RESONANCES AND S-MATRIX APPROACH TO THERMODYNAMICS

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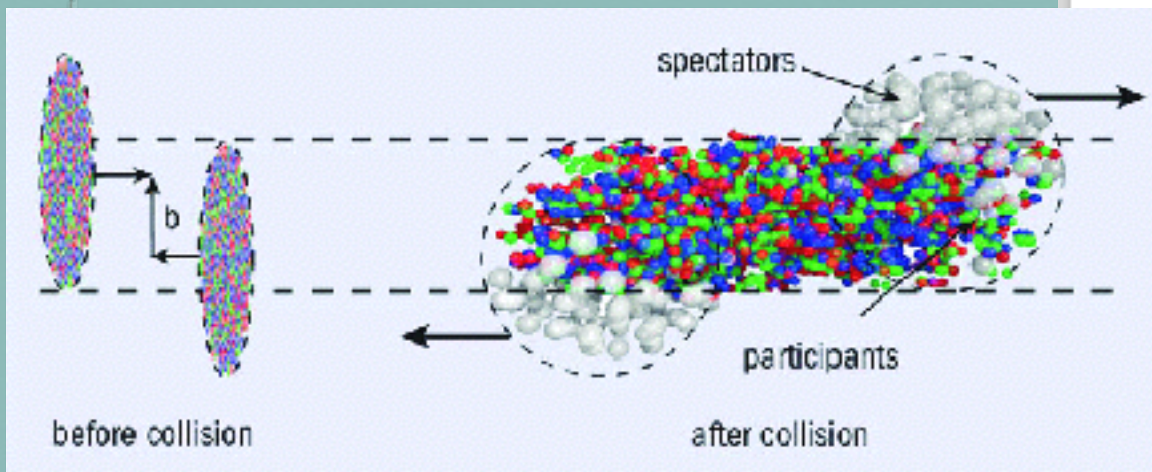
PML, EPJC **77** no.8 533 (2017)

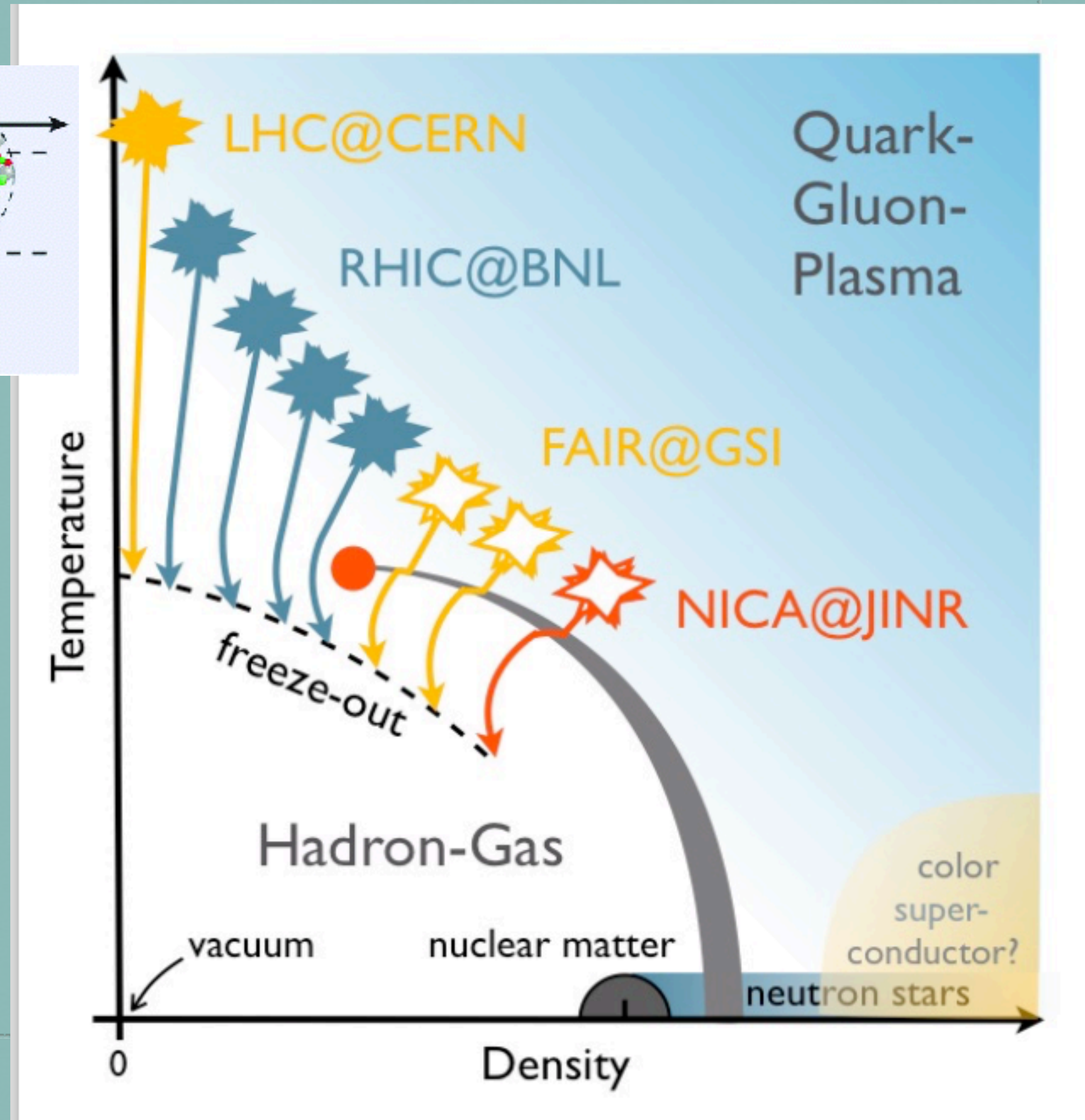
Q&A

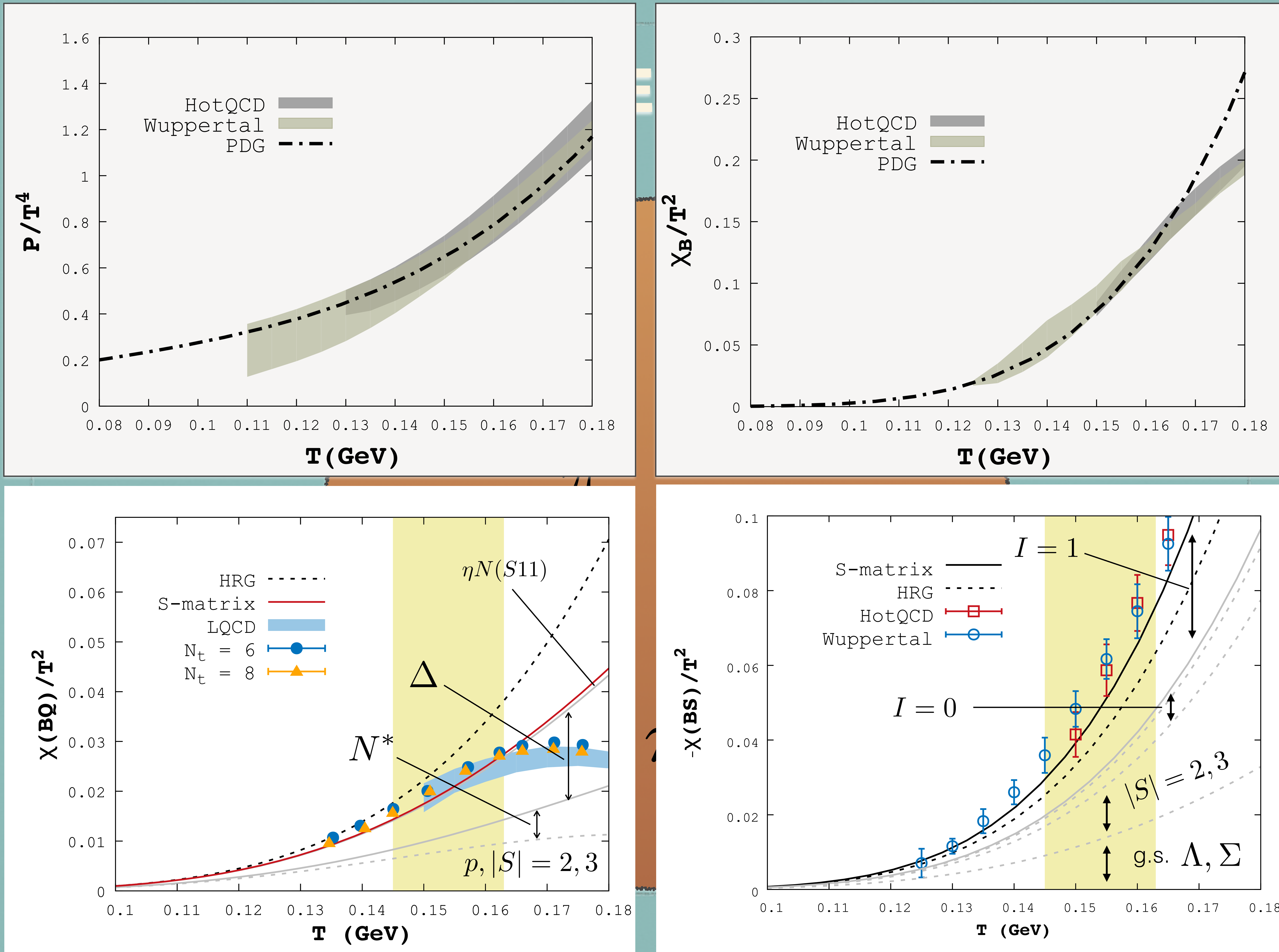
BEAM ENERGY SCAN



$\sqrt{s}, b, (Z, A), \ldots$

$T, \mu, B, \ldots$





HotQCD
Wuppertal
PDG
P/T^4
T(GeV)
HotQCD
Wuppertal
PDG
χB/T^2
T(GeV)
HRG
S-matrix
LQCD
Nt = 6
Nt = 8
ηN(S11)
Δ
N*
p, |S| = 2, 3
χ(BQ)/T^2
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
-χ(BS)/T^2
T (GeV)

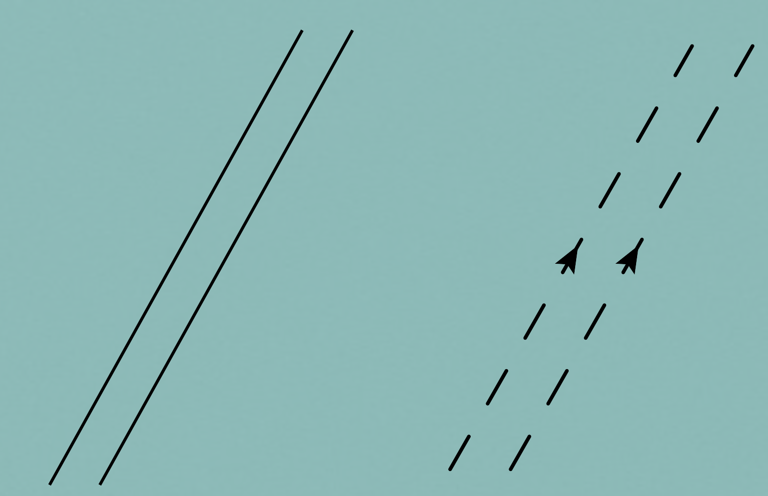
S-MATRIX FORMULATION OF THERMODYNAMICS

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

thermal system:

$$(\pi\pi \leftrightarrow \rho)$$

$$B = A_\rho + \Delta A_{\pi\pi}$$



$|\rho\rangle$ $|\pi\pi\rangle$

$$P = P_\pi + P_\rho + P_{\pi\pi} + \ldots$$

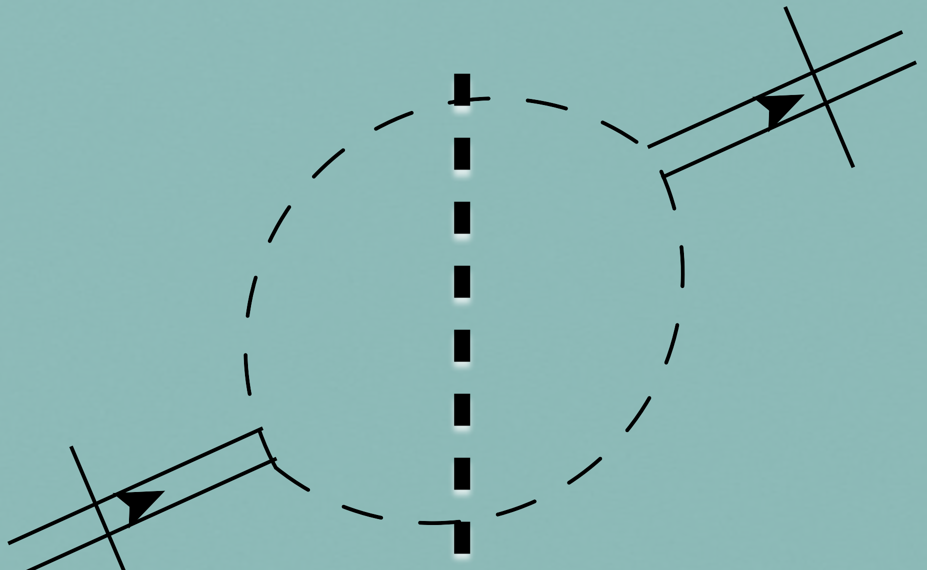
PHYSICS OF B

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

$$B = 2\,\frac{\partial}{\partial E}\delta$$
$$= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1}$$
$$= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E}G_\rho]$$
$$= A_\rho(E) + \Delta A_{\pi\pi}$$

physical interpretation:

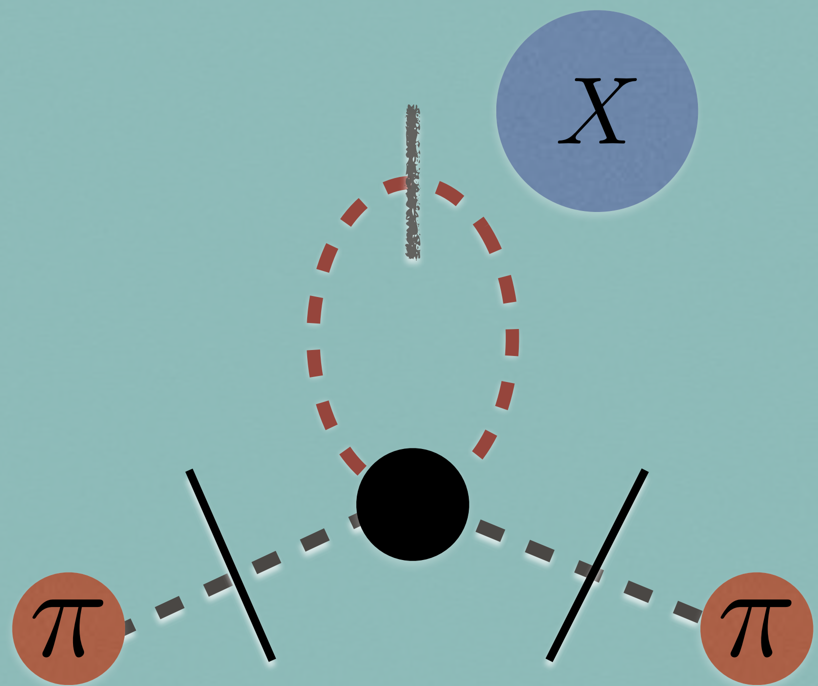
contribution from correlated pi pi pair



$$\frac{\partial \Sigma_\rho}{\partial E}$$

IN-MEDIUM EFFECTS

$$\Sigma_\pi =$$

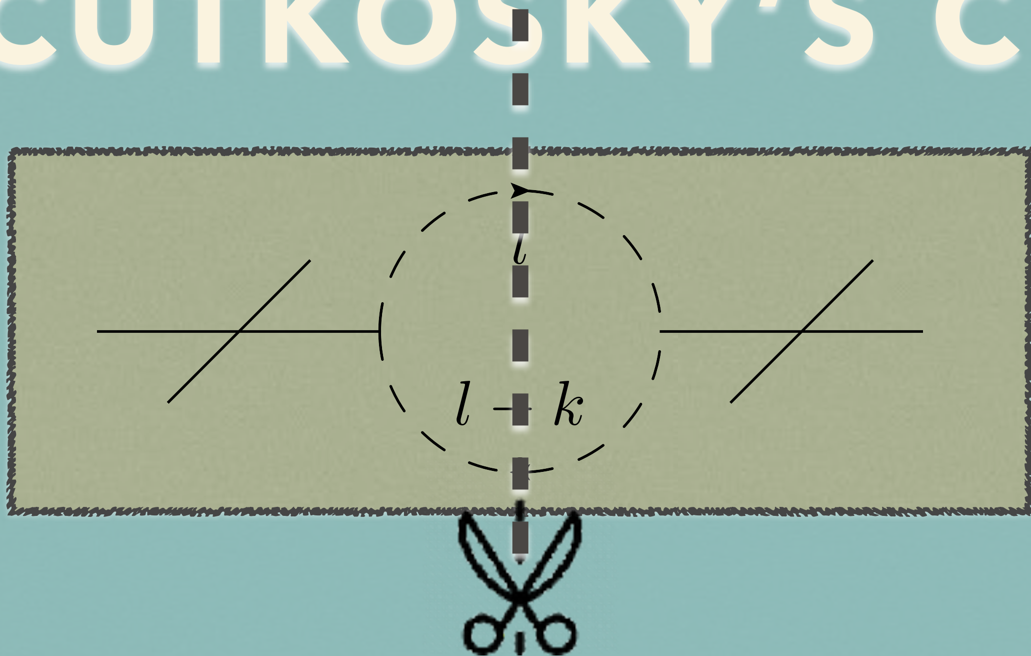


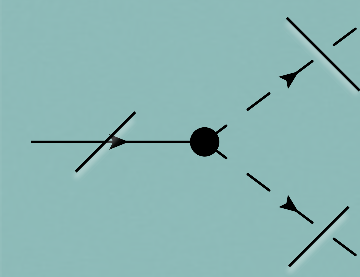
$$\propto \int \frac{d^3q}{\omega_p\,\omega_q}\, n_X \times T_{\pi X}(s)$$

forward amplitude

A. Schenk NPB 363(1991)

CUTKOSKY'S CUTTING RULES

Im 

= $\int d\phi \, \Big|$  $\Big|^2$

Phase space approach

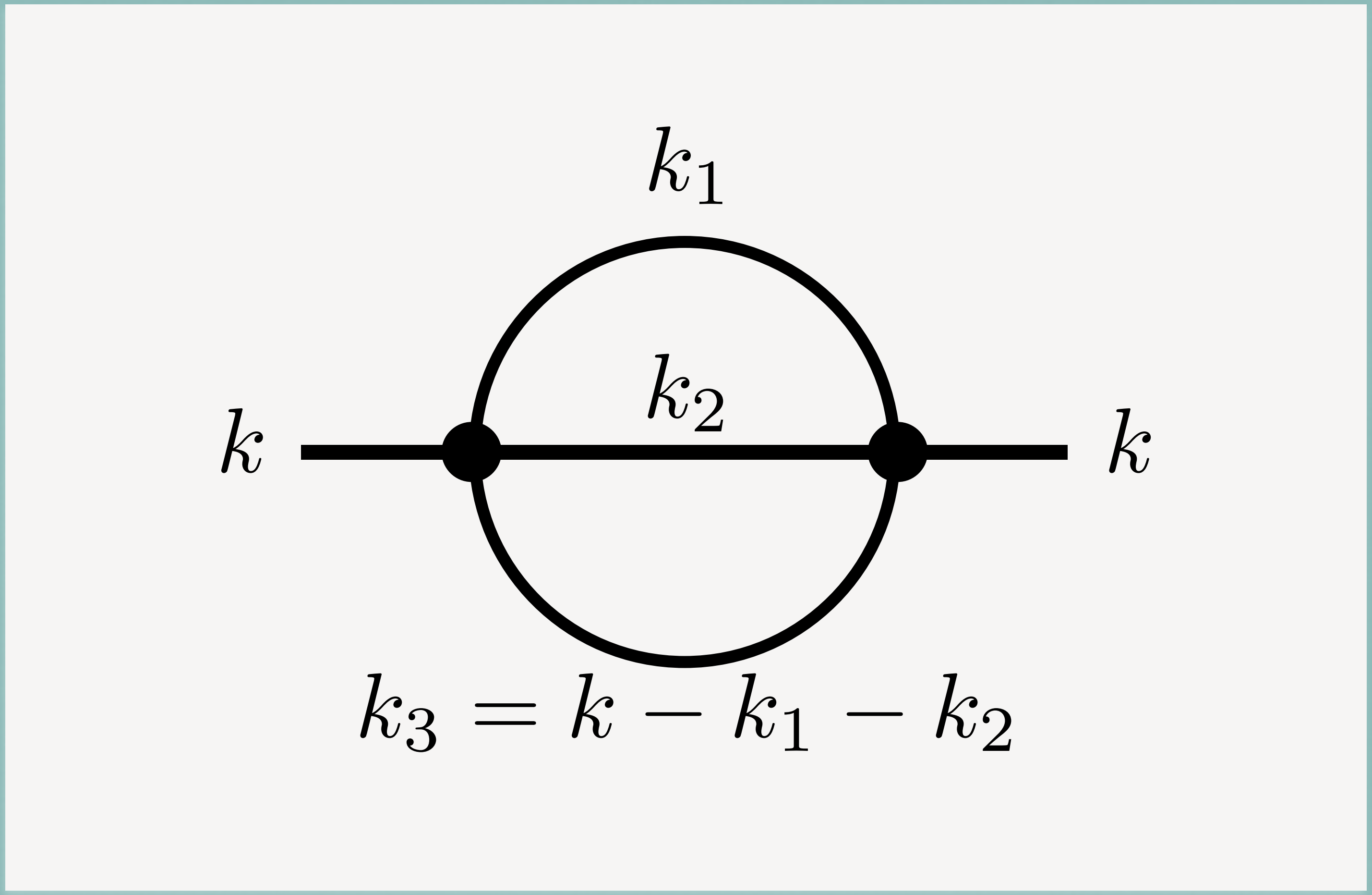
$$-\mathrm{Im}\,\Sigma_\sigma = M\Gamma = \frac{1}{2}\int d\phi_2\, |\Gamma_{\sigma\to\pi\pi}|^2$$

$$\int d\phi_2\,(\cdots) \to \int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\times$$

$$(2\pi)^4\,\delta^4(P_I - \sum_i p_i)\,(\cdots).$$

$$\to \quad \frac{1}{2}\times\frac{q}{4\pi M}\times g^2\times 2$$

SUNSET DIAGRAM

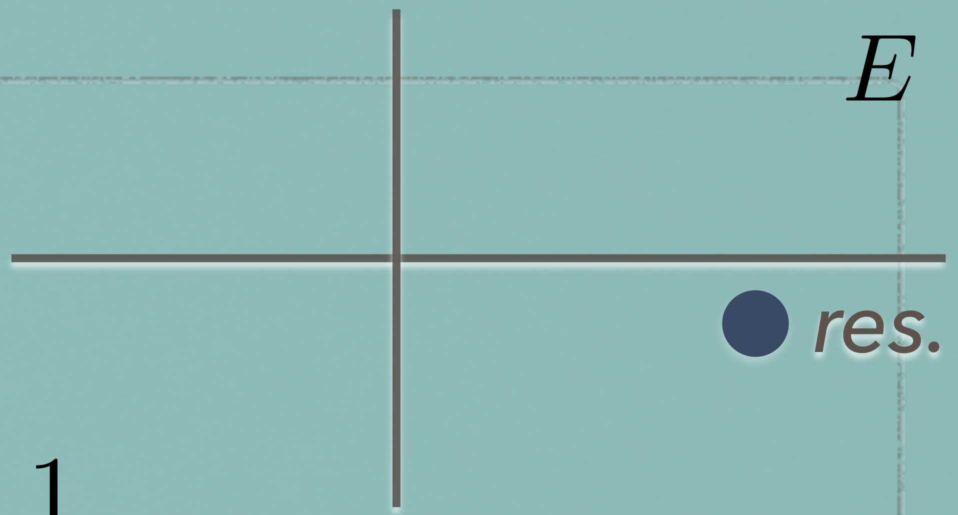


$$\mathrm{Im}\, I \propto \frac{1}{2} \int d\phi_3 \left| \Gamma_{s \to \pi\pi\pi} \right|^2$$

LECTURE III

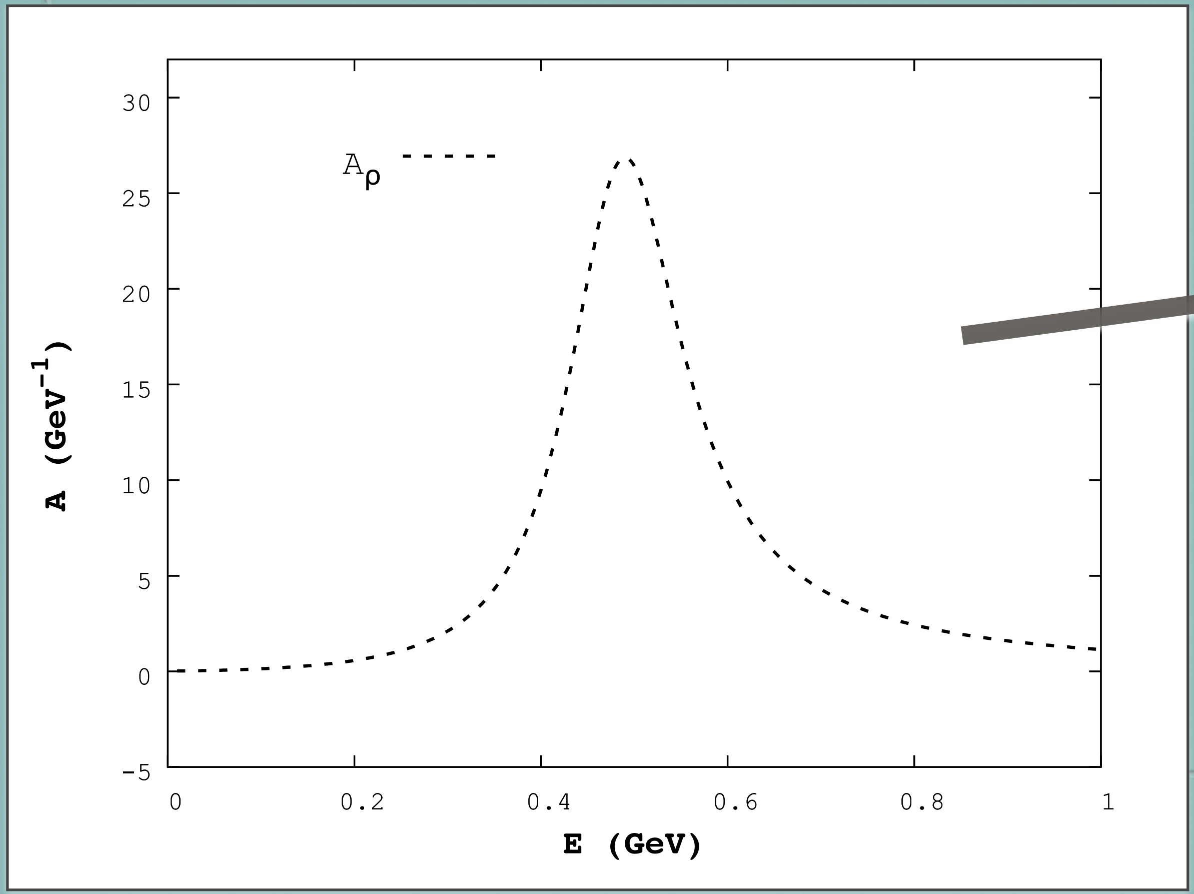
S-MATRIX POLOLOGY

POLE OF S-MATRIX



$$G = \frac{1}{E - m_0 - \Sigma} \rightarrow \frac{1}{E - \epsilon_R + i\,\frac{1}{2}\gamma_E}$$

$$A = -2\,\mathrm{Im}G \rightarrow \frac{\gamma_E}{(E-\epsilon_R)^2 + \frac{1}{4}\gamma_E^2}$$

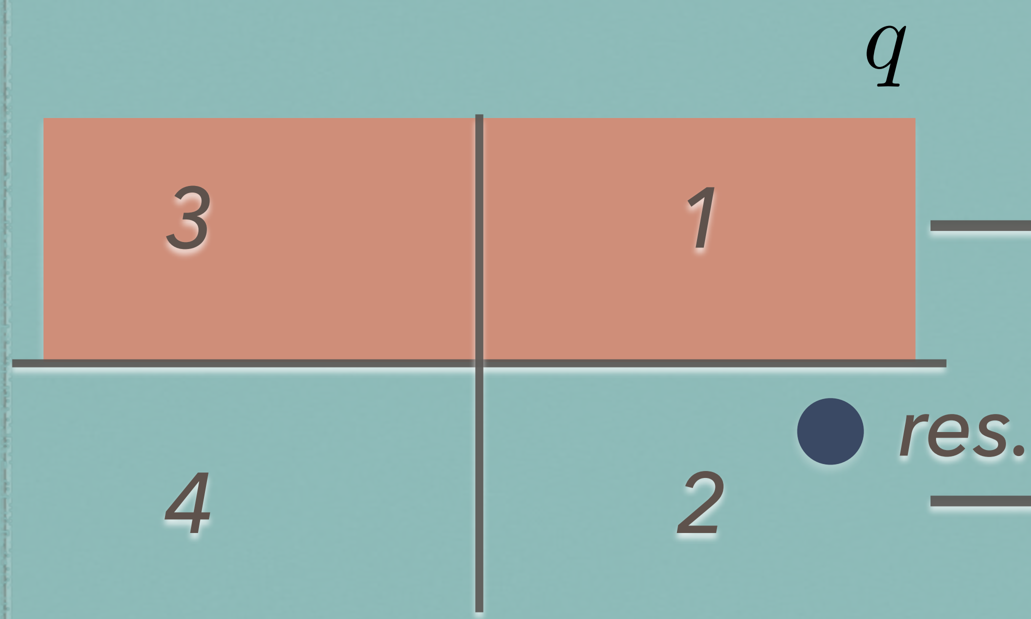


Resonance pole =
Divergence at

$$E = \epsilon_R - i\,\frac{1}{2}\gamma_E$$

RIEMANN SHEET

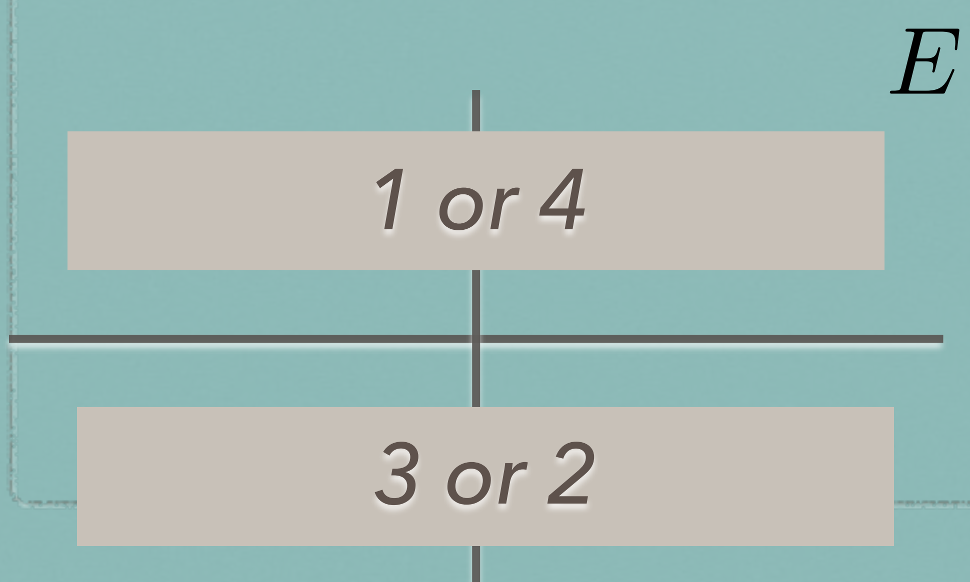
$$E = \frac{q^2}{2m_r}$$



$\mathrm{Im}\, q > 0$ → *physical sheet*

$\mathrm{Re}\, q > 0$ → *2nd sheet*

each sheet can cover the whole complex-E plane → *E1, E2 sheets*



MOVING POLE

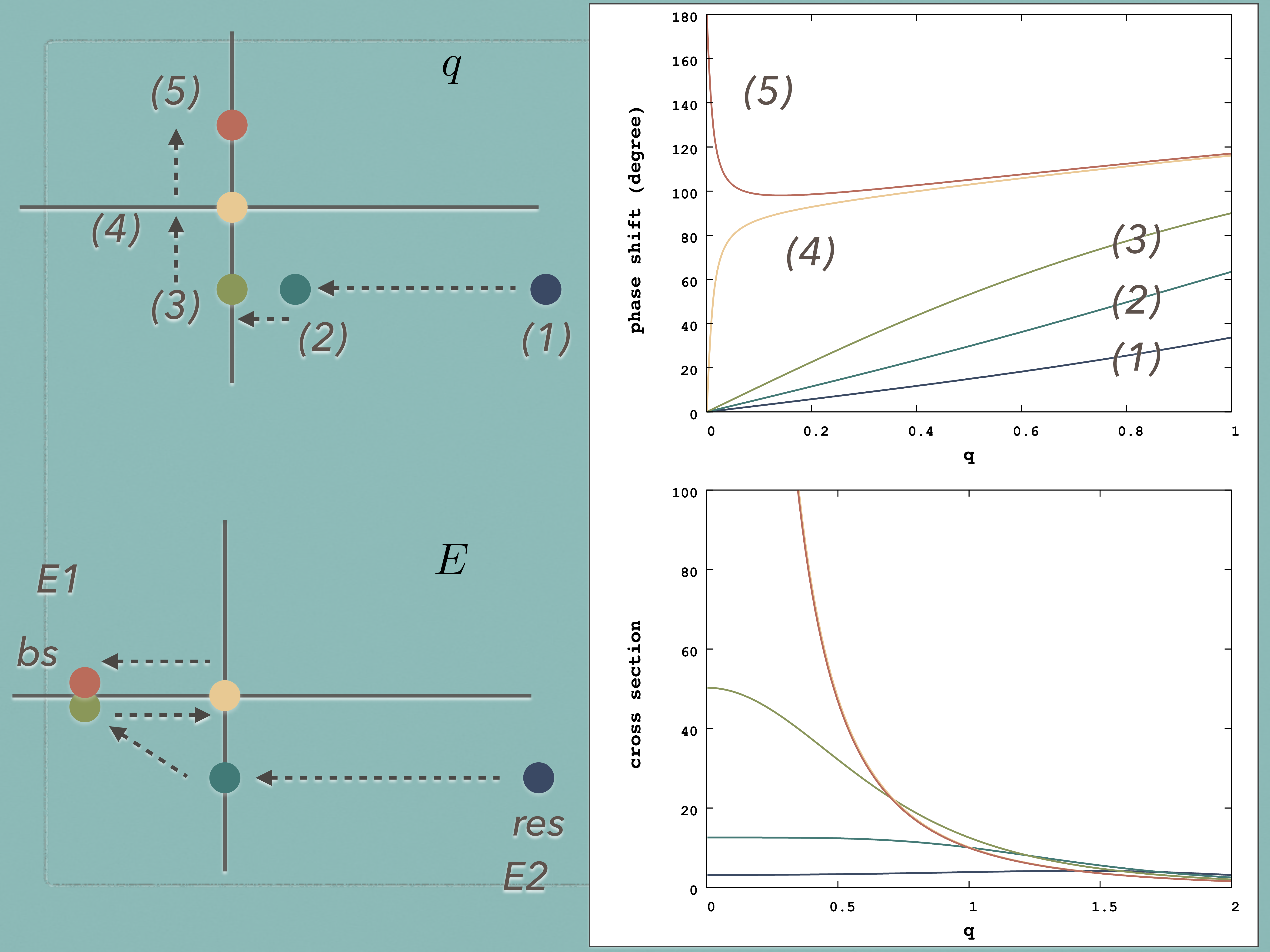
$$G(E) = \frac{1}{E - \epsilon_R + i\,\frac{1}{2}\gamma_E}$$

$G^{-1}(E) = 0$ *at* $E \to E_{\rm pole} = \dfrac{q_{\rm pole}^2}{2m_r}$

s-wave: $\gamma_E \propto g^2 \phi_2 \propto g^2 q$

study $q_{\rm pole}$ *&* $E_{\rm pole}$ *as* ϵ_R *or* g *changes*

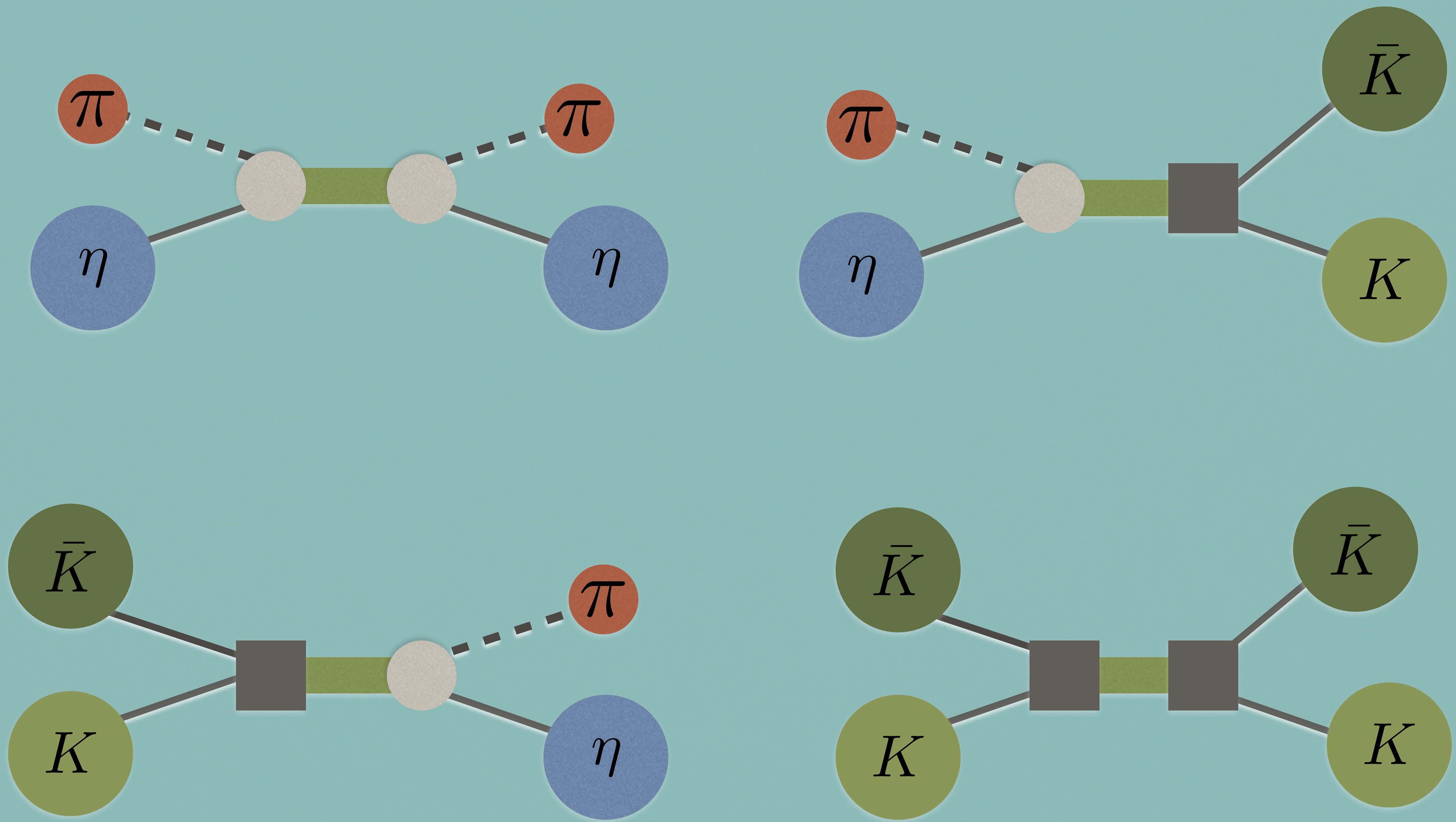
K.W. McVoy, Nucl. Phys. A, 115, 481 (1968)


q
(5)
(4)
(3)
(2)
(1)
E
E1
bs
res
E2
phase shift (degree)
180
160
140
120
100
80
60
40
20
0
0
0.2
0.4
0.6
0.8
1
q
(5)
(4)
(3)
(2)
(1)
cross section
100
80
60
40
20
0
0
0.5
1
1.5
2
q

COUPLED CHANNEL SYSTEM

C. Fernandez-Ramirez, PML, and P. Petreczky,
arXiv:1806.02177 [hep-ph].

1 RES. 2 CHANNELS PROBLEM



COUPLED-CHANNEL PROBLEM

$$\{\gamma_1, \gamma_2, m_{\rm res}\} \longleftrightarrow \{\delta_1, \delta_2, \eta\}$$

$$S(s) = 1 + i\,\hat{T}(s),$$

$$\hat{T}(s) = \frac{-2\sqrt{s}\,\gamma_{\rm res}}{s - m_R^2 + i\sqrt{s}\,\gamma_{\rm res}} \times \hat{t}$$

$$\hat{t} = \frac{1}{g_a^2\,\phi_a + g_b^2\,\phi_b} \begin{pmatrix} g_a^2\,\phi_a & g_a g_b\sqrt{\phi_a\phi_b} \\ g_a g_b\sqrt{\phi_a\phi_b} & g_b^2\,\phi_b \end{pmatrix}.$$

WIGNER, EISENBUD, SMITH, …

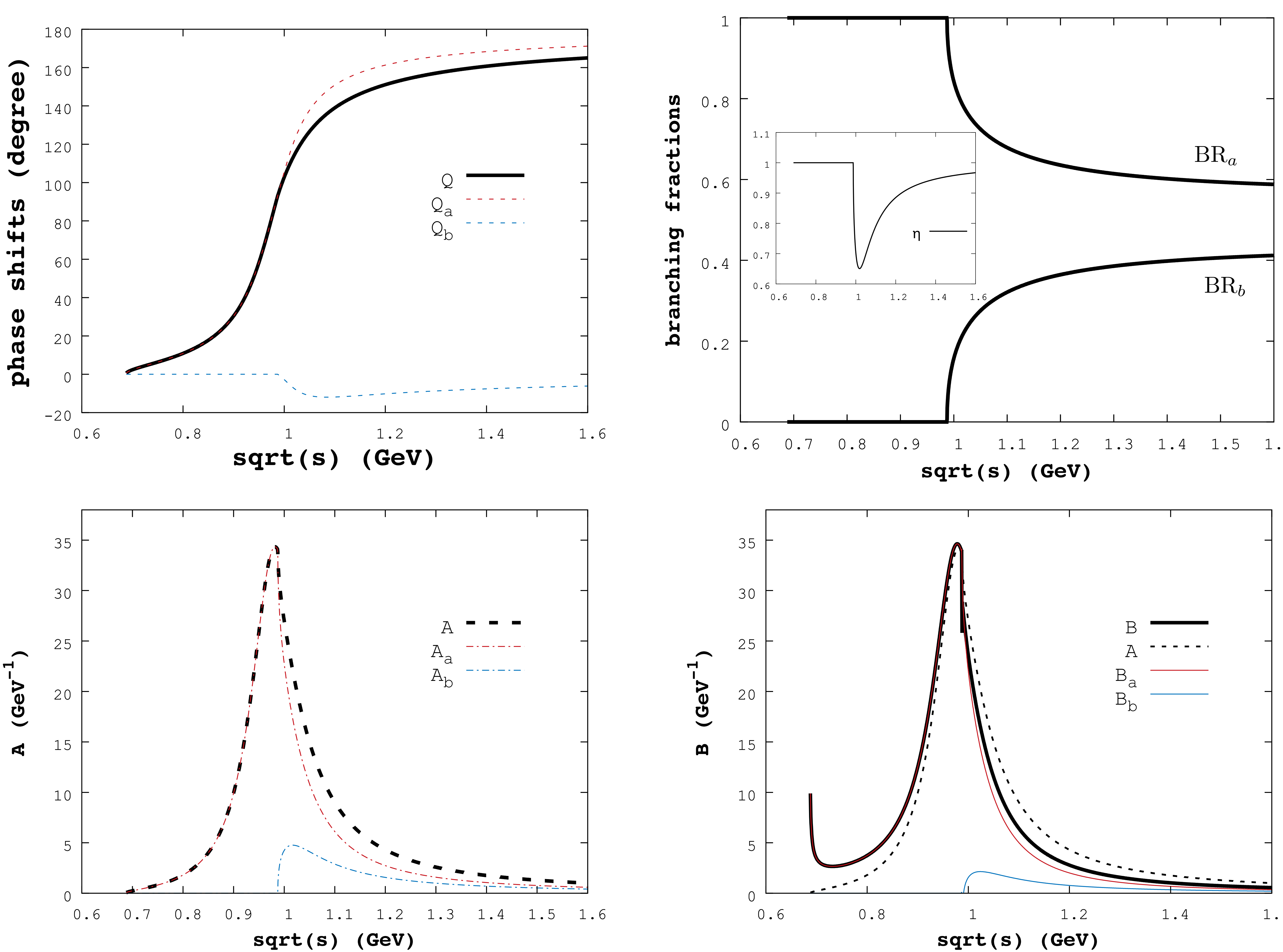
$$S \to U^\dagger S_d U$$

$$S_d = \begin{pmatrix} e^{2i\delta_{\rm res}(s)} & 0 \\ 0 & 1 \end{pmatrix},$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

$$\mathrm{BR}_a = \cos^2\theta = \frac{g_a^2\,\phi_a}{g_a^2\,\phi_a + g_b^2\,\phi_b},$$

$$\mathrm{BR}_b = \sin^2\theta = \frac{g_b^2\,\phi_b}{g_a^2\,\phi_a + g_b^2\,\phi_b}.$$



phase shifts (degree)
sqrt(s) (GeV)
Q
Q_a
Q_b
branching fractions
BR_a
BR_b
η
A (GeV^-1)
A
A_a
A_b
B (GeV^-1)
B
A
B_a
B_b

COUPLED-CHANNEL PROBLEM

$$\{\gamma_1, \gamma_2, m_{\rm res}\} \longleftrightarrow \{\delta_1, \delta_2, \eta\}$$

$$S = \begin{pmatrix} \eta\, e^{2\,i\,\delta_I} & i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} \\ i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} & \eta\, e^{2\,i\,\delta_{II}} \end{pmatrix}$$

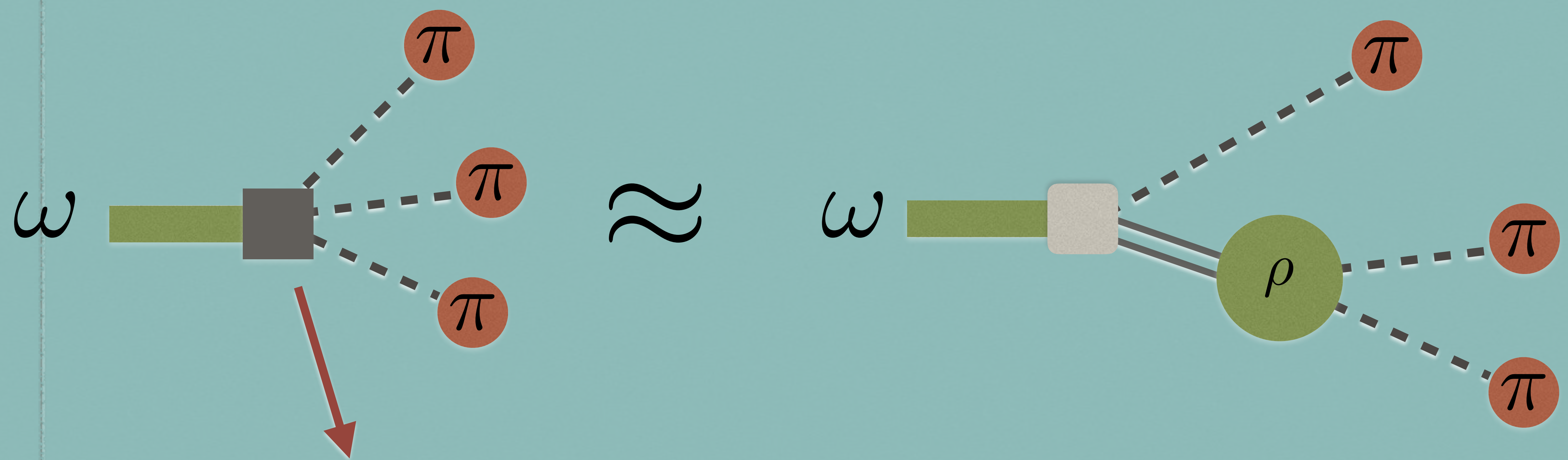
$$\begin{aligned} \mathcal{Q}(M) &\equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln \mathrm{S}) \\ &= \frac{1}{2}\,\mathrm{Im}\,(\ln\,\det\,[\mathrm{S}]) \\ &= \delta_I + \delta_{II}. \end{aligned}$$

$a_0(980)$ *system*

$$\pi\eta \to \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \to \pi\eta$$

$$K\bar{K} \to \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \to K\bar{K}$$

OMEGA TO 3 PI: GSW

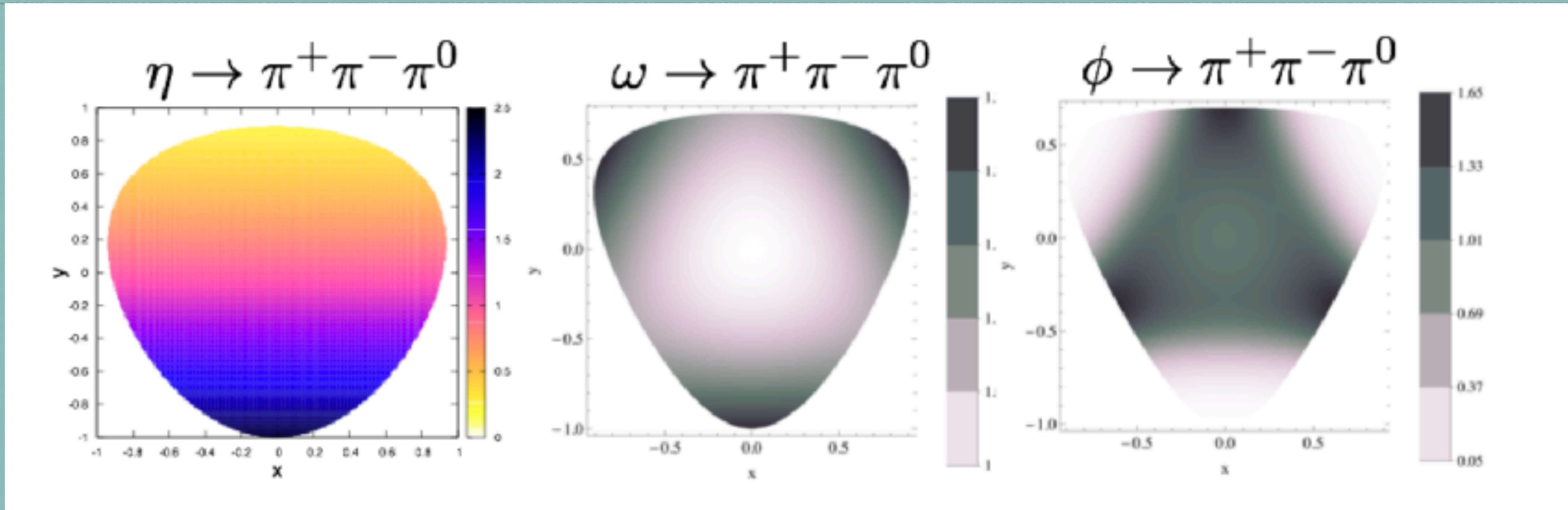


$$|\Gamma_{\omega\to3\pi}|^2 = \mathcal{P} \times |C_{V\to123}|^2$$

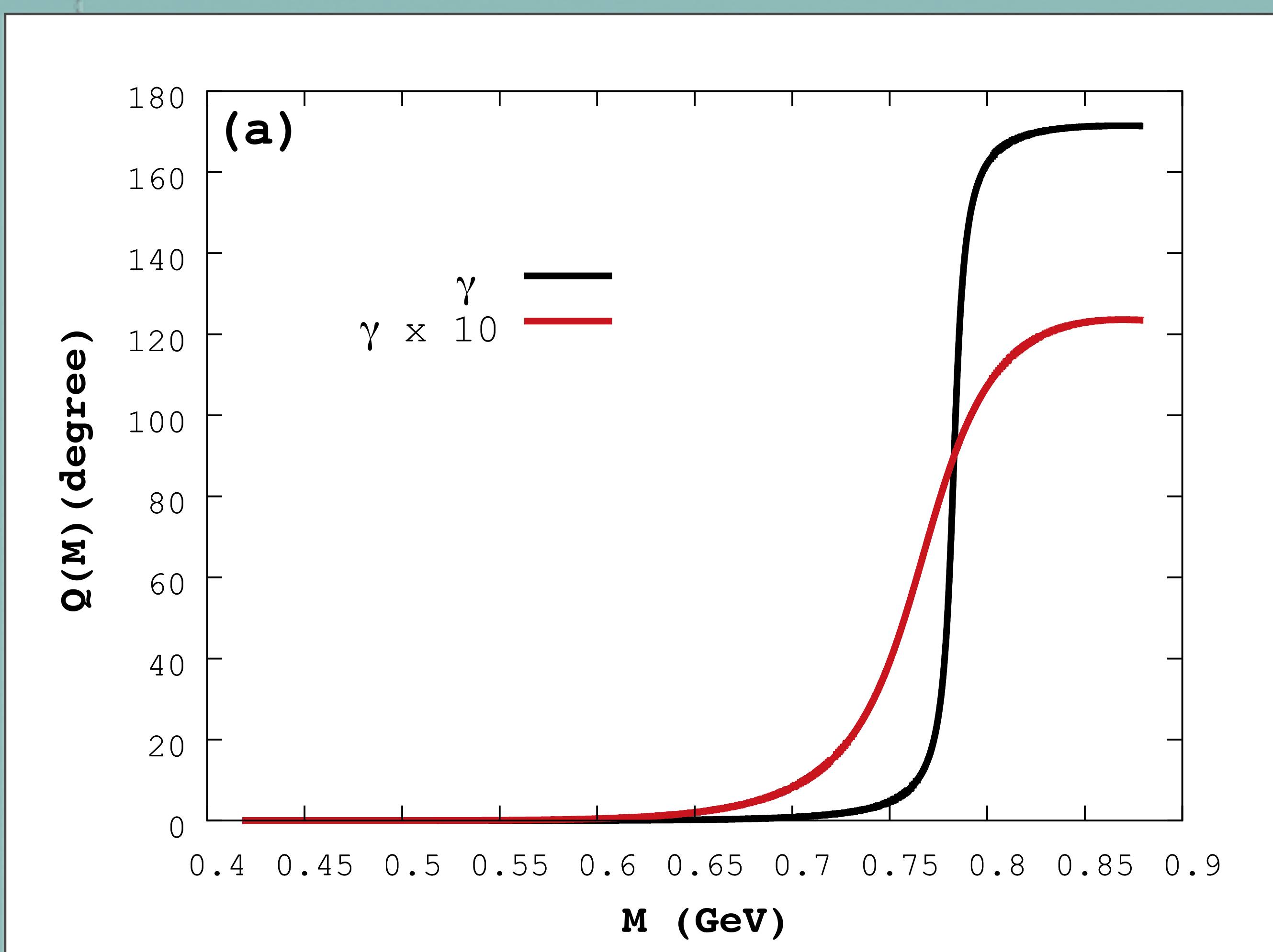
$$\mathcal{P} = -\frac{1}{3}\epsilon_{\mu\nu\alpha\beta}\epsilon_{abcd}P^{\mu}\, p_1^{\nu}\, p_2^{\alpha}\, P^{a}\, p_1^{b}\, p_2^{c}\, g^{\beta d}$$

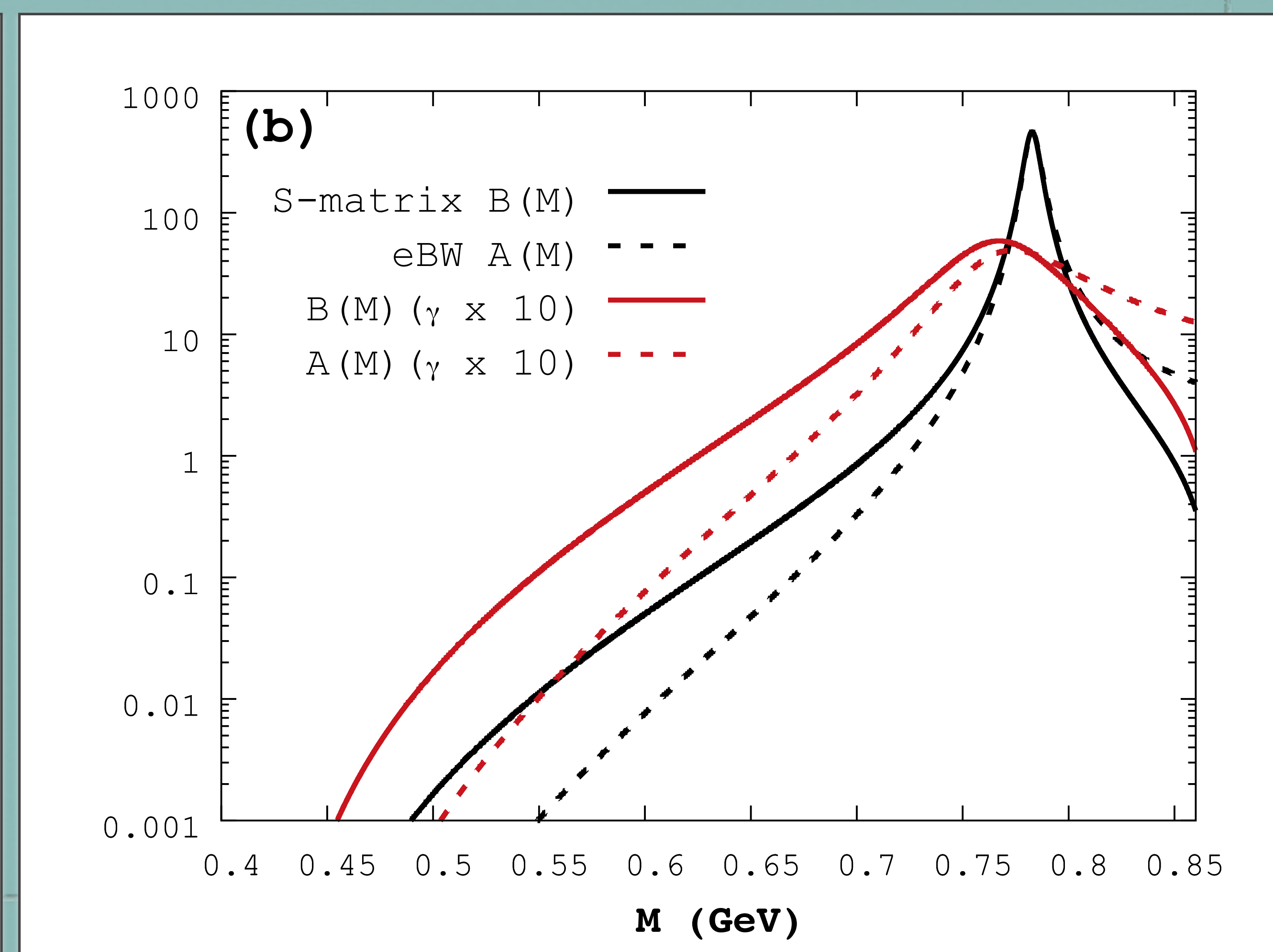
$$= \frac{1}{12} \times \left(s_{12}s_{23}s_{13} - m_\pi^2(m_{\text{res}}^2 - m_\pi^2)^2\right),$$

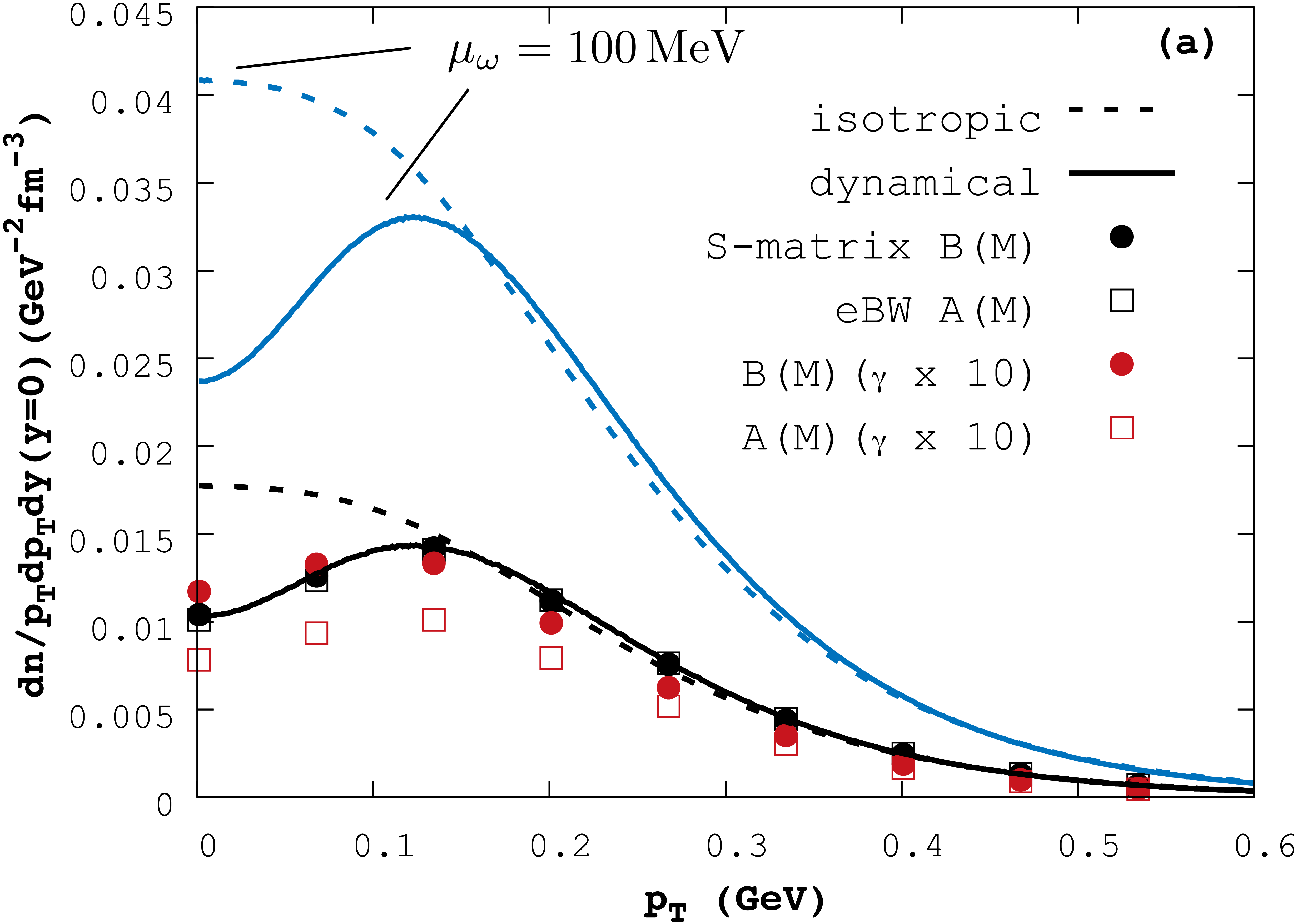
Dalitz info. <=>``3-body phase shift"



JPAC: http://www.indiana.edu/~jpac/







PML, PRC **97**, no. 3, 035210 (2018)

N-BODY SCATTERING

PML, EPJC **77** no.8 533 (2017)

WHY N-BODY? (WHY NOT?)

- EOS for dense system
  -> nuclear matter
  -> ultracold fermi gas (dimer)
- Explore the influence of N-body scatterings on heavy ion collision observables:
  pT-spectra, flow etc.
- phenomenology
  -> model S-matrix element instead…

RECIPE

Feynman amplitude

- generalized phase shift

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$$

$$d\phi_N = \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\cdots\frac{d^3p_N}{(2\pi)^3}\frac{1}{2E_N}\times (2\pi)^4\,\delta^4(P - \sum_i p_i).$$

phase space approach

PHASE SPACE DOMINANCE

$$\mathcal{Q}_N(M) = \frac{1}{2}\,\mathrm{Im}\left[\ln\left(1 + \int d\phi_N\, i\mathcal{M}\right)\right]$$

- structureless scattering

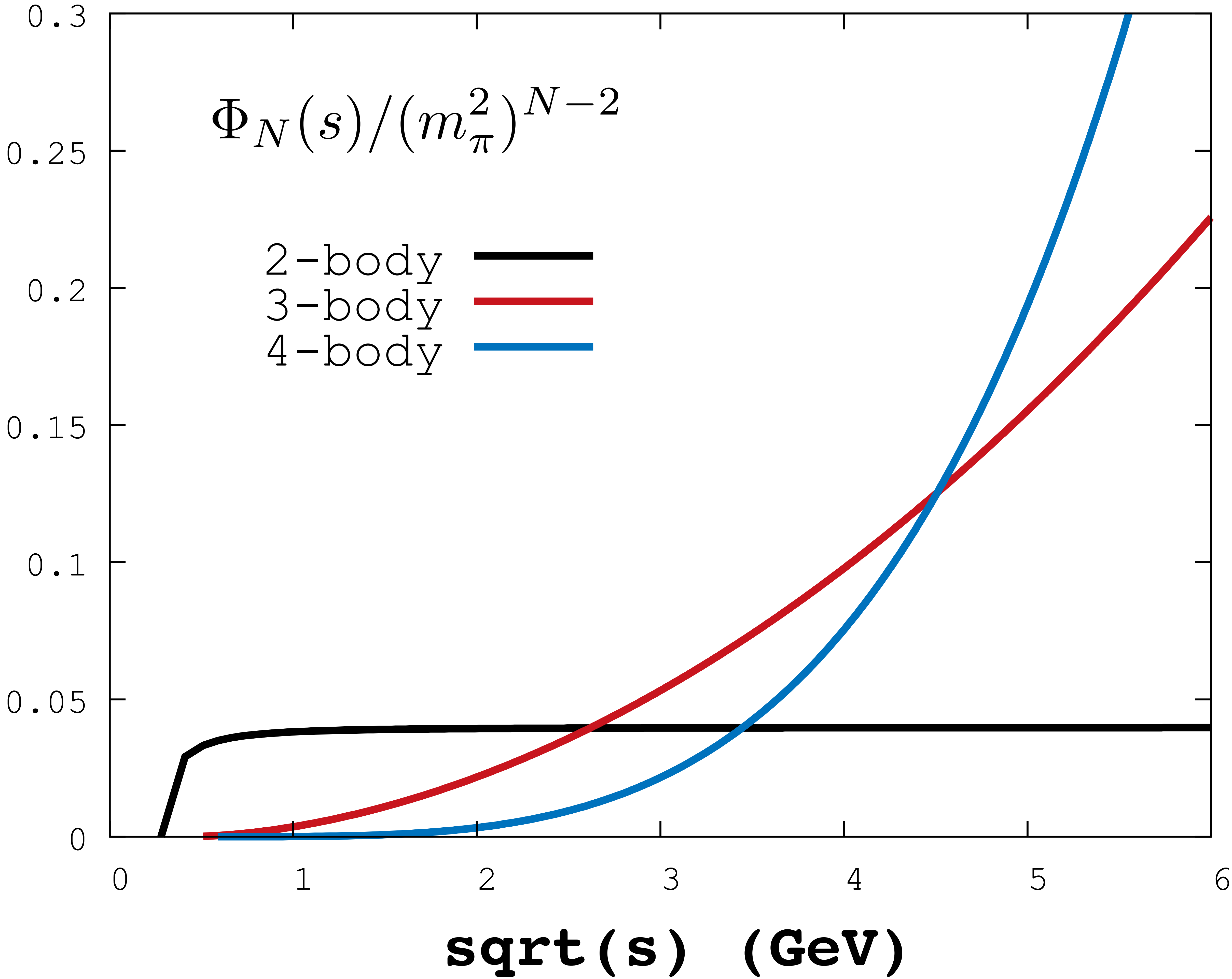
Dimension: $\sim E^{2N-4}$

$$i\mathcal{M} = i\,\lambda_N$$

Källén triangle function

$$\phi_N(s) = \frac{1}{16\,\pi^2 s}\int_{s'_-}^{s'_+} ds'\,\sqrt{\lambda(s, s', m_N^2)}\,\times$$

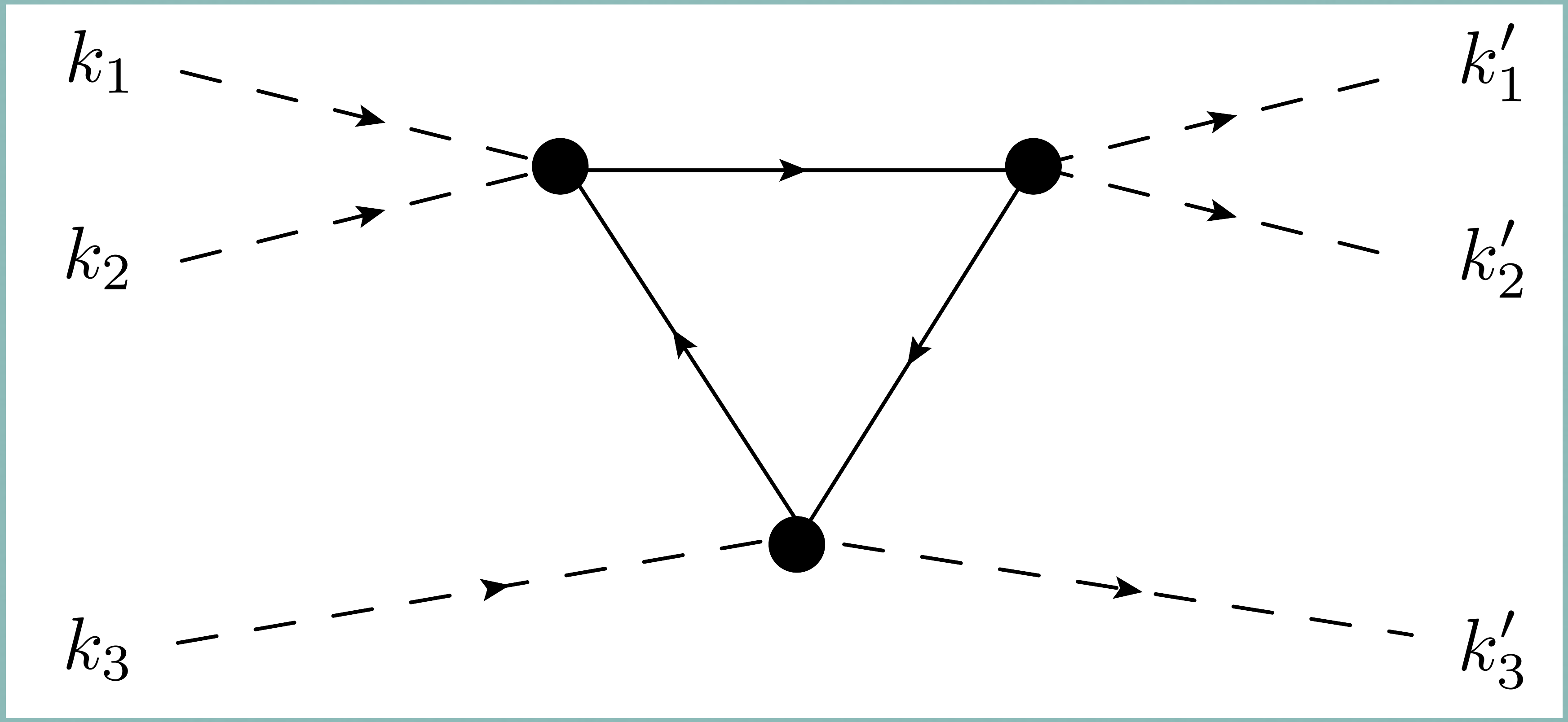
$$\phi_{N-1}(s', m_1^2, m_2^2, ..., m_{N-1}^2)$$



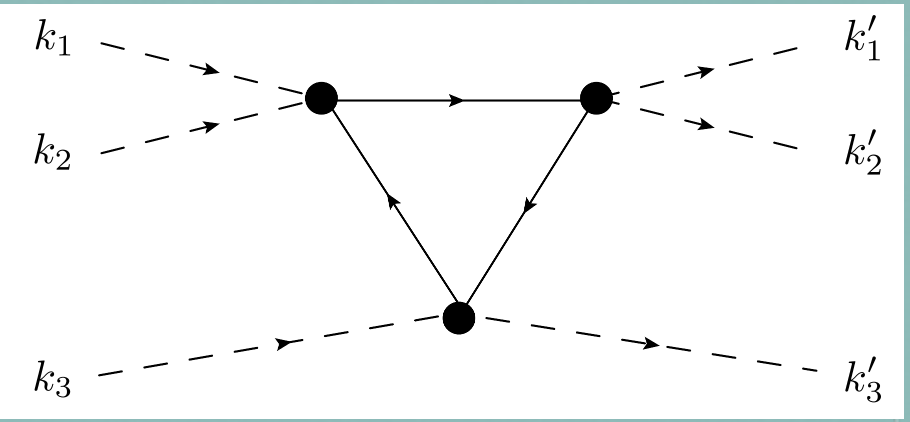
0.3
0.25
0.2
0.15
0.1
0.05
0
$\Phi_N(s)/(m_\pi^2)^{N-2}$
2-body
3-body
4-body
0
1
2
3
4
5
6
sqrt(s) (GeV)

TRIANGLE DIAGRAM

- 3-body diagram



Explicit calculation



$$i\mathcal{M}^{\triangle}(Q_1, Q_2, Q_3) = \int \frac{d^4 l}{(2\pi)^4} \times (-i\,\lambda)^3 \times i\,G(l) \times i\,G(l+Q_1) \times i\,G(l-Q_2)$$

Feynman's trick + dim reg.

$$i\mathcal{M}^{\triangle}(Q_1^2, Q_2^2, s = P_I^2) = -i\frac{\lambda^3}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \, \frac{1}{\Delta(x,y)}$$

$$\Delta(x,y) = m_\pi^2 - x(1-x)\,Q_1^2 - y(1-y)\,Q_2^2 - 2\,xy\,Q_1\cdot Q_2 - i\epsilon.$$

- to lowest order $$\mathcal{Q}(s) \approx \frac{1}{2}\,\mathrm{Im}\left[\int d\phi_3\, i\mathcal{M}^{\text{triangle}}\right],$$

=> only need to deal with on-shell condition

$$k_i' = k_i$$

analytic result:

$$i\mathcal{M}^{\triangle,o.s.}(Q_1^2, s) = -i\frac{\lambda^3}{16\,\pi^2}\,\frac{z}{Q_1^2}\,\ln\frac{1-z}{1+z}$$

$$z = \frac{1}{\sqrt{1-\frac{4m_\pi^2}{Q_1^2}}}.$$

$$\mathcal{Q}(s) \approx \frac{1}{2}\,\mathrm{Im}\left[\int d\phi_3\, i\mathcal{M}^{\mathrm{triangle}}\right],$$

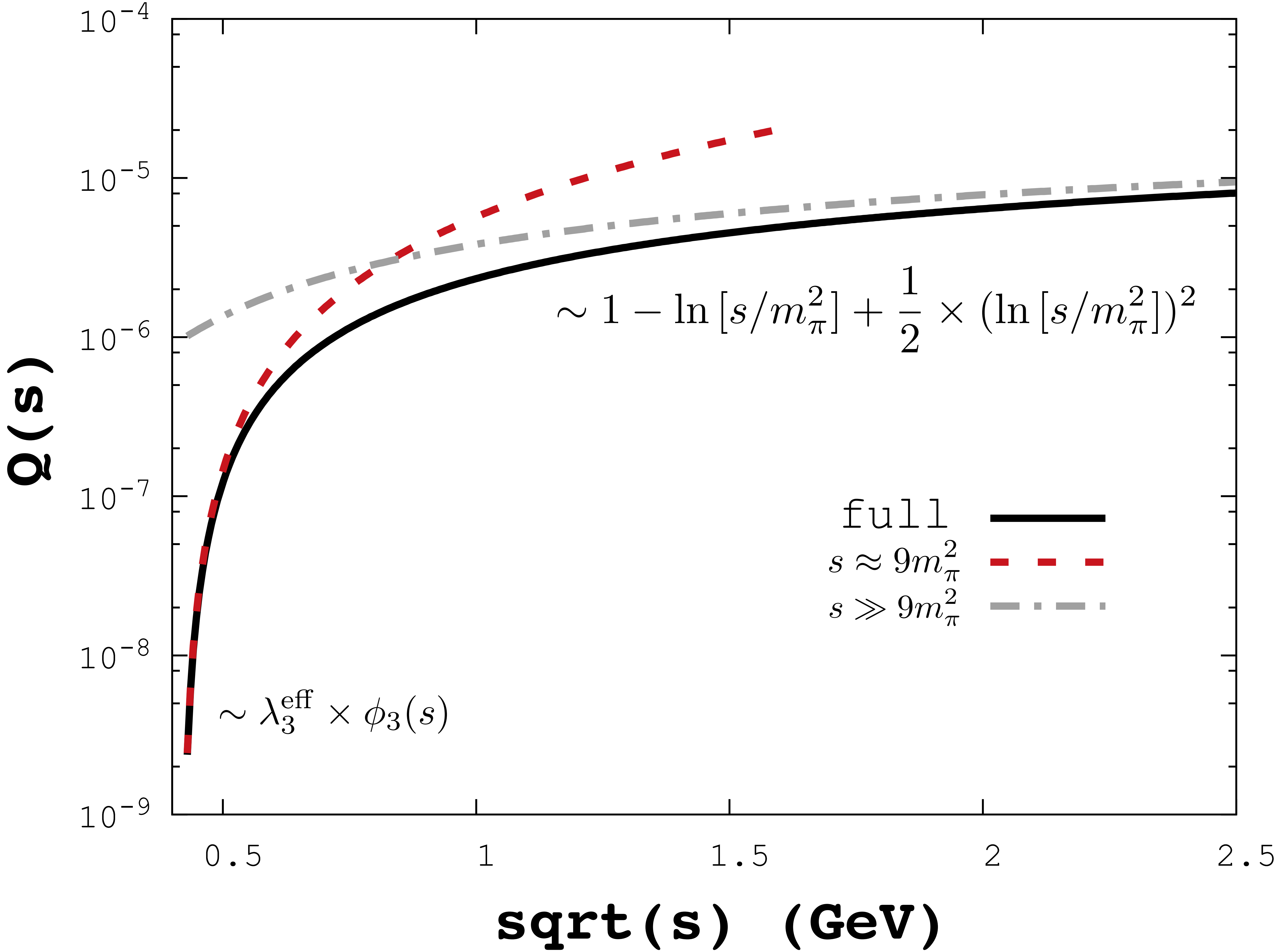
Limits:

$s \to 9m_\pi^2$ $$\mathcal{Q}(s) \approx \frac{1}{2} \times \lambda_3^{\mathrm{eff}} \times \phi_3(s).$$

$s \gg 9m_\pi^2$ $$\mathcal{Q}(s) \approx \frac{\lambda^3}{8192\,\pi^5} \int_{\xi_0}^{1} d\xi\,(\frac{1}{\xi} - 1)\,\left[-z\,\ln\left|\frac{1-z}{1+z}\right|\right]$$

$$\approx \frac{\lambda^3}{4096\,\pi^5} \times \left[1 + \ln\frac{\xi_0}{4} + (\ln\frac{\xi_0}{4})^2\right]$$

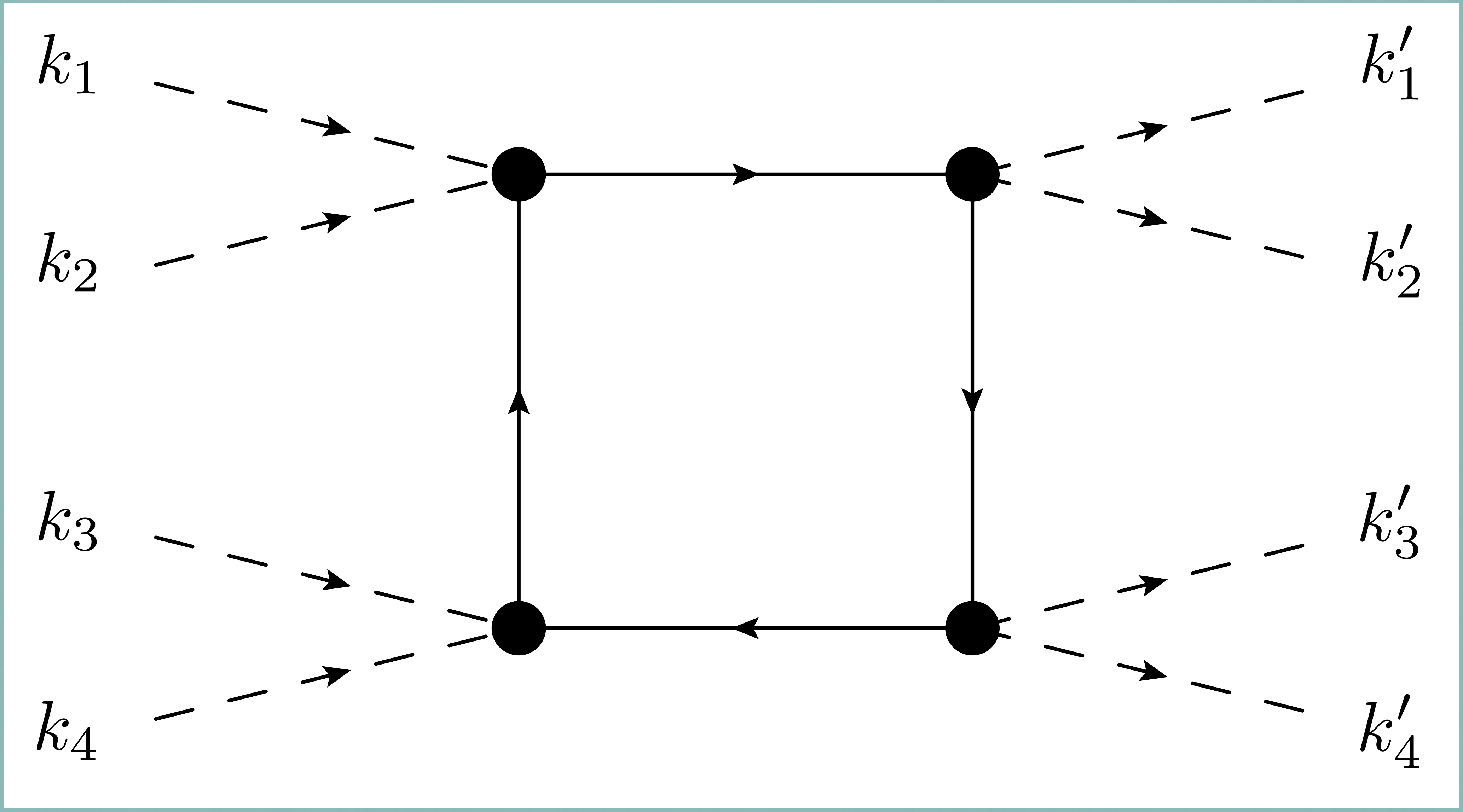
where $$\xi_0 = \frac{4m_\pi^2}{s}.$$



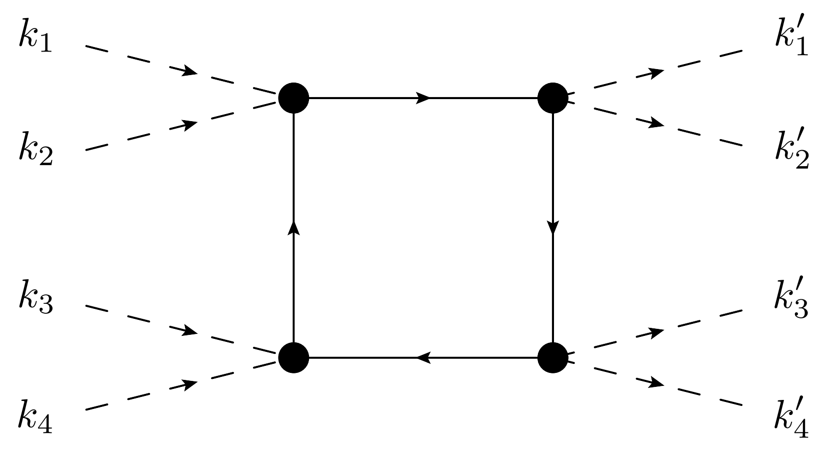
Q(s)
sqrt(s) (GeV)
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0.5
1
1.5
2
2.5
$\sim 1 - \ln[s/m_\pi^2] + \frac{1}{2} \times (\ln[s/m_\pi^2])^2$
$\sim \lambda_3^{\rm eff} \times \phi_3(s)$
full
$s \approx 9m_\pi^2$
$s \gg 9m_\pi^2$

BOX DIAGRAM

- 4-body diagram



Explicit calculation



$$i\mathcal{M}^{\rm box}(Q_1,Q_2,Q_3,Q_4) = \int \frac{d^4 l}{(2\pi)^4}\,(-i\,\lambda)^4 \times i\,G(l) \times i\,G(l+Q_1) \times i\,G(l+Q_1-Q_3) \times i\,G(l-Q_2)$$

Feynman's trick + dim reg.

$$i\mathcal{M}^{\rm box}(Q_1,Q_2,Q_3,Q_4) = i\frac{\lambda^4}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \times \int_0^{1-x-y} dz \times \left(\frac{1}{\Delta(x,y,z)}\right)^2$$

$$\mathcal{Q}(s) \approx \frac{1}{2}\,\mathrm{Im}\left[\int d\phi_4\, i\mathcal{M}^{\mathrm{box,o.s.}}\right].$$

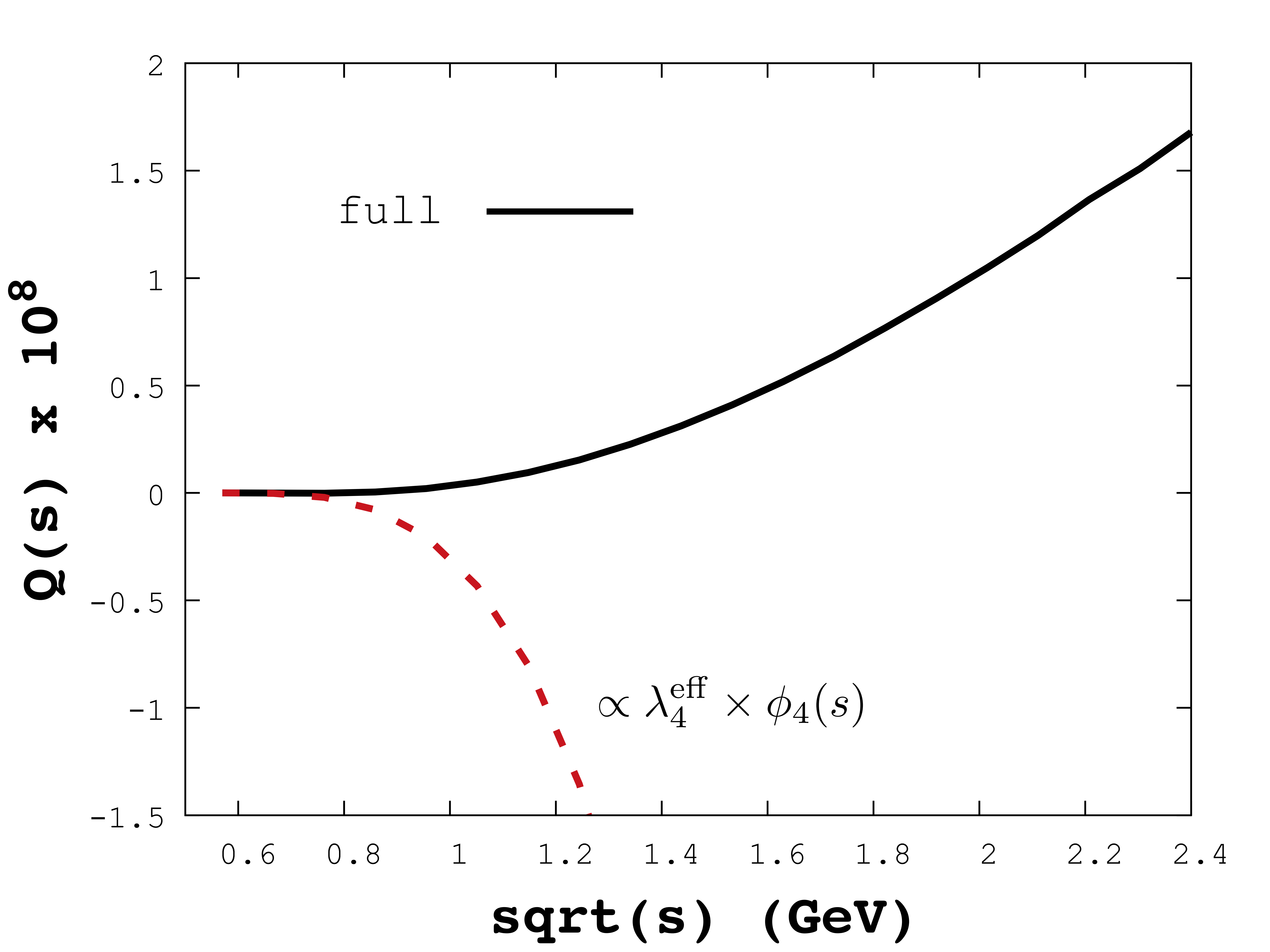
Limits: $s \to 16\, m_\pi^2$

$$\mathrm{Im}\,\left(i\mathcal{M}^{\mathrm{box,o.s.}}(q_1^2, q_2^2, s)\right) \approx \lambda_4^{\mathrm{eff}}$$

$$\lambda_4^{\mathrm{eff}} = \frac{\lambda^4}{256\pi^2}\frac{1}{m_\pi^4} \times (\frac{\sqrt{3}}{2}\,\ln(7 - 4\sqrt{3}) + 2)$$ *Negative!*

$$\mathcal{Q}(s) \approx \frac{1}{2} \times \lambda_4^{\mathrm{eff}} \times \phi_4(s).$$

$s >> 16\, m_\pi^2$???



full
$\propto \lambda_4^{\text{eff}} \times \phi_4(s)$
Q(s) x 10^8
sqrt(s) (GeV)
2
1.5
1
0.5
0
-0.5
-1
-1.5
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4

FUNCTIONAL APPROACH AND SCHWINGER DYSON EQUATIONS

KEY QUANTITIES

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4$$

generators for…

$Z\left[j(x)\right] = \int D\phi\, e^{i\int(\mathcal{L}+j(x)\phi(x))}$	*standard Green's functions*
$W\left[j\right] = -i\ln Z\left[j\right]$	*connected Green's functions*
$\Gamma[\phi] = W - \int j\phi$	*1PI Green's functions*

RECIPE

$$0 = \int D\phi \, \frac{\delta}{\delta\phi} e^{i\int(\mathcal{L}+j\phi)}$$

Master equation

$$\left(\frac{\delta S}{\delta\phi} + j\right) Z[j] = 0$$ *with* $$\phi \to -i\frac{\delta}{\delta j}$$

$$\left(\frac{\delta S}{\delta\phi} + j\right) I = 0$$ *with* $$\phi \to -i\frac{\delta}{\delta j} + \frac{\delta W}{\delta j}$$

$$\left(\frac{\delta S}{\delta\phi} + j\right) I = 0$$ *with* $$\phi \to \int i\,G\,\frac{\delta}{\delta\phi} + \phi$$

RECIPE

$$\frac{\delta S}{\delta \phi} = -(\partial^2 + m^2)\phi_x - \frac{\lambda}{6}\phi_x^3$$

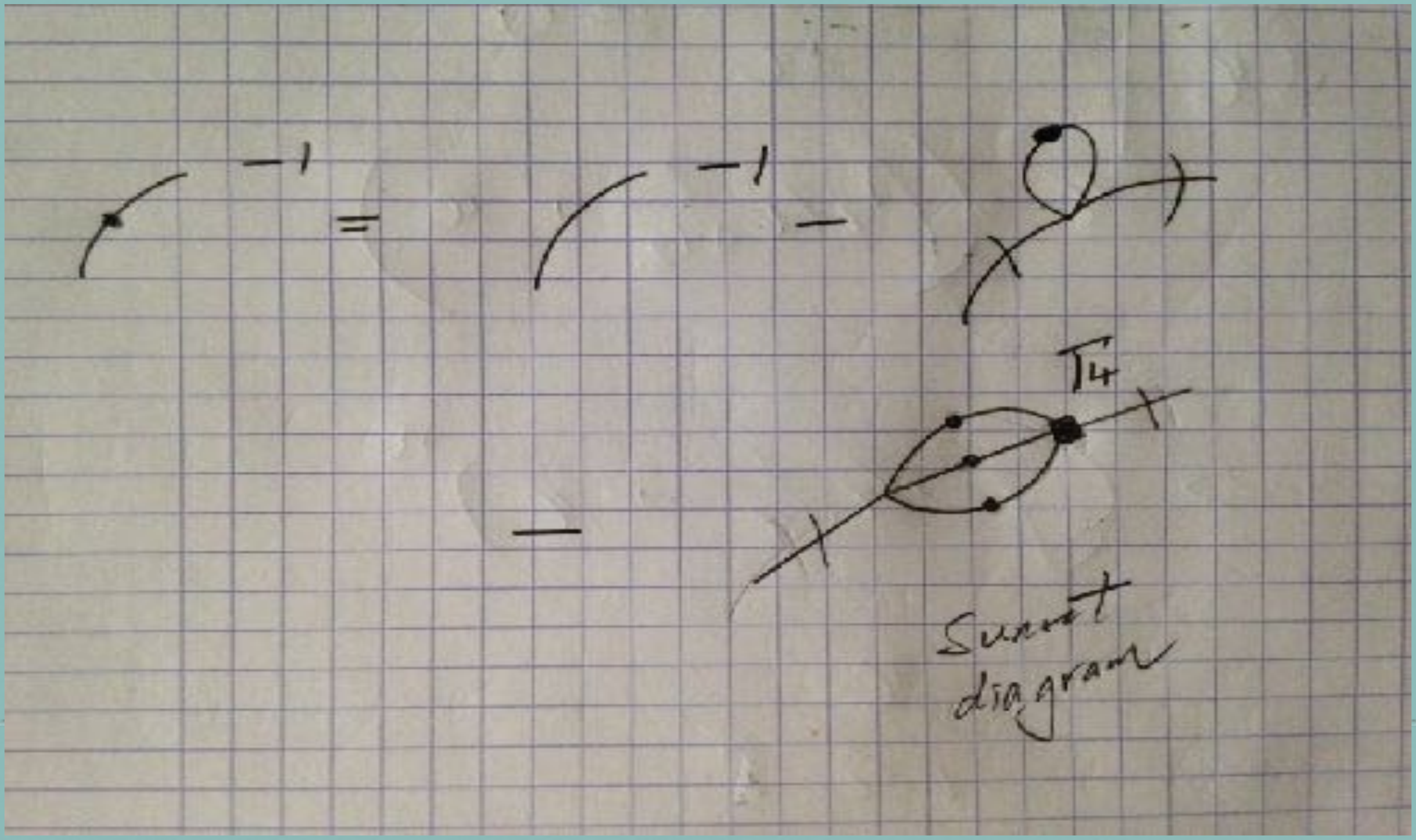
$$\left(\frac{\delta S}{\delta \phi} + j\right) Z[j] = 0 \qquad \textit{with} \qquad \phi \to -i\frac{\delta}{\delta j}$$

$$\left(\frac{\delta S}{\delta \phi} + j\right) I = 0 \qquad \textit{with} \qquad \phi \to -i\frac{\delta}{\delta j} + \frac{\delta W}{\delta j}$$

$$\left(\frac{\delta S}{\delta \phi} + j\right) I = 0 \qquad \textit{with} \qquad \phi \to \int i\,G\,\frac{\delta}{\delta \phi} + \phi$$

FULL EQN. FOR PROPAGATOR

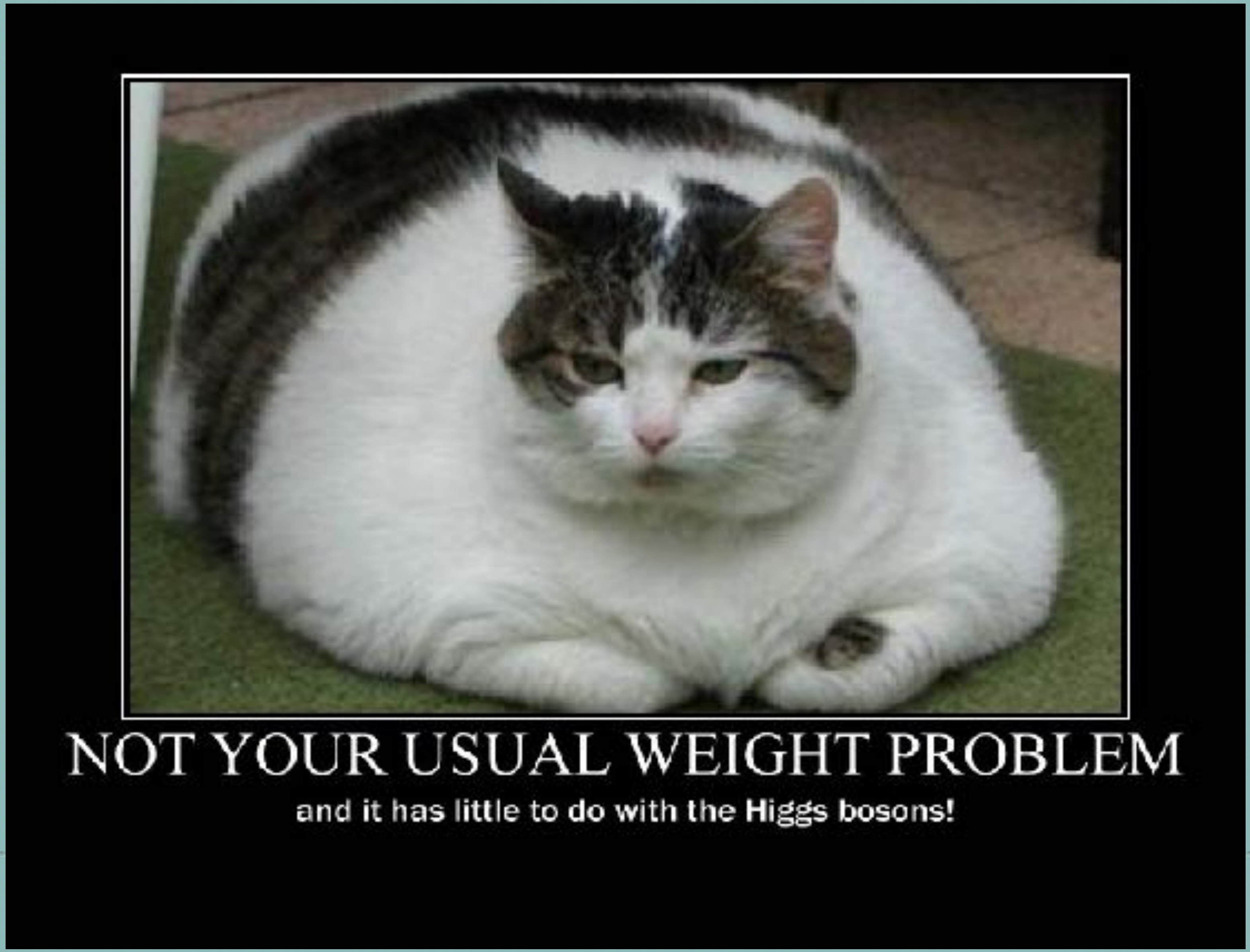
$$G^{-1} = -(\partial^2 + m^2)\delta - \frac{\lambda}{2} iG(x,x)\delta +$$
$$- \frac{\lambda}{6}\int G(x,z)G(x,z)G(x,z)\Gamma_4(z,z,z,y)$$



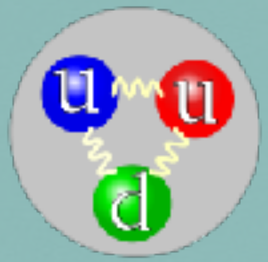
CHIRAL SYMMETRY

ORIGIN OF MASS

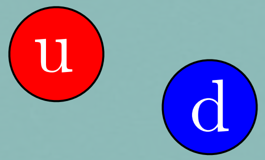
- 99% of the Mass of the visible Universe is not explained by Higgs bosons!



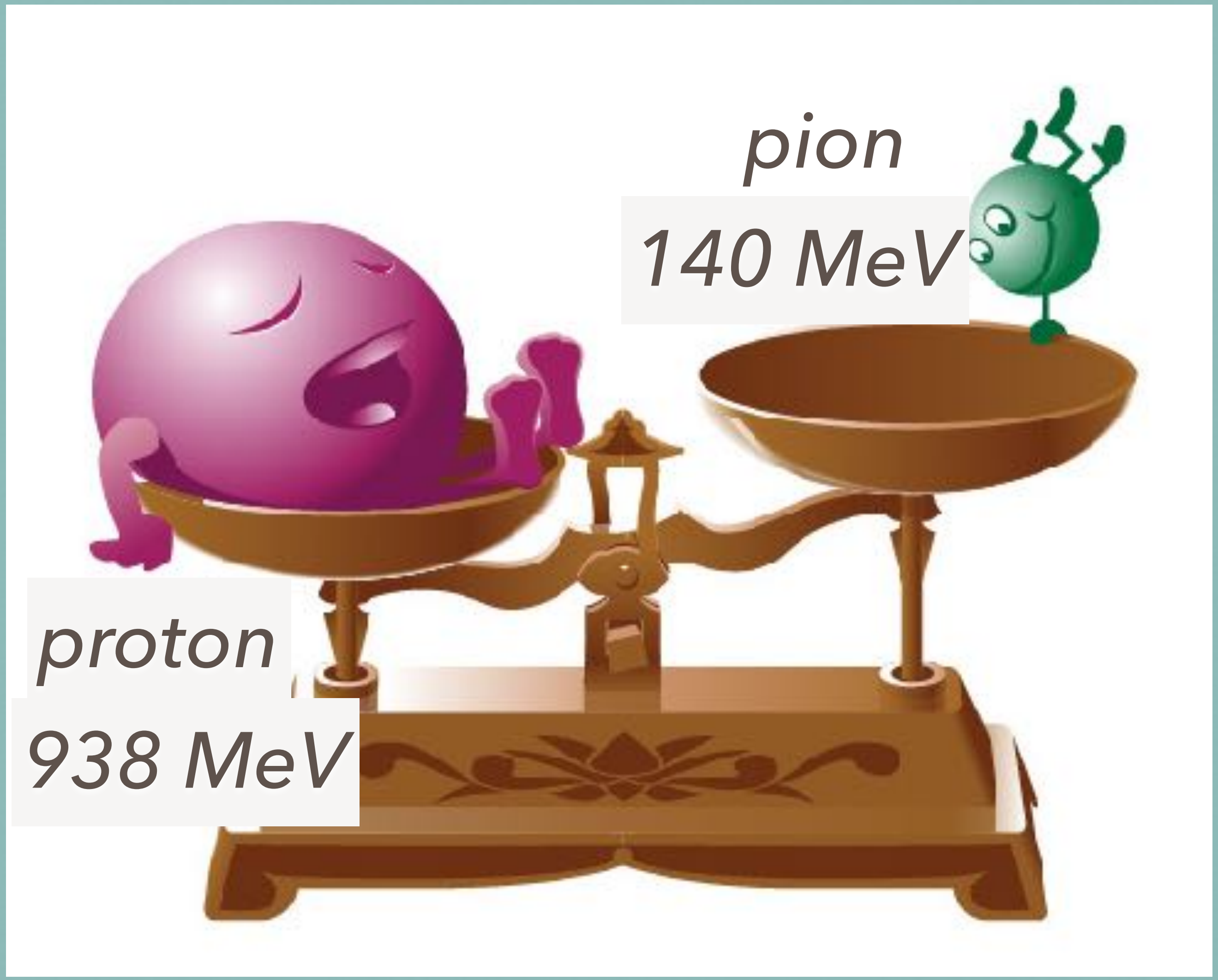
SPONTANEOUS CHIRAL SYMMETRY BREAKING



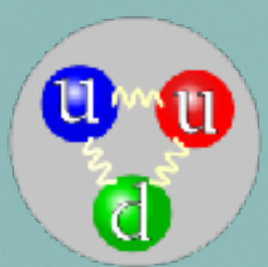
$$M_P = 938\,\mathrm{MeV}$$



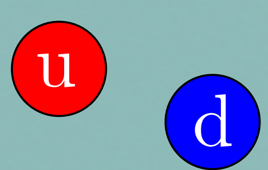
$$m_q \approx \mathrm{few\,MeV}$$



SPONTANEOUS CHIRAL SYMMETRY BREAKING



$$M_P = 938\,\mathrm{MeV}$$



$$m_q \approx \text{few MeV}$$

$m_q \to 0$ ⟶ *QCD is chiral symmetric.*

⟶ **spontaneously** *broken*

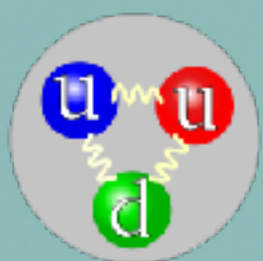
Fermion mass are **dynamically** *generated.*

⟶ $M_q^{\text{consti.}} \approx 300\text{MeV}$

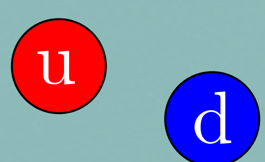
SPONTANEOUS CHIRAL SYMMETRY BREAKING

$\pi \to$ *Goldstone bosons*

$$m_\pi^2 \propto m_q$$



$$M_P = 938\,\text{MeV}$$



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Fermion mass are **dynamically** *generated.*

⟶ $M_q^{\text{consti.}} \approx 300\text{MeV}$

QCD IN COULOMB GAUGE

- An instantaneous potential obtained from QCD
- All degree of freedom are physical
  ghost-free!
- Confining and momentum dependent
  VS NJL

QCD IN COULOMB GAUGE

$$\mathcal{H} = -i\bar{\psi}\vec{\gamma}\cdot\nabla\psi + m\bar{\psi}\psi + \frac{1}{2}(\vec{E}^2 + \vec{B}^2) - g\bar{\psi}\,\vec{\gamma}\,T^a\cdot\vec{A}^a$$

$$+ \frac{1}{2}\,\rho\,[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]\,\rho$$

generalized Coulomb potential

$$D_\mu = \partial_\mu + igA_\mu$$

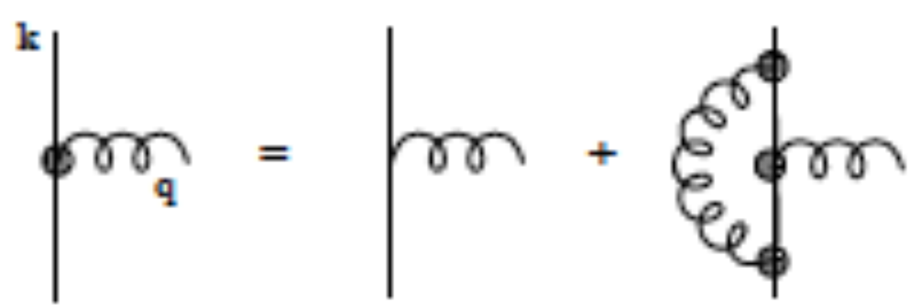
$$\rho^a = \bar{\psi}\gamma^0 T^a\psi + f^{\rm abc}\,A_i^b\,E_c^i$$

both quarks and gluons are color charged!

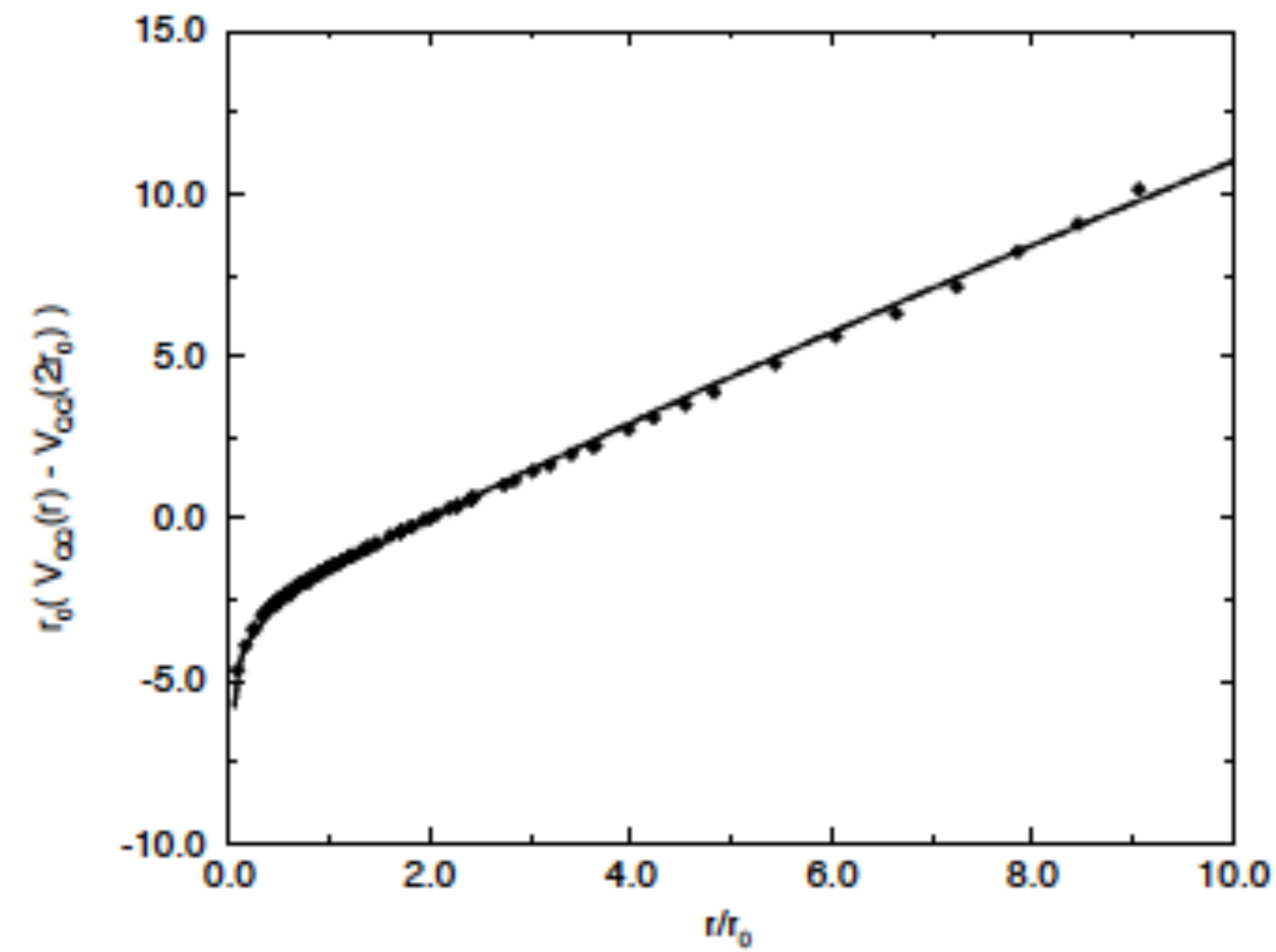
Christ and Lee

POTENTIAL IN COULOMB GAUGE QCD

$$K_{ab}(x,y;A) := \langle x,a|[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]|y,b\rangle$$



A. Szczepaniak and E. Swanson

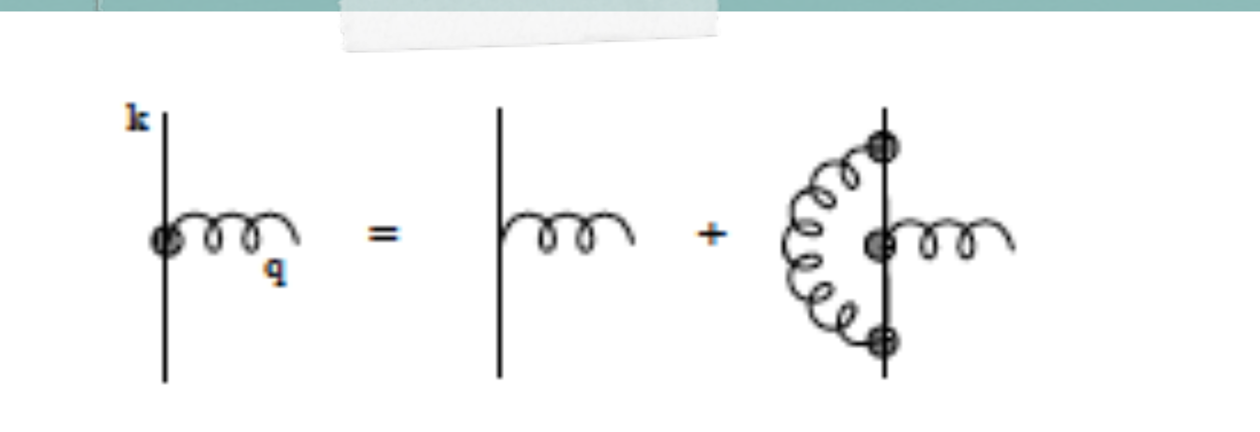


Pure Yang Mills

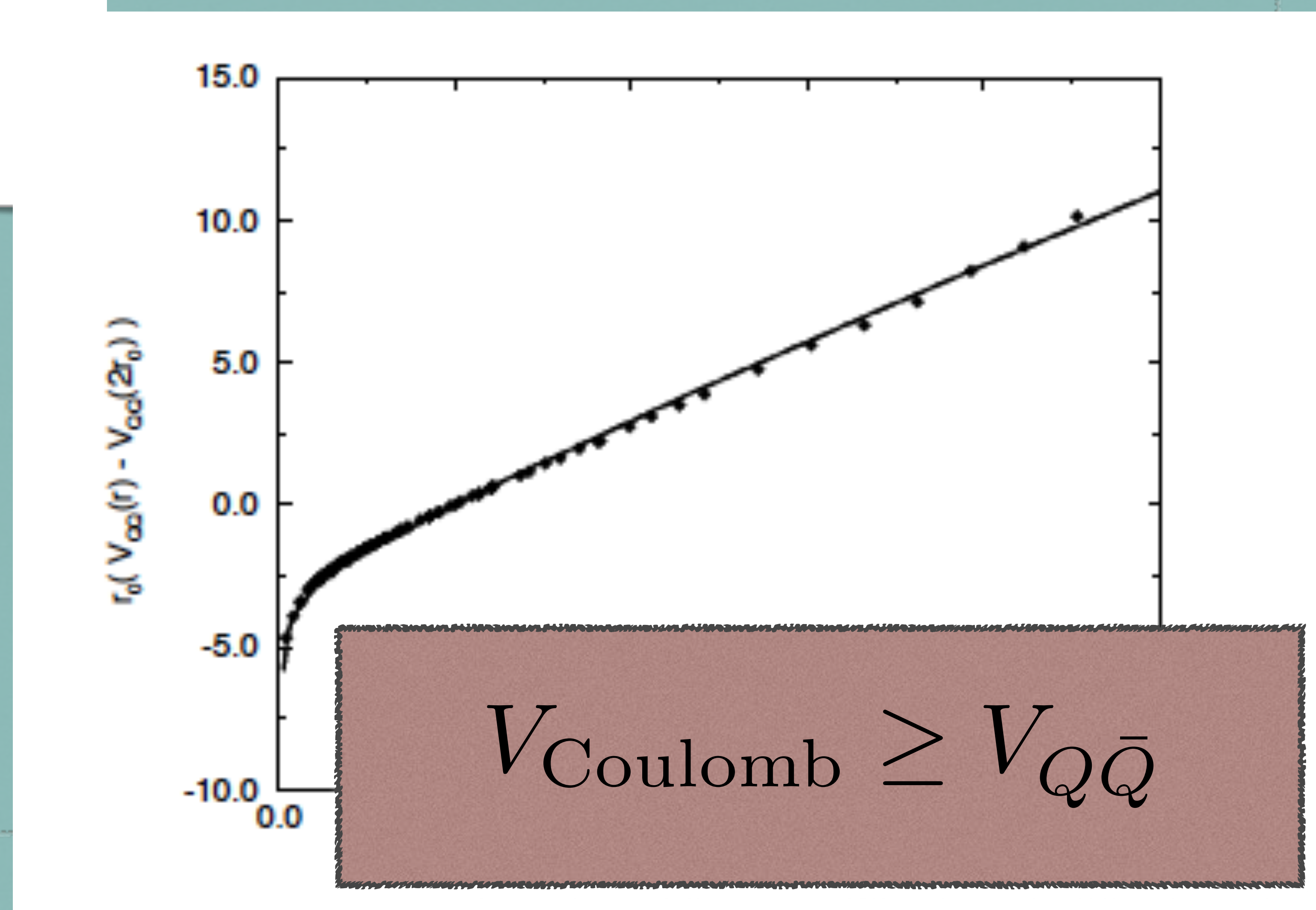
POTENTIAL IN COULOMB GAUGE QCD

$$K_{ab}(x,y;A) := \langle x,a|[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]|y,b\rangle$$

A. Szczepaniak and E. Swanson



Pure Yang Mills



$$V_{\text{Coulomb}} \geq V_{Q\bar{Q}}$$

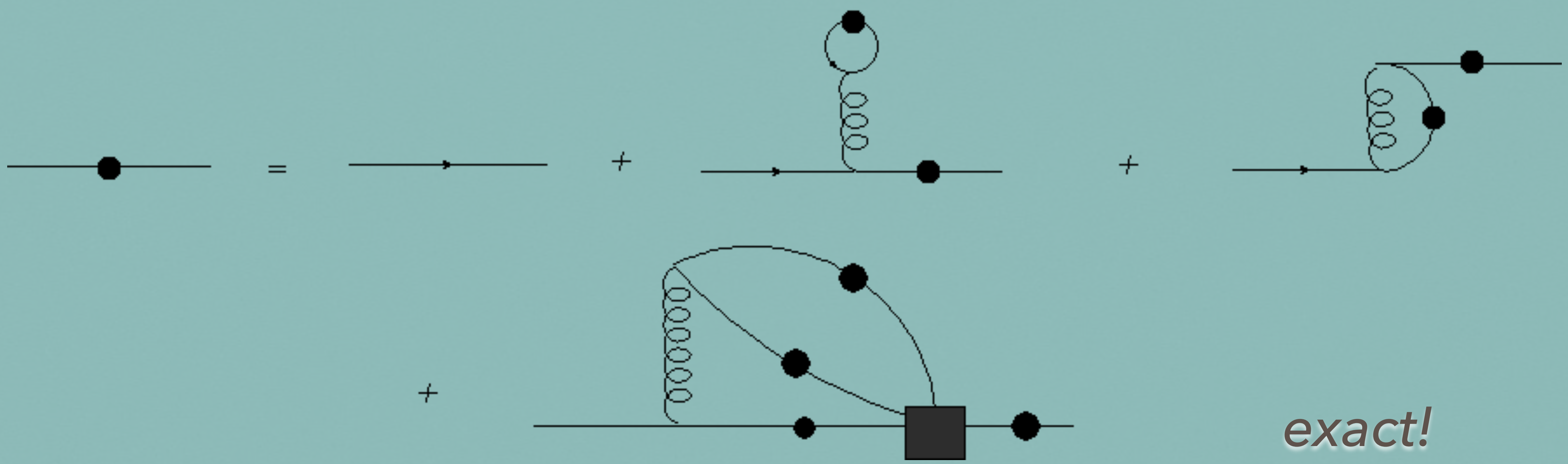
DYSON-SCHWINGER EQUATIONS

- Non-perturbative
- Continuum approach
- No sign problem

- Need to find a good Truncation Scheme

DYNAMICAL MASS GENERATION

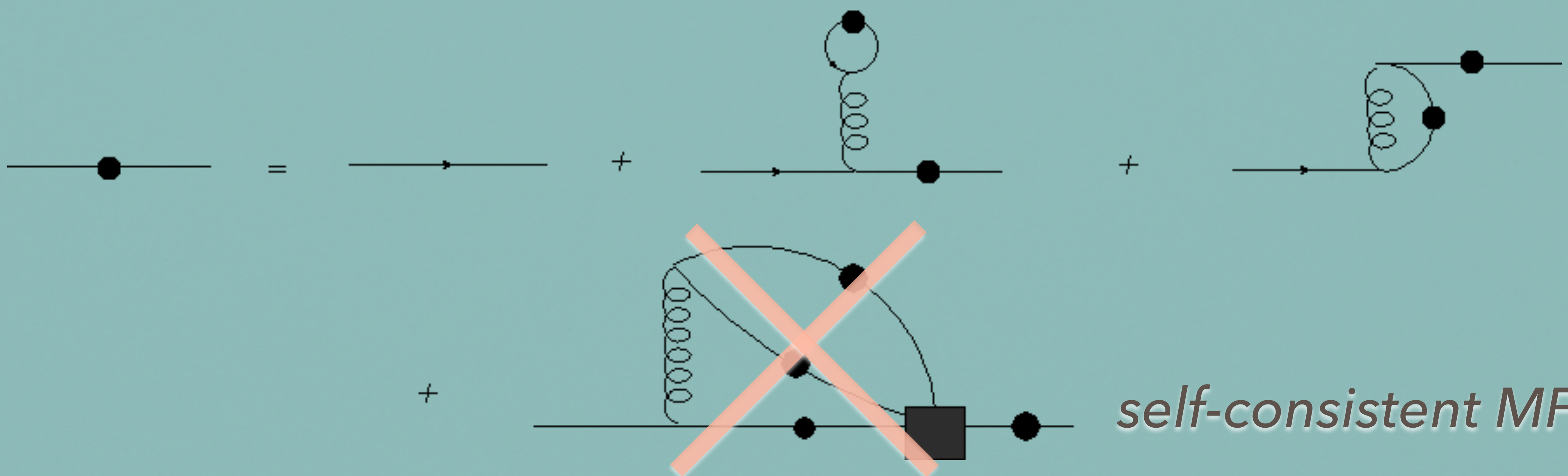
Dyson-Schwinger Equations



exact!

DYNAMICAL MASS GENERATION

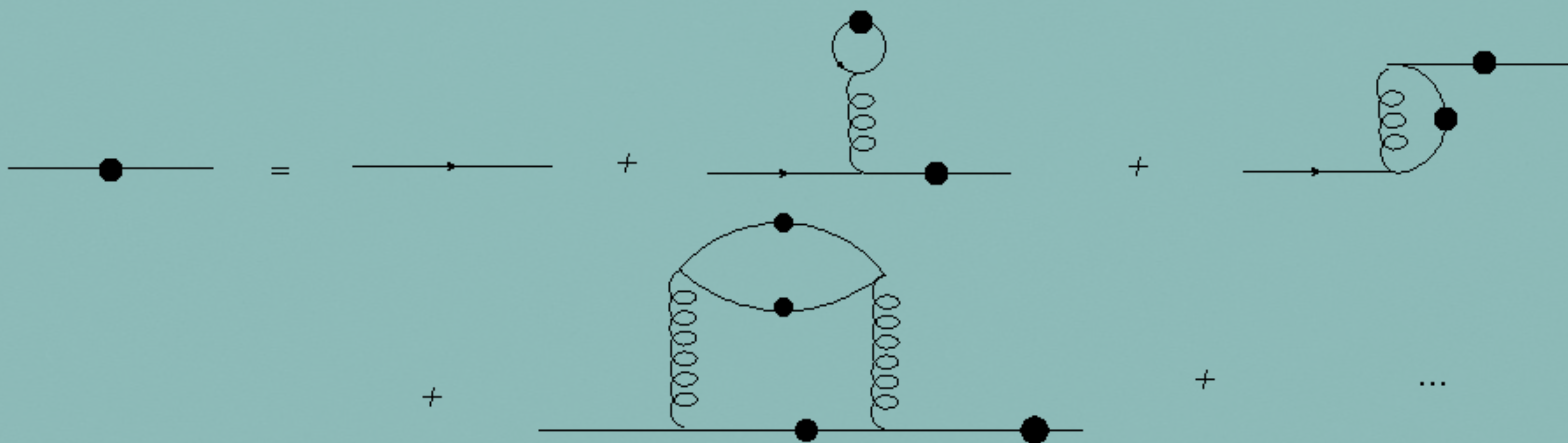
Dyson-Schwinger Equations



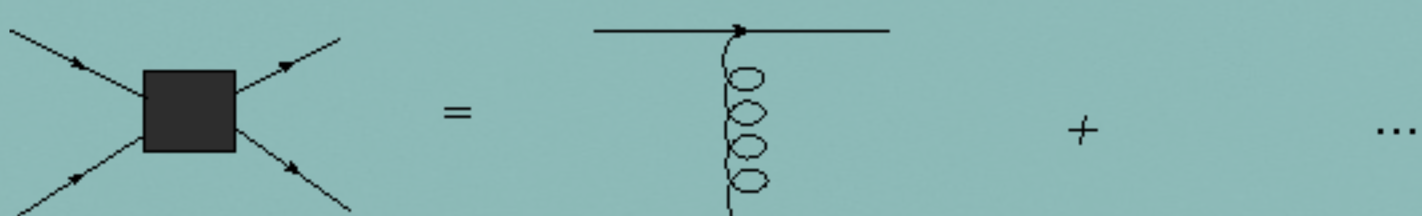
self-consistent MF

DYNAMICAL MASS GENERATION

Dyson-Schwinger Equations



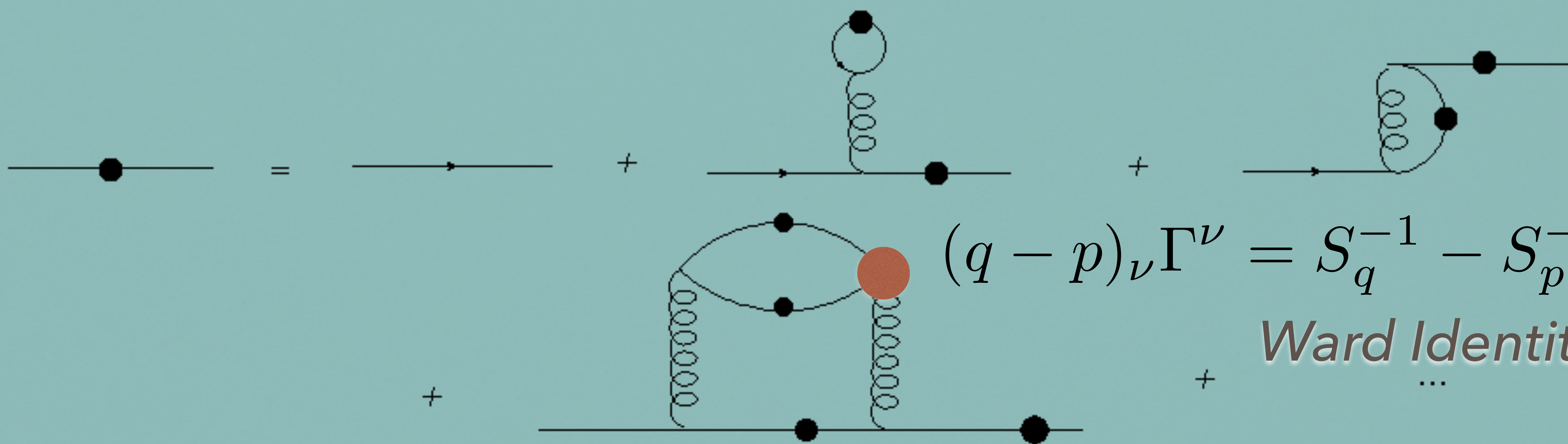
RPA approx



PML and E. S. Swanson, PRD 81 034030 (2010)

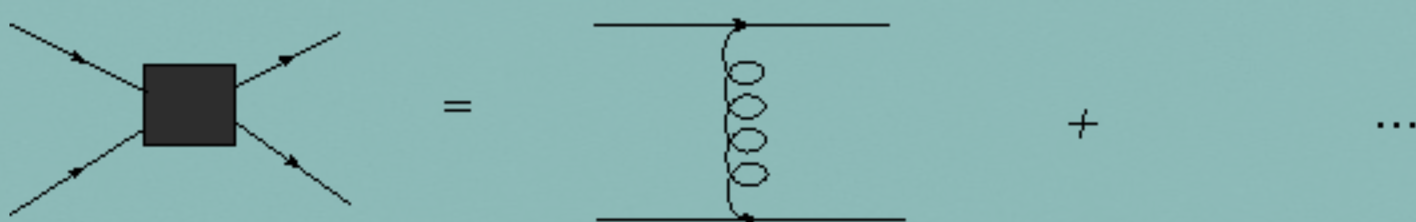
DYNAMICAL MASS GENERATION

Dyson-Schwinger Equations



$$(q-p)_\nu \Gamma^\nu = S_q^{-1} - S_p^{-1}$$

Ward Identity



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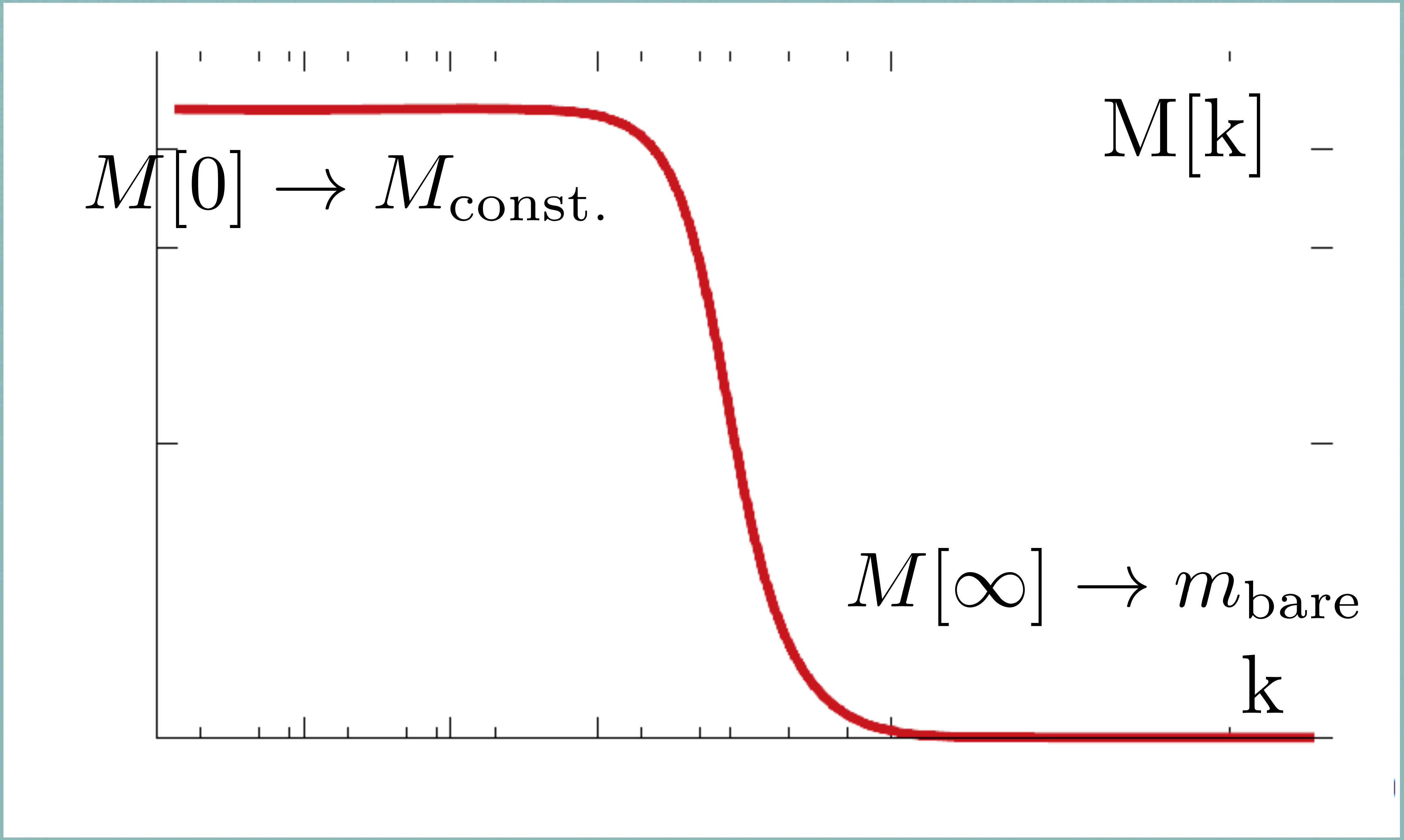
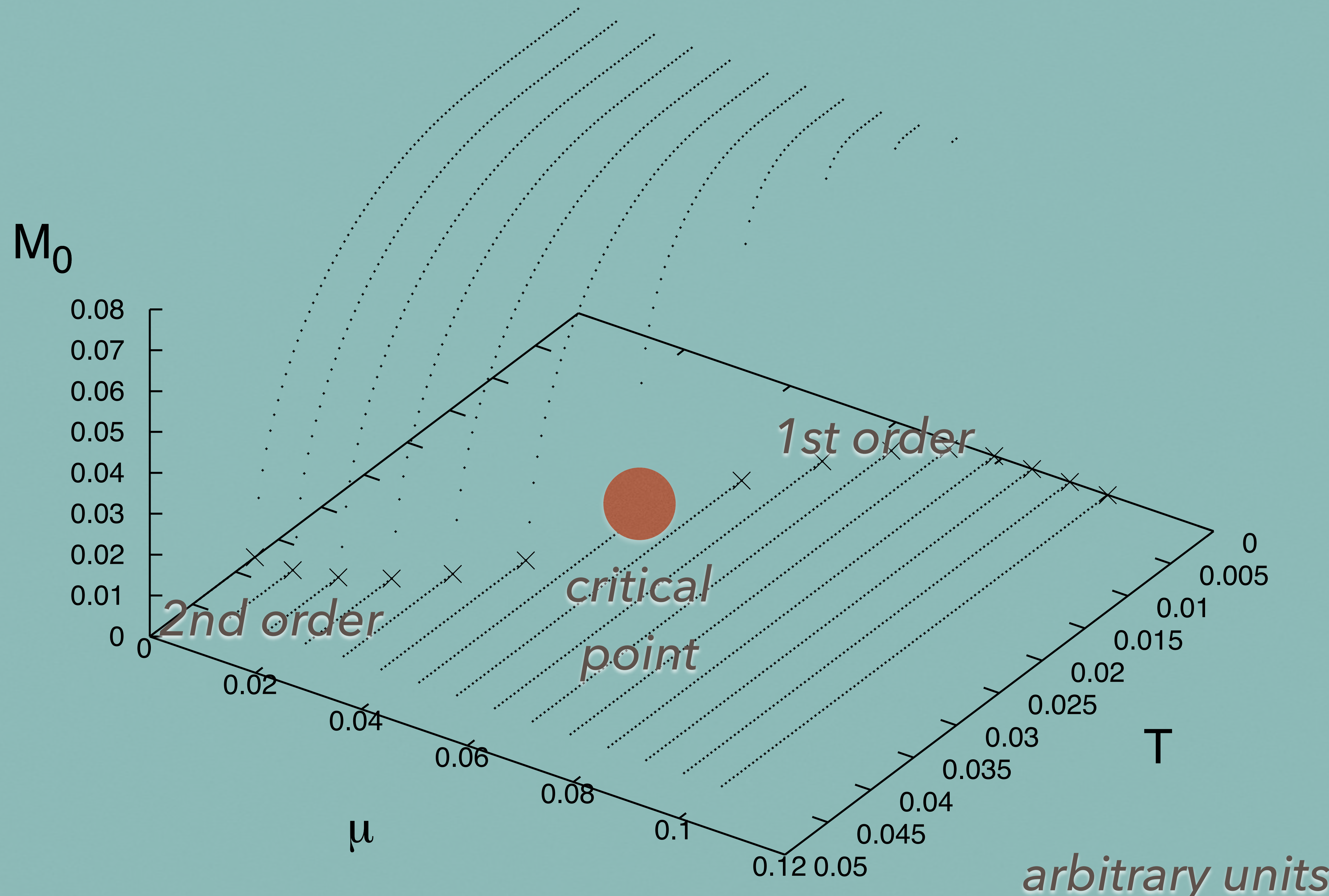
DYNAMICAL MASS GENERATION



CHIRAL PHASE STRUCTURE



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BIG SUMMARY

- S-matrix formulation of statistical mechanics
  factorization of dynamical and statistical parts
- Many-body Techniques
  Dispersion Relations
  Functional methods
- Applications:
  proton puzzle, hyperon spectrum, …

USEFUL REFERENCES

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345.

P. Cvitanovic, R. Artuso, M. R., G. Tanner, and G. Vattay, Chaos: Classical and Quantum(ChaosBook.org, Niels Bohr Institute, Copenhagen, 2016).

John R. Taylor, Scattering Theory

Roman Zwicky, arXiv:1610.06090

Eric Swanson, arXiv:1008.4337v2

THANK YOU
&
ENJOY!

完

END OF LECTURE III