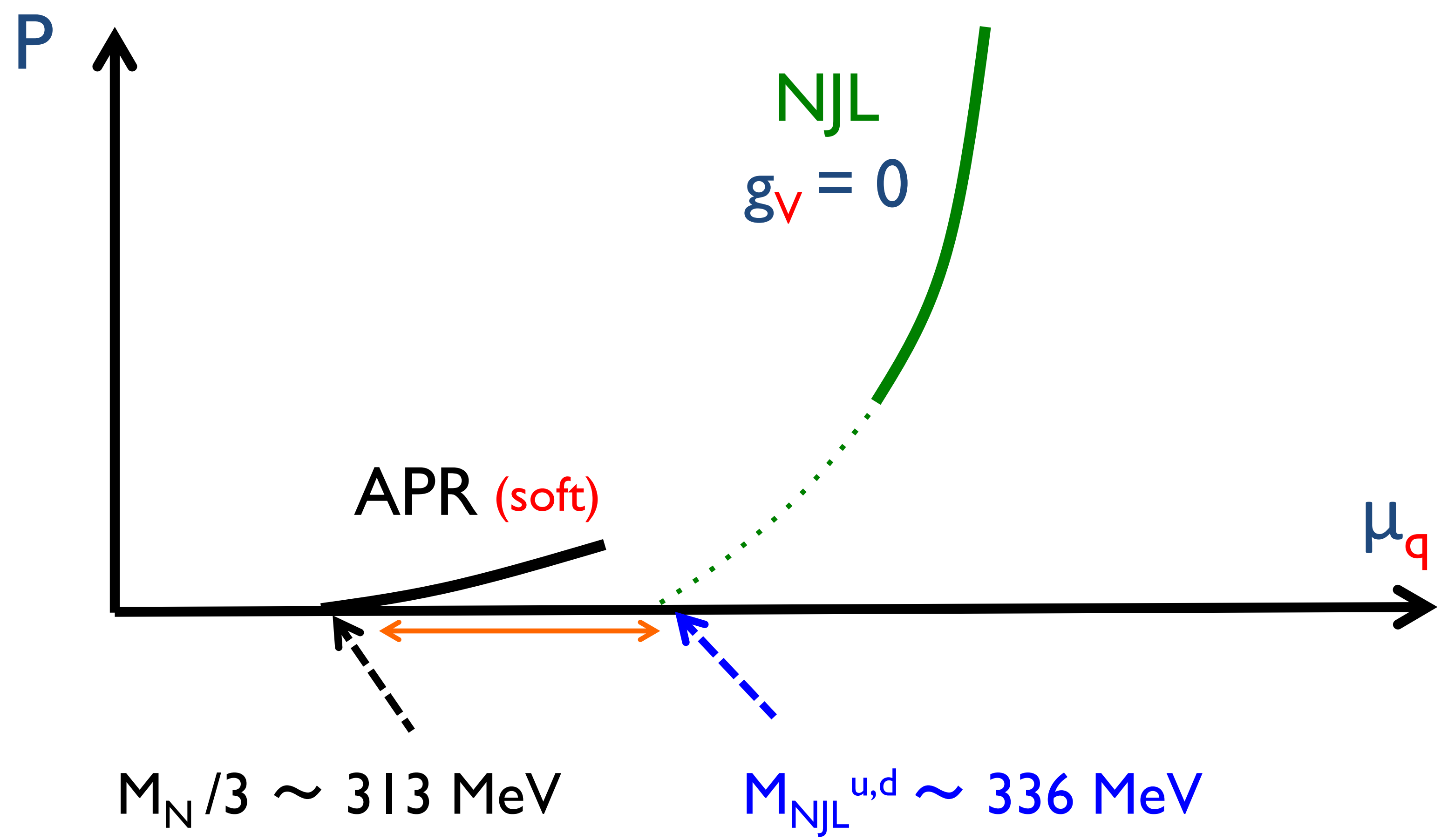
$$+\;\; \mathcal{H}_{\rm nucl}$$ ➡ $$+\; g_V\left(\bar{\psi}\gamma_0\psi\right)^2$$

~ ω-exchange (repulsive)

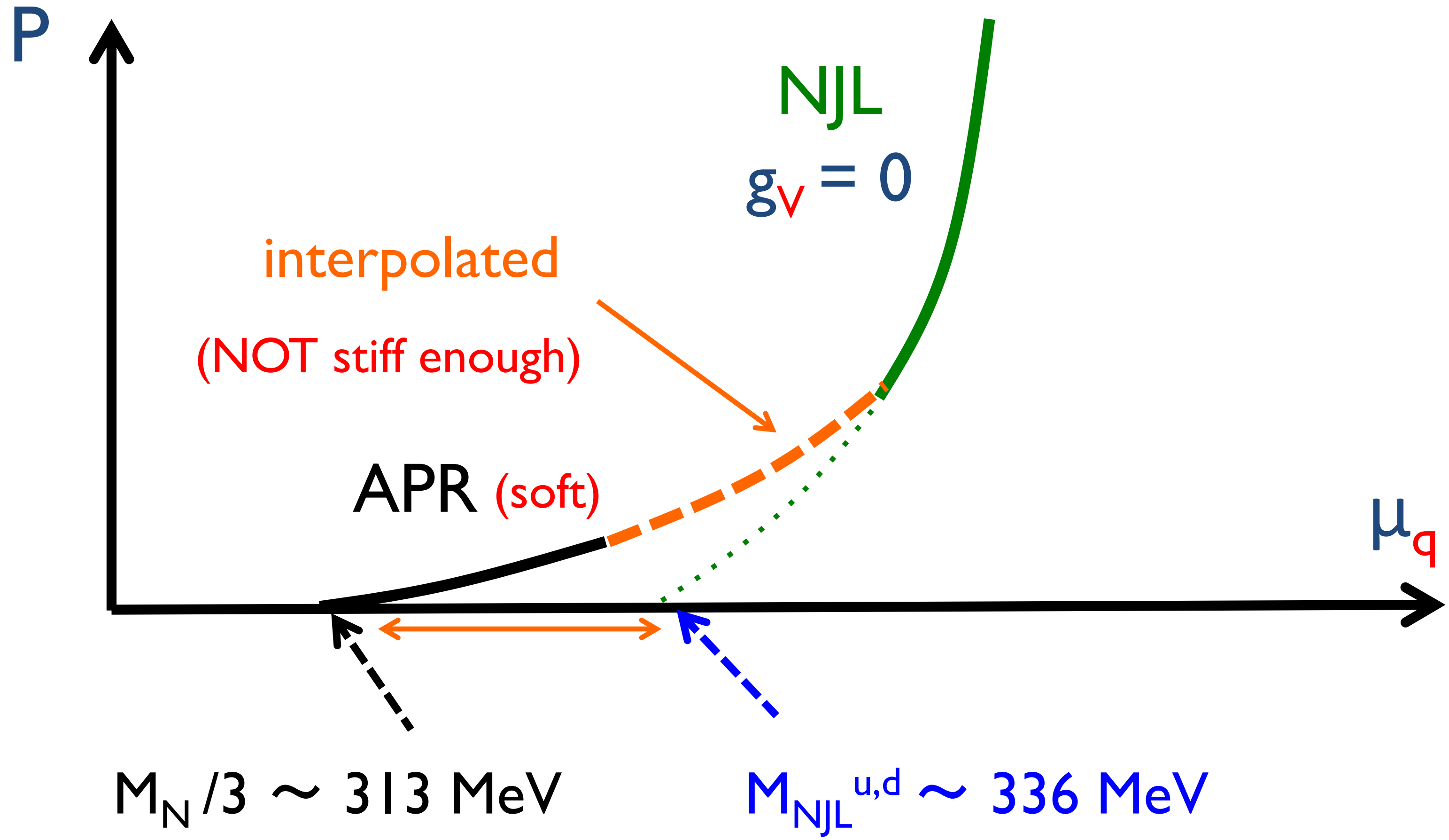
+ **important** constraints (charge neutrality & β- equilibrium & color-neutrality)

Goal: **Delineate** the properties of matter through (G_s, H, g_V)$_{@5\text{-}10n0}$

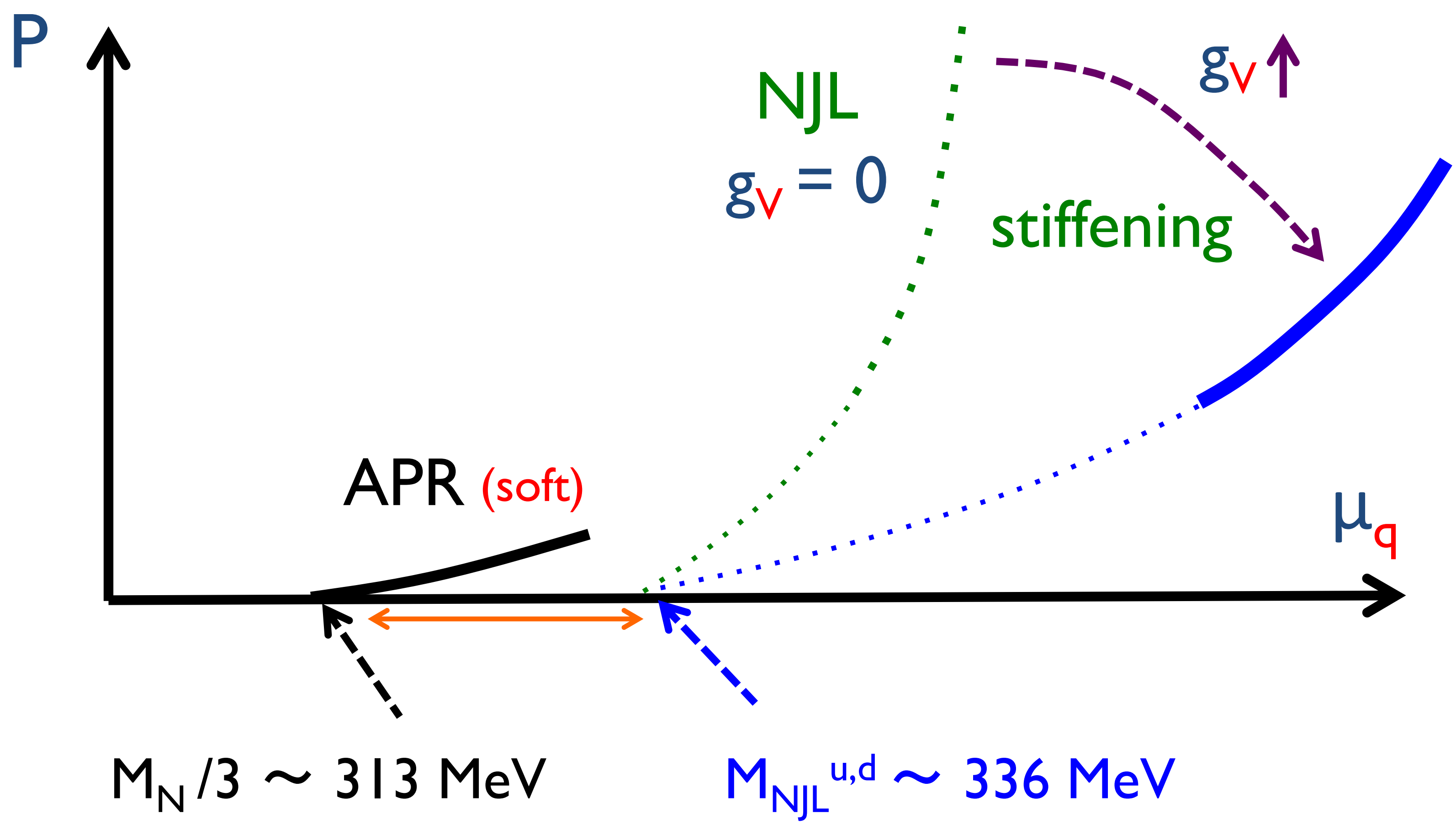
minimal



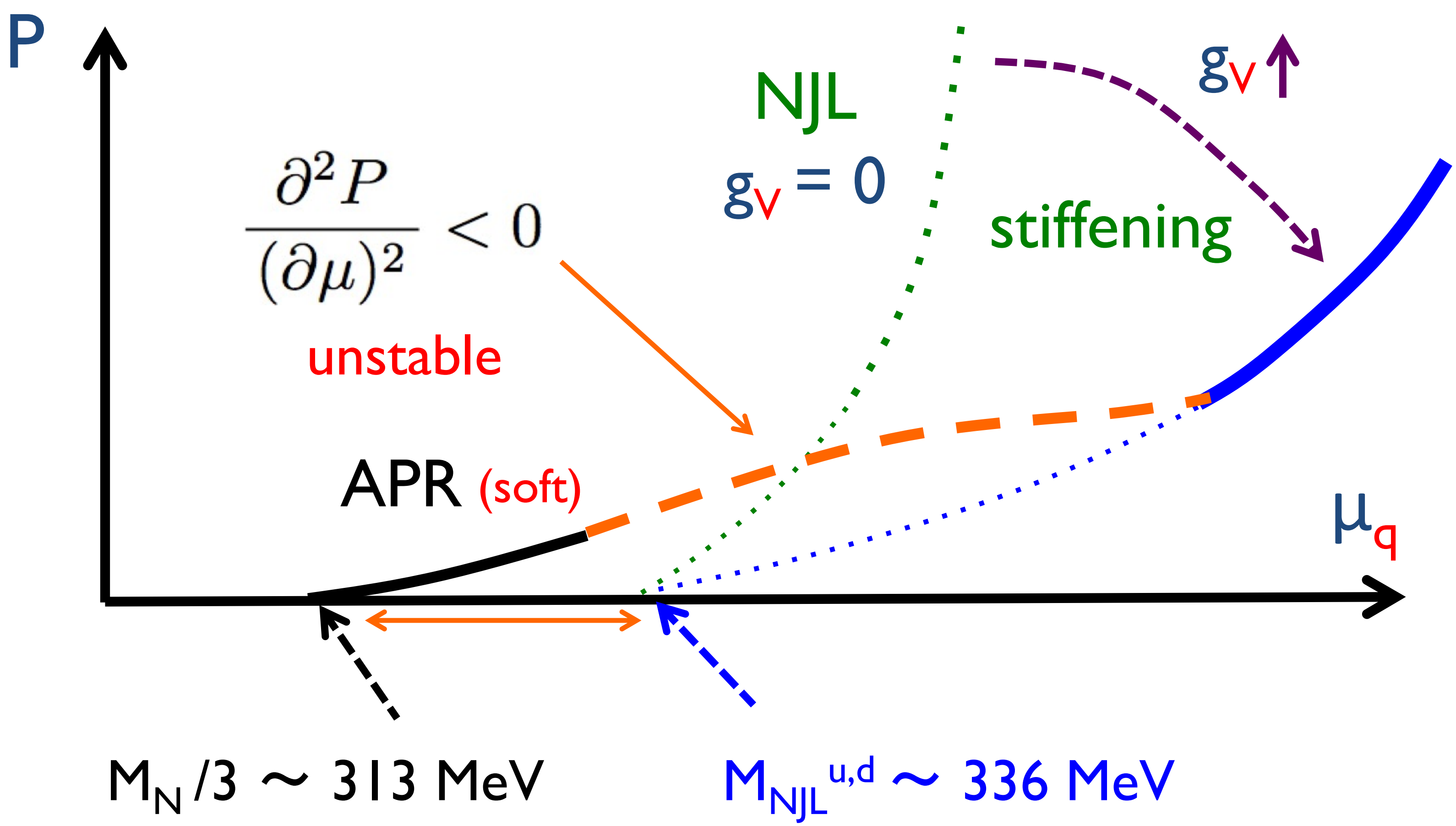
minimal


P
NJL
$g_V = 0$
interpolated
(NOT stiff enough)
APR (soft)
μ_q
$M_N/3 \sim 313$ MeV
$M_{NJL}^{u,d} \sim 336$ MeV

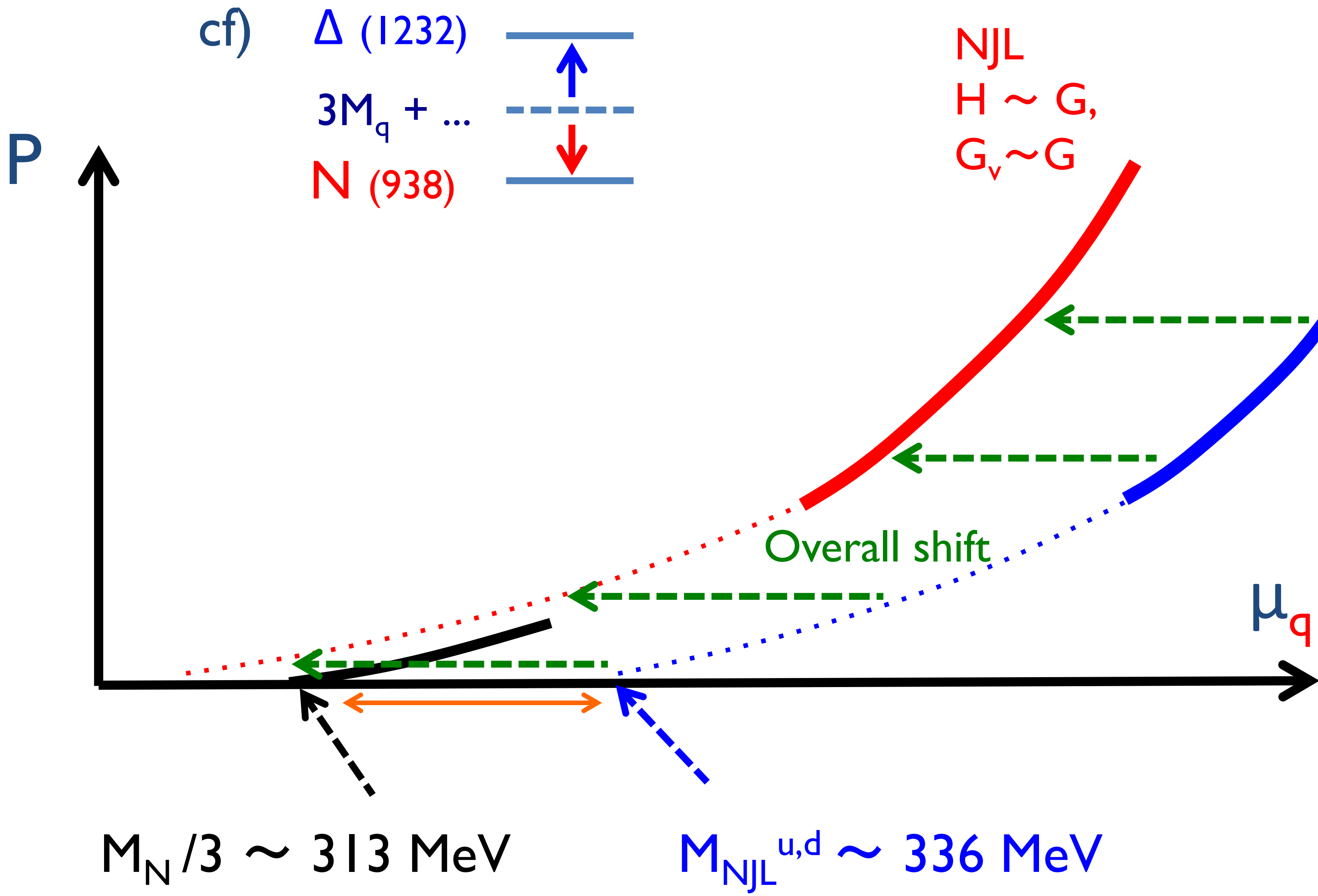
minimal + vector int.



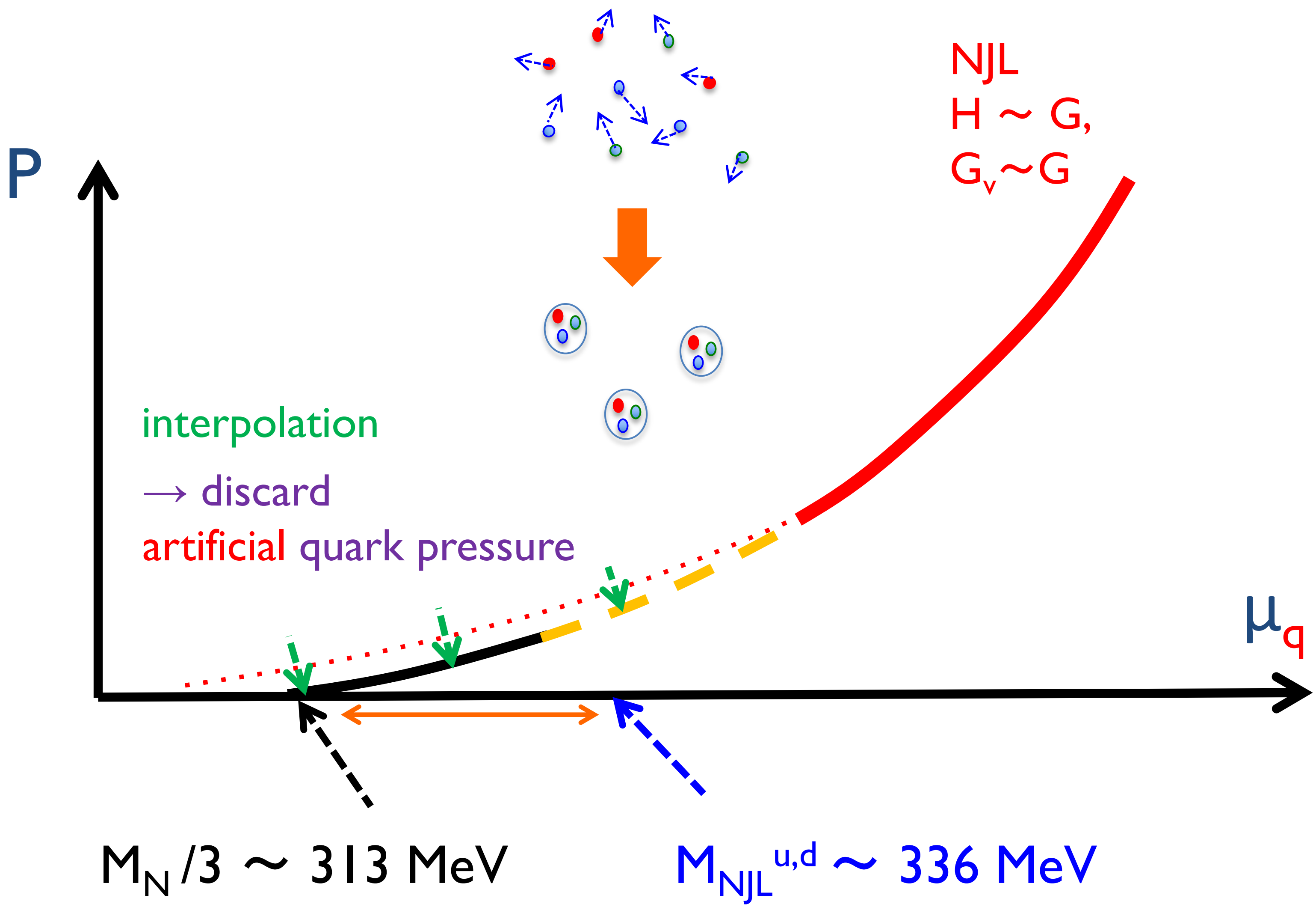
minimal + vector int.



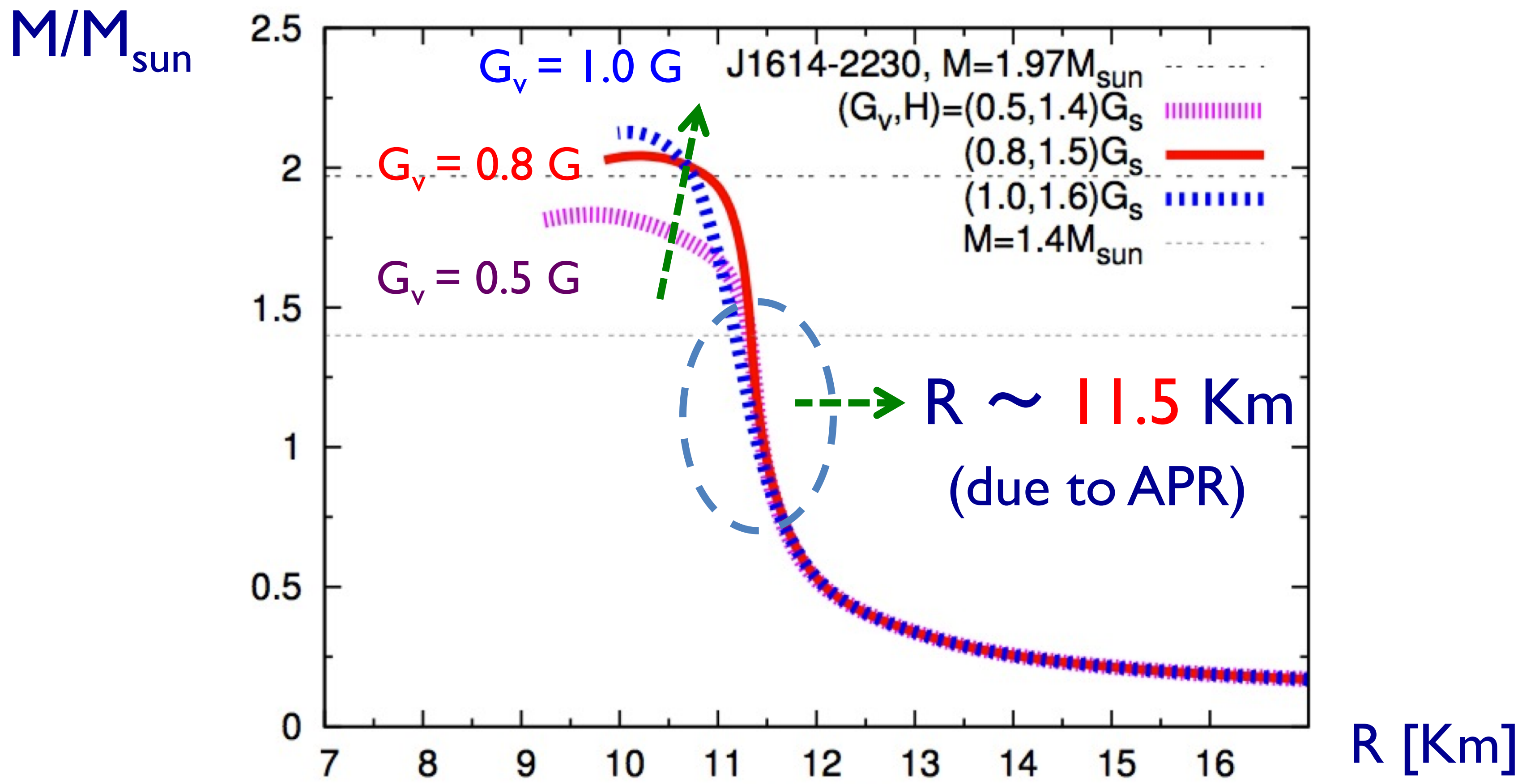
+ attractive color-magnetic int.



+ confinement in dilute matter



M-R curves for QHC18

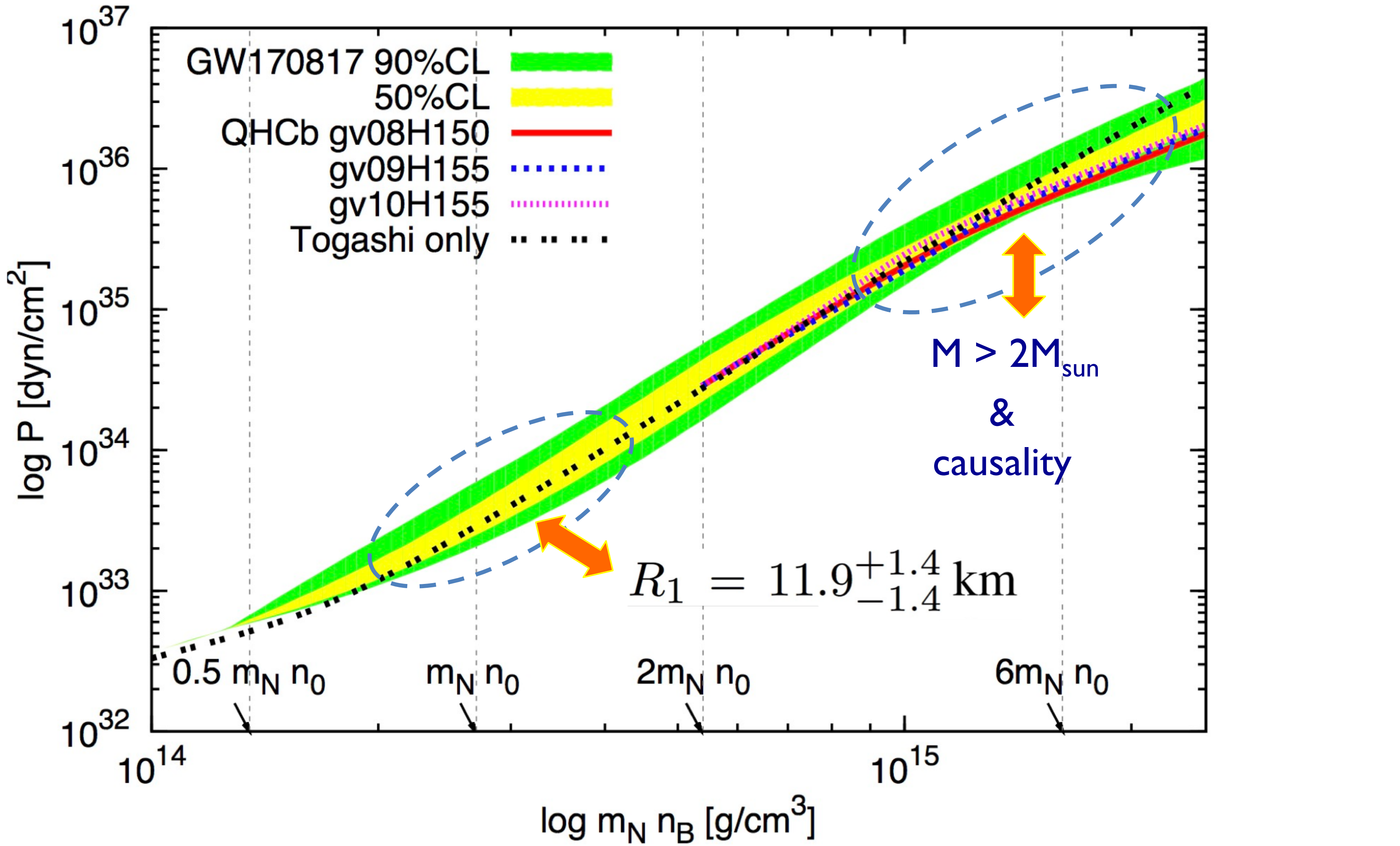


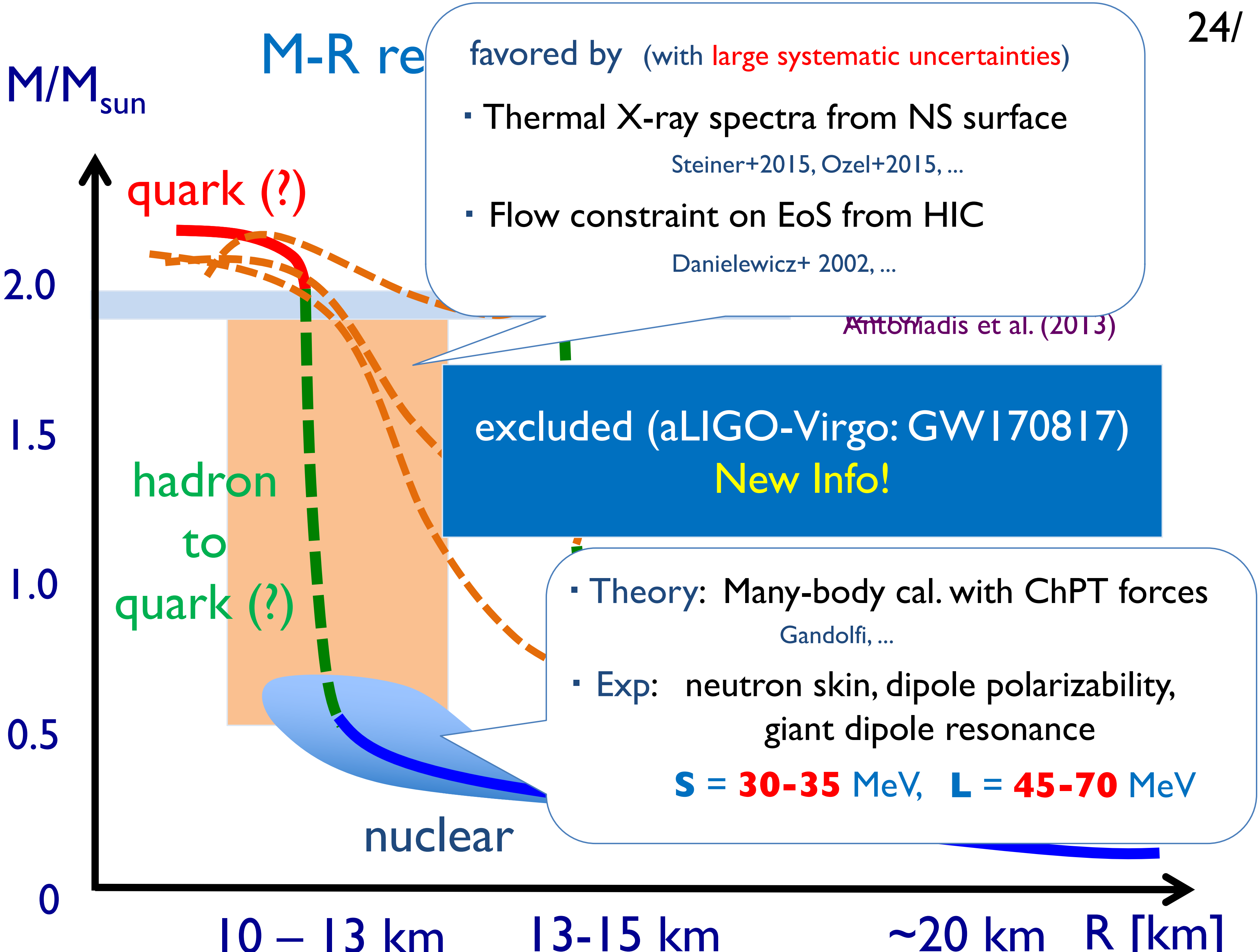
we need :

$G_s \sim G_v \sim H$ @ $n_B = 5\text{-}10\ n_0 \rightarrow O(G_s^{vac})$

EoS from aLIGO vs QHC18b

aLIGO & Virgo new analyses for GW170817 arXiv: 1805.11581 [gr-qc]

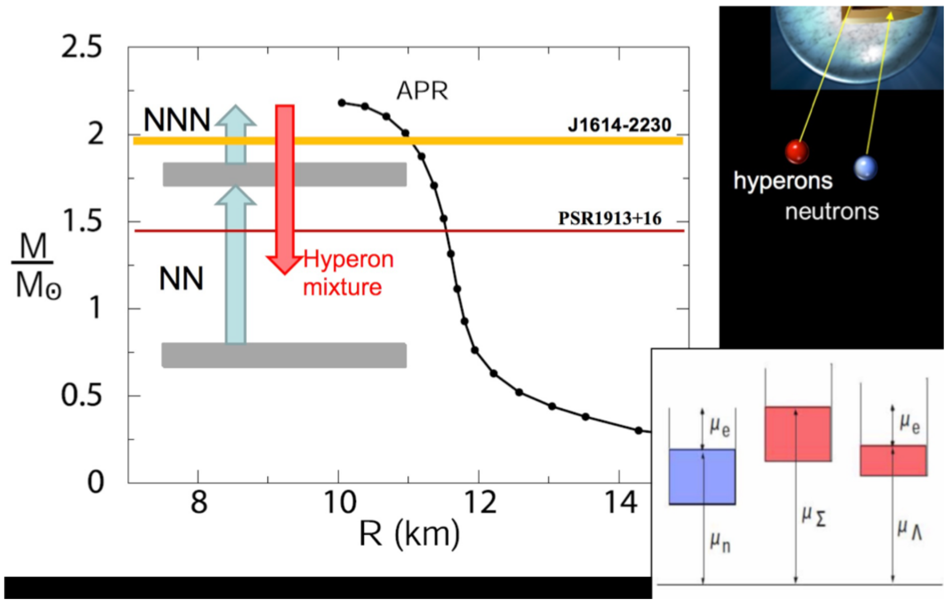



M-R re
M/M_sun
2.0
1.5
1.0
0.5
0
quark (?)
hadron
to
quark (?)
nuclear
10 – 13 km
13-15 km
~20 km
R [km]
favored by (with large systematic uncertainties)
▪ Thermal X-ray spectra from NS surface
Steiner+2015, Ozel+2015, ...
▪ Flow constraint on EoS from HIC
Danielewicz+ 2002, ...
Antoniadis et al. (2013)
excluded (aLIGO-Virgo: GW170817)
New Info!
▪ Theory: Many-body cal. with ChPT forces
Gandolfi, ...
▪ Exp: neutron skin, dipole polarizability,
giant dipole resonance
S = 30-35 MeV, L = 45-70 MeV

So we need dynamical arguments

- Troubles of purely hadronic EoS at $n_B > \sim 2n_0$
  - Convergence: 2-body forces ~ 3-body forces
  - Hyperon problems (softening)

Most typical attempts



Put by hand

Exclusion volume effect for baryons

or

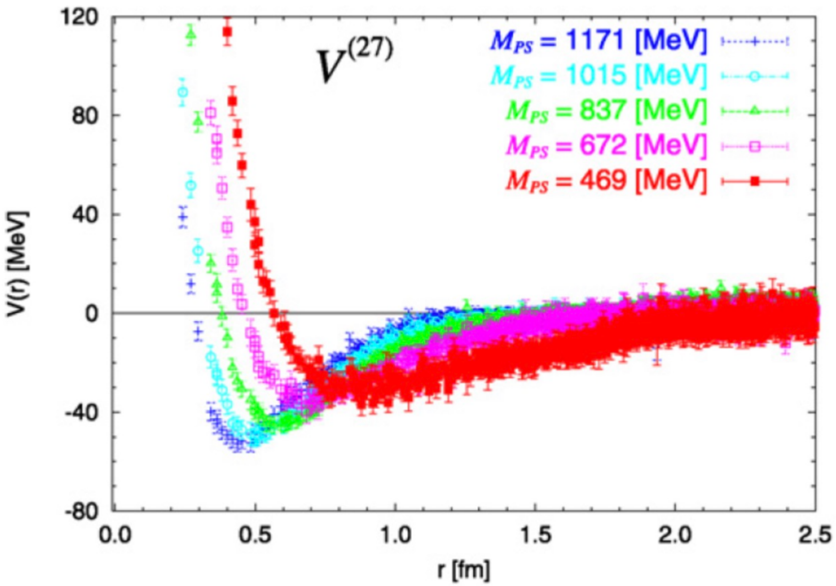
repulsive forces universal for all flavors

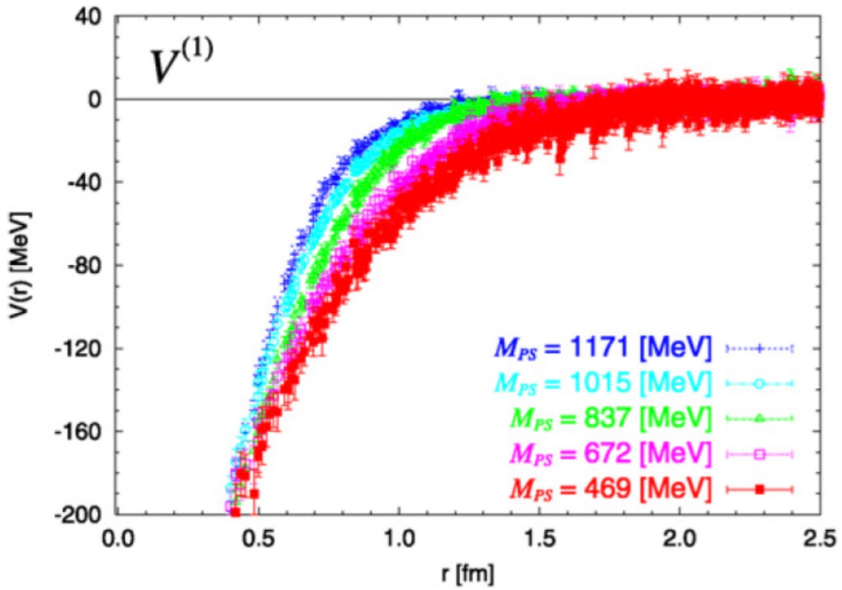
Hard core is not universal

consistent with 6q calculations in constituent quark models;

Pauli-blocking x color magnetic interactions (Oka-Yazaki)

uud-uud Figs. from HAL QCD 2011 uds-uds





Can we block the appearance of the strangeness to $n_B \sim 5n_0$??

Summary

1, Neutron star M-R relations → Direct Info of QCD EoS

2, Hints for **Soft**-**Stiff** EoS

→ crossover or weak 1st order P.T. for 2-5n_0

3, Quark matter EoS can be stiff;

the impression of soft quark EoS was largely biased by traditional hybrid construction...

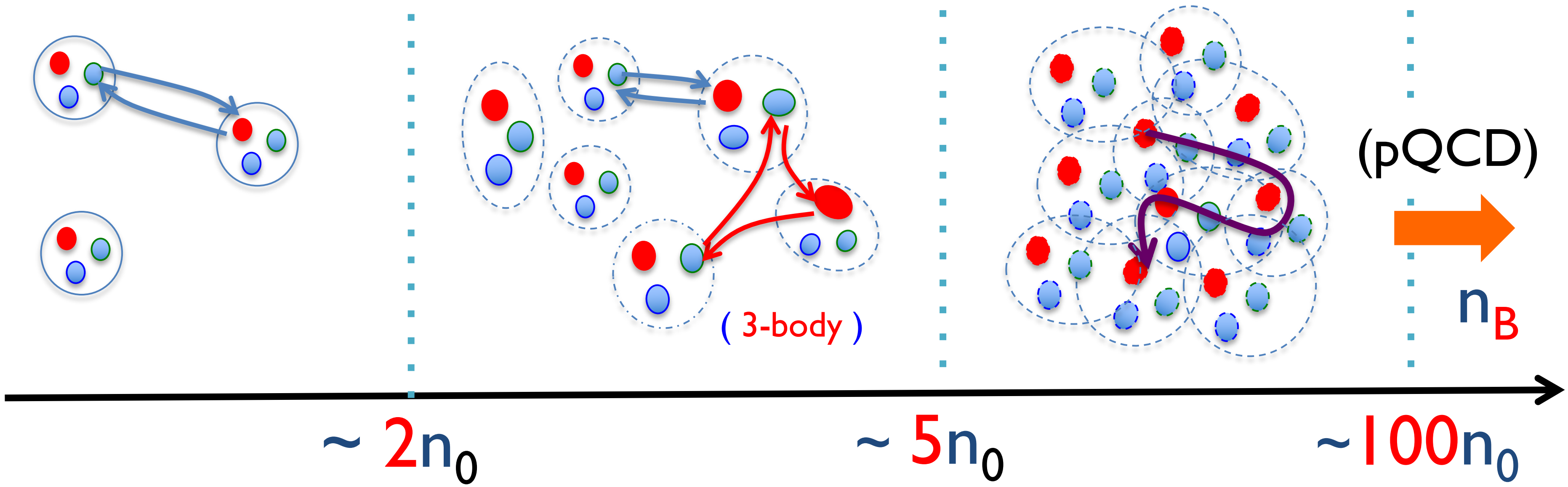
4, $(Gs, G, H)_{@5\text{-}10n0} \sim Gs^{vac}$ → Hints for non-pert. gluons

To Do (work in progress...)

Nuclear matter
+ quark substructure corrections

Hardest part
modeling?

Quark matter
+ hadronic correlations



Then the matter should be heated up → predictions for HMNS

excitation modes ⟺ **the phase structure**

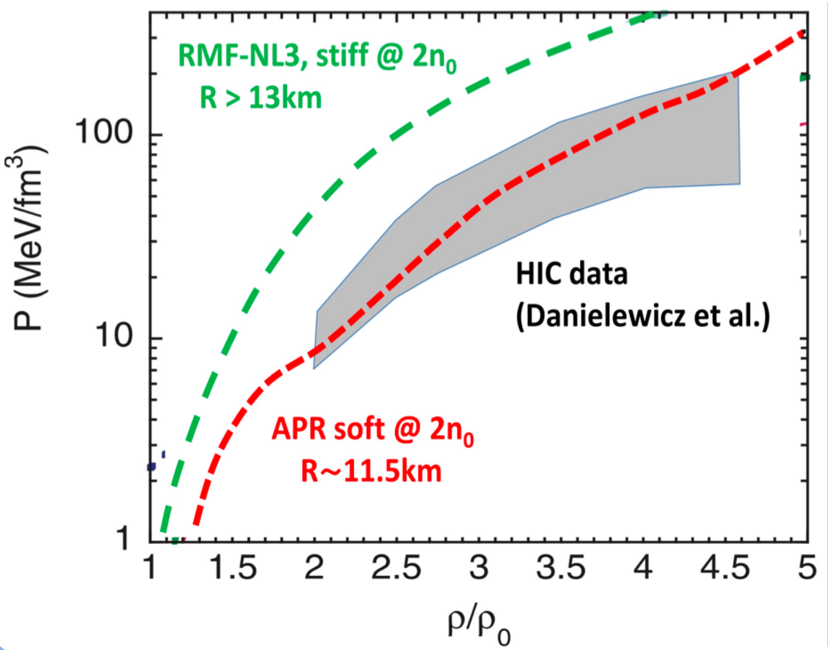
Small $R_{1.4}$ & soft EoS @ 1-2 n_0 ?

▪ Thermal X-rays analyses for NS radii :

- Suleimanov et al (2011) : > 13.9 km
- Guillot et al. (2011) : $9.1^{+1.3}_{-1.5}$ km
- Ozel & Freire (2015) : 10.6 ± 0.6 km
- Steiner et al (2015) : 12.0 ± 1.0 km

systematic uncertainties : distance to NS, atmosphere of NS, uniform T distributions,...

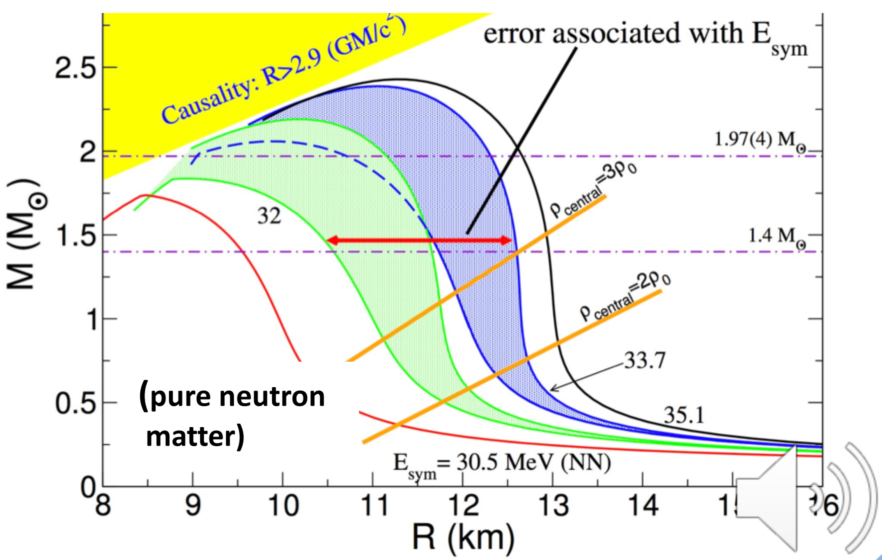
▪ HIC : (Danielewicz et al. 2002)



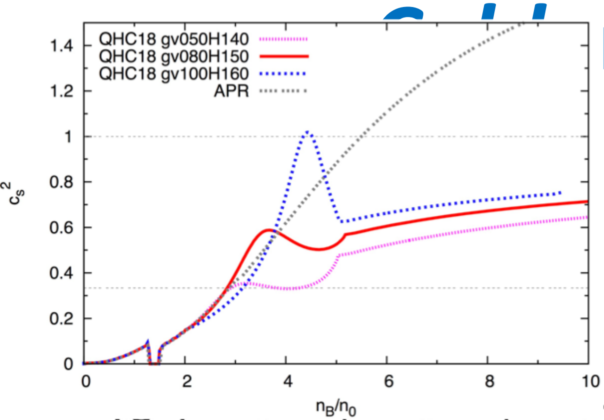
▪ nuclear EoS extrapolation :

(Gandolfi et al. 2015)

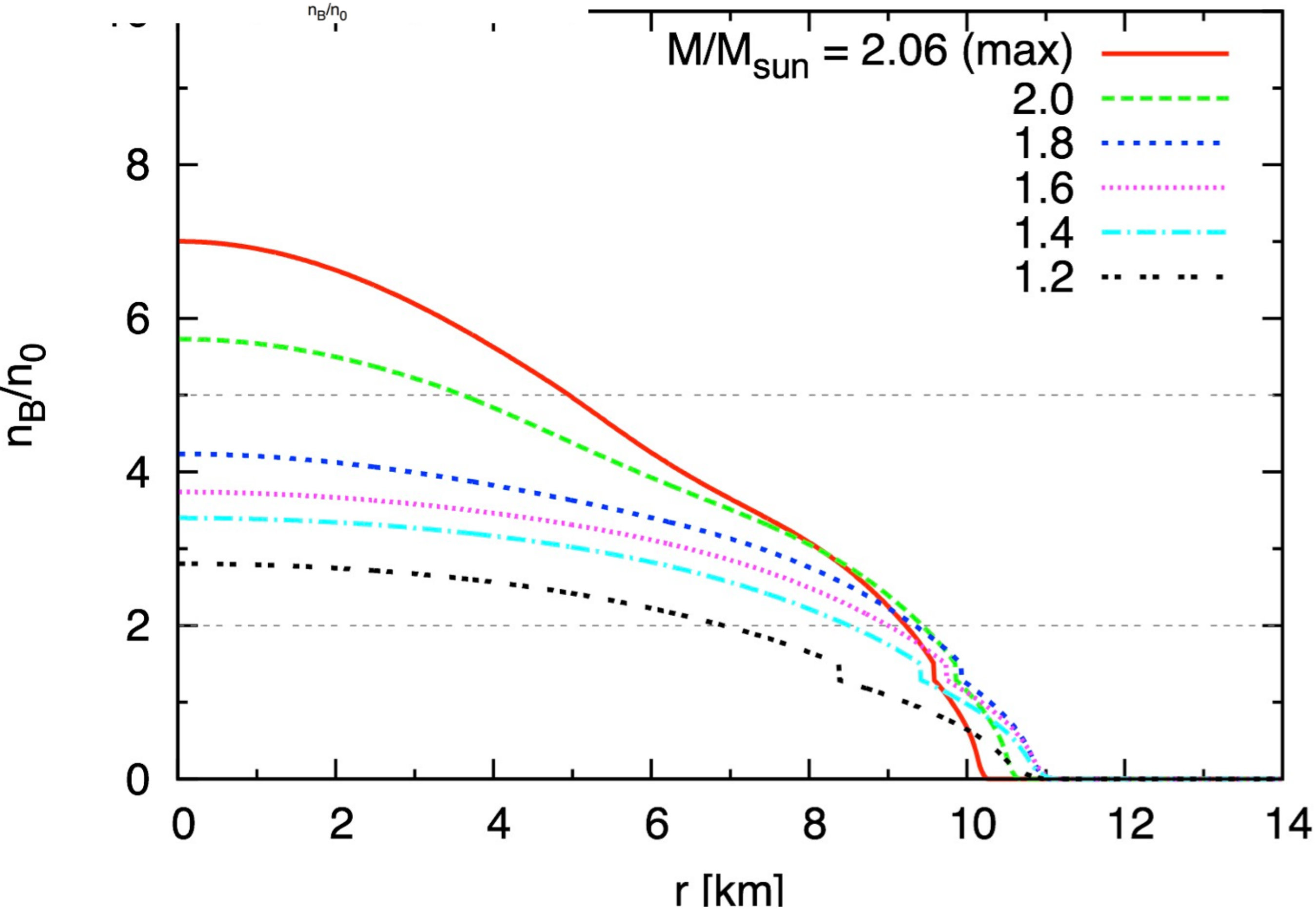
(*sophisticated* potentials & Monte-Carlo)



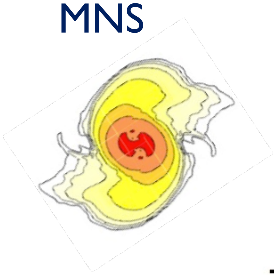
dense EoS : *Low* density



$$R_1 = 11.9^{+1.4}_{-1.4}\,\mathrm{km}$$

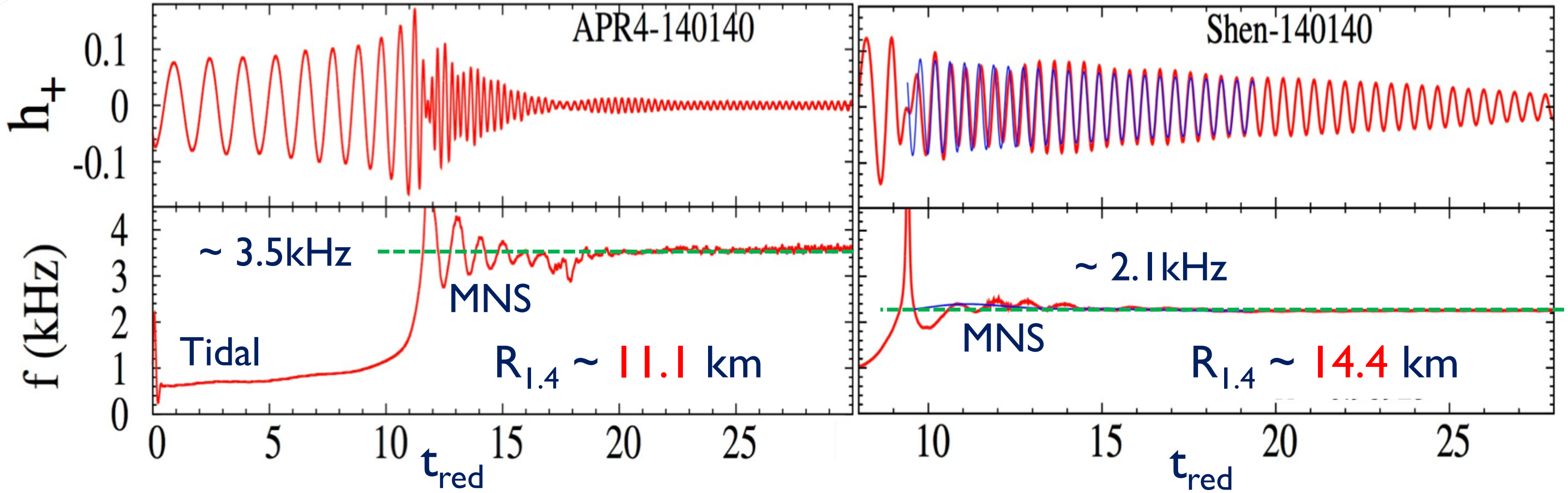


MNS



Merger & HMNS: $f_{GW} \rightarrow R_{NS}$

Figs from Hotokezaka+ 2013



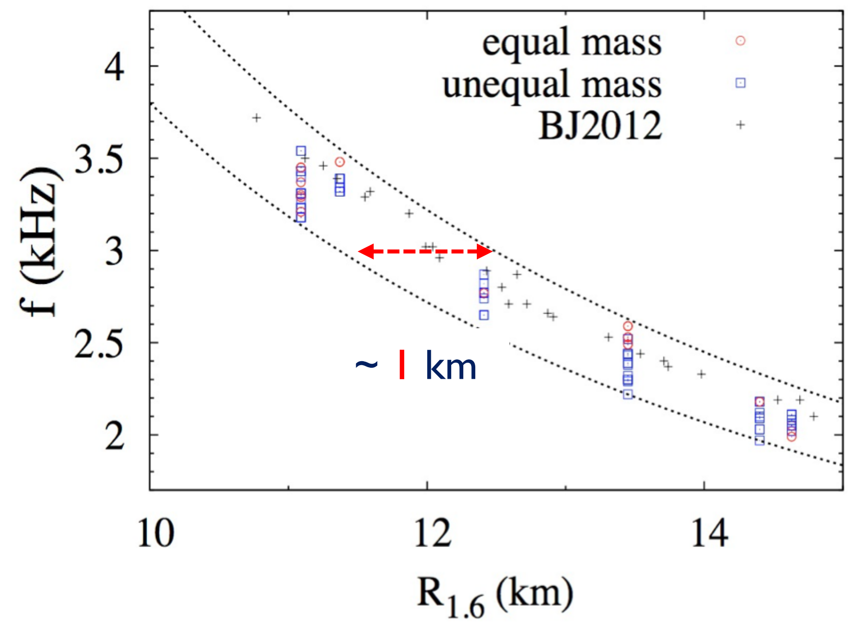
compact stars → high frequency GW

smaller R_{NS} → larger f_{GW}

(Bauswein and Janka 2012)

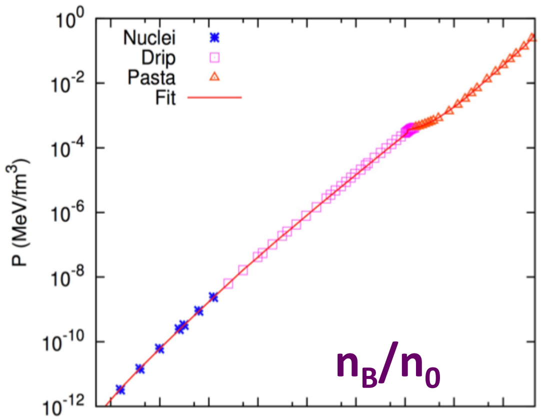
For **GW170817** :

f_{GW} is **NOT measured yet**;
high frequency region → smaller S/N



EoS & M-R relation

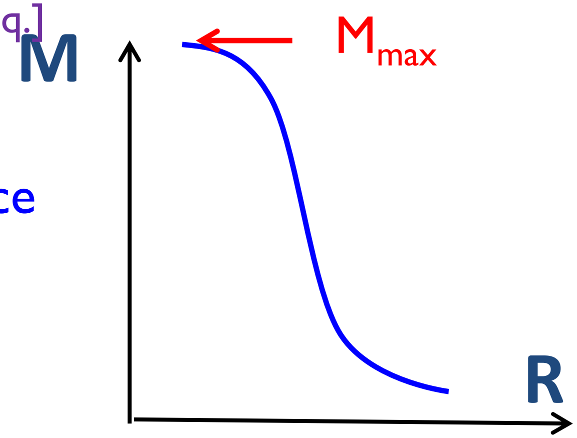
Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ QCD EoS



[for spherical NS : TOV eq.]

1-to-1 correspondence

Lindblom (1992)



1) non-rotating, spherical NS : TOV equation — $M_{TOV} > 2M_{sun}$

2) uniformly rotating NS : e.g. Hartle-Thorne — $M_{uni} \sim 1.2\ M_{TOV}$

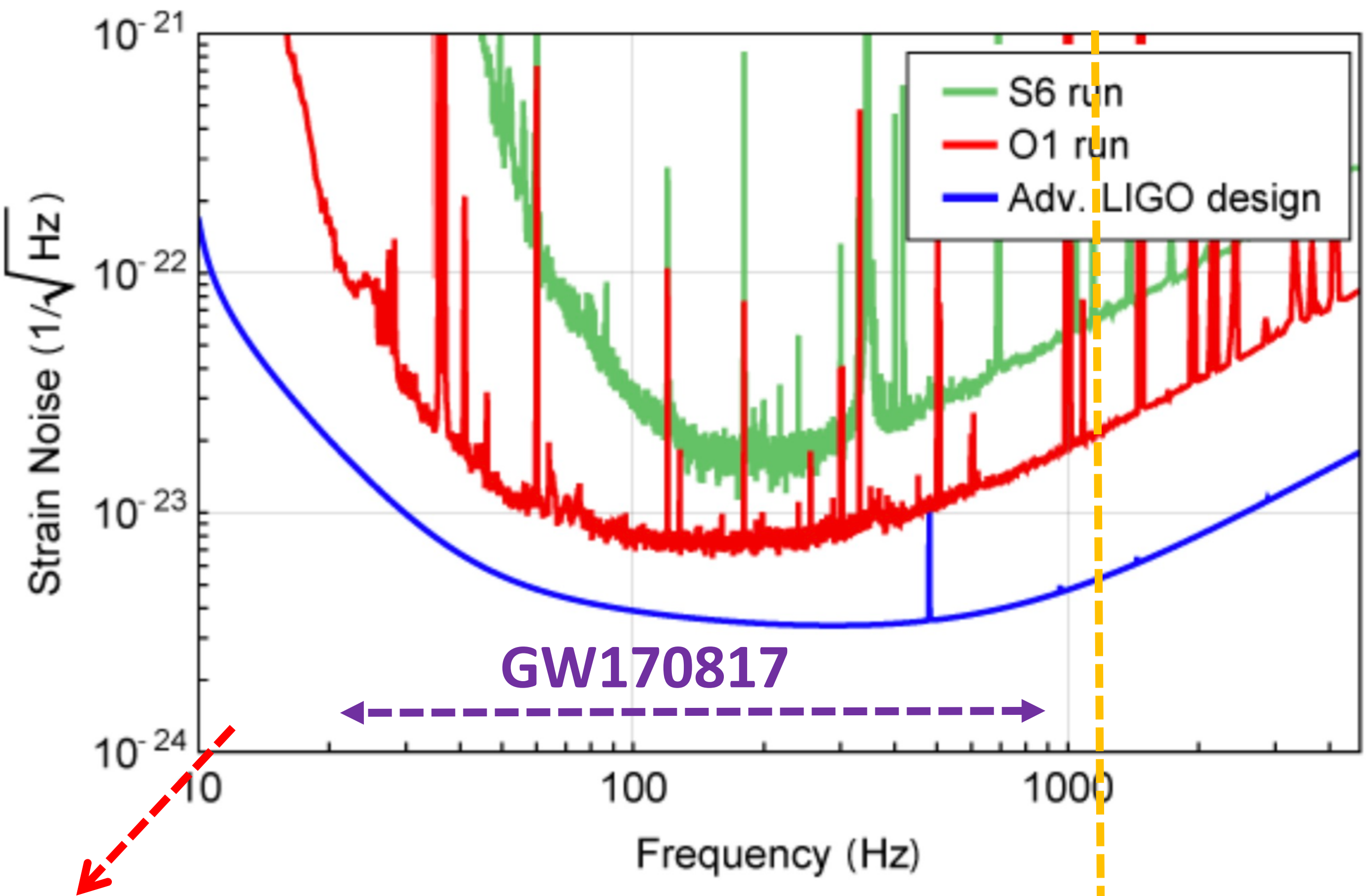
(stable if rotation is slow enough)

3) differentially rotating NS : Numerical GR — $M_{diff} \sim 1.5\ M_{TOV}$

(short-live; dissipation and magnetic braking → collapse)

Baryon number density

Design sensitivity



inspiral

(noise: seismology)

tidally deformed phase

(noise: mirror)

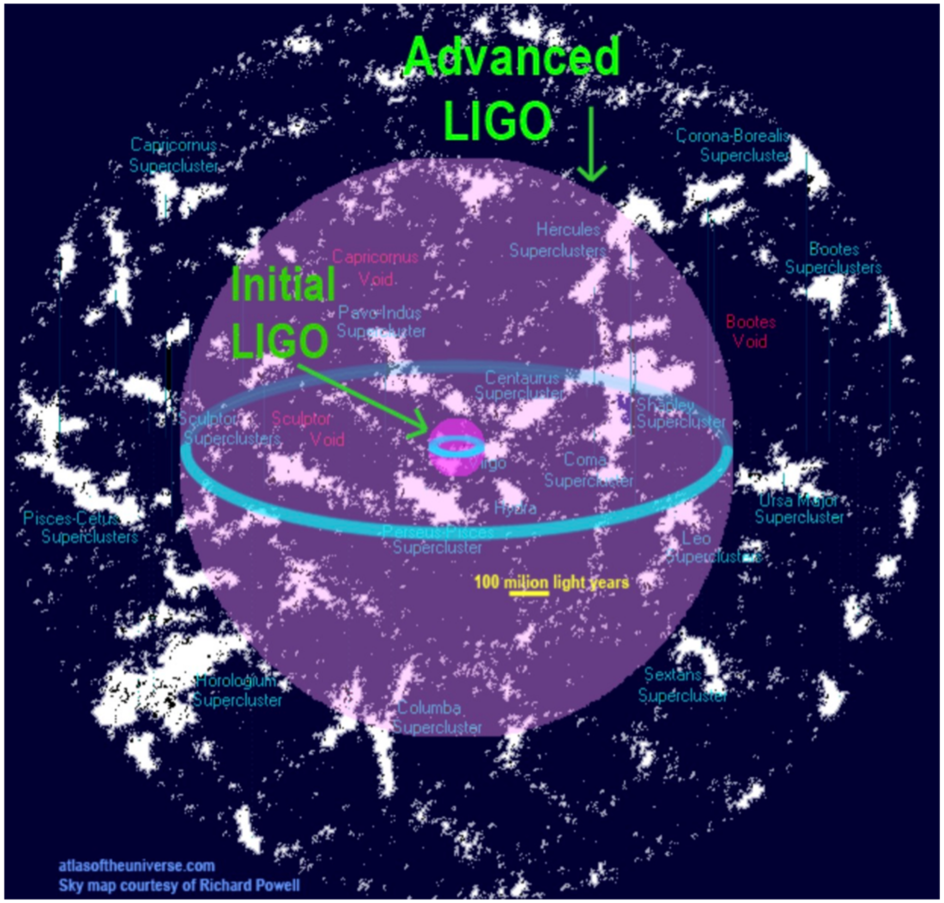
~ post-merger

HMNS or BH

(quantum noise: laser)

To detect rare events

1pc = 3.26 lyr



- our galaxy (milky-way) ~ 31-55 kpc
- to the edge of universe ~ 14 Gpc
- *detector horizon*
  - aLIGO
    Livingston ~ 218 Mpc
    Hanford ~ 107 Mpc
  - Virgo ~ 58 Mpc
- *expected detection rate*
  0.1 – 100 events/year

- GW170817 happened at 40^{+8}_{-14} Mpc

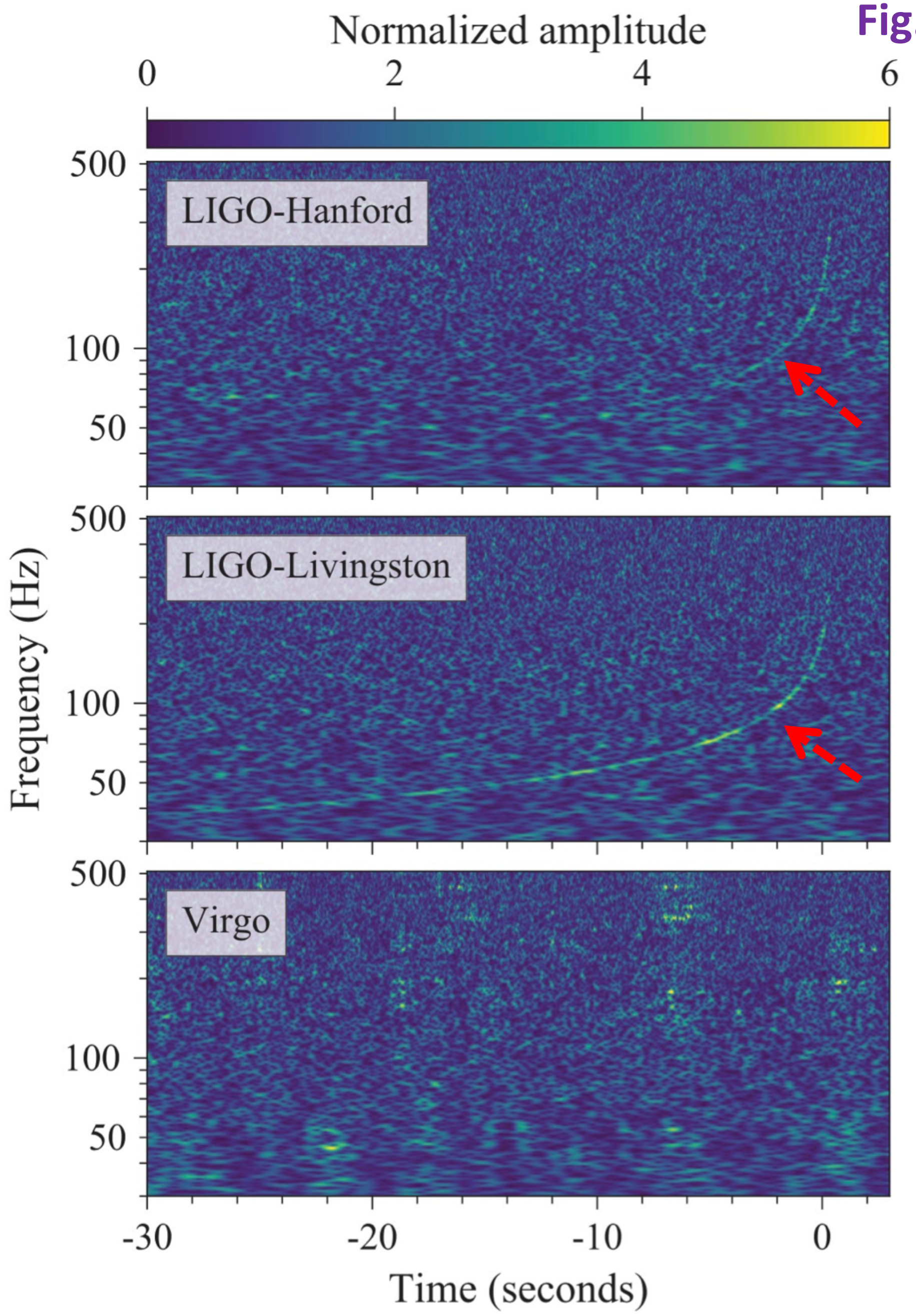


Fig. from PRL 119, 161101 (2017)

- **aLIGO: signal-to-noise = 32.4 !**

  (largest GW signal ever)

- **Virgo did not find it**

  GWs from the blind spot of Virgo

  → strongly constrain the location

  → trigger follow-up EM studies

- **clear signal 20 Hz - 1kHz**

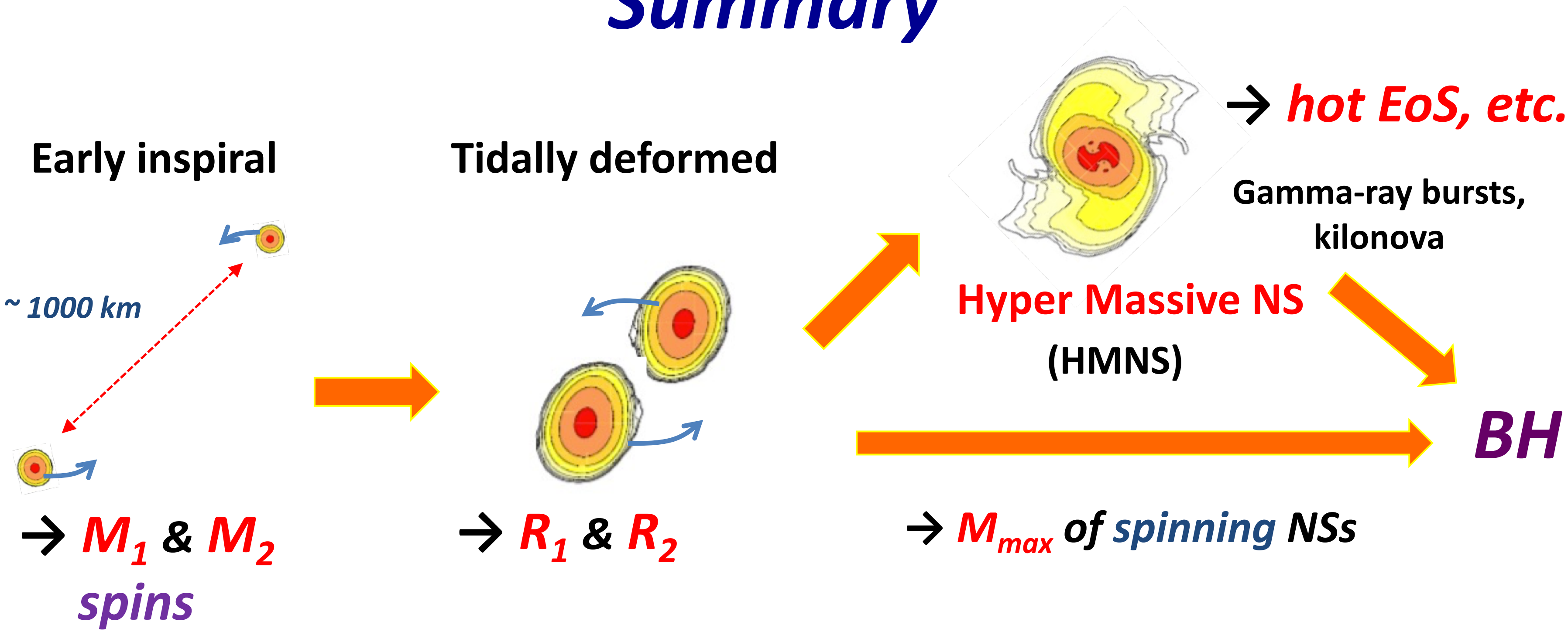
  ***inspiral* – *tidal deformed* phases**

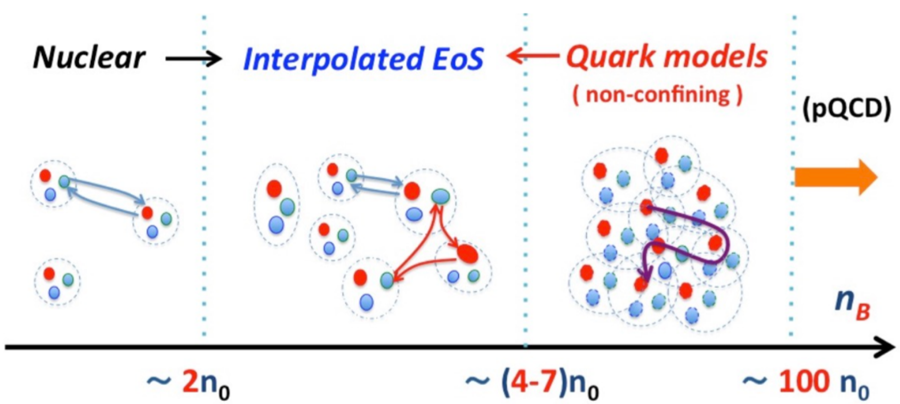
  ***BH ring-down* not measured**

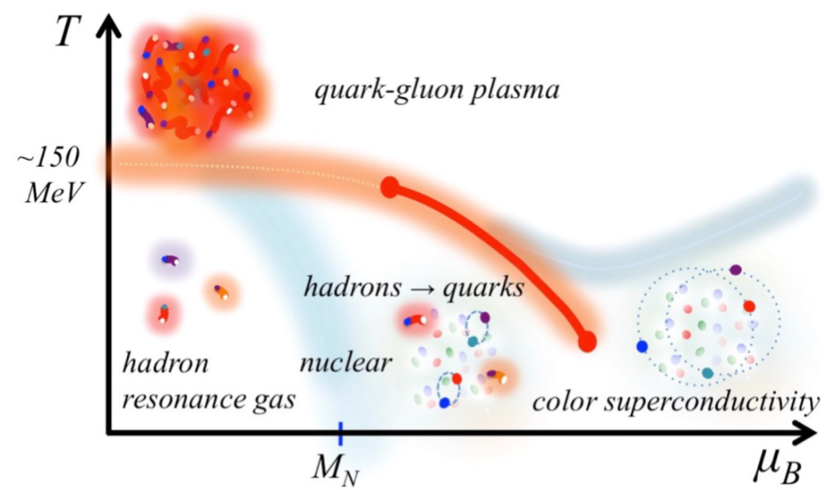
  (larger noise at higher frequency)

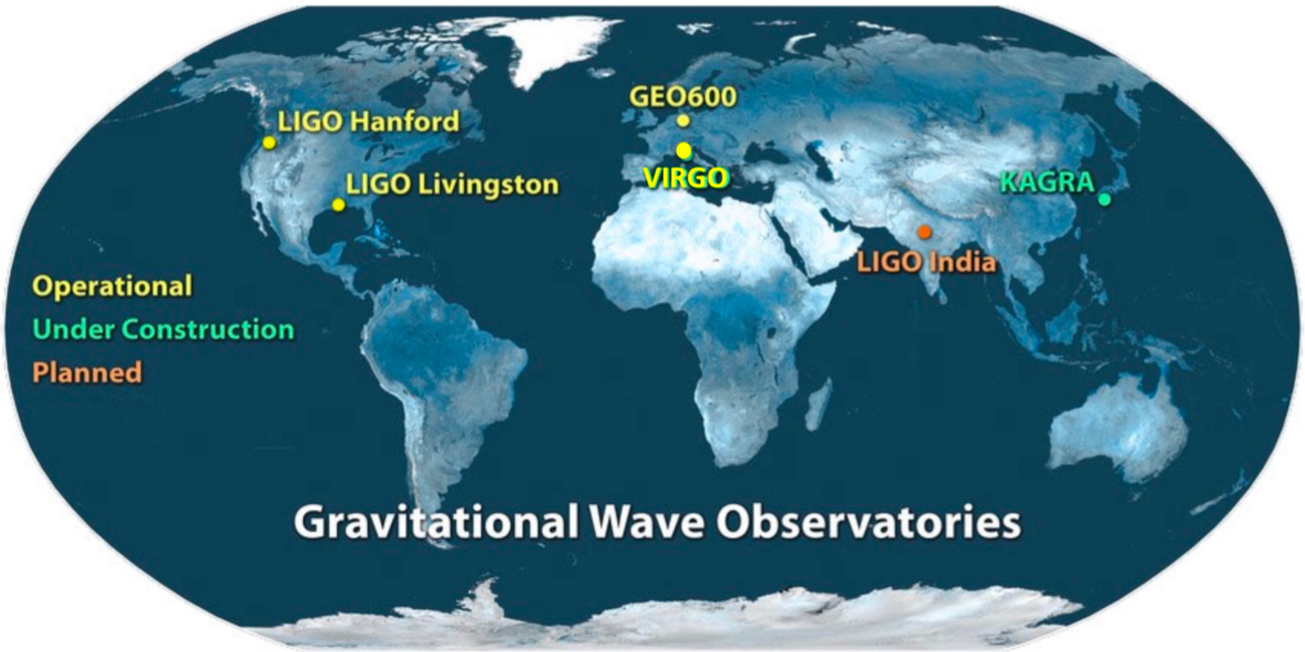
- **EM signals from objects just after merger**

Summary


Early inspiral
~ 1000 km
→ M_1 & M_2
spins
Tidally deformed
→ R_1 & R_2
→ hot EoS, etc.
Gamma-ray bursts, kilonova
Hyper Massive NS
(HMNS)
BH
→ M_{max} of spinning NSs


Nuclear
Interpolated EoS
Quark models
(non-confining)
(pQCD)
n_B
~ $2n_0$
~ $(4\text{-}7)n_0$
~ $100\ n_0$

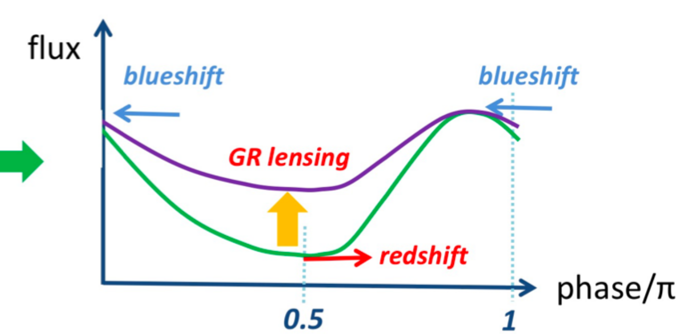

T
quark-gluon plasma
~150 MeV
hadrons → quarks
hadron resonance gas
nuclear
color superconductivity
M_N
μ_B

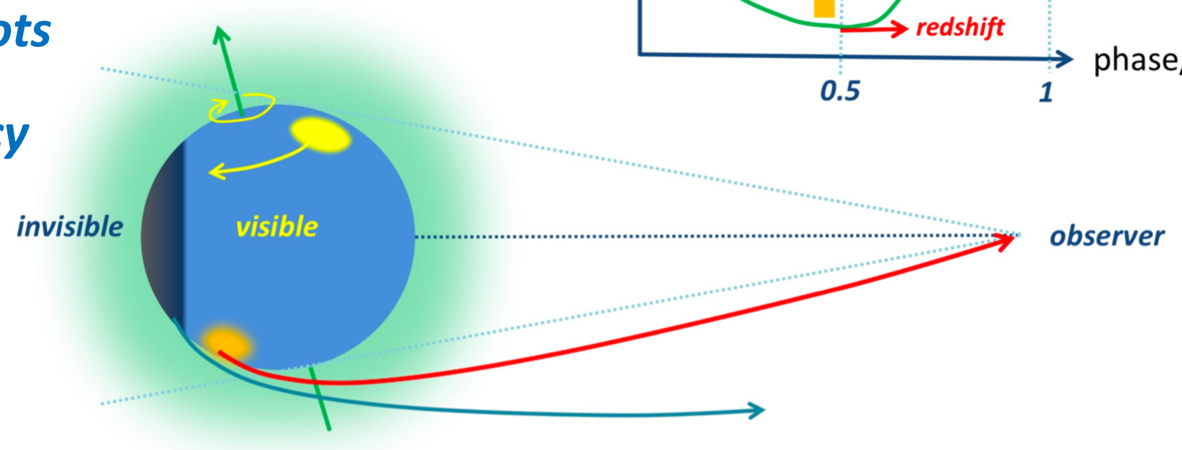


- **GW detectors :**
  - aLIGO (O3)
  - VIRGO
  - KAGRA
  - LIGO India, …

- **NICER (2017~) :**
  - *timing analyses of hot spots*
  - *R & M/R → 5-10 % accuracy*

Modeling



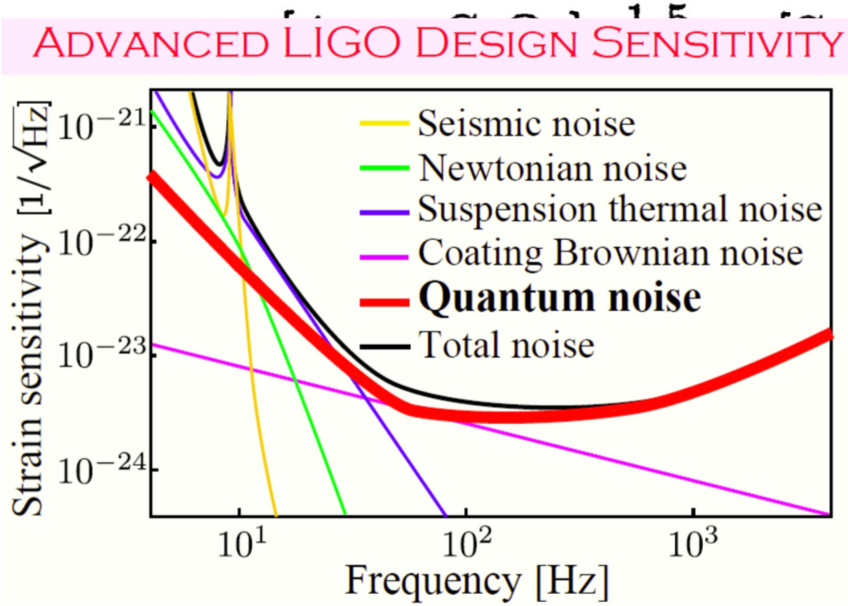


Template 1: post-Newtonian for f < ~1kHz

Cutler et al., PRL70, 2984 (1993)

$$\frac{d\mathcal{N}_{\rm cyc}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1+\left(\frac{743}{336}+\frac{11}{4}\frac{\mu}{M}\right)x\right.$$

$$\left.\mathrm{S.}]x^2 + O(x^{2.5})\right\}.$$



Delayed vs prompt collapse → $(M^{TOV})_{max}$

Lattimer, talk at INT, 2018

M_{rem}

BH

prompt collapse

(life << 1s)

~ 1.5 $(M^{TOV})_{max}$

Hyper Massive

differential rotation

(short life ~ **1s**;
viscous & mag. braking)

~ 1.2 $(M^{TOV})_{max}$

Supra Massive

uniform rotation

(long life >> 1s)

$(M^{TOV})_{max}$ > ~ $2M_{sun}$

Non-rotating

(stable)

For **GW170817** :

- collapse to BH after ~ **1s**
- ~2.28 < M_{rem}/M_{sun} < ~2.53 (estimated)

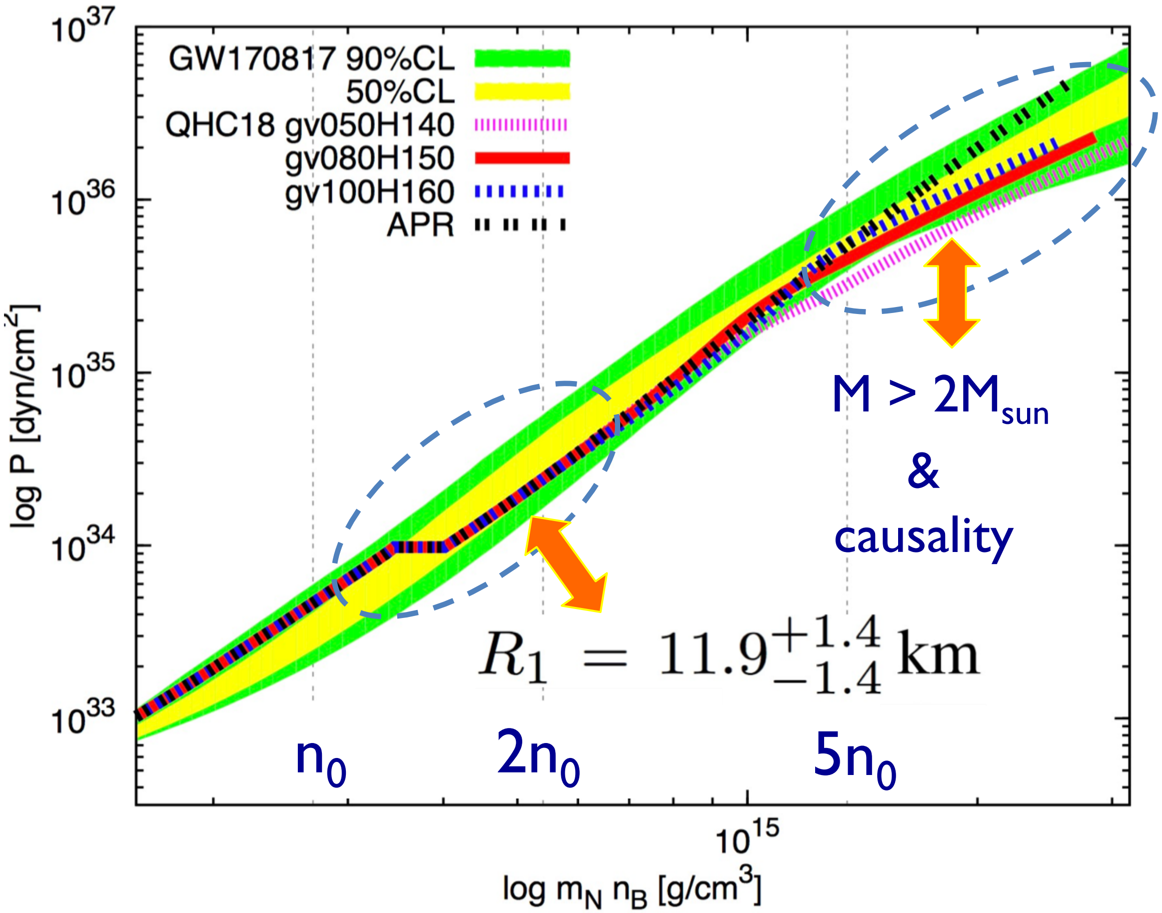
1.2 $(M^{TOV})_{max}$ < 2.53 M_{sun}

→ $(M^{TOV})_{max}$ < 2.11 M_{sun}

- If thermal effects are included, the constraint may be even stronger

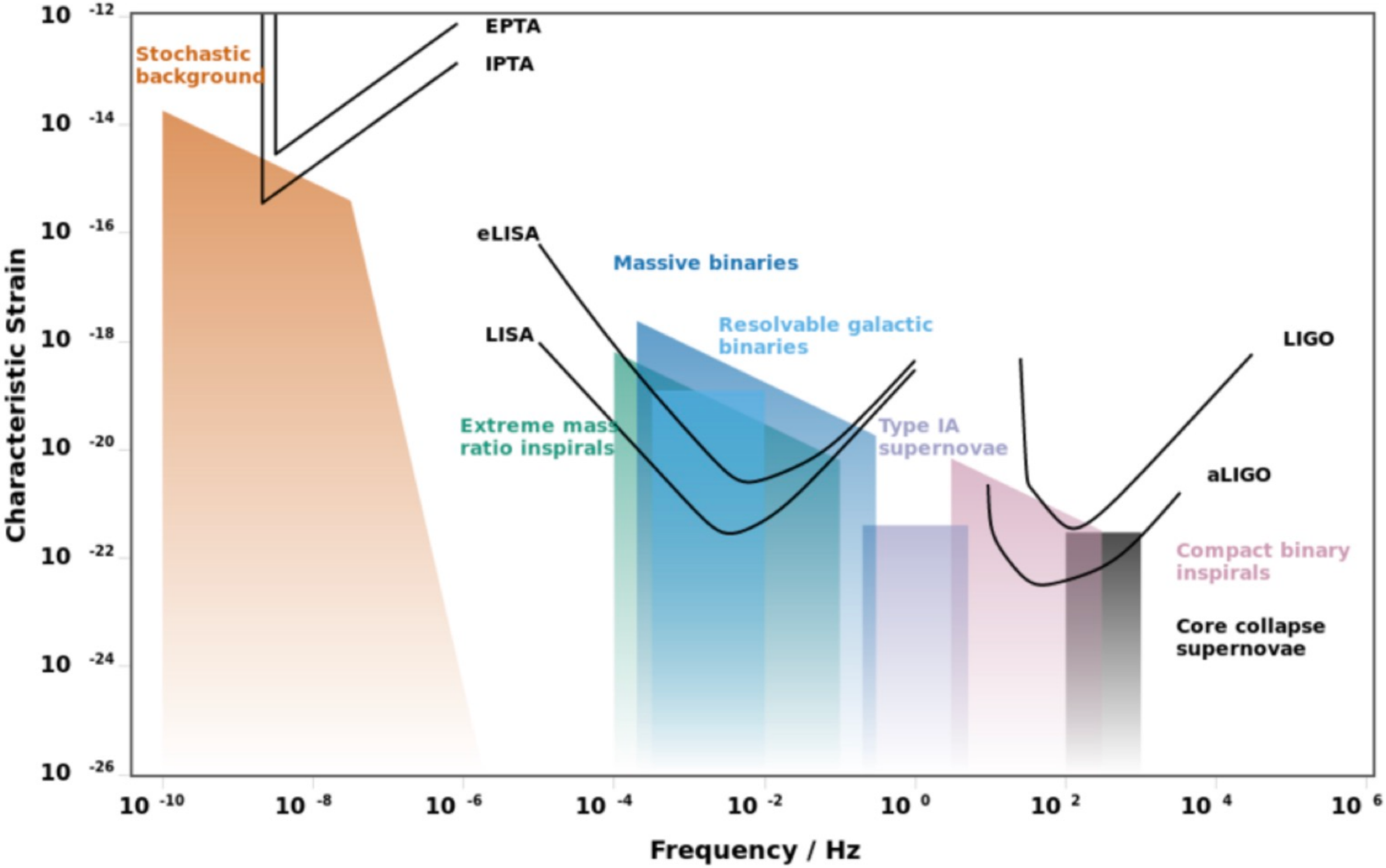
EoS from aLIGO vs QHC18

aLIGO & Virgo new analyses for GW170817 arXiv: 1805.11581 [gr-qc]



EoS constraints with

- $M > 2M_{sun}$
- tidal deformability
- causality



Characteristic Strain
10 -12
10 -14
10 -16
10 -18
10 -20
10 -22
10 -24
10 -26
Stochastic background
EPTA
IPTA
eLISA
LISA
Massive binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Type IA supernovae
LIGO
aLIGO
Compact binary inspirals
Core collapse supernovae
10 -10
10 -8
10 -6
10 -4
10 -2
10 0
10 2
10 4
10 6
Frequency / Hz

APR~11.1km, H4~13.6km, MS1~14.5km

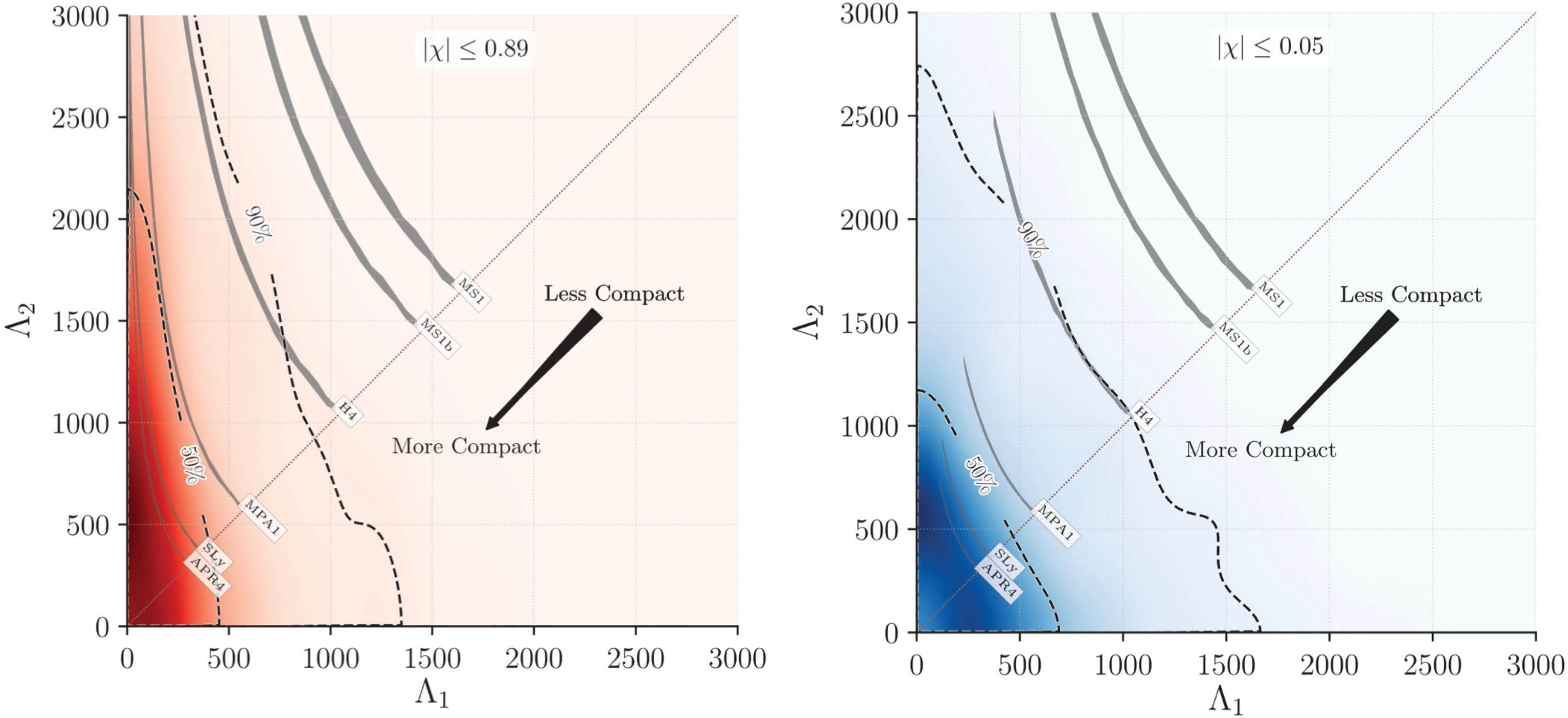


Table 1: Key Properties of GW170817

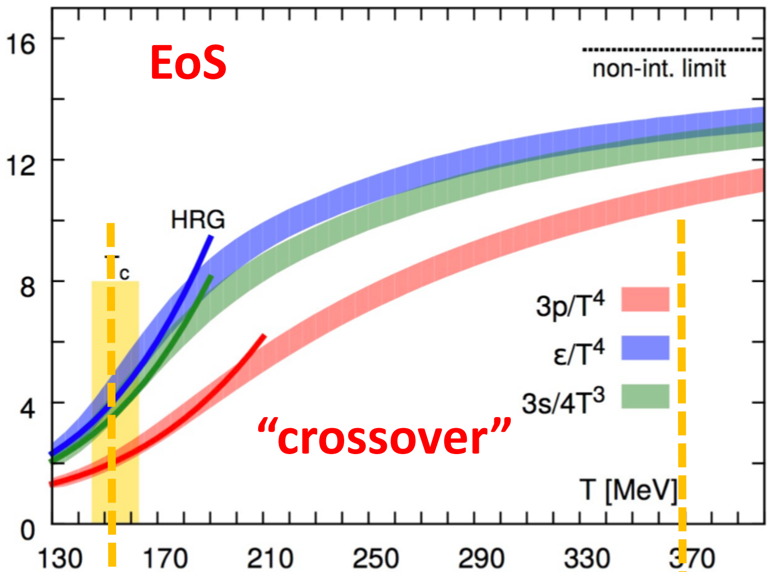
Property	Value	Reference
Chirp mass, $\mathcal{M}$ (rest frame)	$1.188^{+0.004}_{-0.002} M_\odot$	1
First NS mass, M_1	$1.36 - 1.60 M_\odot$ (90%, low spin prior)	1
Second NS mass, M_2	$1.17 - 1.36 M_\odot$ (90%, low spin prior)	1
Total binary mass, $M_{\rm tot} = M_1 + M_2$	$\approx 2.74^{0.04}_{-0.01} M_\odot$	1
Observer angle relative to binary axis, $\theta_{\rm obs}$	$11 - 33^\circ$ (68.3%)	2
Blue KN ejecta ($A_{\rm max} \lesssim 140$)	$\approx 0.01 - 0.02 M_\odot$	e.g., 3,4,5
Red KN ejecta ($A_{\rm max} \gtrsim 140$)	$\approx 0.04 M_\odot$	e.g., 3,5,6
Light r-process yield ($A \lesssim 140$)	$\approx 0.05 - 0.06 M_\odot$	
Heavy r-process yield ($A \gtrsim 140$)	$\approx 0.01 M_\odot$	
Gold yield	$\sim 100 - 200 M_\oplus$	8
Uranium yield	$\sim 30 - 60 M_\oplus$	8
Kinetic energy of off-axis GRB jet	$10^{49} - 10^{50}$ erg	e.g., 9, 10, 11, 12
ISM density	$10^{-4} - 10^{-2}$ cm^{-3}	e.g., 9, 10, 11, 12

(1) LIGO Scientific Collaboration et al. 2017c; (2) depends on Hubble Constant, LIGO Scientific Collaboration et al. 2017d; (3) Cowperthwaite et al. 2017; (4) Nicholl et al. 2017; (5) Kasen et al. 2017; (6) Chornock et al. 2017; (8) assuming heavy r-process ($A > 140$) yields distributed as solar abundances (Arnould et al., 2007); (9)Margutti et al. 2017; (10) Troja et al. 2017; (11) Fong et al. 2017; (12) Hallinan et al. 2017

Delineating QCD matter from HOT EoS

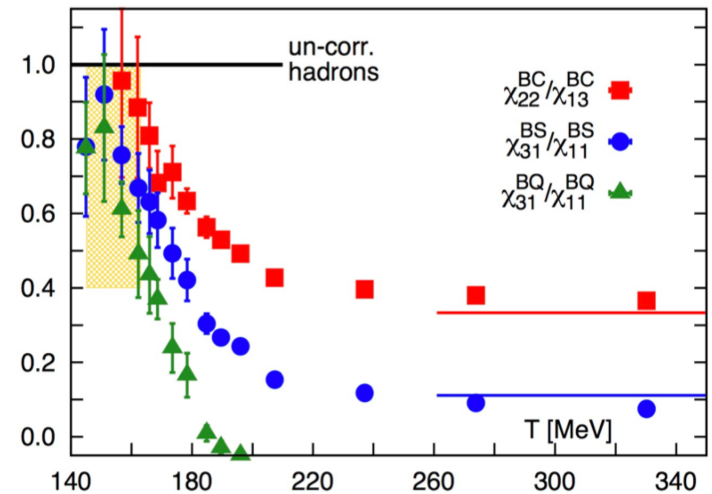
lattice calculations

(Ding-Karsch-Makherjee, review 2015)



Hadron resonance gas | semi-QGP | pQGP

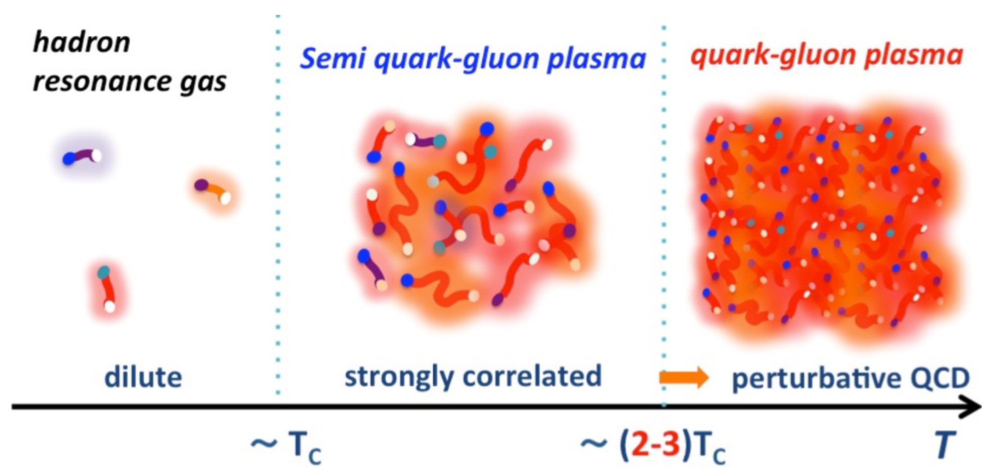
derivatives of EoS



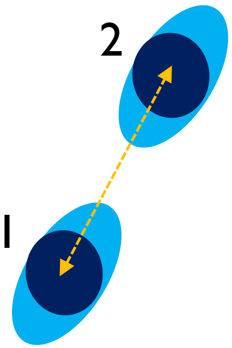
→ T_c: **universal** for different flavors

plausible picture

Fig. from Baym et al. 2018



Dimensionless tidal deformability → R_{NS}



more common to use

$$\Lambda(M) = 32\frac{\lambda G}{R^5} = \frac{2}{3}k_2\left(\frac{R}{GM}\right)^5$$

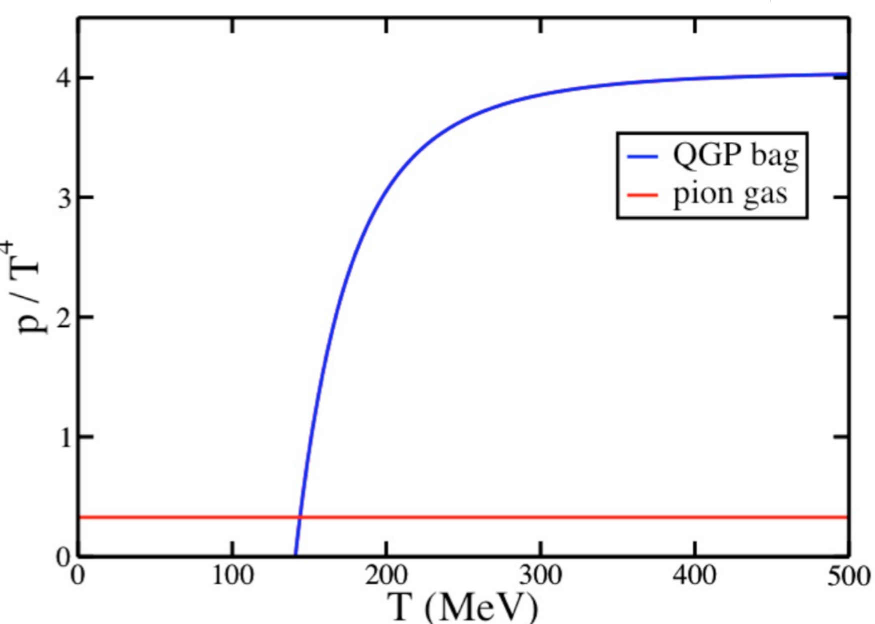
(k_2 : Love number)

What GW analyses measure: combination of Λ for star 1 & 2 :

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$

(measured)

2-parameters: M_1 & M_2



$$= 32\frac{\lambda G}{R^5}$$