Hadron formation & dissociation in dense matter

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1. Mott dissociation of pions in a Polyakov - NJL model

2. Thermodynamics of Mott-HRG and lattice QCD data

3. Mott-Anderson localization model for chemical freeze-out

HISS “Matter under Extreme Conditions in HIC & Astro”, Dubna, 20.-31.08.18

JINR DUBNA · MEPhI · Uniwersytet Wrocławski · Narodowe Centrum Nauki

Mott Dissociation of Hadrons in Hadron Matter

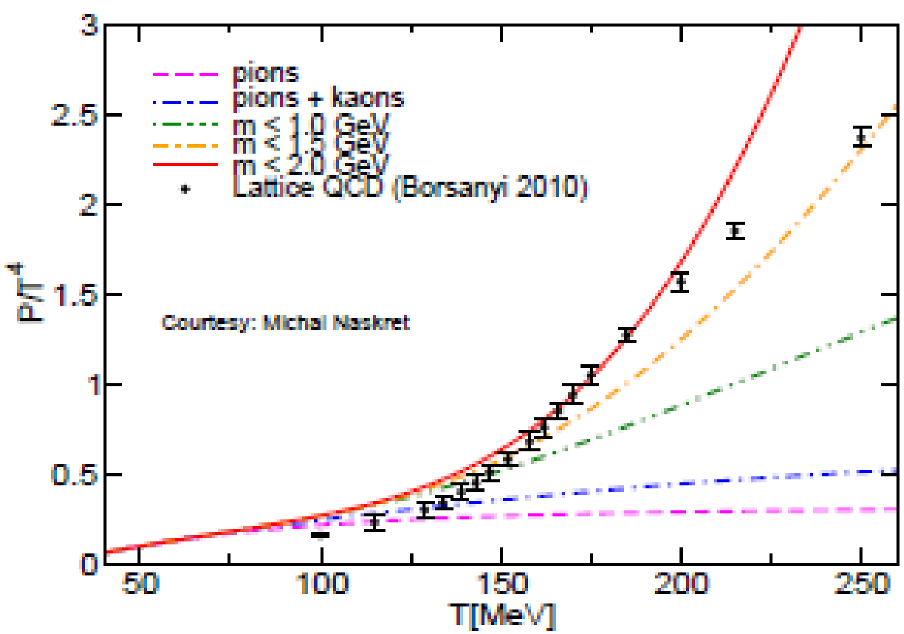
- Partition function as a Path Integral (imaginary time $\tau = i\,t,\, 0 \le \tau \le \beta = 1/T$)

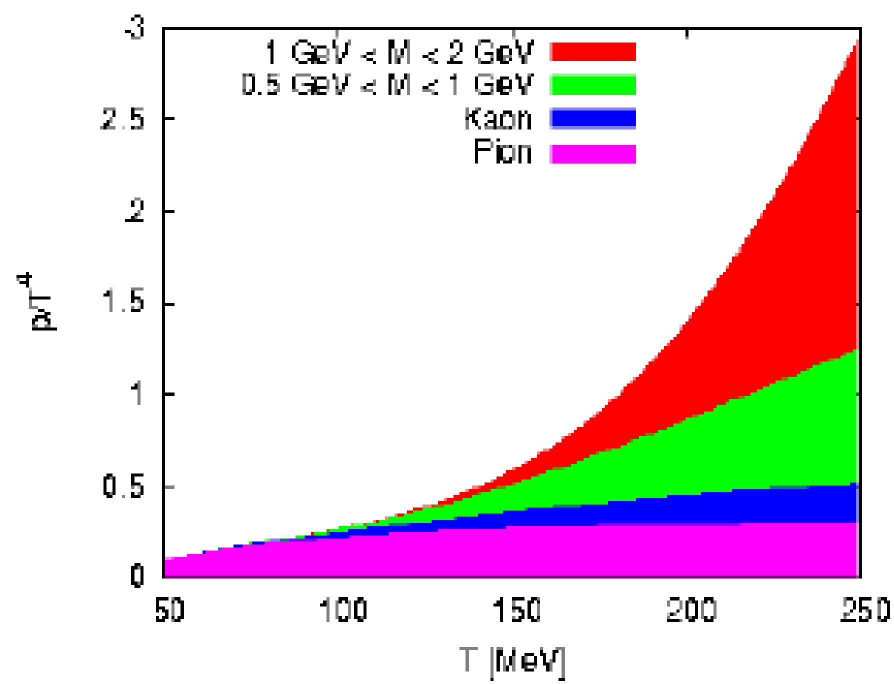
$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \exp\left\{-\int_0^\beta d\tau \int_V d^3x\, \mathcal{L}_{QCD}(\psi,\bar{\psi},A)\right\}$$

- QCD Lagrangian, non-Abelian gluon field strength: $F^a_{\mu\nu}(A) = \partial_\mu A^a\nu - \partial_\nu A^a_\mu + g\, f^{abc}[A^b_\mu, A^c_\nu]$

$$\mathcal{L}_{QCD}(\psi,\bar{\psi},A) = \bar{\psi}[i\gamma^\mu(\partial_\mu - igA_\mu) - m - \gamma^0\mu]\psi - \frac{1}{4}F^a_{\mu\nu}(A)F^{a,\mu\nu}(A)$$

- Numerical evaluation: Lattice gauge theory simulations (hotQCD, Wuppertal-Budapest)

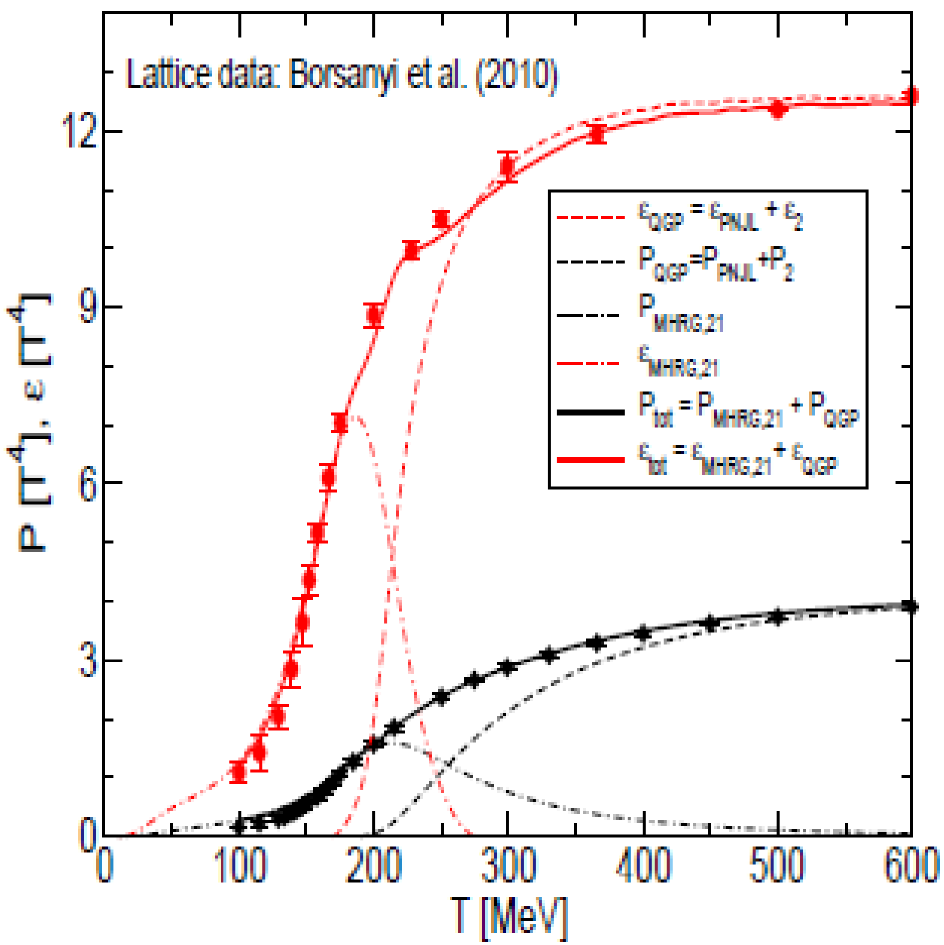




Mott Dissociation of Hadrons in Hadron Matter

Intuitive “guess”: Hadron gas with spectral broadeing (lifetime) + PNJL model for q-g sector

$$P_{\rm tot}(T,\{\mu_j\}) = P_{\rm PNJL}(T,\{\mu_i\}) + \sum_{r=M,B} \delta_r g_r \int ds\, A_r(s,m_r;T) \int \frac{d^3p}{(2\pi)^3} T \ln\left\{1+\delta_r \exp\left(\frac{\sqrt{p^2+s}-\mu_r}{T}\right)\right\}$$



Spectral function for hadronic resonances:

$$A_r(s,m;T) = N_s \frac{m\Gamma_r(T)}{(s-m^2)^2 + m^2\Gamma_r^2(T)}$$

Ansatz motivated by chemical freeze-out model:

$$\Gamma_r(T) = \tau_r^{-1}(T) = \sum_h \lambda < r_r^2 >_T < r_h^2 >_T n_h(T)$$

Apparent phase transition at $T_c \sim 165$ MeV

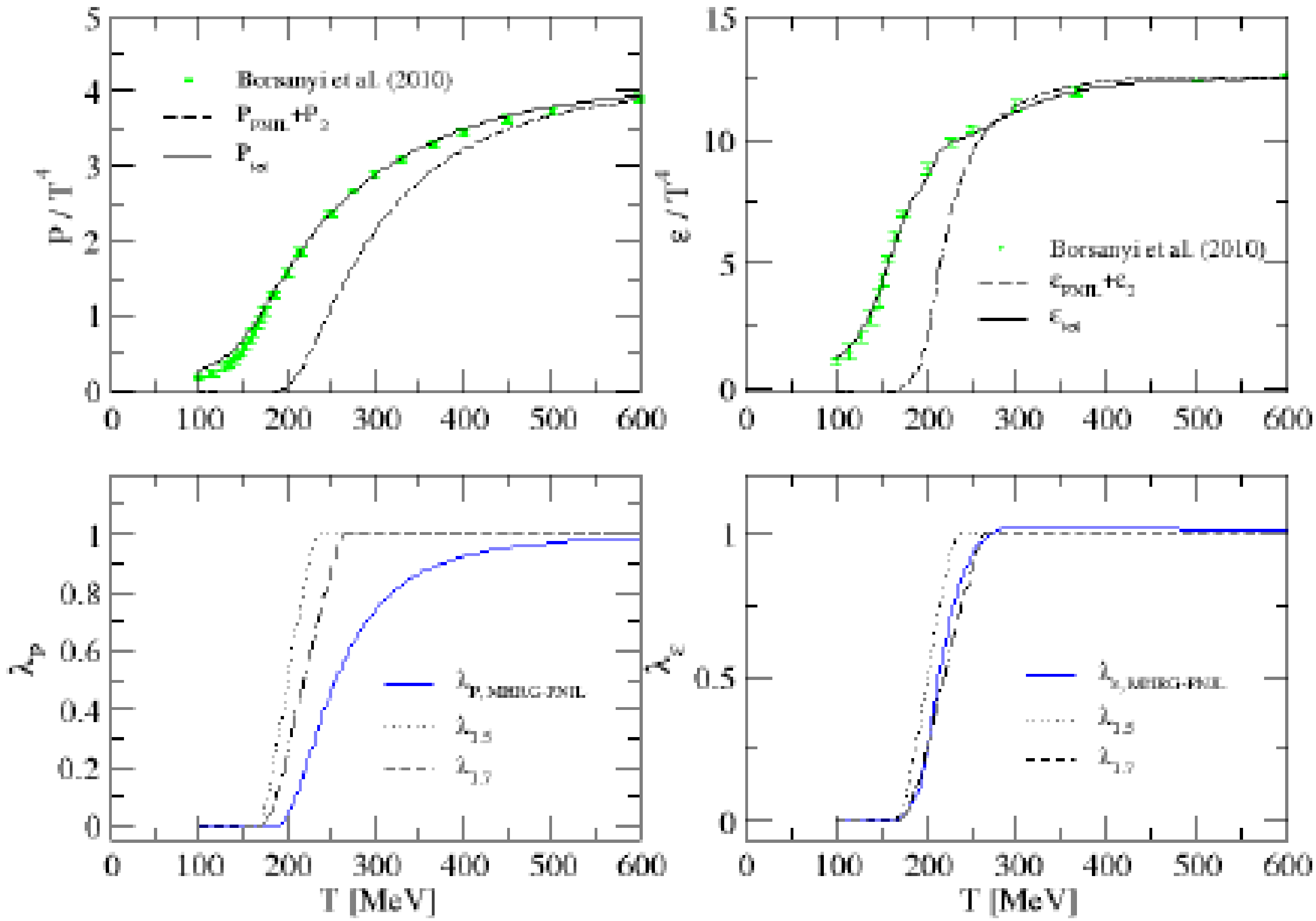
Hadron resonances present up to $T_{\rm max} \sim 250$ MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)
Prog. Part. Nucl. Phys. 53, 197 (2004)
Turko, Blaschke, Prorok & Berdermann,
APPS 5, 485 (2012); J. Phys. Conf. Ser. 455, 012056 (2013)

Hadronic states above T_c ! See also: Ratti, Bellwied et al., arXiv:1109.6243 [hep-ph]

Mott Dissociation of Hadrons in Hadron Matter

Possible application: parton fraction in the EoS at the hadronization transition



L. Turko et al. “Effective degrees of freedom in QCD …”, EPJ Web Conf. 71 (2014) 00134

Compare:

M. Nahrgang et al. “Influence of hadronicbound states above Tc …”, PRC 89 (2014) 014004

Mott Dissociation of Mesons in Quark Matter

D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki, Ann. Phys. 348, 228 (2014)

- Partition function as a Path Integral (imaginary time $\tau = i\ t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{-\int^{\beta} d\tau \int_V d^3x [\bar{q}(i\gamma^\mu\partial_\mu - m_0 - \gamma^0\mu)q + \sum_{M=\pi,\sigma} G_M(\bar{q}\Gamma_M q)^2]\right\}$$

- Couplings: $G_\pi = G_\sigma = G_S$ (chiral symmetry)
- Vertices: $\Gamma_\sigma = \mathbf{1}_D \otimes \mathbf{1}_f \otimes \mathbf{1}_c$; $\Gamma_\pi = i\gamma_5 \otimes \vec{\tau} \otimes \mathbf{1}_c$
- Bosonization (Hubbard-Stratonovich Transformation)

$$\exp\left[G_S(\bar{q}\Gamma_\sigma q)^2\right] = \text{const.} \int \mathcal{D}\sigma \exp\left[\frac{\sigma^2}{4G_S} + \bar{q}\Gamma_\sigma q\sigma\right]$$

- Integrate out quark fields $\longrightarrow$ bosonized partition function

$$Z[T,V,\mu] = \int \mathcal{D}\sigma\mathcal{D}\pi \exp\left\{-\frac{\sigma^2+\pi^2}{4G_S} + \frac{1}{2}\mathrm{Tr}\ \ln S^{-1}[\sigma,\pi]\right\}$$

- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)

Mott Dissociation of Mesons in Quark Matter

- Separate the mean-field part of the quark determinant

$$\mathrm{Tr}\,\ln S^{-1}[\sigma,\pi] = \mathrm{Tr}\,\ln S^{-1}_{\mathrm{MF}}[m] + \mathrm{Tr}\,\ln[1+(\sigma+i\gamma_5\vec{\tau}\vec{\pi})S_{\mathrm{MF}}[m]]$$

- Mean-field quark propagator

$$S_{\mathrm{MF}}(\vec{p},i\omega_n;m) = \frac{\gamma_0(i\omega_n+\mu)-\vec{\gamma}\cdot\vec{p}+m}{(i\omega_n+\mu)^2-E_p^2}$$

- Expand the logarithm: $\ln(1+x) = -\sum_{n=1}^{\infty}(-1)^n x^n/n = x - x^2/2 + \ldots$
- Thermodynamic potential in Gaussian approximation

$$\Omega(T,\mu) \;=\; -T\ln Z(T,\mu) = \Omega_{\mathrm{MF}}(T,\mu) + \sum_M \Omega^{(2)}_{\mathrm{M}}(T,\mu) + \mathcal{O}[\phi_M^3]$$

$$\Omega^{(2)}_{\mathrm{M}}(T,\mu) \;=\; \frac{N_M}{2}\int\frac{d^2p}{(2\pi)^3}\frac{1}{\beta}\sum_n \mathrm{e}^{i\nu_n\eta}\ln S_M^{-1}(\vec{p},i\nu_n)\;,\quad N_\sigma=1,\; N_\pi=3$$

- Meson propagator $S_M(\vec{p},i\nu_n) = 1/\left[1/(2G_S)-\Pi_M(\vec{p},i\nu_n)\right]$
- Mesonic polarization loop

$$\Pi_M(\vec{p},i\nu_n) = -\frac{1}{\beta}\sum_{n'}\mathrm{e}^{i\nu_{n'}\eta}\int\frac{d^2k}{(2\pi)^3}\mathrm{Tr}\left[\Gamma_M S_{\mathrm{MF}}(-\vec{k},-i\omega_{n'})\Gamma_M S_{\mathrm{MF}}(\vec{k}+\vec{p},i\omega_{n'}+i\nu_n)\right]$$

Mott Dissociation of Mesons in Quark Matter

- Polar representation of the analytically continued quark propagator

$$S_{\rm M} = |S_{\rm M}| e^{i\delta_{\rm M}} = S_R + iS_I \ ,$$

- Phase shift $\delta_{\rm M}(\omega, \mathbf{q}) = -{\rm Im} \ln S_{\rm M}^{-1}(\omega - \mu_M + i\eta, \mathbf{q})$
- Thermodynamic potential for mesonic modes

$$\begin{aligned}\Omega_{\rm M}(T,\mu) &= {\rm Tr} \ln S_{\rm M}^{-1}(iz_n, \mathbf{q}) = d_{\rm M} T \sum_n \int \frac{{\rm d}^3q}{(2\pi)^3} \ln S_{\rm M}^{-1}(iz_n, \mathbf{q}) \ , \\ &= -d_{\rm M} T \sum_n \int \frac{{\rm d}^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{{\rm d}\omega}{2\pi} \frac{1}{iz_n - \omega} {\rm Im} \ln S_{\rm M}^{-1}(\omega + i\eta, \mathbf{q})\end{aligned}$$

- Perform Matsubara summation $\Omega_{\rm M}(T,\mu) = d_{\rm M} \int \frac{{\rm d}^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{{\rm d}\omega}{2\pi} n_M^-(\omega) \delta_M(\omega, \mathbf{q})$
- Using symmetries of Bose function $n_M^-(-\omega) = -[1 + n_M^+(\omega)]$ and polarization loop

$$\Omega_{\rm M}(T,\mu) = d_{\rm M} \int \frac{{\rm d}^3q}{(2\pi)^3} \int_0^{\infty} \frac{{\rm d}\omega}{2\pi} [1 + n_M^-(\omega) + n_M^+(\omega)] \delta_{\rm M}(\omega, \mathbf{q})$$

- Partial integration gives field theoretic Beth-Uhlenbeck formula

$$\Omega_{\rm M} = -d_{\rm M} \int \frac{{\rm d}^3q}{(2\pi)^3} \int_0^{\infty} \frac{{\rm d}\omega}{2\pi} \left[\omega + T \ln\left(1 - {\rm e}^{-(\omega - \mu_{\rm M})/T}\right) + T \ln\left(1 - {\rm e}^{-(\omega + \mu_{\rm M})/T}\right)\right] \frac{d\delta_{\rm M}(\omega, \mathbf{q})}{d\omega}$$

Mott Dissociation of Mesons in Quark Matter

- When polarization loop integral can be expressed in the form

$$\Pi_{\rm M}(z,\mathbf{q}) = \Pi_{M,0} + \Pi_{M,2}(z,\mathbf{q})$$

- Factorization of two-particle propagator possible with $R_M(z,\mathbf{q}) = \frac{1-G_{\rm M}\Pi_{M,0}}{G_{\rm M}\Pi_{M,2}(z,\mathbf{q})}$

$$S_{\rm M}(z,\mathbf{q}) = \frac{1}{G_{\rm M}^{-1} - \Pi_{M,0} - \Pi_{M,2}(z,\mathbf{q})} = \frac{1}{\Pi_{M,2}(z,\mathbf{q})}\frac{1}{R_M(z,\mathbf{q})-1}$$

- This entails $\ln S_{\rm M}(z,\mathbf{q})^{-1} = \ln \Pi_{M,2}(z,\mathbf{q}) + \ln[R_M(z,\mathbf{q})-1]$
  and thus a separation of the phase shift in two contributions

$$\delta_{\rm M}(\omega,\mathbf{q}) = \delta_{X,c}(\omega,\mathbf{q}) + \delta_{X,R}(\omega,\mathbf{q})$$

- They correspond to continuum (state independent) and resonant phases

$$\delta_{M,c}(\omega,\mathbf{q}) = -\arctan\left(\frac{{\rm Im}\Pi_{M,2}(\omega-\mu_M+i\eta,\mathbf{q})}{{\rm Re}\Pi_{M,2}(\omega-\mu_M+i\eta,\mathbf{q})}\right)$$

$$\delta_{M,R}(\omega,\mathbf{q}) = \arctan\left(\frac{{\rm Im}R_M(\omega-\mu_M+i\eta,\mathbf{q})}{1-{\rm Re}R_M(\omega-\mu_M+i\eta,\mathbf{q})}\right)$$

Mott Dissociation of Mesons in Quark Matter

- Suppose $\delta_{X,R}(\omega,\mathbf{q})$ corresponds to a resonance at $\omega = \omega_M = \sqrt{\mathbf{q}^2 + M_{\rm M}^2}$, then the propagator shall have the representation with a complex pole at $z = z_M = \omega_M + i\Gamma_M/2$, where $\Gamma_{\rm M}$ is the width of the resonance.
- The position of the pole is found from the condition ${\rm Re}R_M(z_M,\mathbf{q}) = 1$, where $\delta_{M,R}(\omega \to \omega_M) \to \pi/2$ since $\tan\delta_{M,R}(\omega \to \omega_M) \to \infty$
- Expanding $R_M(z,\mathbf{q})$ at the complex pole z_M for small width, one obtains

$$1 - {\rm Re}R_M(z_M,\mathbf{q}) = -(\omega^2 - \omega_M^2)\frac{dR_M(z,\mathbf{q})}{d\omega^2}\Big|_{z=z_M} \quad , \quad {\rm Im}R_M(z_M,\mathbf{q}) = \omega_M\Gamma_M\frac{dR_M(z,\mathbf{q})}{d\omega^2}\Big|_{z=z_M} \quad (1)$$

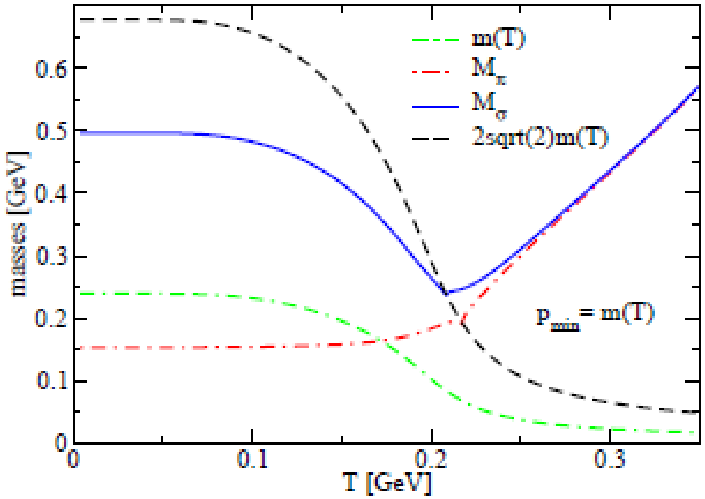
- The resonant shift becomes $\delta_{M,R}(\omega,\mathbf{q}) = -\arctan\left(\frac{\omega_{\rm M}\Gamma_{\rm M}}{\omega^2-\omega_M^2}\right)$ corresponding to a Breit-Wigner form of the spectral density in the Beth-Uhlenbeck EoS

$$\frac{{\rm d}\delta_{M,R}}{{\rm d}\omega} = \frac{2\omega\omega_{\rm M}\Gamma_{\rm M}}{(\omega^2-\omega_M^2)^2 + \omega_{\rm M}^2\Gamma_{\rm M}^2} .$$

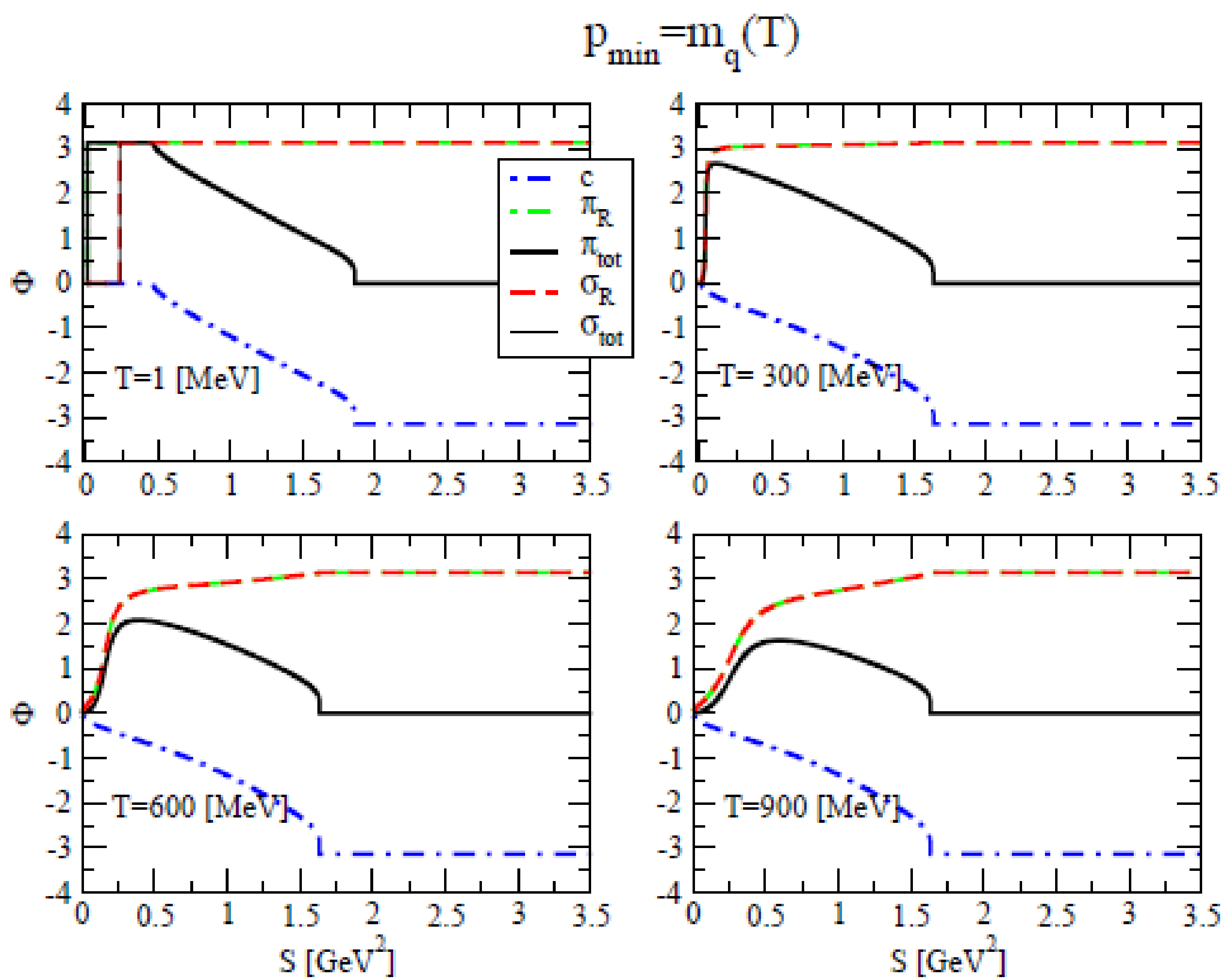
- This takes the form of a bound state spectral density for $\Gamma_M \to 0$

$$\lim_{\Gamma_{\rm M}\to 0} \delta'_{M,R}(\omega) = \pi\left[\delta(\omega-\omega_M) + \delta(\omega+\omega_M)\right]$$

Mott Dissociation of Mesons in Quark Matter

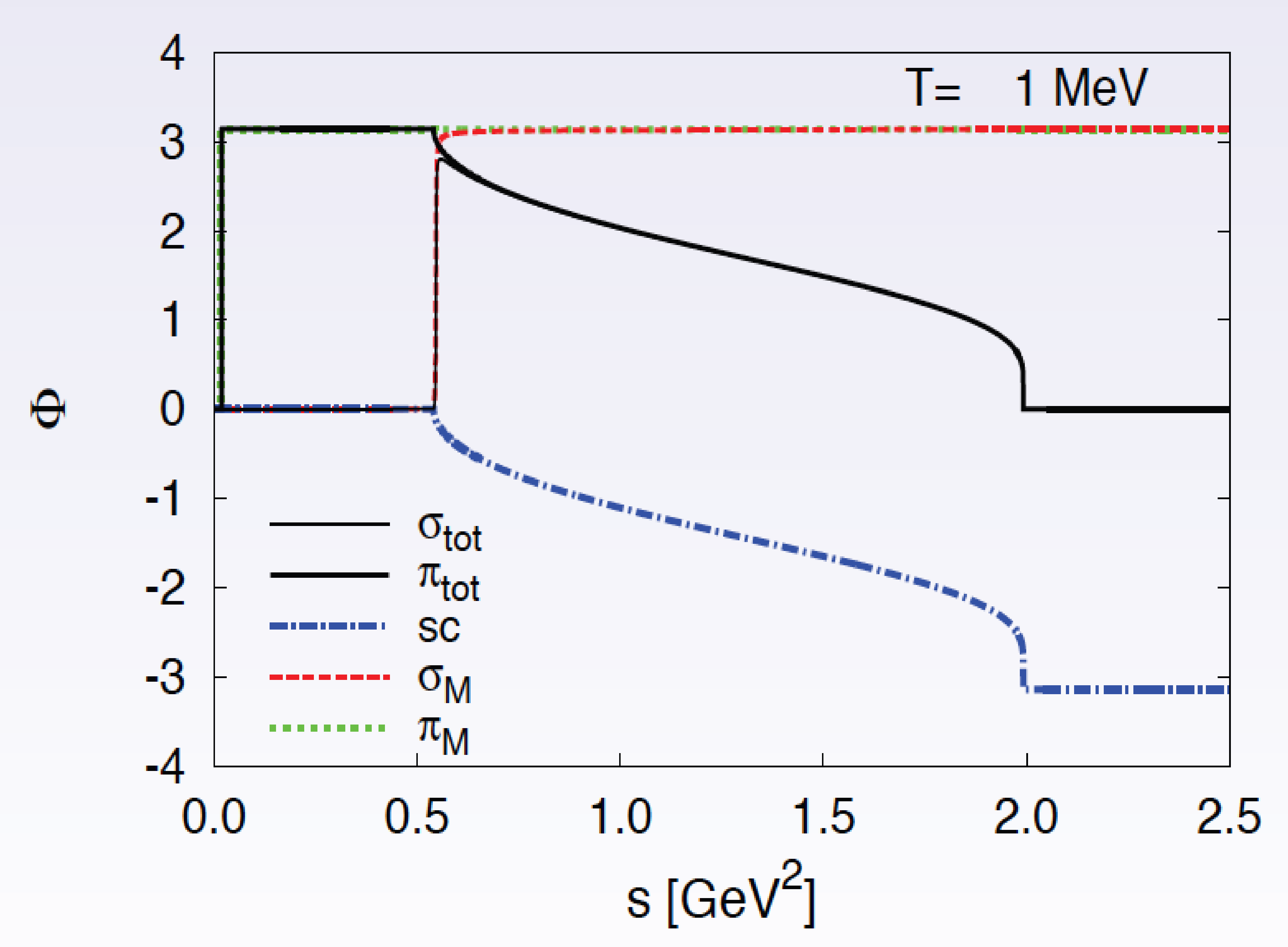


D. Blaschke, A. Dubinin, Yu. Kalinovsky,
Acta Phys. Pol. Suppl. 7 (2014)



XXXI. Max Born Symposium,
Wroclaw (2013)

Phase shifts



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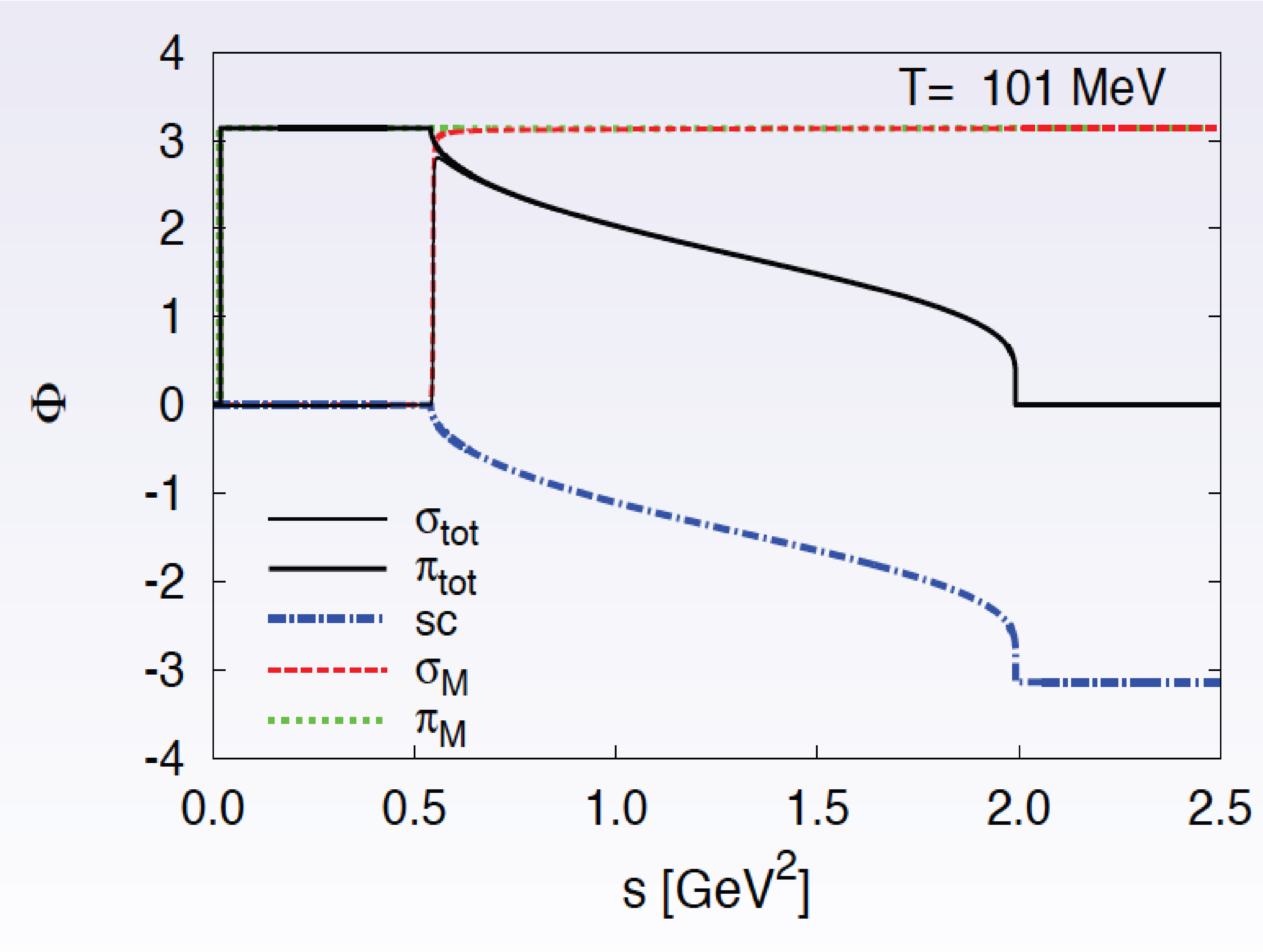

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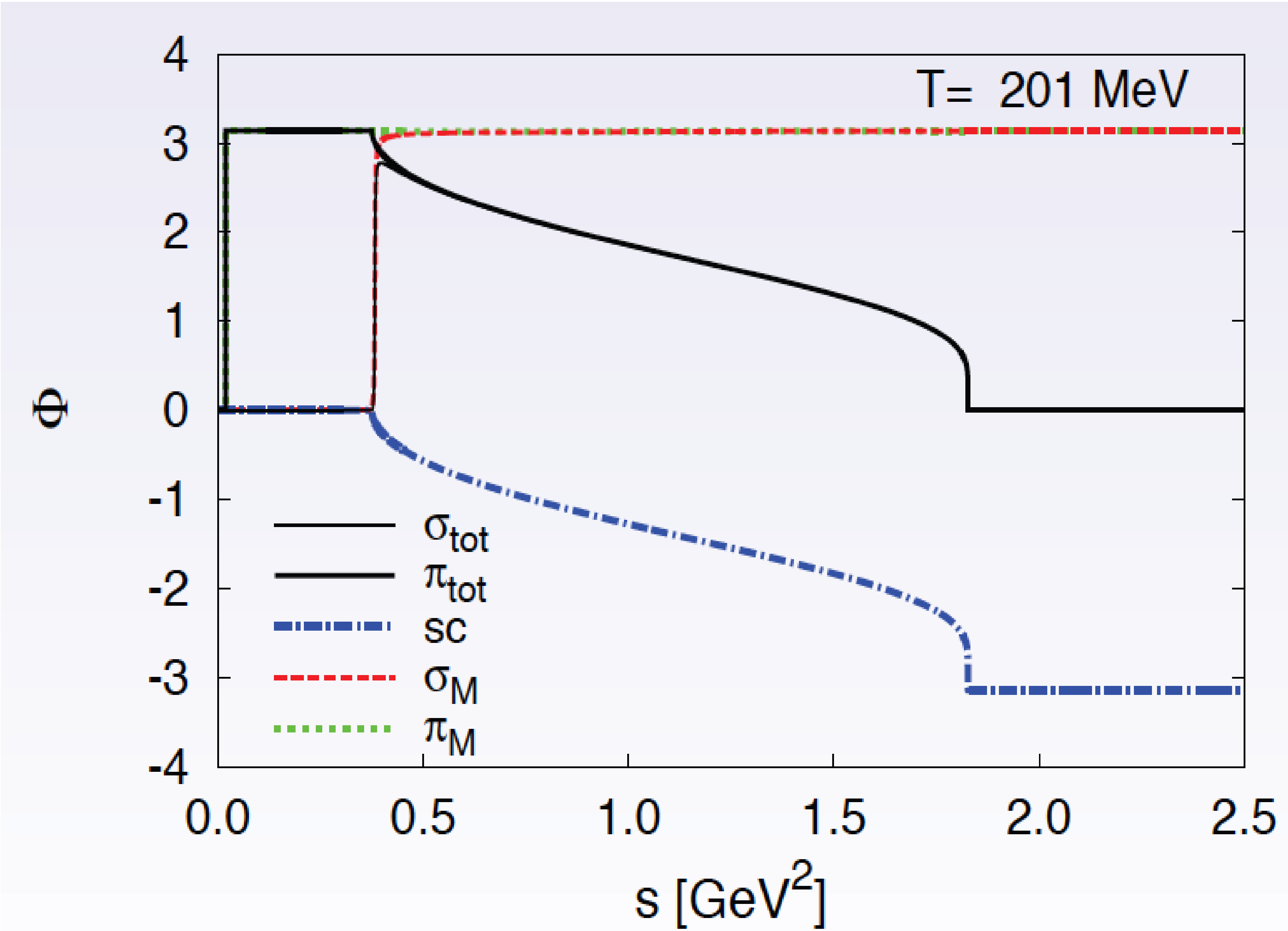

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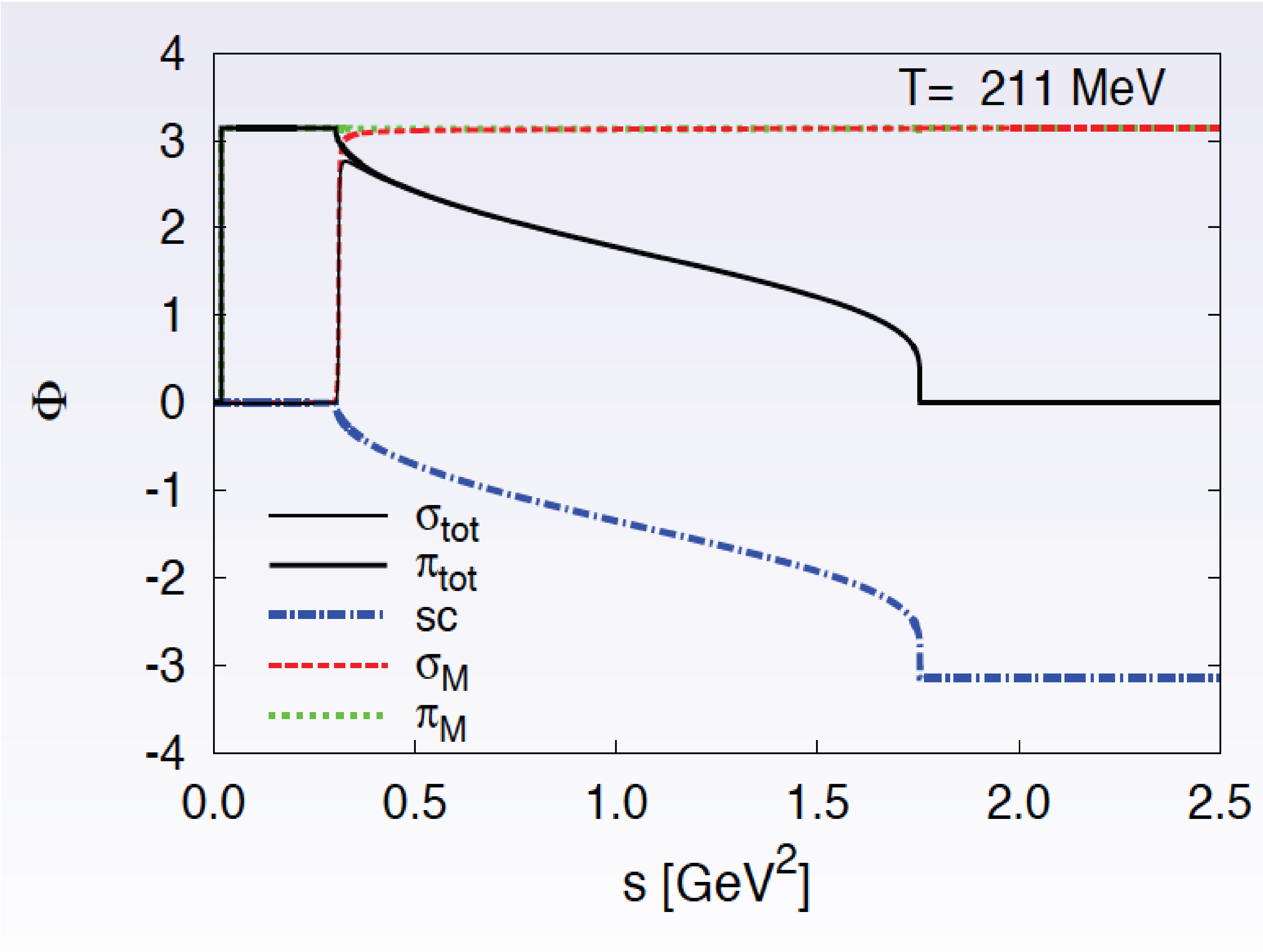

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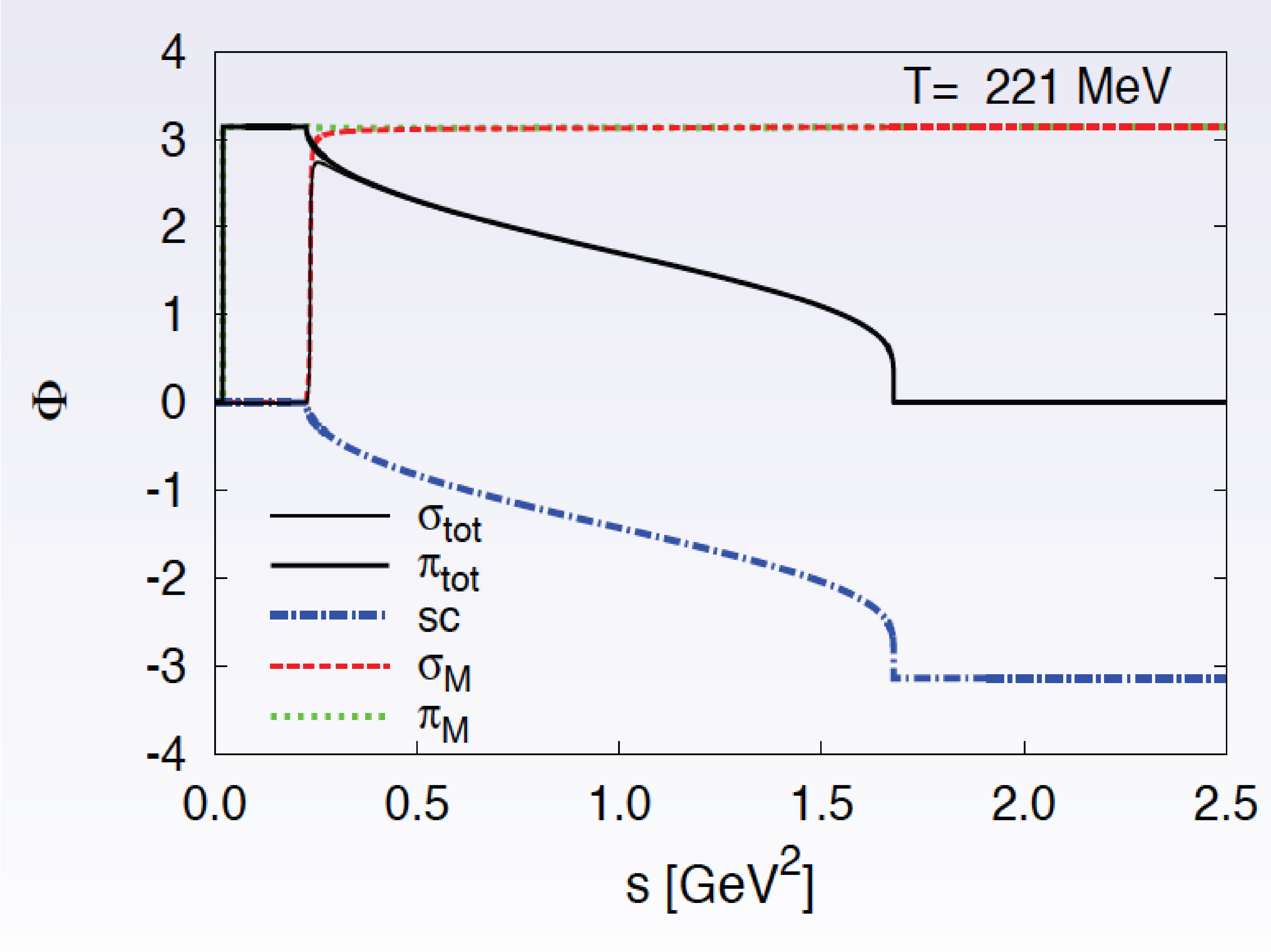

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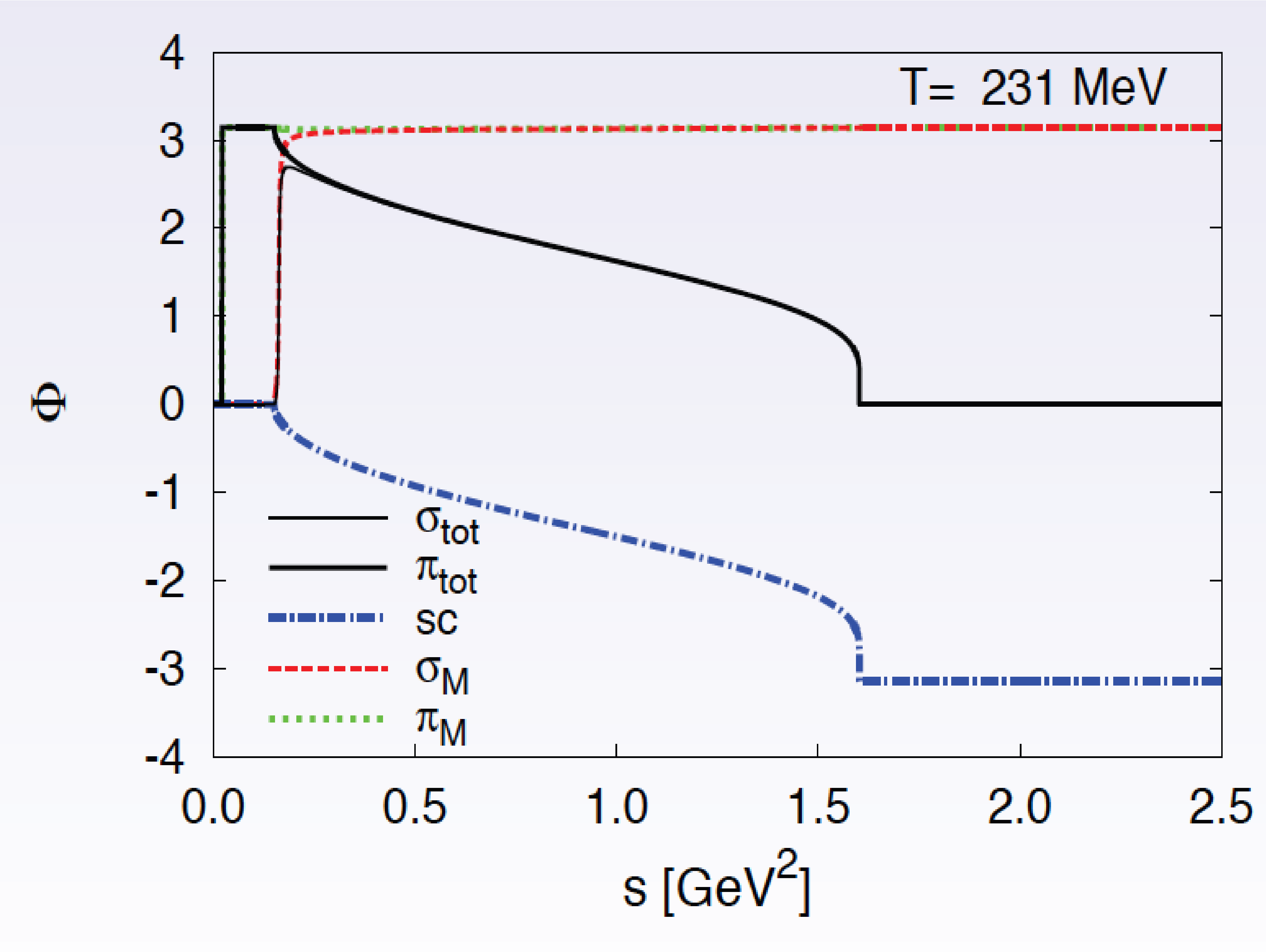

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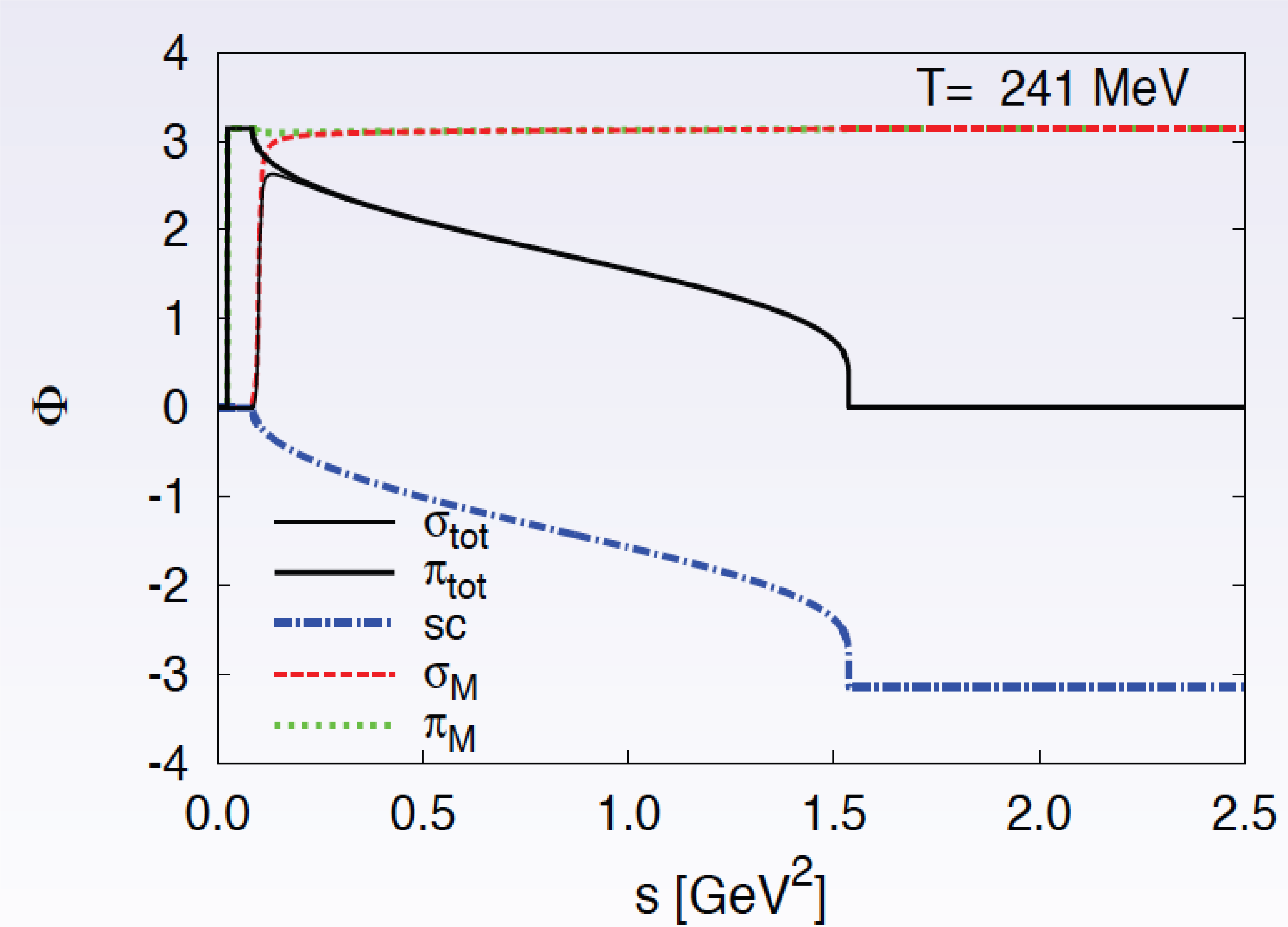
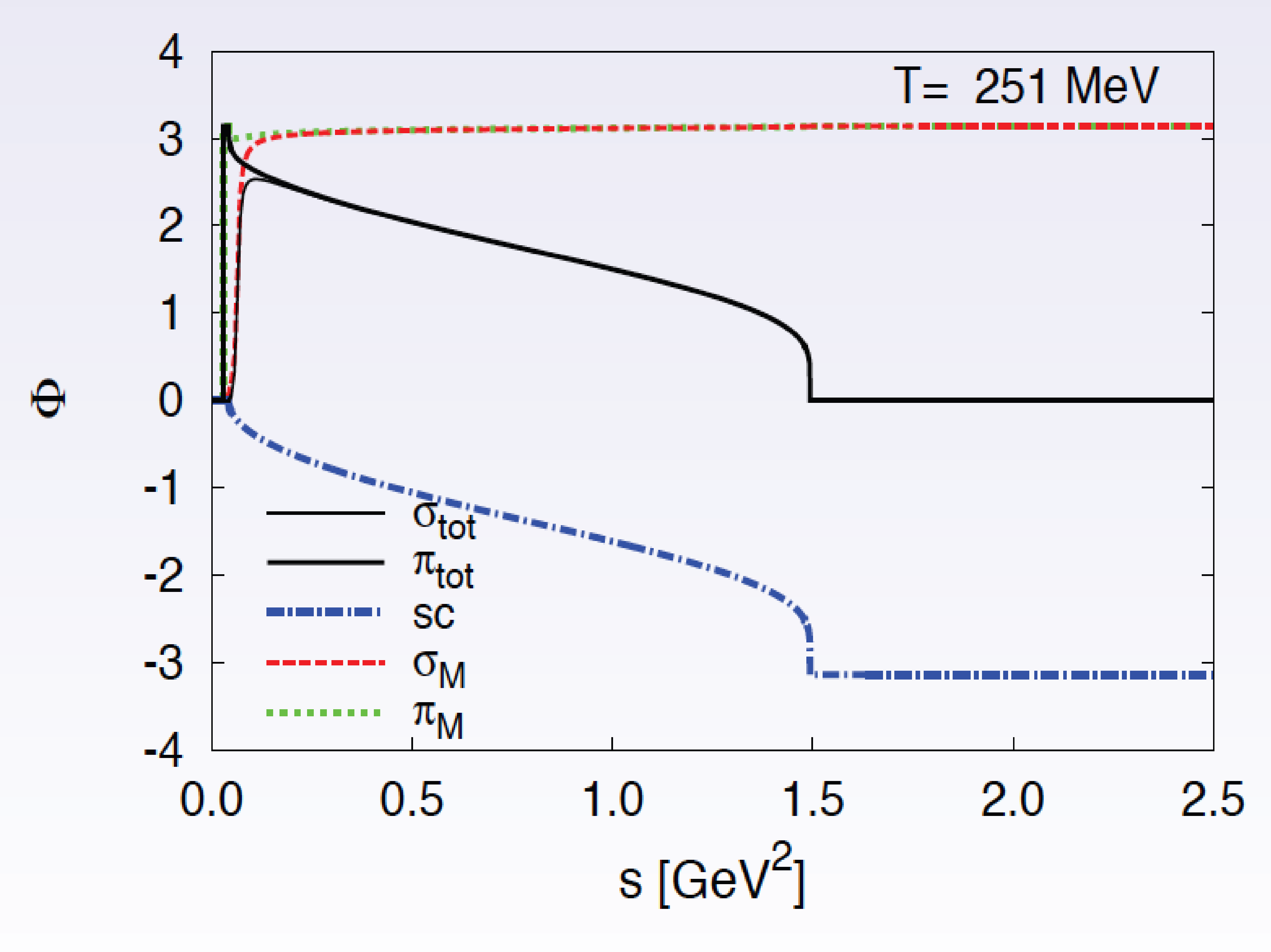


Phase shifts



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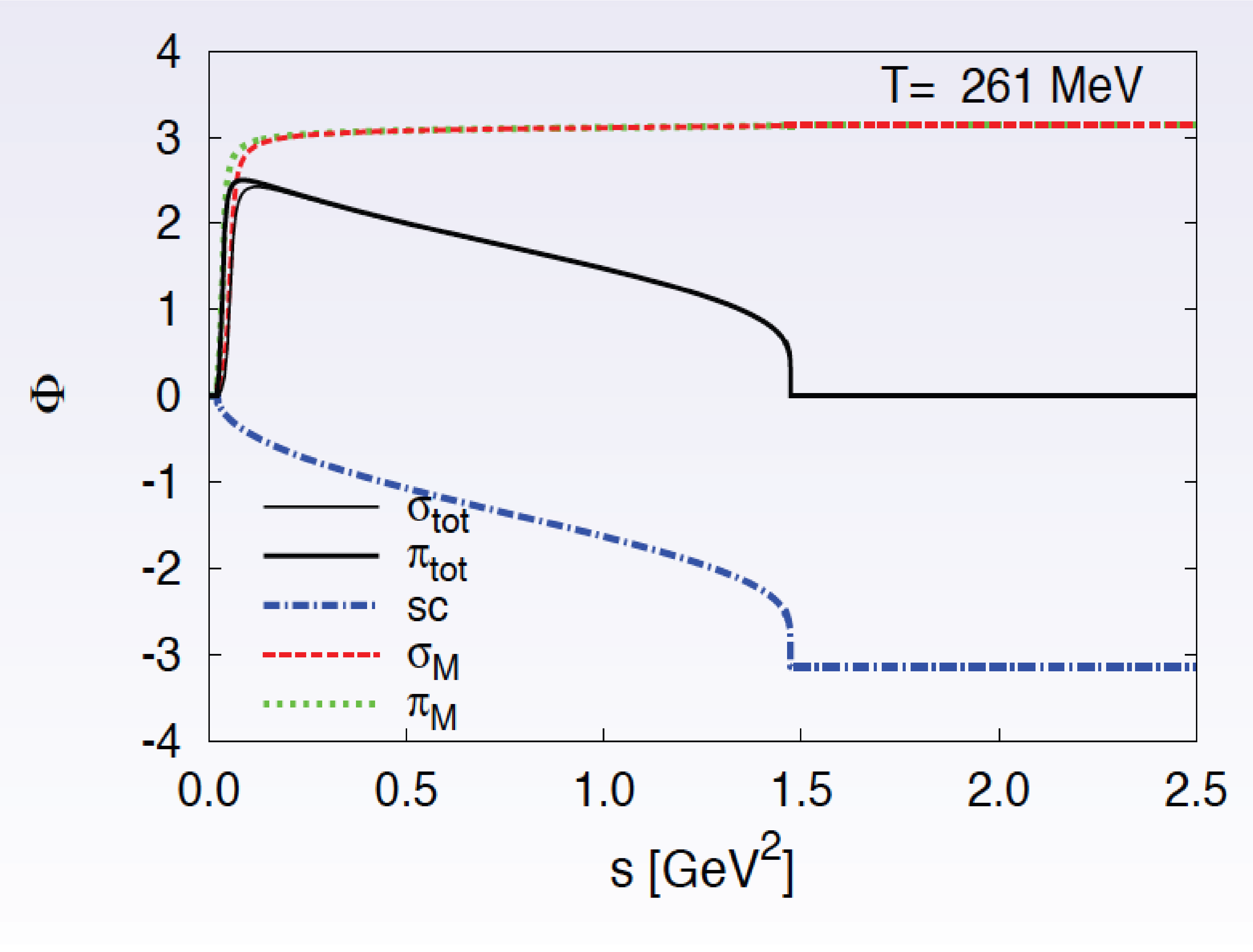

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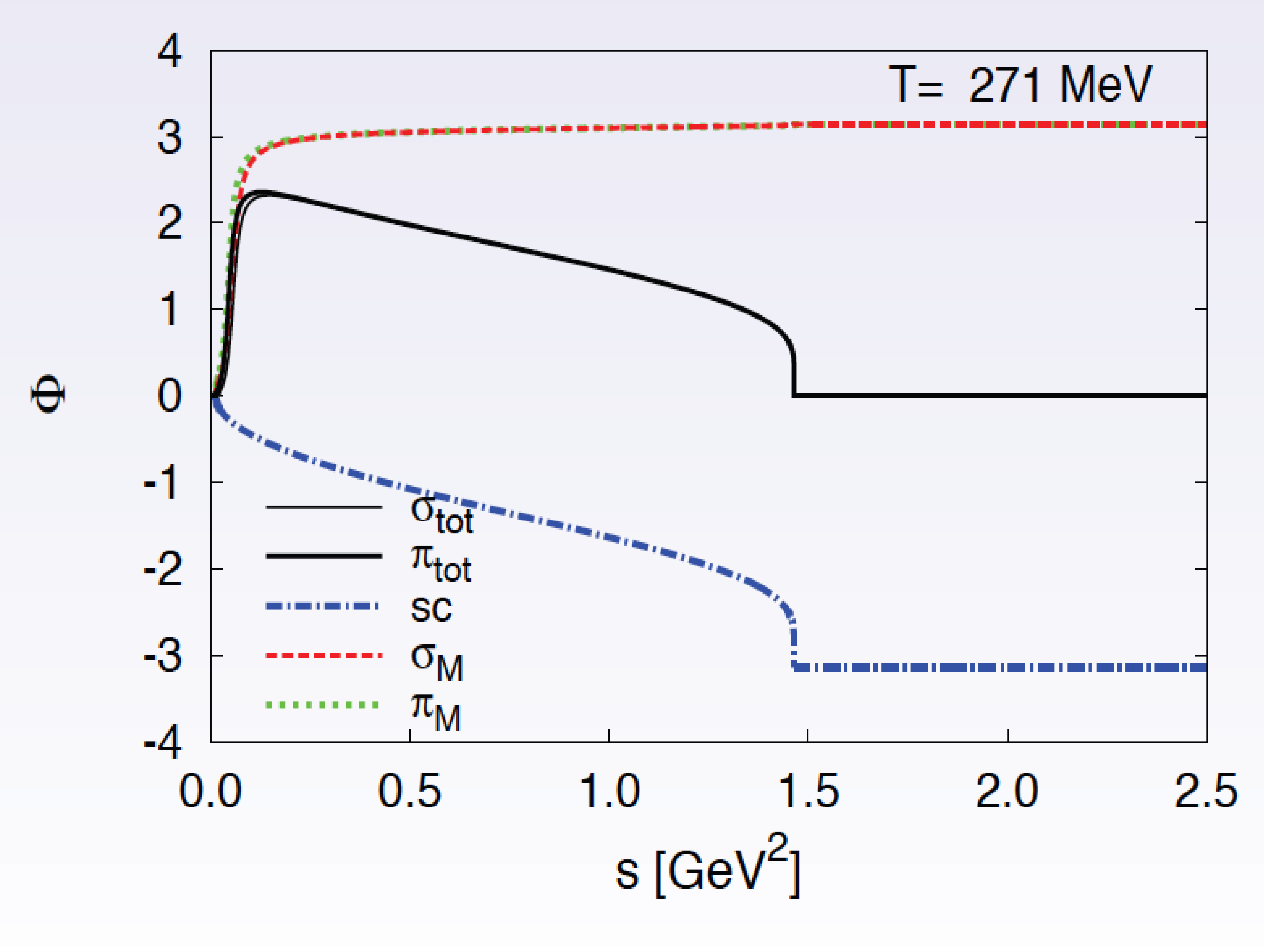

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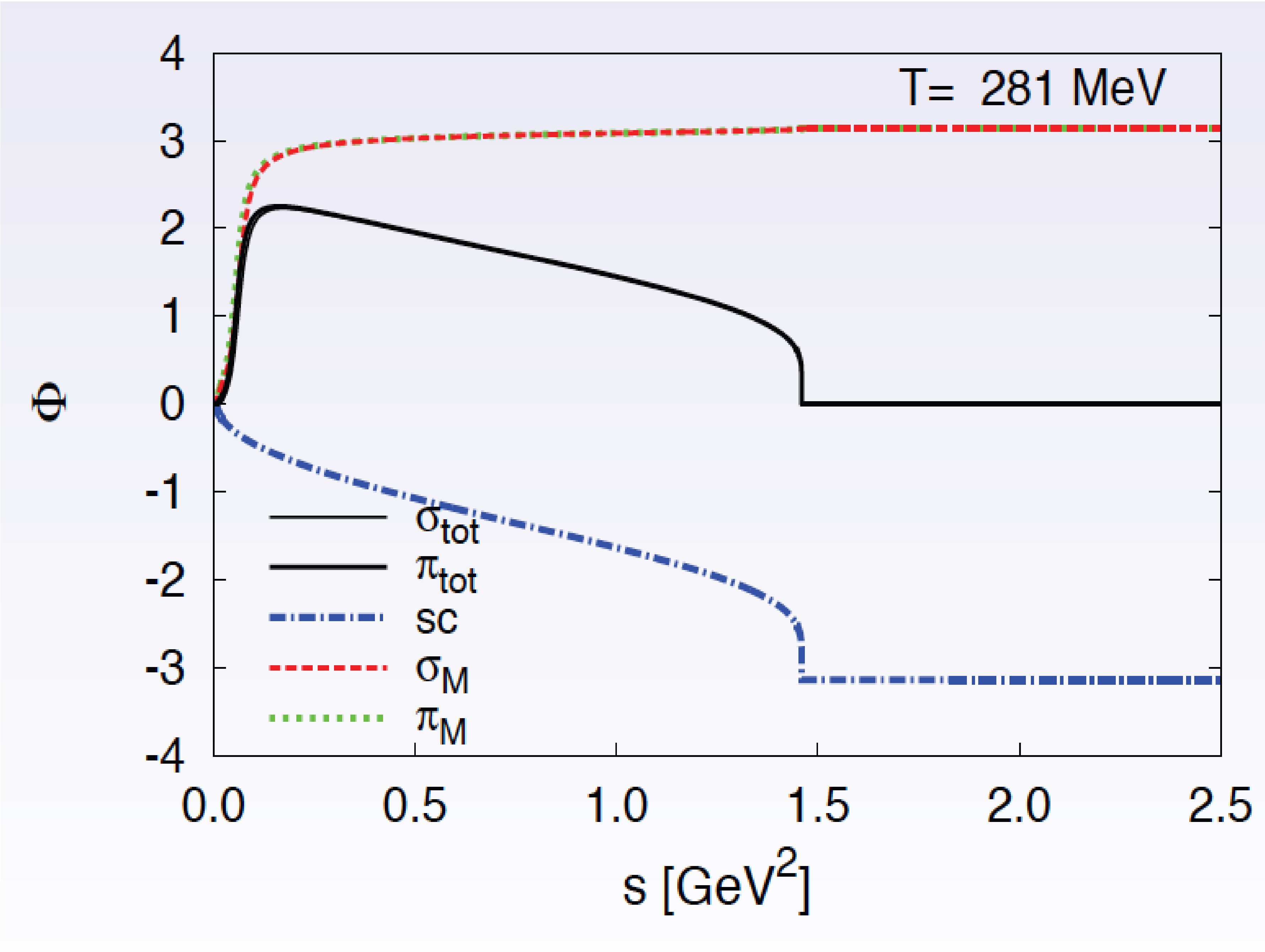

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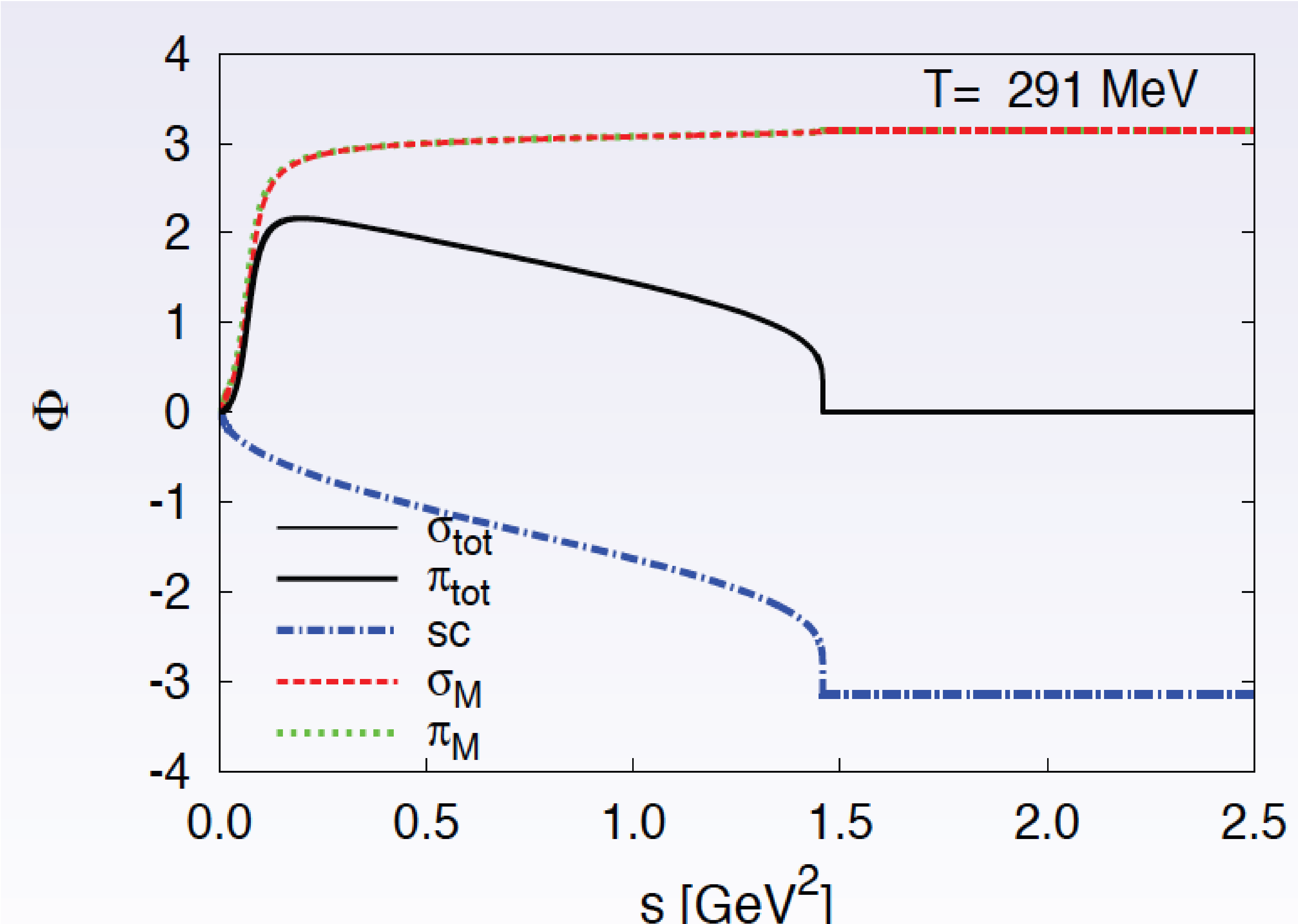

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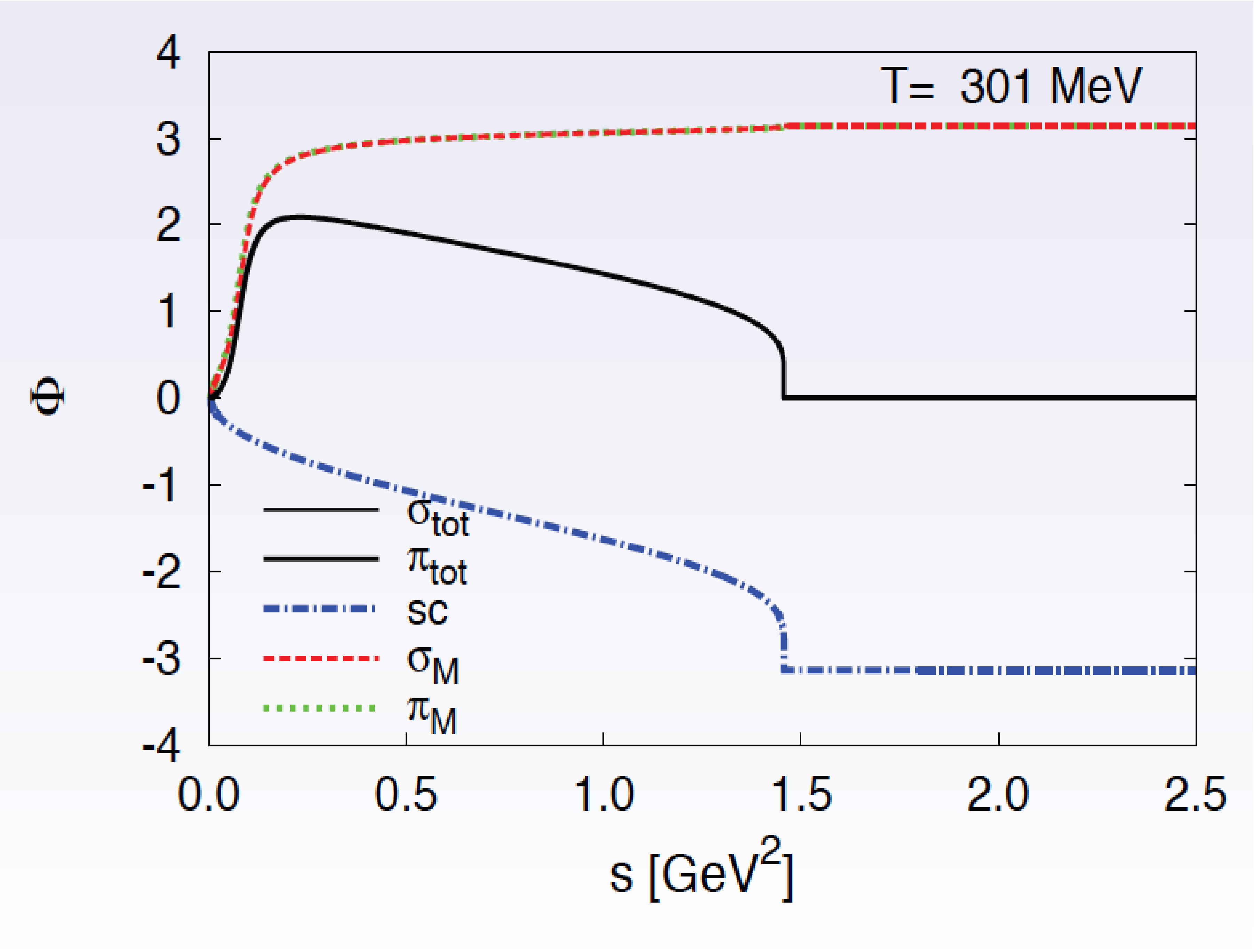

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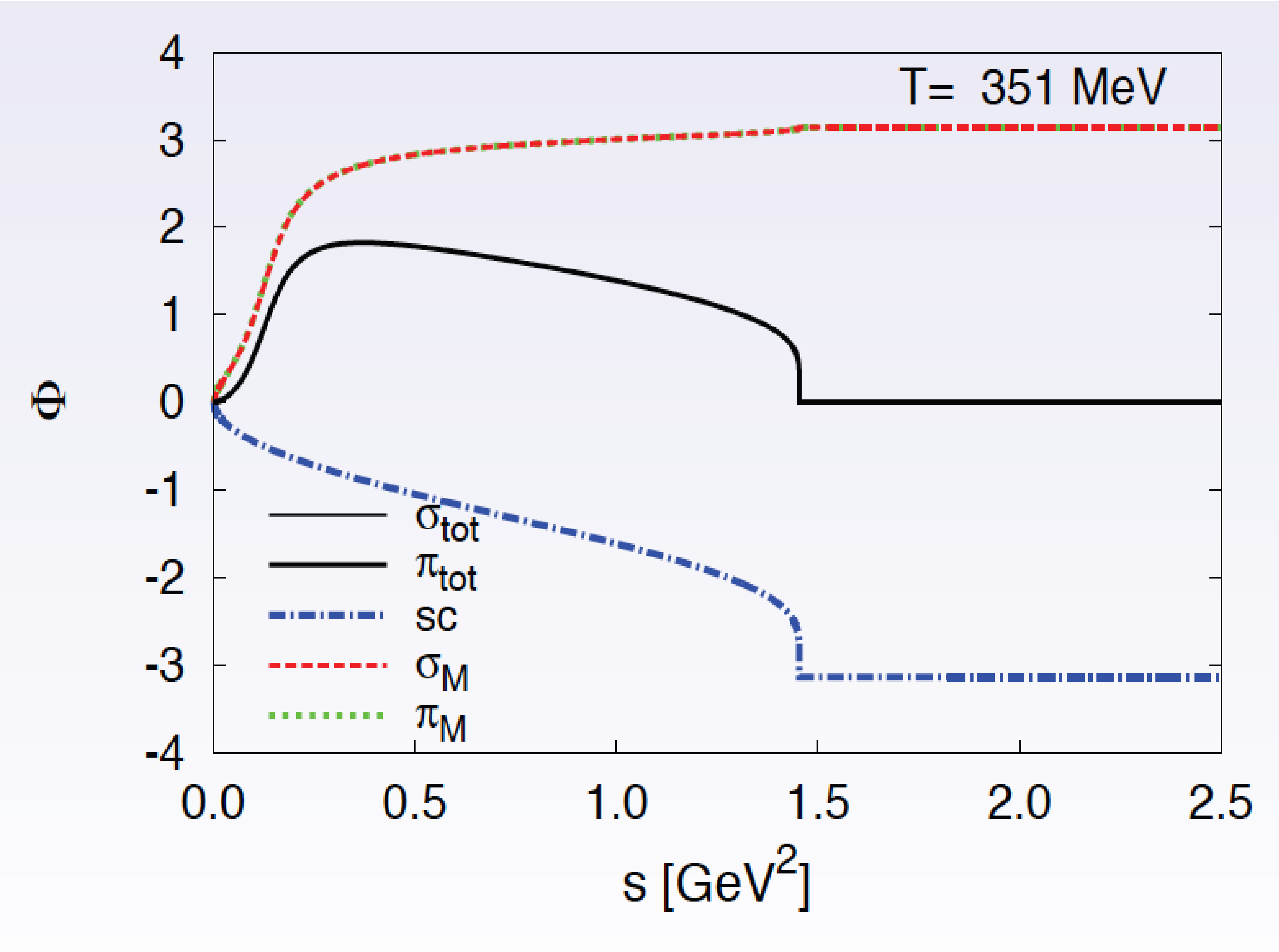

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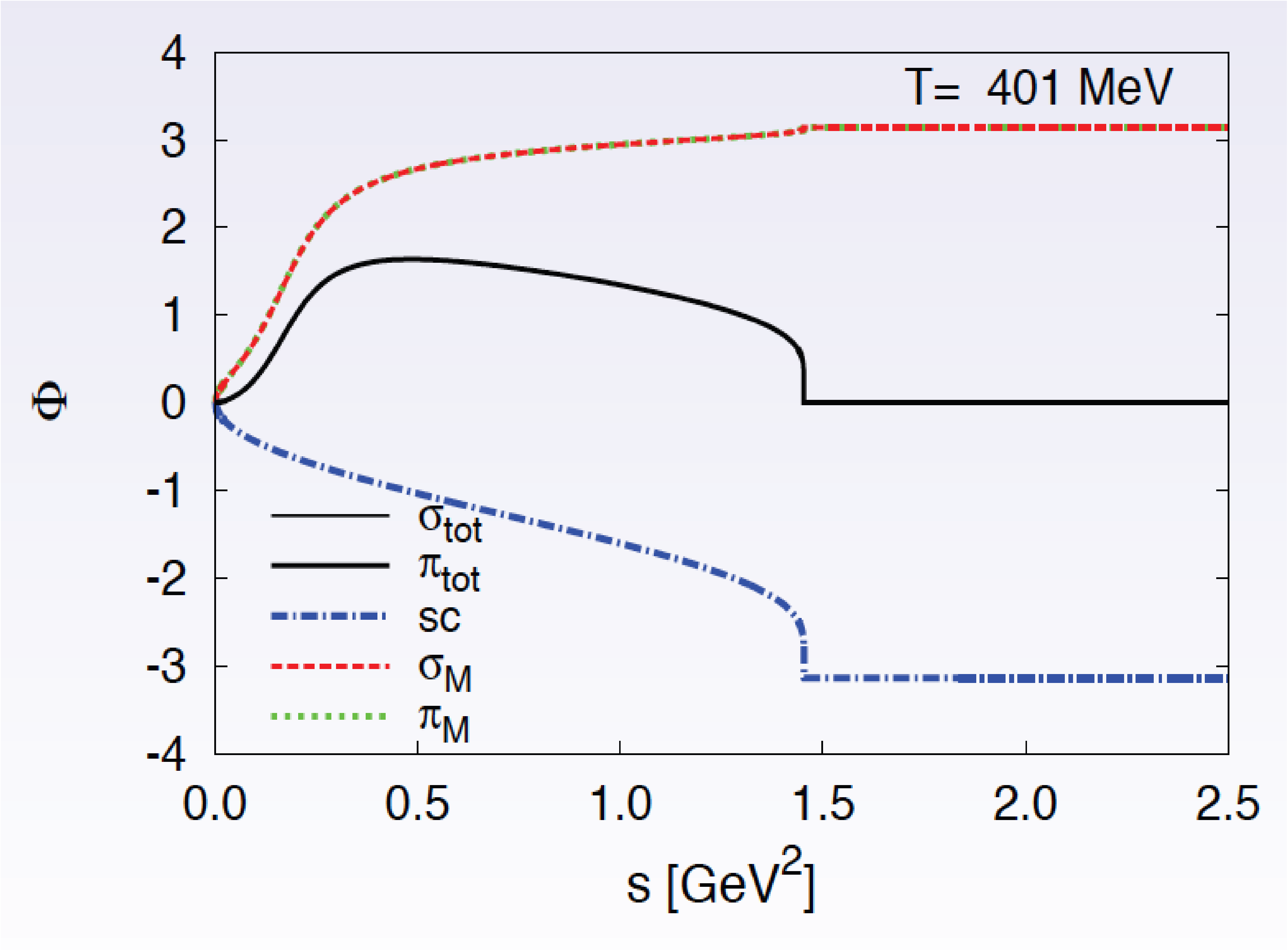

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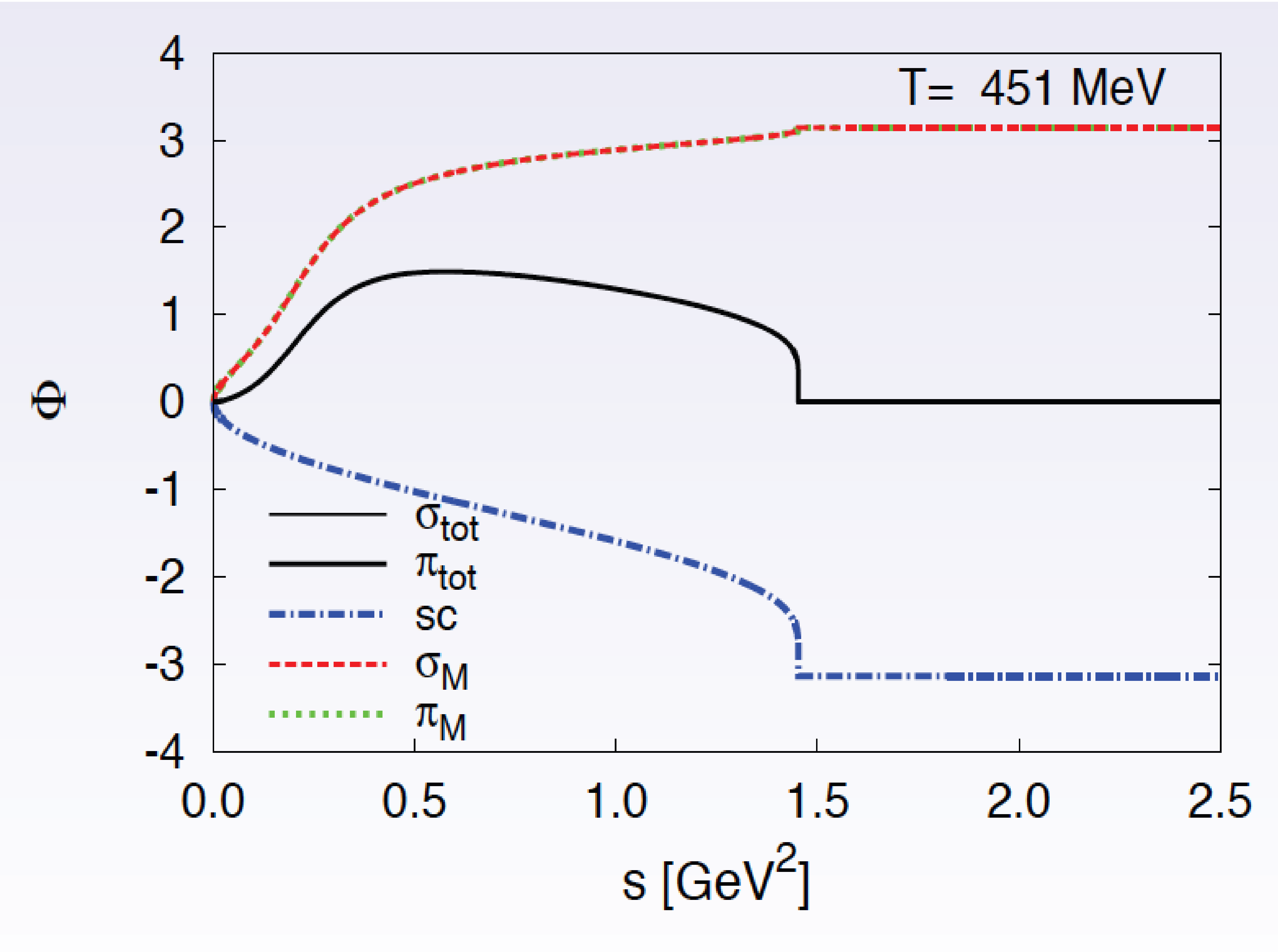

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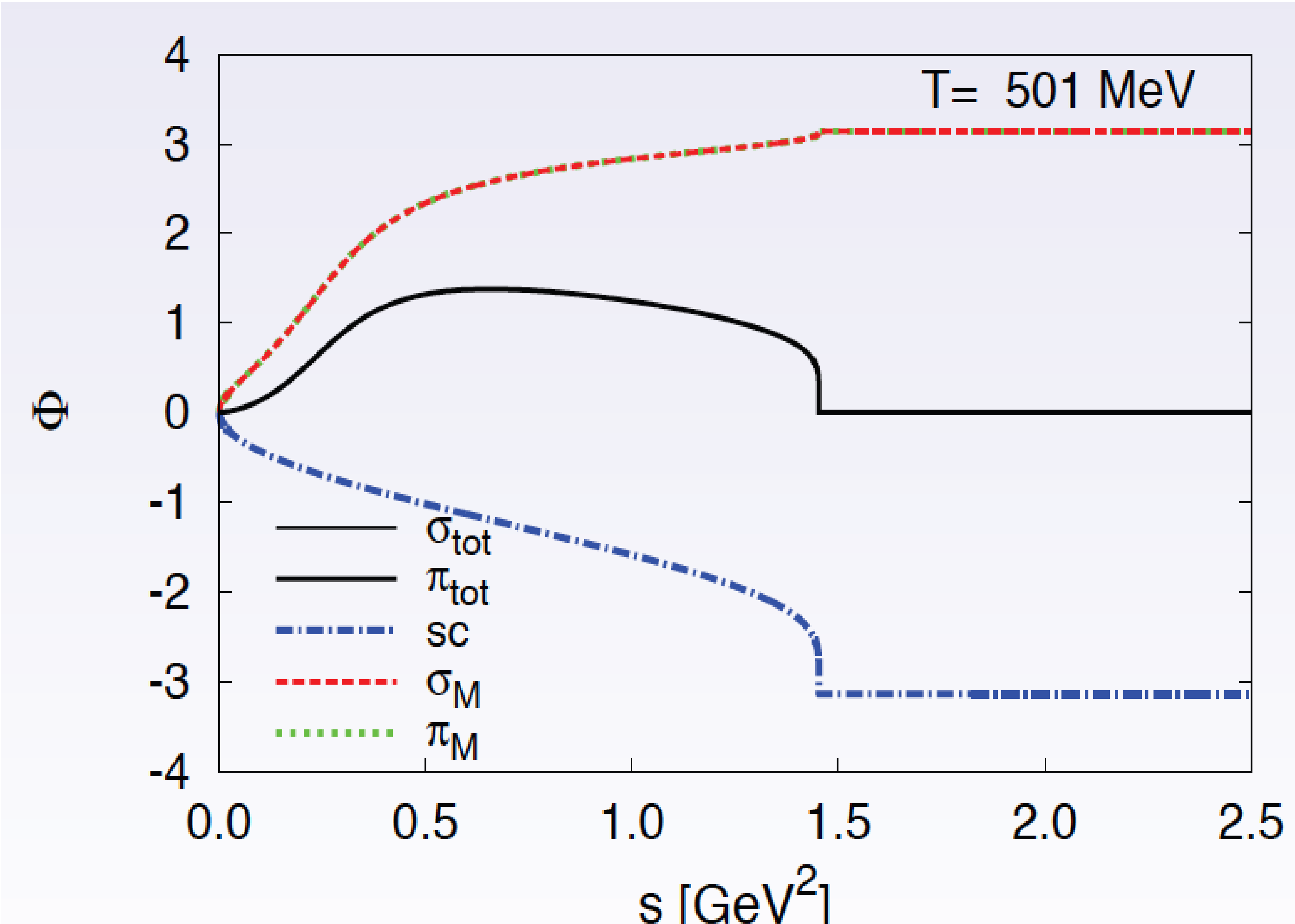

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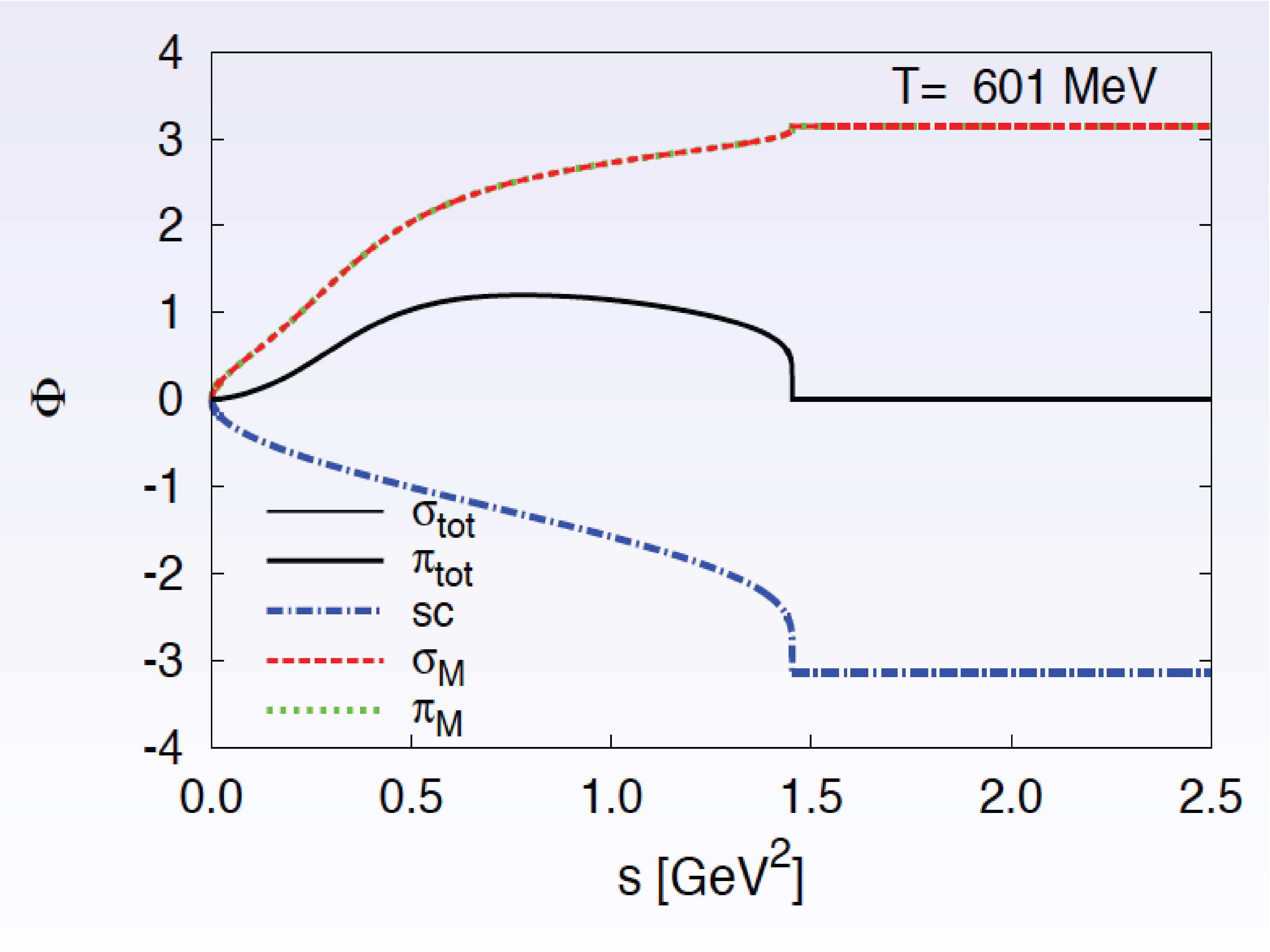

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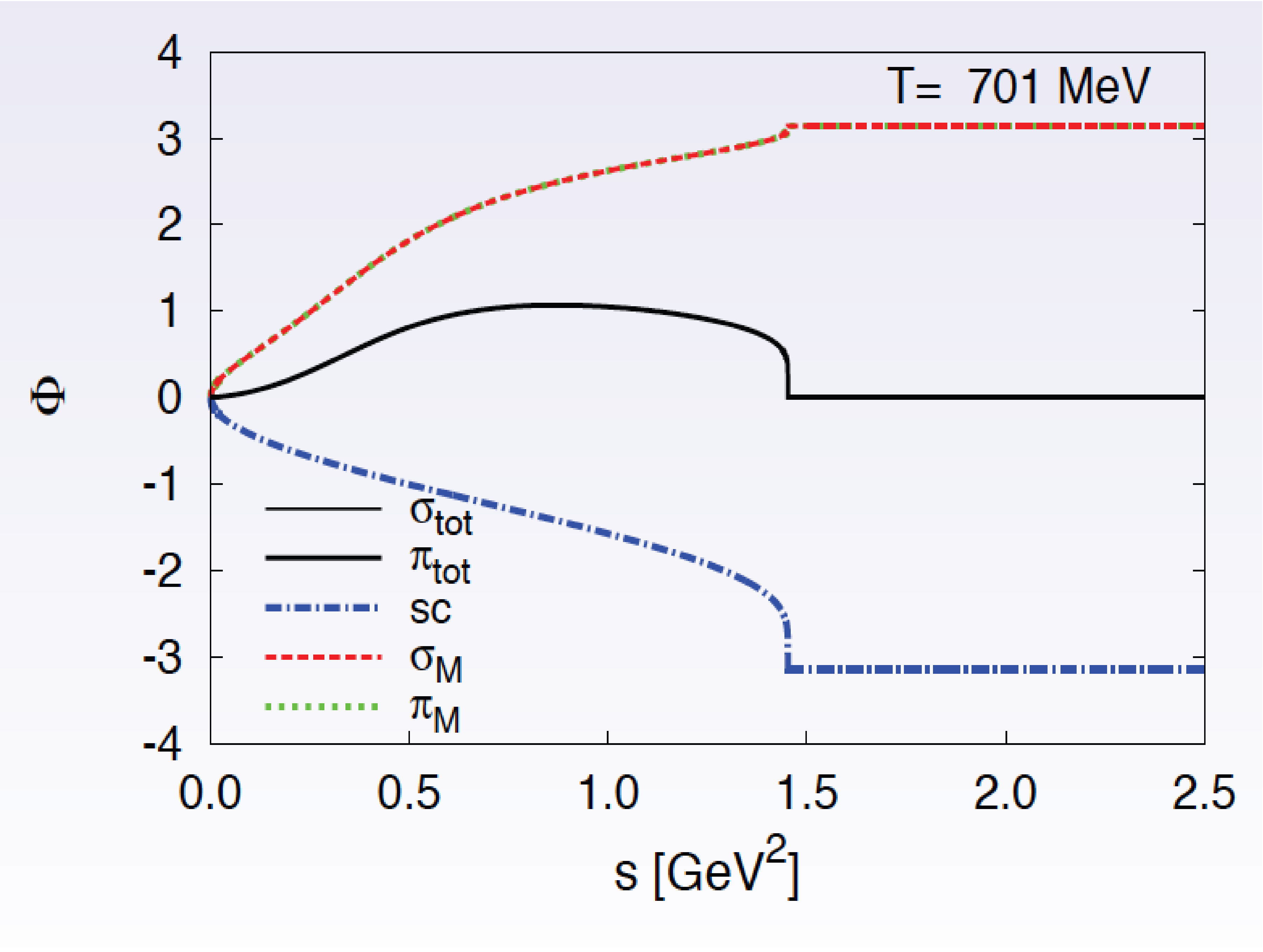

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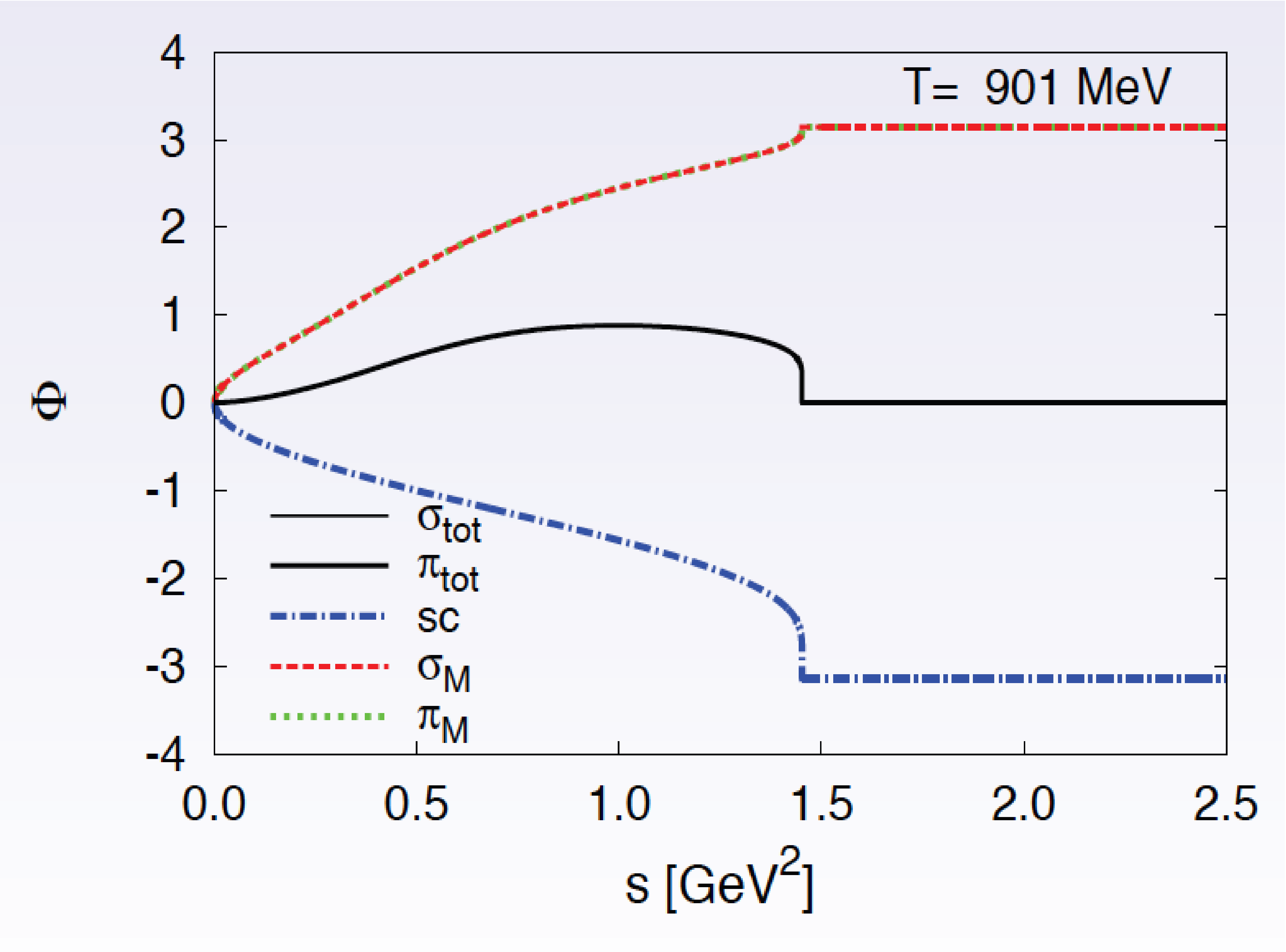

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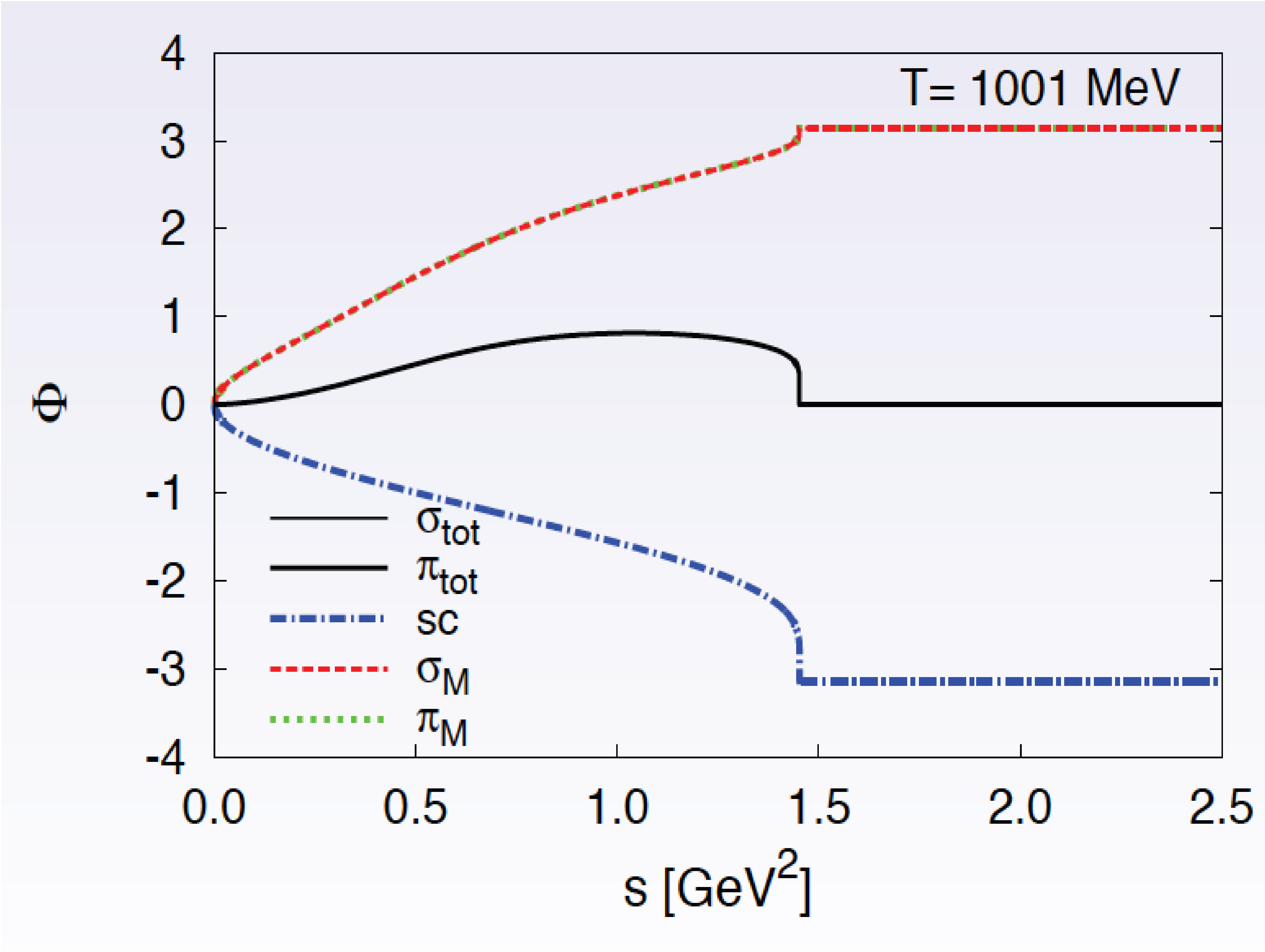

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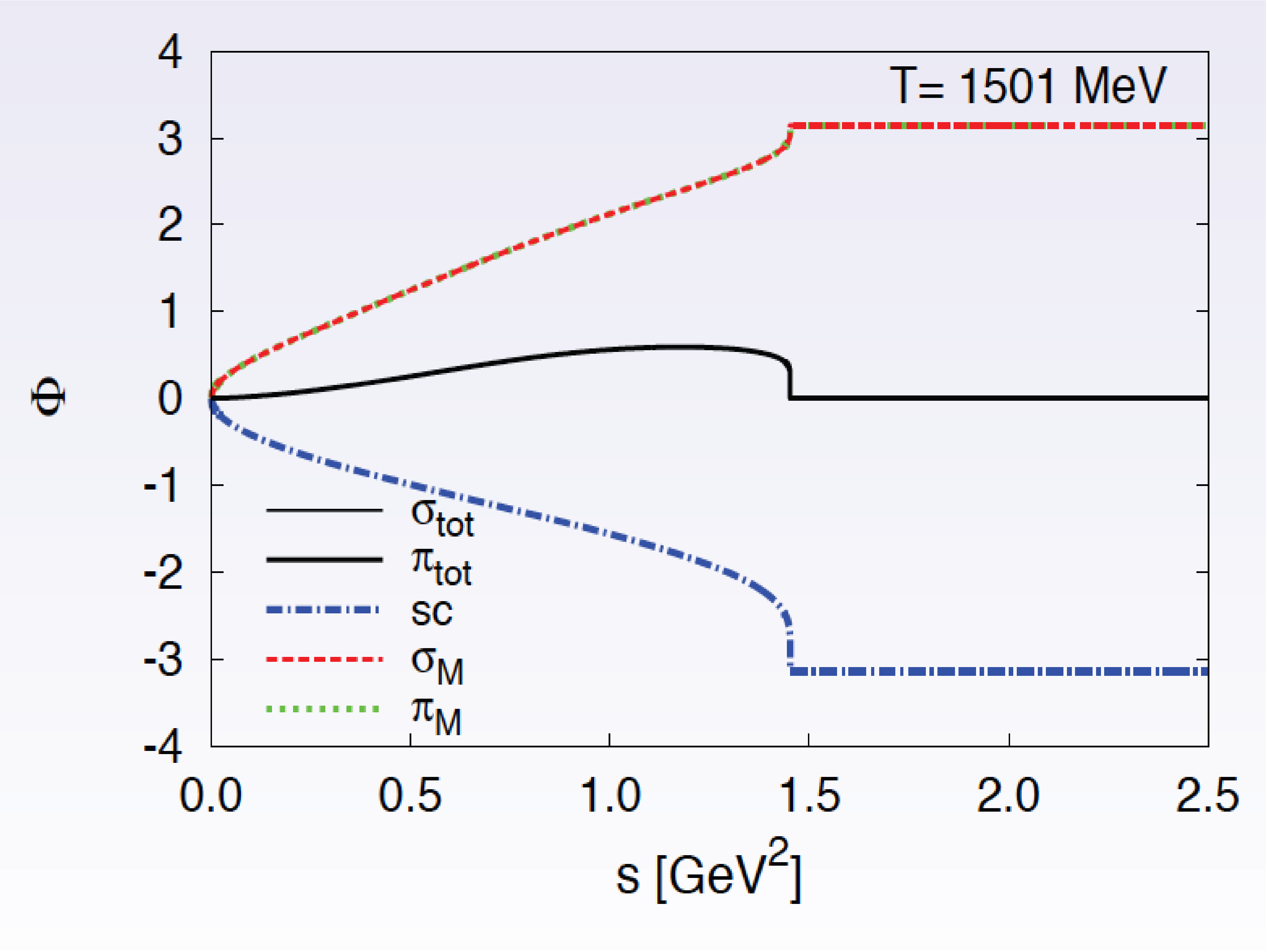

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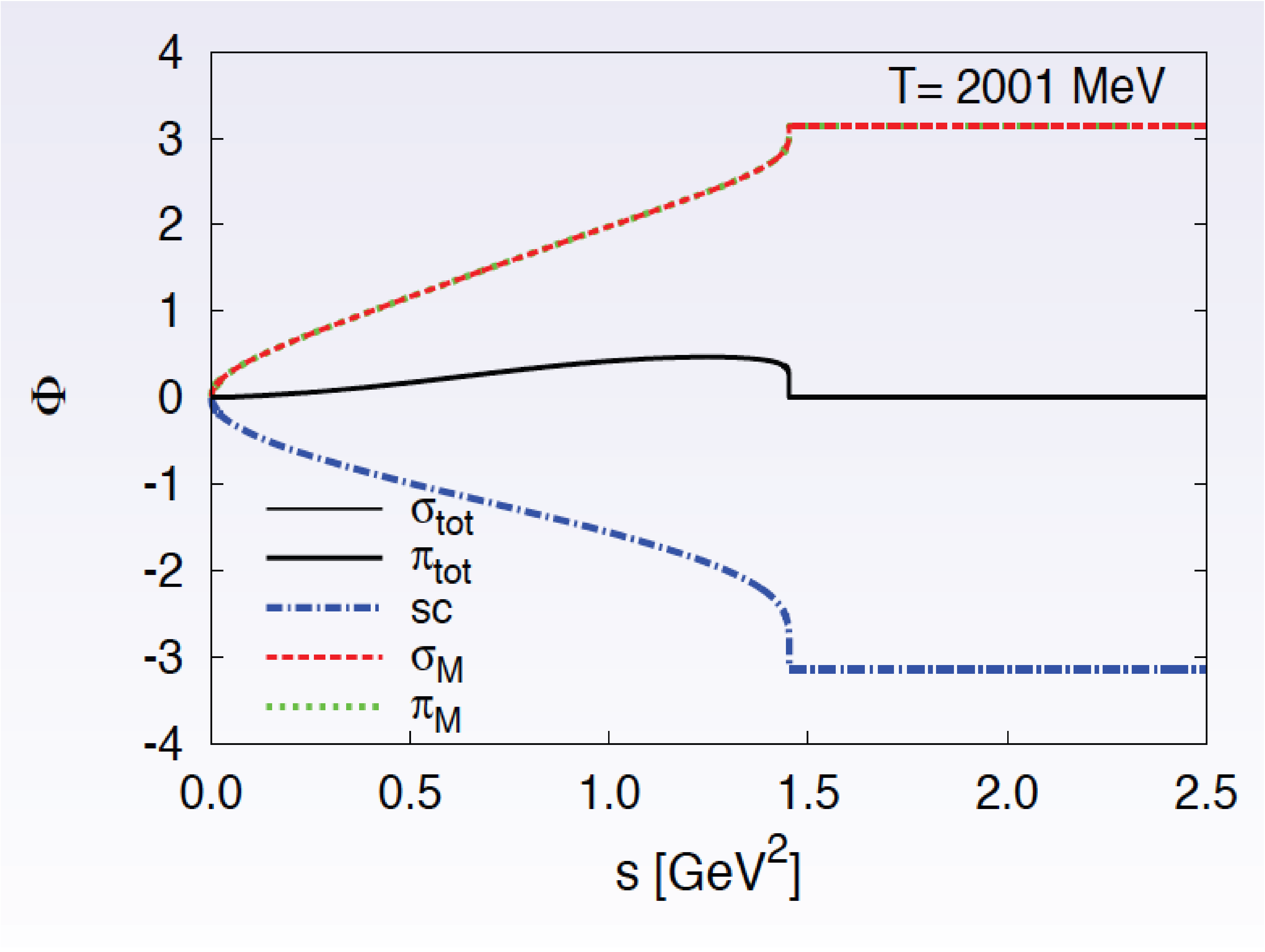

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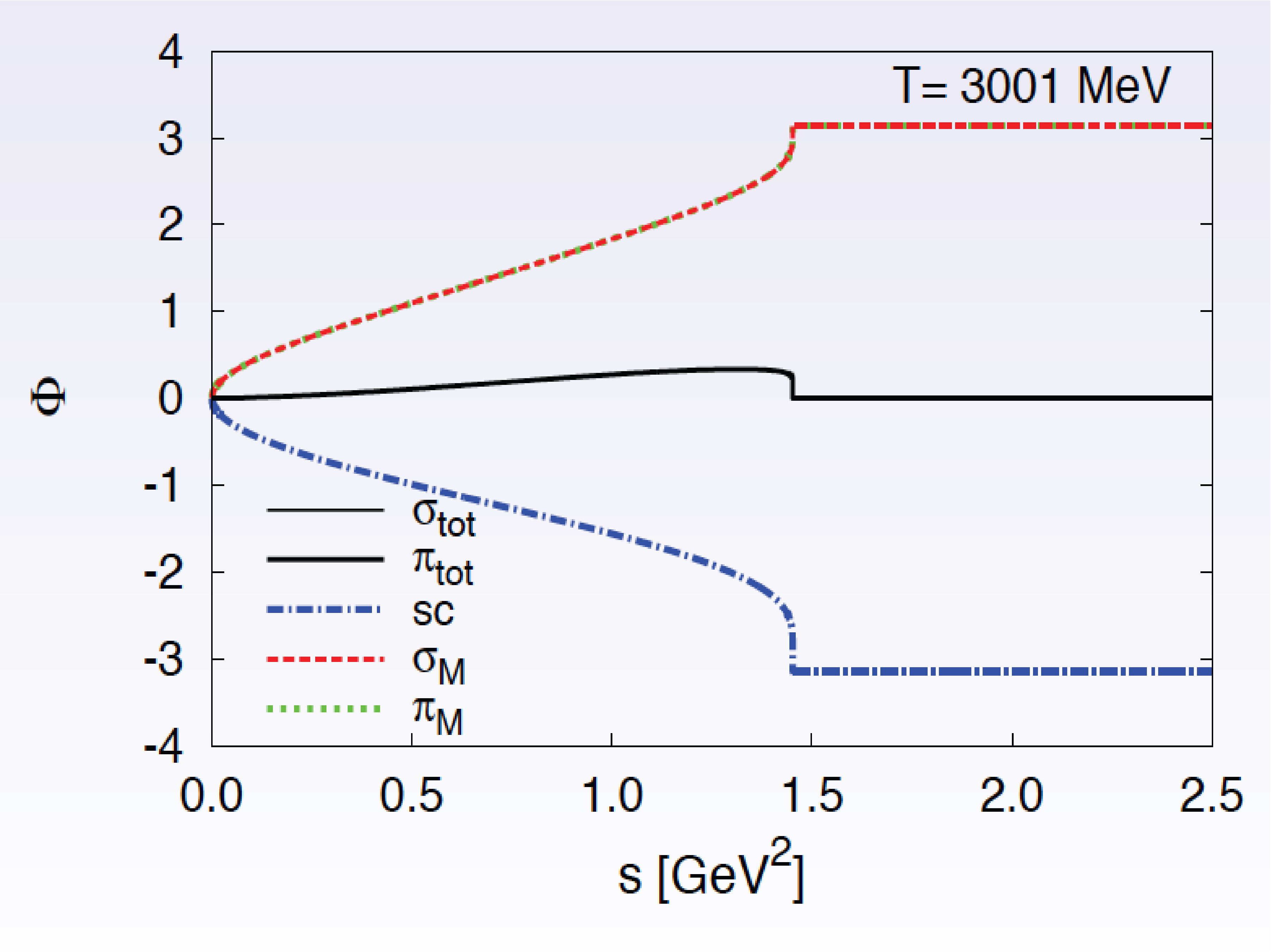
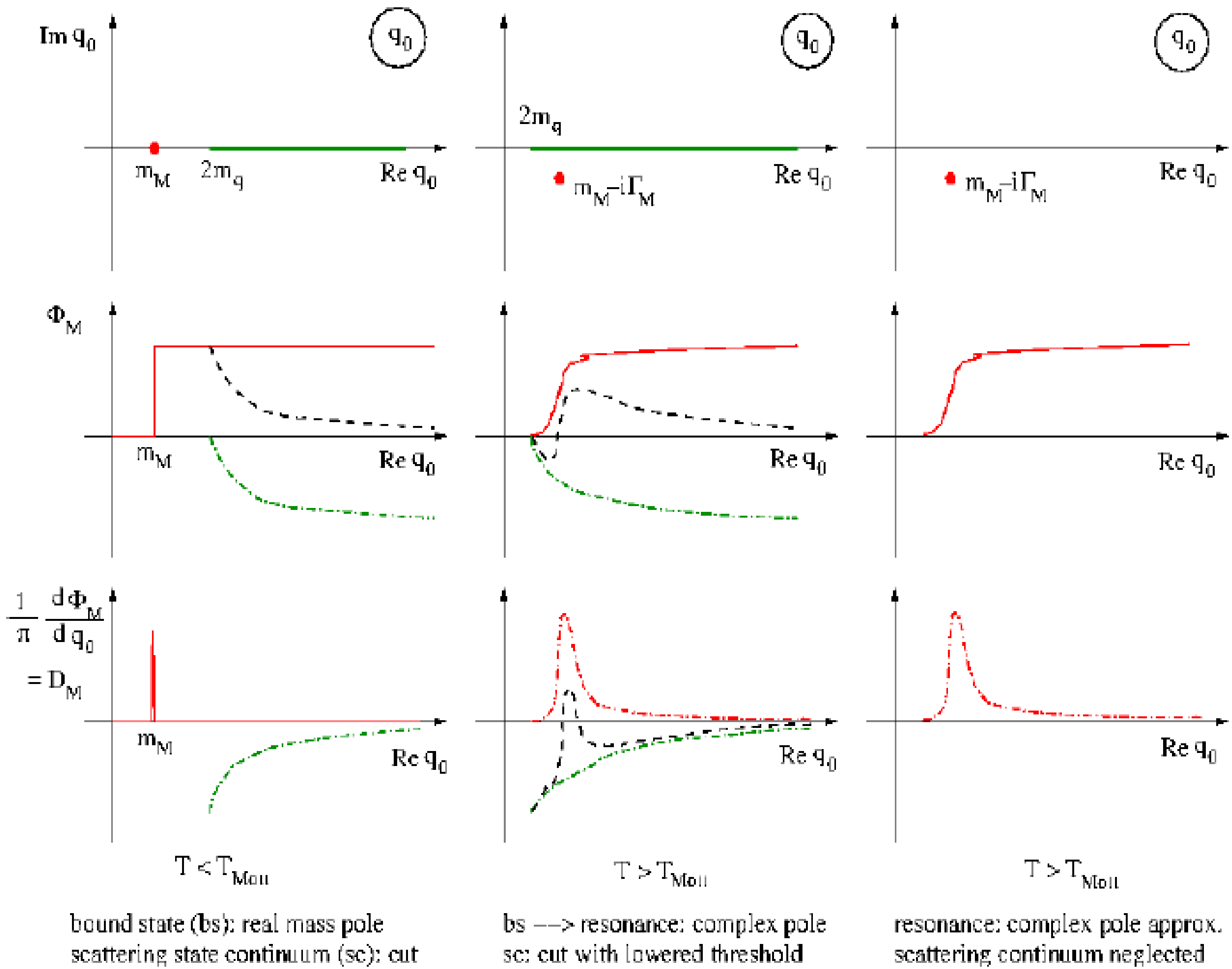
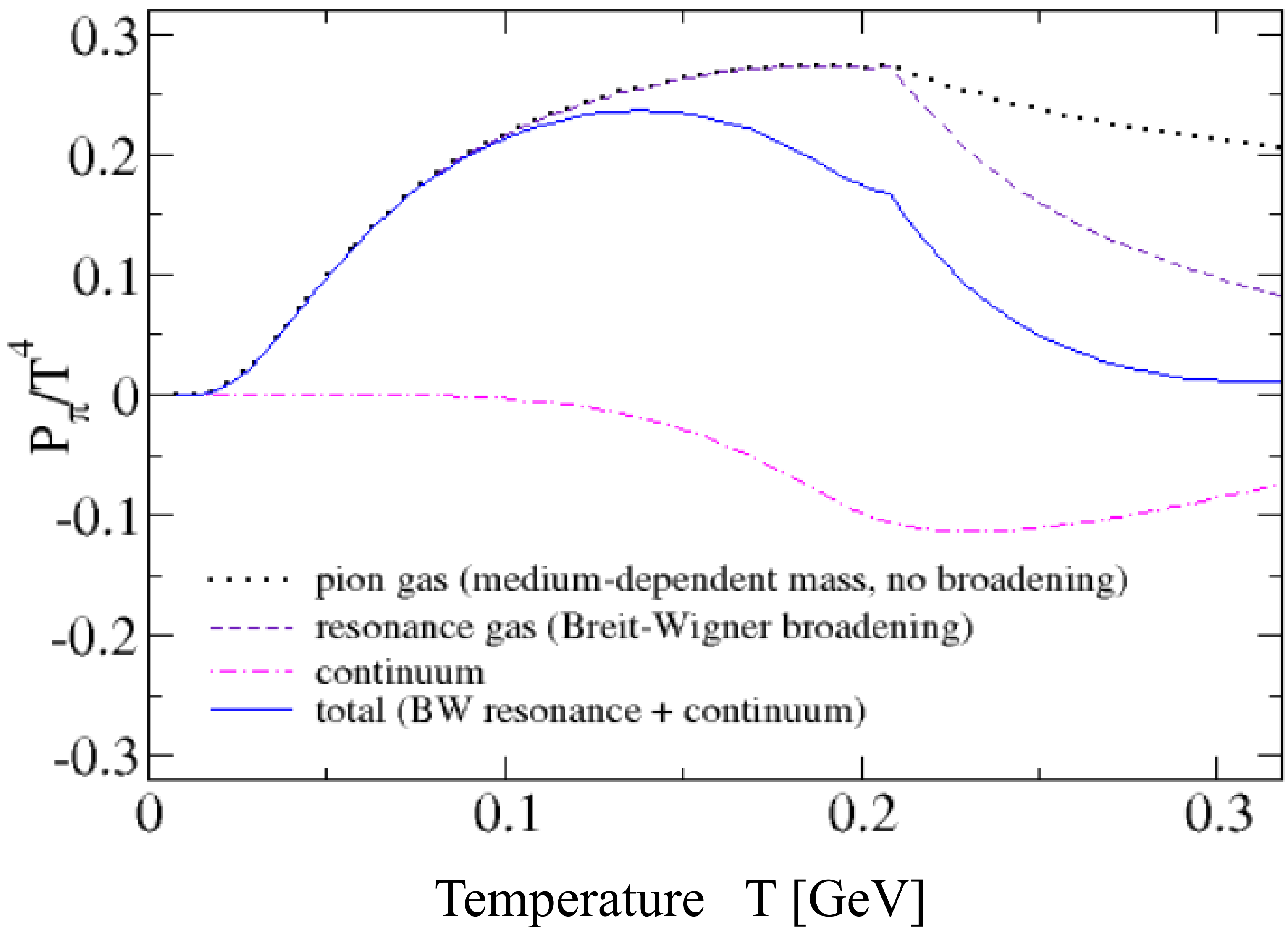


Summary: Levinson's Theorem & analytical properties



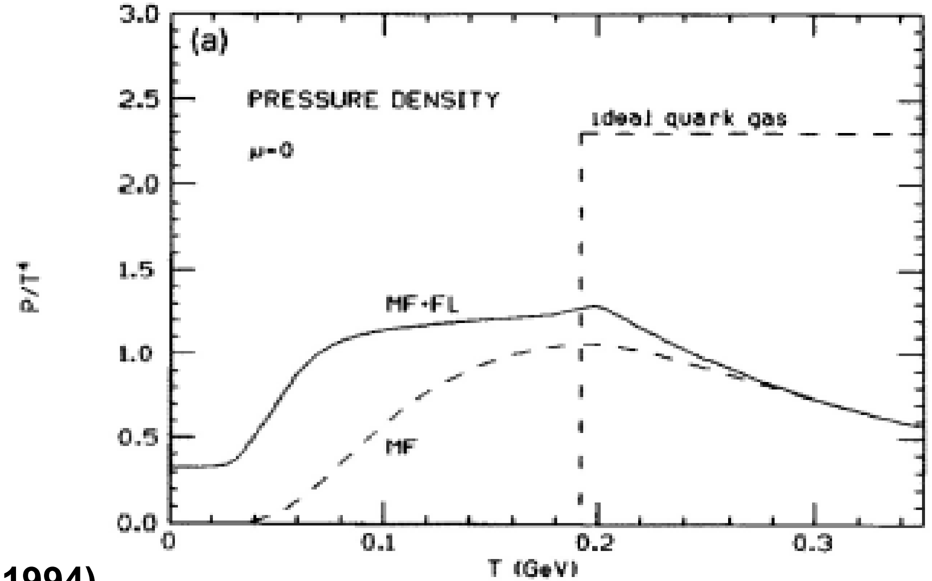
Pion pressure

Role of scattering continuum (Levinson theorem!) for pressure:

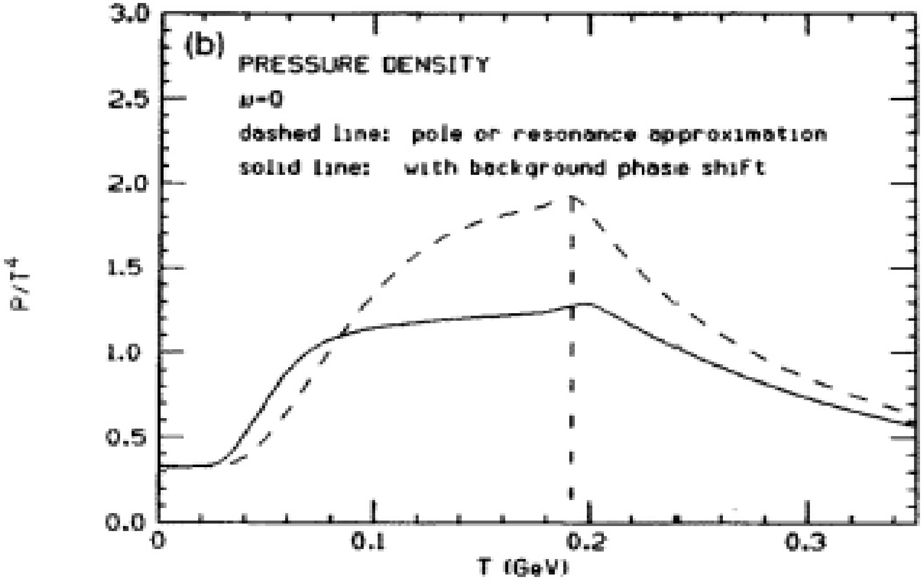


Mott Dissociation of Mesons in Quark Matter

J. Huefner, S.P. Klevansky, P. Zhuang, H. Voss, Ann. Phys. 234, 225 (1994)



P. Zhuang, J. Huefner, S.P. Klevansky, NPA 576, 525 (1994)



Mott Dissociation of Mesons in Quark Matter

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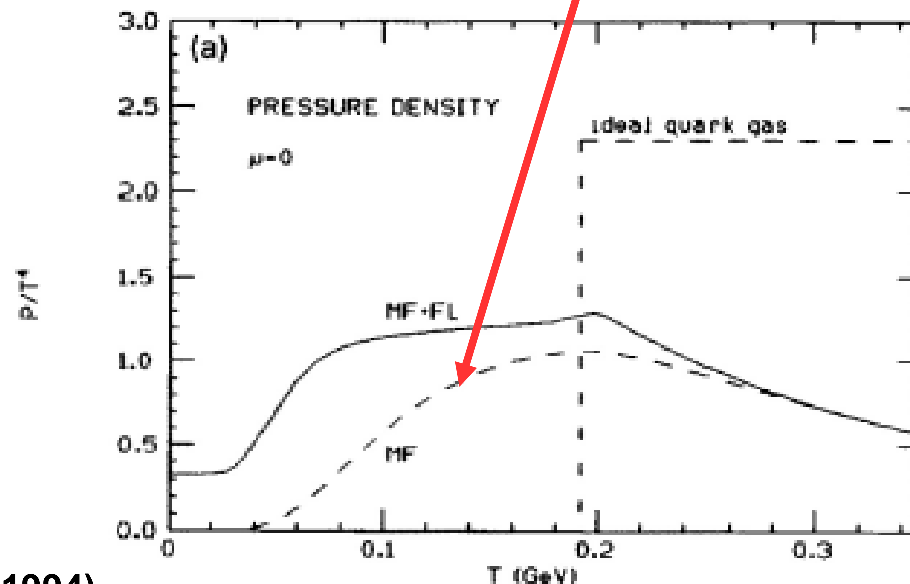
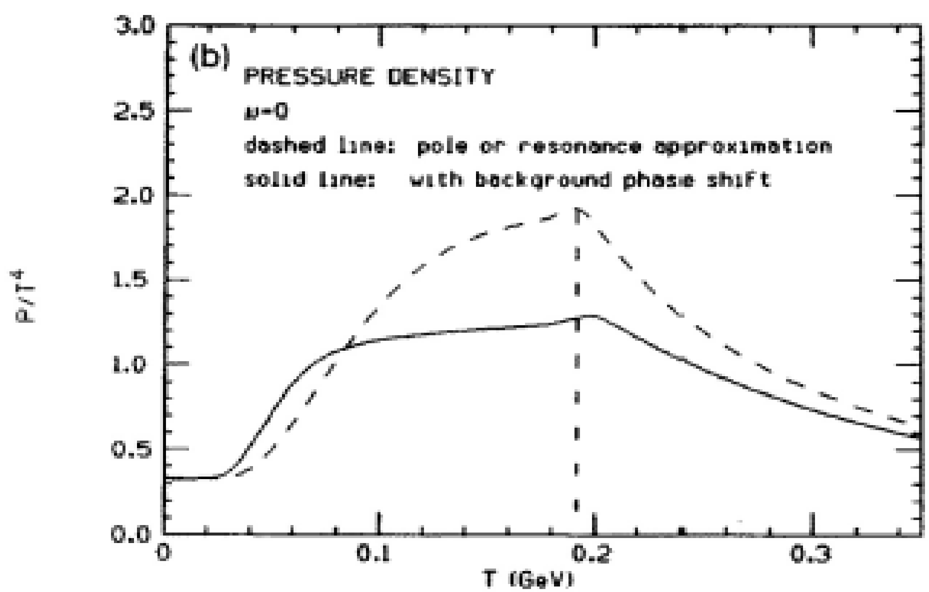
Problem:
No Quark Confinement !



P. Zhuang, J. Huefner, S.P. Klevansky, NPA 576, 525 (1994)



Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\,t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}[i\gamma^\mu\partial_\mu - m - \gamma^0(\mu + \lambda_8\mu_8 + i\lambda_3\phi_3]\psi - \mathcal{L}_{\rm int} + U(\Phi)]\right\}$$

Polyakov loop: $\Phi = N_c^{-1}{\rm Tr}_c[\exp(i\beta\lambda_3\phi_3)]$ Order parameter for deconfinement

- Current-current interaction (4-Fermion coupling) and KMT determinant interaction

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2 - K[\det{}_f(\bar{q}(1+\gamma_5)q) + \det{}_f(\bar{q}(1-\gamma_5)q)]$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D\, {\rm e}^{-\sum_{M,D}\frac{M_M^2}{4G_M} - \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}{\rm Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\},\Phi] + U(\Phi) + V_{\rm KMT}}$$

- Collective quark fields: Mesons (M_M) and Diquarks (Δ_D); Gluon mean field: Φ
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (I)

$SU(N_c)$ pure gauge sector: Polyakov line

$$L(\vec{x}) \equiv \mathcal{P} \exp \left[i \int_0^\beta d\tau \, A_4(\vec{x}, \tau) \right] \; ; \quad A_4 = iA^0 = \lambda_3 \phi_3 + \lambda_8 \phi_8$$

Polyakov loop

$$l(\vec{x}) = \frac{1}{N_c} \mathrm{Tr} L(\vec{x}) \, , \quad \langle l(\vec{x}) \rangle = e^{-\beta \Delta F_Q(\vec{x})} .$$

$\mathbf{Z}_{N_c}$ symmetric phase: $\langle l(\vec{x}) \rangle = 0 \quad \Longrightarrow \Delta F_Q \to \infty$: Confinement !

Polyakov loop field:

$$\Phi(\vec{x}) \equiv \langle\langle l(\vec{x}) \rangle\rangle = \frac{1}{N_c} \mathrm{Tr}_c \langle\langle L(\vec{x}) \rangle\rangle$$

Potential for the PL-meanfield $\Phi(\vec{x})$ =const., which fits quenched QCD lattice thermodynamics

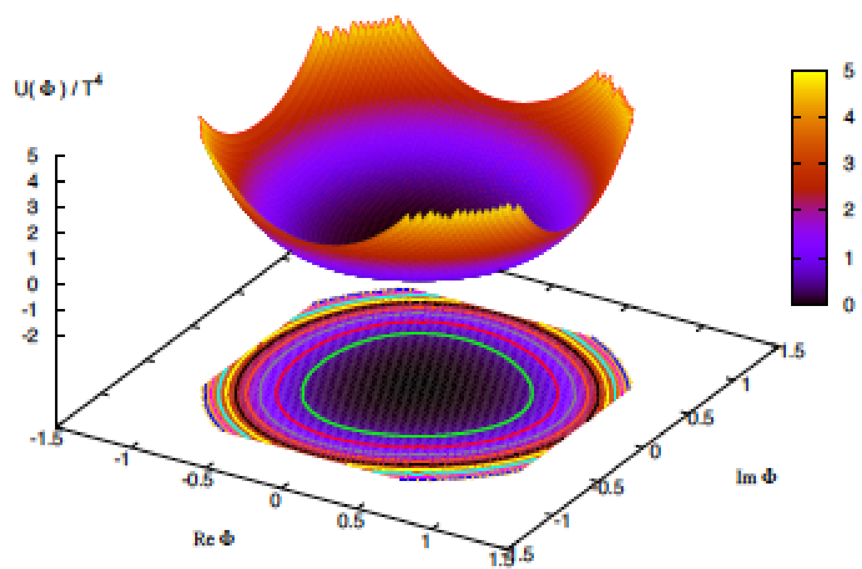
$$\frac{\mathcal{U}(\Phi, \bar{\Phi}; T)}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6} \left(\Phi^3 + \bar{\Phi}^3 \right) + \frac{b_4}{4} \left(\bar{\Phi}\Phi \right)^2 \, ,$$

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3 .$$

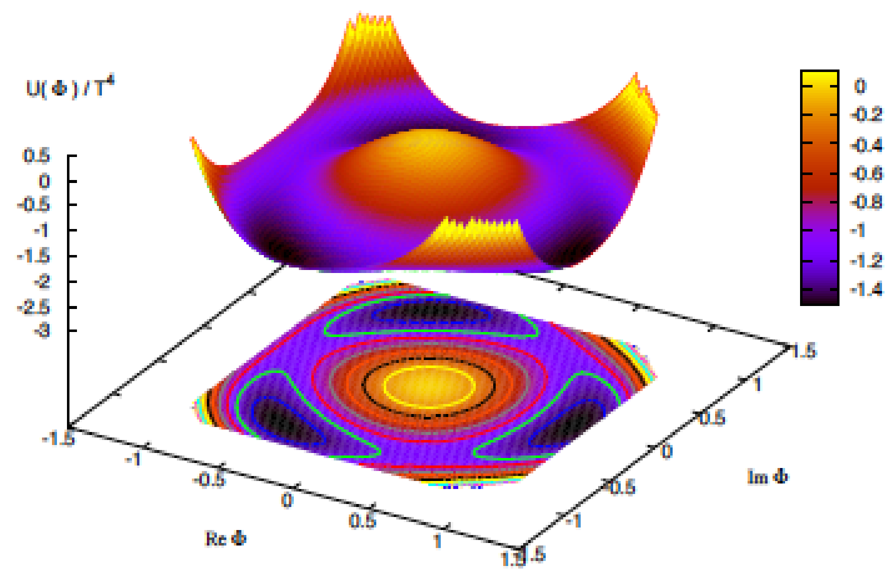
a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (II)

Temperature dependence of the Polyakov-loop potential $U(\Phi, \bar{\Phi}; T)$



$T = 0.26$ GeV$< T_0$
"Color confinement"



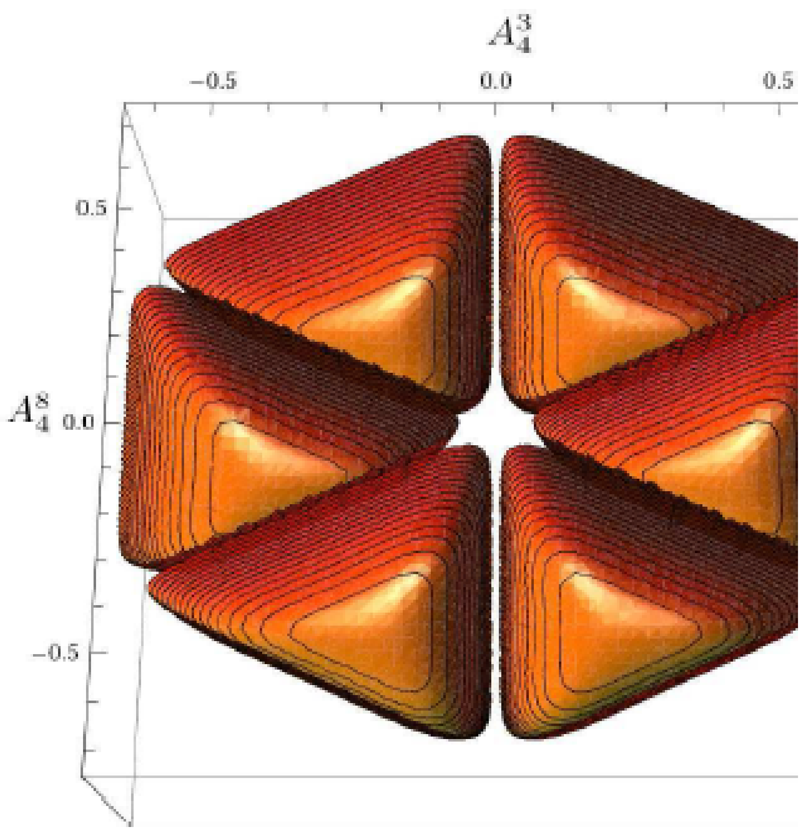
$T = 1.0$ GeV$> T_0$
"Color deconfinement"

Critical temperature for pure gauge $SU_c(3)$ lattice simulations: $T_0 = 270$ MeV.

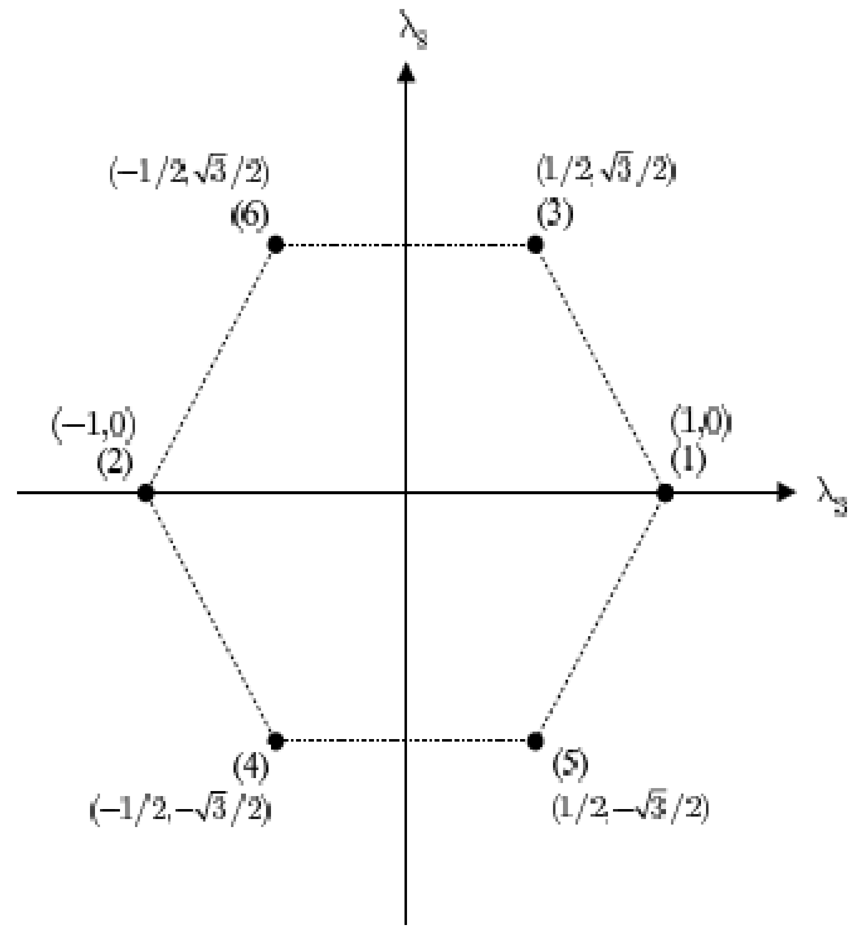
Hansen et al., Phys.Rev. D75, 065004 (2007)

POLYAKOV-LOOP VARIABLE Φ

Degeneracy in $\Phi = Tr_c\{\exp[i\beta A_4]\}/N_c$; $A_4 = \lambda_3\phi_3 + \lambda_8\phi_8$; Internal Z(3) Symmetry



Hell et al., 0810.1099 [hep-ph]



Abuki et al., 0811.1512 [hep-ph]

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (III)

Lagrangian for $N_f = 2$, $N_c = 3$ quark matter, coupled to the gauge sector

$$\mathcal{L}_{PNJL} \; = \; \bar{q}(i\gamma^\mu D_\mu - \hat{m} + \gamma_0 \mu)q + G_1 \left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 \right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right),$$

$D^\mu = \partial^\mu - iA^\mu$; $A^\mu = \delta^\mu_0 A^0$ (Polyakov gauge), with $A^0 = -iA_4$

Diagrammatic Hartree equation: 

$$S_0(p) = \text{———} = -(\not{p} - m_0 + \gamma^0(\mu - iA_4))^{-1}\, ; \quad S(p) = \textbf{———} = -(\not{p} - m + \gamma^0(\mu - iA_4))^{-1}$$

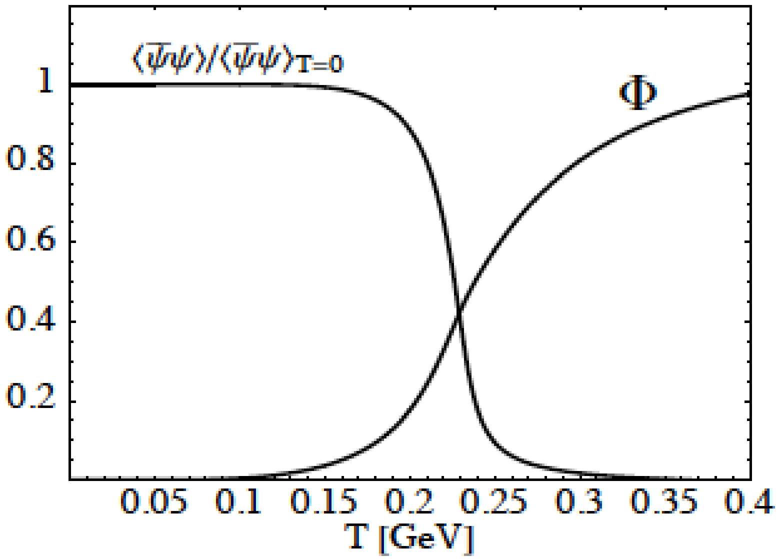
Dynamical chiral symmetry breaking $\sigma = m - m_0 \neq 0$? Solve Gap Equation! ($E = \sqrt{p^2 + m^2}$)

$$\begin{aligned} m - m_0 \; &= \; 2G_1 T \,\mathrm{Tr} \sum_{n=-\infty}^{+\infty} \int_\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{-1}{\not{p} - m + \gamma^0(\mu - iA_4)} \\ &= \; 2G_1 N_f N_c \int_\Lambda \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{2m}{E} [1 - f^+_\Phi(E) - f^-_\Phi(E)] \end{aligned}$$

Modified quark distribution functions ($\Phi = \bar{\Phi} = 0$: "poor man's nucleon": $E_N = 3E$, $\mu_N = 3\mu$)

$$f^\pm_\Phi(E) = \frac{\left(\Phi + 2\bar{\Phi} e^{-\beta\left(E_p \mp \mu\right)}\right) e^{-\beta\left(E_p \mp \mu\right)} + e^{-3\beta\left(E_p \mp \mu\right)}}{1 + 3\left(\Phi + \bar{\Phi} e^{-\beta\left(E_p \mp \mu\right)}\right) e^{-\beta\left(E_p \mp \mu\right)} + e^{-3\beta\left(E_p \mp \mu\right)}} \longrightarrow f^\pm_0(E) = \frac{1}{1 + e^{\beta(E_N \mp \mu_N)}}$$

Polyakov-loop Nambu–Jona-Lasinio Model (IV)

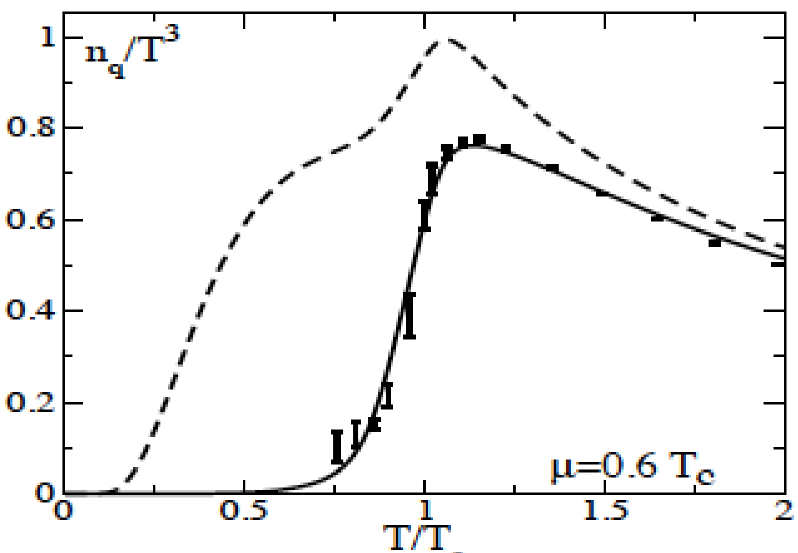


Grand canonical thermodynamical potential

$$\begin{aligned}\Omega(T,\mu;\Phi,m) &= \frac{\sigma^2}{2G} - 6N_f \int \frac{d^3p}{(2\pi)^3} E\,\theta\left(\Lambda^2 - \vec{p}^{\,2}\right) \\ &- 2N_f T \int \frac{d^3p}{(2\pi)^3} \left\{ \mathrm{Tr}_c \ln\left[1 + L\,e^{-(E-\mu)/T}\right]\right. \\ &+ \left.\mathrm{Tr}_c \ln\left[1 + L^\dagger e^{-(E+\mu)/T}\right]\right\} + \mathcal{U}\left(\Phi, \bar{\Phi}, T\right)\end{aligned}$$

Appearance of quarks below T_c largely suppressed:

$$\begin{aligned}&\ln\det\left[1 + L\,e^{-(E-\mu)/T}\right] + \ln\det\left[1 + L^\dagger e^{-(E+\mu)/T}\right] \\ &= \ln\left[1 + 3\left(\Phi + \bar{\Phi}e^{-(E-\mu)/T}\right)e^{-(E-\mu)/T} + e^{-3(E-\mu)/T}\right] \\ &+ \ln\left[1 + 3\left(\bar{\Phi} + \Phi e^{-(E+\mu)/T}\right)e^{-(E+\mu)/T} + e^{-3(E+\mu)/T}\right].\end{aligned}$$

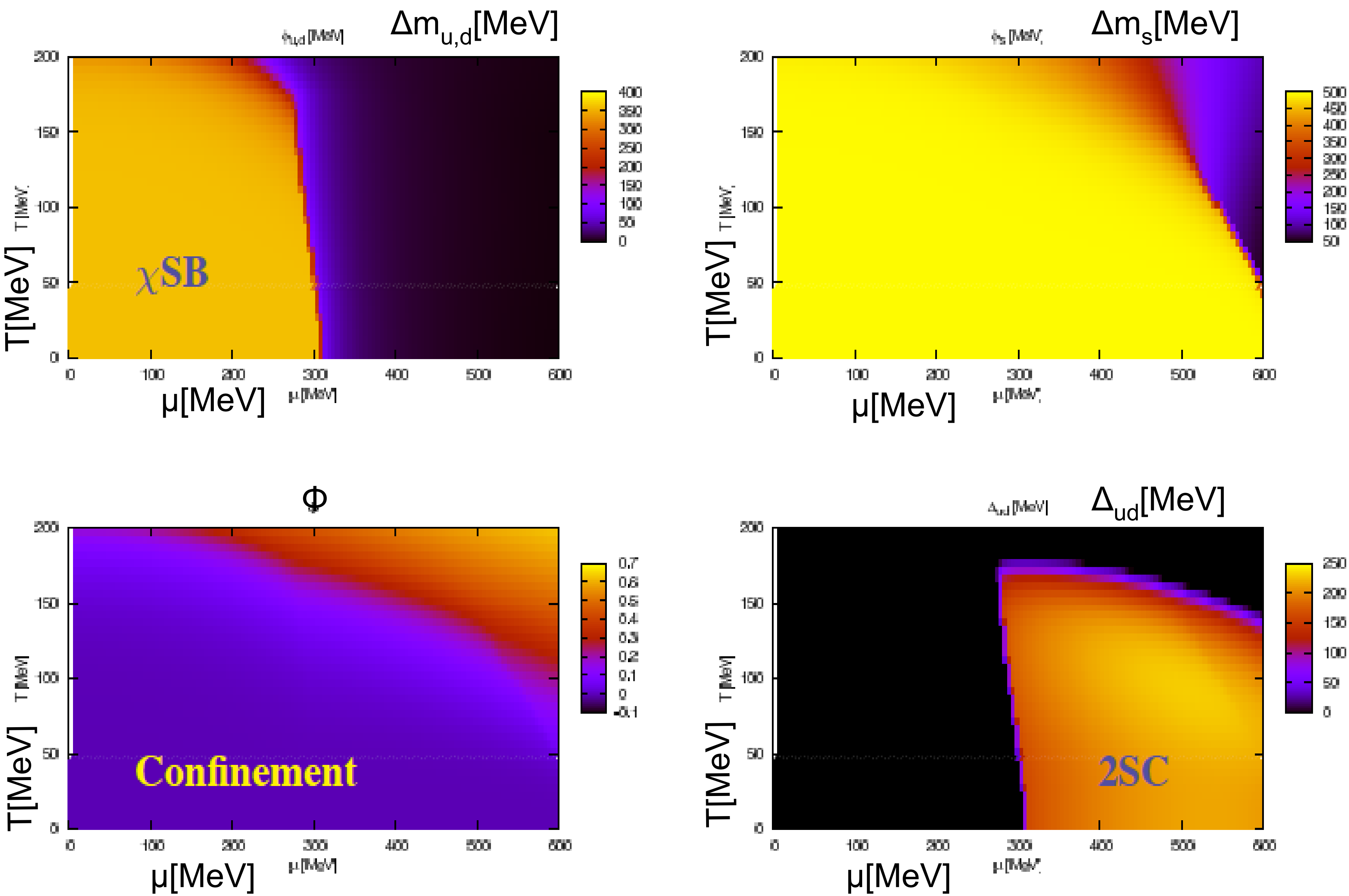


Accordance with QCD lattice susceptibilities! Example:

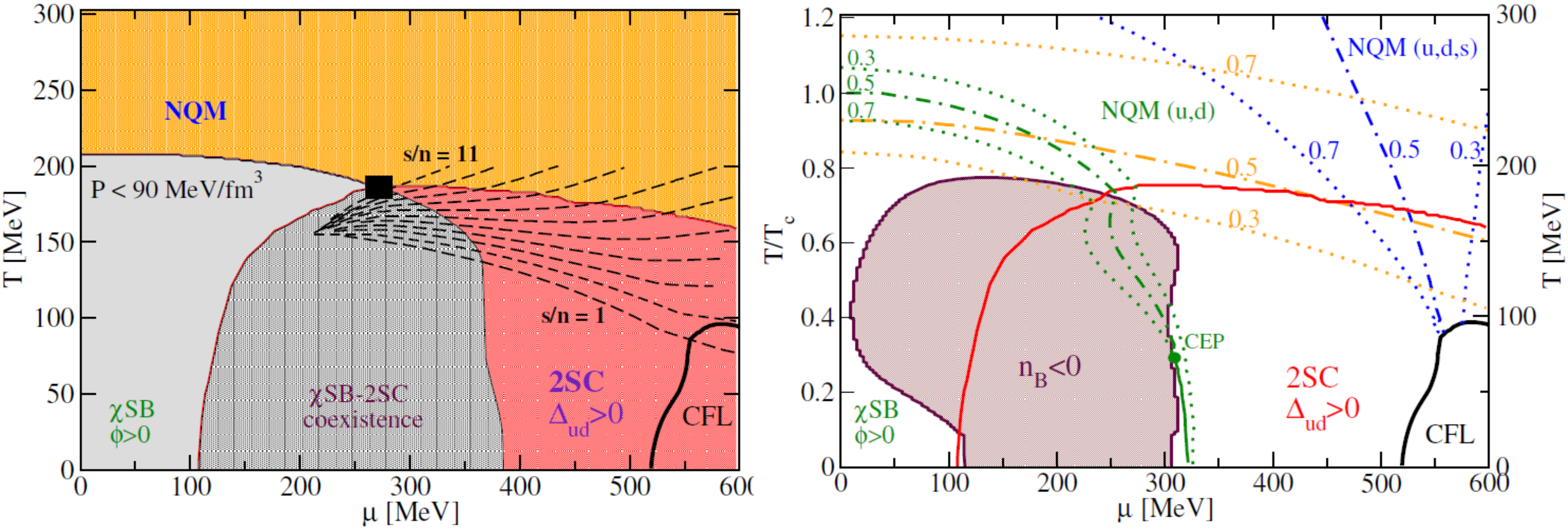
$$\frac{n_q(T,\mu)}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega(T,\mu)}{\partial\mu},$$

Ratti, Thaler, Weise, PRD 73 (2006) 014019.

PHASES OF QCD @ EXTREMES: NO COLOR NEUTRALITY



Phase diagram for the color superconducting PNJL model (with color neutrality)

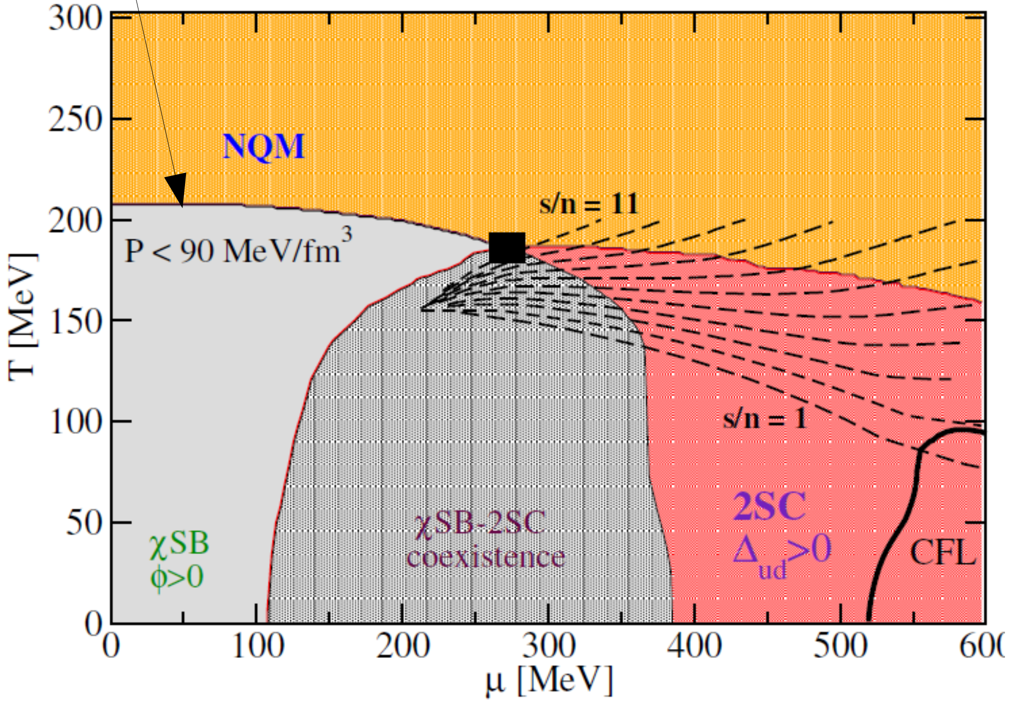


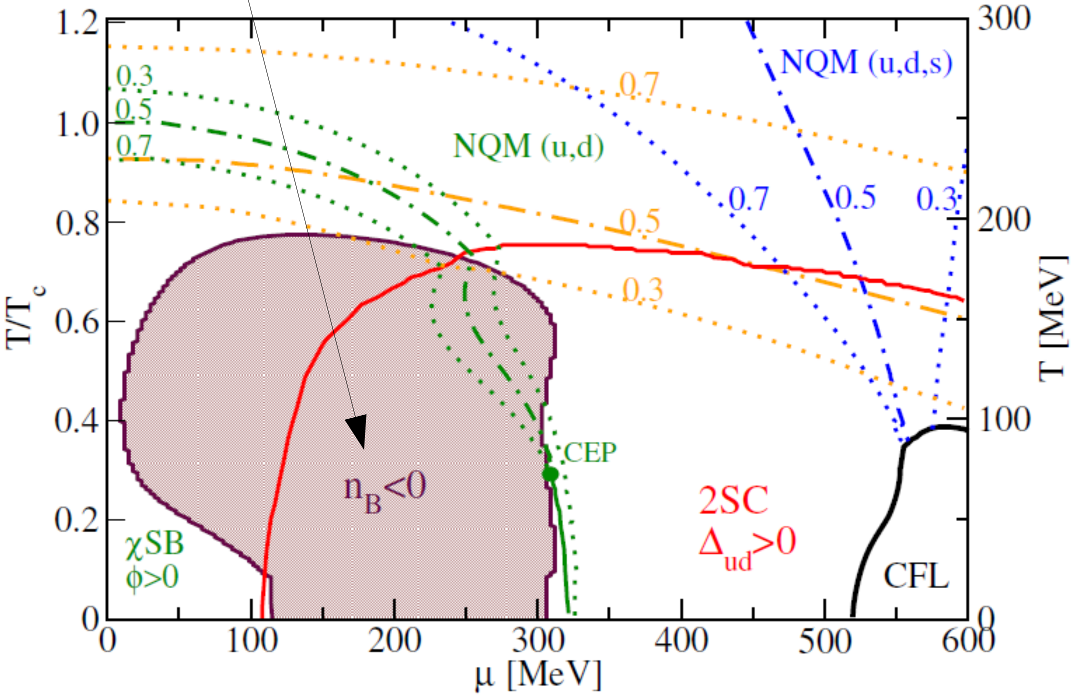
A. Ayriyan, J. Berdermann, D. Blaschke, R. Lastowiecki, arxiv:1608.07875 [hep-ph]

Phase diagram for the color superconducting PNJL model (with color neutrality)

Universal pressure at quark-hadron border?
Petran & Rafelski, Phys. Rev. C 88 (2013)

Instability, related to color neutrality (confinement)





A. Ayriyan, J. Berdermann, D. Blaschke, R. Lastowiecki, arxiv:1608.07875 [hep-ph]

Polyakov-loop Nambu–Jona-Lasinio Model (V)

Mesonic currents

$$J_P^a(x) = \bar{q}(x) i\gamma_5 \tau^a q(x) \quad \text{(pion)} \; ; \quad J_S(x) = \bar{q}(x)q(x) - \langle \bar{q}(x)q(x)\rangle \quad \text{(sigma)}$$

... and correlation functions

$$C_{ab}^{PP}(q^2) \equiv i \int d^4x e^{iq.x} \langle 0| T\left(J_P^a(x) J_P^{b\dagger}(0)\right)|0\rangle = C^{PP}(q^2)\delta_{ab}$$

$$C_{ab}^{SS}(q^2) \equiv i \int d^4x e^{iq.x} \langle 0| T\left(J_S(x) J_S^{\dagger}(0)\right)|0\rangle$$

Schwinger-Dyson Equations, $T = \mu = 0$

$$C^{MM}(q^2) = \Pi^{MM}(q^2) + \sum_{M'} \Pi^{MM'}(q^2)(2G_1)C^{M'M}(q^2)$$

One-loop polarization functions

$$\Pi^{MM'}(q^2) \equiv \int_\Lambda \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(\Gamma_M S(p+q)\Gamma_{M'} S(q)\right)$$

Hartree quark propagator $S(p)$

Polyakov-loop Nambu–Jona-Lasinio Model (VI)

Example of the pion channel:

$$\Pi^{PP}(q^2) = -4iN_cN_f\int_\Lambda \frac{d^4p}{(2\pi)^4}\frac{m^2-p^2+q^2/4}{[(p+q/2)^2-m^2][(p-q/2)^2-m^2]} = 4iN_cN_fI_1 - 2iN_cN_fq^2I_2(q^2)$$

Loop Integrals:

$$I_1 = \int_\Lambda \frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2} \quad ; \quad I_2(q^2) = \int_\Lambda \frac{d^4p}{(2\pi)^4}\frac{1}{[(p+q)^2-m^2]\,[p^2-m^2]}$$

With pseudoscalar decay constant (f_P) and gap equation for I_1

$$f_P^2(q^2) = -4iN_cm^2I_2(q^2) \;\; ; \;\; I_1 = \frac{m-m_0}{8iG_1mN_cN_f},$$

One obtains $\Pi^{PP}(q^2) = \frac{m-m_0}{2G_1m} + f_P^2(q^2)\frac{q^2}{m^2}$; $\Pi^{SS}(q^2) = \frac{m-m_0}{2G_1m} + f_P^2(q^2)\frac{q^2-4m^2}{m^2}$. In the chiral limit ($m_0 = 0$), the correlation functions

$$C^{MM}(q^2) = \Pi^{MM}(q^2) + \Pi^{MM}(q^2)(2G_1)C^{MM}(q^2) = \frac{\Pi^{MM}(q^2)}{1-2G_1\Pi^{MM}(q^2)} \;\; , \;\; M = P, S\,,$$

have poles at $q^2 = M_P^2 = 0$ (Pion) and $q^2 = M_S^2 = (2m)^2$ (Sigma meson) $\Longrightarrow$ Check !

Polyakov-loop Nambu–Jona-Lasinio Model (VII)

Finite T, μ: $p = (p_0, \vec{p}) \to (i\omega_n + \mu - iA_4, \vec{p})$; $i\int_\Lambda \frac{d^4p}{(2\pi)^4} \to -T\frac{1}{N_c}\mathrm{Tr}_c \sum_n \int_\Lambda \frac{d^3p}{(2\pi)^3}$

$$I_1 = -i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1 - f(E_p - \mu) - f(E_p + \mu)}{2E_p}$$

$$I_2(\omega, \vec{q}) = i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p 2E_{p+q}}\frac{f(E_p + \mu) + f(E_p - \mu) - f(E_{p+q} + \mu) - f(E_{p+q} - \mu)}{\omega - E_{p+q} + E_p}$$
$$+i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1 - f(E_p - \mu) - f(E_{p+q} + \mu)}{2E_p 2E_{p+q}}\left(\frac{1}{\omega + E_{p+q} + E_p} - \frac{1}{\omega - E_{p+q} - E_p}\right)$$

For a meson at rest in the medium ($\vec{q} = 0$)

$$I_2\left(\omega, \vec{0}\right) = -i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1 - f(E_p + \mu) - f(E_p - \mu)}{E_p\left(\omega^2 - 4E_p^2\right)}$$

which develops an imaginary part

$$\Im m\ (-iI_2(\omega, 0)) = \frac{1}{16\pi}\left(1 - f\left(\frac{\omega}{2} - \mu\right) - f\left(\frac{\omega}{2} + \mu\right)\right)\sqrt{\frac{\omega^2 - 4m^2}{\omega^2}} \times \Theta(\omega^2 - 4m^2)\Theta(4(\Lambda^2 + m^2) - \omega^2)$$

with the Pauli-blocking factor: $N(\omega, \mu) = \left(1 - f\left(\frac{\omega}{2} - \mu\right) - f\left(\frac{\omega}{2} + \mu\right)\right)$

Polyakov-loop Nambu–Jona-Lasinio Model (VIII)

Spectral function

$$F^{MM}(\omega,\vec{q}) \equiv \Im m\, C^{MM}(\omega+i\eta,\vec{q}) = \Im m \frac{\Pi^{MM}(\omega+i\eta,\vec{q})}{1-2G_1\Pi^{MM}(\omega+i\eta,\vec{q})}.$$

$$F^{MM}(\omega) = \frac{\pi}{2G_1}\frac{1}{\pi}\frac{2G_1\Im m\,\Pi^{MM}(\omega+i\eta)}{\left(1-2G_1\Re e\,\Pi^{MM}(\omega)\right)^2+\left(2G_1\Im m\,\Pi^{MM}(\omega+i\eta)\right)^2}.$$

For $\omega < 2m(T,\mu)$, $\Im m\,\Pi = 0$: decay channel closed $\to$ bound state!

$$F^{MM}(\omega) = \frac{\pi}{2G_1}\delta\left(1-2G_1\Re e\,\Pi^{MM}(\omega)\right) = \frac{\pi}{4G_1^2\left|\frac{\partial \Re e\,\Pi^{MM}}{\partial\omega}\right|_{\omega=m_M}}\delta(\omega-m_M)\,.$$

The meson mass m_M is the solution of

$$1-2G_1\Re e\,\Pi^{MM}(m_M)=0$$

The decay width (inverse lifetime) is

$$\Gamma_M = 2G_1\Im m\,\Pi^{MM}(m_M)$$

NONLOCAL POLYAKOV-LOOP CHIRAL QUARK MODEL

$$\Omega(T) = = \mathcal{U}(\Phi,\bar{\Phi}) - T\mathrm{Tr}_{\vec{p},n,\alpha,f,D}\left[\ln\{S_f^{-1}(p_n^\alpha,T)\} - \frac{1}{2}\Sigma_f(p_n^\alpha,T)\cdot S_f(p_n^\alpha,T)\right] ,$$

where the full quark propagator for the flavor $f = u, d, s$,

$$\begin{aligned} S_f^{-1}(p_n^\alpha,T) &= S_{f,0}^{-1}(p_n^\alpha,T) - \Sigma_f^{-1}(p_n^\alpha,T) \\ &= i\vec{\gamma}\cdot\vec{p}\,A_f((p_n^\alpha)^2,T) + i\gamma_4\omega_n\,C_f((p_n^\alpha)^2,T) + B_f((p_n^\alpha)^2,T) , \end{aligned}$$

is defined by the DSE for the quark selfenergy Σ, see below. The Polyakov-loop potential is:

$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln\left[1 - 6\Phi^*\Phi + 4(\Phi^{*3}+\Phi^3) - 3(\Phi^*\Phi)^2\right] .$$

The Matsubara 4-momenta are defined as $(p_n^\alpha)^2 = (\omega_n^\alpha)^2 + \vec{p}^2, \quad \omega_n^\alpha = \omega_n + \alpha\phi_3, \quad \alpha = -1, 0, +1$, and are coupled to the Polyakov-loop variable $\Phi = \bar{\Phi} = \frac{1}{N_c}\left(1 + e^{i\frac{\phi_3}{T}} + e^{-i\frac{\phi_3}{T}}\right) = \frac{1}{N_c}\left(1 + 2\cos\left(\frac{\phi_3}{T}\right)\right)$. via the parameter ϕ_3. Employing for the effective gluon propagator in a Feynman-like gauge, $g^2 D_{\mu\nu}^{\mathrm{eff}}(p-q) = \delta_{\mu\nu} D(p^2, q^2, p\cdot q)$, a rank-2 separable ansatz

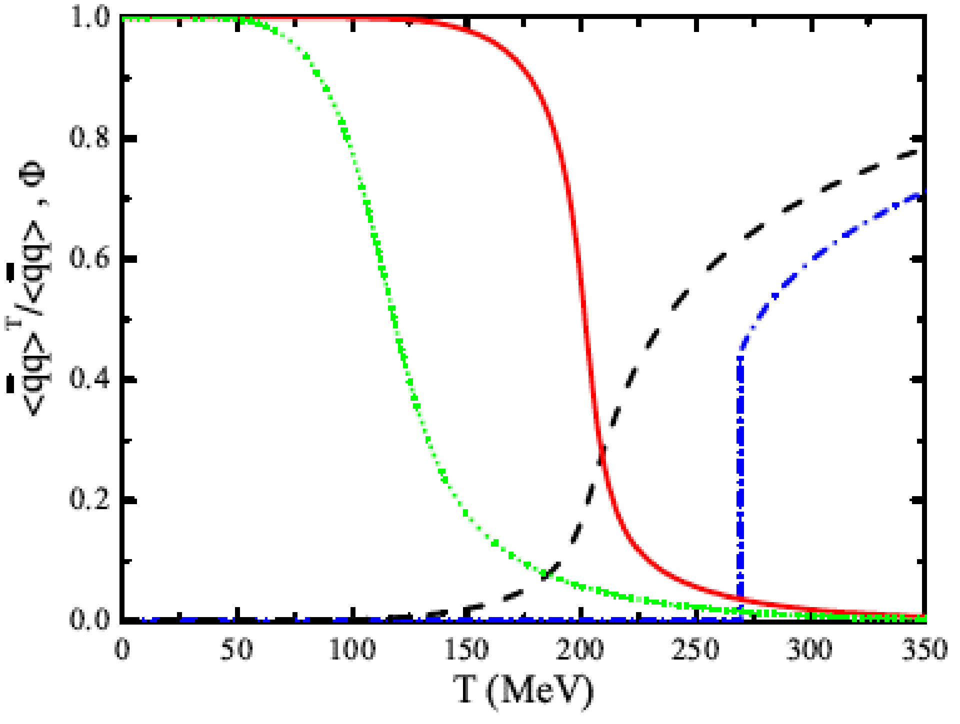
$$D(p^2,q^2,p\cdot q) = D_0\mathcal{F}_0(p^2)\mathcal{F}_0(q^2) + D_1\mathcal{F}_1(p^2)(p\cdot q)\mathcal{F}_1(q^2) ,$$

the propagator amplitudes are given by

$$\begin{aligned} B_f(p_n^2,T) &= \tilde{m}_f + b_f(T)\mathcal{F}_0(p_n^2) , \\ A_f(p_n^2,T) &= 1 + a_f(T)\mathcal{F}_1(p_n^2) , \\ C_f(p_n^2,T) &= 1 + c_f(T)\mathcal{F}_1(p_n^2) , \end{aligned}$$

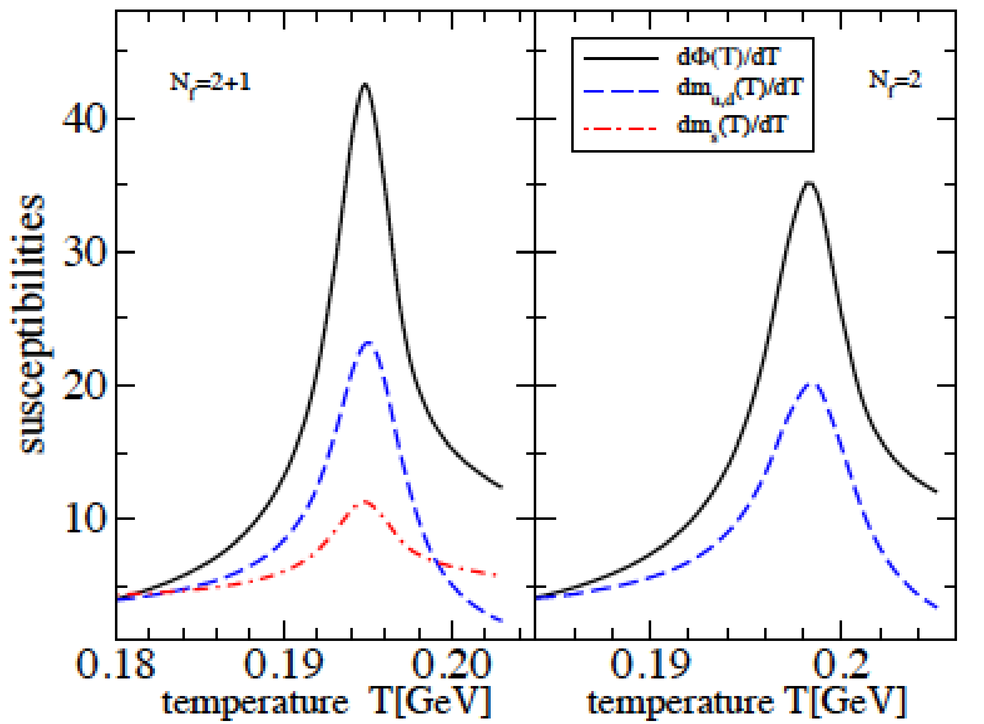
NONLOCAL POLYAKOV LOOP CHIRAL QUARK MODEL

2-flavor, rank-1, 4D separable
order parameters:



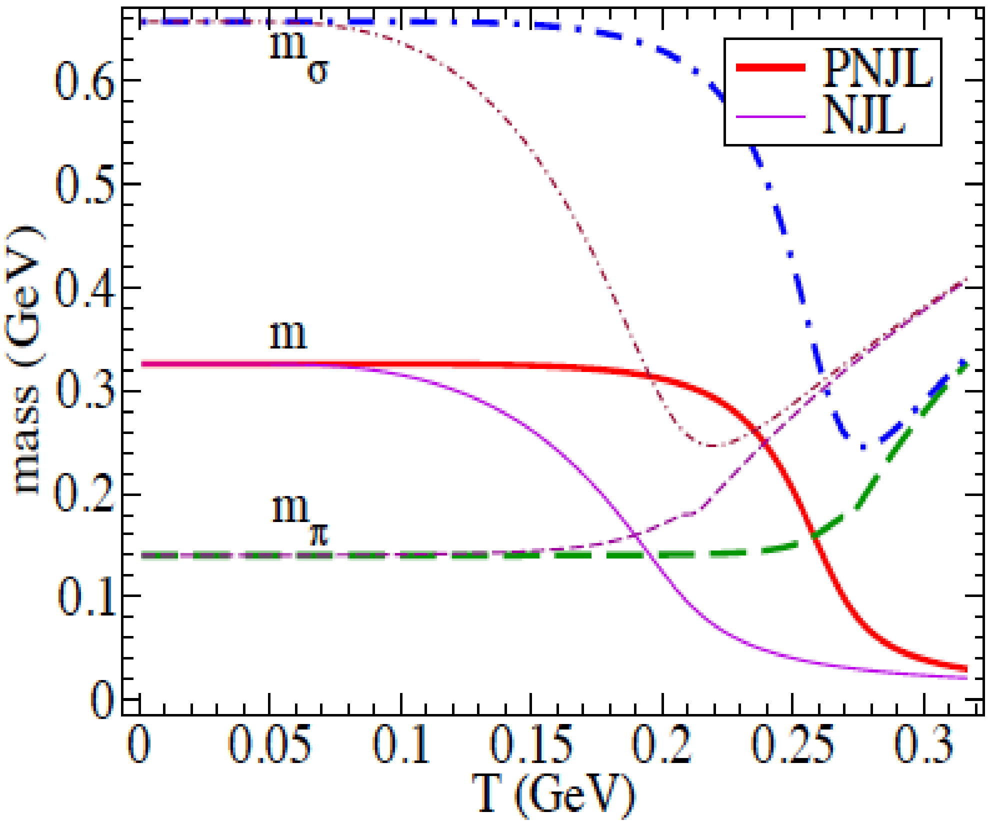
D.B., Buballa, Radzhabov, Volkov,
Yad. Fiz. 71 (2008); arXiv:0705.0384

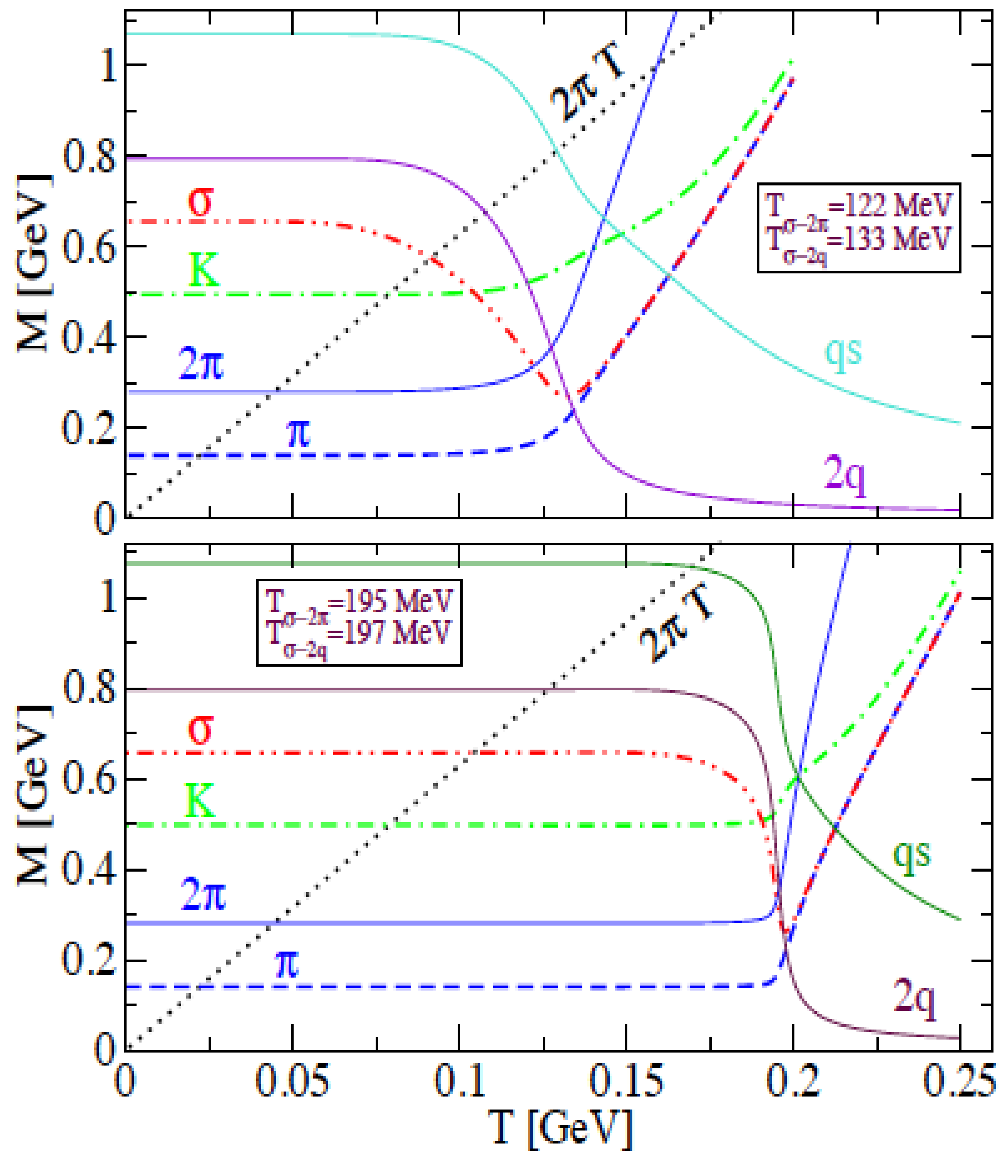
3-flavor, rank-2, 4D separable
susceptibilities:



Horvatic, D.B., Klabucar, Kaczmarek, PRD 84 (2011)

PNJL vs. NJL model: mass spectrum

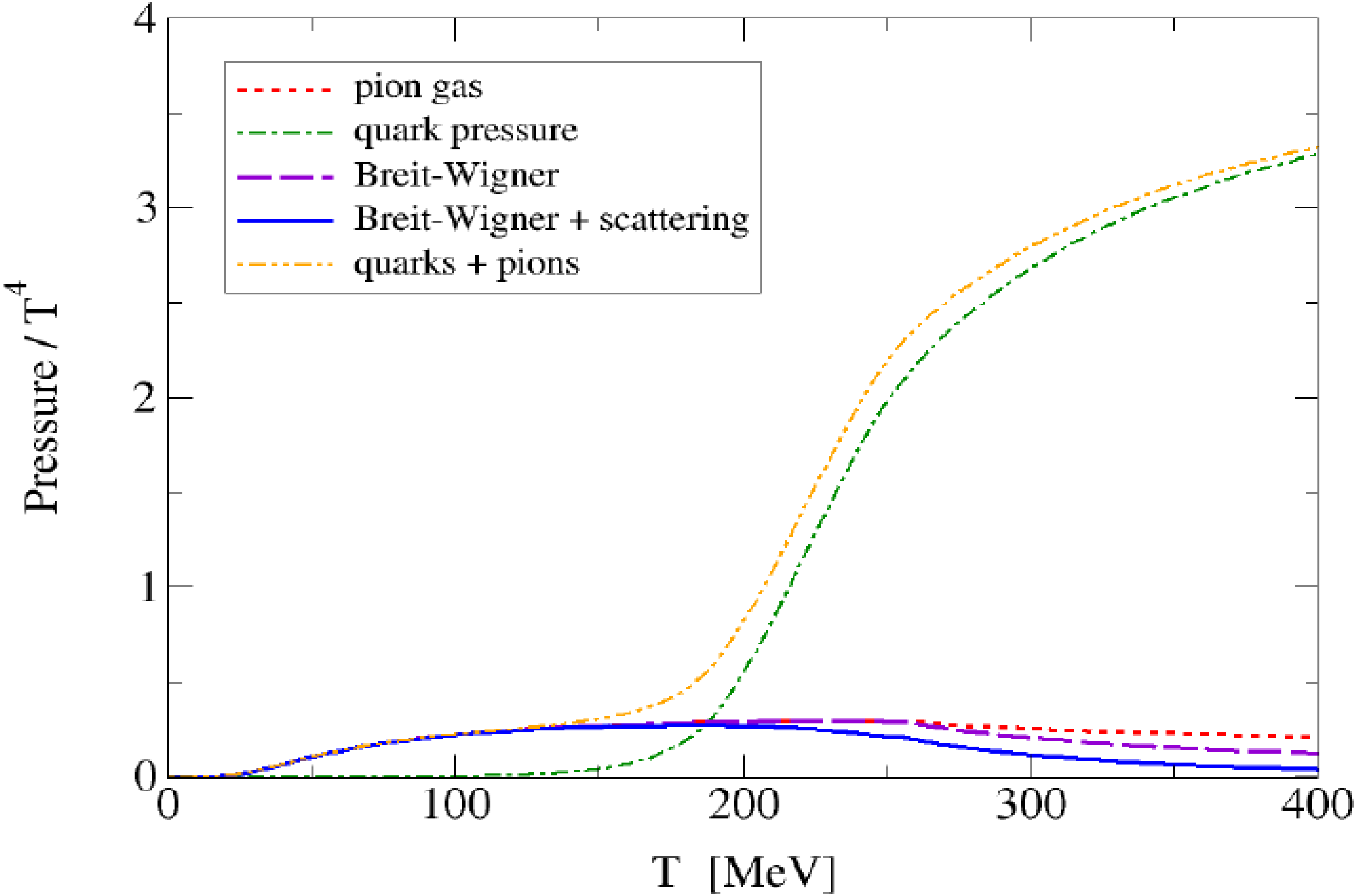




H. Hansen et al., PRD 75, 065004 (2007) ↑

D. Horvatic et al., PRD 84, 016005 (2011) →

Quark + pion pressure



A fantastic result !!

D.B., Agnieszka Wergieluk, Ludwik Turko

A fantastic result !!

Agnieszka Wergieluk, Aleksandr Dubinin, Pok Man Lo, .., Larry McLerran, ...

Mesons & Diquarks in PNJL Quark Matter

$$\mathcal{L} = \bar{q}[\mathrm{i}\partial\!\!\!/ - m_0 + \gamma_0(\mu - \mathrm{i}A_4)]q \; + \mathcal{L}_{\mathrm{int}} - \mathcal{U}(\Phi, \bar{\Phi}; T), \quad \mathcal{L}_{\mathrm{int}} = G_{\mathrm{S}}[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma_5\tau q)^2] + G_{\mathrm{D}} \sum_{A=2,5,7} (\bar{q}\mathrm{i}\gamma_5\tau_2\lambda_A q^c)(\bar{q}^c\mathrm{i}\gamma_5\tau_2\lambda_A q).$$

$$\Omega_{\mathrm{Gauß}} = \mathcal{U}(\Phi, \bar{\Phi}; T) + \frac{\sigma_{\mathrm{MF}}^2}{4G_{\mathrm{S}}} + \Omega_{\mathrm{Q}} + \Omega_{\mathrm{M}} + \Omega_{\mathrm{D}} + \Omega_{\bar{\mathrm{D}}},$$

$$\Omega_{\mathrm{Q}} = -\frac{1}{2}\frac{T}{V}\mathrm{Tr}\ln[\beta S_{\mathrm{Q}}^{-1}], \qquad S_{\mathrm{Q}}^{-1} = \begin{pmatrix} (\mathrm{i}z_n + \hat{\mu})\gamma_0 - \boldsymbol{\gamma}\cdot\mathbf{p} - m & \Delta_{\mathrm{MF}}\mathrm{i}\gamma_5\tau_2\lambda_2 \\ \Delta_{\mathrm{MF}}^*\mathrm{i}\gamma_5\tau_2\lambda_2 & (\mathrm{i}z_n - \hat{\mu})\gamma_0 - \boldsymbol{\gamma}\cdot\mathbf{p} - m \end{pmatrix}$$

$$m = m_0 + \sigma_{\mathrm{MF}}, \quad \hat{\mu} = \mu - \mathrm{i}A_4 = \mathrm{diag}(\mu - \mathrm{i}\phi_3 - \mathrm{i}\phi_8, \mu + \mathrm{i}\phi_3 - \mathrm{i}\phi_8, \mu + 2\mathrm{i}\phi_8) = \mathrm{diag}(\mu_r, \mu_g, \mu_b)$$

$$\Omega_{\mathrm{Q}} = -2N_cN_f\int^{\Lambda}\frac{d^3p}{(2\pi)^3}E_p - 2N_fT\int\frac{d^3p}{(2\pi)^3}\{\mathrm{tr}_{c=r,g,b}\ln[1 + \mathrm{e}^{-(E_p-\mu_c)/T}] + \mathrm{tr}_{c=r,g,b}\ln[1 + \mathrm{e}^{-(E_p+\mu_c)/T}]\},$$

$$= -2N_cN_f\int^{\Lambda}\frac{d^3p}{(2\pi)^3}E_p - 2N_fT\int\frac{d^3p}{(2\pi)^3}\{\ln[(1 + Y\mathrm{e}^{-\mathrm{i}\beta(\phi_3+\phi_8)})(1 + Y\mathrm{e}^{\mathrm{i}\beta(\phi_3-\phi_8)})(1 + Y\mathrm{e}^{2\mathrm{i}\beta\phi_8})]$$
$$+ \ln[(1 + \bar{Y}\mathrm{e}^{\mathrm{i}\beta(\phi_3+\phi_8)})(1 + \bar{Y}\mathrm{e}^{-\mathrm{i}\beta(\phi_3-\phi_8)})(1 + \bar{Y}\mathrm{e}^{-2\mathrm{i}\beta\phi_8})]\},$$

$$= -2N_cN_f\int^{\Lambda}\frac{d^3p}{(2\pi)^3}E_p - 2N_fT\int\frac{d^3p}{(2\pi)^3}\{\ln[1 + 3\bar{\Phi}Y + 3\Phi Y^2 + Y^3] + \ln[1 + 3\Phi\bar{Y} + 3\bar{\Phi}\bar{Y}^2 + \bar{Y}^3]\},$$

$$\Omega_{\mathrm{Q}} = -\frac{2N_cN_f}{3}\int\frac{dp}{2\pi^2}\frac{p^4}{E_p}[f_{\Phi}^+(E_p) + f_{\bar{\Phi}}^-(E_p)]. \qquad f_{\Phi}^+(E_p) = \frac{(\bar{\Phi} + 2\Phi Y)Y + Y^3}{1 + 3(\bar{\Phi} + \Phi Y)Y + Y^3}, \qquad Y = \mathrm{e}^{-(E_p-\mu)/T}$$

Mesons & Diquarks in PNJL Quark Matter

$$\Omega_X = \frac{1}{2}\frac{T}{V}\mathrm{Tr}\ln[\beta^2 S_X^{-1}], \qquad X = \mathrm{M}, \mathrm{D}, \bar{\mathrm{D}}, \qquad S_X^{-1}(iz_n, \mathbf{q}) = \frac{1}{G_X} - \Pi_X(iz_n, \mathbf{q}), \quad S_X(\omega + i\eta, \mathbf{q}) = |S_X(\omega, \mathbf{q})| \exp[i\delta_X(\omega, \mathbf{q})]$$

$$\Omega_M = d_M T \int \frac{d^3 q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \{\ln(1 - e^{-(\omega-\mu_M)/T}) + \ln(1 - e^{-(\omega+\mu_M)/T})\} \frac{d\delta_M(\omega, \mathbf{q})}{d\omega}$$

Three color antitriplet diquark channels D_A, A=2, 5, 7; correspondingly, chemical potentials are:

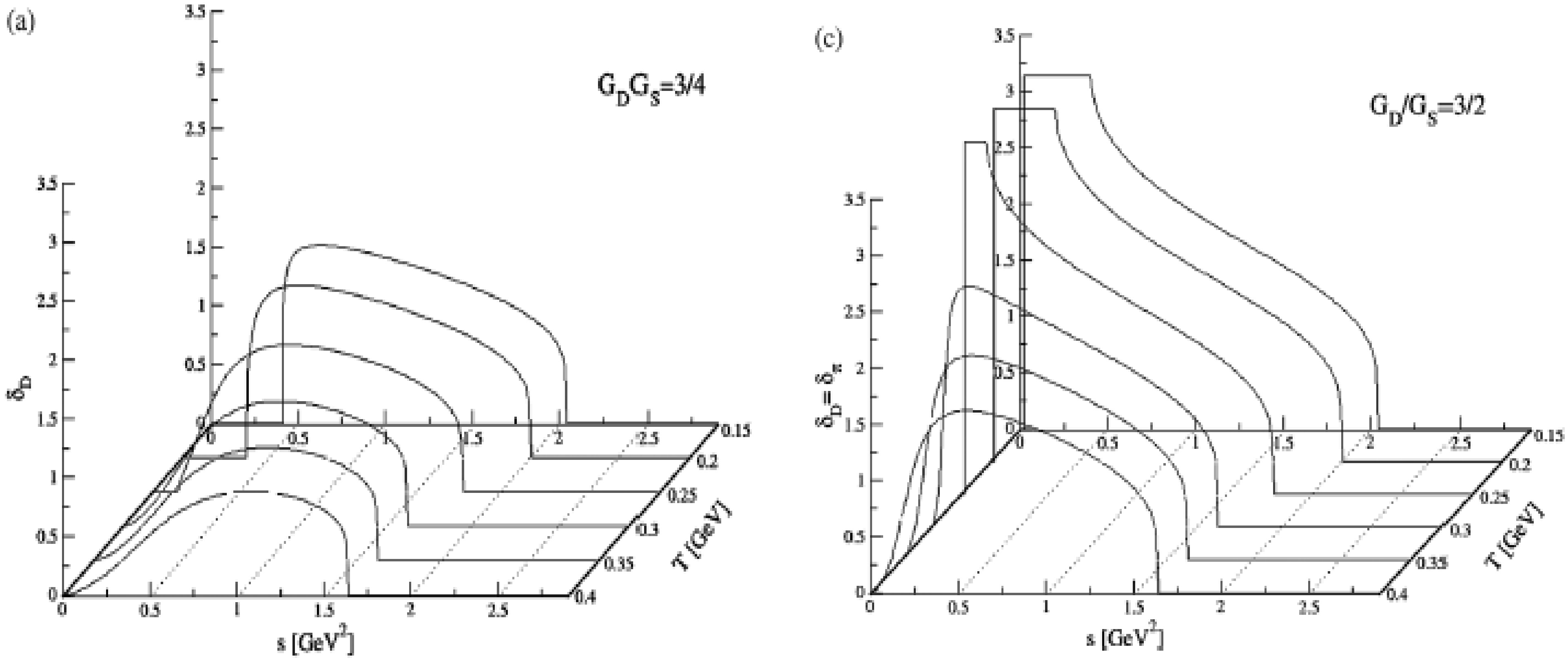
$$\mu_2 = \mu_r + \mu_g = 2\mu - 2i\phi_8 \qquad \mu_5 = \mu_r + \mu_b = 2\mu - i(\phi_3 - \phi_8) \qquad \mu_7 = \mu_r + \mu_g = 2\mu + i(\phi_3 + \phi_8)$$

$$\begin{aligned}
\Omega_D &= \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \{3\omega + T\mathrm{tr}_{A=2,5,7} \ln[1 - e^{-(\omega-\mu_A)/T}] + T\mathrm{tr}_{A=2,5,7} \ln[1 - e^{-(\omega+\mu_A)/T}]\} \frac{d\delta_D(\omega)}{d\omega}, \\
&= \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \{3\omega + T\ln[(1 - Xe^{-2i\beta\phi_8})(1 - Xe^{-i\beta(\phi_3-\phi_8)})(1 - Xe^{i\beta(\phi_3+\phi_8)})] \\
&\quad + T\ln[(1 - \bar{X}e^{2i\beta\phi_8})(1 - \bar{X}e^{i\beta(\phi_3-\phi_8)})(1 - \bar{X}e^{-i\beta(\phi_3+\phi_8)})]\} \frac{d\delta_D(\omega)}{d\omega}, \\
&= \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \{3\omega + T\ln[1 - 3\Phi X + 3\bar{\Phi}X^2 - X^3] + T\ln[1 - 3\bar{\Phi}\bar{X} + 3\Phi\bar{X}^2 - \bar{X}^3]\} \frac{d\delta_D(\omega)}{d\omega},
\end{aligned}$$

$$\Omega_D = -3\int \frac{d^3 p}{(2\pi)^3} \int \frac{d\omega}{2\pi} [g_\Phi^+(\omega) + g_\Phi^-(\omega)]\delta_D(\omega), \quad g_\Phi^\pm(\omega) = \frac{(\Phi - 2\bar{\Phi}X)X + X^3}{1 - 3(\Phi - \bar{\Phi}X)X - X^3},$$

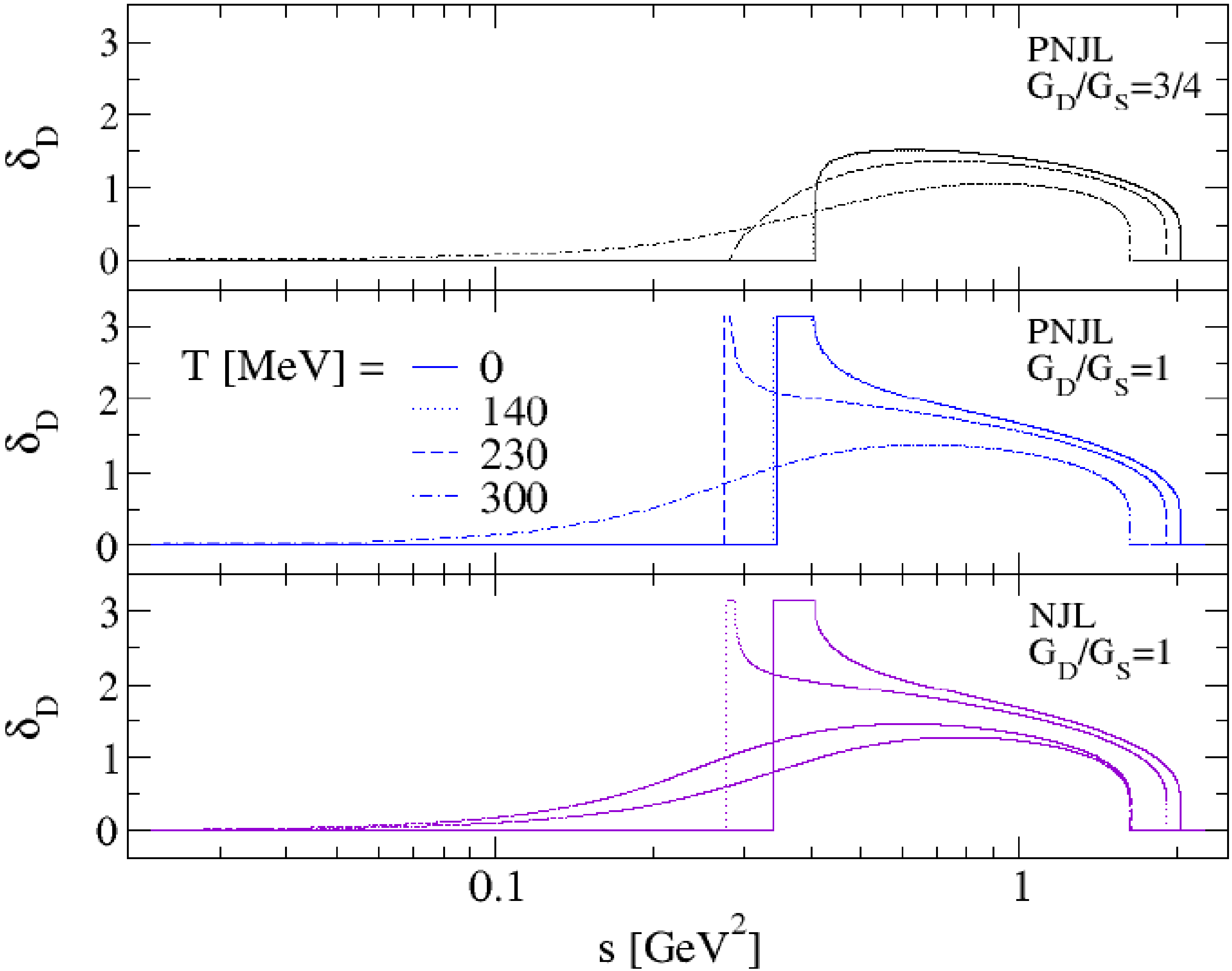
$$g_\Phi^\pm(\omega)|_{\Phi=0} = \frac{1}{\exp[3(\omega \mp 2\mu)/T] - 1}, \qquad g_\Phi^\pm(\omega)|_{\Phi=1} = \frac{1}{\exp[(\omega \mp 2\mu)/T] - 1},$$

Mesons & Diquarks in PNJL Quark Matter

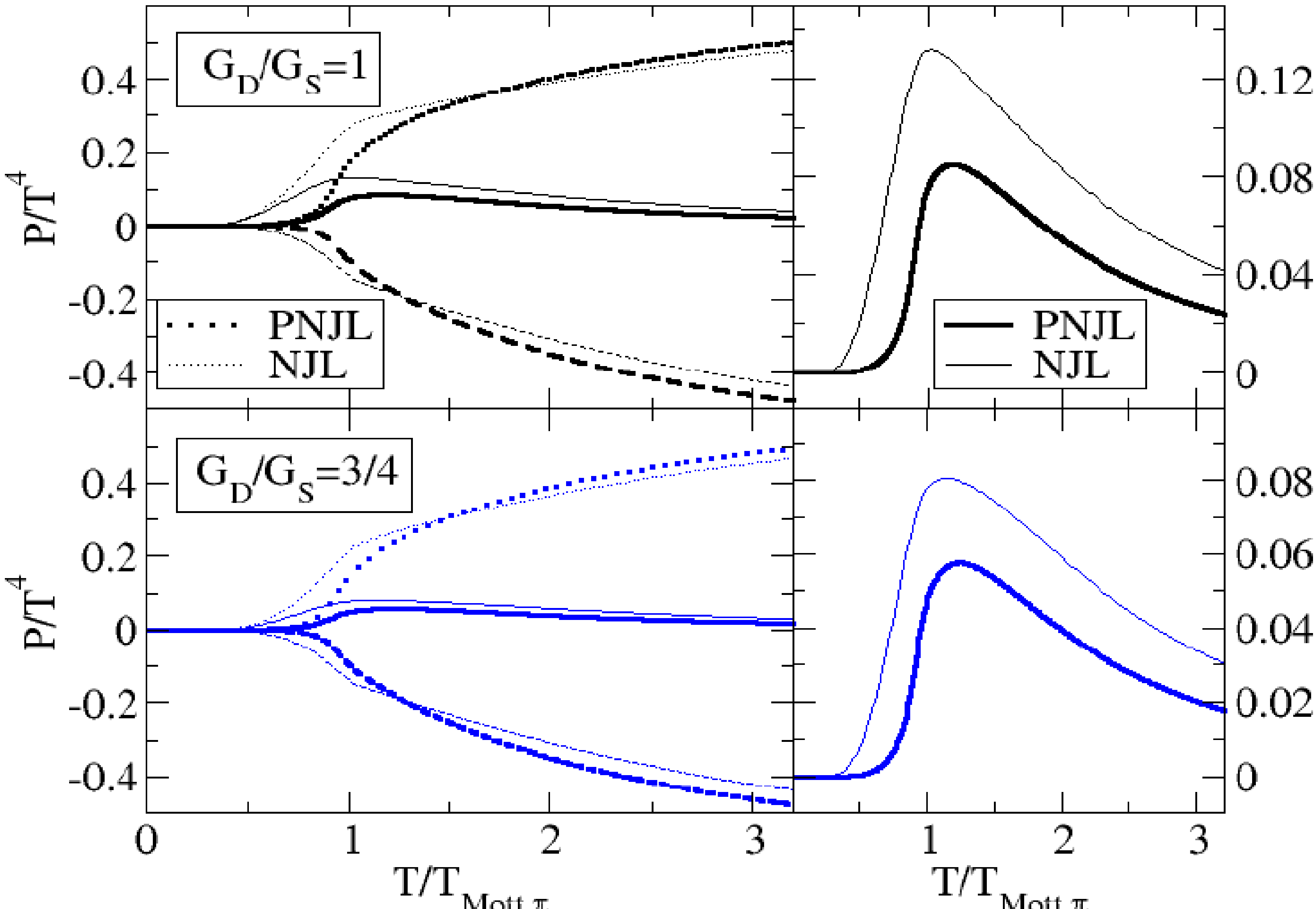


D.B., A. Dubinin, M. Buballa, Phys. Rev. D 91 (2015) 125040

Diquark phase shifts at finite temperature

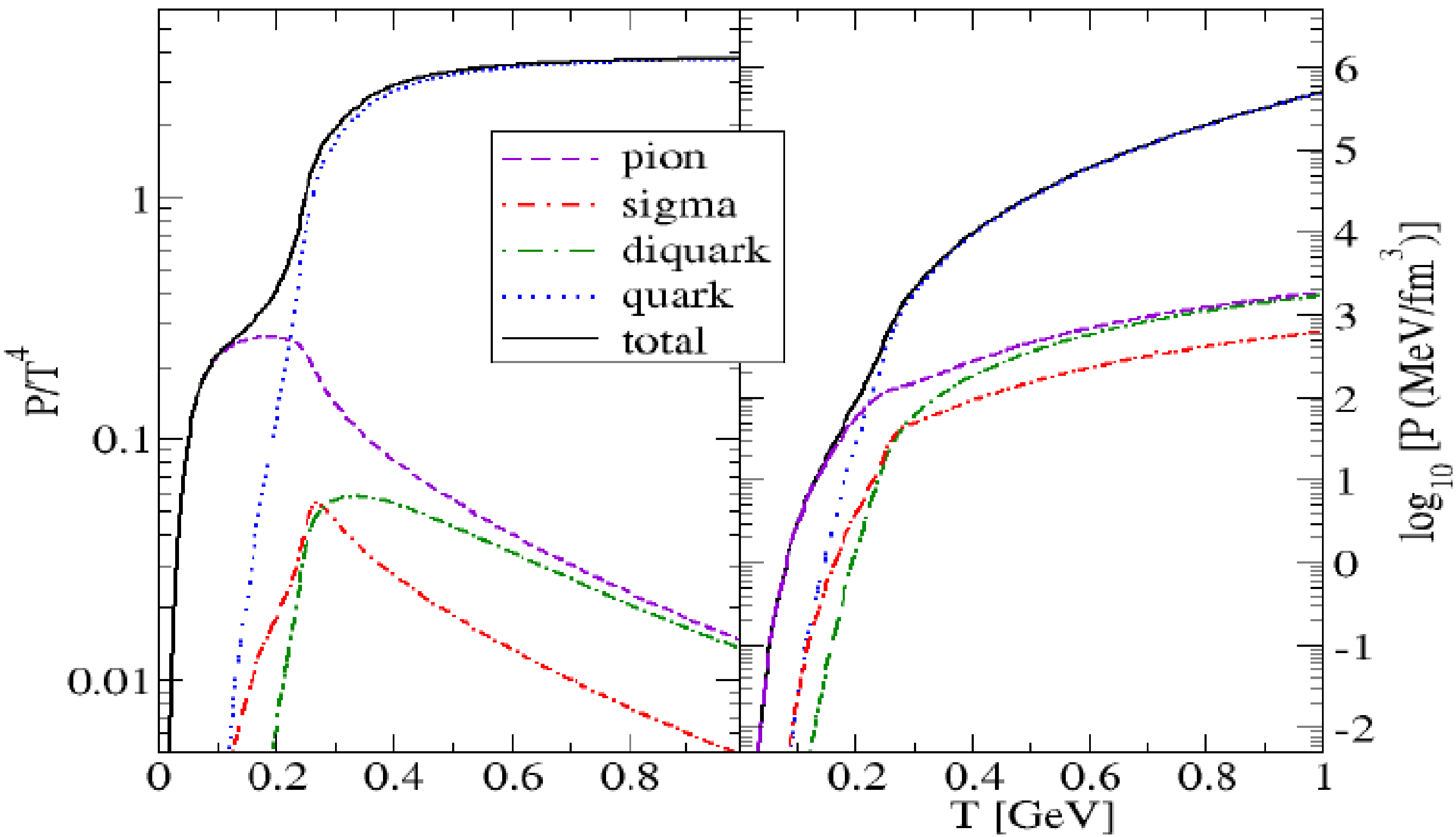


Polyakov-loop suppression of diquark pressure



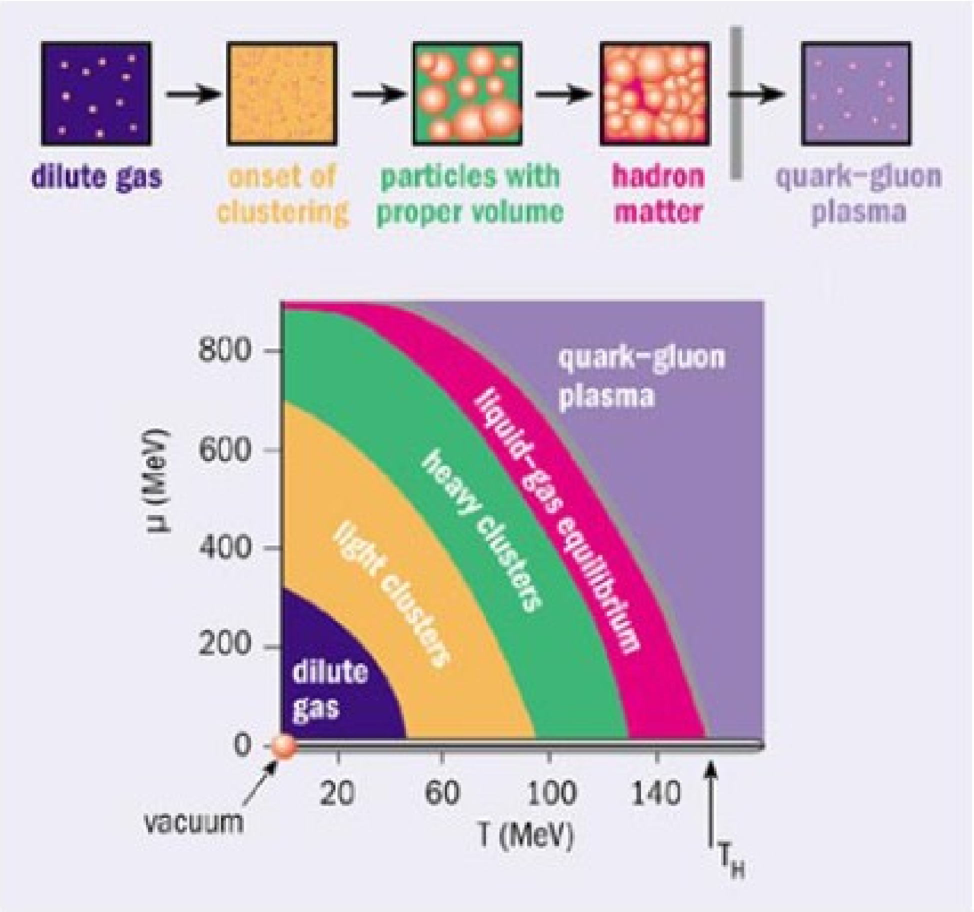
D.B., A. Dubinin, M. Buballa, Phys. Rev. D 91 (2015) 125040

Partial pressures in a quark-meson-diquark system



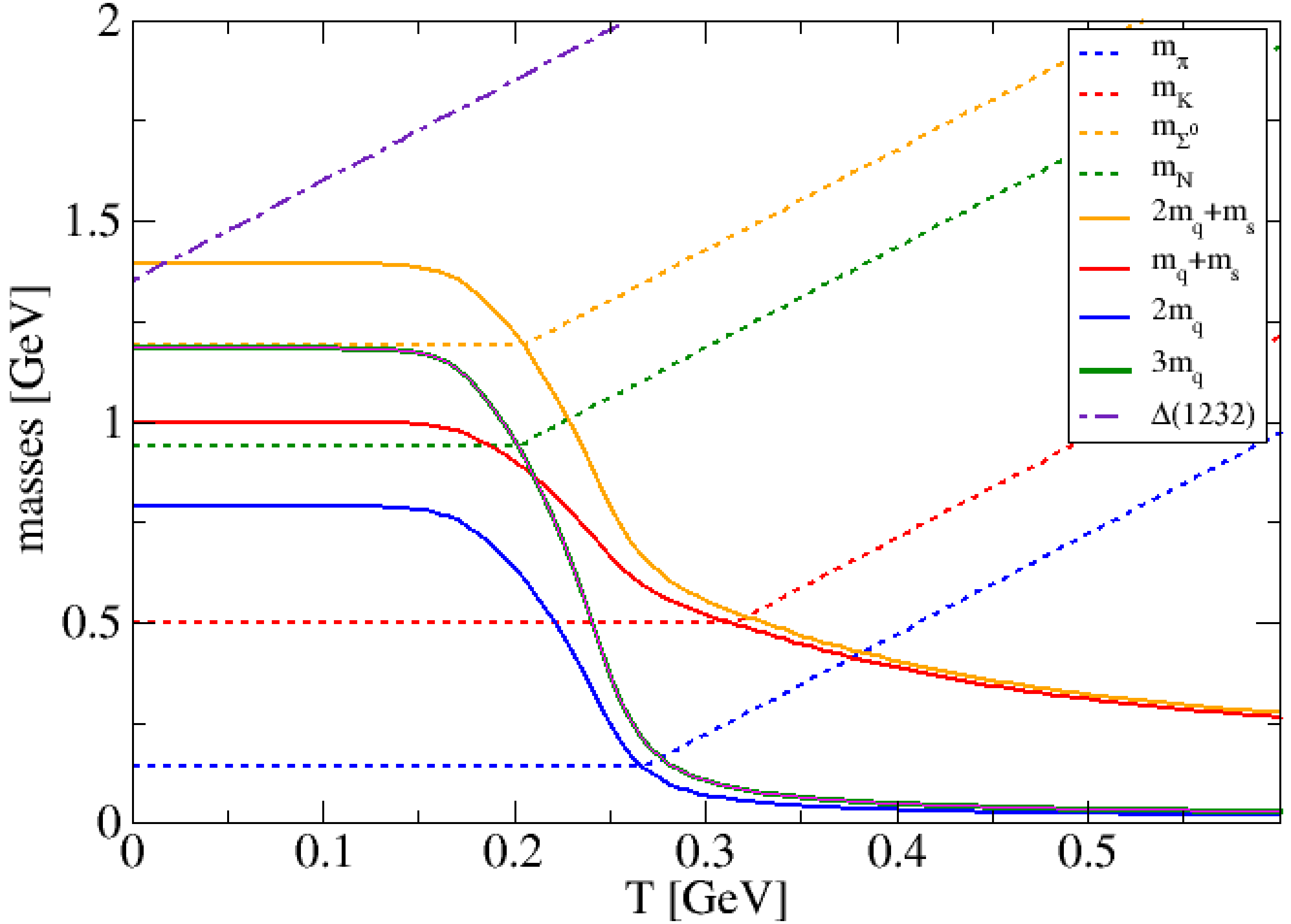
D.B., A. Dubinin, M. Buballa, Phys. Rev. D 91 (2015) 125040

Rolf Hagedorn - Statistical model of particle production



Hadron Resonance Gas with Mott Dissociation

D. Blaschke, A. Dubinin, in preparation

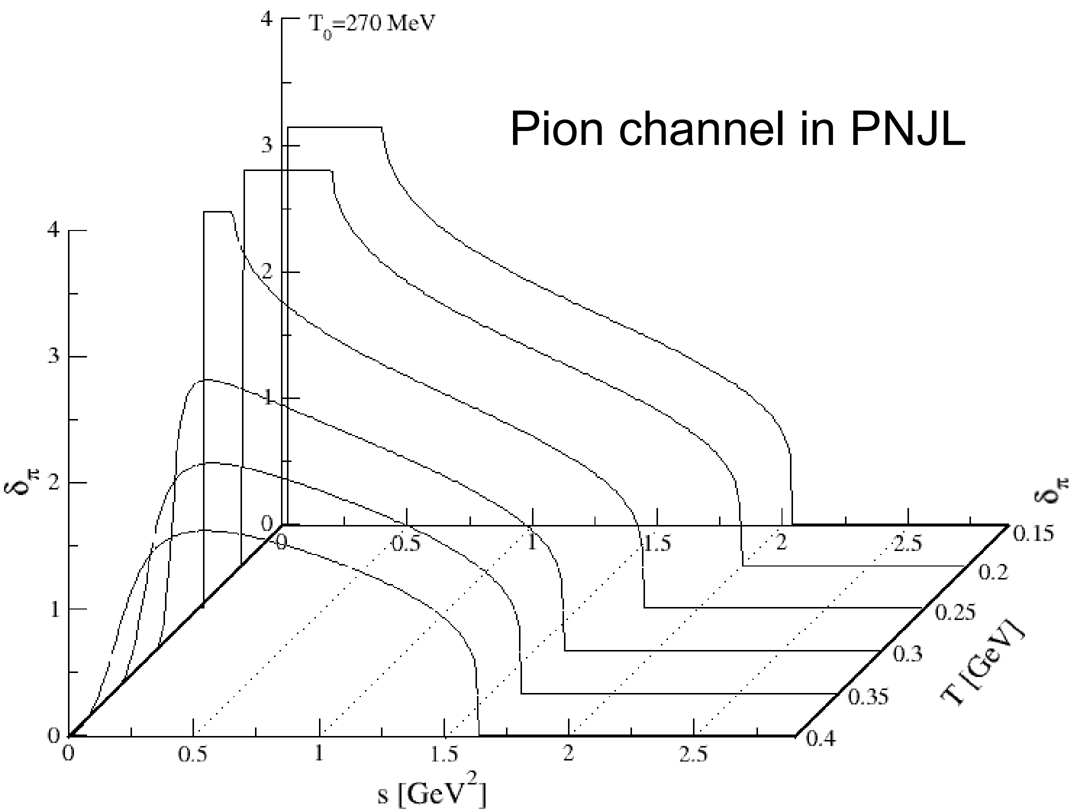


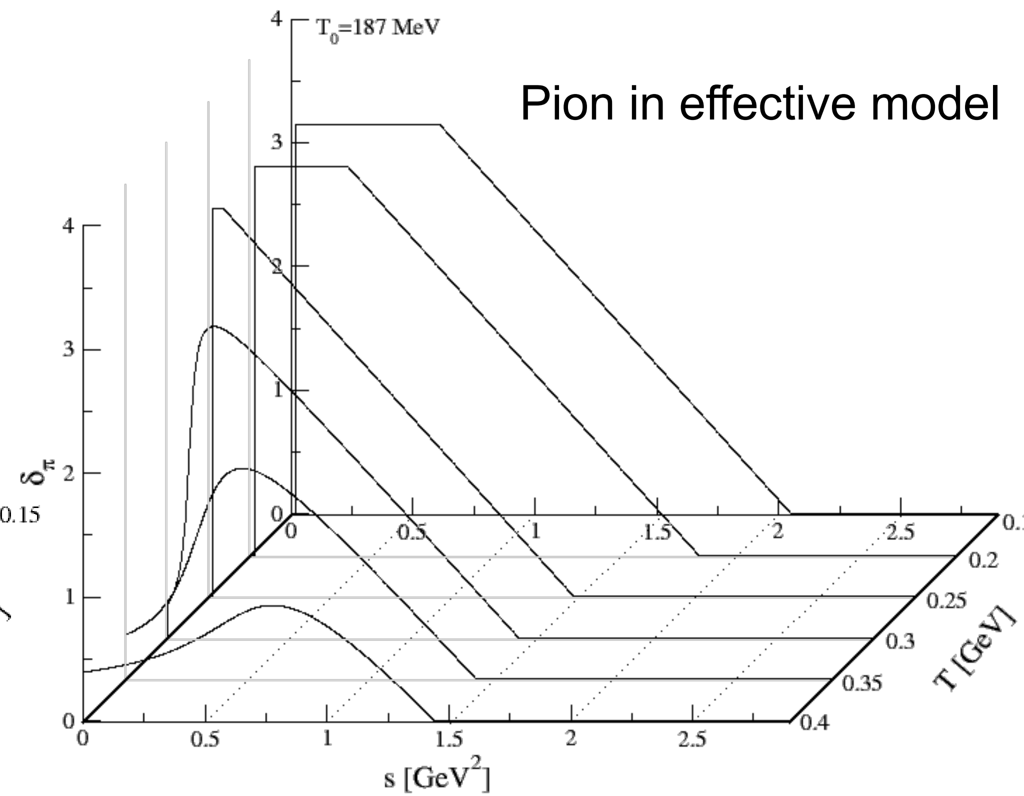
$$M_i(T_{\rm Mott,i}) = m_{\rm thr,i}(T_{\rm Mott,i}) ,$$
$$m_{\rm thr,M}(T) = (2-N_s)m(T) + N_s m_s(T)$$
$$m_{\rm thr,B}(T) = (3-N_s)m(T) + N_s m_s(T)$$

$$M_i(T) = M_i(0) + \Gamma_i(T) ,$$
$$\Gamma_i(T) = a\,(T - T_{\rm Mott,i})\Theta(T - T_{\rm Mott,i})$$

Hadron Resonance Gas with Mott Dissociation

D. Blaschke, A. Dubinin, in preparation



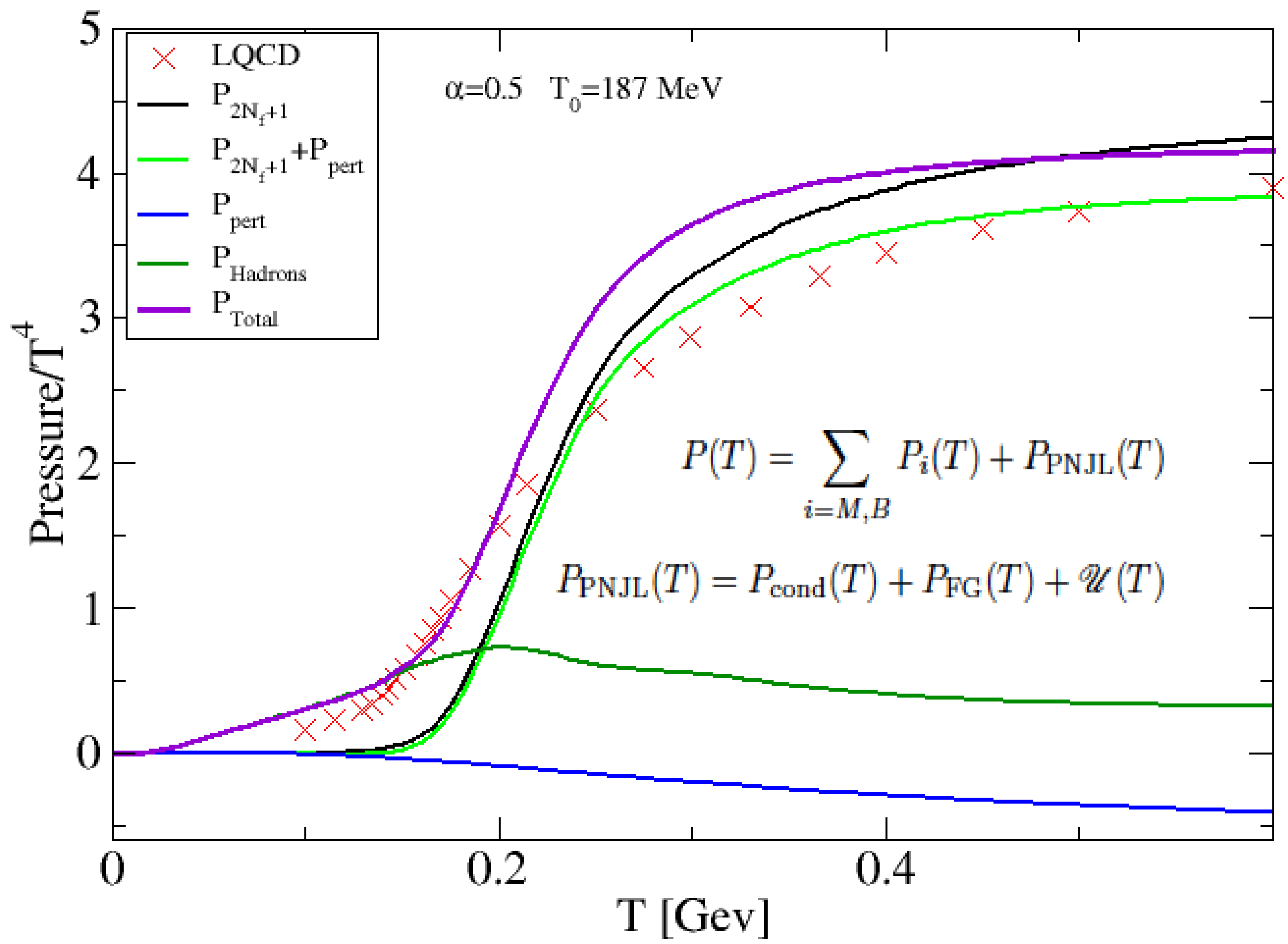


Effective model for in-medium hadron phase shifts

$$\delta_i(s;T) = \left[\frac{\pi}{2} + \arctan\left(\frac{s - M_i^2(T)}{M_i(T)\Gamma_i(T)}\right)\right]\left\{\Theta[m_{\mathrm{thr,i}}^2 - s] + \Theta[s - m_{\mathrm{thr,i}}^2]\Theta[m_{\mathrm{thr,i}}^2 + N_i^2\Lambda^2 - s]\left[\frac{[m_{\mathrm{thr,i}}^2 + N_i^2\Lambda^2 - s}{N_i^2\Lambda^2}\right]\right\} \quad (7)$$

Hadron Resonance Gas with Mott Dissociation

D. Blaschke, A. Dubinin, L. Turko, arxiv:1612.09556 [hep-ph]



$$P_i(T) \;=\; d_i \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} f_i(\omega)\delta_i(\omega;T) \;=\; d_i \int_0^\infty \frac{dp\; p^2}{2\pi^2} \int_0^\infty \frac{ds}{2\pi} \frac{1}{\sqrt{p^2+s}} \; f_i(\sqrt{p^2+s})\delta_i(s;T)$$

Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

The basic idea: Localization of (certain) multiquark states ("cluster") = hadronization;
Reverse process = delocalization by quark exchange between hadrons

Freeze-out criterion:

$$\tau_{\exp}(T,\mu) = \tau_{\rm coll}(T,\mu)$$

$$\tau^{-1}_{\rm coll}(T,\mu) = \sum_{i,j}\sigma_{ij}n_j$$

Povh-Huefner law, PRC 46 (1992) 990

$$\sigma_{ij} = \lambda\langle r_i^2\rangle\langle r_j^2\rangle$$

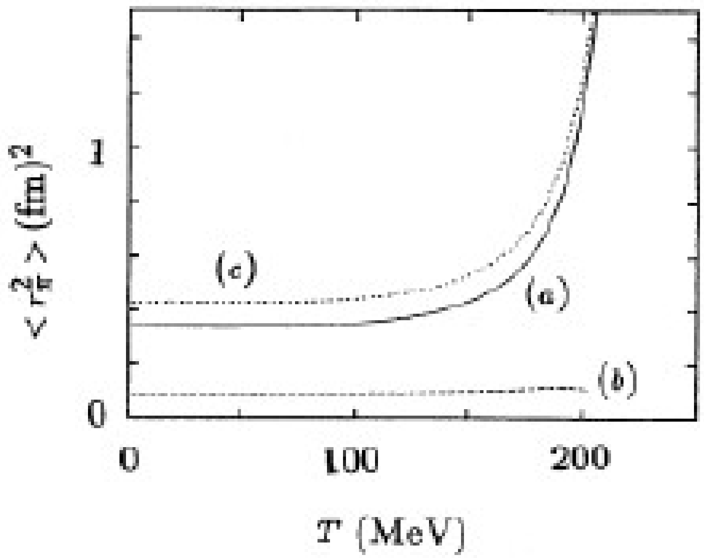
$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2}f_\pi^{-2}(T,\mu)$$

$$f_\pi^2(T,\mu) = -m_0\langle\bar{q}q\rangle_{T,\mu}/M_\pi^2$$

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}\left|\langle\bar{q}q\rangle_{T,\mu}\right|^{-1}$$

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_{\rm MF}\left[1-\frac{T^2}{8f_\pi^2(T,\mu)}-\frac{\sigma_N n_{s,N}(T,\mu)}{M_\pi^2 f_\pi^2(T,\mu)}\right]$$

Hippe & Klevansky, PRC 52 (1995) 2172



Mott-Anderson localization model for chemical freeze-out

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Povh-Huefner law behaviour for quark exchange between hadrons

PHYSICAL REVIEW C VOLUME 51, NUMBER 5 MAY 1995

Quark exchange model for charmonium dissociation in hot hadronic matter

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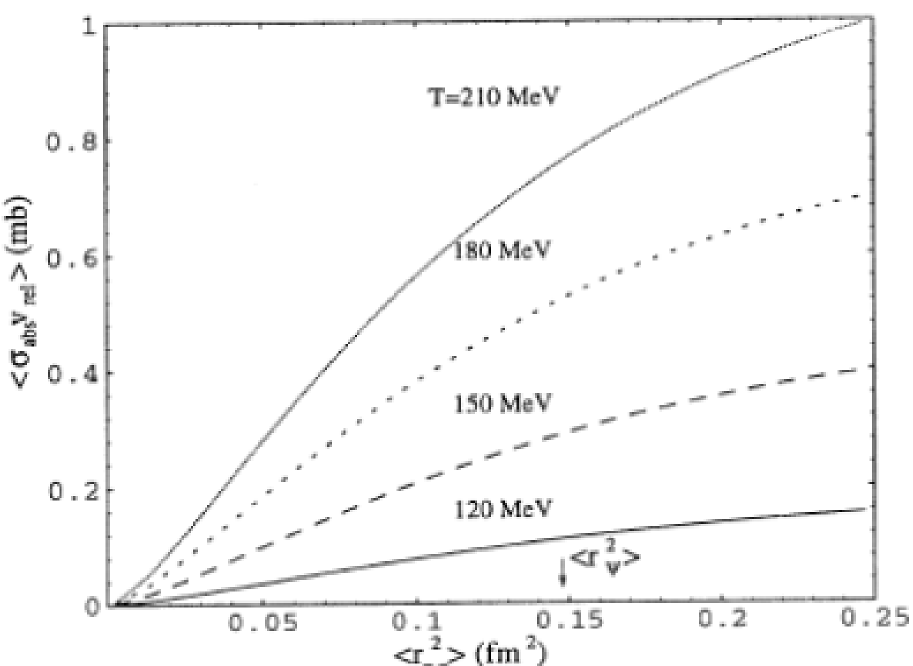
Gesellschaft für Schwerionenforschung mbH, Postfach 11 05 52, D-64220 Darmstadt, Germany
(Received 15 November 1994)



$$\langle \sigma_{\rm abs} v_{\rm rel} \rangle \propto \langle r^2 \rangle_{Q\bar{Q}} \langle r^2 \rangle_{q\bar{q}}$$

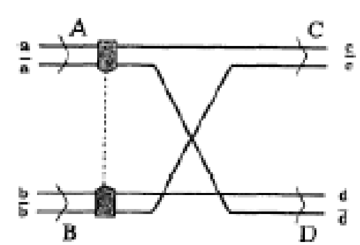
Flavor exchange processes

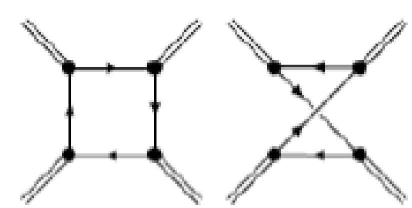
$$\phi + \pi \rightarrow K + \bar{K},$$

$$K^- + p \rightarrow \Lambda + X,$$
$$K^+ + p \rightarrow \Lambda + X,$$

Nonrelativistic → rel. quark loop integrals

M_{fi} =





Mott-Anderson localization model for chemical freeze-out

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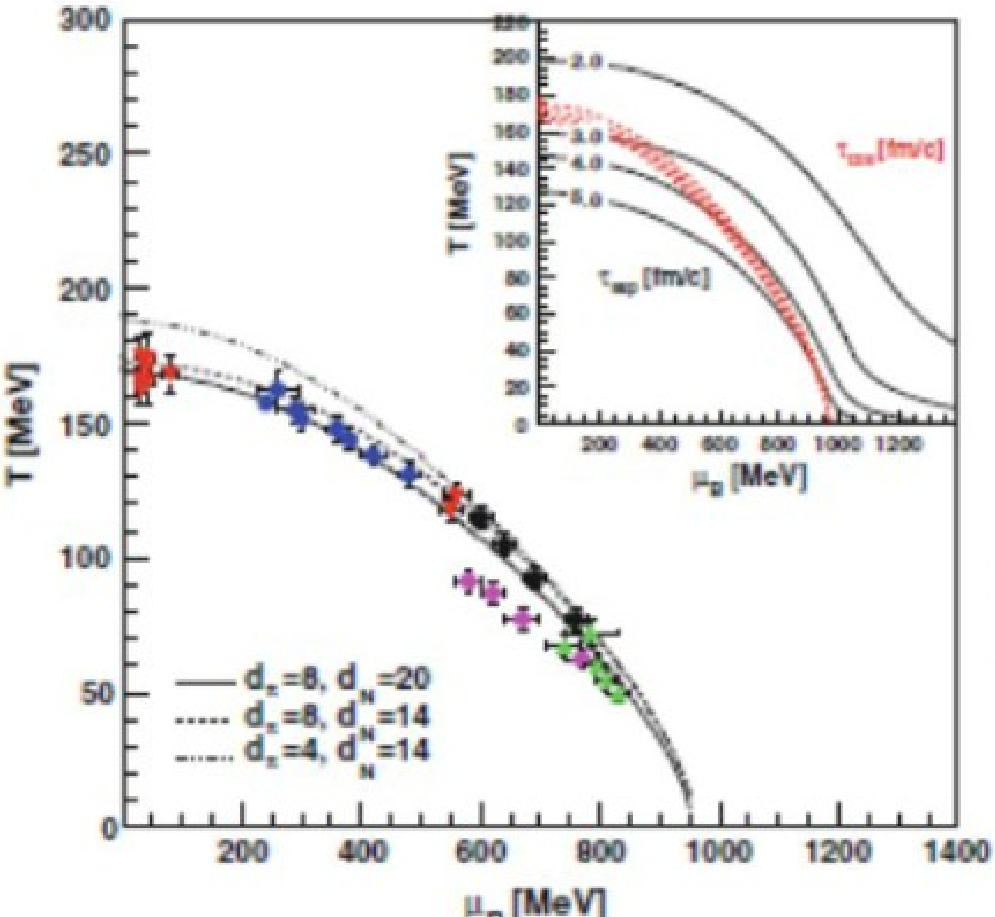
Model results:

$$\tau_{exp}(T, \mu) = \tau_{coll}(T, \mu)$$

Collision time strongly T, mu dependent !

Schematic resonance gas: dp pions, dN nucleons

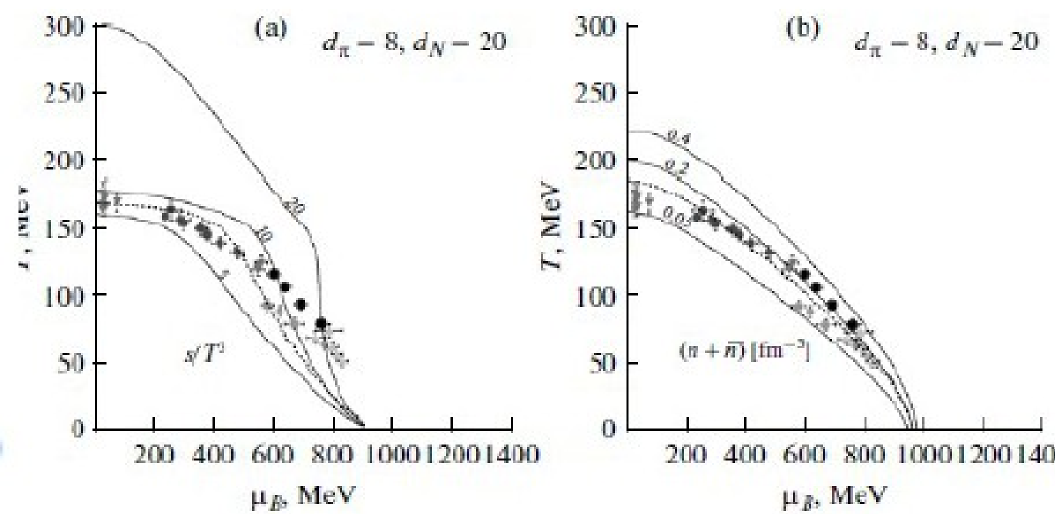


Expansion time scale from entropy conservation:

$$s(T, \mu)\ V(\tau_{exp}) = \text{const}$$

$$\tau_{exp}(T, \mu) = a s^{-1/3}(T, \mu),$$

Thermodynamics consistent with phenomenological Freeze-out rules:



Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Few Body Syst. 53 (2012) 99

Model results:

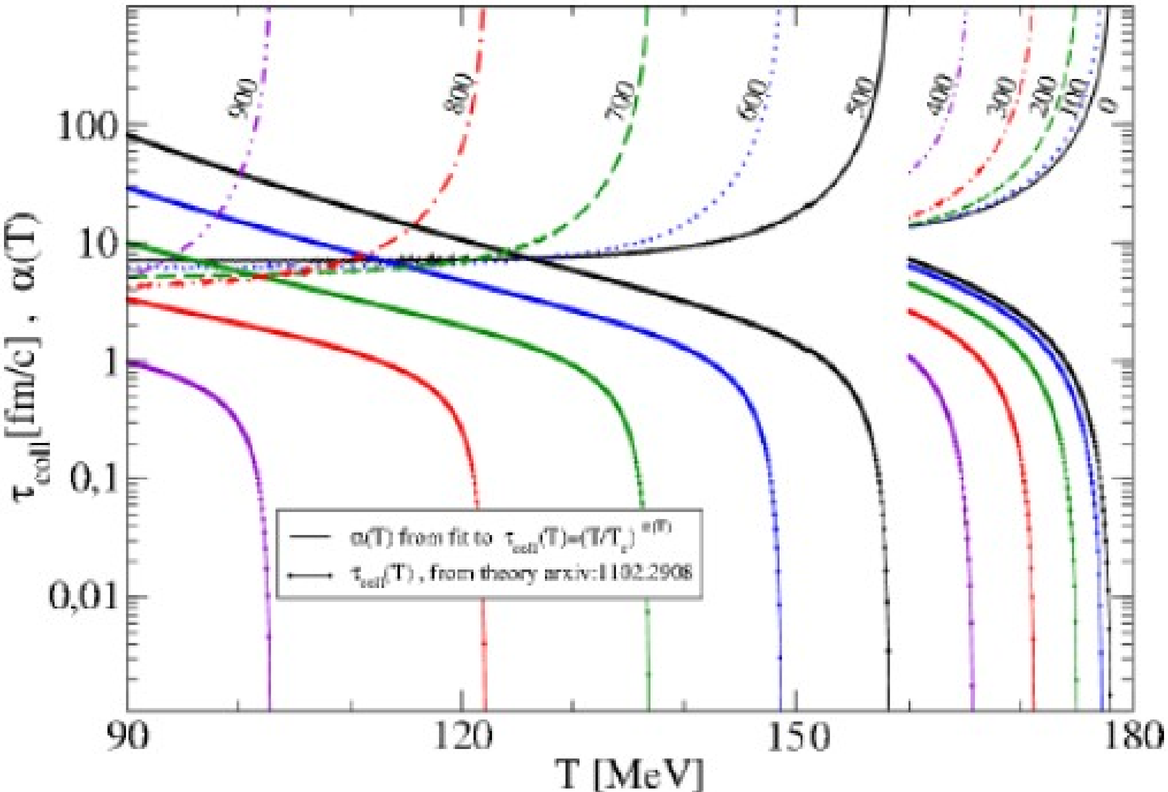
Full hadron resonance gas model

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle \ ;$$

$$r_\pi^2(T,\mu) = \frac{3 M_\pi^2}{4\pi^2 m_q} \left| \langle \bar{q} q \rangle_{T,\mu} \right|^{-1}$$

$$r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu)$$

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\rm vac}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \Bigg\{ 4N_c \int \frac{dp\, p^2}{2\pi^2} \frac{m}{\varepsilon_p} \left[f_\Phi^+ + f_\Phi^- \right] + \sum_{M=f_0,\omega,\ldots} d_M (2 - N_s) \int \frac{dp\, p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) + \sum_{B=N,\Lambda,\ldots} d_B (3 - N_s) \int \frac{dp\, p^2}{2\pi^2} \frac{m_B}{E_B(p)} \left[f_B^+(E_B(p)) + f_B^-(E_B(p)) \right] \Bigg\} - \sum_{G=\pi,K,\eta,\eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)).$$



Collision time follows a power law
t_coll ~ (T/Tc)^a
with a large exponent a ~ 20

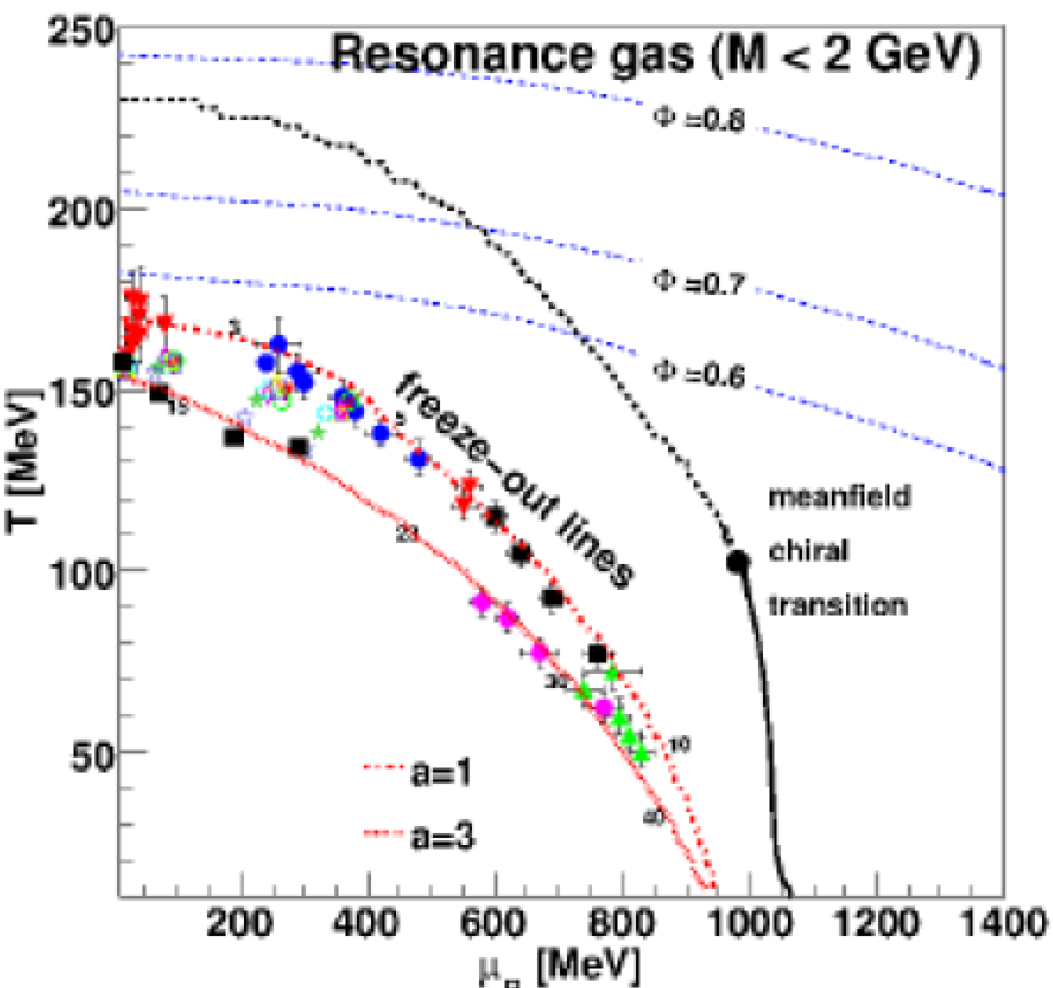
See also: P. Braun-Munzinger, J. Stachel, C. Wetterich, PLB (2004)

Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Few Body Syst. 53 (2012) 99

Model results:
Full hadron resonance gas model

See also: S. Leupold, J. Phys. G (2006)



$$\frac{\langle\bar{q}q\rangle}{\langle\bar{q}q\rangle_{vac}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2}\Big\{4N_c\int\frac{dp\,p^2}{2\pi^2}\frac{m}{\varepsilon_p}[f_\Phi^+ + f_\Phi^-]$$
$$+ \sum_{M=f_0,\omega,\ldots} d_M(2-N_s)\int\frac{dp\,p^2}{2\pi^2}\frac{m_M}{E_M(p)}f_M(E_M(p))$$
$$+ \sum_{B=N,\Lambda,\ldots} d_B(3-N_s)\int\frac{dp\,p^2}{2\pi^2}\frac{m_B}{E_B(p)}[f_B^+(E_B(p)) + f_B^-(E_B(p))]\Big\}$$
$$- \sum_{G=\pi,K,\eta,\eta'}\frac{d_G r_G}{4\pi^2 F_G^2}\int dp\,\frac{p^2}{E_G(p)}f_G(E_G(p)).$$

$$\sigma_{ij} = \lambda\langle r_i^2\rangle\langle r_j^2\rangle \quad ; \quad r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu)$$

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}|\langle\bar{q}q\rangle_{T,\mu}|^{-1}$$

The coeff cient a stands for the inverse system size in the formula

$$\tau_{exp}(T,\mu) = \tau_{coll}(T,\mu)$$

for the 3D expansion time scale assuming entropy conservation

Summary

- Generalized Beth-Uhlenbeck approach as microphysical basis to account for hadron dissociation (Mott effect) at extreme temperatures and densities
- Benchmark: pion and sigma Mott effect within NJL model, revised within nonlocal PNJL model
- Nonlocal PNJL model calibrated with lattice quark propagator data, EoS at finite T and μ, Phase diagram with critical point
- Application of GBU to interprete chemical freeze-out as Mott-Anderson localization
- Effective GBU model description: Mott-Hagedorn resonance gas + PNJL model describes Lattice QCD thermodynamics

Outlook

- RMF (Walecka) model as limit of the PNJL model: chiral transition effects in nuclear EoS
- Prospects for HIC (CBM & NICA) and Supernovae:
  color superconducting (quarkyonic) phases accessible!

Solving the Puzzles of Compact Star Interiors

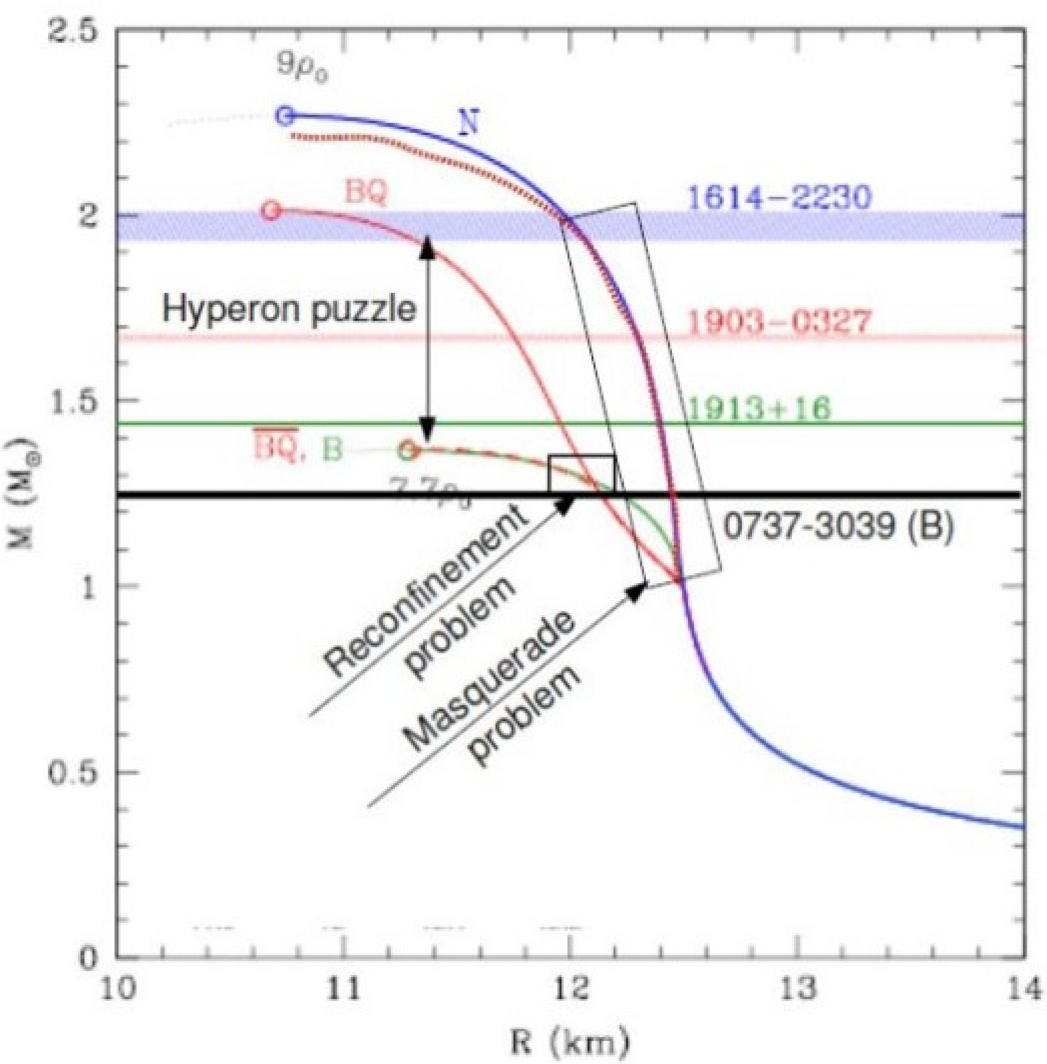
David Blaschke (University of Wroclaw, Poland & JINR Dubna, Russia)

1. The Puzzles:
- Hyperon puzzle
- Reconfinement
- Masquerade

2. The Solution:
Baryon finite size (compositeness
→ Excluded volume Appr. (EVA)

3. The Mechanism:
Quark Pauli Blocking

4. Outlook:
- High-Mass Twins (next talk)
- Supernova explosion mechanis



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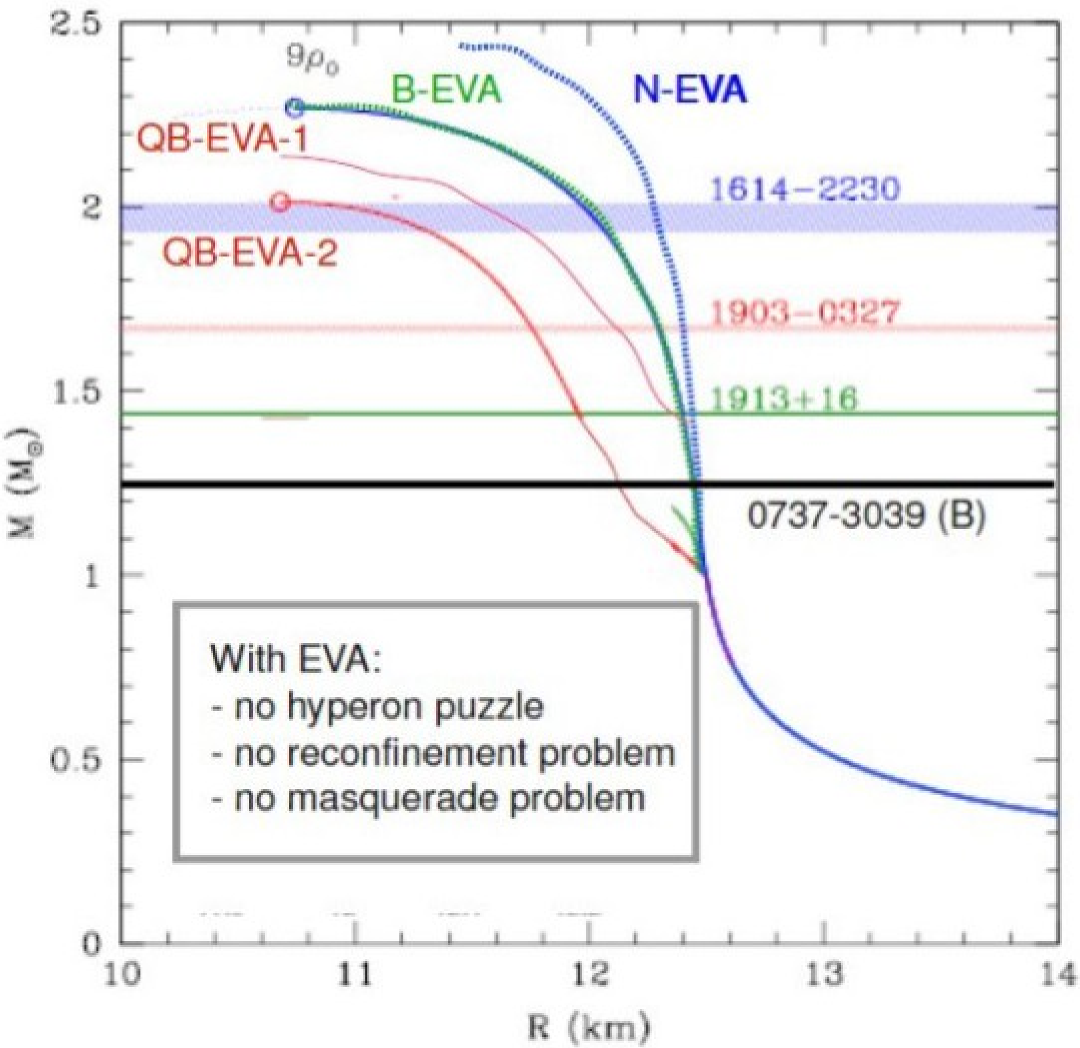

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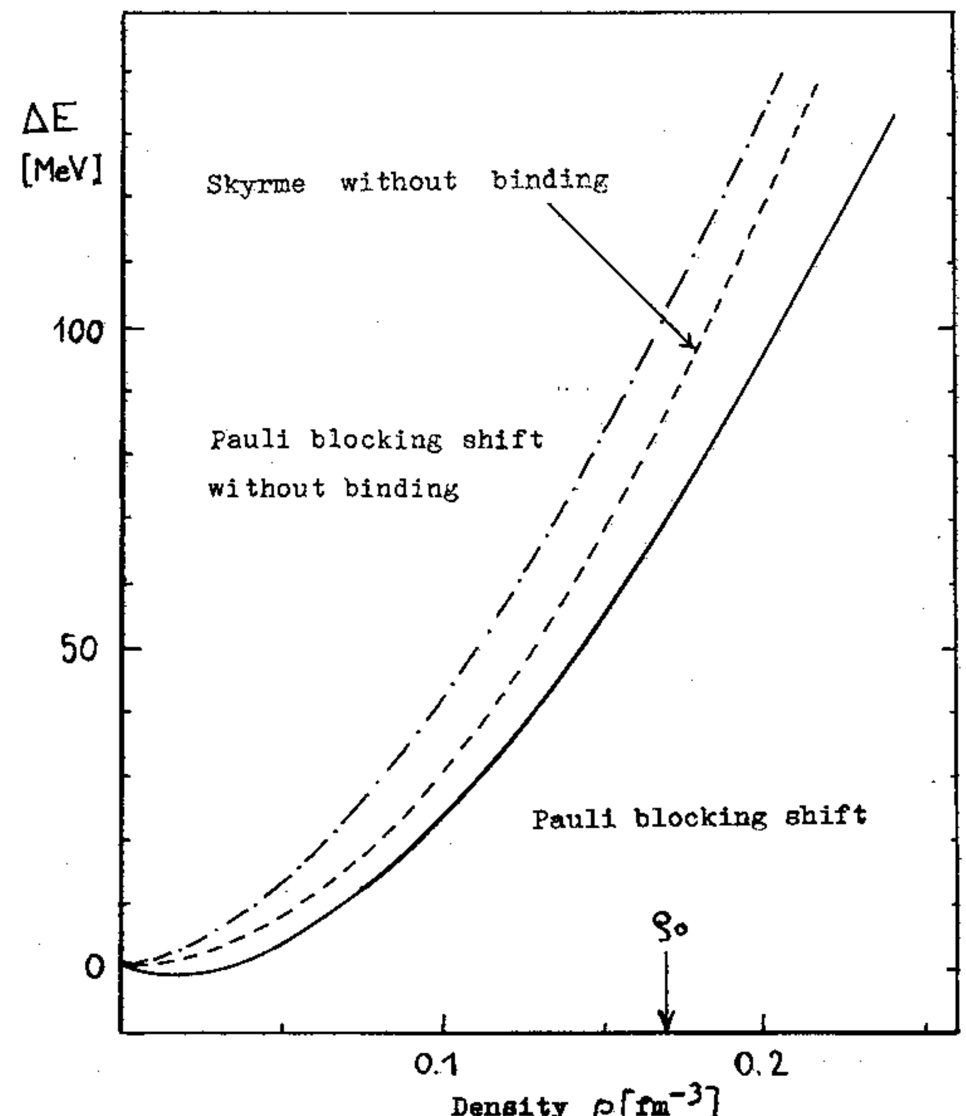

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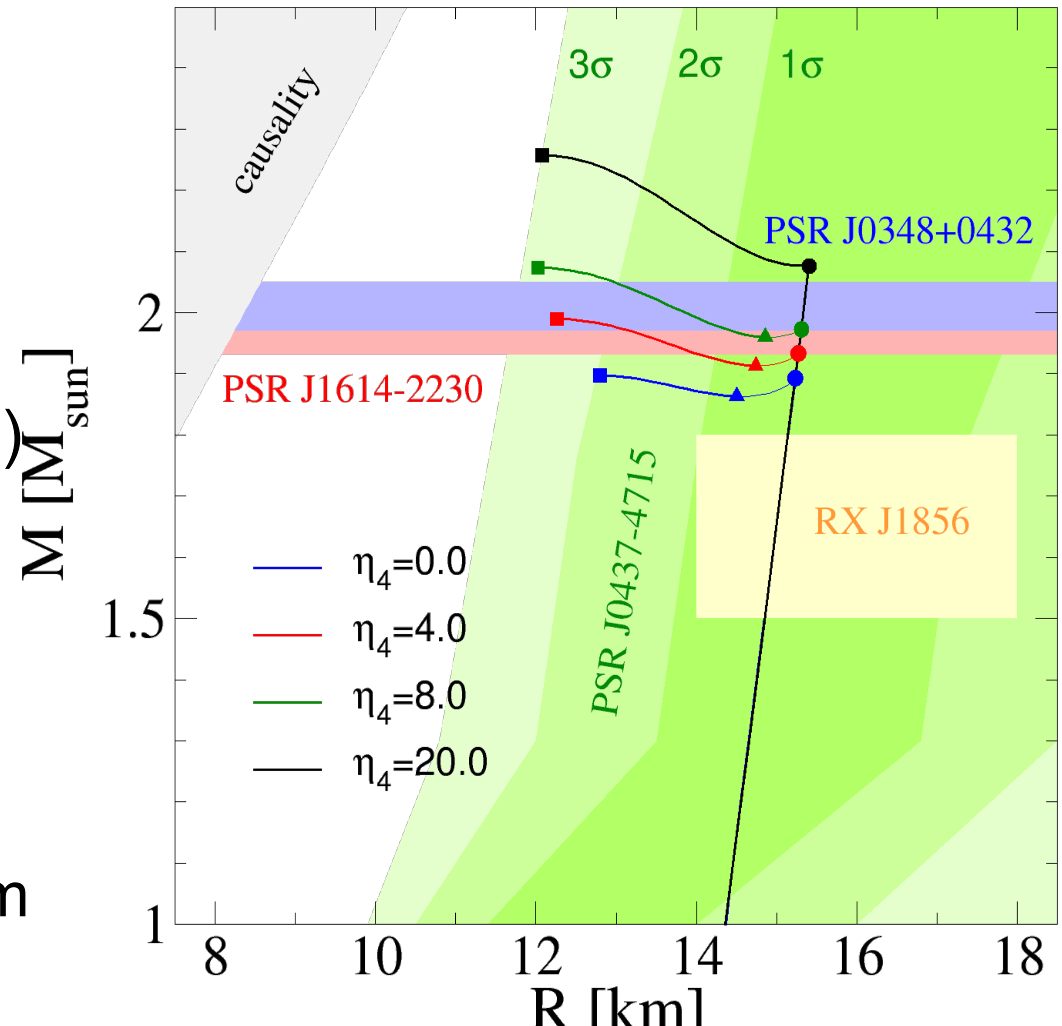

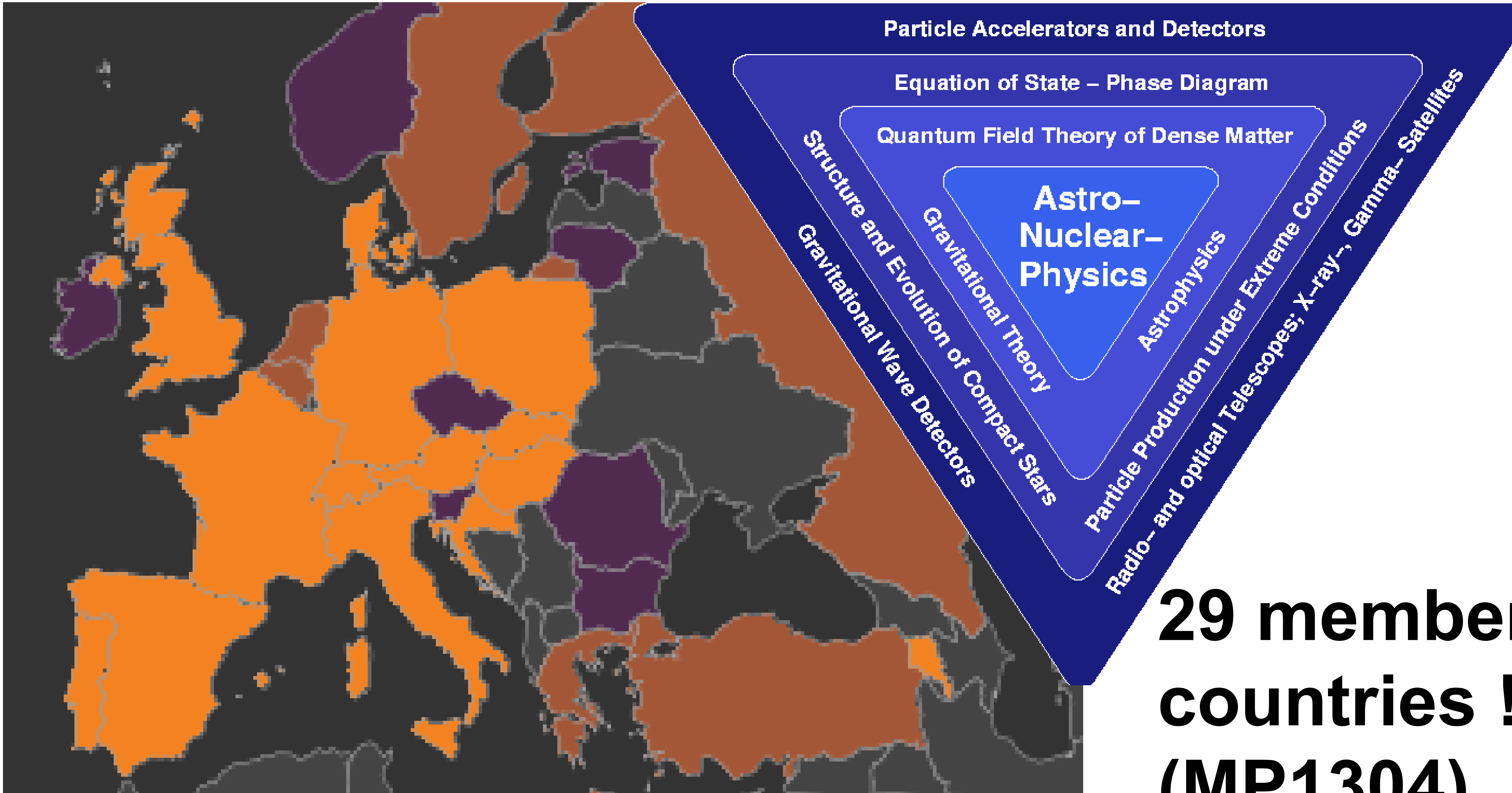

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IN SCIENCE AND TECHNOLOGY

Kick-off: Brussels, November 25, 2013