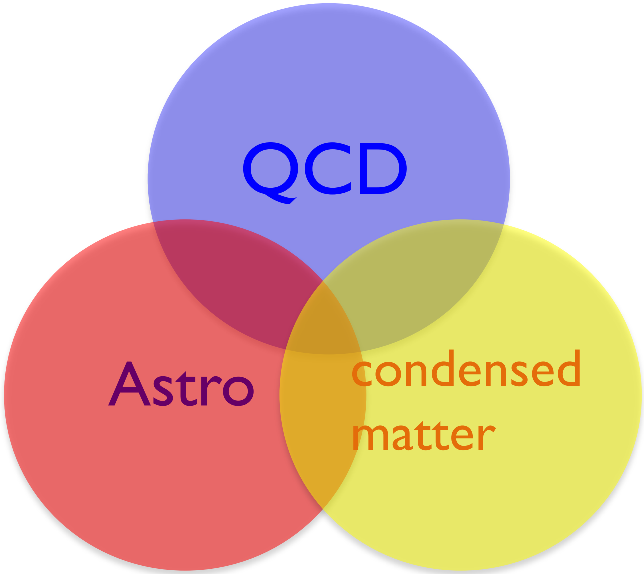
Three-window approach to (cold) dense matter

Toru Kojo (CCNU, Wuhan)



References (3-window modeling)

- Masuda-Hatsuda-Takatsuka (2012, 2013) : the 1st version
- Kojo-Powell-Song-Baym (2014), Fukushima-Kojo (2015) : extension
- Kojo (2015) : concise **review** of 3-window modeling
- Baym-Hatsuda-Kojo-Powell-Song-Takatsuka (2017) : comprehensive **review**

  Rept. Prog. Phys. 81 (2018) no.5, 056902 (arXiv: 1707.04966 [astro-ph])

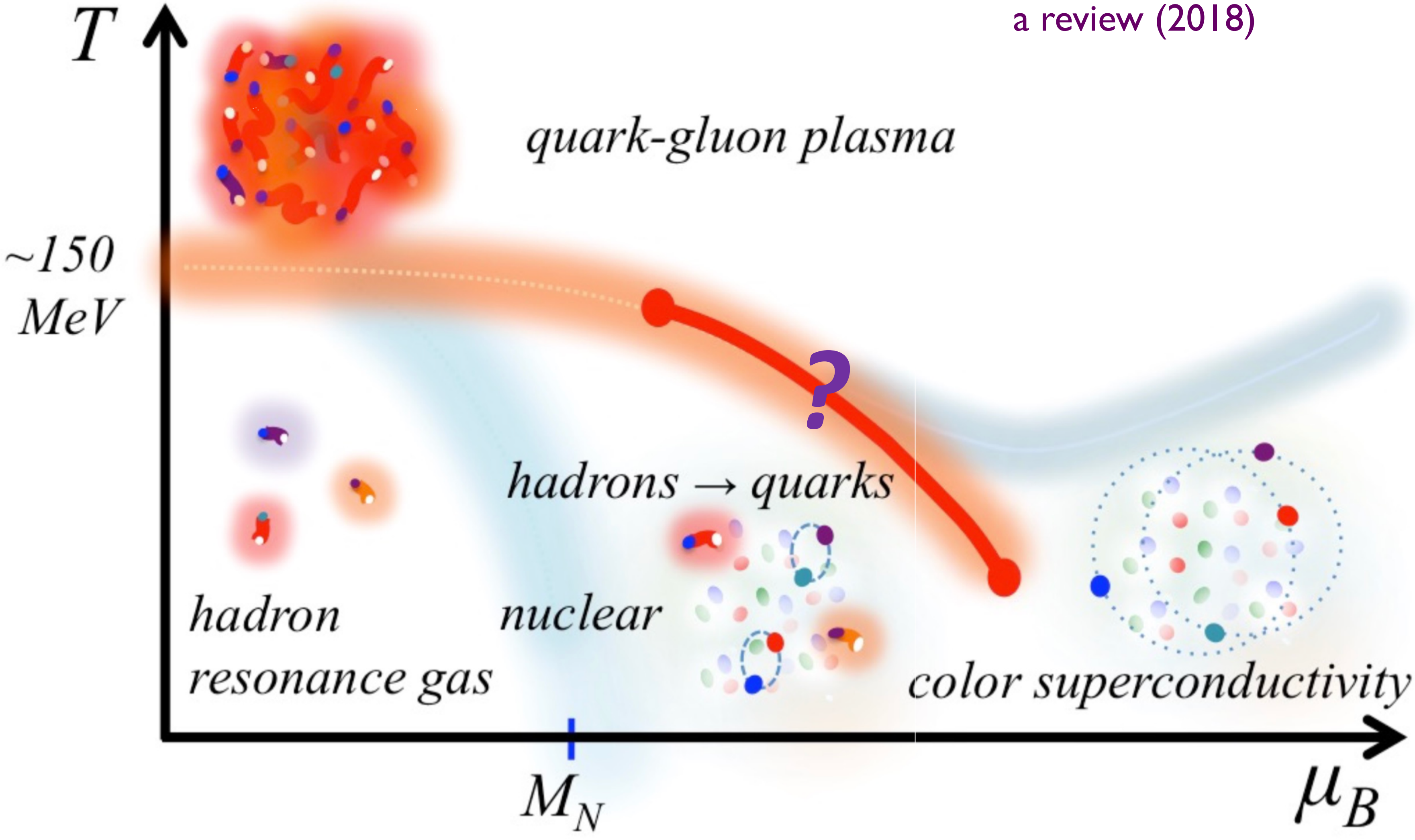
  including EoS: Quark-Hadron-Crossover (**QHC18**)

- Annala-Gorda-Kurkela-Vuorinen (2017) & refs. therein

  More conservative use of 3-window modeling

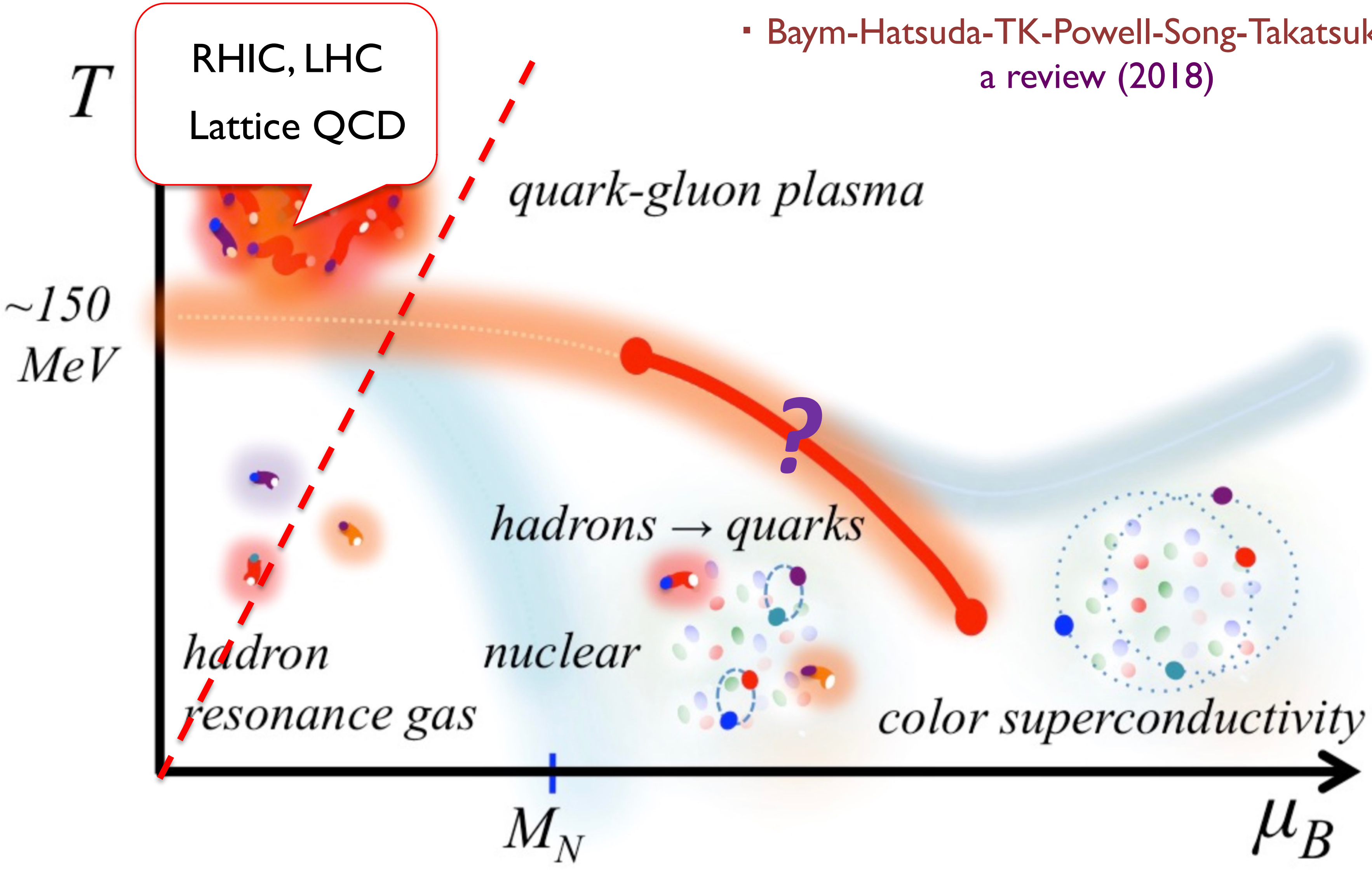
Lattice + HIC + HIC + Astro

- Baym-Hatsuda-TK-Powell-Song-Takatsuka: a review (2018)



Lattice + HIC + HIC + Astro

- Baym-Hatsuda-TK-Powell-Song-Takatsuka: a review (2018)



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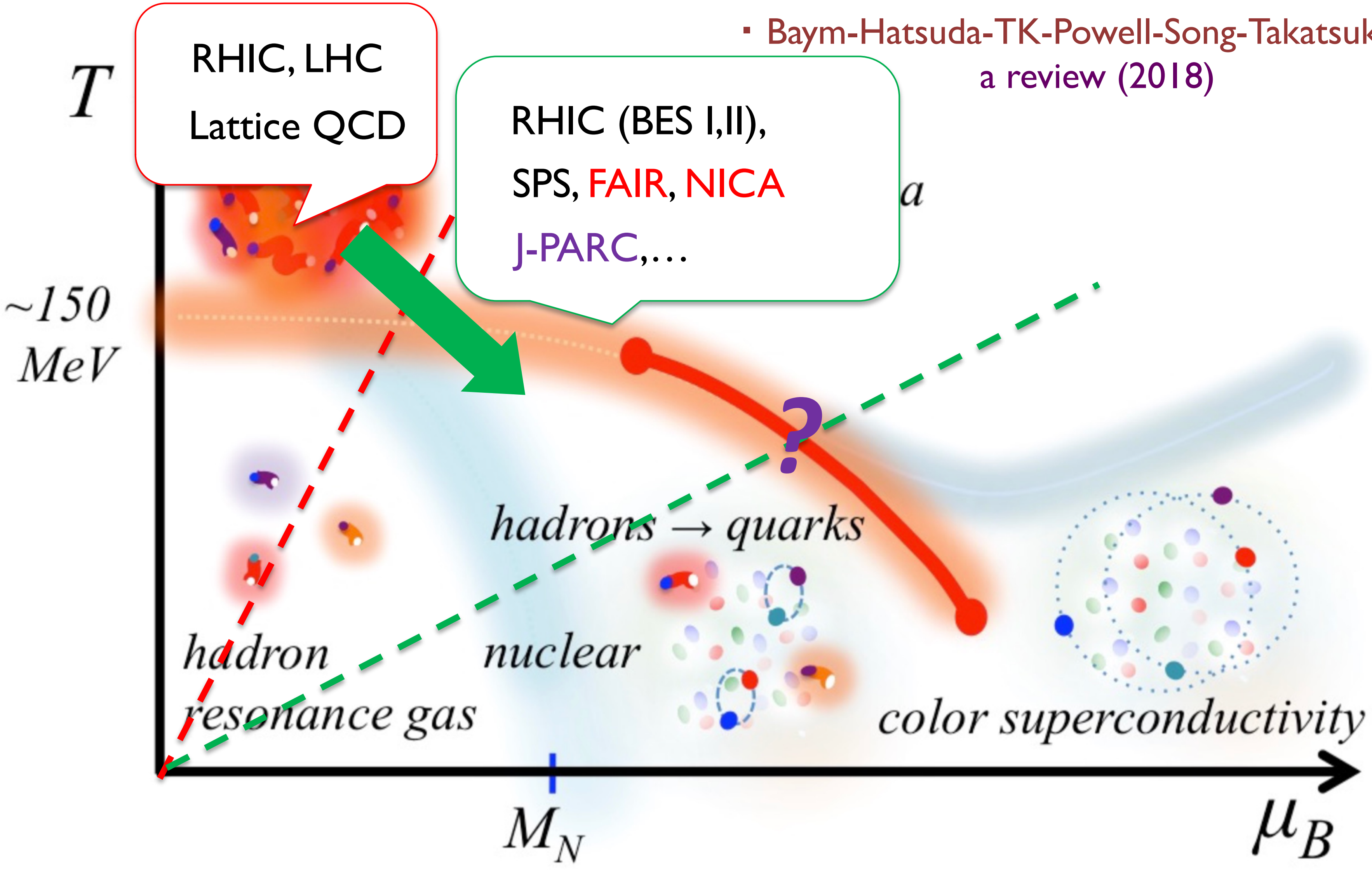


RHIC, LHC

Lattice QCD

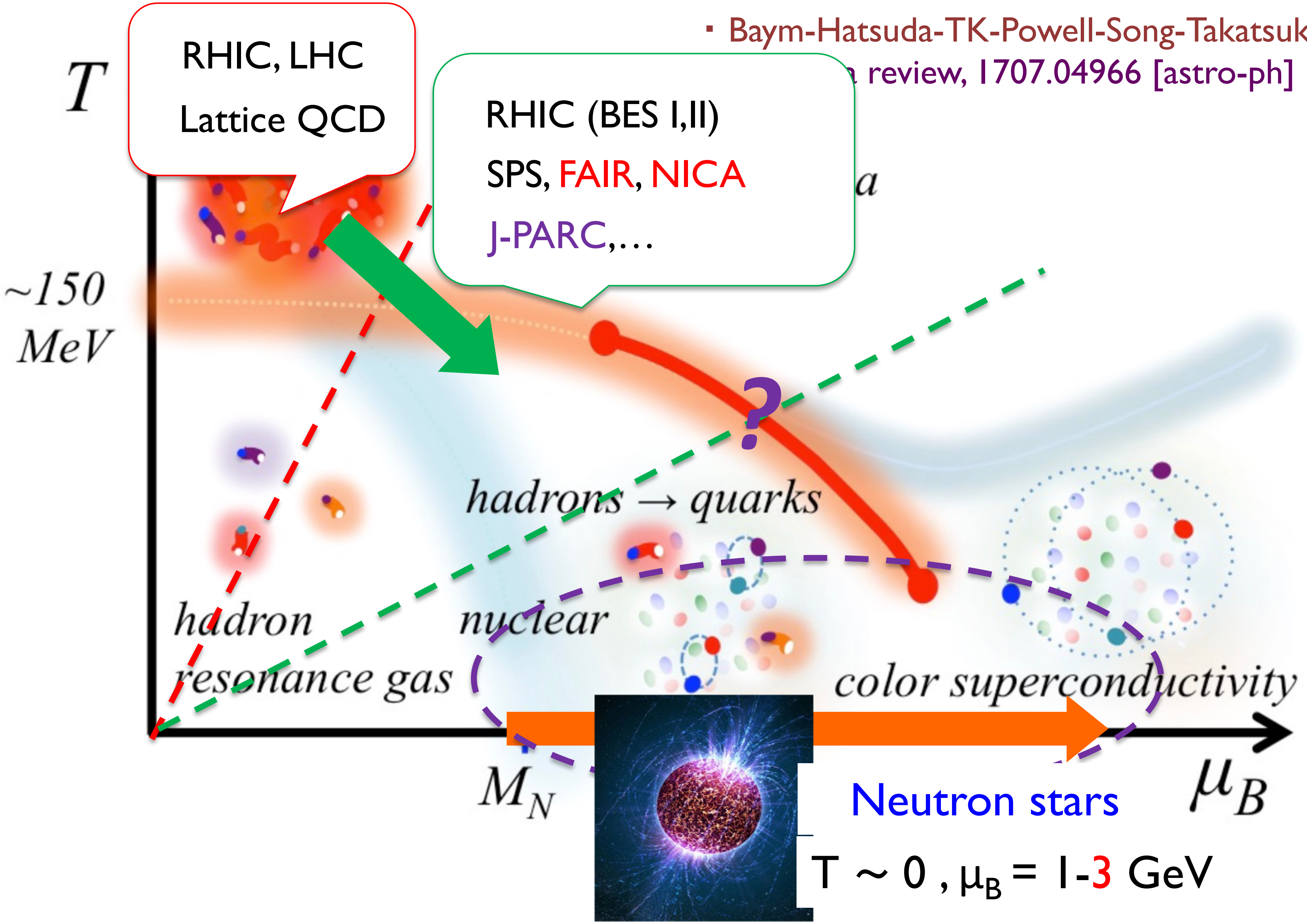
RHIC (BES I,II),

SPS, FAIR, NICA

J-PARC,…

Lattice + HIC + HIC + Astro

- Baym-Hatsuda-TK-Powell-Song-Takatsuka: review, 1707.04966 [astro-ph]



RHIC, LHC
Lattice QCD

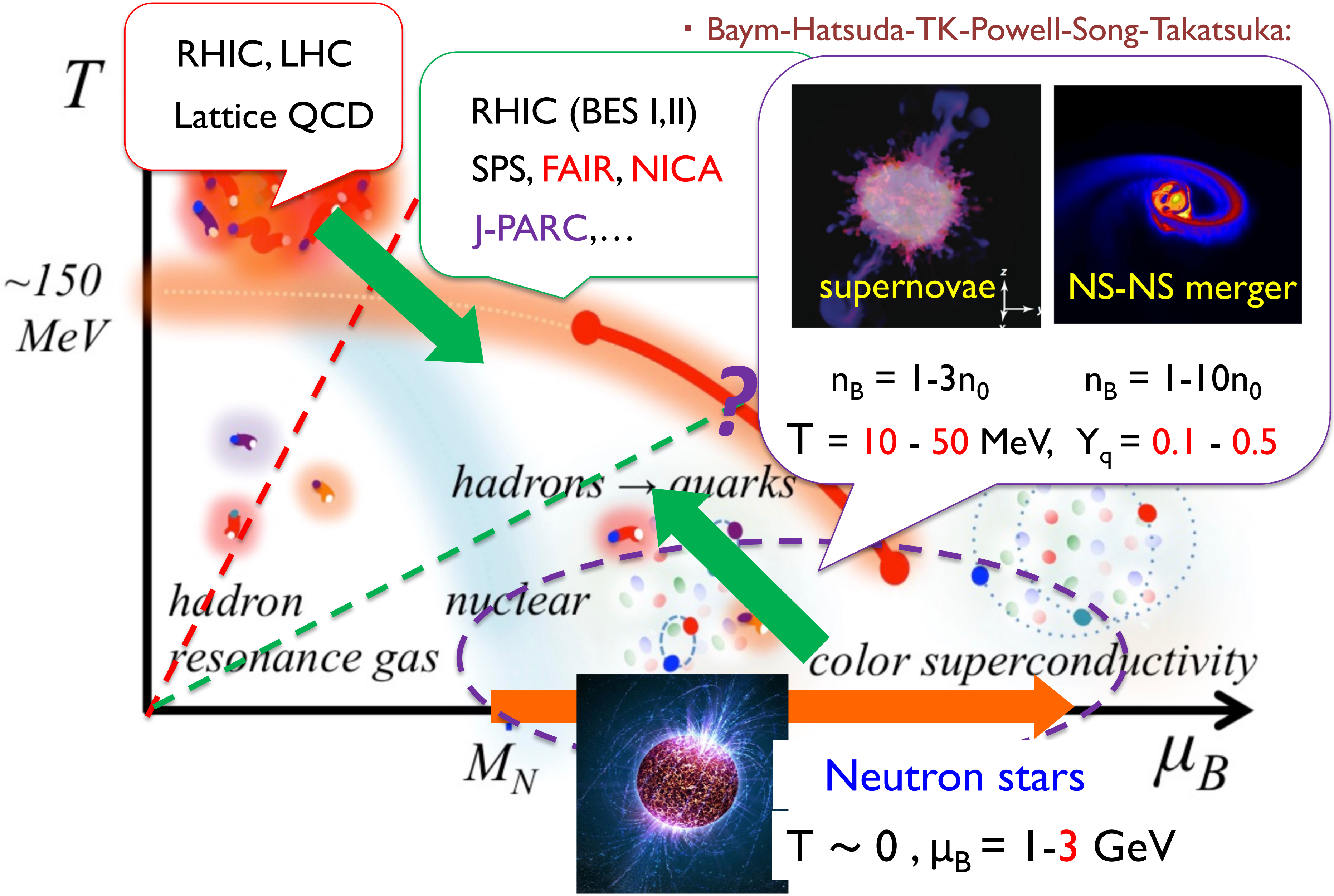
RHIC (BES I,II)
SPS, FAIR, NICA
J-PARC,…

Neutron stars

$T \sim 0$, μ_B = 1-3 GeV

Lattice + HIC + HIC + Astro

▪ Baym-Hatsuda-TK-Powell-Song-Takatsuka:



RHIC, LHC
Lattice QCD

RHIC (BES I,II)
SPS, FAIR, NICA
J-PARC,…

n_B = 1-3n_0 n_B = 1-10n_0
T = 10 - 50 MeV, Y_q = 0.1 - 0.5

Neutron stars
T ~ 0 , μ_B = 1-3 GeV

3-characteristic regimes in QCD matter

1, **Low** density regime (hadrons **dilute**)

Theory reliable : EFT with exp. inputs (hadron spectroscopy)

Effective d.o.f : hadrons

2, **High** density regime (hadrons **overlapped**)

Theory reliable : weak coupling computations

Effective d.o.f : quarks & gluons

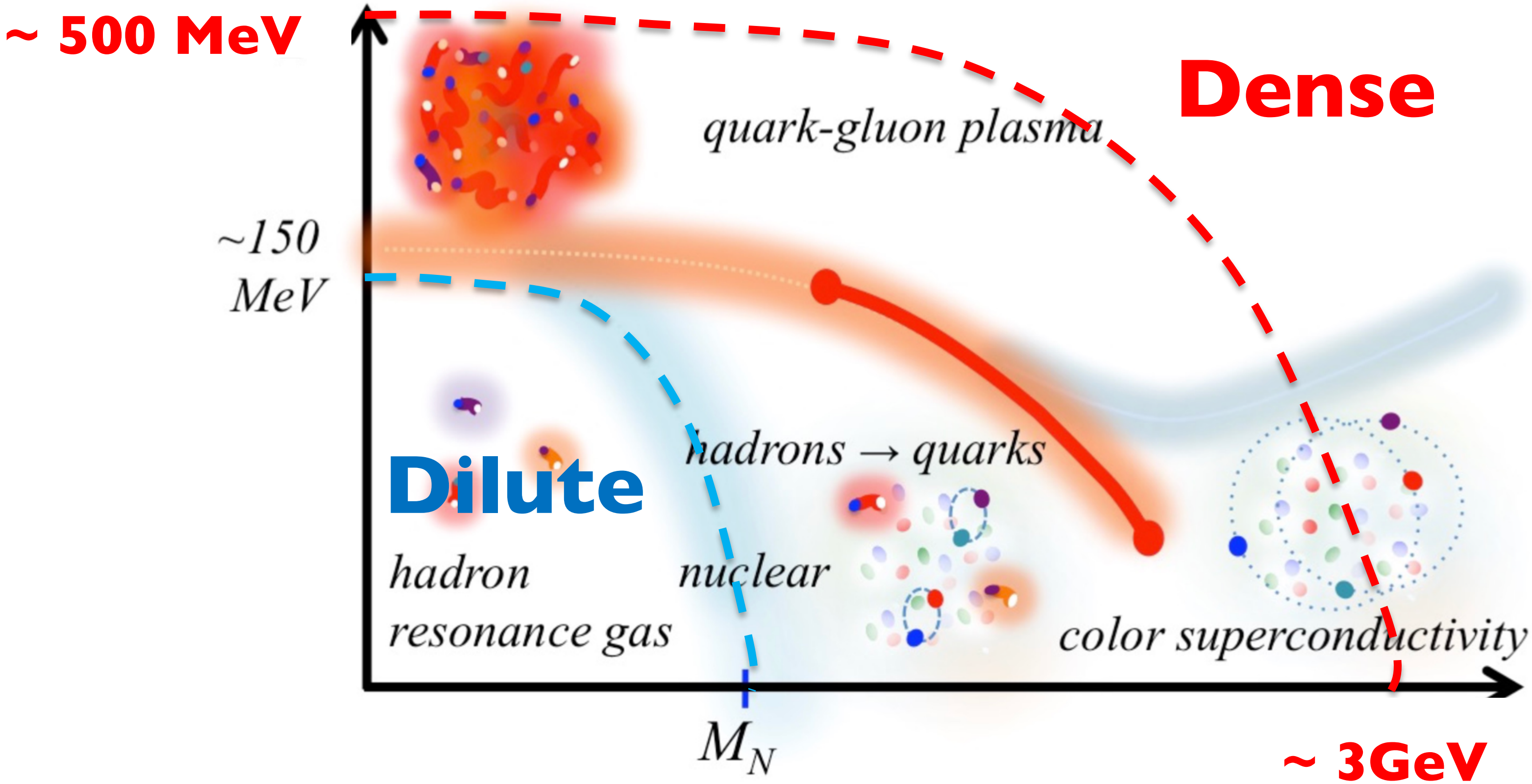
3, **Intermediate** regime

Theoretically most difficult, most important in phenomenology

Effective d.o.f : **NOT clear-cut**, collective something?

Difficulties in the QCD case

The domains of (theoretically reliable)
low & high density regimes DO NOT overlap !



Difficulties in predicting physics around phase transitions

What we will discuss

- the **gross pictures** on the QCD phase diagram
- **effective** d.o.f. & interactions
- how to use the thermodynamic relations **in practice**
- how to use the astrophysical data **in practice**
- **orientations** toward the **future** QCD computations

Many details remain to be worked out

Plan of lectures

1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

3, NS constraints on EoS : hints for **soft**-**stiff** EoS

4, Crossover scenario: chiral restorations, etc.

5, A quark model : delineating the properties of matter

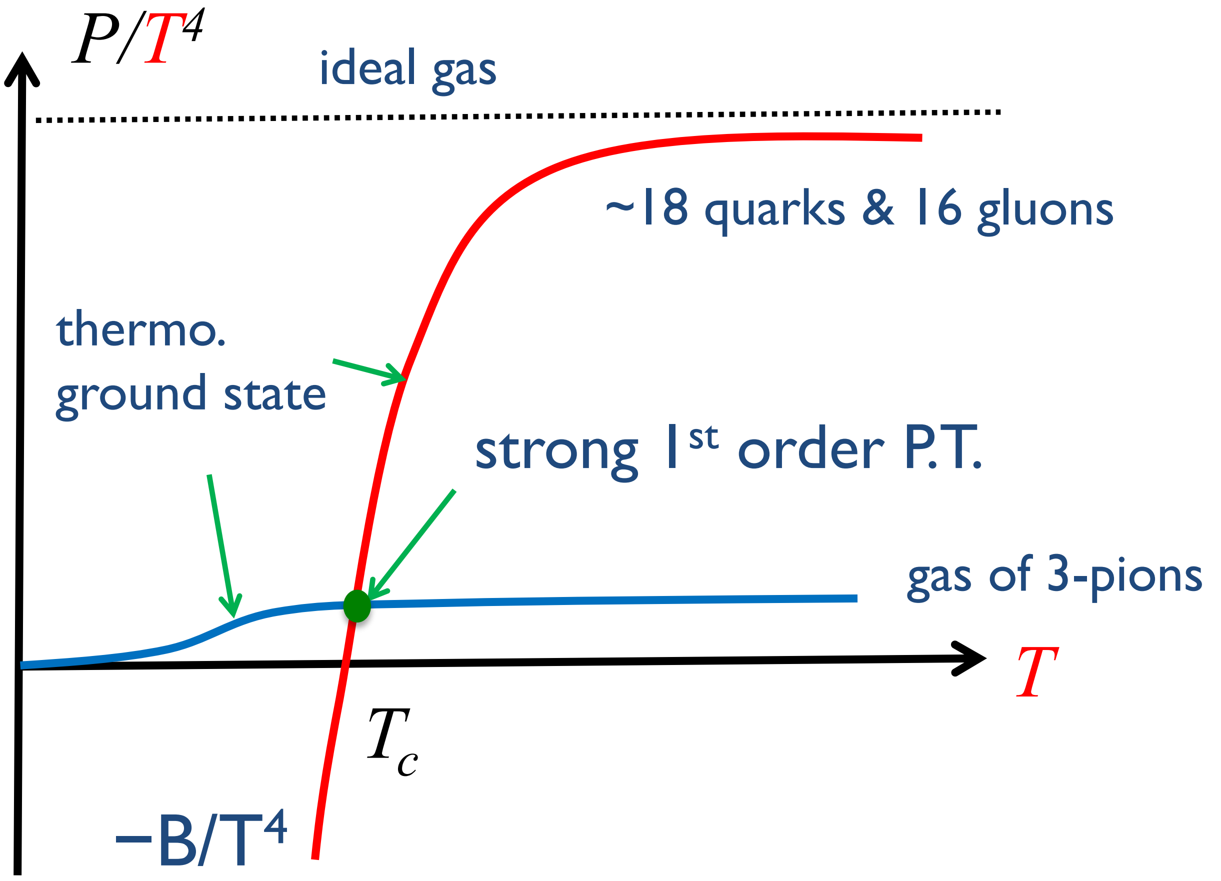
6, The astrophysical results from EoS QHC18

7-, Other topics: warm EoS, beyond-MF, etc.

Hot QCD case I : textbook example

Textbook example: **pion gas** vs **bag model QGP**

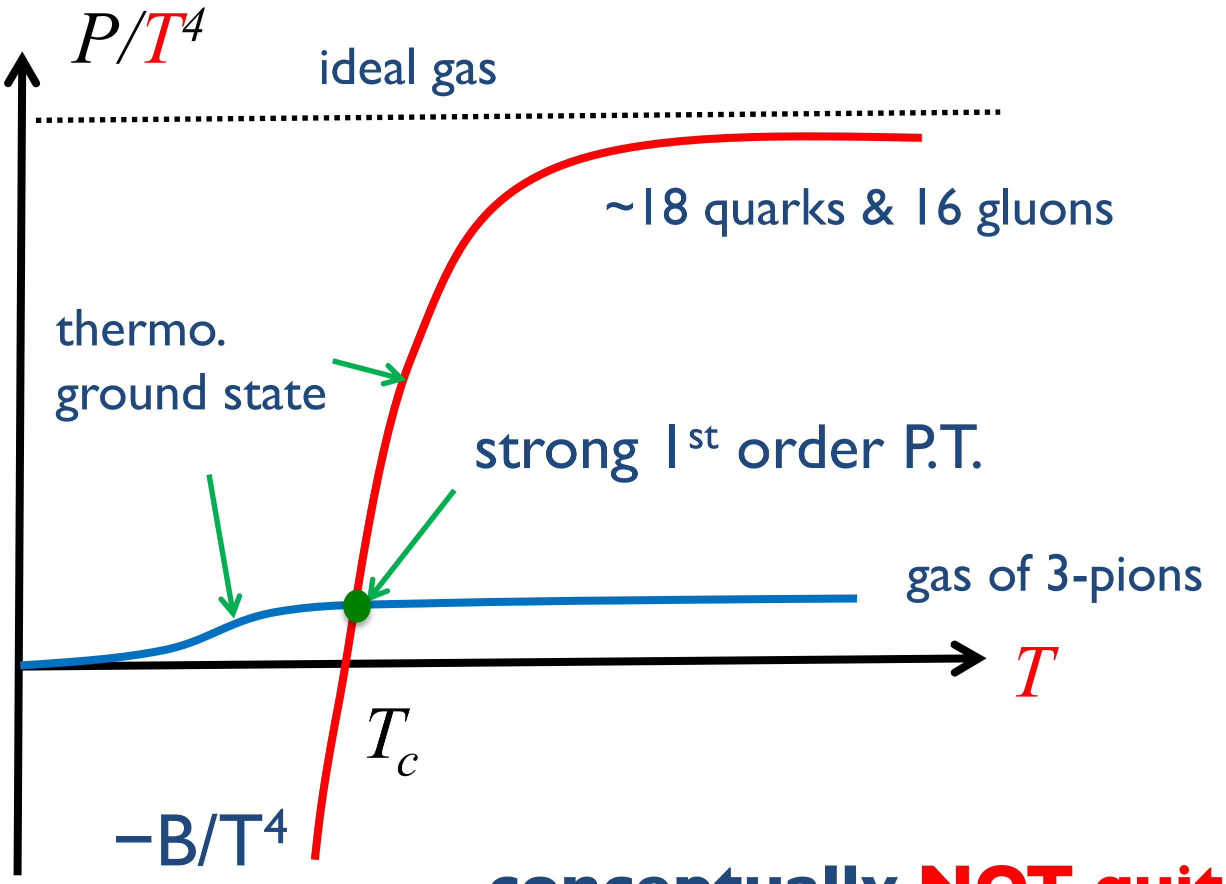
$(P_{bag} = P_{ideal} - B)$



Hot QCD case 1 : textbook example

Textbook example: **pion gas** vs **bag model QGP**

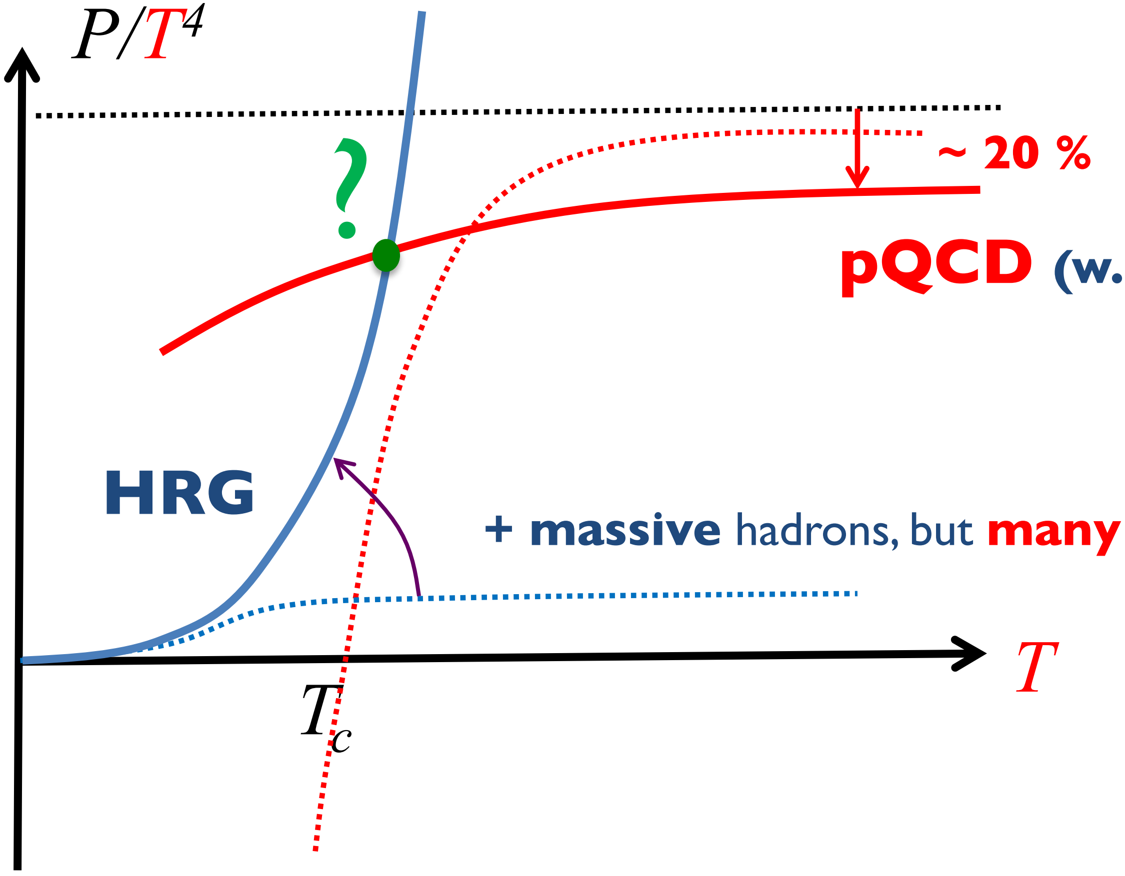
($P_{bag} = P_{ideal} - B$)



conceptually NOT quite correct

Hot QCD case 2 : improved low & high T EoS

modern version: **HRG** (hadron resonance gas) vs **pQCD** **(resummed)**



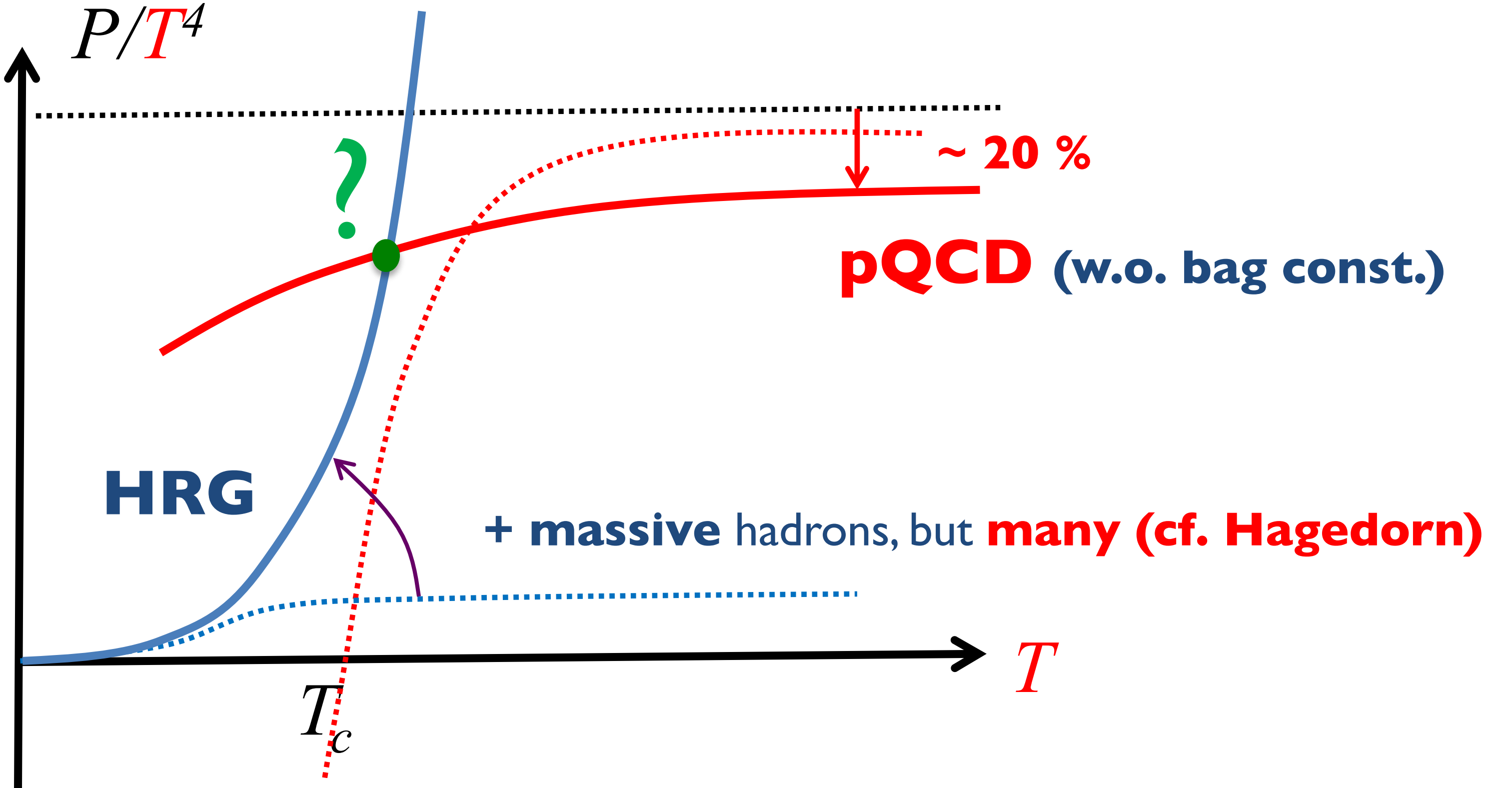
pQCD **(w.o. bag const.)**

HRG

+ massive hadrons, but **many (cf. Hagedorn)**

Hot QCD case 2 : improved low & high T EoS

modern version: **HRG** (hadron resonance gas) vs **pQCD** **(resummed)**

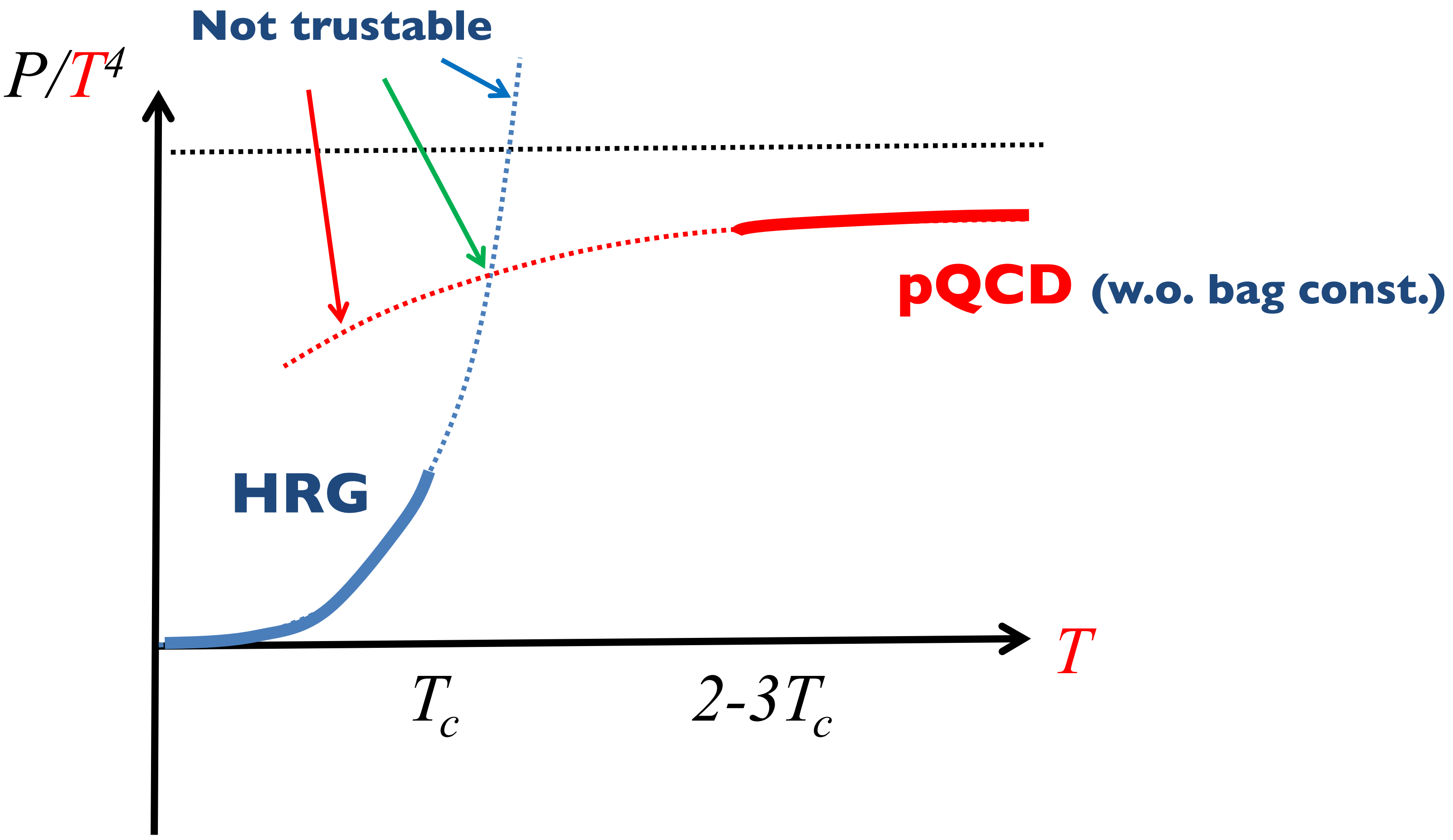


improved descriptions at dilute & dense regimes,

but wrong picture in the **intermediate** region

Hot QCD case 3 : 3-window modeling

use **only** trustable parts of **HRG** & **pQCD** **(resummed)**



Hot QCD case 3 : 3-window modeling

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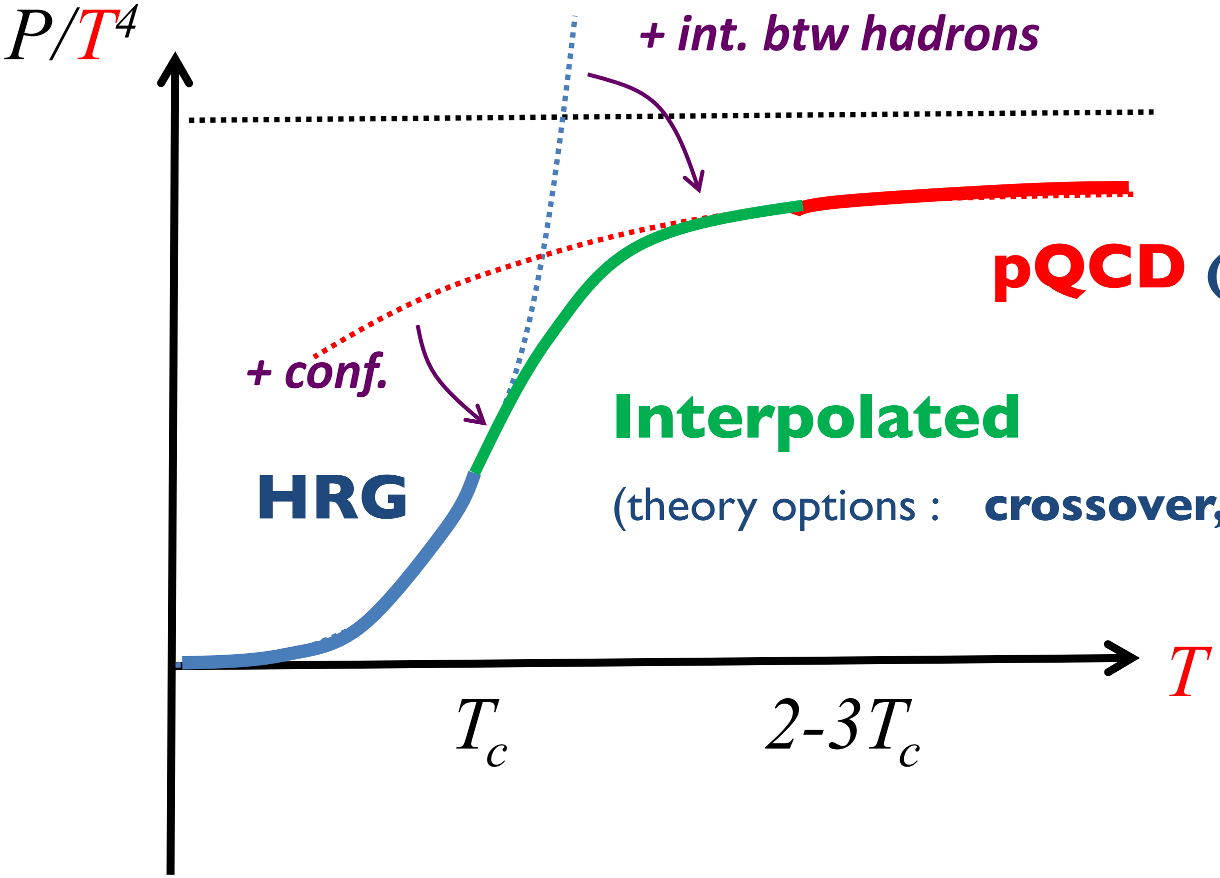


pQCD (w.o. bag const.)

Interpolated

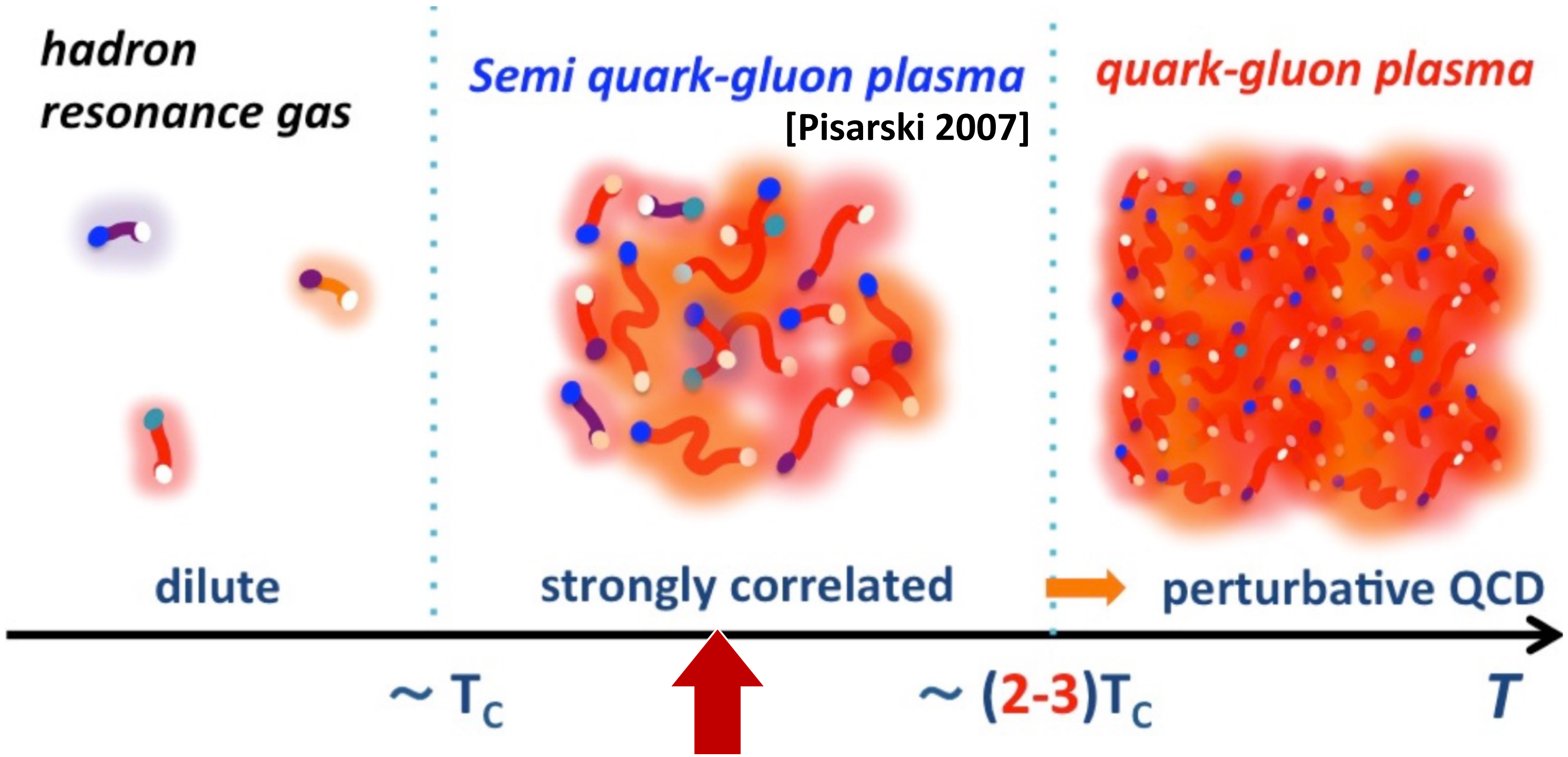
HRG

(theory options : **crossover, 1st order P.T.,** etc.)

picture reasonably consistent with **lattice & exp. data**

Plausible picture for HOT QCD

Fig. from Baym et al. 2018



new state of matter

rather than a mixed state of HRG & pQCD gas

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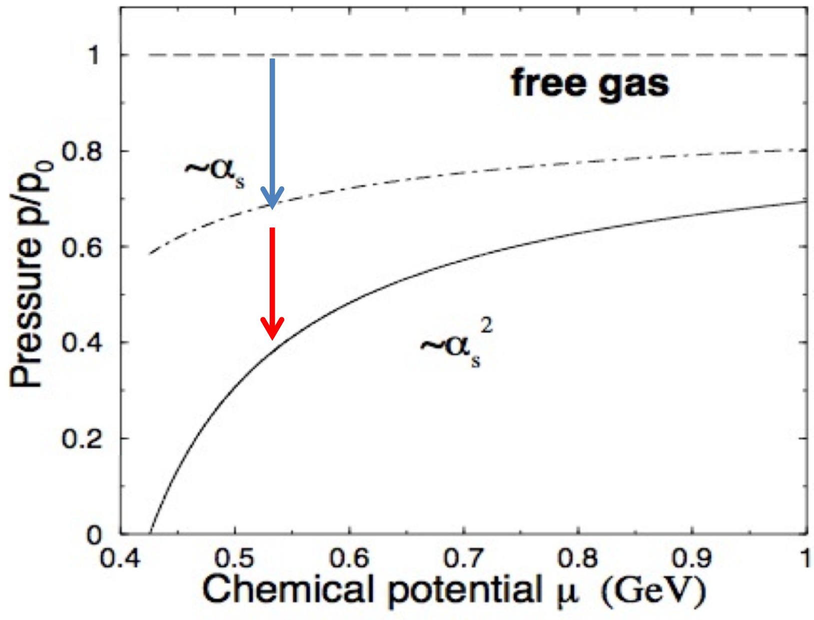
Cold, dense EoS : High density

3-loop pQCD : Freedman-McLerran 78; Baluni 78; Kurkela-Romatschke-Vuorinen 09

[some **4-loop** contributions: E. Sappi et al.]

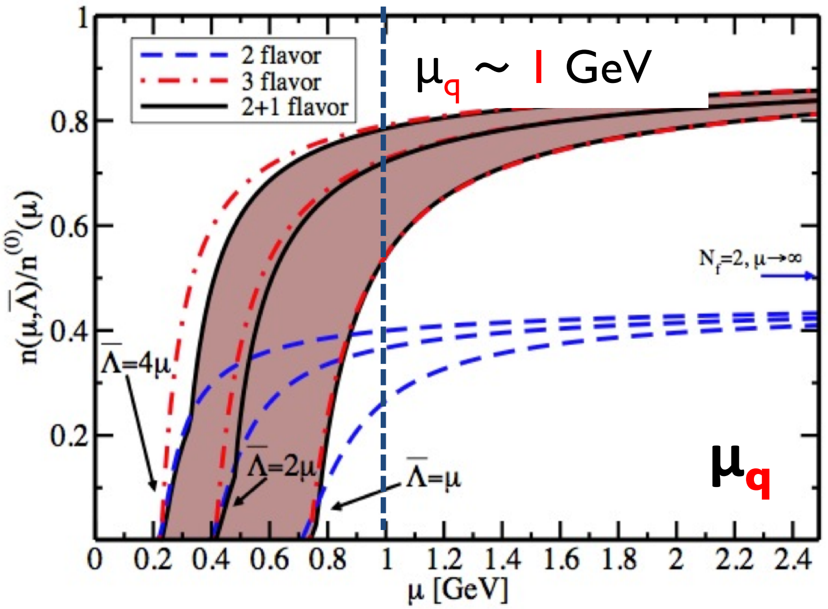
check of convergence

(Fraga-Pisarski-Schaffner-Bielich 01)



check of renorm. scale dep.

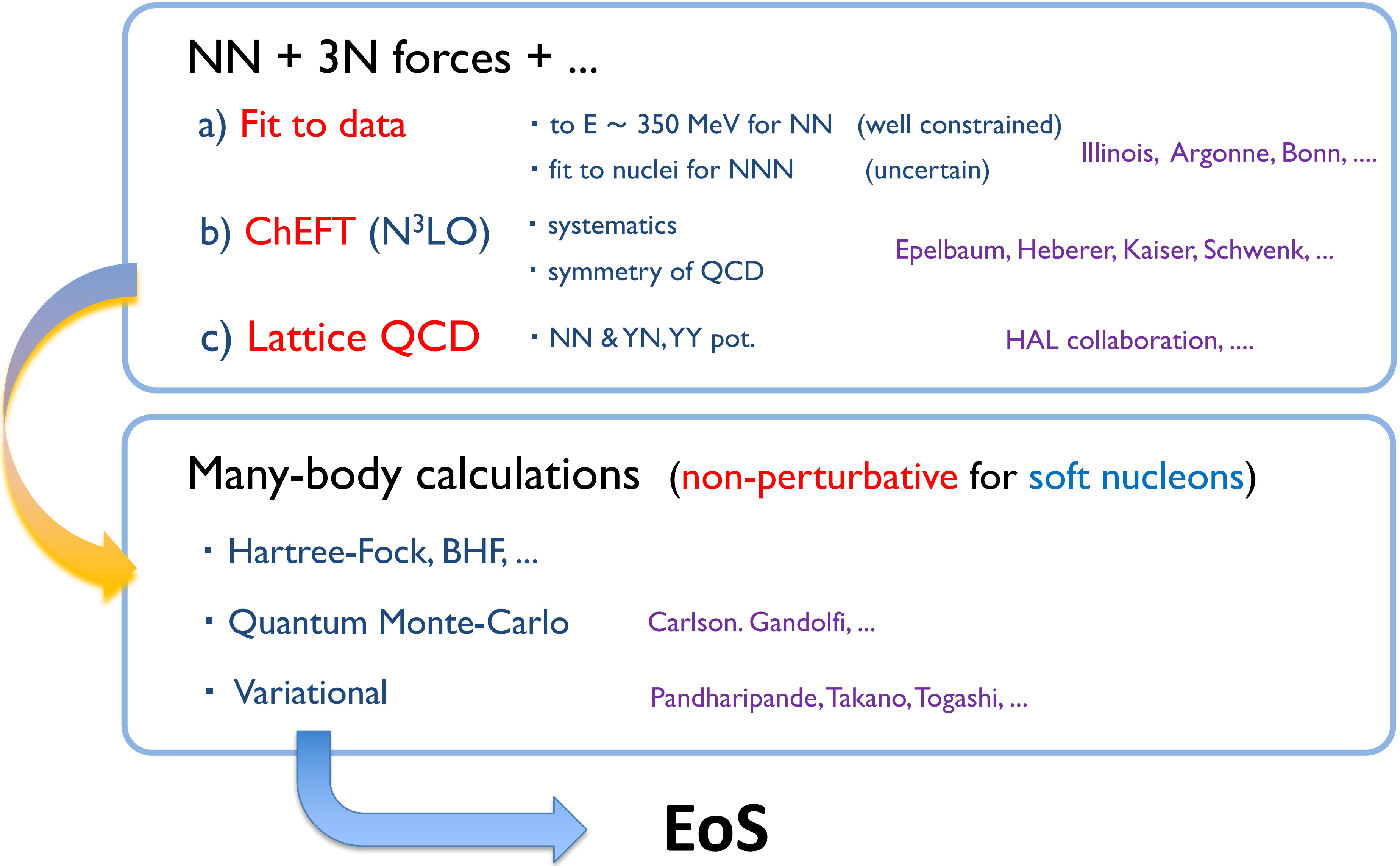
(Kurkela-Romatschke-Vuorinen 09)



- Interactions crucial for $\mu_q < \sim$ **1** GeV or $n_B < \sim$ **50**n_0
- Hints for effective **repulsion** (more μ needed to reach n_{ideal})

Cold, dense EoS : Low density

calculations based on microscopic interactions



NN + 3N forces + ...

a) Fit to data
- to E ~ 350 MeV for NN (well constrained)
- fit to nuclei for NNN (uncertain)

Illinois, Argonne, Bonn, ...

b) ChEFT (N^{3}LO)
- systematics
- symmetry of QCD

Epelbaum, Heberer, Kaiser, Schwenk, ...

c) Lattice QCD
- NN & YN, YY pot.

HAL collaboration, ...

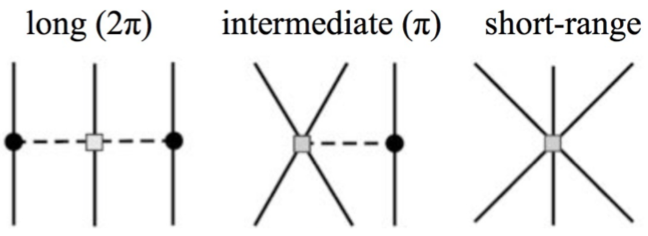
Many-body calculations (non-perturbative for soft nucleons)

- Hartree-Fock, BHF, ...
- Quantum Monte-Carlo — Carlson. Gandolfi, ...
- Variational — Pandharipande, Takano, Togashi, ...

EoS

Cold, dense EoS : Low density

- short range part of 3N forces is uncertain

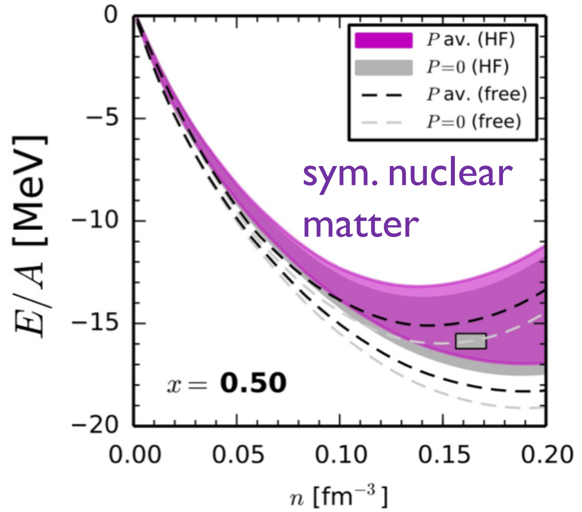


known

uncertain

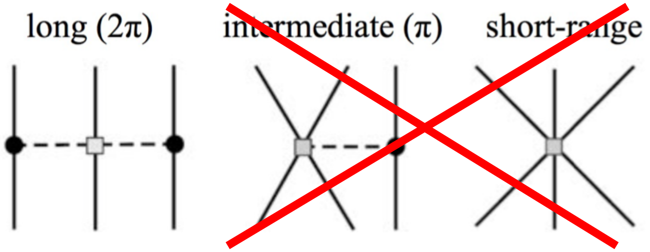
many-body cal.

Drischler-Hebeler-Schwenk, 2016



- pure neutron matter is less uncertain:

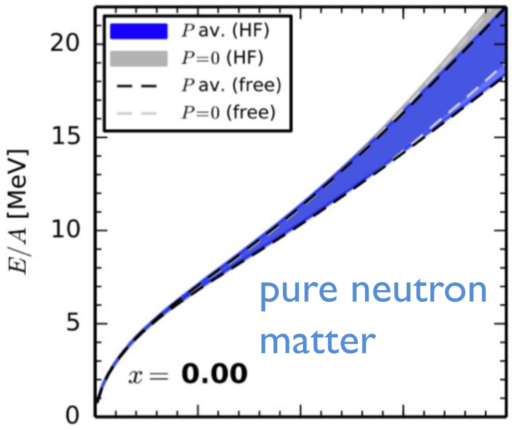
(Good for NS community)



known

forbidden

many-body cal.



microscopic calculations at n_B = 1-2 n_0 : consistent with empirical facts

Cold, dense EoS : Low density

For NS applications (n_B=1-10n_0), the fundamental question is:

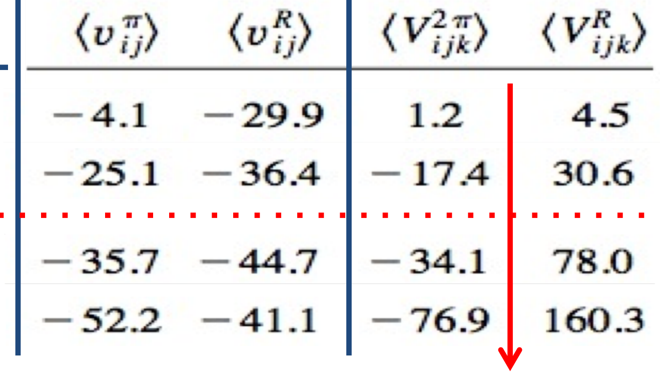
convergence of **many-body forces**

e.g.1) parameterized **pure neutron** matter EoS [Gandolfi+, 2009]

~kin. + 2-body ~3-body

$$\varepsilon = n_0 \left[(12 \pm 1\,\mathrm{MeV}) \left(\frac{n_B}{n_0}\right)^{1.45\pm0.05} + (4 \pm 2\,\mathrm{MeV}) \left(\frac{n_B}{n_0}\right)^{3.3\pm0.3} \right]$$

e.g.2) Akmal-Pandharipande-Ravenhall EoS (**APR 98**) [Table V of APR paper]



pure neutron matter

n_B	2 –body int.		3 –body int.	
	$\langle v_{ij}^{\pi} \rangle$	$\langle v_{ij}^{R} \rangle$	$\langle V_{ijk}^{2\pi} \rangle$	$\langle V_{ijk}^{R} \rangle$
n_0	-4.1	-29.9	1.2	4.5
2 n_0	-25.1	-36.4	-17.4	30.6
3 n_0	-35.7	-44.7	-34.1	78.0
4 n_0	-52.2	-41.1	-76.9	160.3

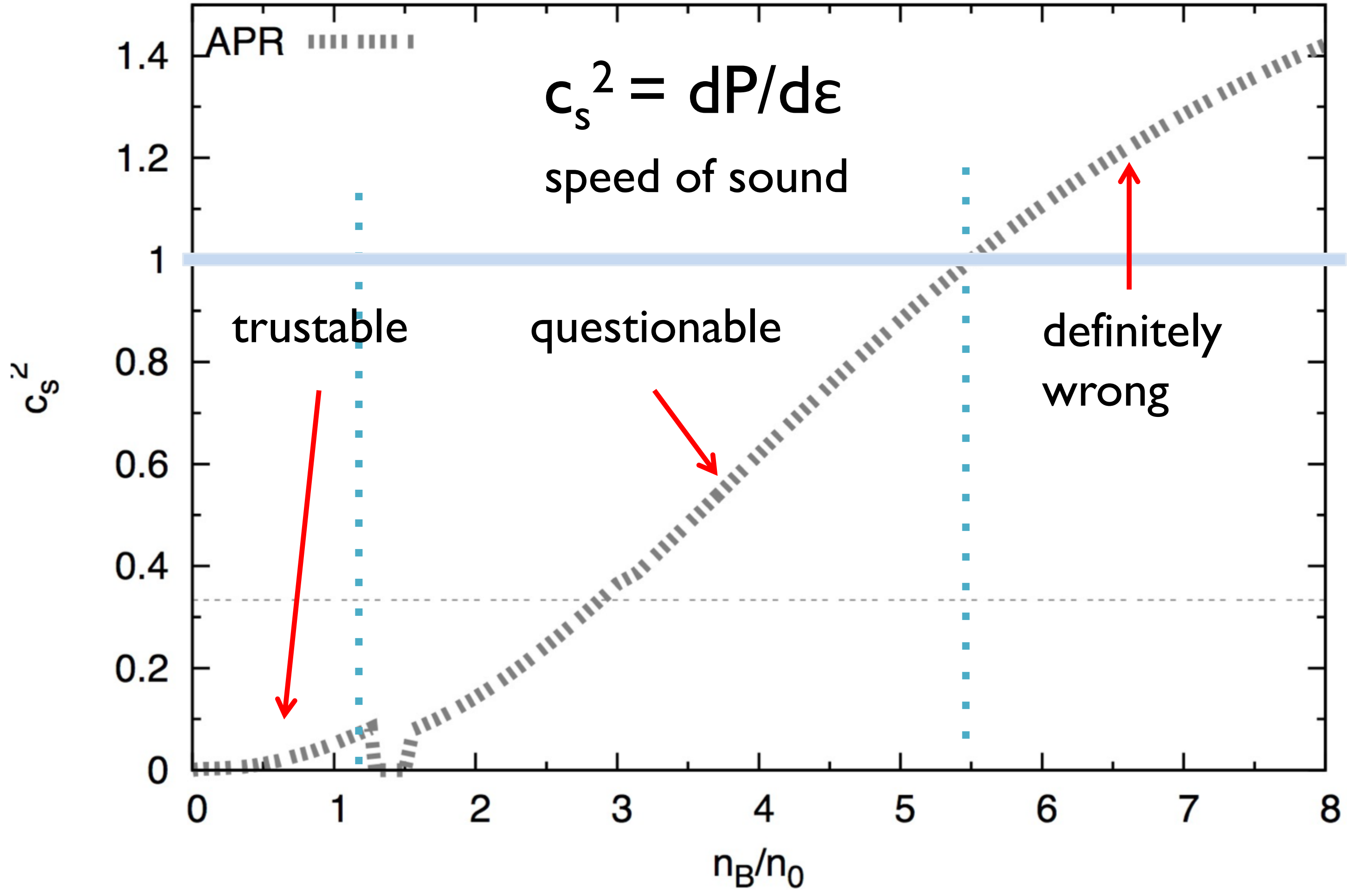
grow rapidly!

4-, 5- or more-body forces should be important as well beyond ~ **$2n_0$**

$$\langle V_{N\text{-body}} \rangle \sim (n_B/n_0)^N$$

Cold, dense EoS : Low density

Akmal-Pandharipande-Ravenhall EoS (**APR 98**)



$c_s{}^2 = dP/d\varepsilon$

speed of sound

trustable

questionable

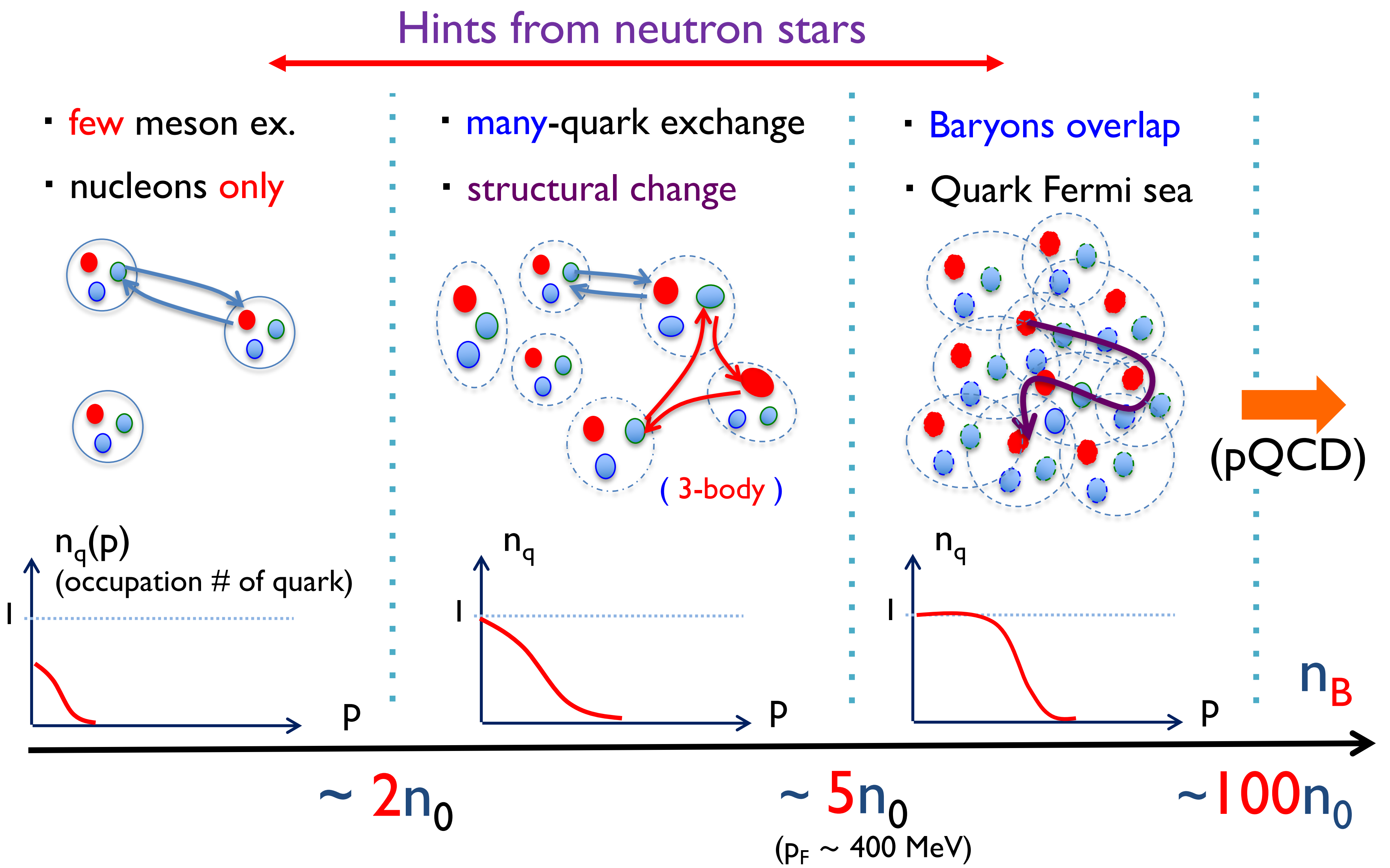
definitely wrong

light velocity

ideal gas

Picture to be developed

Masuda-Hatsuda-Takatsuka 2012
TK-Powell-Song-Baym 2014



Hints from neutron stars

- few meson ex.
- nucleons only

- many-quark exchange
- structural change

- Baryons overlap
- Quark Fermi sea

(3-body)

(pQCD)

$n_q(p)$
(occupation # of quark)

$\sim 2n_0$

$\sim 5n_0$
($p_F \sim 400$ MeV)

$\sim 100n_0$

Plan of lectures

1, Lessons from hot QCD: how it works

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3, NS constraints on EoS : hints for **soft-stiff** EoS

……

4, Crossover scenario: chiral restorations, etc.

5, A quark model : delineating the properties of matter

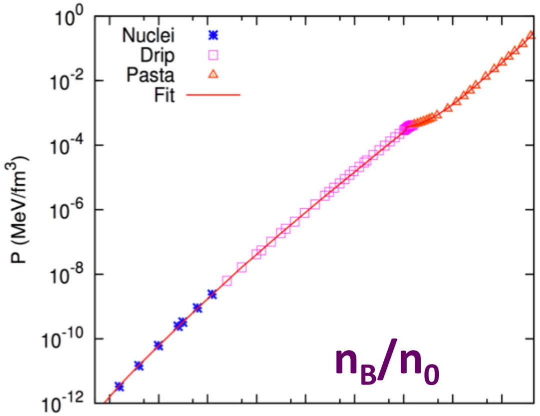
6, The astrophysical results from QHC18

……

7-, Other topics: warm EoS, beyond-MF, etc.

EoS & M-R relation

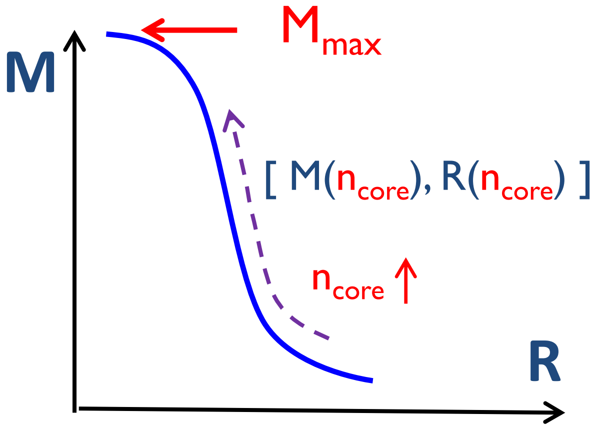
Einstein eq. : $$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$ ······· QCD EoS



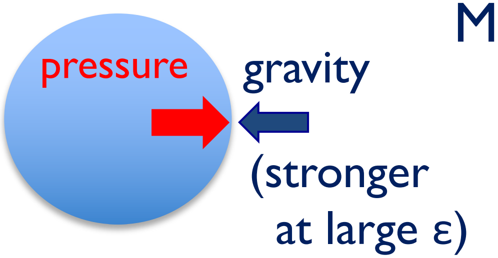
[for spherical NS → TOV eq.]

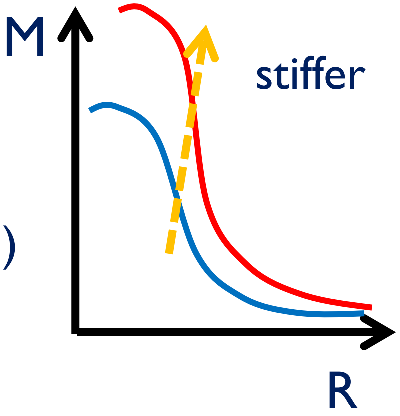
1-to-1 correspondence

Lindblom (1992)



Terminology (my convention)



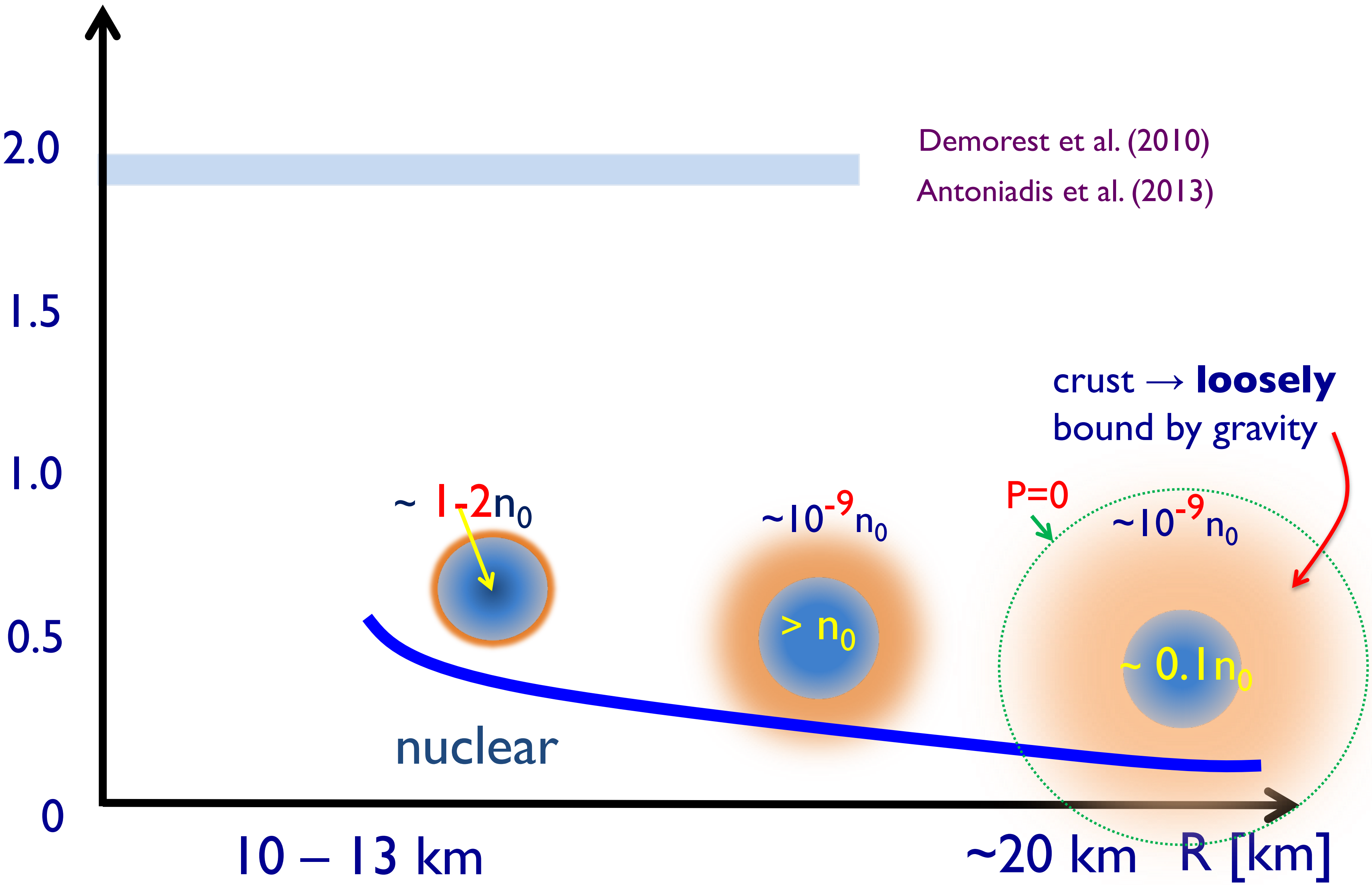


1) **Stiff** EoS : P is large at given ε

2) **Soft-Stiff** EoS : Soft at $n_B < \mathbf{2} n_0$ & Stiff at $n_B > \mathbf{5} n_0$

M-R relation & baryon density

Ref) Lattimer & Prakash (2001)



Demorest et al. (2010)

Antoniadis et al. (2013)

crust → **loosely** bound by gravity

M-R relation & baryon density

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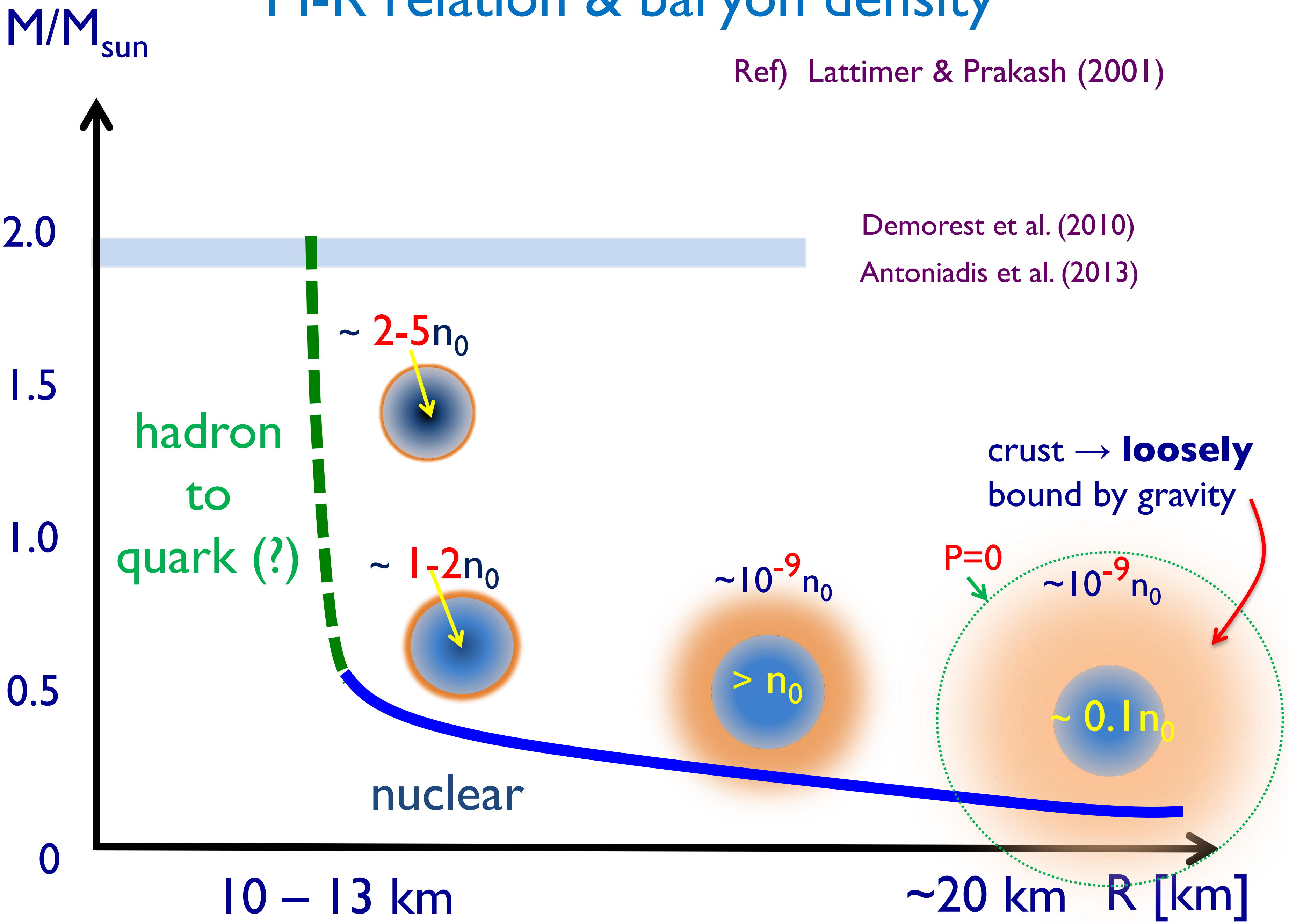

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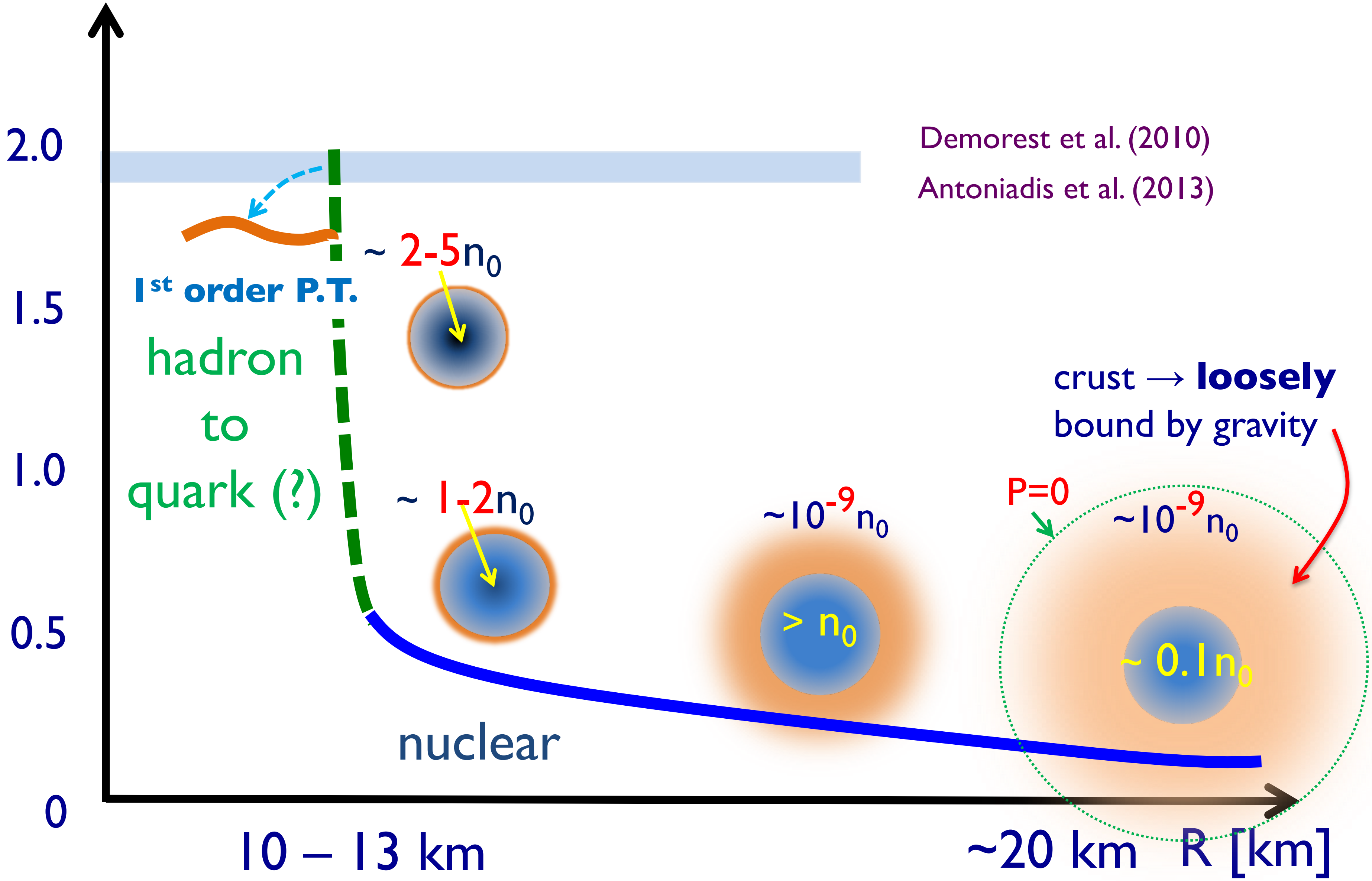
M/M_{sun}



Demorest et al. (2010)

Antoniadis et al. (2013)

~ 2-5n_0

1st order P.T.

hadron
to
quark (?)

~ 1-2n_0

nuclear

$\sim 10^{-9} n_0$

$> n_0$

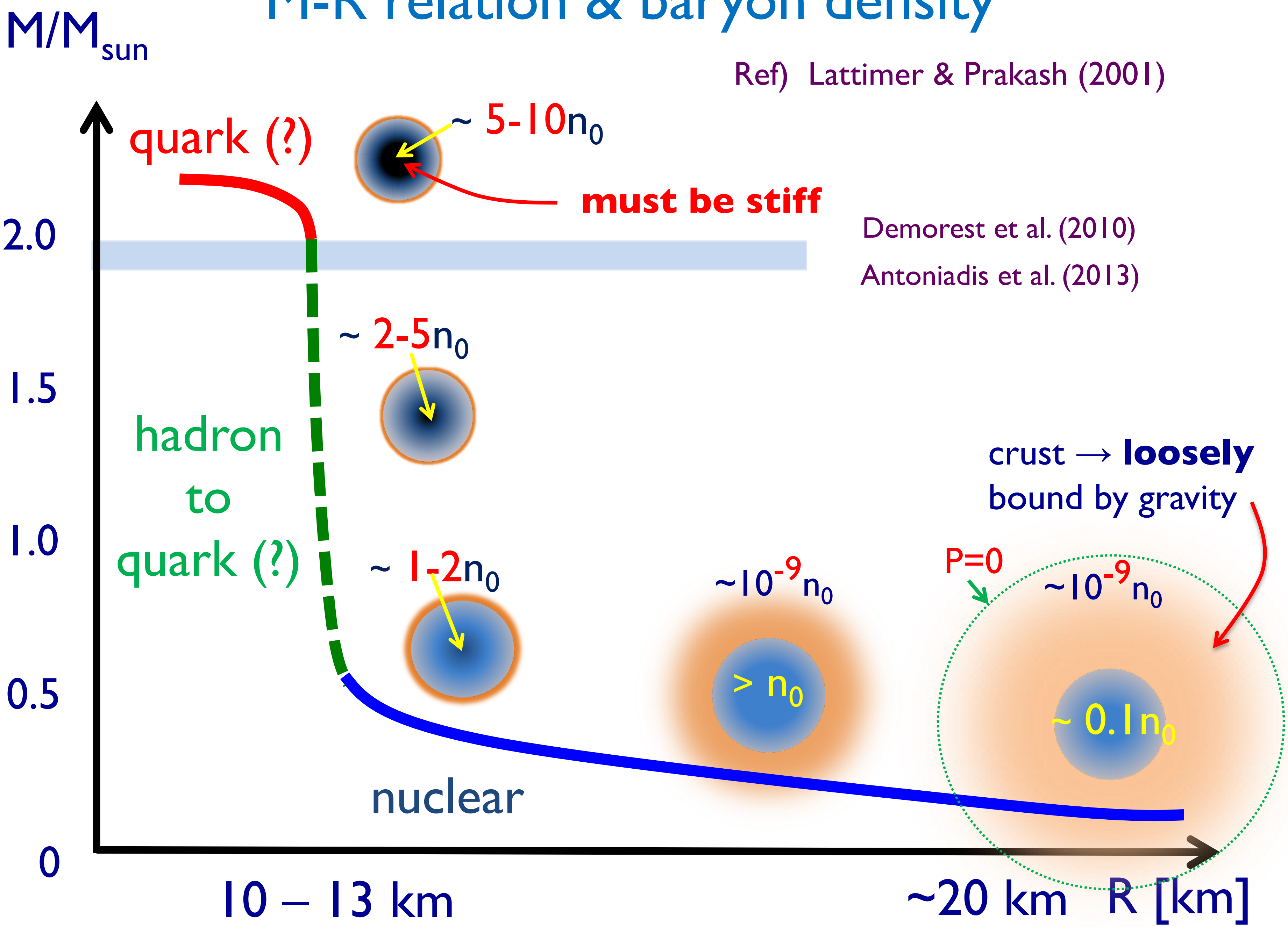
P=0

$\sim 10^{-9} n_0$

~ 0.1n_0

crust → **loosely** bound by gravity

M-R relation & baryon density



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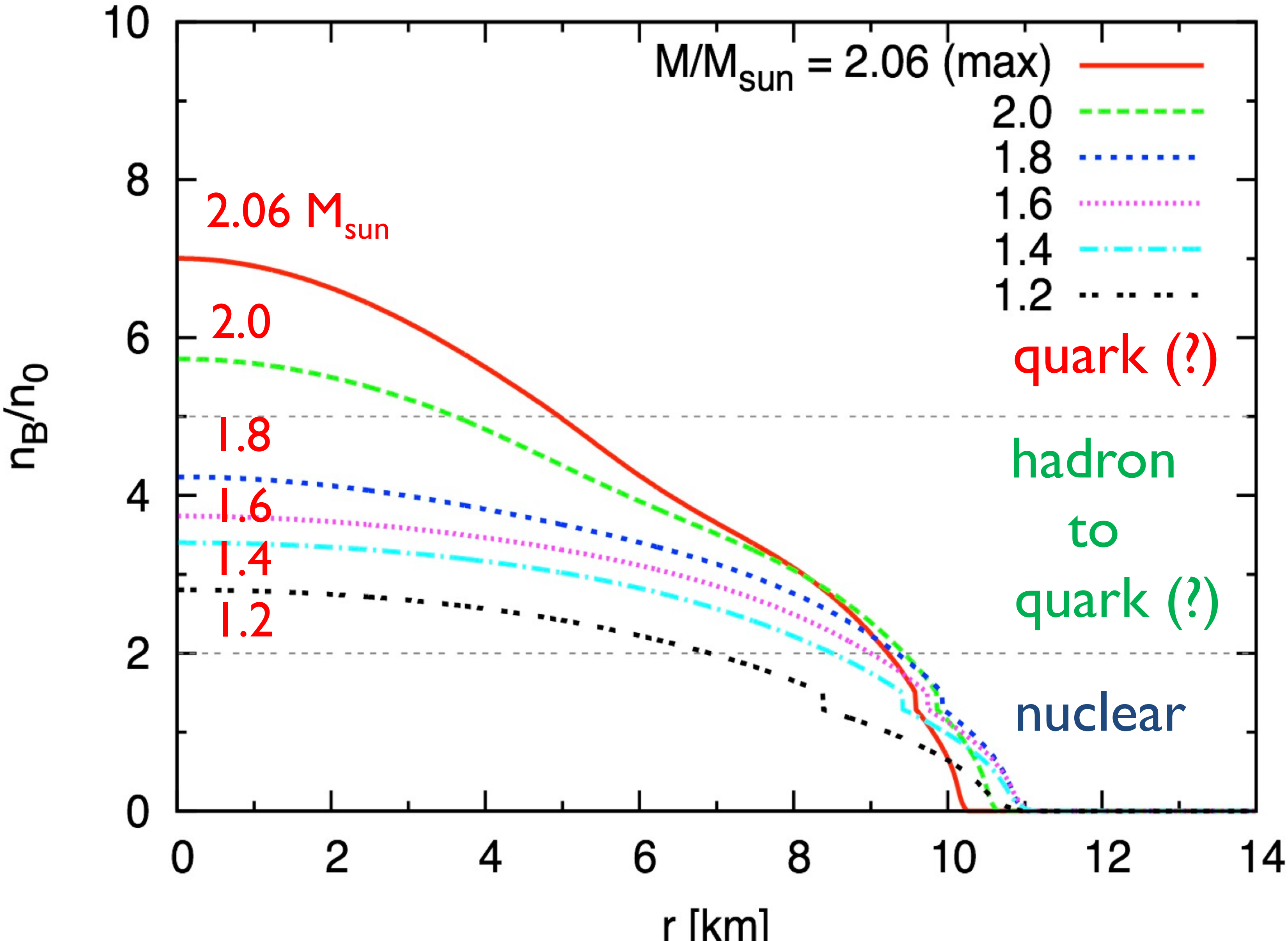
Baryon density in a neutron star (QHC18)



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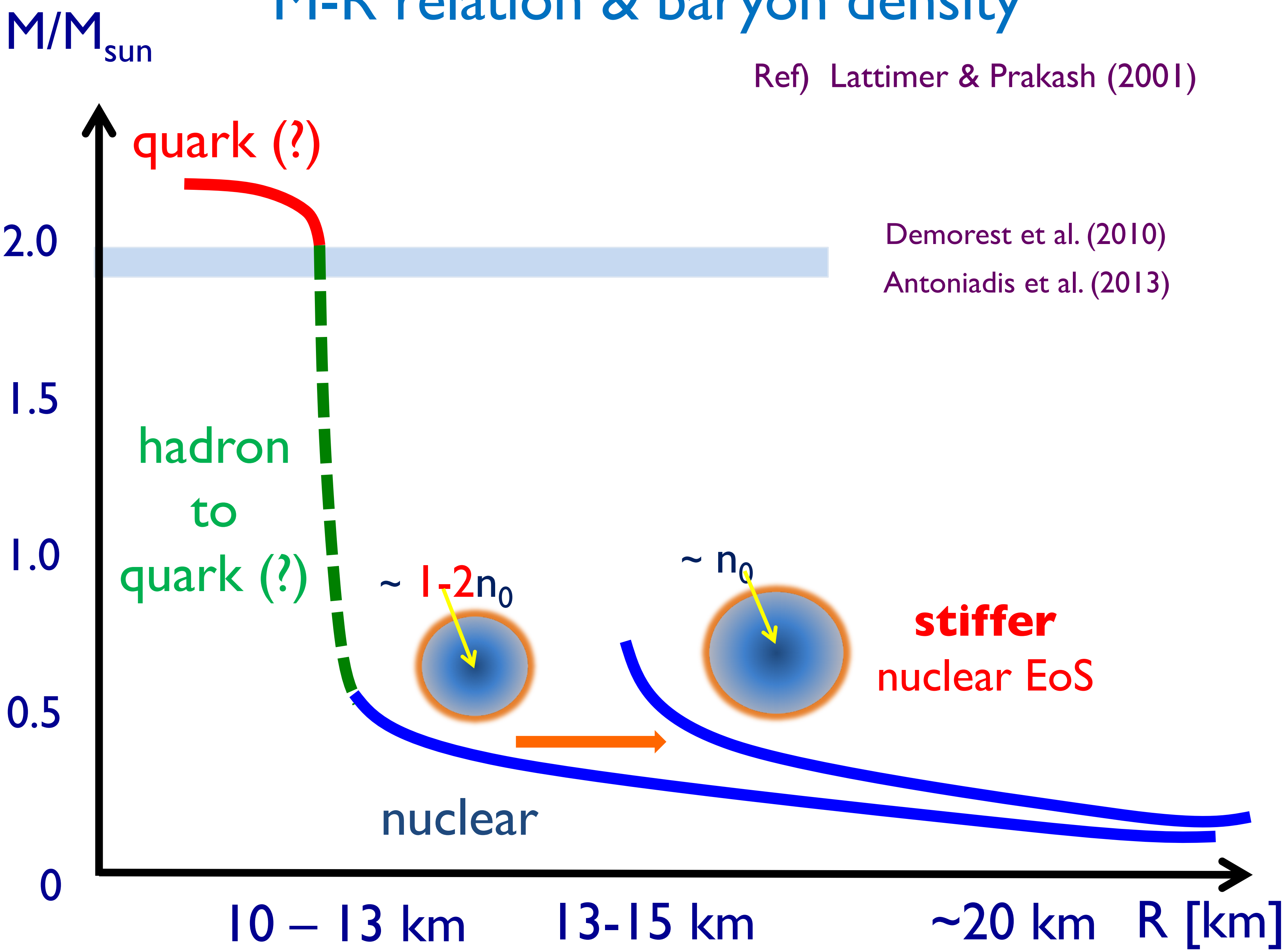

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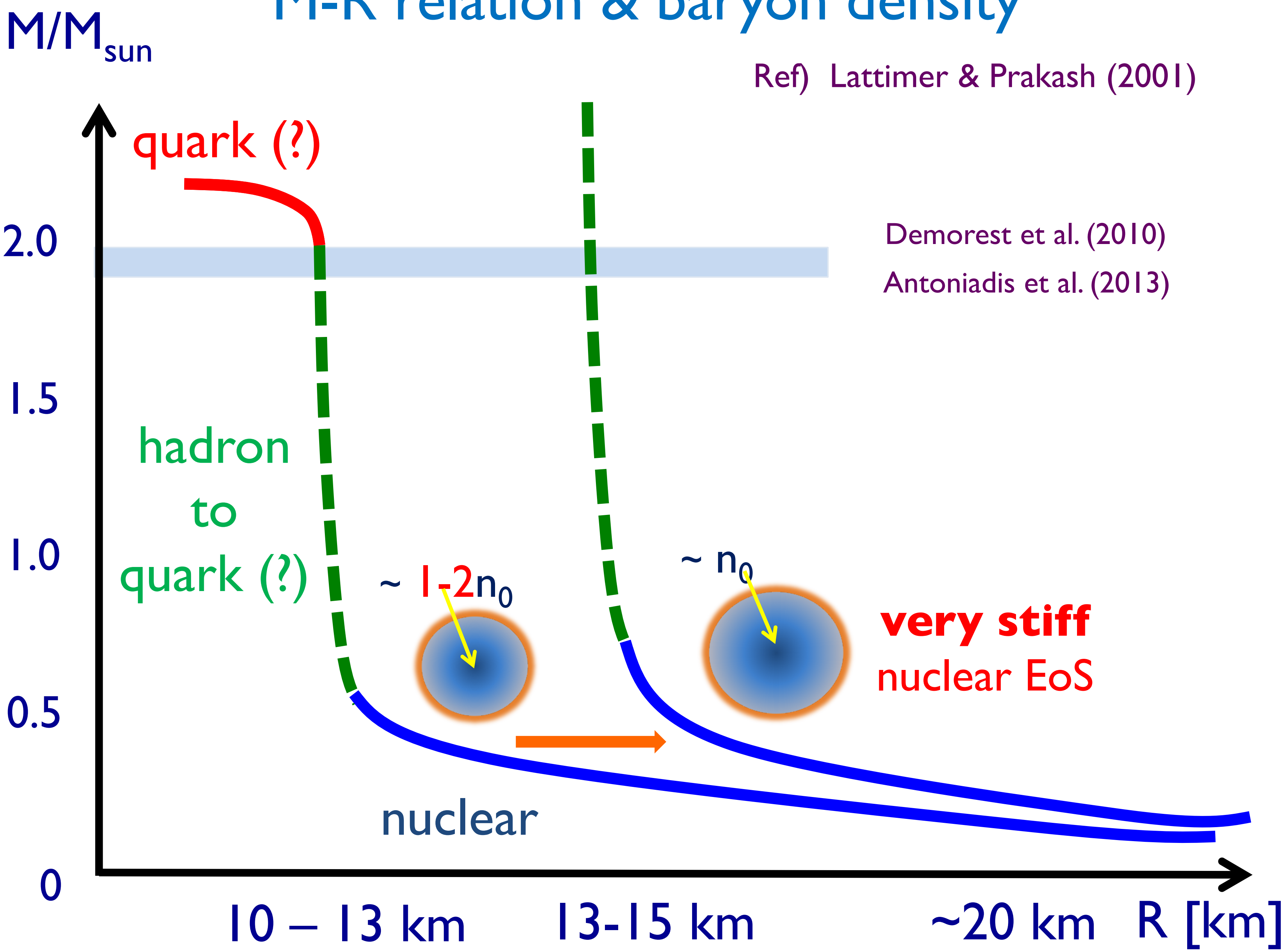

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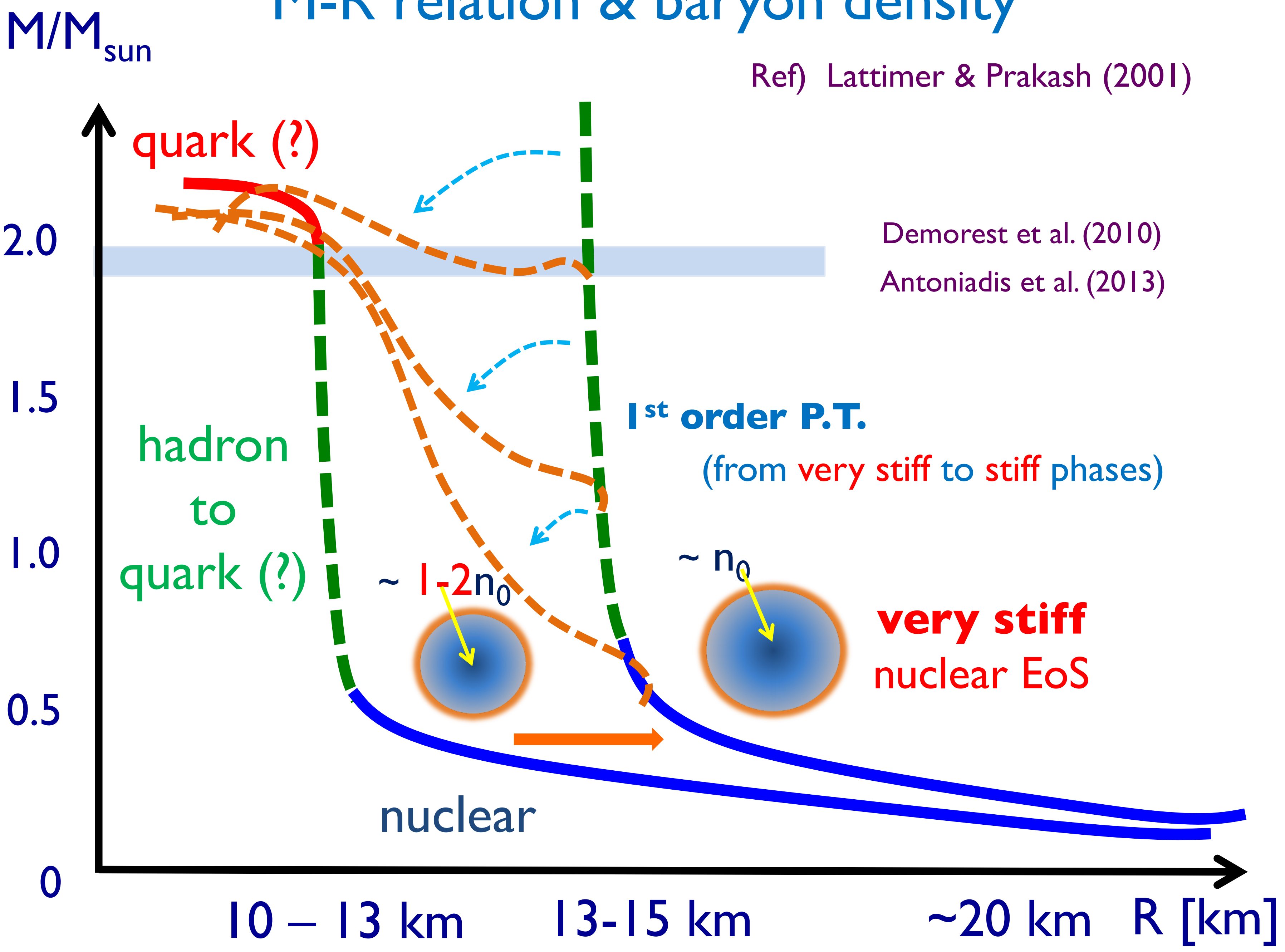


M-R re

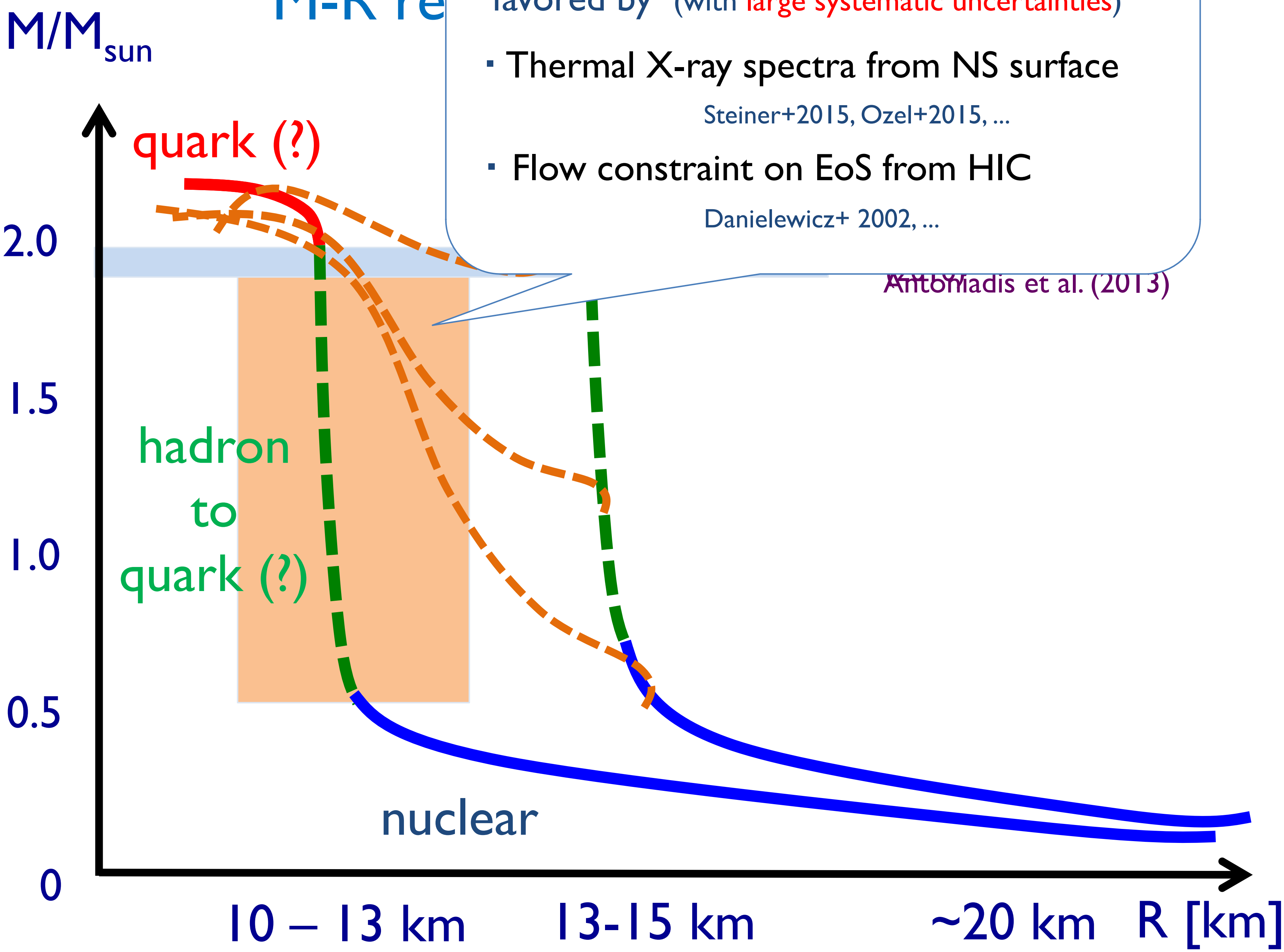
favored by (with large systematic uncertainties)

- Thermal X-ray spectra from NS surface

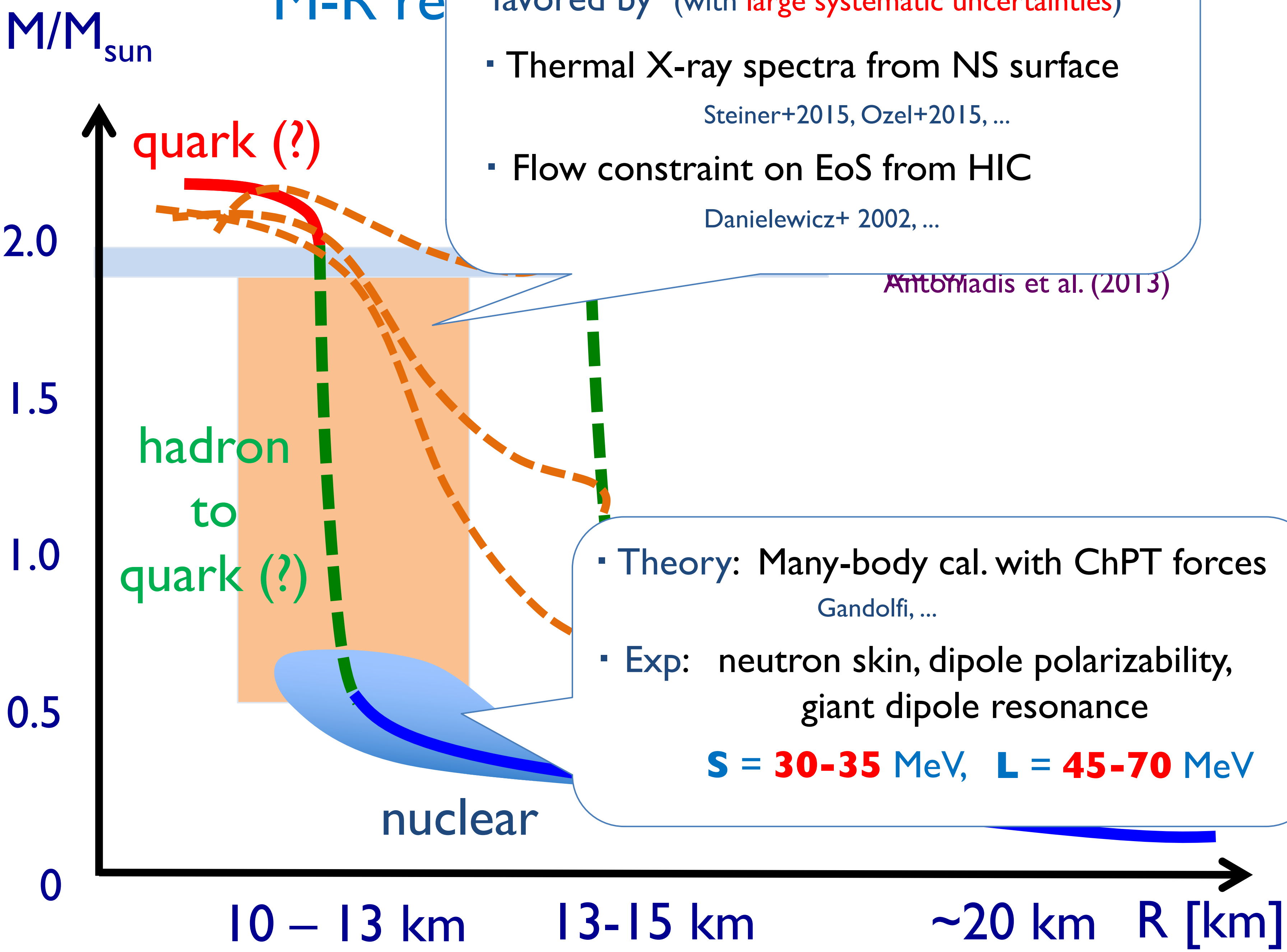
  Steiner+2015, Ozel+2015, ...

- Flow constraint on EoS from HIC

  Danielewicz+ 2002, ...



M-R re



favored by (with large systematic uncertainties)

- Thermal X-ray spectra from NS surface

  Steiner+2015, Ozel+2015, ...

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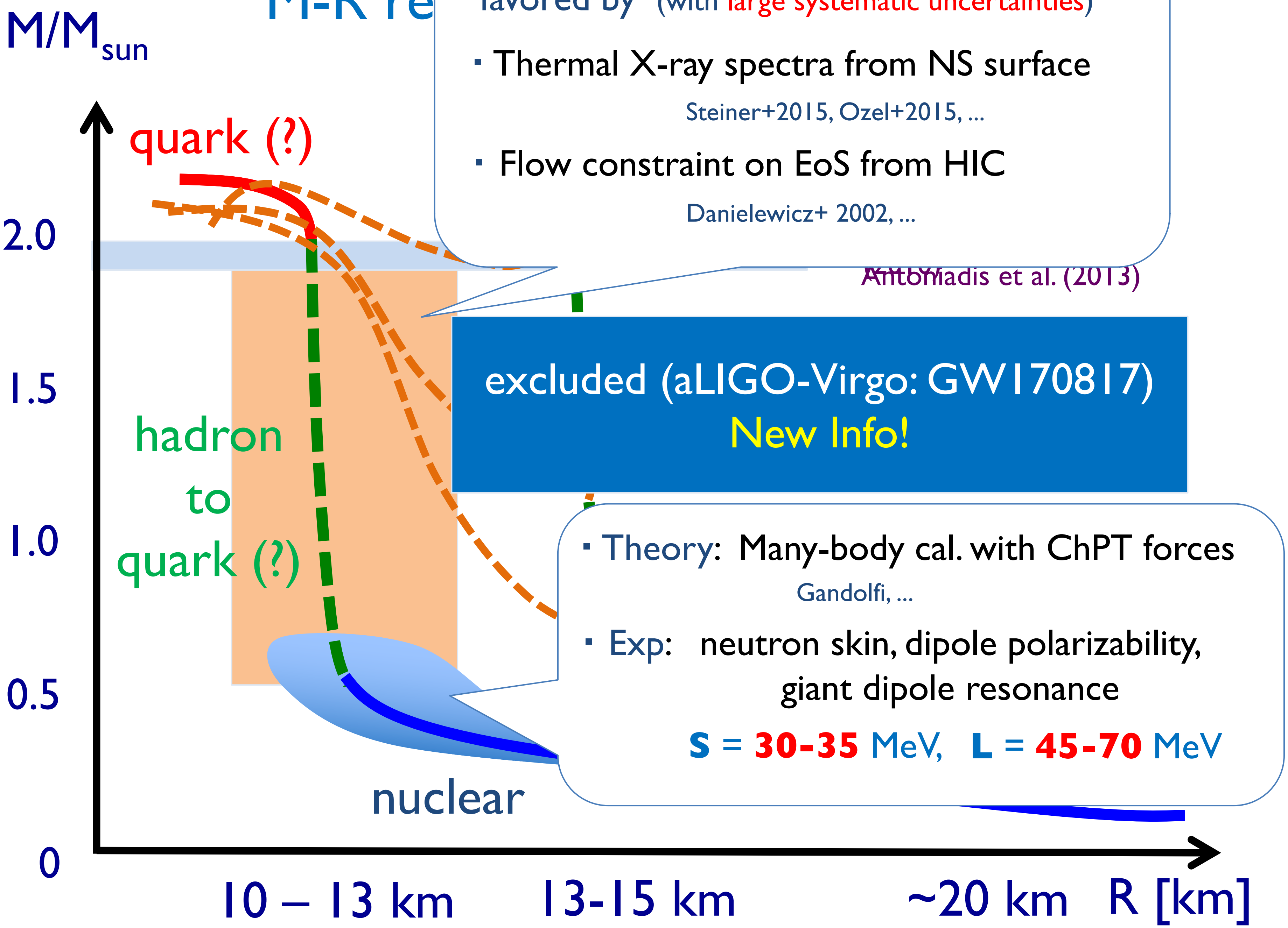
- Theory: Many-body cal. with ChPT forces

  Gandolfi, ...

- Exp: neutron skin, dipole polarizability, giant dipole resonance

  S = **30-35** MeV, **L** = **45-70** MeV

M-R re



favored by (with large systematic uncertainties)

- Thermal X-ray spectra from NS surface

Steiner+2015, Ozel+2015, ...

- Flow constraint on EoS from HIC

Danielewicz+ 2002, ...

excluded (aLIGO-Virgo: GW170817)
New Info!

- Theory: Many-body cal. with ChPT forces

Gandolfi, ...

- Exp: neutron skin, dipole polarizability, giant dipole resonance

S = **30-35** MeV, **L** = **45-70** MeV

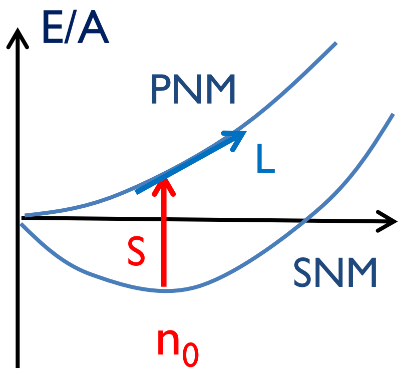
Hints for **soft** EoS at $n_B < 2n_0$

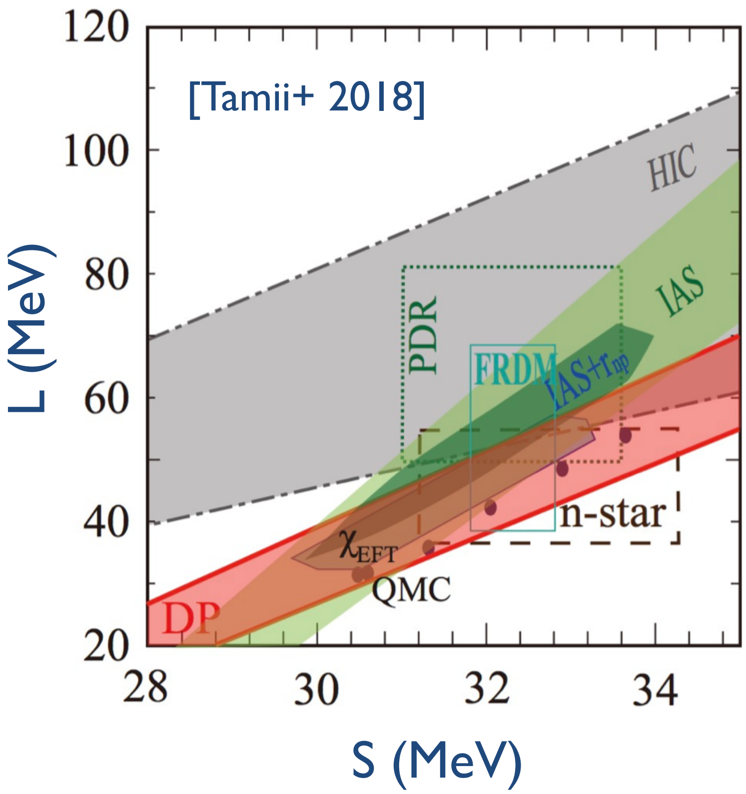
$$\frac{E}{A} = -16\,\mathrm{MeV} + S + \frac{L}{3}\,\frac{n-n_0}{n_0} + \cdots$$

pure neutron
matter EoS

sym. energy

density dep.





Theory · Many-body cal. with ChPT forces

Exp.
- Neutron skin
- Dipole polarizability
- Giant dipole resonance

T ~ 0 MeV

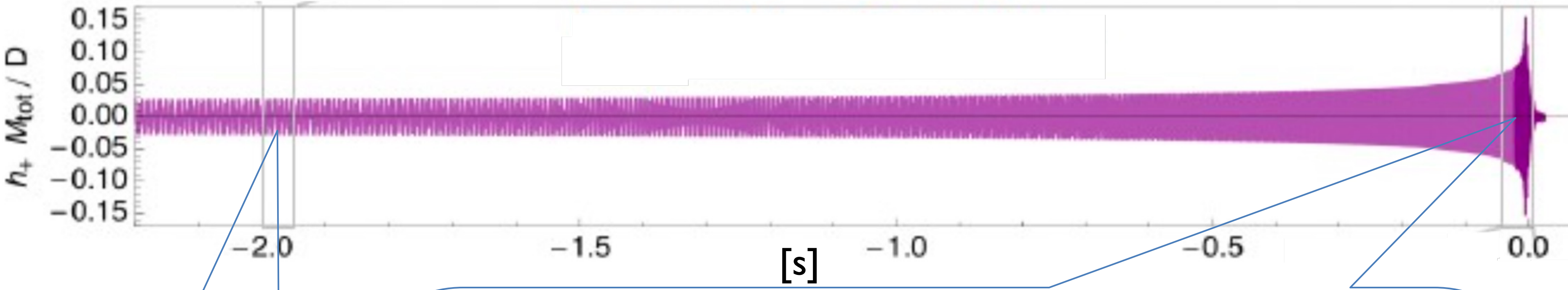
- Heavy ion (E_{lab}/A ~ 200 MeV)

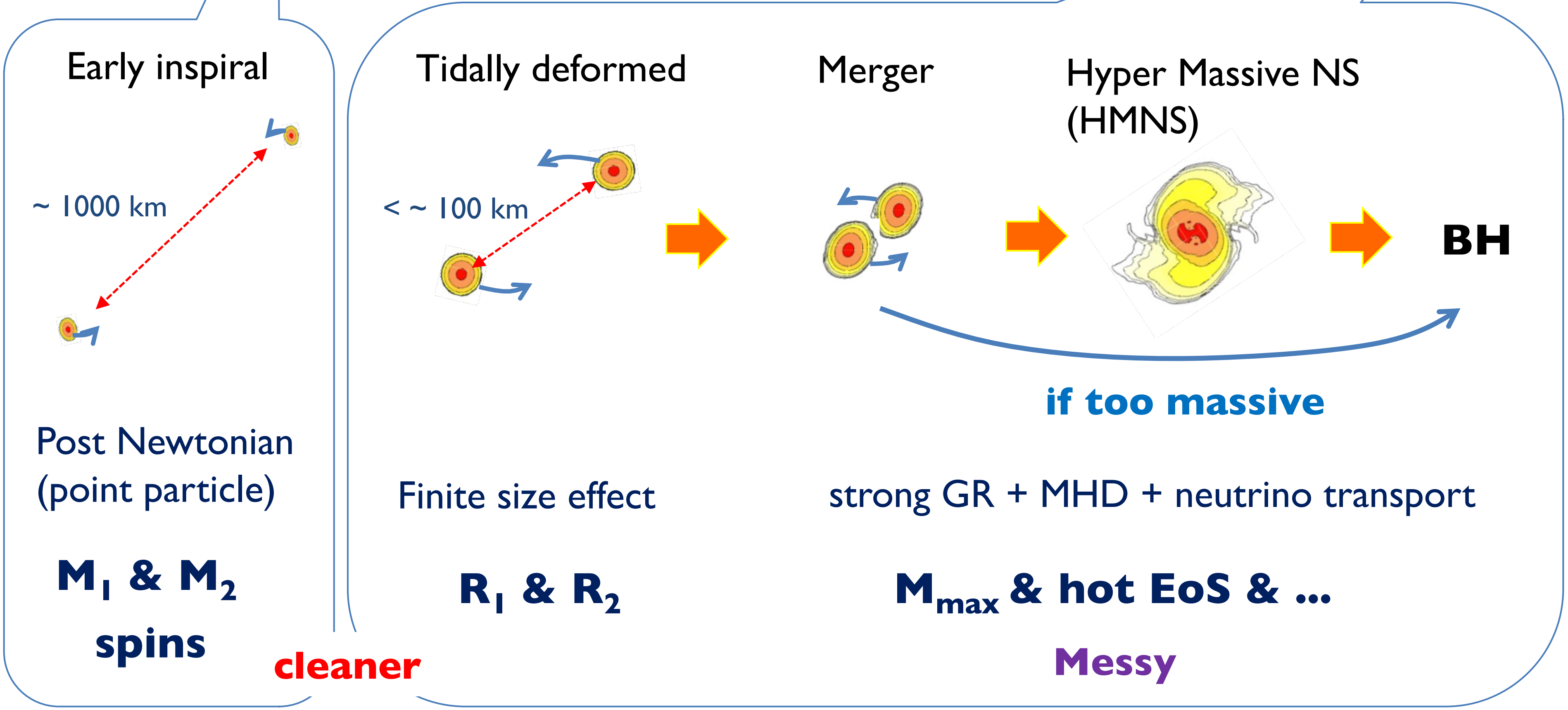
S = **30-35** MeV, **L** = **45-70** MeV

Then, EoS extrapolated to $2n_0$ leads to

$R_{1.4}$ = 11-13 km

GWs from NS-NS mergers





Early inspiral

~ 1000 km

Post Newtonian (point particle)

M_1 & M_2

spins

cleaner

Tidally deformed

< ~ 100 km

Merger

Hyper Massive NS (HMNS)

BH

if too massive

Finite size effect

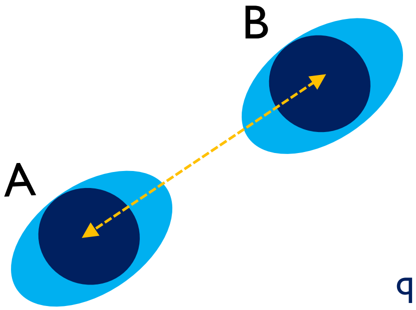
strong GR + MHD + neutrino transport

R_1 & R_2

M_{max} & hot EoS & ...

Messy

Tidal deformation → accelerated phase evolution



1) grav. fields from star B → the deformation of star A

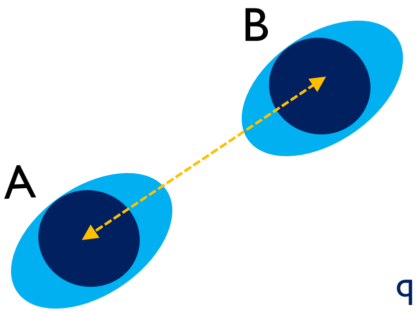
2) deformed energy density → quadrupole grav. fields

polarizability

quadrupole moment $Q_{ij} = -\lambda(M)E_{ij}$ external field

$$E_{ij} = -\frac{\partial^2 V}{\partial x_i \partial x_j}$$

Tidal deformation → accelerated phase evolution



1) grav. fields from star B → the deformation of star A
2) deformed energy density → quadrupole grav. fields

polarizability

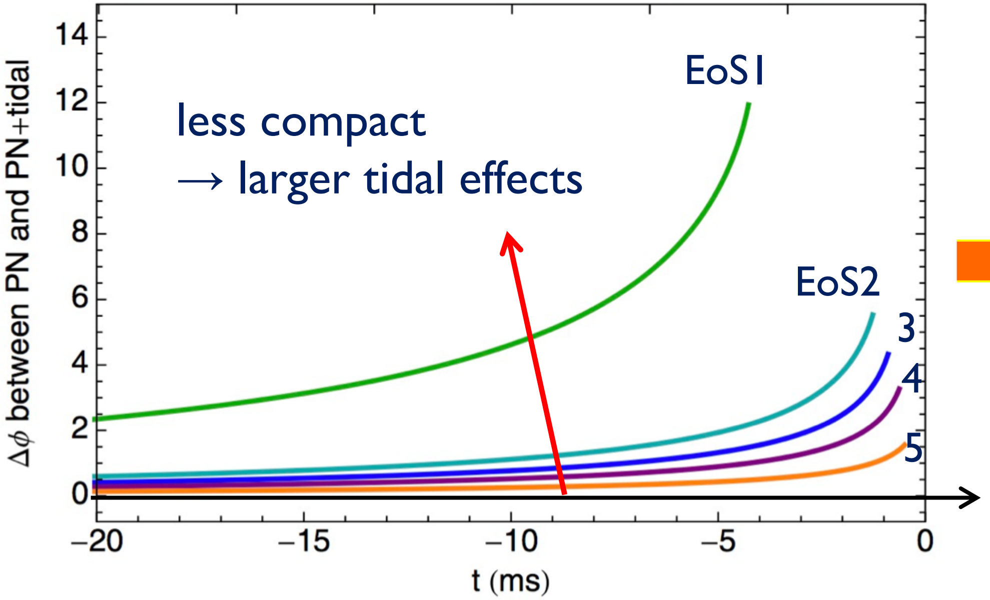
quadrupole moment $Q_{ij} = -\lambda(M) E_{ij}$ external field $E_{ij} = -\dfrac{\partial^2 V}{\partial x_i \partial x_j}$

gravitational pot. from the star A

$$V_A(r) \simeq -\frac{GM_A}{r} - \underline{\frac{GQ_{AB}}{r^3}} \simeq -\frac{GM_A}{r} - \underline{\frac{G}{r^3}\left(\frac{\lambda G M_B}{r^3}\right)}$$

attractive
→ acceleration

Read+ 2012

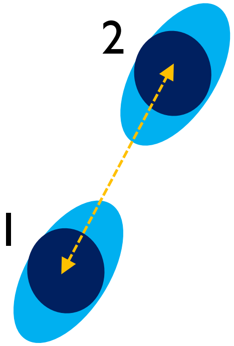


less compact
→ larger tidal effects

point particle

upperbound on **λ** & **R**

Dimensionless tidal deformability → R_{NS}



more common to use $$\Lambda(M) = 32\frac{\lambda G}{R^5}$$

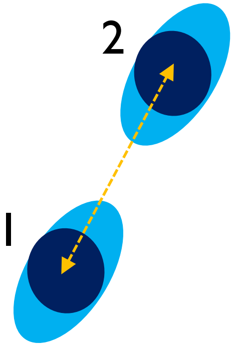
What GW analyses measure: combination of Λ for star 1 & 2 :

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$

(measured)

2-parameters: M_1 & M_2

Dimensionless tidal deformability → R_{NS}



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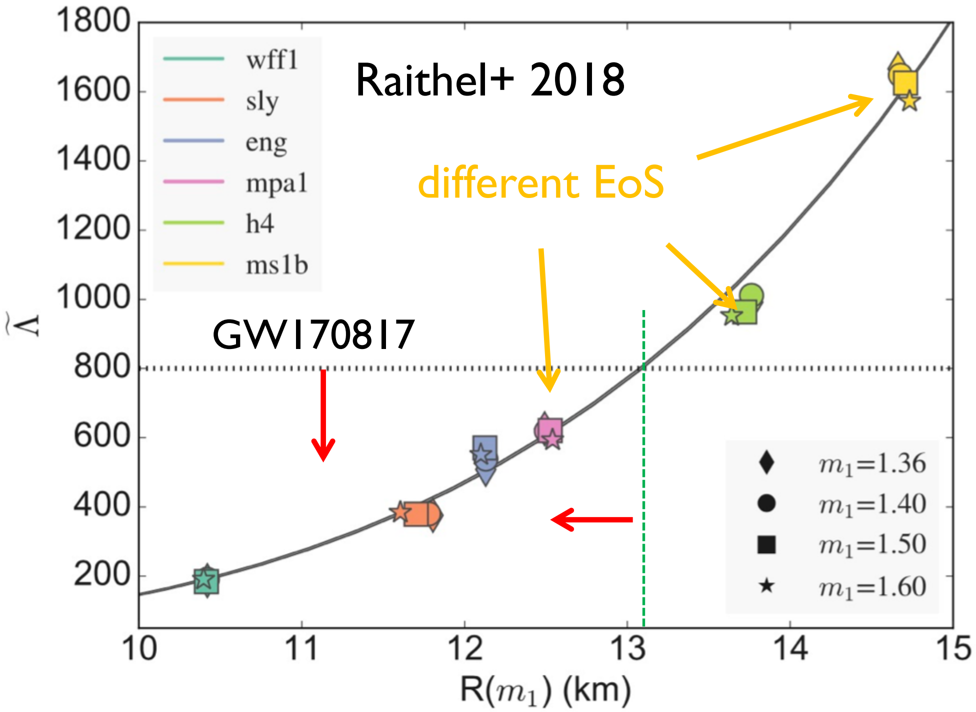


For **GW170817** :

chirp mass (1.188 M_{sun}) (determined)

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = m_1\frac{q^{3/5}}{(1+q)^{1/5}}$$

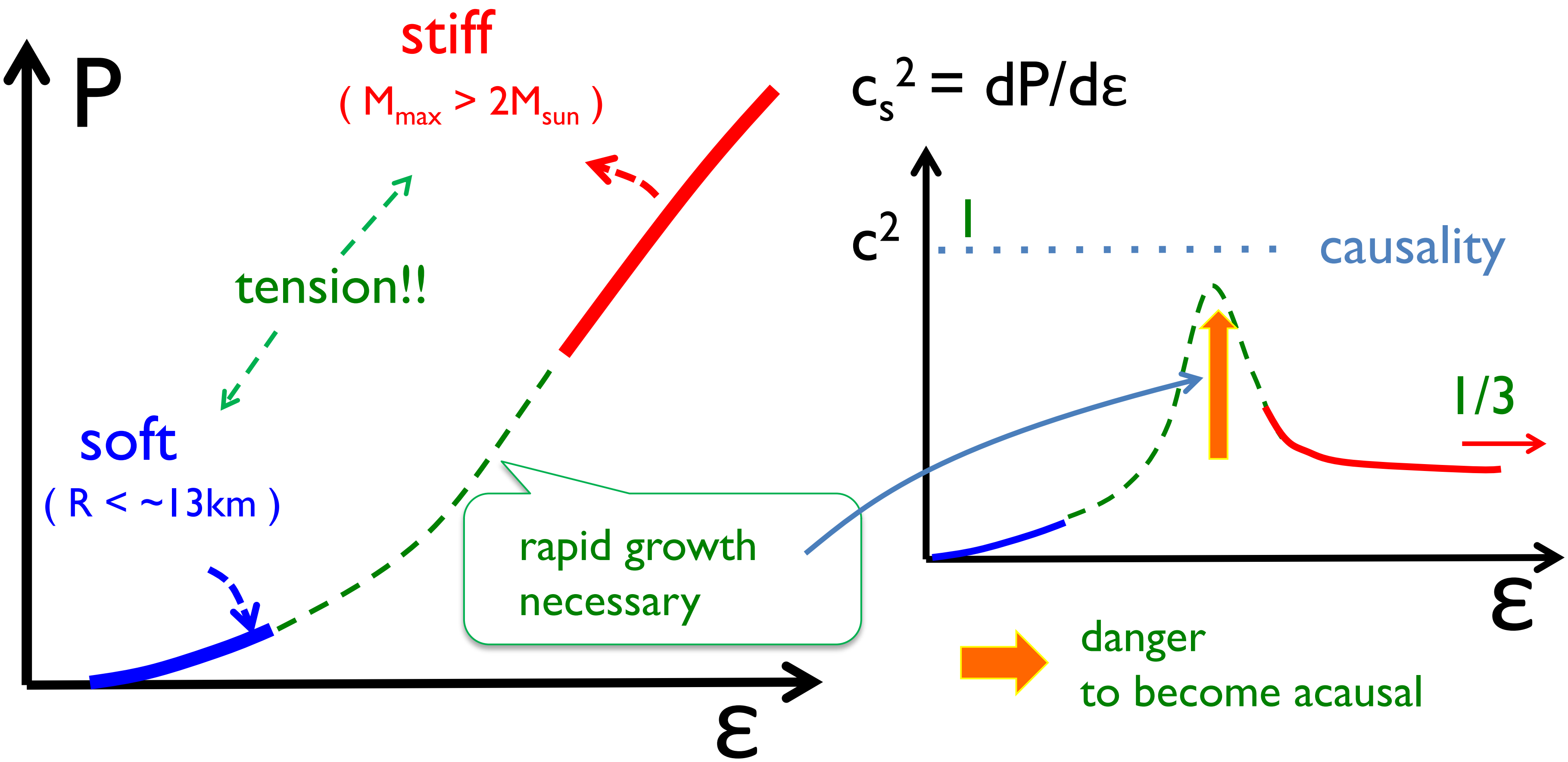
mass ratio

$q = M_2/M_1$ (undetermined)

- different q degenerate !
- R < ~ 13 km

Causality constraint on $2n_0$-$5n_0$ region

assume: R < 13km & $M_{max} > 2M_{sun}$

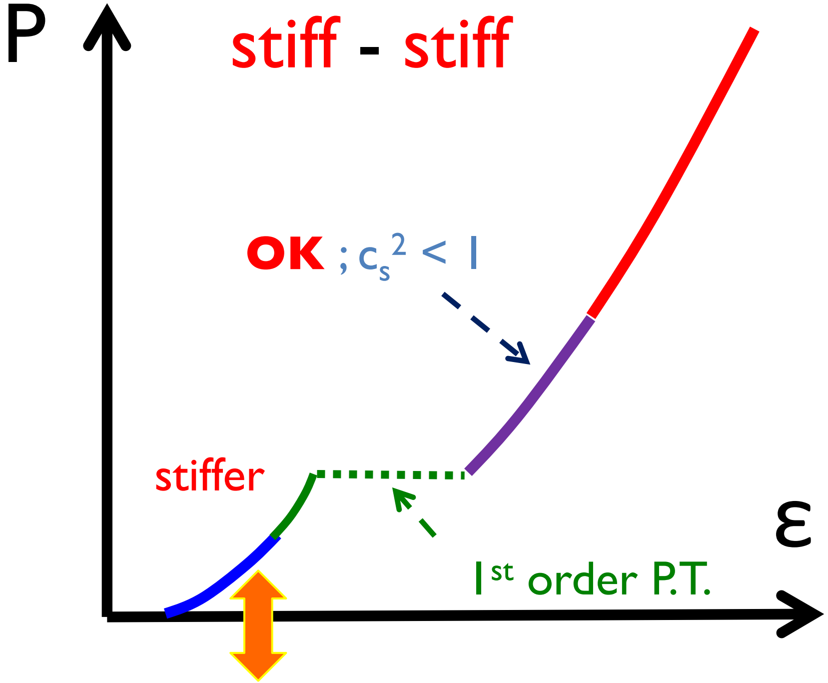


rapid growth
necessary

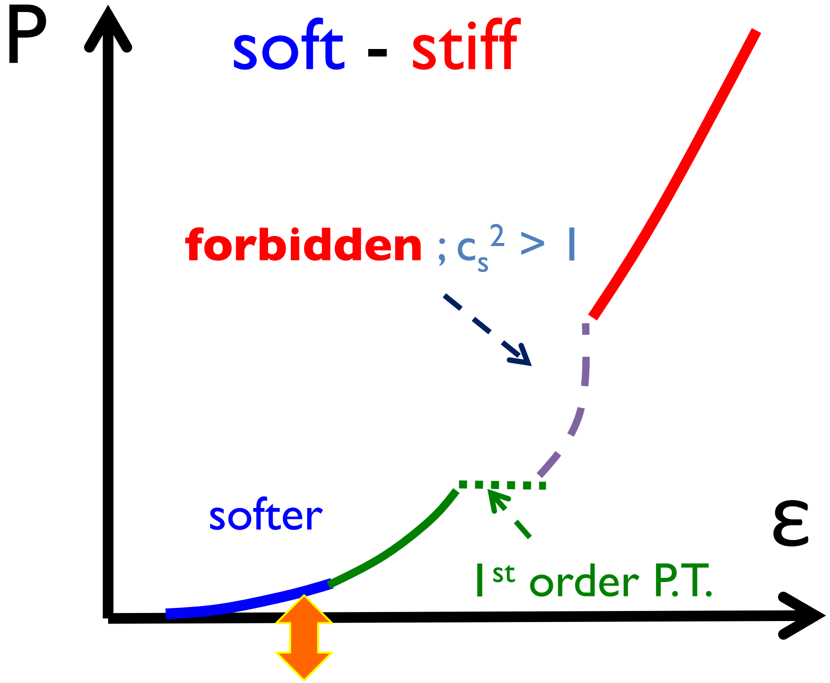
danger
to become acausal

Stiff-Stiff v.s. Soft-Stiff EoS

[more quantitative analyses → Han-Alford-Prakash 13]



larger R (> ~13 km)



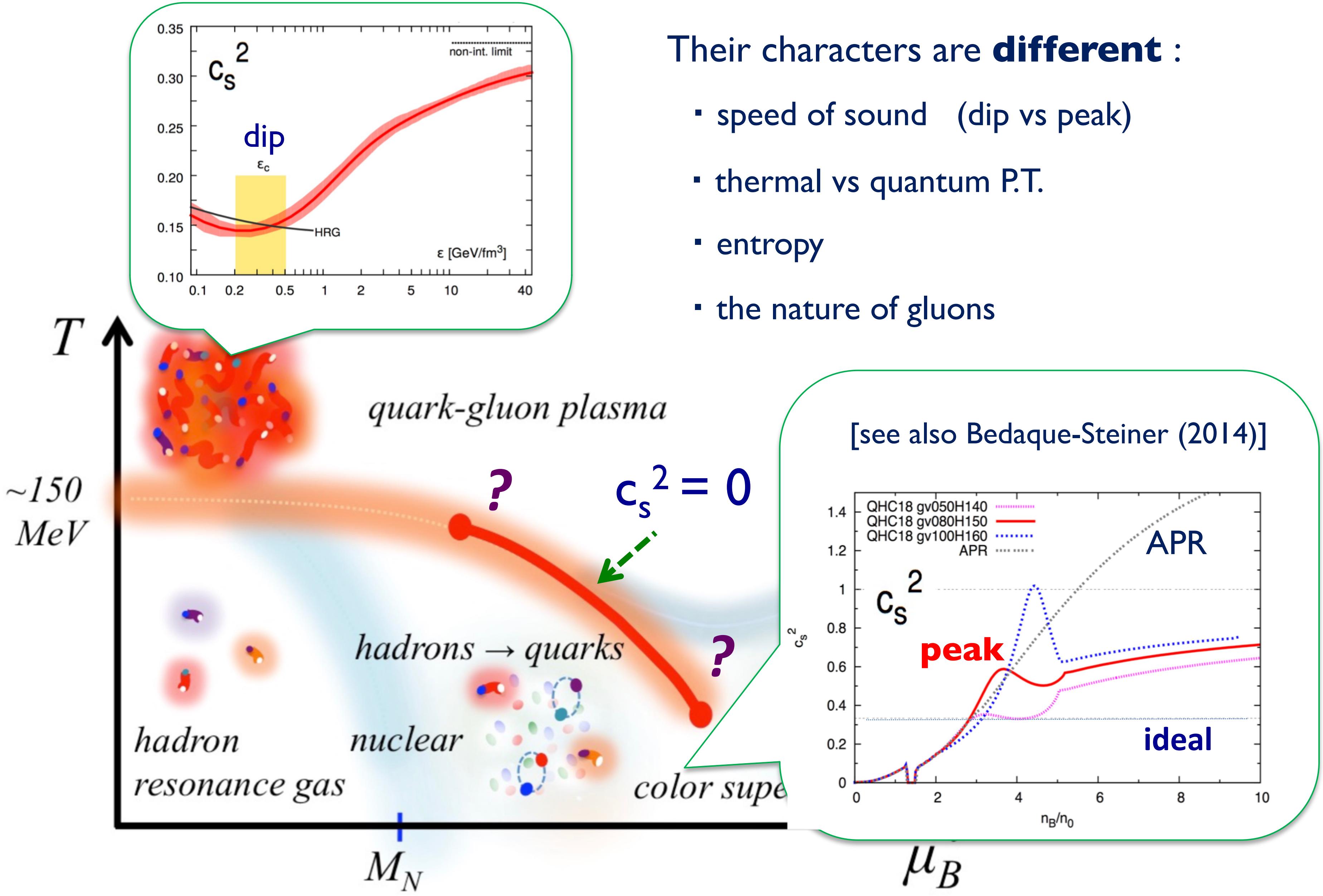
smaller R (< ~13 km)

→ we consider a **soft-stiff** EoS with **crossover** (or weak 1st order)

Finite T vs low T crossover

Their characters are **different** :

- speed of sound (dip vs peak)
- thermal vs quantum P.T.
- entropy
- the nature of gluons



[see also Bedaque-Steiner (2014)]

Summary of lecture 1

1, QCD has reliable high & low density limits,
but be careful in extrapolating these results:

interpolation of these limits are **much safer** procedure

2, 1st principle methods
→ the validity range of quasi-particle pictures

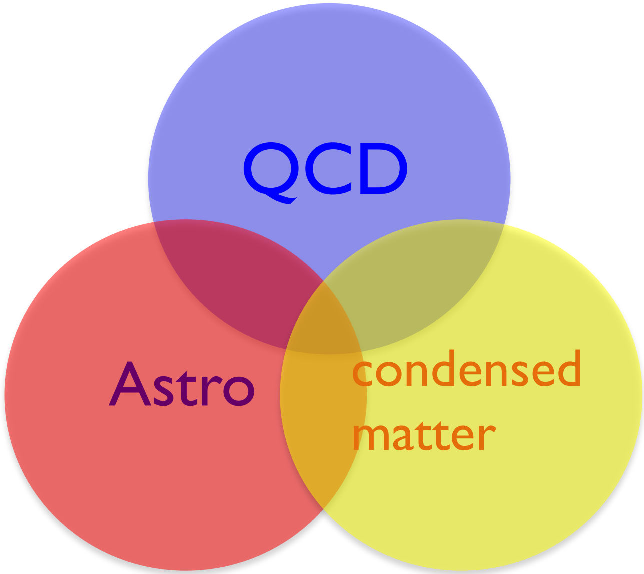
3, NS observations → Hints for **soft**-**stiff** EoS
\+ **causality**

→ Hadron-quark P.T. : **crossover** or **weak 1st order**

Lecture 2 : **Discussions from microscopic point of view**

Three-window approach to (cold) dense matter: Lect. 2

Toru Kojo (CCNU, Wuhan)



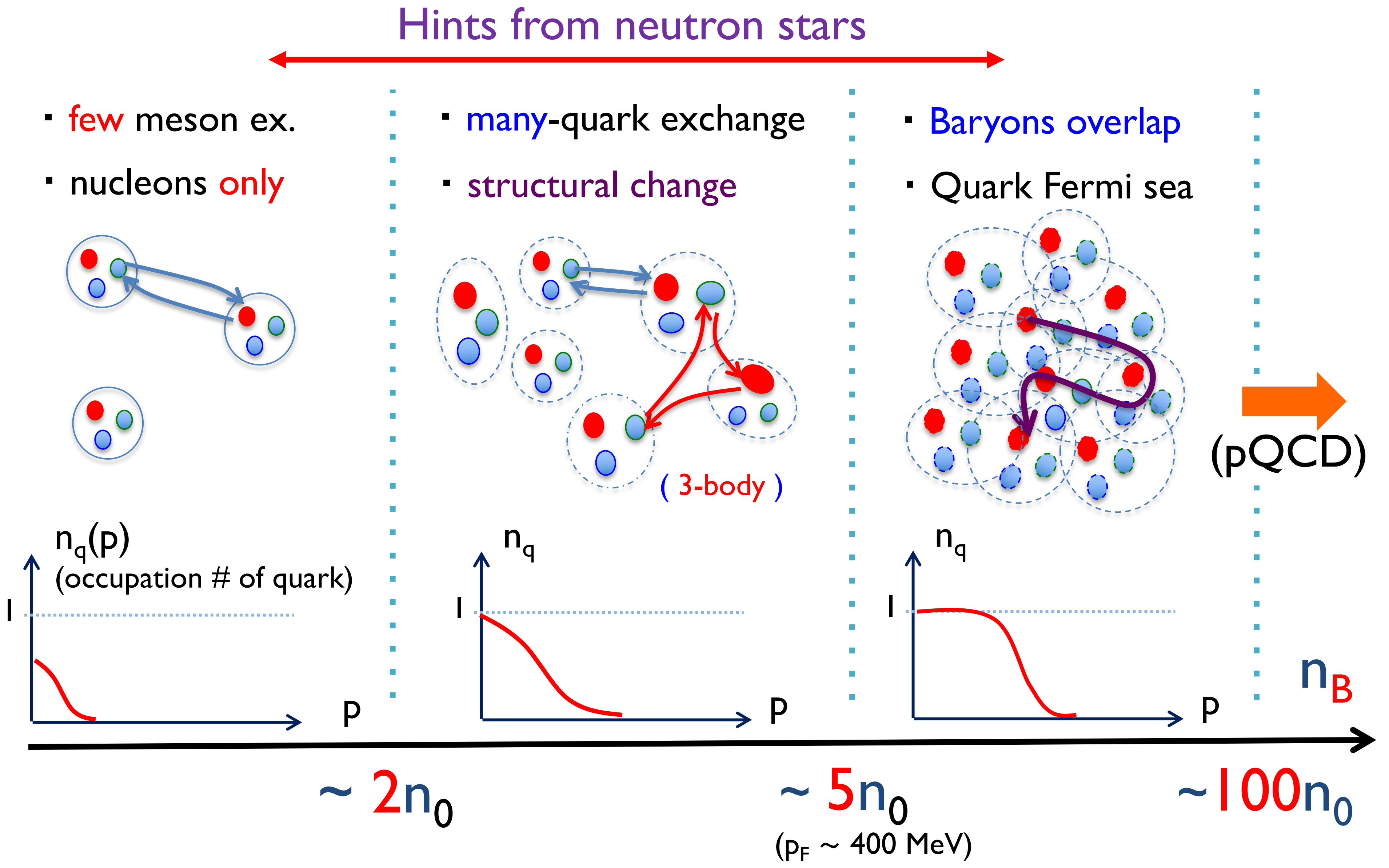
QCD

Astro

condensed matter

Picture to be developed

Masuda-Hatsuda-Takatsuka 2012
TK-Powell-Song-Baym 2014



Hints from neutron stars

- few meson ex.
- nucleons only

- many-quark exchange
- structural change

- Baryons overlap
- Quark Fermi sea

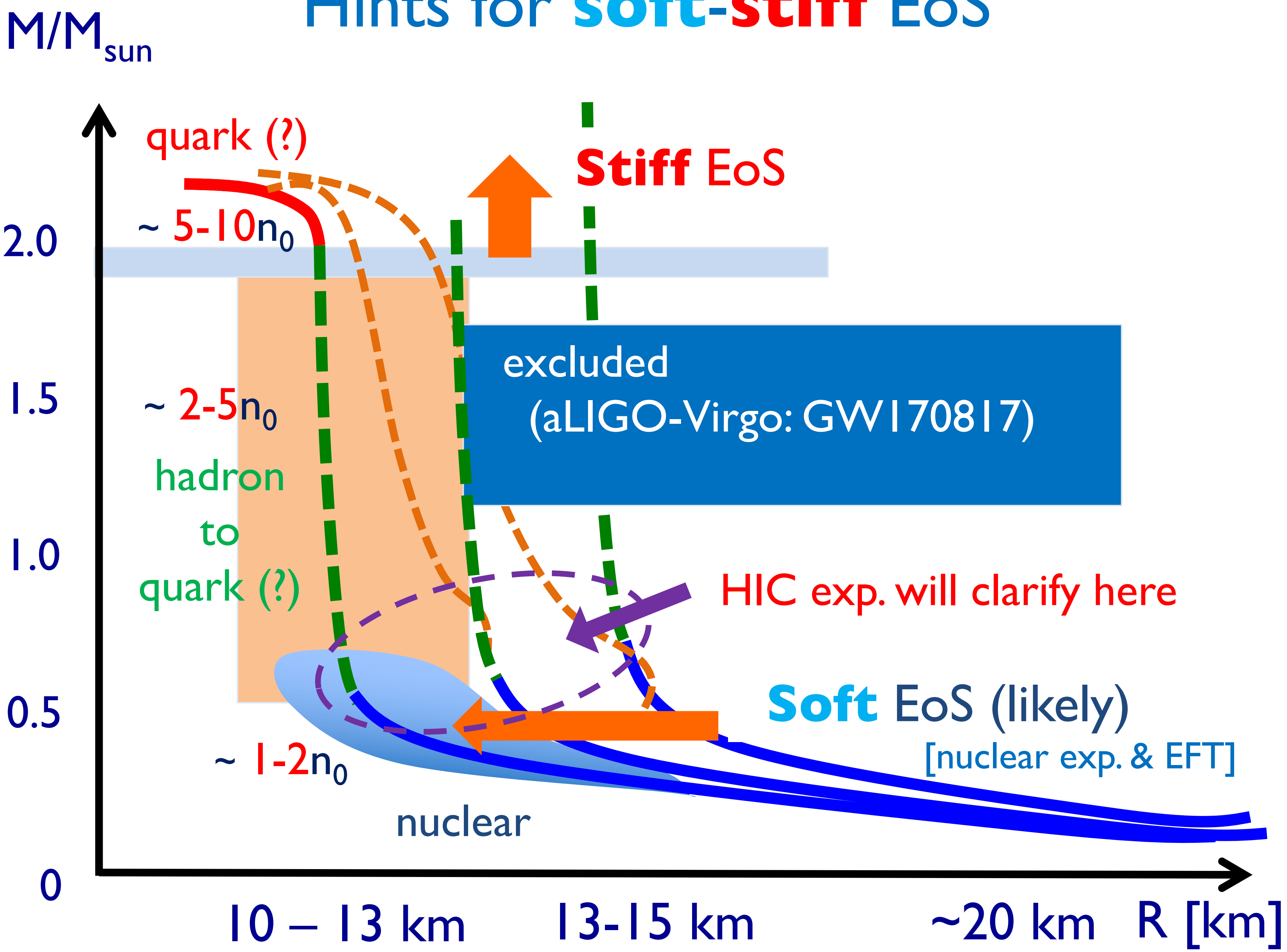
~ $2n_0$

~ $5n_0$

(p_F ~ 400 MeV)

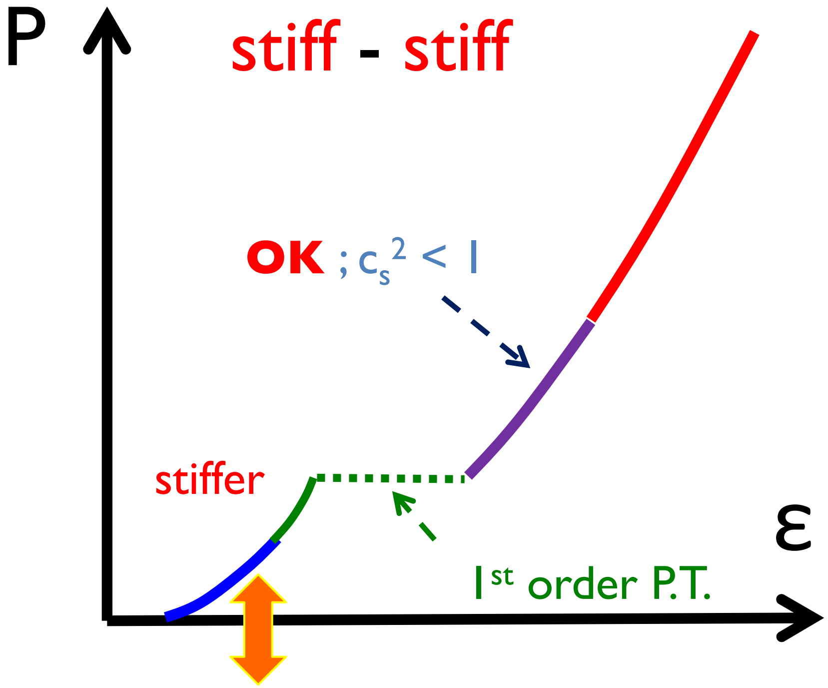
~$100n_0$

Hints for **soft-stiff** EoS

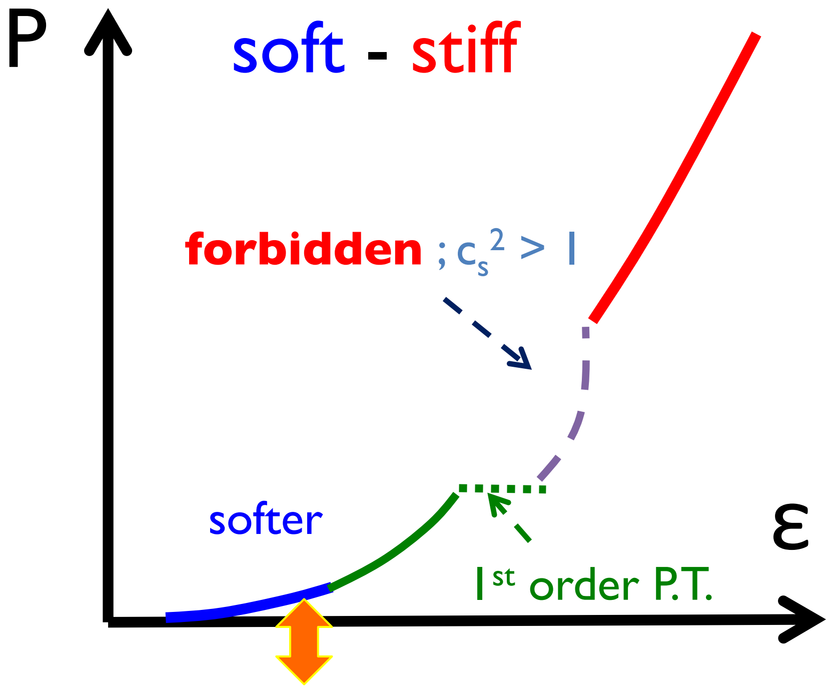


Stiff-Stiff v.s. Soft-Stiff EoS

$c_s^2 = dP/d\varepsilon < c^2$ (**causality**)



larger R (> ~13 km)

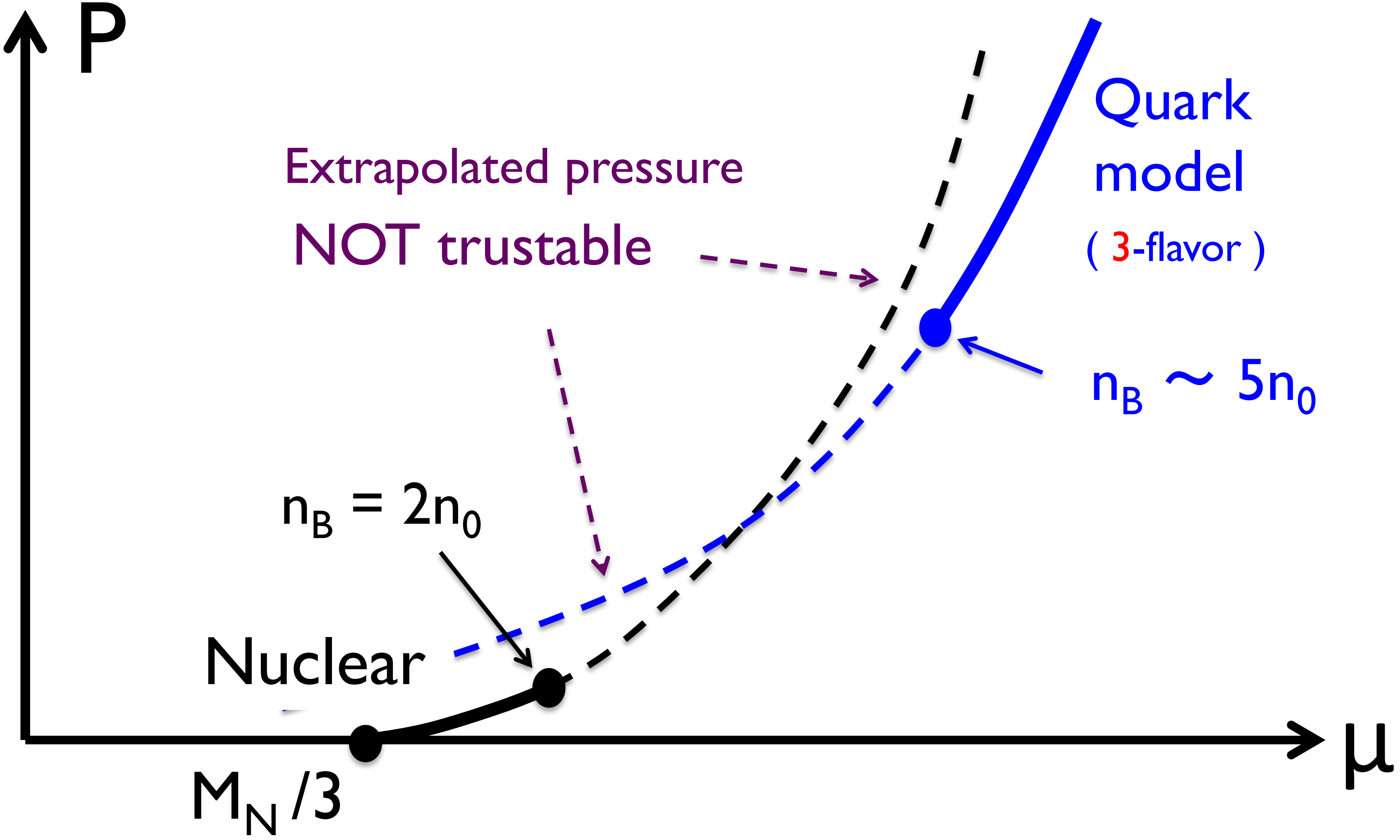


smaller R (< ~13 km)

→ we consider a **soft-stiff** EoS with **crossover** (or weak 1^{st} order)

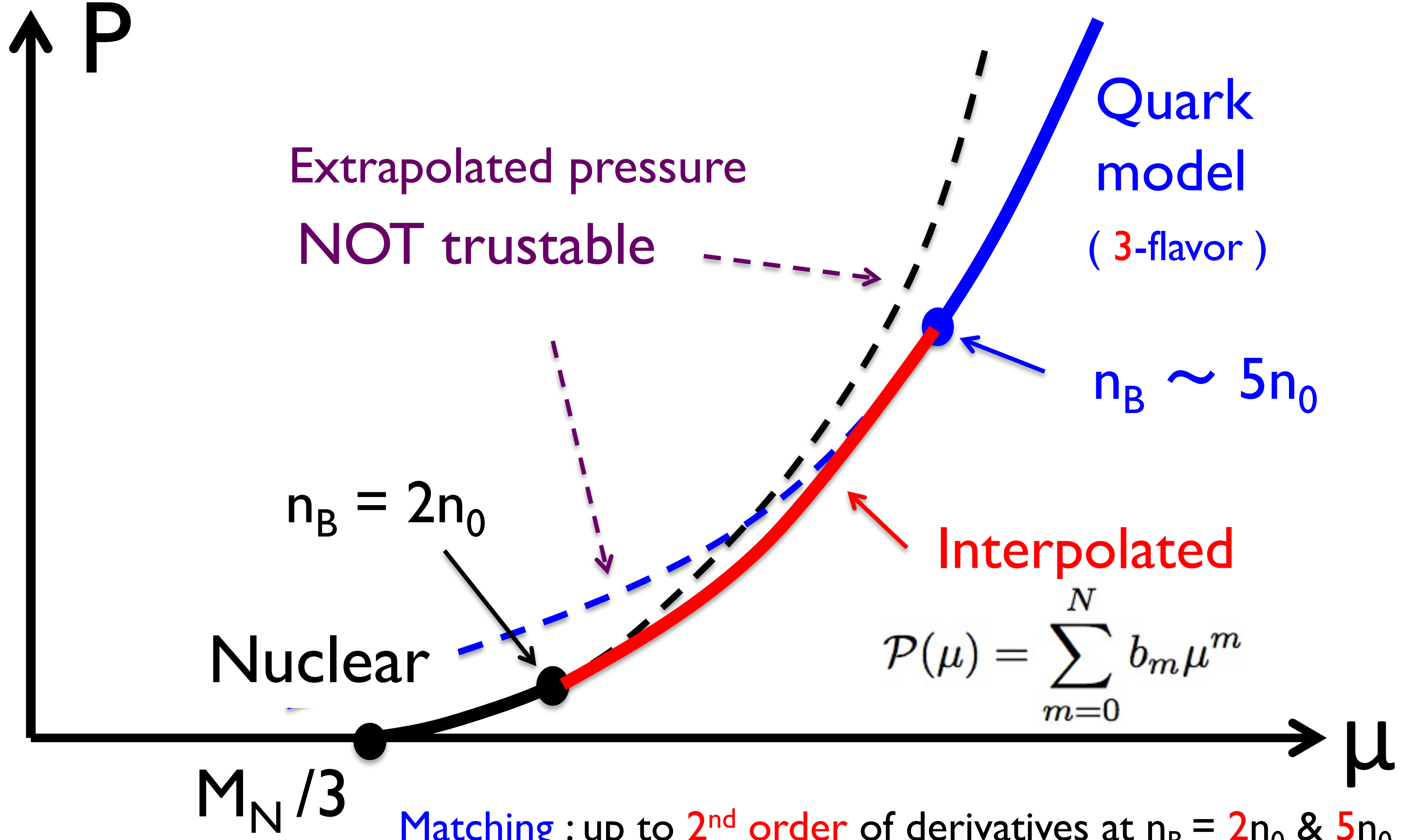
3-window modeling : P vs μ

Masuda+2012, Kojo+2014, ...



3-window modeling : P vs μ

Masuda+2012, Kojo+2014, ...



$$\mathcal{P}(\mu) = \sum_{m=0}^{N} b_m \mu^m$$

Matching : up to 2nd order of derivatives at $n_B = 2n_0$ & $5n_0$

(if you wish, put a small kink for weak 1st order P.T.)

Caveats

Matching or interpolation of 2-EoSs look innocent,
but actually it is NOT a trivial task at all.

(especially when underlying microphysics are different)

EoS must be
- thermodynamically consistent
- causal ($dP/d\varepsilon|_s = c_s^2 < c^2$)

Otherwise numerical simulations easily stop by instability.

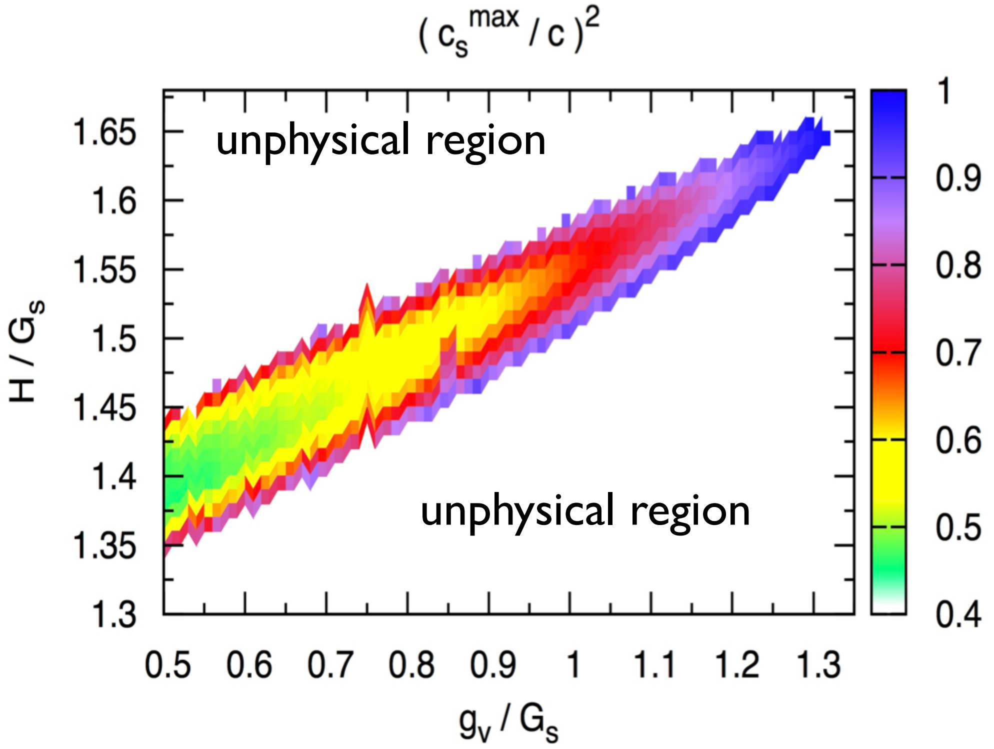
(In fact some EoS tables in the website are not usable...)

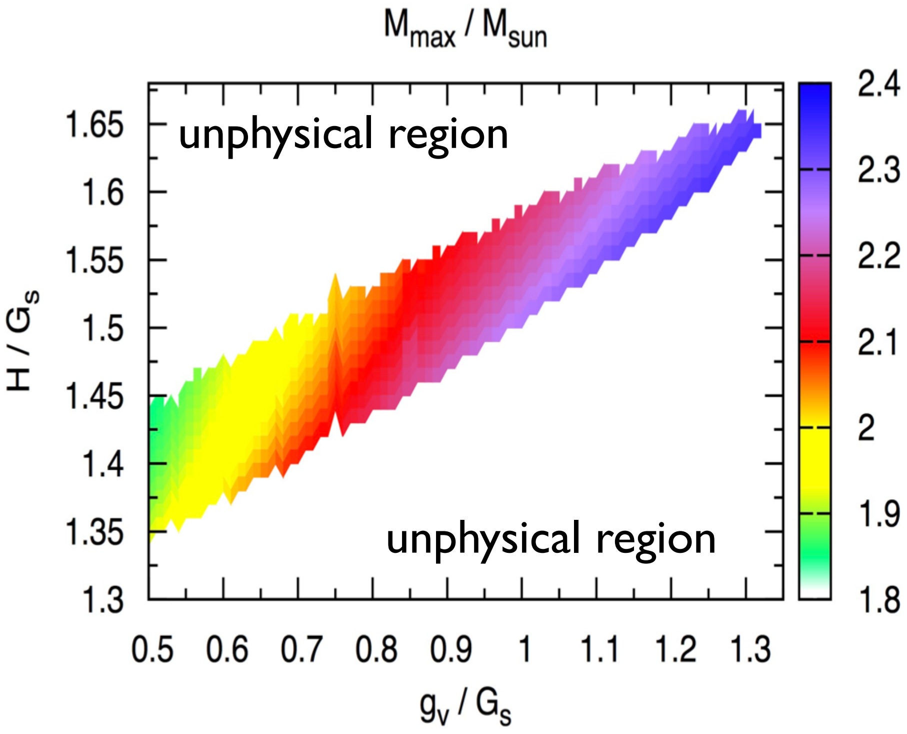
In addition, we have nuclear & astrophysical constraints :

Stronger constraints for **softer**-**stiffer** EoS.

(→ more chances to select out the correct EoS)

Constraints -> quark model parameters





The allowed range of (gv, H) is constrained;

-> predictions for other domains, e.g. (Ye, T, ...)

(explanations for plots -> Lect.3)

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1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

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5, Order parameter & symmetry

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From $\varepsilon(n)$ to $\Omega(\mu)$ $(= -P(\mu))$ (at $T=0$)

astro. people prefer n QCD people prefer μ

energy density at a given number density

$$\varepsilon(n):\quad d\varepsilon(n) = \mu\, dn \qquad\qquad \mu(n) = d\varepsilon(n)/dn$$

For QCD calculations, more common to work at fixed μ

change of variables : **Legendre transf.**

$$-P = \Omega(\mu) = \varepsilon - \mu n$$

$$\rightarrow d\Omega(\mu) = d\varepsilon - \mu\, dn - n\, d\mu = -\, n\, d\mu$$

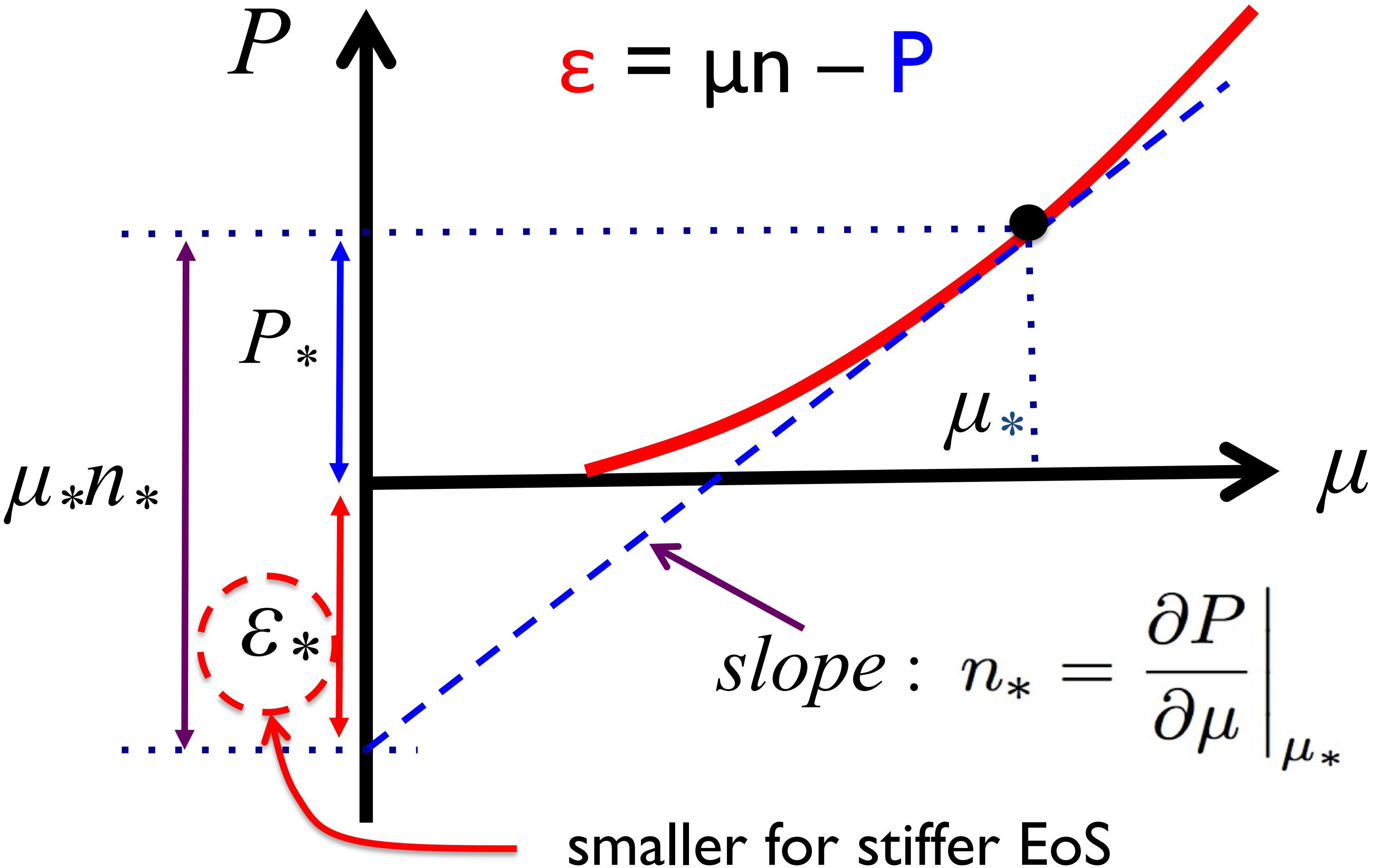
With the expression of $P(\mu)$ given ➡

$$n(\mu) = dP/d\mu$$

$$\varepsilon(\mu) = \mu\, n(\mu) - P(\mu)$$

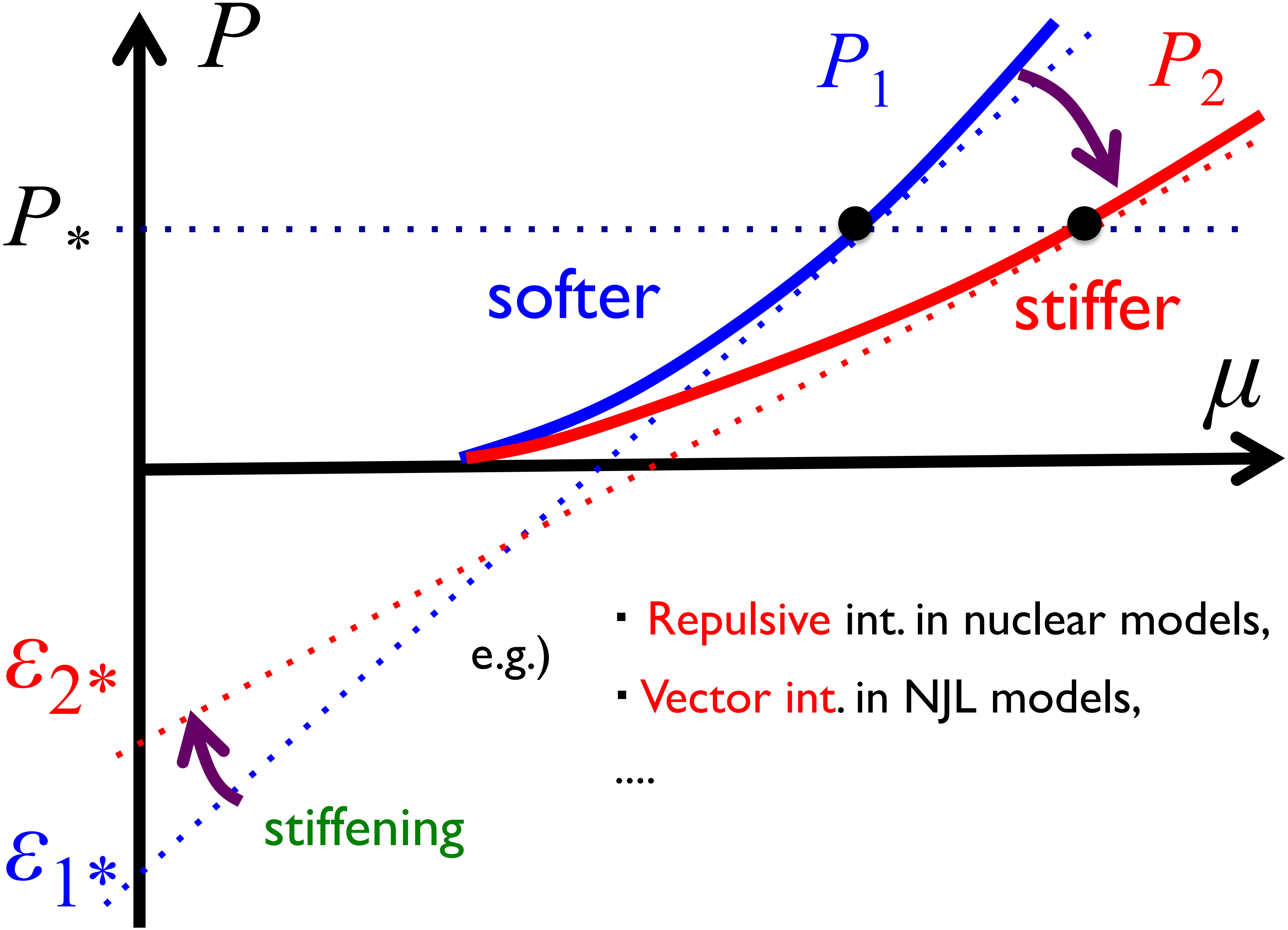
all info about EoS included

How stiff EoS looks like in P(μ) curves



smaller for stiffer EoS

Stiffening I : Rotation

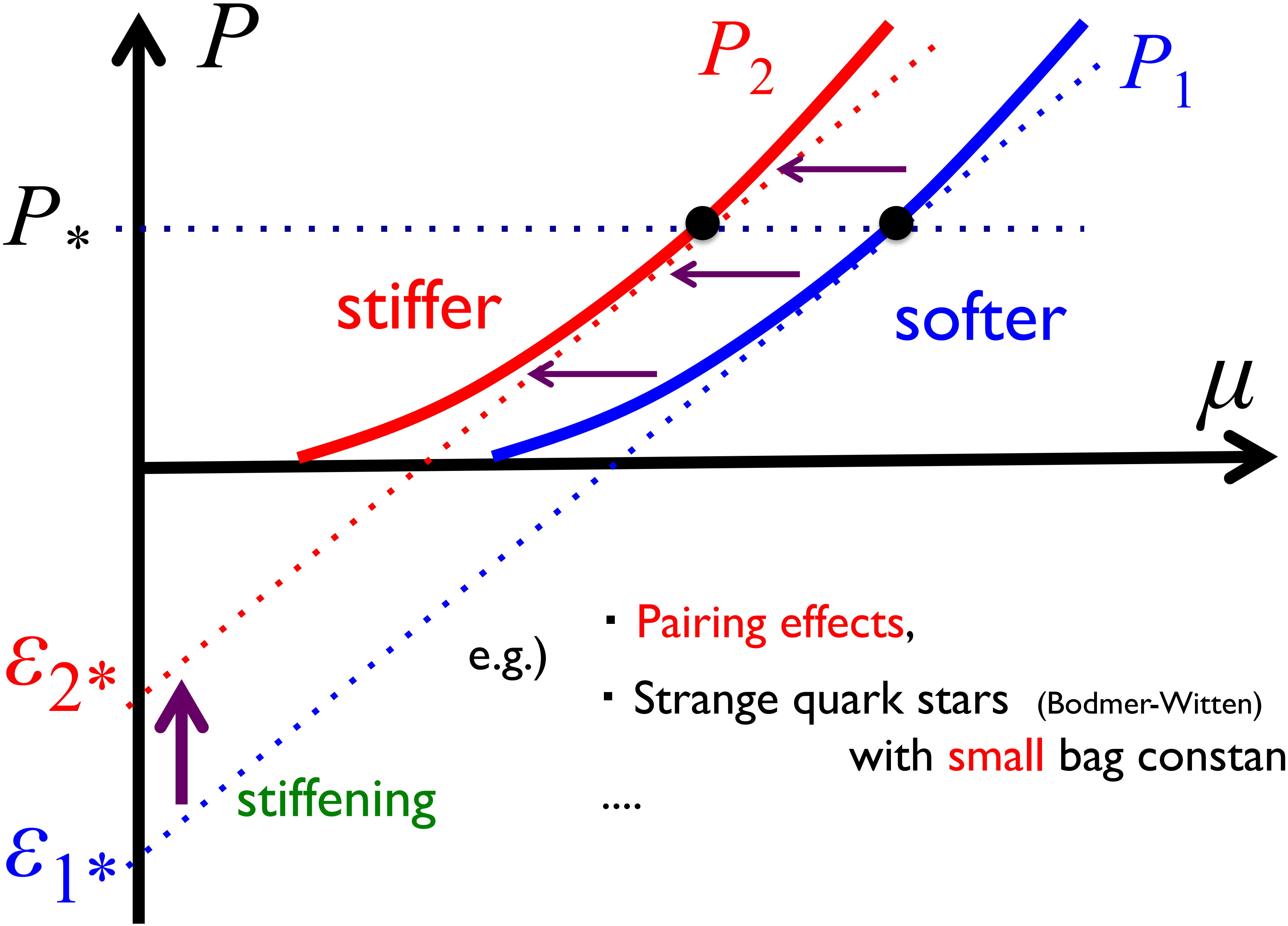


e.g.)

- Repulsive int. in nuclear models,
- Vector int. in NJL models,

....

Stiffening 2 : Parallel shift



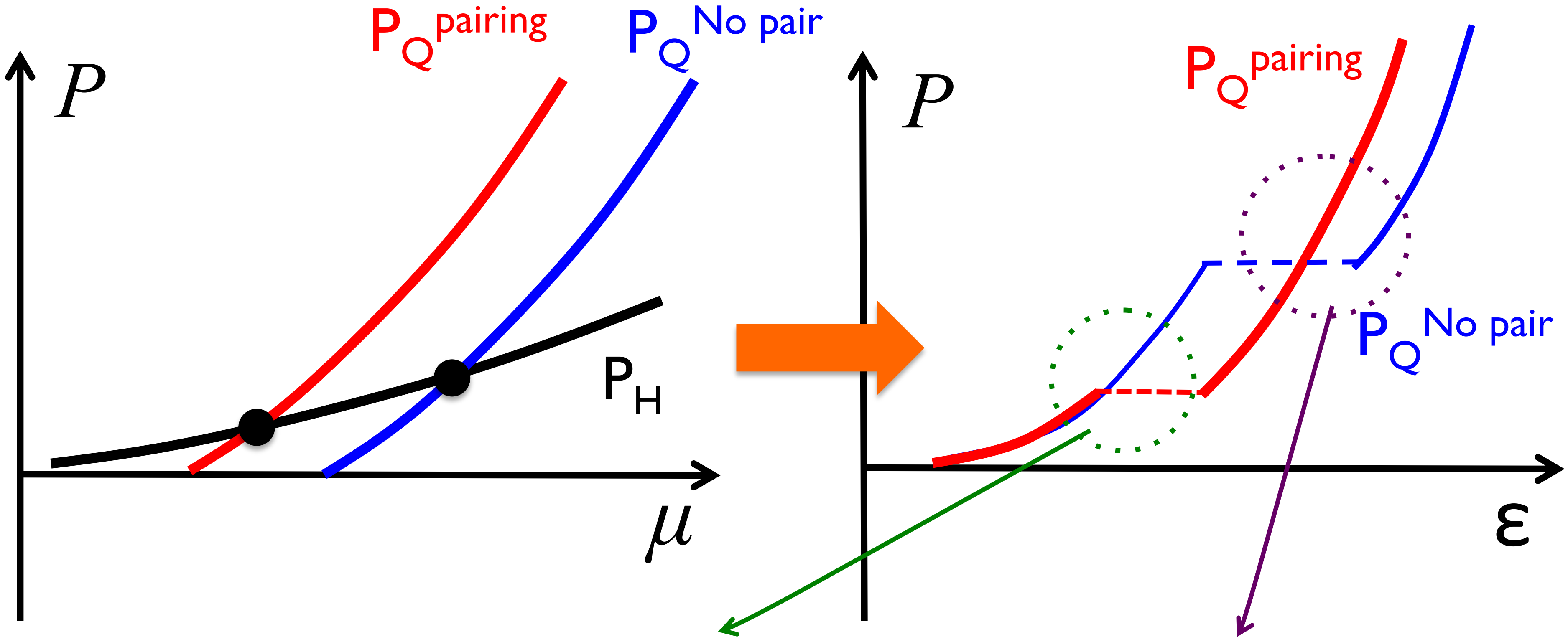
e.g.)

- Pairing effects,
- Strange quark stars (Bodmer-Witten)
  with small bag constant,

....

App.1 :“Pairing” can stiffen EoS

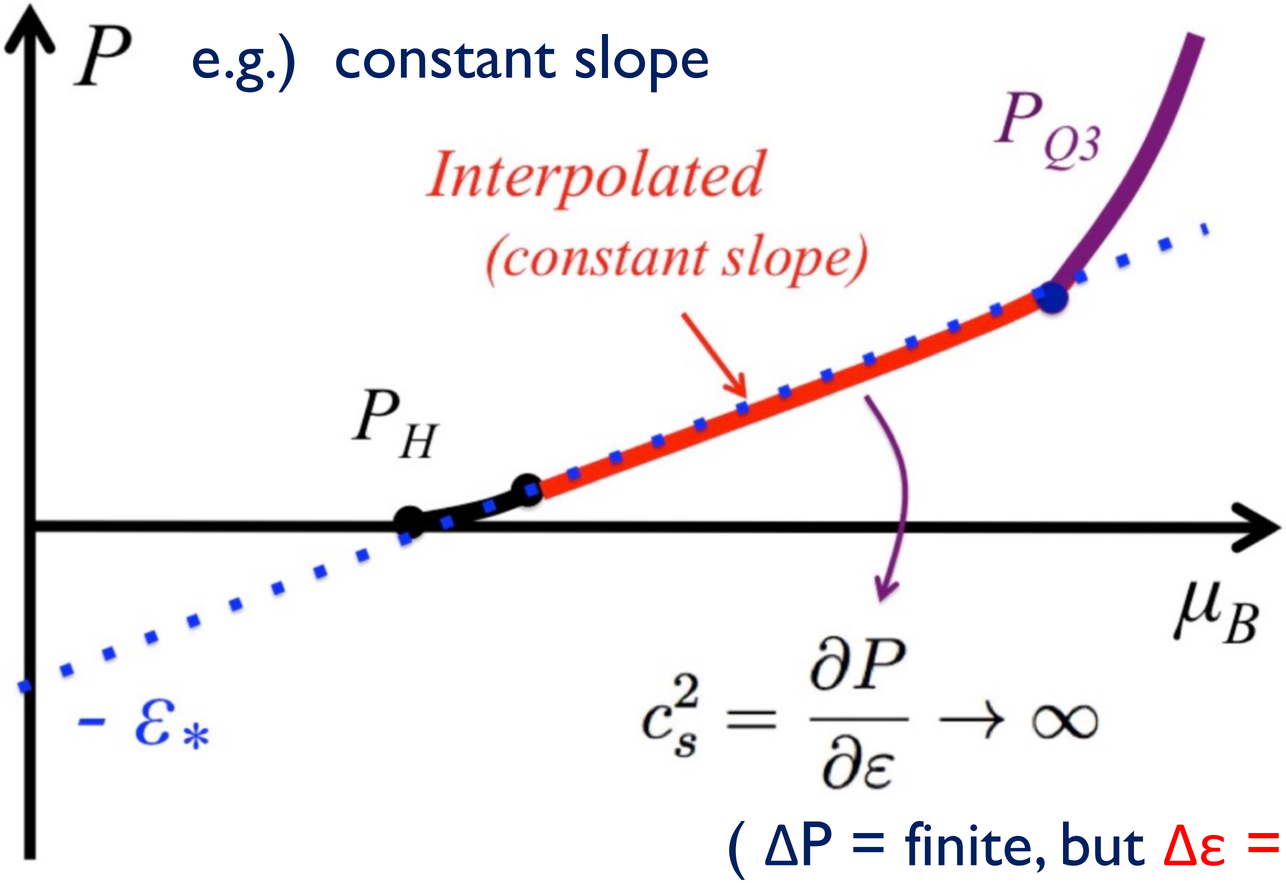
Do exotic phases always give softening? → Not necessarily



→ Softening at low n_B & stiffening at high n_B

App.2 : P(μ) must grow sufficiently fast

(otherwise the speed of sound becomes superluminal)



e.g.) constant slope

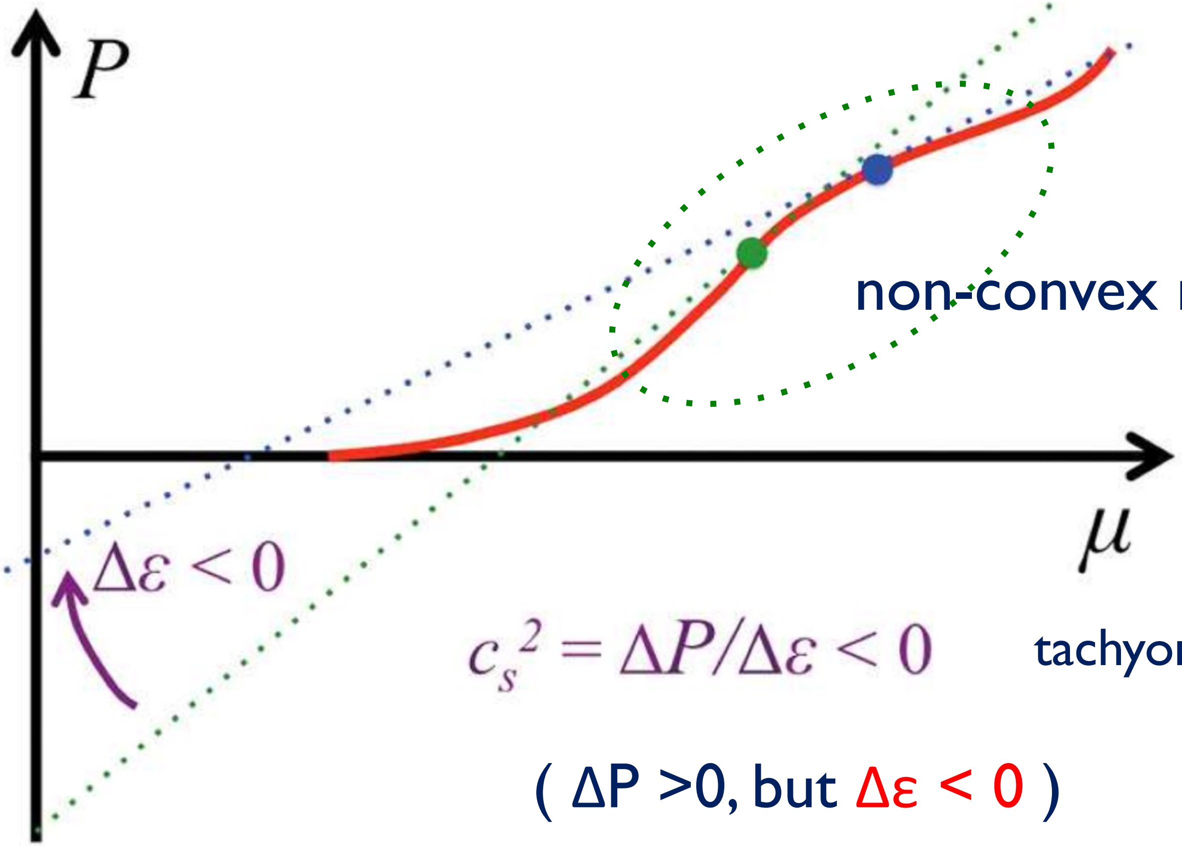
$$c_s^2 = \frac{\partial P}{\partial \varepsilon} \to \infty$$

(ΔP = finite, but $\Delta\varepsilon = 0$)

(more analyses → P(μ) should grow faster than μ^2)

App.3 : P(μ) must NOT have inflection points

(or P(μ) must be convex)

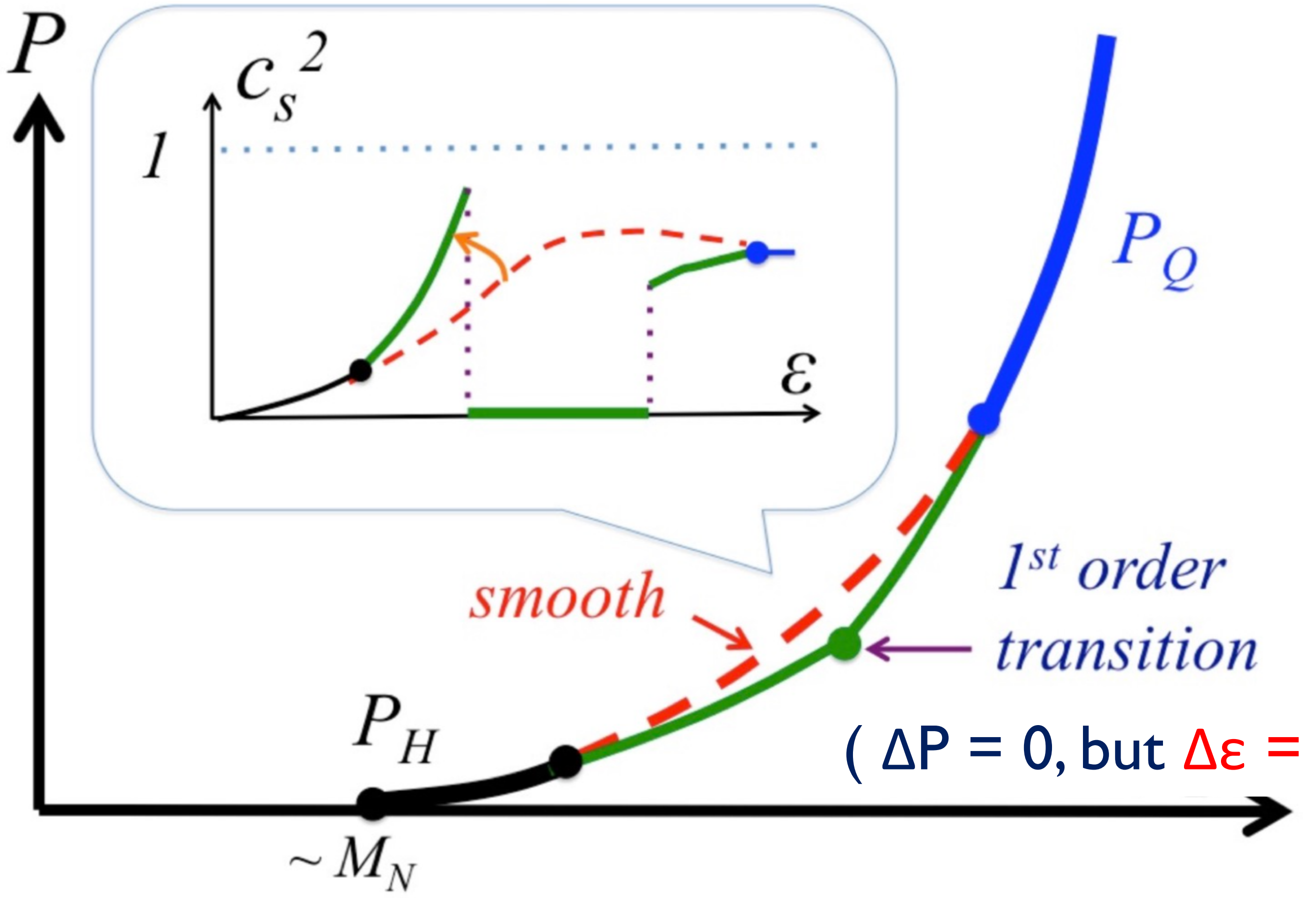


non-convex region

tachyonic sound mode

(ΔP >0, but Δε < 0)

App.4 : 1st order P.T. & speed of sound



($\Delta P = 0$, but $\Delta\varepsilon$ = finite)

$c_s{}^2$ grows rapidly before P.T., then suddenly reduces to zero.

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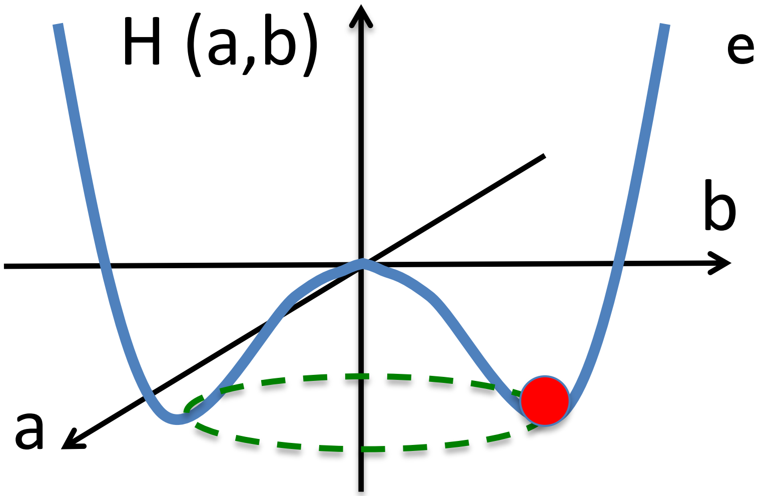
Symmetry & Order parameters I

Spontaneous Symmetry Breaking (**SSB**)

(Heisenberg, Landau, Nambu 60, Goldstone 61)

Sym. of **Hamiltonian** $\neq$ Sym. of **States**



e.g.) A ball in a wine bottle (**classical**)

- Hamiltonian:
  rotational symmetric
- Ground state:
  NOT rotational symmetric

Symmetry & Order parameters 2 (**Quantum**)

Symmetry of H → symmetry generator Q

$$\mathrm{e}^{\mathrm{i}\theta\hat{Q}}\,\hat{H}\,\mathrm{e}^{-\mathrm{i}\theta\hat{Q}} = \hat{H}$$ "rotation" invariant

Suppose the G.S. is $|0\rangle$. "Rotate" the G.S. as

$$|\theta\rangle = \mathrm{e}^{-\mathrm{i}\theta\hat{Q}}|0\rangle$$

The "rotated" state has the **same energy** as $|0\rangle$.

$|\theta\rangle = \mathrm{e}^{\mathrm{i}\delta}|0\rangle$ **trivial** ($|0\rangle$ & $|\theta\rangle$ are the **same** state)

$|\theta\rangle \neq \mathrm{e}^{\mathrm{i}\delta}|0\rangle$ **SSB** ($|0\rangle$ & $|\theta\rangle$ can be G.S., but $|0\rangle$ was **chosen**)

How to check? We look for **order parameter** :

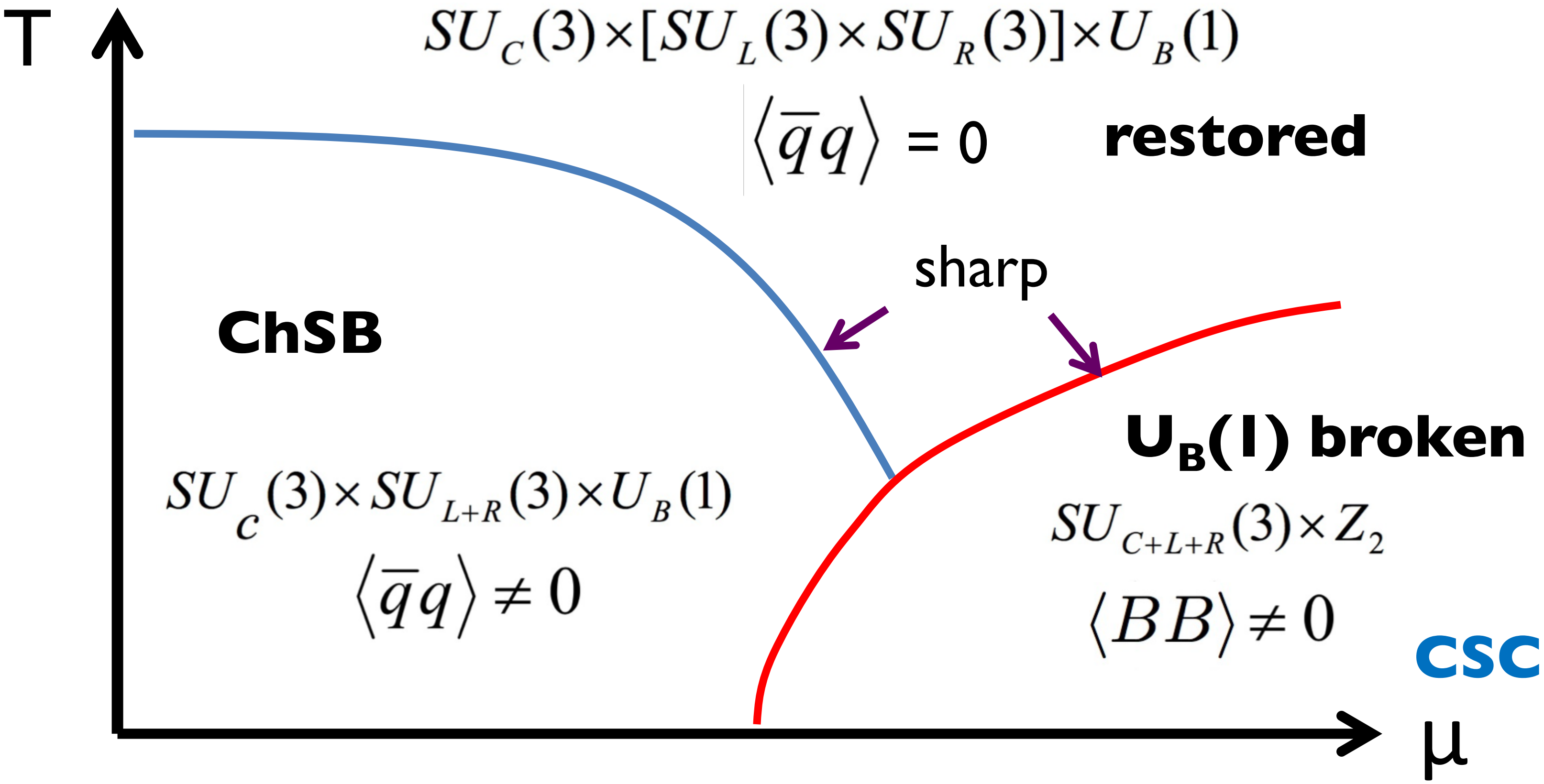
$$\langle\theta|\hat{O}|\theta\rangle - \langle 0|\hat{O}|0\rangle = \mathrm{i}\theta\underline{\langle 0|\delta_Q\hat{O}|0\rangle} + \cdots$$

If $\langle\delta O\rangle$ is nonzero, we can say $|0\rangle$ & $|\theta\rangle$ are different.

Symmetry & Order parameters 3

Sym. **unambiguously** distinguishes the phases

If chiral sym. of QCD **were exact**...



$SU_C(3)\times[SU_L(3)\times SU_R(3)]\times U_B(1)$

$\langle\bar{q}q\rangle = 0$ **restored**

sharp

ChSB

$SU_C(3)\times SU_{L+R}(3)\times U_B(1)$

$\langle\bar{q}q\rangle \neq 0$

$U_B(1)$ broken

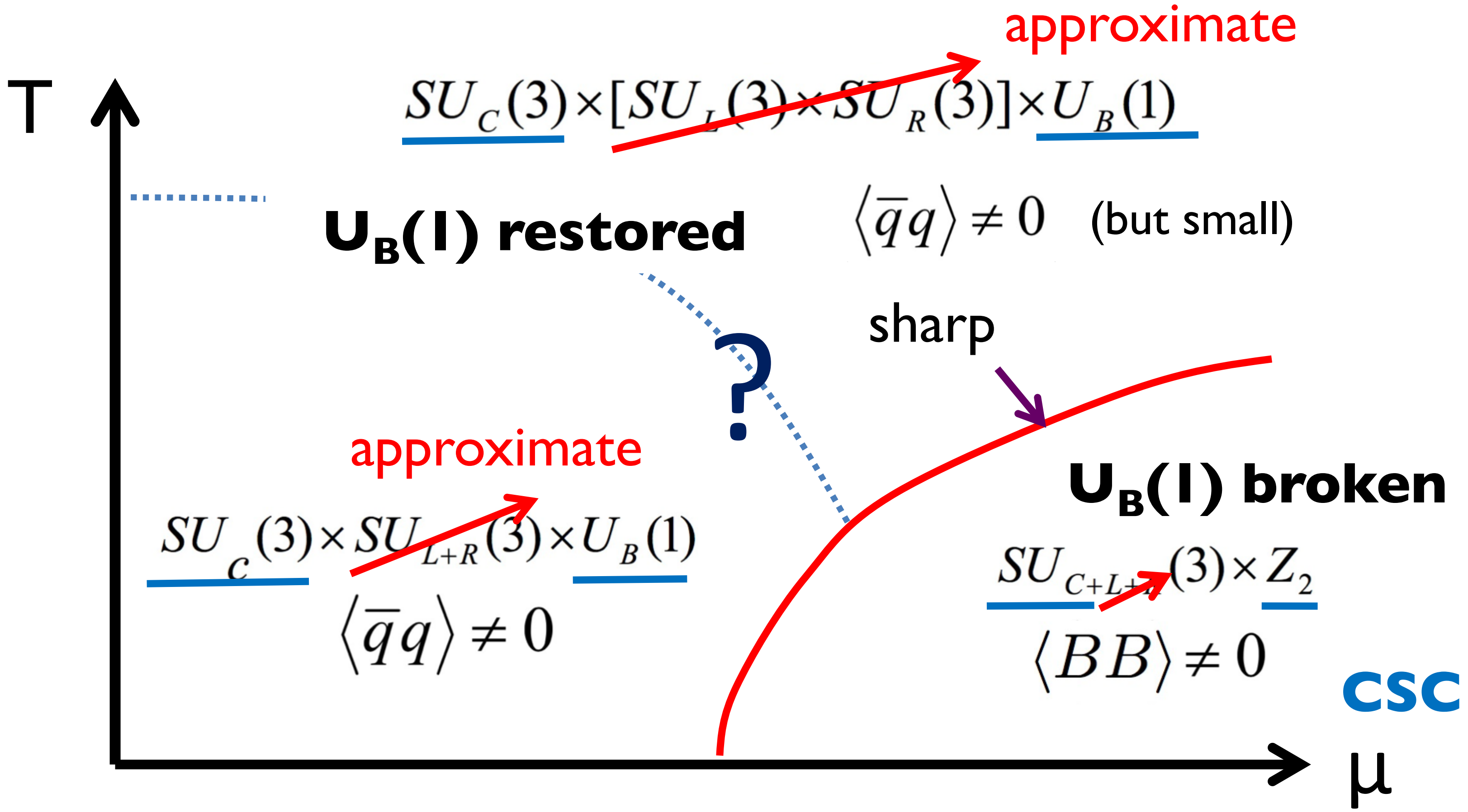
$SU_{C+L+R}(3)\times Z_2$

$\langle BB\rangle \neq 0$

CSC

Symmetry & Order parameters 4

In reality: $m_u \neq m_d \neq m_s$



approximate

$\underline{SU_C(3)}\times\cancel{[SU_L(3)\times SU_R(3)]}\times\underline{U_B(1)}$

$U_B(1)$ restored $\langle\bar{q}q\rangle\neq 0$ (but small)

?

sharp

approximate

$\underline{SU_c(3)}\times\cancel{SU_{L+R}(3)}\times\underline{U_B(1)}$

$\langle\bar{q}q\rangle\neq 0$

$U_B(1)$ broken

$\underline{SU_{C+L+R}}(3)\times\underline{Z_2}$

$\langle BB\rangle\neq 0$

CSC

Symmetry & Order parameters 5

Because of explicit sym. breaking, the possible
chiral phase transitions can be **any**. (crossover, 1st, 2nd, ...)

Also, rigorous order parameters for confinement are **NOT** known.
(except for pure YM)

Hadron-quark P.T. is difficult to define in a formal way.
(even Confinement-Higgs is difficult to distinguish; Fradkin-Shenkar 79)

Then what can we do?

1, If we are lucky, we can find **abrupt changes**
(e.g. 1st order P.T. or radical crossover)

2, If not, need to examine the validity of **effective d.o.f.**
(if proper d.o.f are used, calculations converge quickly)

Dynamics must be discussed

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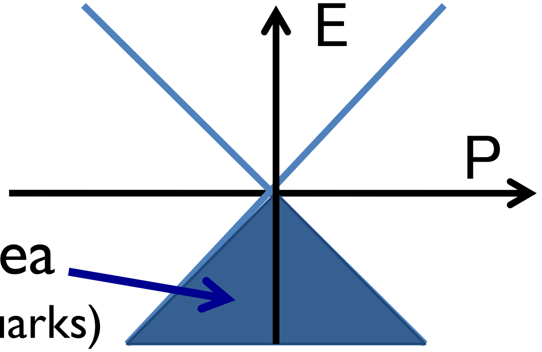
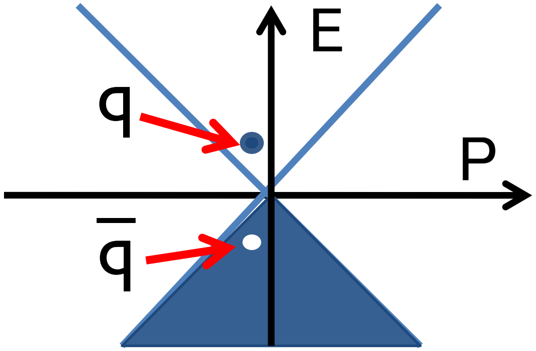
Generation of the chiral condensate



Dirac Sea

(full of quarks)



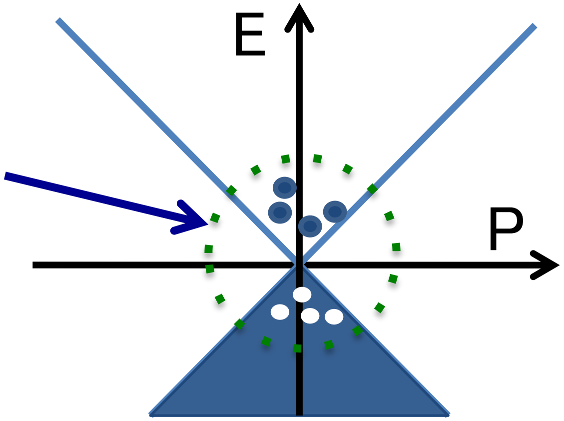
Usually : $E_0 < E_0 + E_q + E_{\bar{q}}$

But with (strong) attractive interaction :

$$E_0 > E_0 + E_{int} + E_q + E_{\bar{q}}$$

Then **macroscopic** number of $q\bar{q}$ are kept excited :

" Condensation "

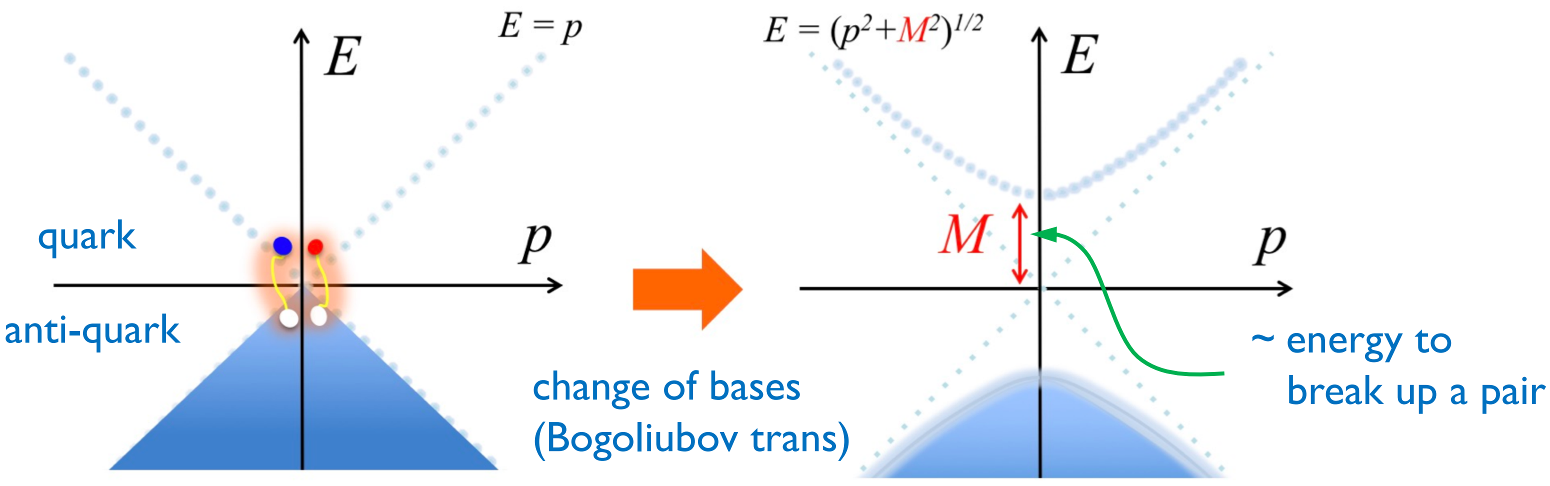


quantified by

Chiral Condensate : $\langle 0|\bar{q}q|0\rangle$

Chiral sym. breaking & restoration

vac



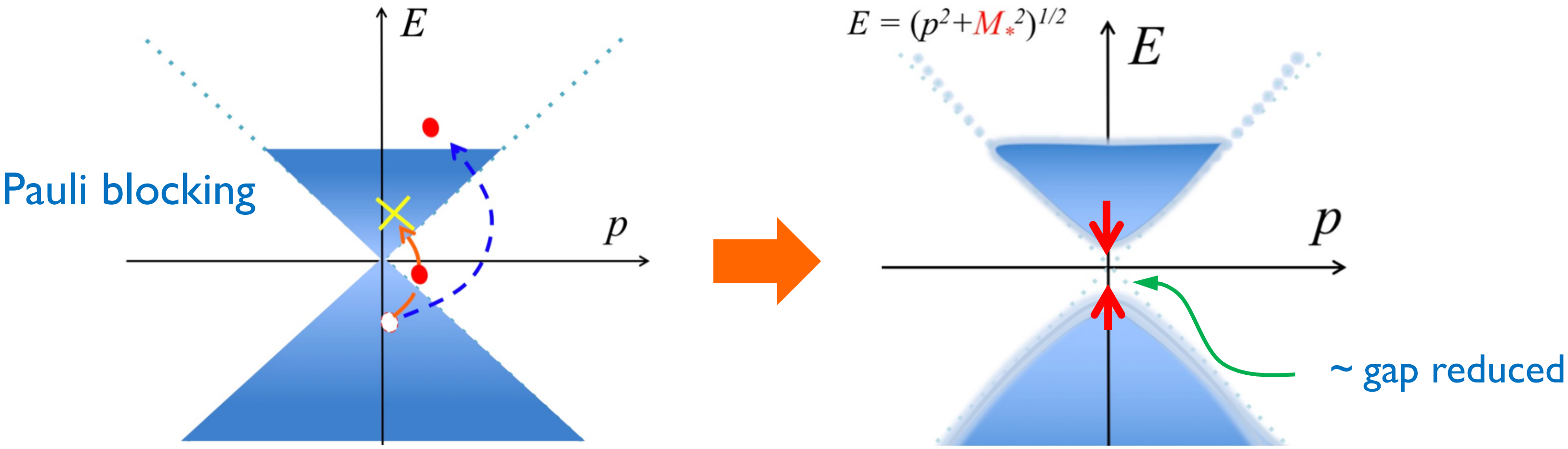
quark

anti-quark

change of bases
(Bogoliubov trans)

~ energy to
break up a pair

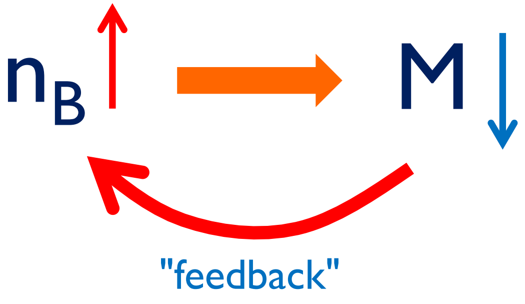
finite density



Pauli blocking

~ gap reduced

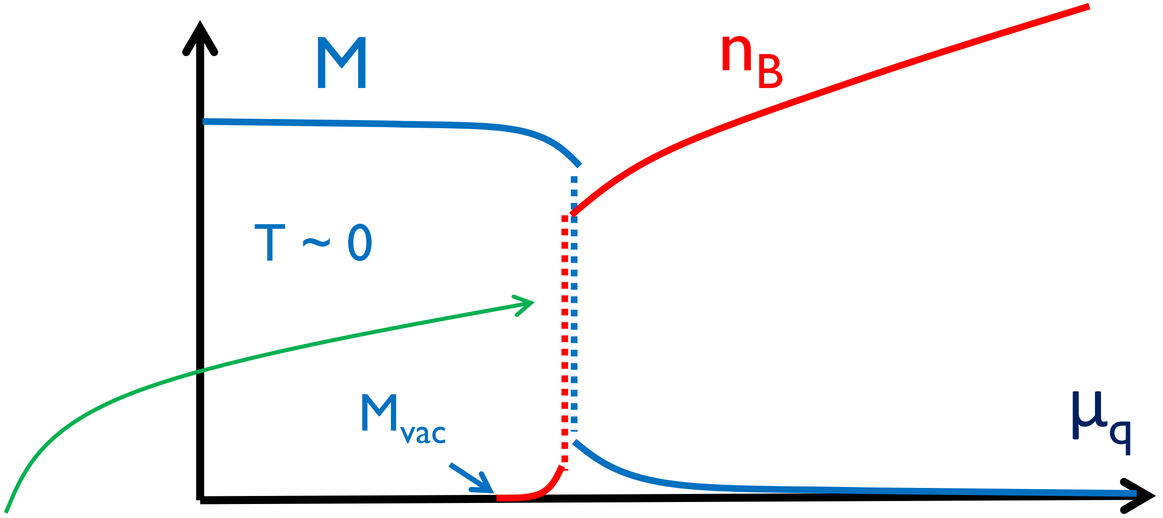
I st order chiral transition (typical quark **models**)

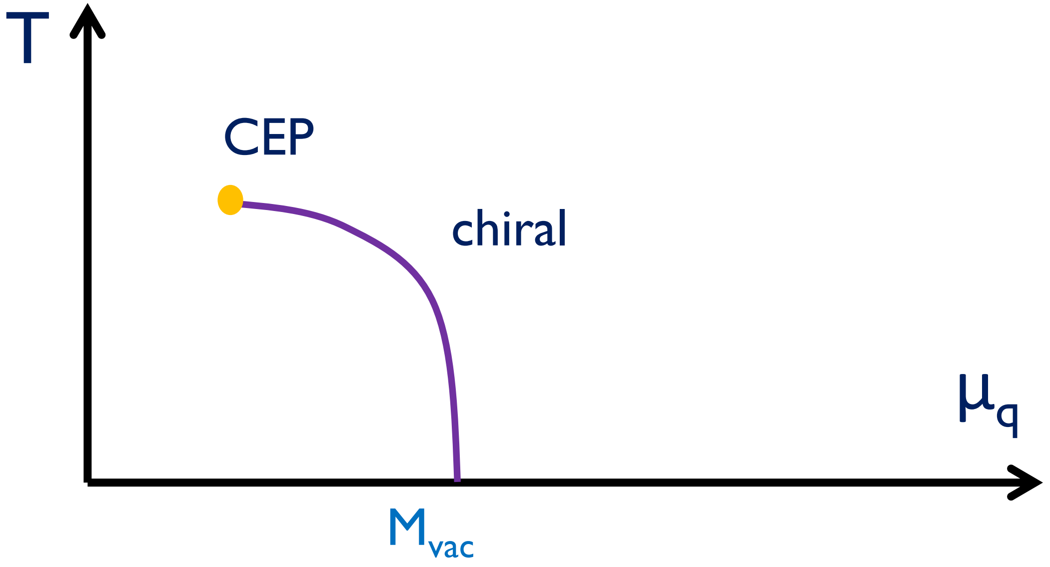


"feedback"

more phase space

→ radical changes in n_B & M





Braking density evolution: Ist → crossover

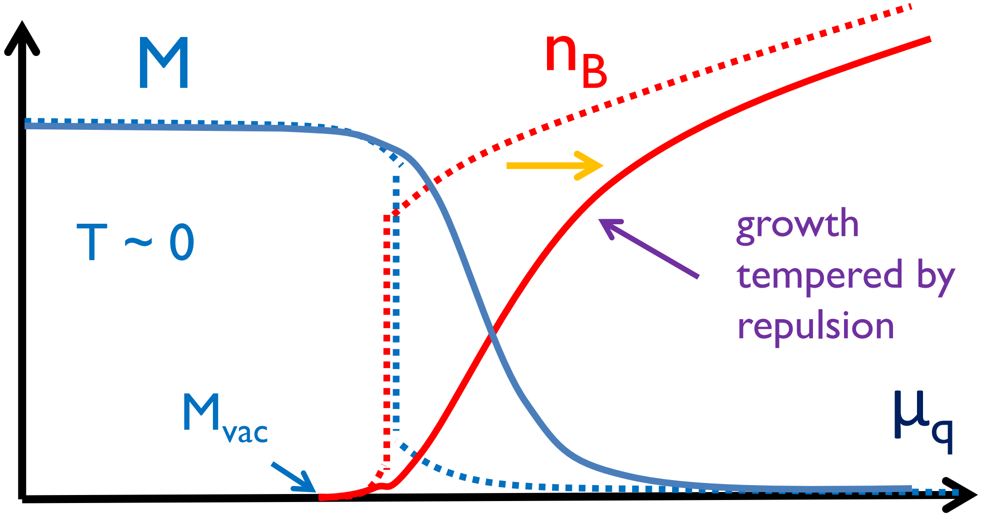
Now add
density-density repulsion

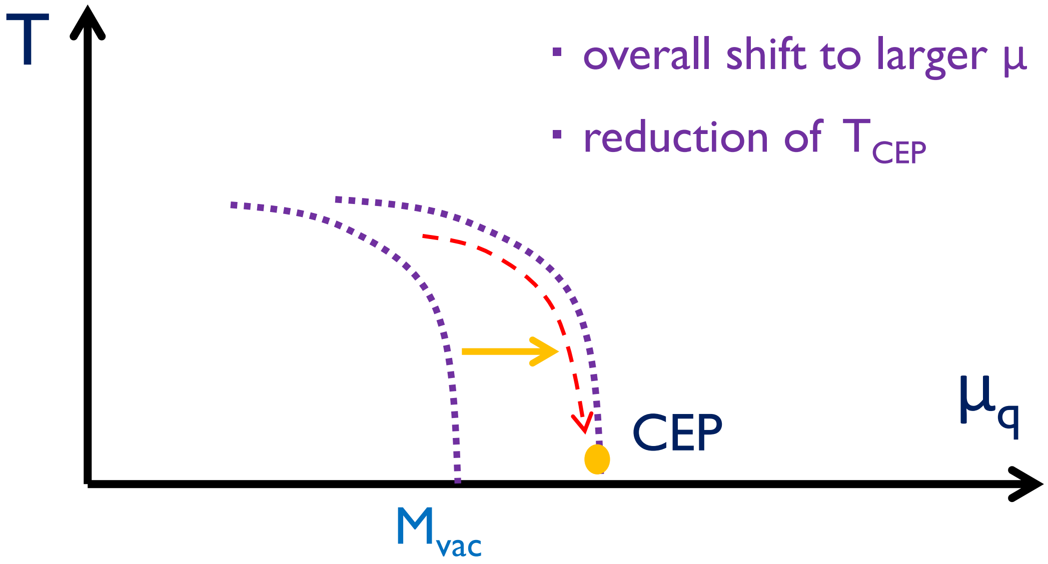
$$\Delta H \sim g_V\,(n_B)^2$$

braking the evolution of n_B

→ milder changes in M

Details of int. are crucial



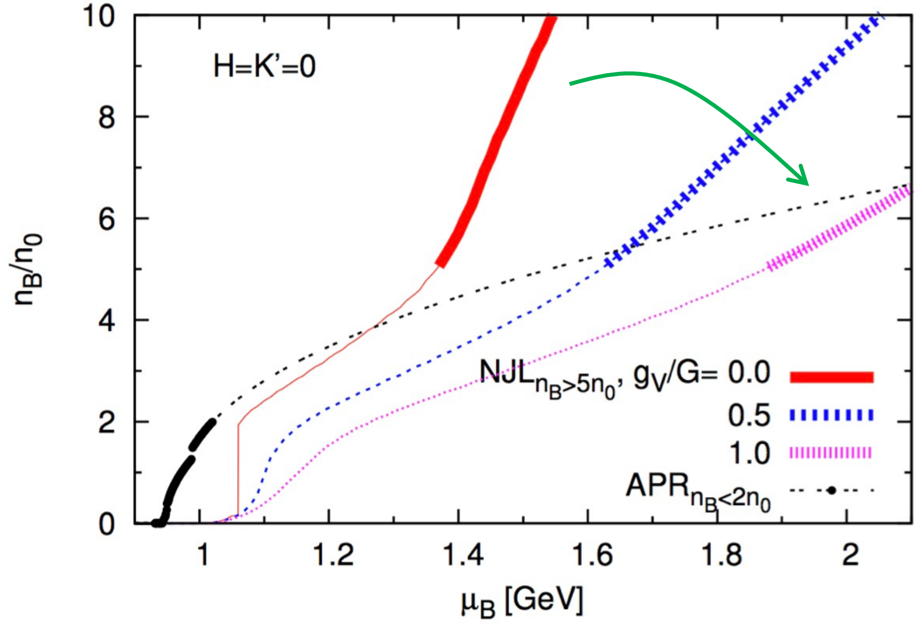


- overall shift to larger μ
- reduction of T_{CEP}

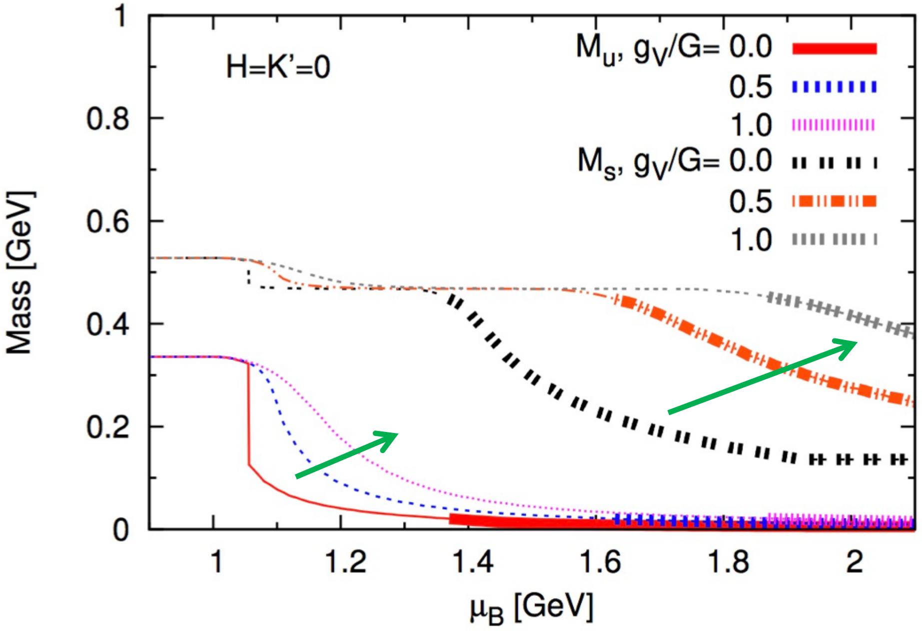
Some quark model results

$$\Delta H \sim g_V \, (n_B)^2$$

number density



Dirac mass



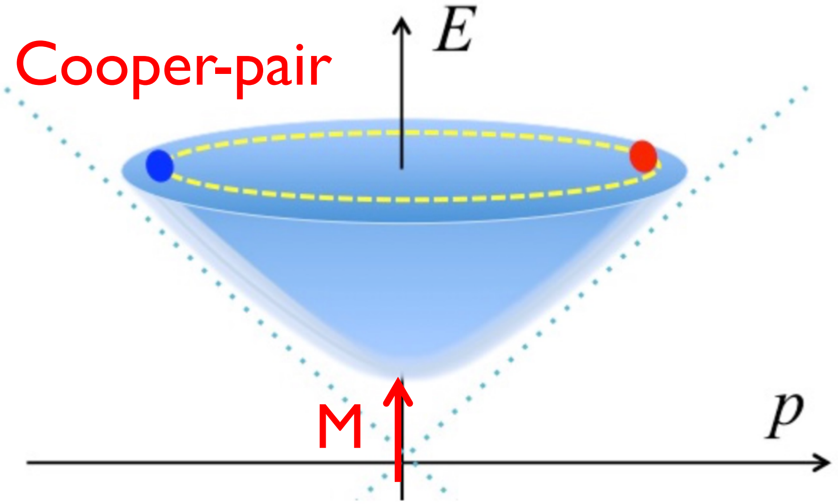
repulsion tempers the growth of n_B

→ milder chiral phase transition (1st -> crossover)

Di-fermion pairing

As density increases, another kind of condensation takes place:

(particle-particle & hole-hole pairing)



di-baryon or di-quark

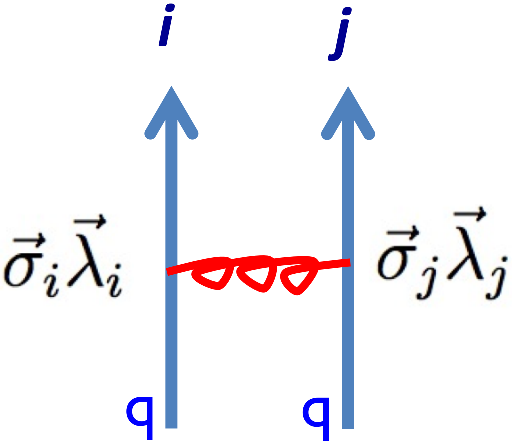
Key elements for condensations

- Fermi surface : large phase space for **gapless** excitations
  (many pairs can be formed)
- attractive interactions (small int. is already enough)

Diquark pairing : quantum number

[Bailin-Love, Alford, Rajagopal, Wilzcek, Schafer, ...]

qq-pairing



less (more) color charges

color: $\mathbf{3} \otimes \mathbf{3} = \underline{\bar{\mathbf{3}}} \oplus \underline{\mathbf{6}}$

So we consider **color anti-symmetric** channel.

Next consider a color-magnetic interaction.

$$\sim \alpha_s \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{M_i M_j} \delta(\vec{r}_{ij})$$

(at short distance)

Most attractive for **spin-singlet** & **S-wave**

& fermion statistics -> **flavor anti-symmetric**

qq-"condensate"

color-antisym

$$\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle = \epsilon^{\alpha\beta A} \epsilon_{ijB} \Delta_B^A$$

scalar 0+

flavor-antisym

CFL & 2SC pairing

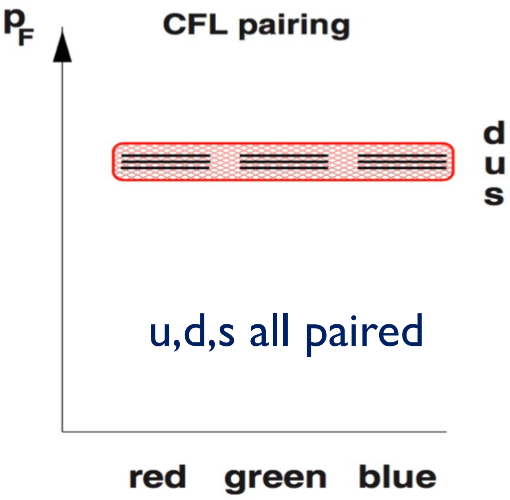
$$\langle \psi_i^\alpha C\gamma_5 \psi_j^\beta \rangle = \epsilon^{\alpha\beta A}\epsilon_{ijB}\Delta^A_B$$

many strange quarks → **Color-Flavor-Locked**

$$\Delta^A_B = \Delta_{\mathrm{CFL}}\underline{\delta^A_B}$$

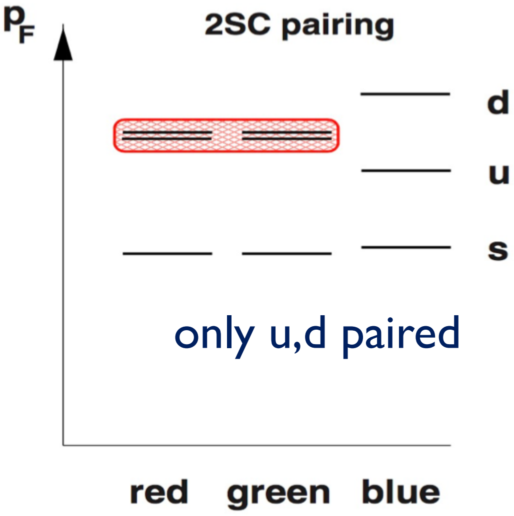
little strange quarks → **2SC**

$$\Delta^A_B = \Delta_{\mathrm{2SC}}\underline{\delta^A_3\delta^3_B}$$



u,d,s all paired

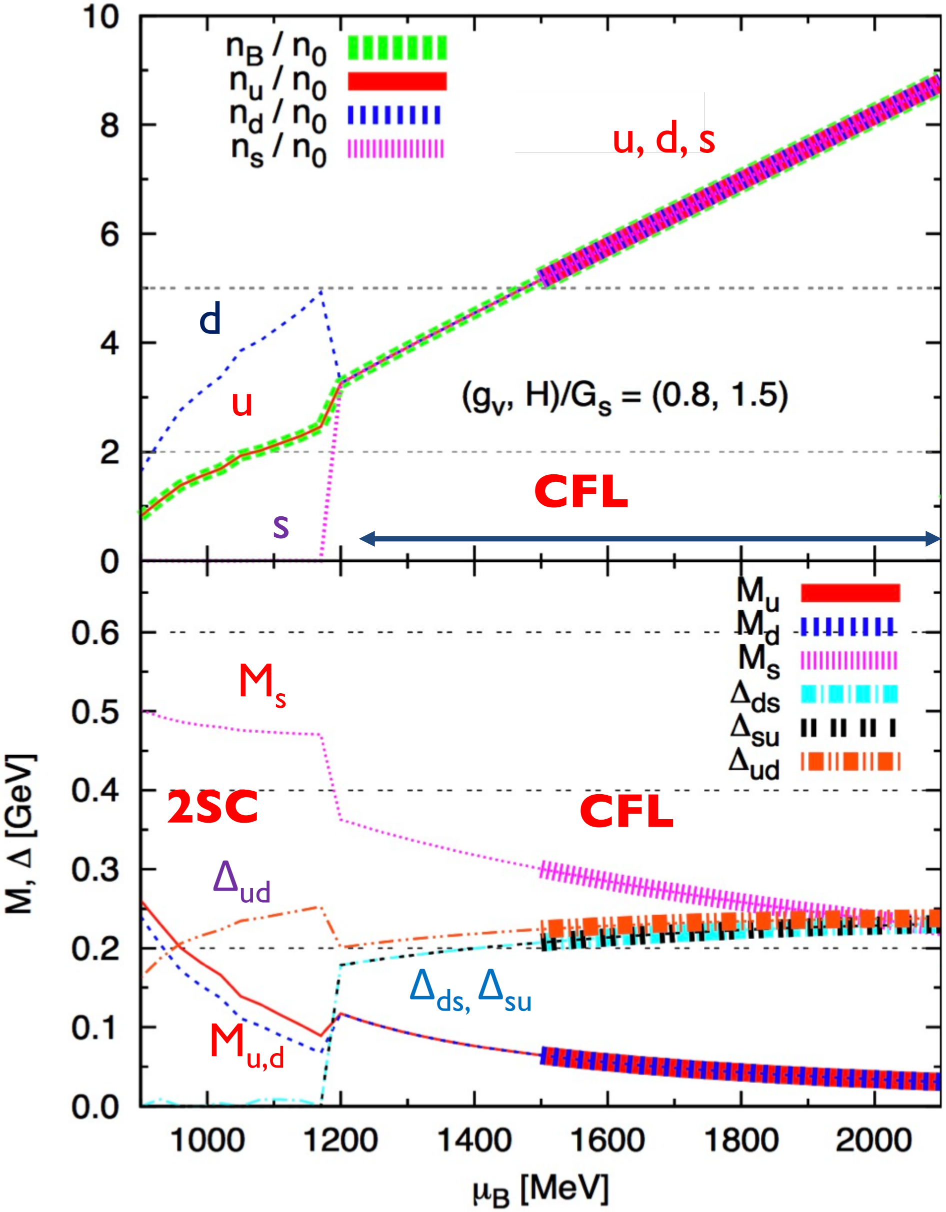
All quarks & gluons are gapped



only u,d paired

gapped and gapless quarks & gluons

Some quark model results [β-equilibrium]



($n_B < 5n_0$ is not trustable)

Remark:

- repulsive forces included
- chiral & diquark condensates coexist
- pairing favors $n_u = n_d = n_s$

many strange quarks!

[more will be explained in Lect. 3]

Summary of lecture 2

1, Interpolation procedure looks innocent,

but many constraints must be taken care.

(thermo., causality, astro & nuclear)

2, How to graphically extract EoS info from P(μ)

3, The nature of chiral restoration strongly depends on

the presence of repulsive interactions;

should be examined when we build models

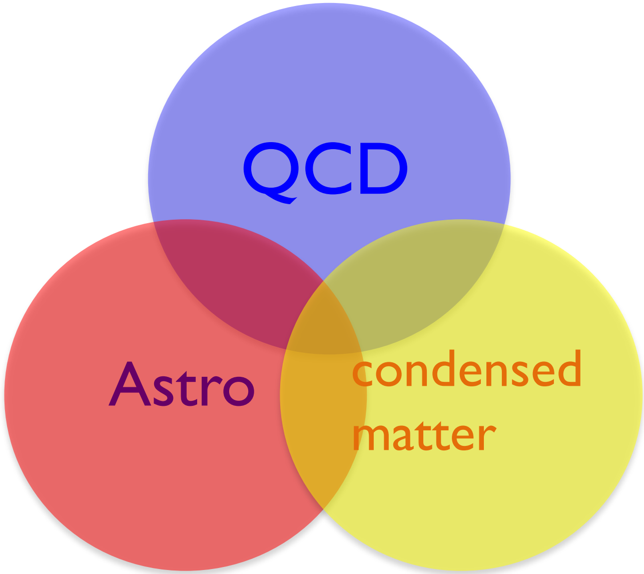
4, Color-superconductivity; theoretically well-motivated

we should include, or should explain why it is not generated...

Lecture 3 : **A quark model & impacts on observables**

Three-window approach to (cold) dense matter: Lect. 3

Toru Kojo (CCNU, Wuhan)



Plan of lectures

1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

3, NS constraints on EoS : hints for **soft**-**stiff** EoS

4, The constraints on P(μ) curves

5, Order parameter & symmetry

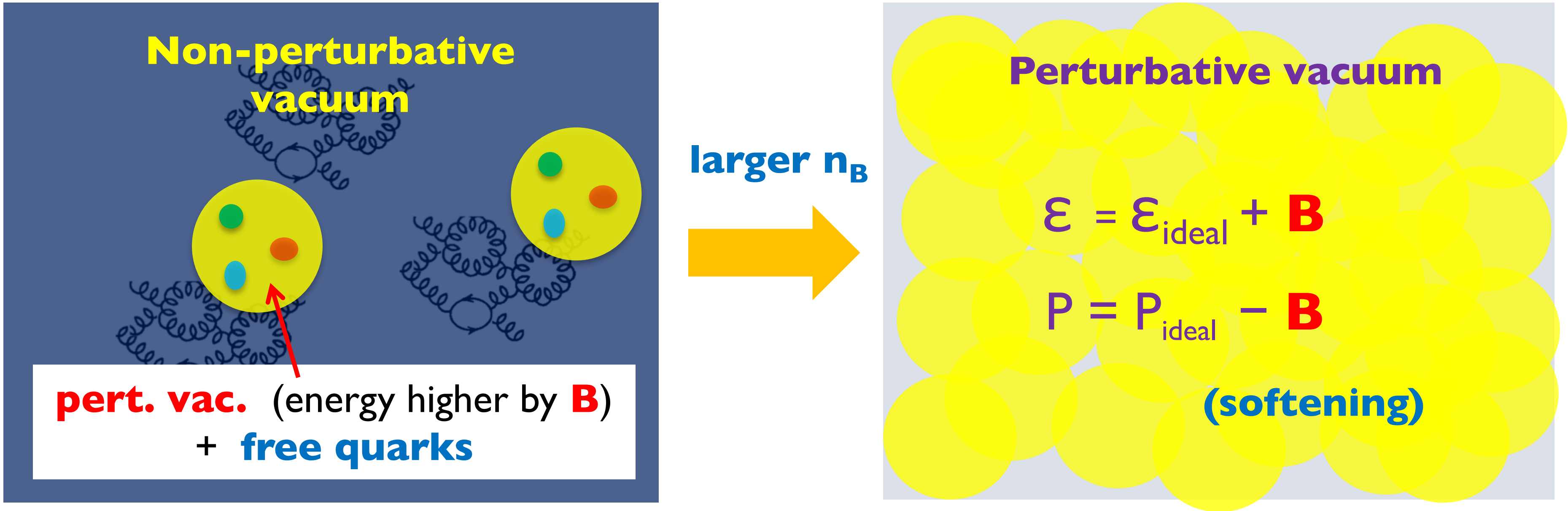
6, Chiral sym. restoration & color-superconductivity

7, A quark model : delineating the properties of matter

8, The astrophysical results from EoS QHC18

A quark model traditionally used in astro-EoS

cf) Chodos et al (1974), MIT bag model



Non-perturbative vacuum

pert. vac. (energy higher by **B**)
\+ **free quarks**

larger n_B

Perturbative vacuum

$$\varepsilon = \varepsilon_{\text{ideal}} + \mathbf{B}$$

$$P = P_{\text{ideal}} - \mathbf{B}$$

(softening)

Several over-simplifications

1, Inside of hadrons is NOT like free media: **ChSB** & constituent quark mass

2, Even in the quark matter regime, interactions are **critically important**

(n_B ~ 100 n_0 is not enough for free gas picture)

"3-window": constituent quarks for hadrons

cf) Manohar-Georgi (1983), Weinberg (2010)

> 1-2 GeV

(< 0.2 fm)

Perturbative

weakly coupled quarks & gluons

~ 1 GeV

(~ 0.2-1 fm)

$\Lambda_\chi \sim 4\pi f_\pi$

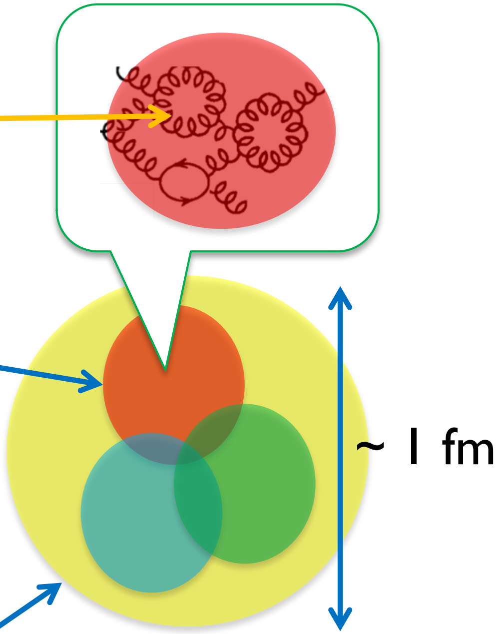
Chiral + OGE

(one-gluon-exchange)

ChSB -> constituent quark mass ~300 MeV

OGE -> int. b.t.w **constituent** quarks

based on **quasi-particle** picture



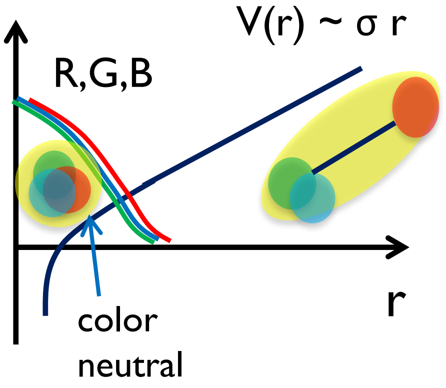
< ~ 0.2 GeV

(> ~ 1 fm)

Confinement

trap quarks to keep color white

quasi-particle gluons
→ **unlikely** generate confining forces



Constituent quark models for hadrons I

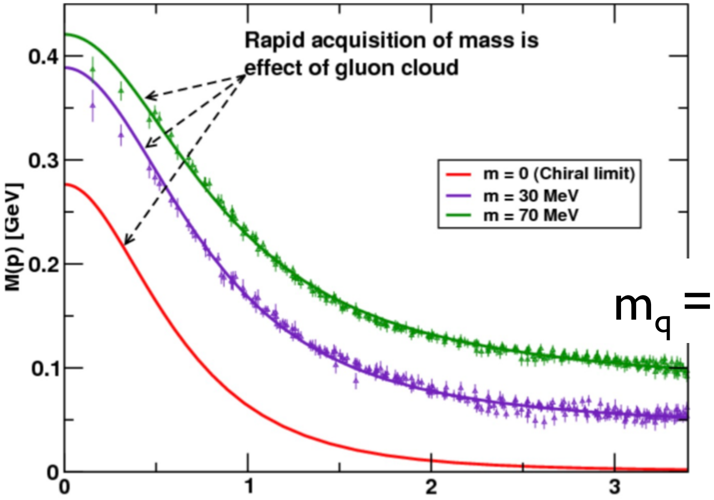
cf) DeRujula-Georgi-Glashow (1975), Isgur-Karl (1978), ...

1, Confining potential put by hand

Even now, no satisfactory analytic derivation... Main info from lattice

2, Constituent quarks assumed: $M_u \sim 350$ MeV, $M_s \sim 540$ MeV

In modern language, produced by dynamical chiral sym. breaking



M(p)

lattice & Dyson-Schwinger

m_q = 70 MeV

30 MeV

0 MeV

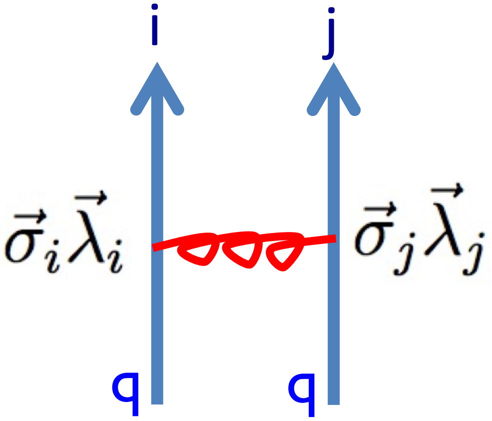
p [GeV]

3, OGE -> **semi-short** range color-**electric** & **magnetic** int.

Constituent quark models for hadrons 2

cf) DeRujula-Georgi-Glashow (1975), Isgur-Karl (1978), ...

Color-**magnetic** interactions : responsible for **level splitting**



non-rela.

color-color **spin-spin**

$$\alpha_s \lambda_i \frac{\vec{\sigma}_i \cdot \vec{q}}{m_i} \frac{1}{q^2} \lambda_j \frac{\vec{\sigma}_j \cdot \vec{q}}{m_j} \simeq \frac{\alpha_s}{\boxed{m_i m_j}} \underline{(\lambda_i \cdot \lambda_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j)}\delta(\vec{r}_{ij})$$

channel dependence

& Fermi statistics → **flavor-flavor correlation**

mag. int. is **enhanced** in **relativistic** regimes

non-rela.

coupling ∝ **velocity** ~ **p/E** (→ **p/M** << 1.)

(sensitive to the quark mass)

Constituent quark models for hadrons 3

cf) DeRujula-Georgi-Glashow (1975), Isgur-Karl (1978), ...

S-wave baryons (octet, decuplet)

constituent quark mass + color-mag. int.

$$M_N = 3M_{u,d} + \langle V_{\mathrm{CM}} \rangle_N \simeq (3 \times 360 - 150)\ \mathrm{MeV} = 930\,\mathrm{MeV}$$

exp) 939 MeV

$$M_\Delta = 3M_{u,d} + \langle V_{\mathrm{CM}} \rangle_\Delta \simeq (3 \times 360 + 150)\ \mathrm{MeV} = 1230\,\mathrm{MeV}$$

exp) 1232 MeV

$$M_{\Sigma,\Lambda} = 2M_{u,d} + M_s + \langle V_{\mathrm{CM}} \rangle_{\Sigma,\Lambda} \simeq (2 \times 360 + 540 - 90)\ \mathrm{MeV} = 1170\,\mathrm{MeV}$$

exp) 1189, 1115 MeV

magnetic moment (octet)

Baryon	Magnetic Moment	quark-model expression	fit
p	2.793 ± 0.000	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_d$	input
n	-1.913 ± 0.000	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	input
Λ	-0.613 ± 0.004	μ_s	input
Σ^+	2.458 ± 0.010	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_s$	2.67
Σ^-	-1.160 ± 0.025	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_s$	-1.09
Σ^0	unknown	$\frac{2}{3}(\mu_u + \mu_d) - \frac{1}{3}\mu_s$	0.79
Ξ^0	-1.250 ± 0.014	$-\frac{1}{3}\mu_u + \frac{4}{3}\mu_s$	-1.43
Ξ^-	-0.651 ± 0.003	$-\frac{1}{3}\mu_d + \frac{4}{3}\mu_s$	-0.49

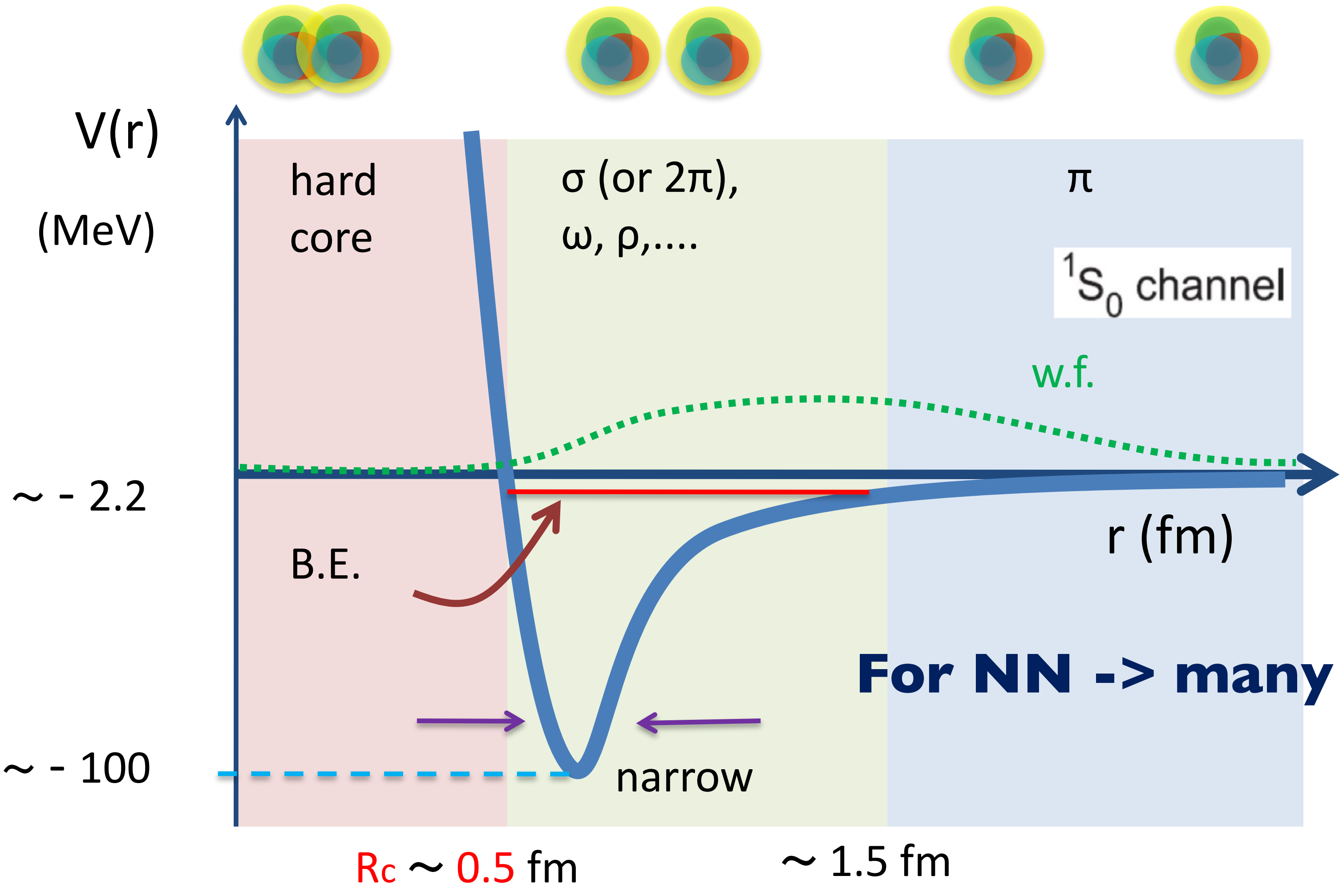
leptonic decay (octet)

Decay	Axial coupling	SU(3) expression	fit
$n \to p\ell\nu$	1.2664 ± 0.0065	$F + D$	1.266
$\Sigma \to \Lambda\ell\nu$	0.602 ± 0.014	$\sqrt{2/3}D$	0.602
$\Lambda \to p\ell\nu$	$-.890 \pm 0.015$	$-\sqrt{3/2}F - \sqrt{1/6}D$	-0.896
$\Sigma \to n\ell\nu$	0.341 ± 0.015	$-F + D$	0.341
$\Xi \to \Lambda\ell\nu$	0.306 ± 0.061	$\sqrt{3/2}F - \sqrt{1/6}D$	0.306
$\Xi \to \Sigma\ell\nu$	0.929 ± 0.0012	$\sqrt{1/2}(F + D)$	0.929

Capture the gross properties of (S-wave) baryons

(~10% accuracy)

"3-window": N-N interactions



For NN -> many data

How about other channels, NY, YY, NΔ, ... ??

Hyperon problems ?

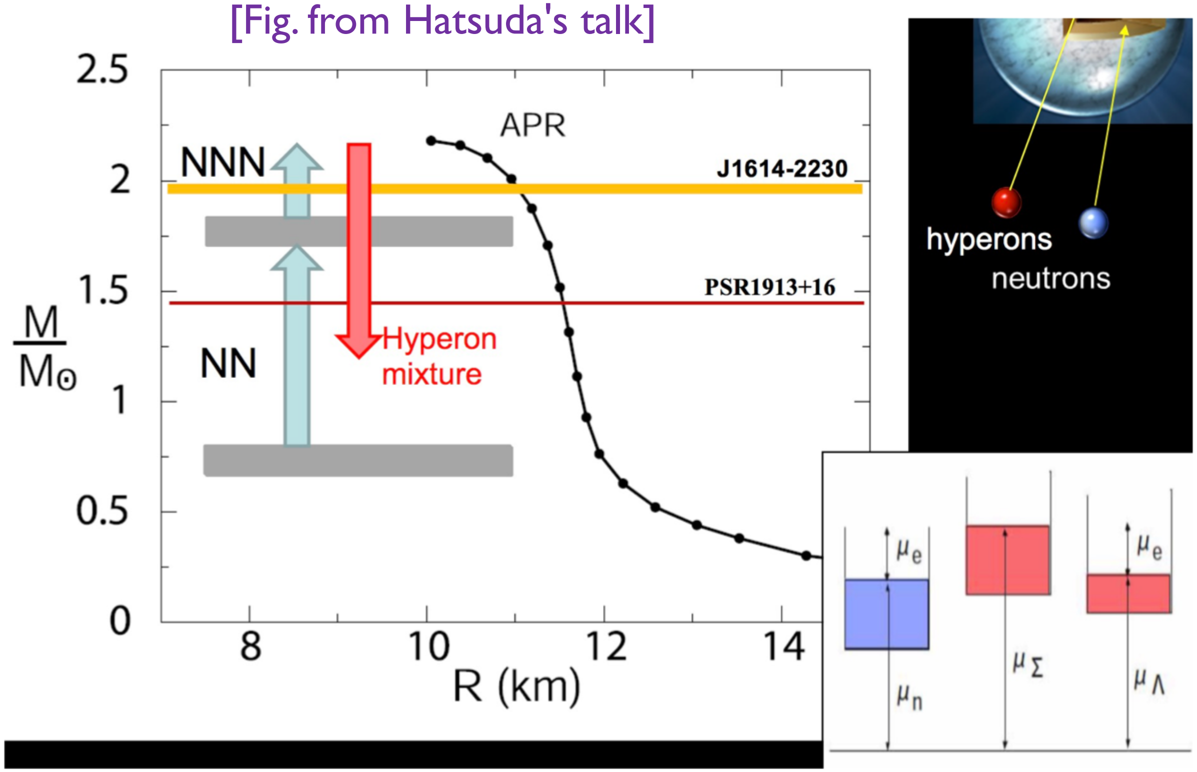
- In **hadronic** regime:

Nuclear matter EoS
with 2- + 3- body int.
pass the $2M_{sun}$ constraint

Naive inclusion **hyperons**
-> add
large rest mass density,
but small pressure

-> **Softening**

[Fig. from Hatsuda's talk]



Typical attempts to avoid the softening

Assume 2- & 3-body YN, YY, YNN,... forces to be **repulsive**

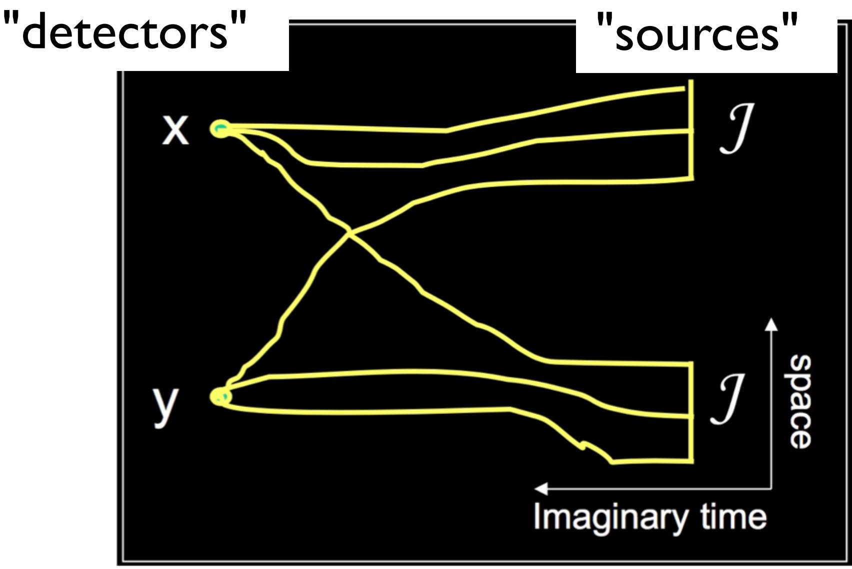
→ forbids hyperons to appear in the range of NSs

Baryon – Baryon interactions

Hyperons, Δ,... are unstable particles -> difficult to prepare in exp.

Lattice QCD : **can switch off** weak int. & decay channels

Scattering experiments on the lattice



$$\langle N_1(\mathbf{x},t) N_2(\mathbf{y},t) \mathcal{J}_1^\dagger(0) \mathcal{J}_2^\dagger(0) \rangle$$

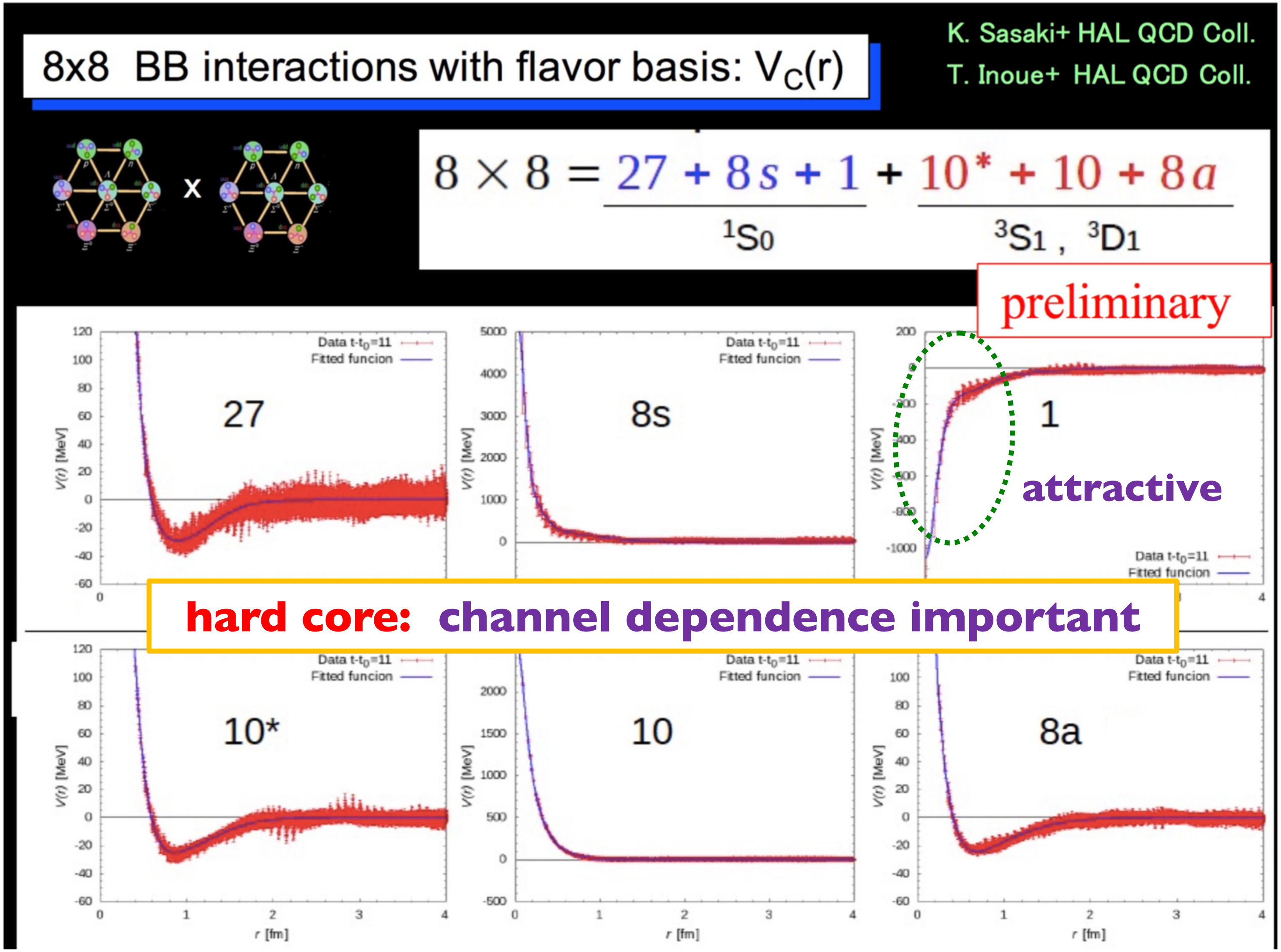
$$= \sum_n \langle 0|N_1(\mathbf{x}) N_2(\mathbf{y})|n\rangle a_n e^{-E_n t}$$

$$\xrightarrow{t>t^*} \phi(\mathbf{r},t) = \sum_{n<n^*} b_n \phi_n(\mathbf{r}) e^{-E_n t}$$

Measure **wave functions at (x,y)** -> **potentials**

Baryon-Baryon int. on a lattice (**$SU_f(3)$ limit**)

[Hatsuda's talk at NFQCD2018]



8x8 BB interactions with flavor basis: $V_C(r)$

K. Sasaki+ HAL QCD Coll.
T. Inoue+ HAL QCD Coll.

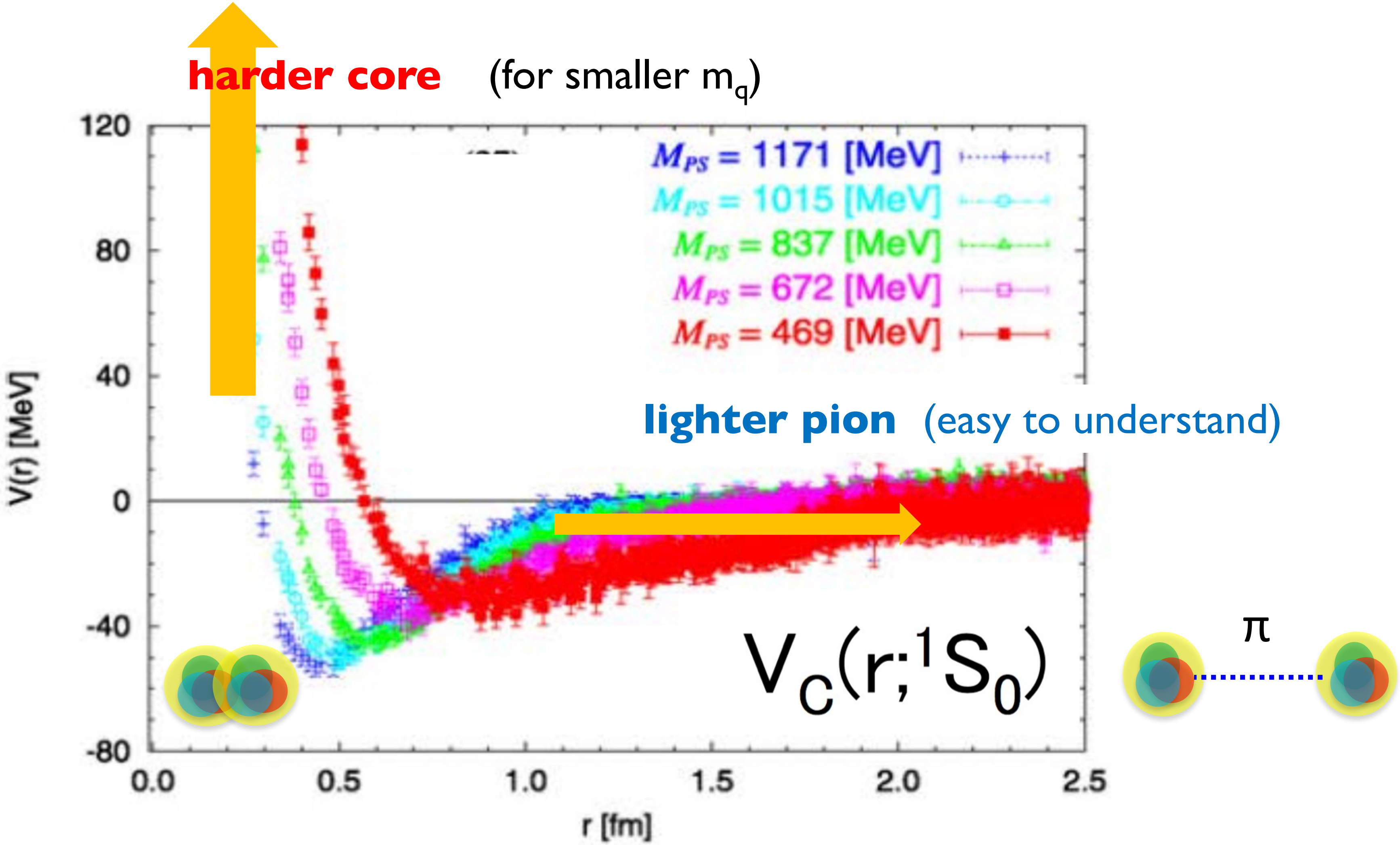
$$8\times 8 = \underset{{}^1S_0}{\underline{27 + 8s + 1}} + \underset{{}^3S_1\,,\ {}^3D_1}{\underline{10^* + 10 + 8a}}$$

preliminary

attractive

hard core: channel dependence important

Mass dependence of NN interactions



harder core (for smaller m_q)

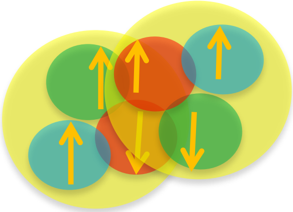
lighter pion (easy to understand)

$V_C(r;{}^1S_0)$

Hard core → due to some relativistic effects?

Quark descriptions for the hard core

cf) Oka-Yazaki (1980),...



6q problem in constituent quark models

Resonating group method (RGM) [Wheeler 1937, Hill-Wheeler 1953]

$$\int \phi_A^{+}(\boldsymbol{\xi}_A)\phi_B^{+}(\boldsymbol{\xi}_B)(H-E)\,\Psi(\boldsymbol{\xi}_A,\boldsymbol{\xi}_B,\boldsymbol{R}_{AB})\,d\boldsymbol{\xi}_A\,d\boldsymbol{\xi}_B=0$$

$$\Psi(\boldsymbol{\xi}_A,\boldsymbol{\xi}_B,\boldsymbol{R}_{AB})=\mathcal{A}[\phi_A(\boldsymbol{\xi}_A)\phi_B(\boldsymbol{\xi}_B)\chi(\boldsymbol{R}_{AB})]$$

scattering problems → phase shift

Findings

1, Quark Pauli blocking : NOT enough for the hard core

2, Color-magnetic interaction is crucial (enhanced at small mass)

3, Hard cores are not universal: attractive for some channels

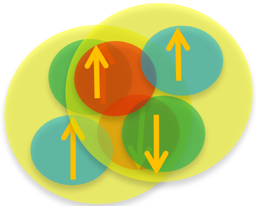
Recent quark model studies for hard cores

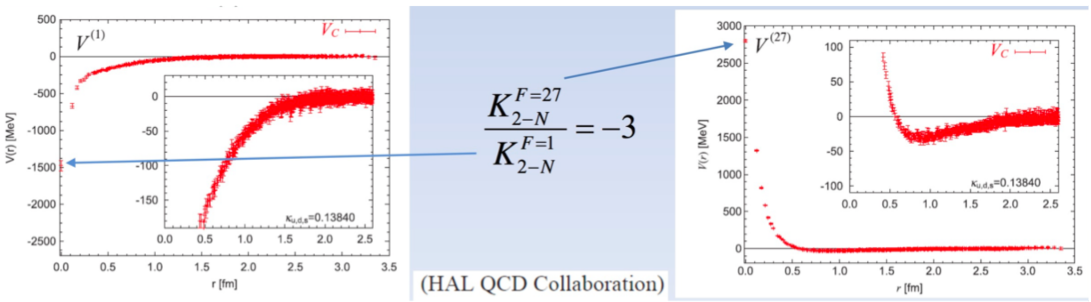
cf) A.Park-W-Park-SuHoungLee (2016),...

evaluate matrix for color-mag. int. for overlapped baryons

→ Channel dep. of the height of the hard core

2-body)





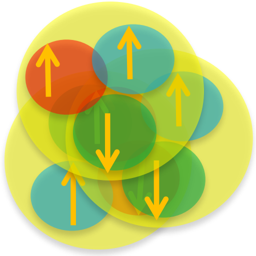
$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$

(HAL QCD Collaboration)

→ semi-quantitative agreement with lattice

3-body)

NNN, NNY are partially investigated



→ overall repulsion, though not universal

See, Su Houng Lee's talk in NFQCD2018 (3rd week), Kyoto

Implications for dense matter

If one accepts the quark model description for hard cores,

the implications would be:

1, Hard core repulsions are weaker for YN & YY than for NN

color-mag. ~ $1/M_iM_j$ $M_{u,d}/M_s \sim 3/5$

2, Short-range int. can be attractive (though relatively rare)

e.g.) H-dibaryon (uds-uds); double Ω (sss-sss),....

Can we block strangeness to $n_B \sim 5n_0$??

3, Mass reduction -> overall enhancement of hard core repulsion

chiral restoration is delayed by the repulsion?

Plan of lectures

1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

3, NS constraints on EoS : hints for **soft**-**stiff** EoS

4, The constraints on P(μ) curves

5, Order parameter & symmetry

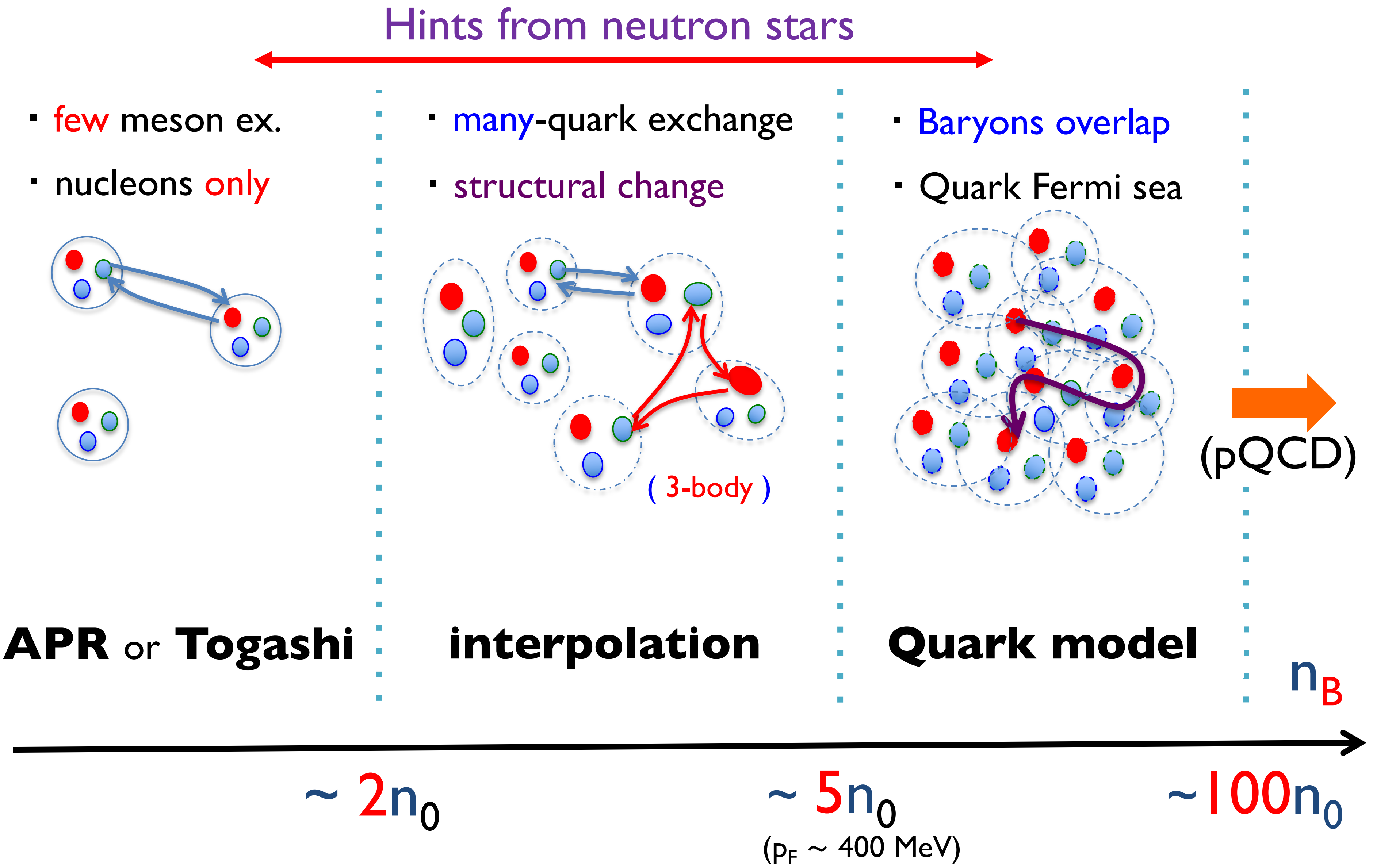
6, Chiral sym. restoration & color-superconductivity

7, A quark model : delineating the properties of matter

8, The astrophysical results from EoS QHC18

Model EoS

Masuda-Hatsuda-Takatsuka 2012
TK-Powell-Song-Baym 2014



Hints from neutron stars

- few meson ex.
- nucleons only

- many-quark exchange
- structural change

(3-body)

- Baryons overlap
- Quark Fermi sea

(pQCD)

APR or **Togashi**

interpolation

Quark model

n_B

$\sim 2n_0$

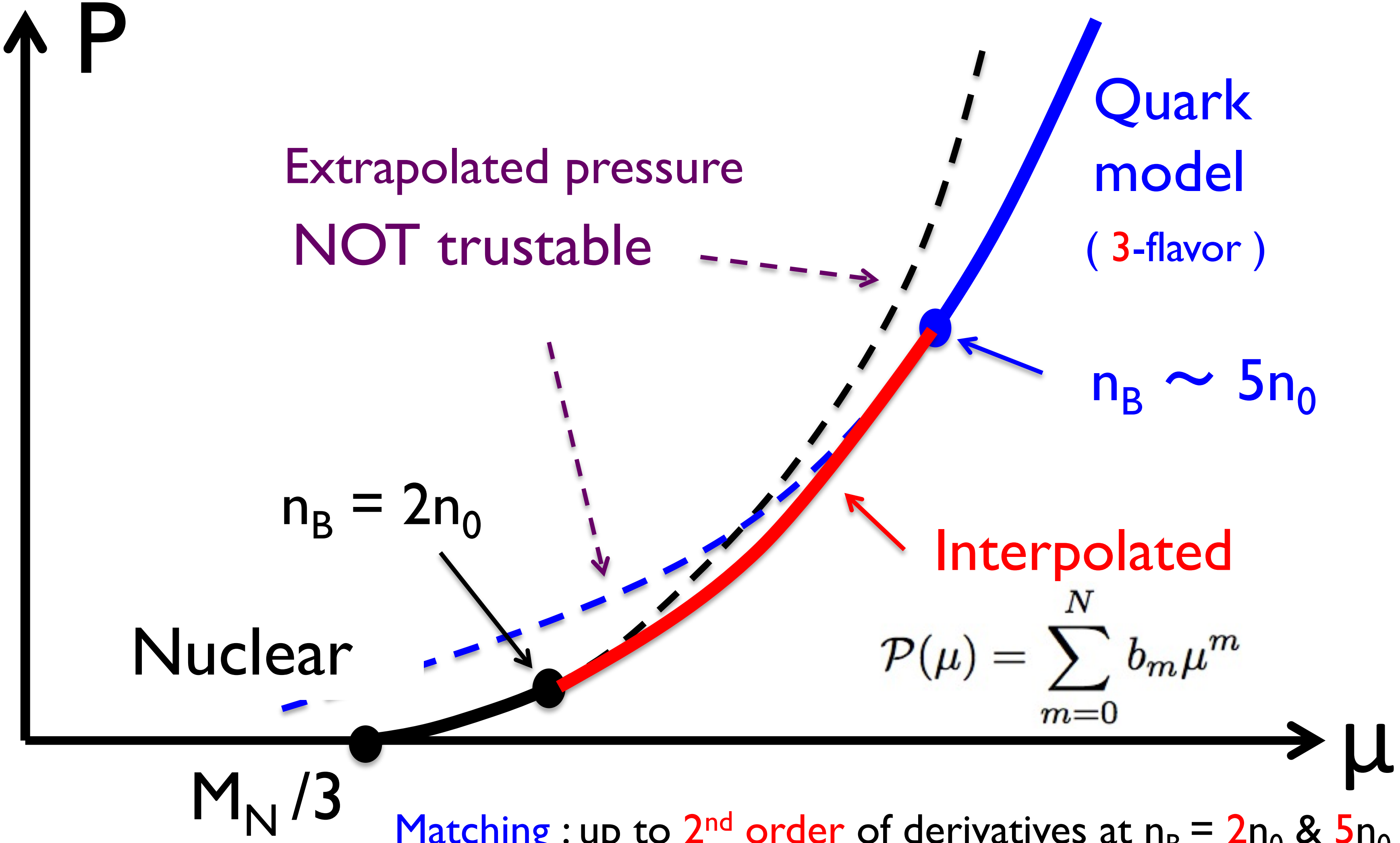
$\sim 5n_0$

$(p_F \sim 400$ MeV)

$\sim 100n_0$

3-window modeling : P vs μ

Masuda+2012, Kojo+2014, ...



$$\mathcal{P}(\mu) = \sum_{m=0}^{N} b_m \mu^m$$

Matching : up to 2nd order of derivatives at $n_B = 2n_0$ & $5n_0$

(if you wish, put a small kink for weak 1st order P.T.)

Nuclear EoS from microscopic potentials

Akmal-Pandheripande-Ravenhall (1998), Togashi et al. (2017)

2N int. : based on exp. NN scattering (**well-determined**)

3N int. : long-range part (2π) & short-range (**phen.**)

Variational calculations with trial many-body w.f.s

We use Togashi-EoS :

(Strategy similar to APR, several important differences)

Unified description from the **crust** to **nuclear liquid**
(no matching procedure needed)

High quality EoS tables

Cover the **wide range** of (Y_e, T)

A **bit softer** than APR to be consistent with sym. energy

Interpolation schemes?

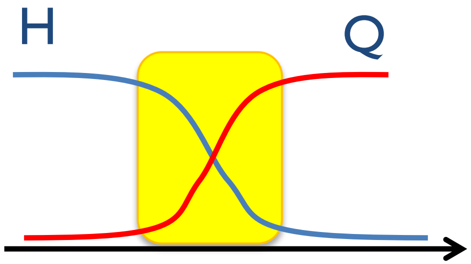
Must choose right variables : P(μ) or ε(n)

scheme 1) use weight functions, **average** P_H and P_Q

$$P_{\rm full}(\mu;\{w\}) = w(\mu) P_H(\mu) + (1-w(\mu))\, P_Q(\mu)$$

advantage: technically easy to get smooth functions

disadvantage: less clear physical interpretations



$$|\Psi\rangle = \sqrt{\alpha}\,|H\rangle + \sqrt{1-\alpha}\,|Q\rangle$$

~ **superposition** of w.f.

scheme 2) use P_H and P_Q **only** as the **boundary conditions**

$$\mathcal{P}(\mu) = \sum_{m=0}^{N} b_m \mu^m$$

for domains b.t.w. $2n_0$ and $5n_0$

(B.C. → coefficients)

$$|\Psi\rangle = |\text{neither } H \text{ nor } Q\rangle$$

~ **single** quantum state

For constituent quarks for matter

cf) Manohar-Georgi (1983), Weinberg (2010)

> 1-2 GeV

(< 0.2 fm)

Perturbative

weakly coupled quarks & gluons

~ 1 GeV

(~ 0.2-1 fm)

$\Lambda_\chi \sim 4\pi f_\pi$

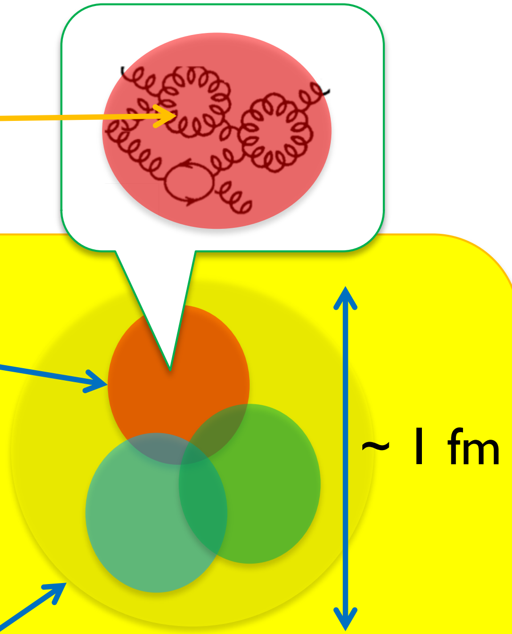
Chiral + OGE

(one-gluon-exchange)

ChSB -> constituent quark mass ~300 MeV

OGE -> int. b.t.w **constituent** quarks

based on **quasi-particle** picture



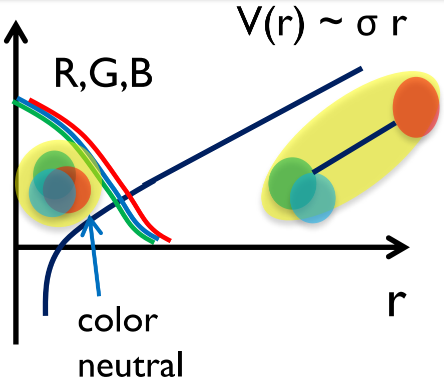
< ~ 0.2 GeV

(> ~ 1 fm)

Confinement

trap quarks to keep color white

quasi-particle gluons
→ **unlikely** generate confining forces



3-flavor quark MF model : template

Kojo+2014

$$\mathcal{H}_{\rm eff} \sim \bar{\psi}\left[-\mathrm{i}\vec{\alpha}\cdot\vec{\partial}+m\right]\psi \;+\; \mathcal{H}_{\rm NJL}^{\rm 4Fermi+KMT}$$

→ ChSB

$+ \;\; \mathcal{H}_{\rm conf}^{3q\to B}$ ➡ will be ignored at $n_B > \sim 5n_0$

mag. part

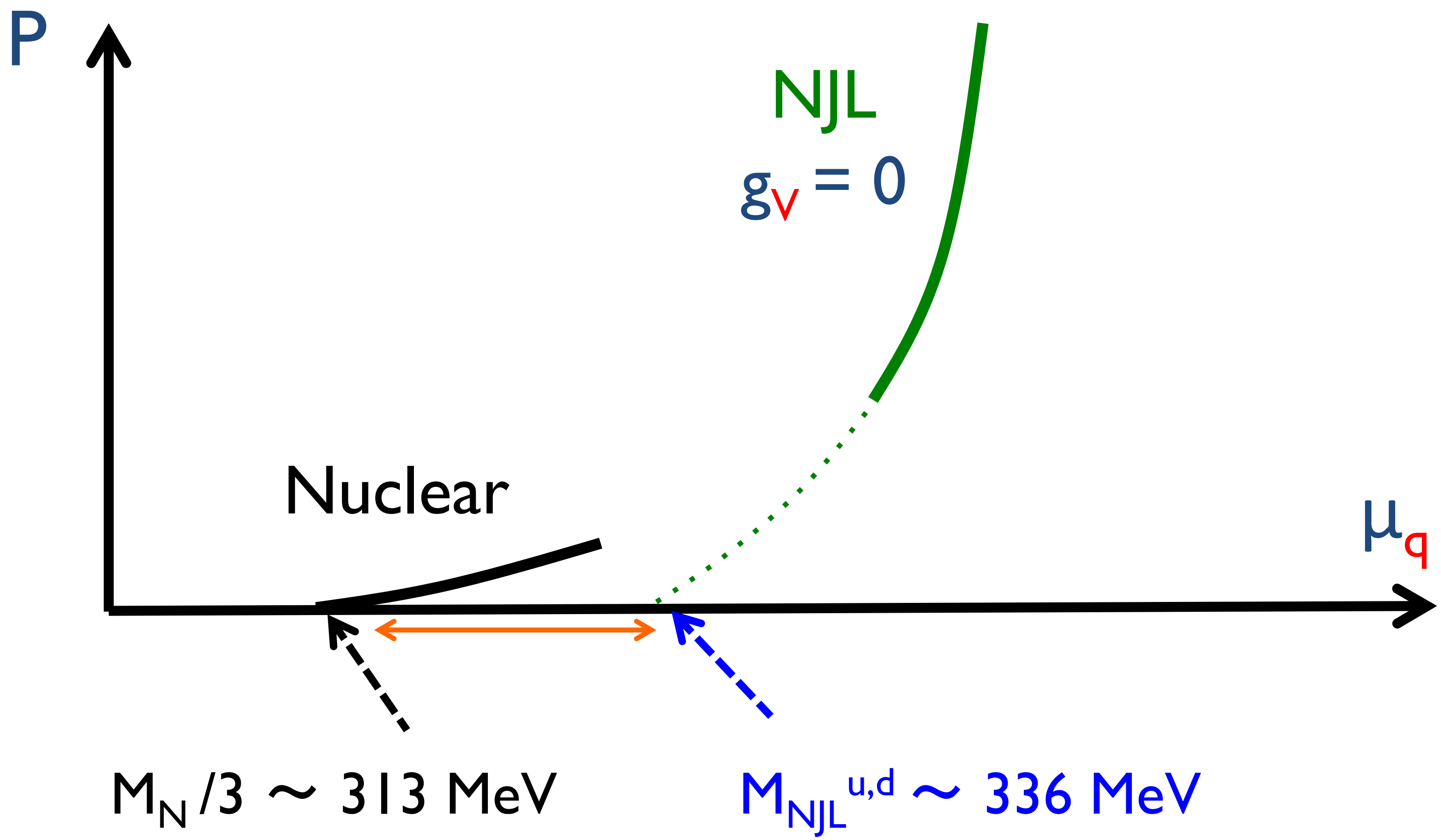
$+ \;\; \mathcal{H}_{\rm OGE}$ ➡ $- H \sum_{A,A'=2,5,7} \left(\bar{\psi}\mathrm{i}\gamma_5\lambda_A\tau_{A'}\psi_c\right)^2$ (attractive)

$+ \;\; \mathcal{H}_{\rm nucl}$ ➡ $+ \; g_V\left(\bar{\psi}\gamma_0\psi\right)^2$ (repulsive)

+ **important** constraints (charge neutrality & β- equilibrium & color-neutrality)

Goal: **Delineate** the properties of matter through (G_s, H, g_V)$_{\textbf{@5-10n0}}$

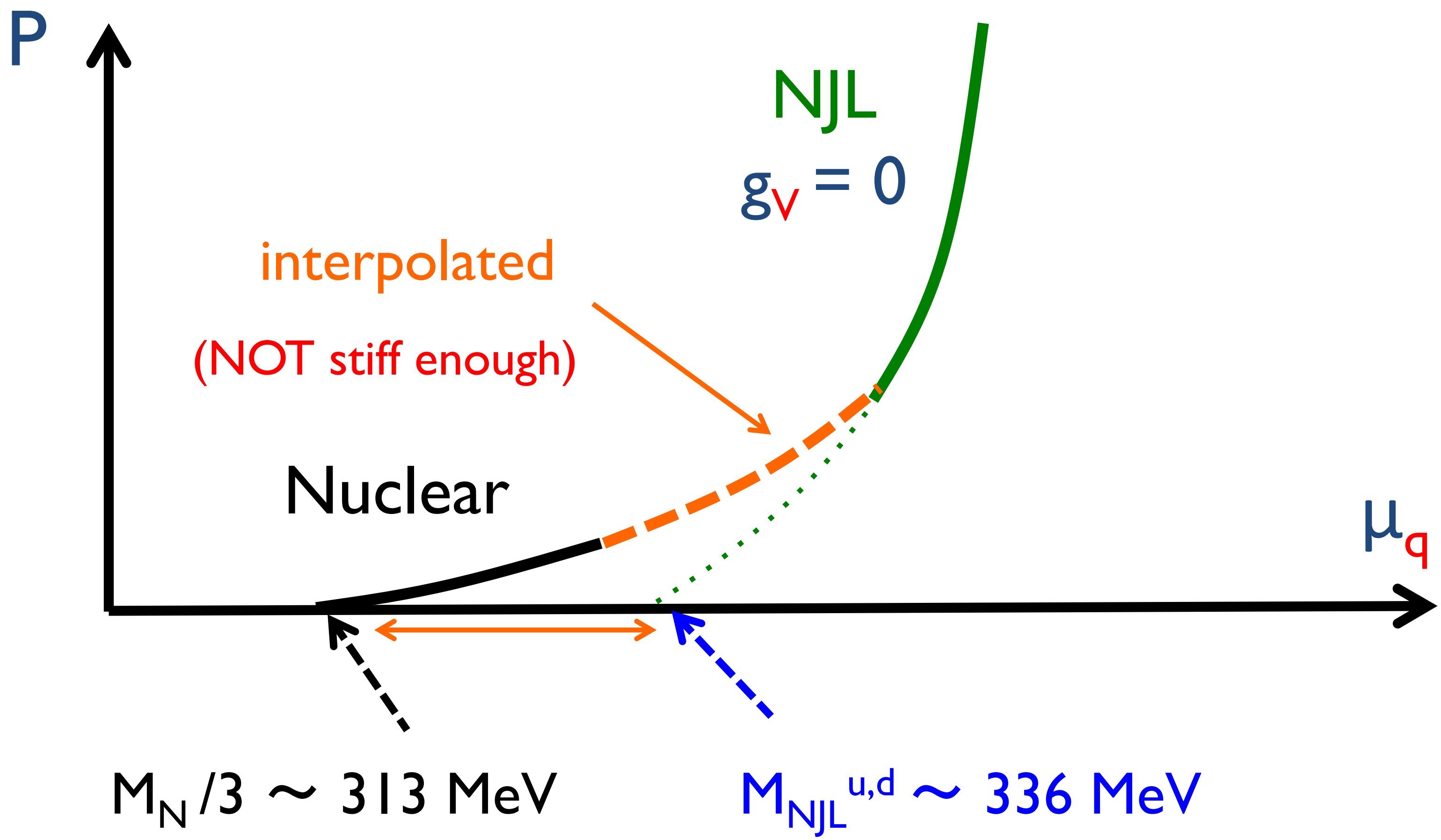
minimal



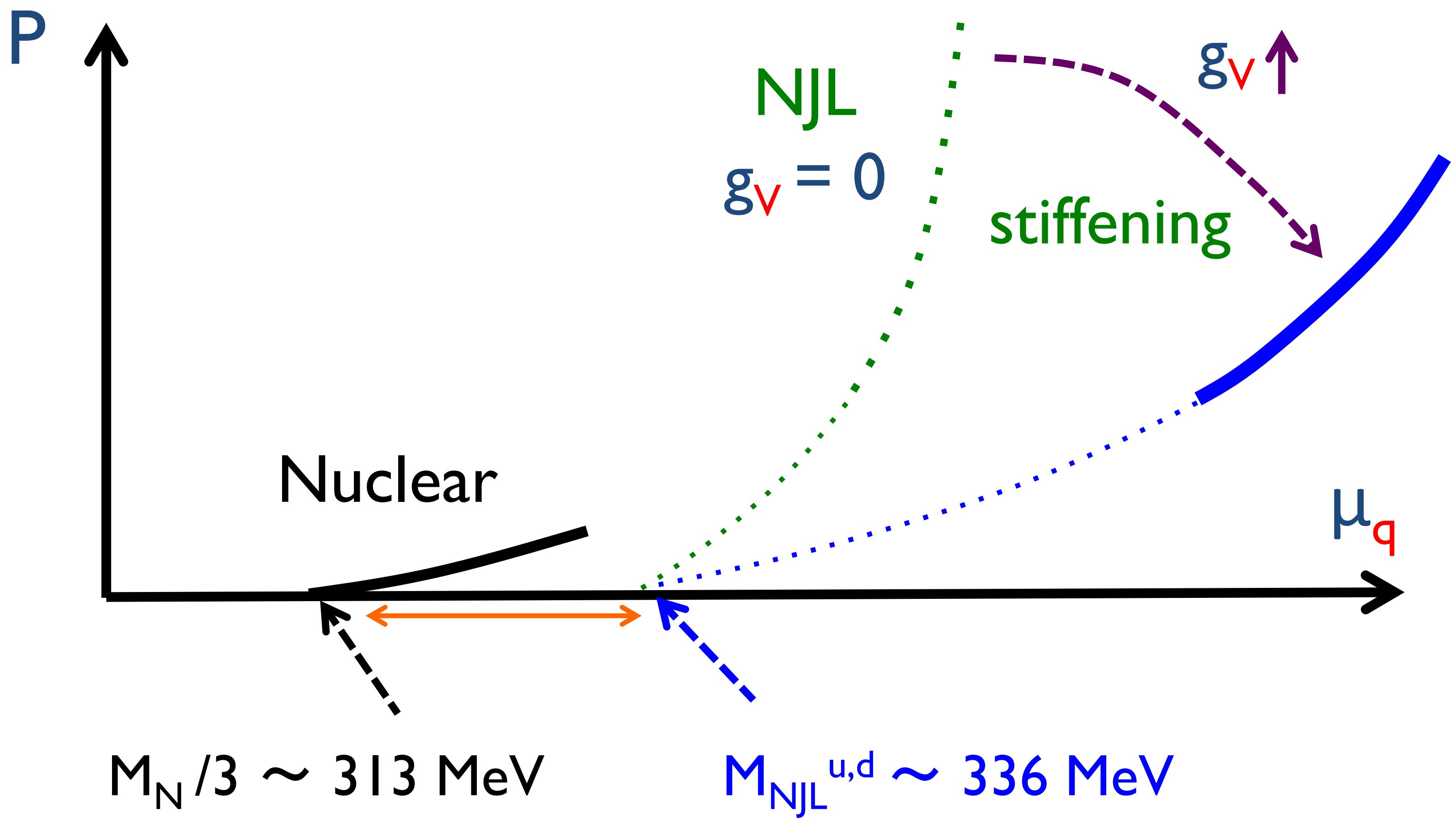
$M_N/3 \sim 313$ MeV

$M_{NJL}^{u,d} \sim 336$ MeV

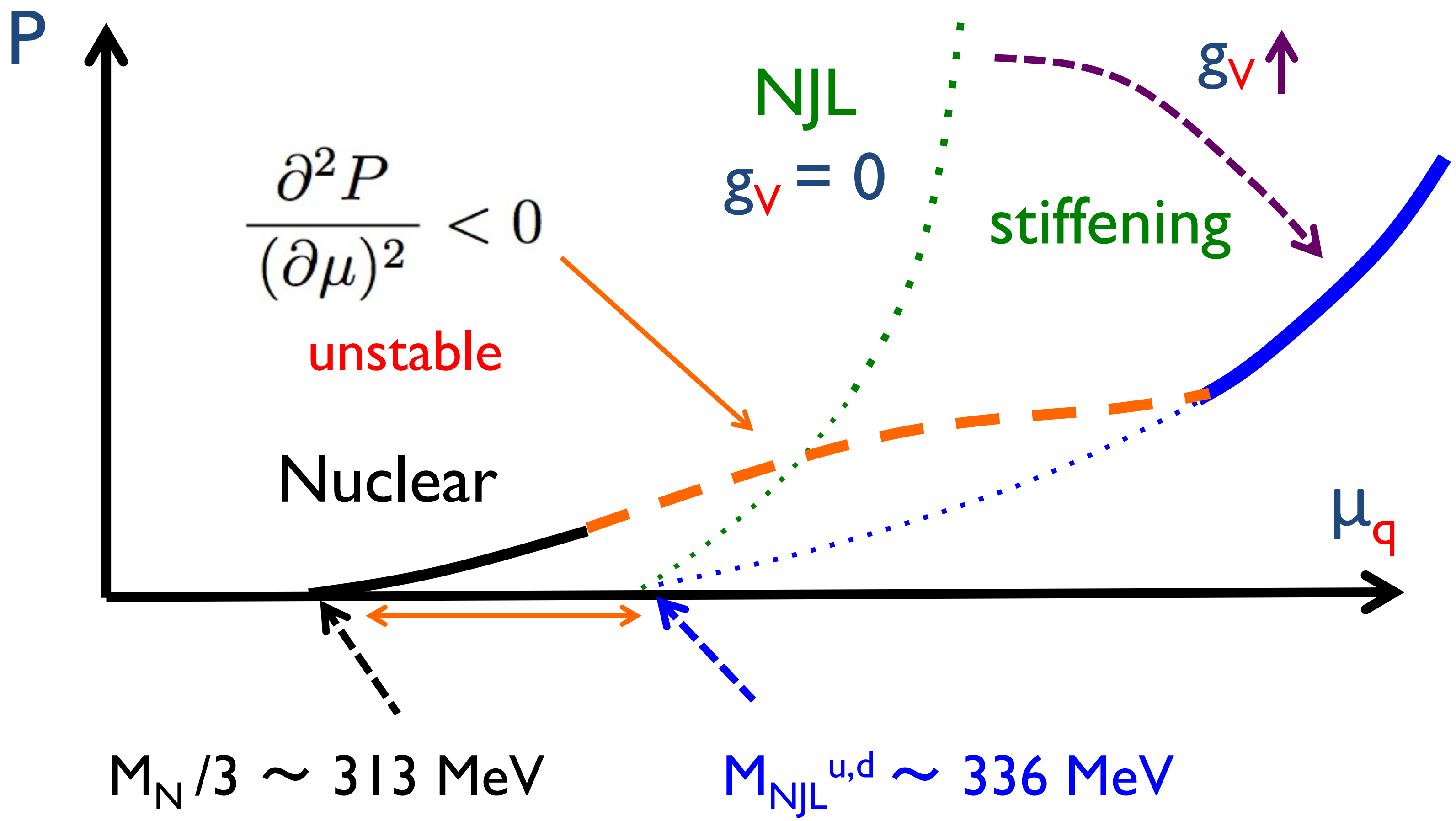
minimal


P
NJL
$g_V = 0$
interpolated
(NOT stiff enough)
Nuclear
μ_q
$M_N /3 \sim 313$ MeV
$M_{NJL}{}^{u,d} \sim 336$ MeV

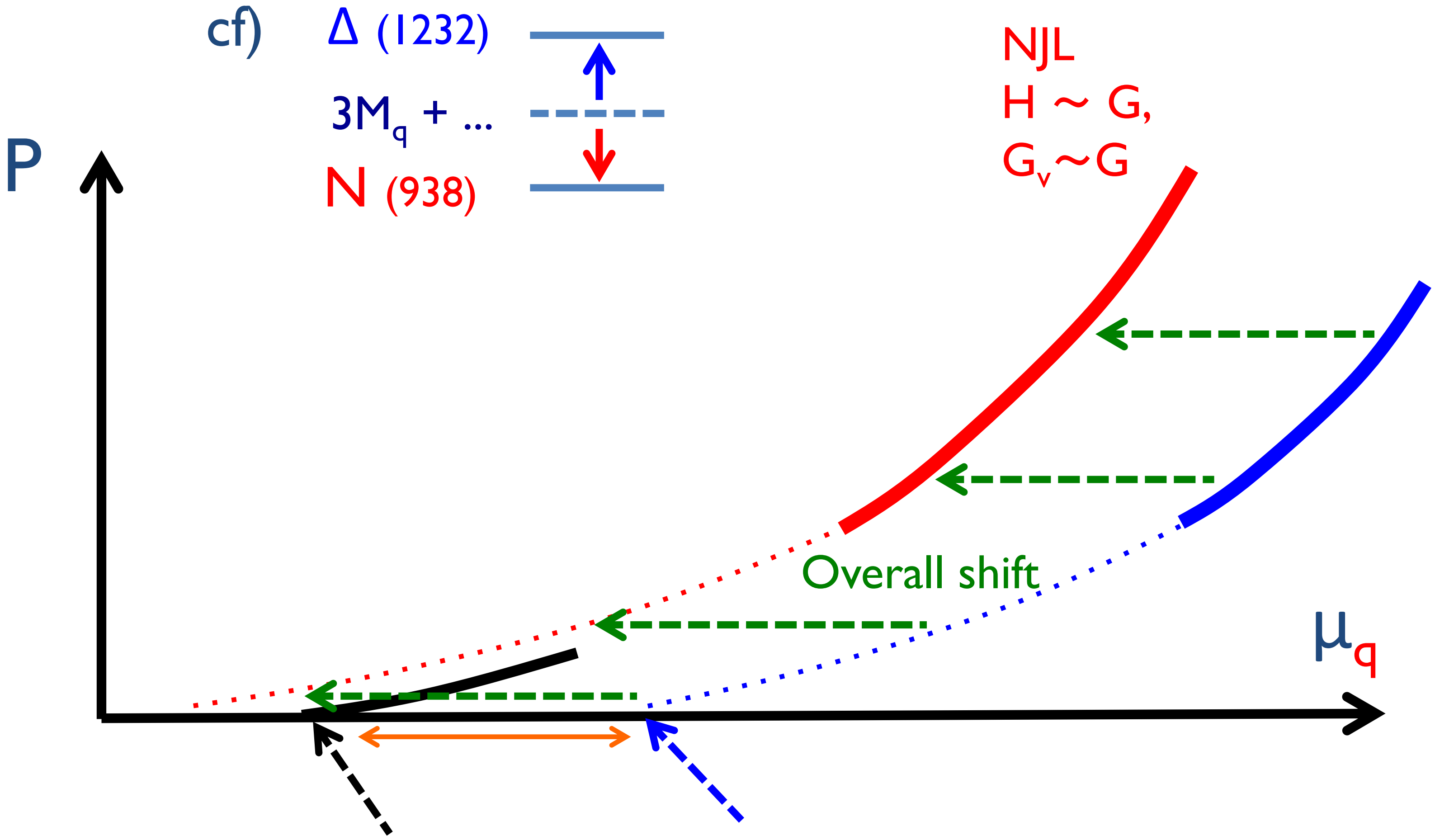
minimal + vector int.



minimal + vector int.



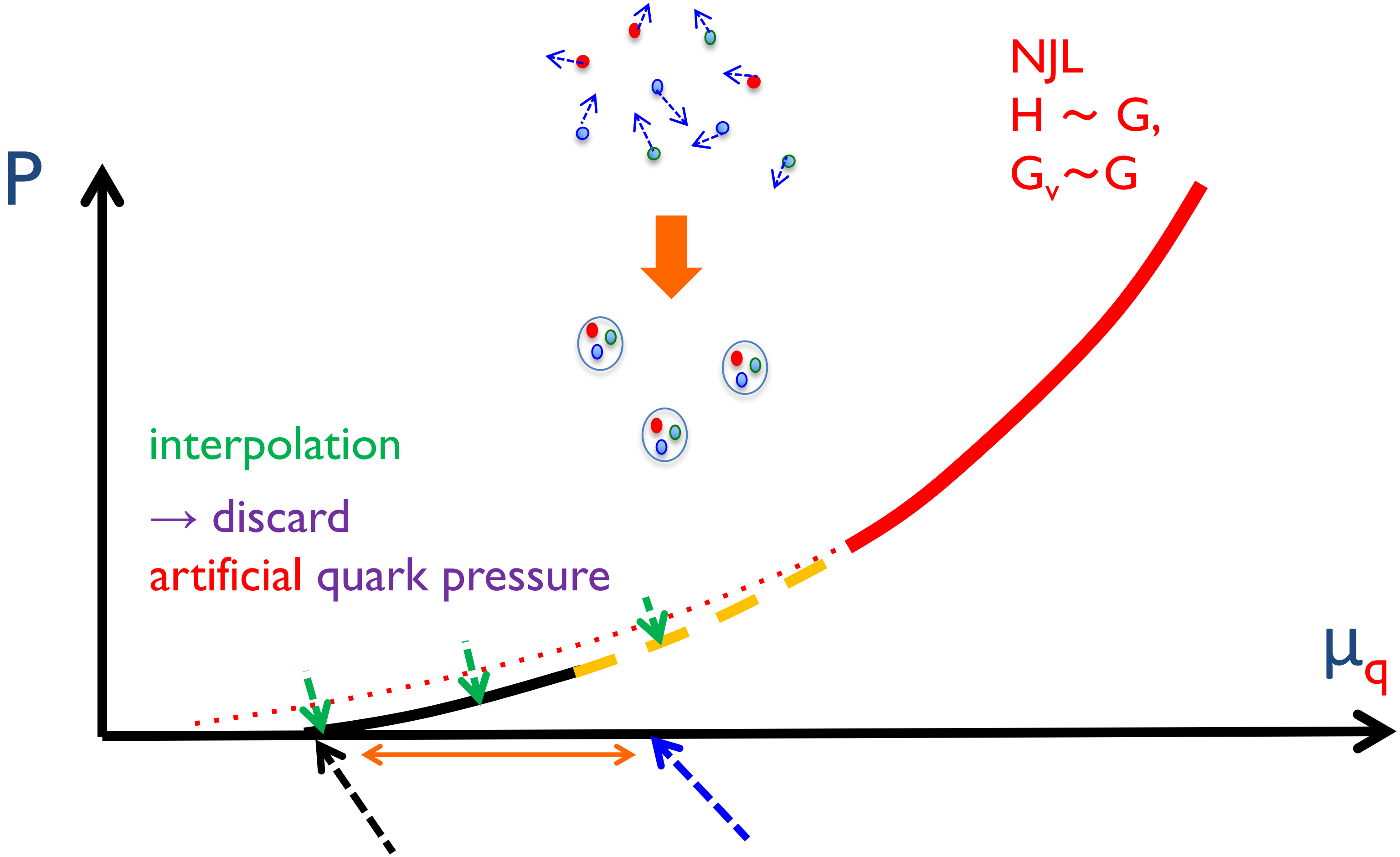
+ attractive color-magnetic int.



$M_N/3 \sim 313$ MeV

$M_{NJL}^{u,d} \sim 336$ MeV

+ confinement in dilute matter



NJL
$H \sim G$,
$G_v \sim G$

interpolation

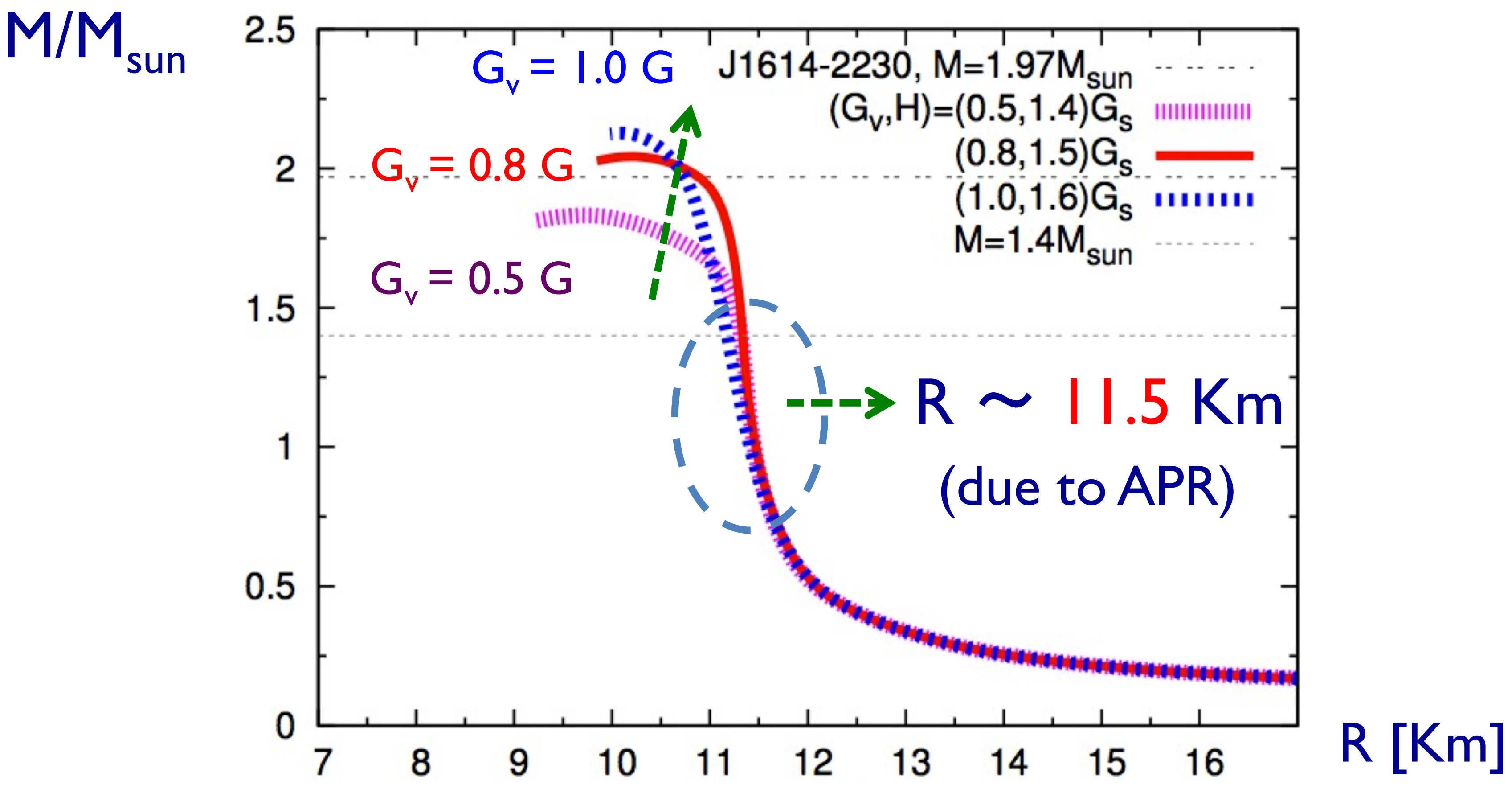
→ discard

artificial quark pressure

$M_N/3 \sim 313$ MeV

$M_{NJL}^{u,d} \sim 336$ MeV

M-R curves for QHC18



R ∼ 11.5 Km

(due to APR)

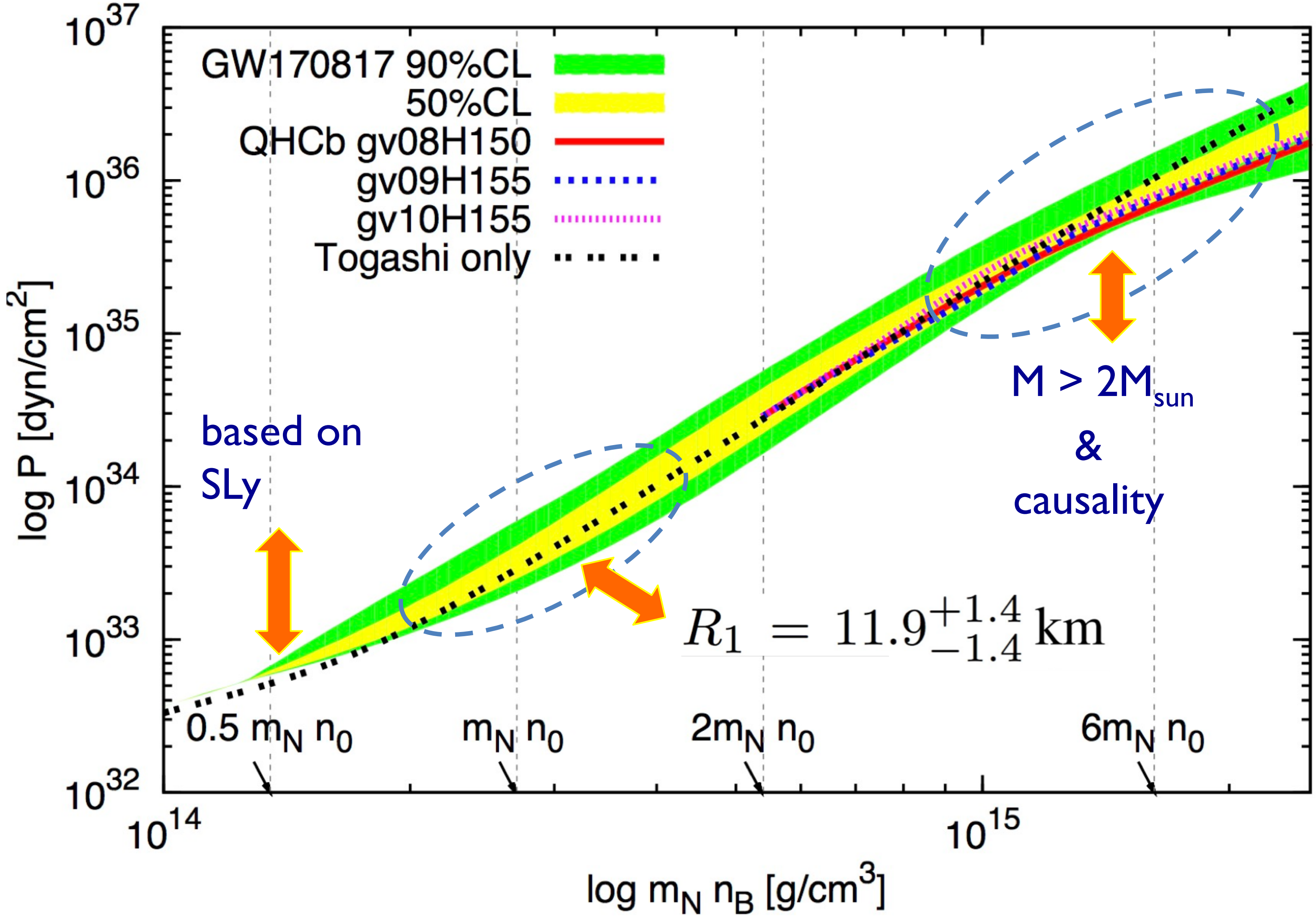
we need :

$$G_s \sim G_v \sim H \quad @ \quad n_B = 5\text{-}10\; n_0 \quad \rightarrow \quad O(G_s^{vac})$$

EoS from aLIGO vs QHC18b

aLIGO & Virgo new analyses for GW170817

arXiv: 1805.11581 [gr-qc]



based on SLy

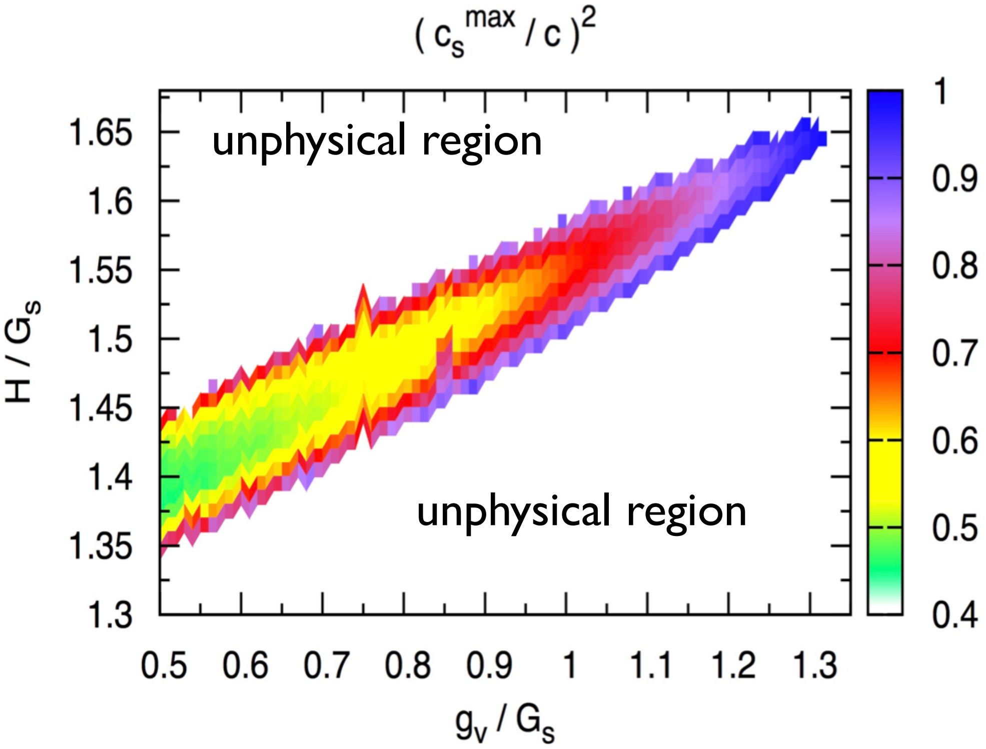
$M > 2M_{sun}$

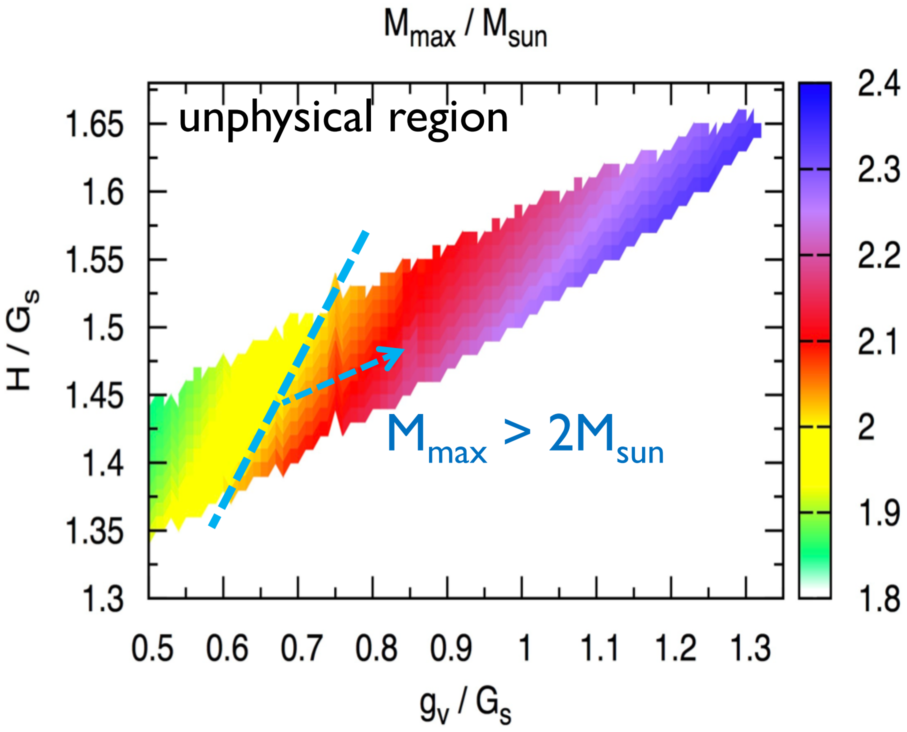
&

causality

$$R_1 = 11.9^{+1.4}_{-1.4}\,\mathrm{km}$$

Constraints -> quark model parameters



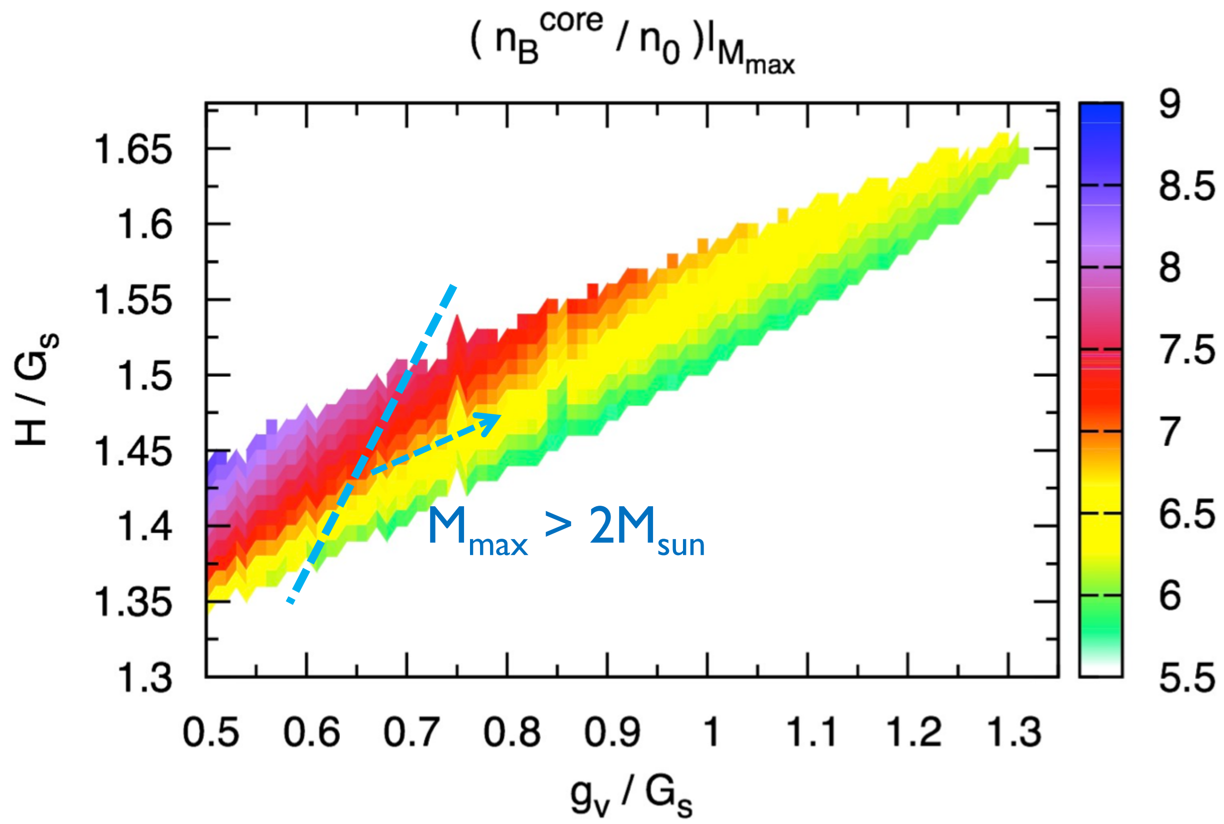


$0.7 < g_v/G_s < 1.3$, $1.4 < H/G_s < 1.65$

(the range of gv & H are tightly correlated)

$M_{max} <$ **2.35** M_{sun} @ $(g_v, H)/G_s = (1.3, 1.65)$

Core baryon density at M_{max} (g_v, H)

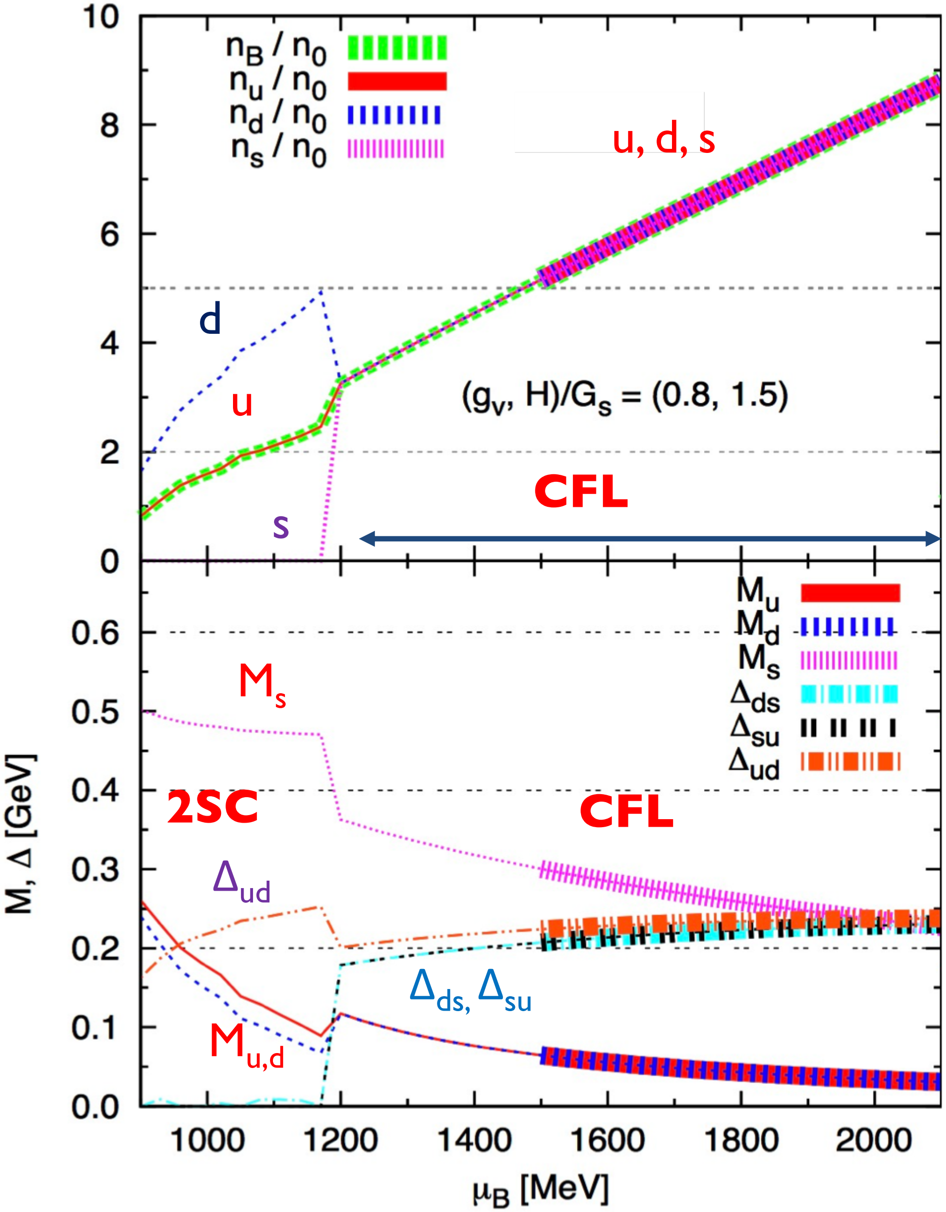


$$\mathbf{5.5} < n_B^{core} / n_0 < \mathbf{7.8}$$

Chance to have quark matter in NSs

Some quark model results

[β-equilibrium]



($n_B < 5n_0$ is not trustable)

Remark:

- pairing favors

$$n_u = n_d = n_s$$

many strange quarks!

- 2SC appears only at $< 5n_0$

Summary of lecture 3

1, 3-window picture of quark models for hadrons.

0.2 – 1.0 GeV : strong coupling (-> ChSB) but quasi-particle picture.

2, Quark models for the short-distance behavior of BB int.

reasonable descriptions by Fermi statistics + color-mag int.

hard core repulsion is not universal

3, Correlations b.t.w Chiral + OGE int. & NS structures

4, Mapping out NS constraints onto microscopic parameters
-> $g_v \sim H \sim G_s$ at 5-10 n_0 : strongly correlated as expected

5, at M_{max} ; the core density -> 5.5 – 7.8 n_0

Summary of lecture 1-3

1, NS & nuclear studies -> Hints for soft-stiff EoS.

(**stay tuned**)

2, Quasi-particle picture unlikely for 2-5n_0.

3, Unified picture necessary : BB-int, hadron physics, matter physics.

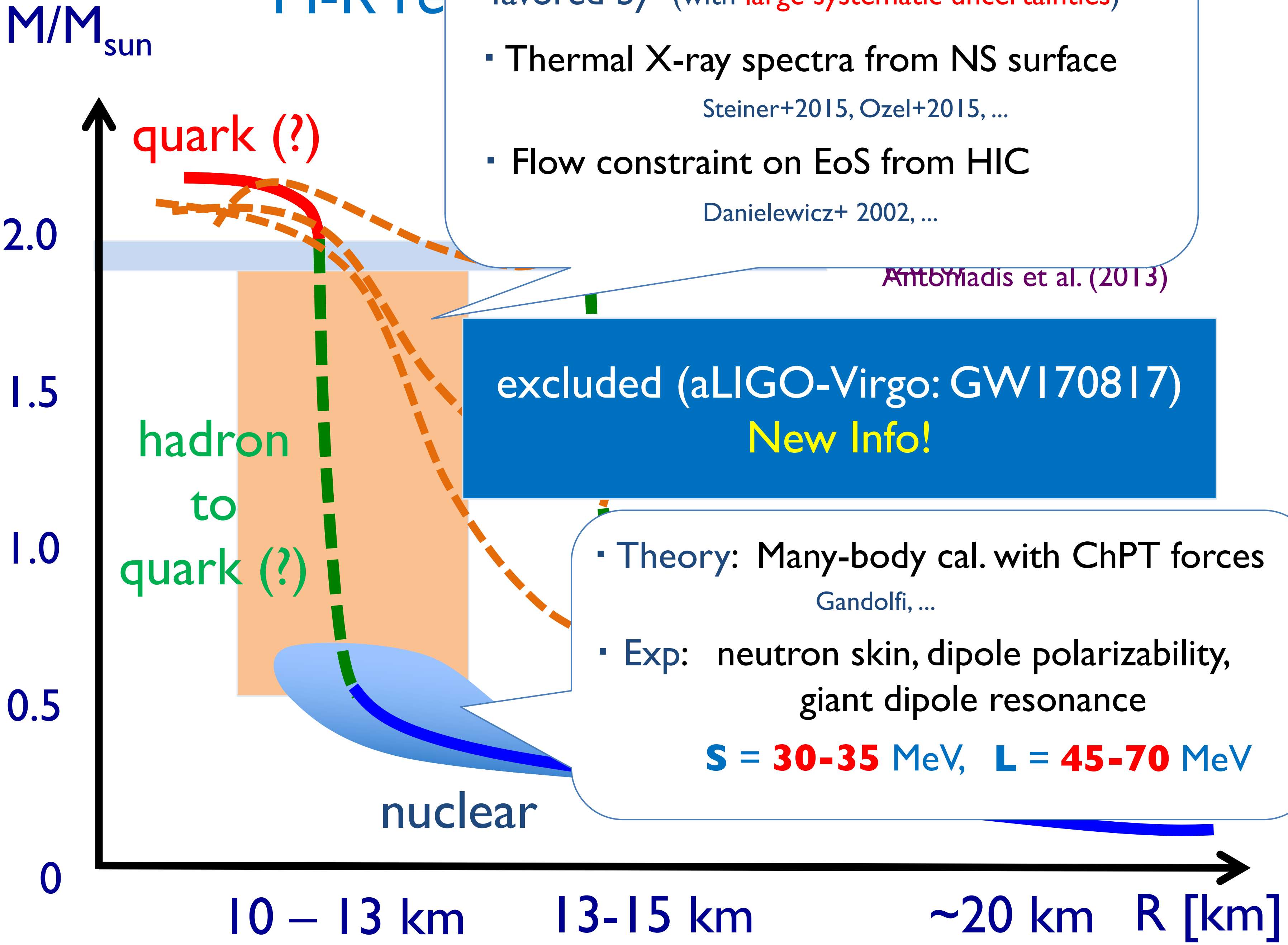
Topics important but not addressed

Beyond mean-field calculations & renormalization of UV divergences

Goldstone modes

Warm EoS & impact of lepton fractions

M-R re



favored by (with large systematic uncertainties)

- Thermal X-ray spectra from NS surface

Steiner+2015, Ozel+2015, ...

- Flow constraint on EoS from HIC

Danielewicz+ 2002, ...

excluded (aLIGO-Virgo: GW170817)
New Info!

- Theory: Many-body cal. with ChPT forces

Gandolfi, ...

- Exp: neutron skin, dipole polarizability, giant dipole resonance

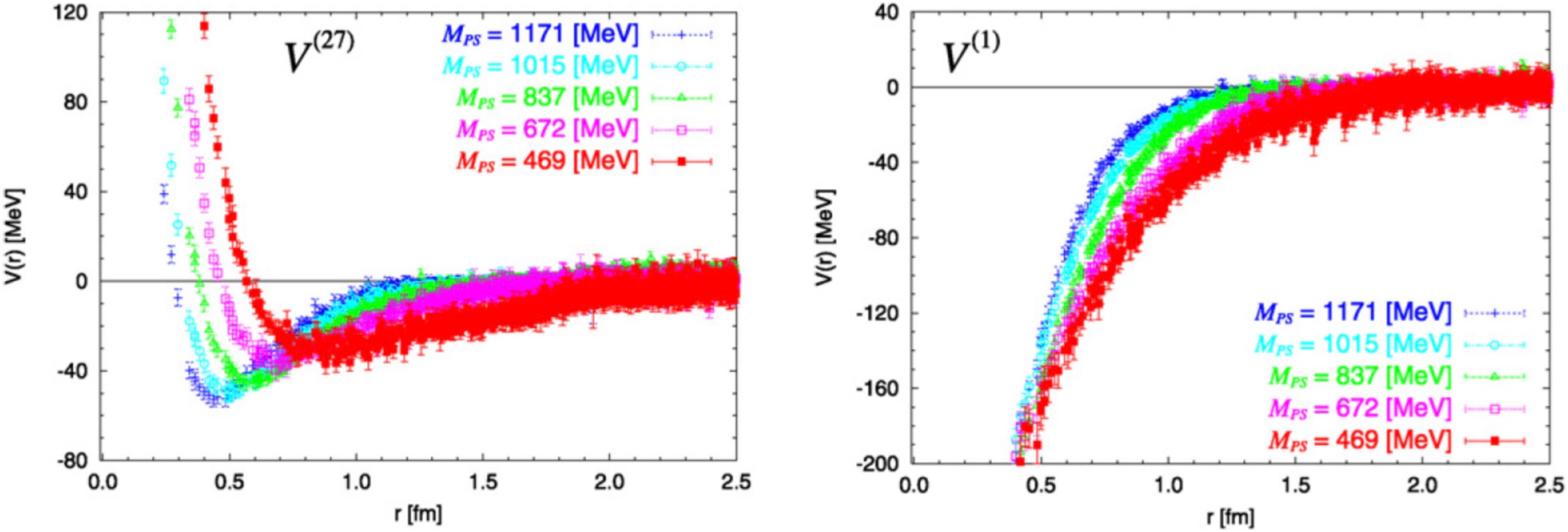
S = **30-35** MeV, **L** = **45-70** MeV

Hard core is not universal

consistent with 6q calculations in constituent quark models;

Pauli-blocking x color magnetic interactions (Oka-Yazaki)

uud-uud Figs. from HAL QCD 2011 **uds-uds**



Can we block the appearance of
the strangeness to $n_B \sim 5n_0$??

"3-window" picture for quark model

Manohar-Georgi (1983), Weinberg

A model of quarks, gluons & pions

Perturbative ($\Lambda_{pert} > 1\text{-}2$ GeV)

Chiral ($\Lambda_\chi \sim 4 \pi f_\pi \sim 1$ GeV)

Confinement ($\Lambda_{conf} \sim 0.2$ GeV)

Quark-Hadron continuity (some history)

- 1, Percolation picture Baym-Chin 1978; Satz-Karsch 1979,...

- 2, In the context of color-superconductivity (CSC) Schafer-Wilczek 1998

symmetry: hadron super fluidity ~ color-flavor-locked (CFL) phases

same order parameters : $\langle BB \rangle \sim \langle (qqq)^2 \rangle$

color singlet, but break $U(1)_B$; chiral sym. is also broken

confinement-Higgs complementarity Fradkin-Shenkar 1979

dynamics: the interplay between chiral & diquark

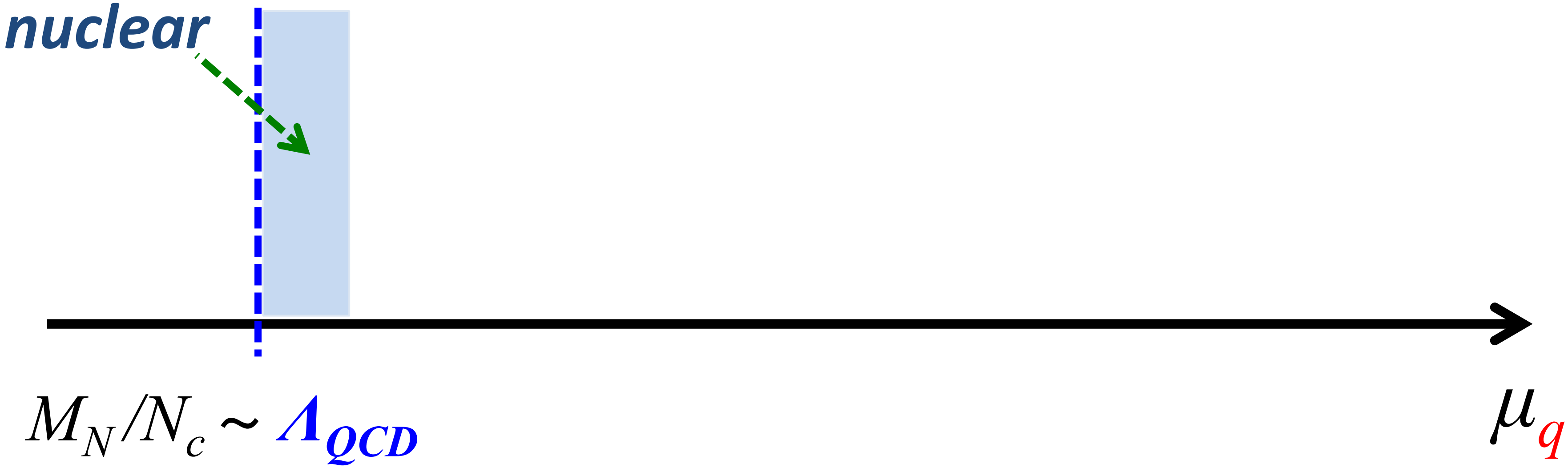
proposal of double CEP Kitazawa+ 2002; Hatsuda+2006; Zhang+ 2009, ...

- 3, Inferred from the NS constraints (for $2n_0 - 5n_0$) Masuda+2012, Kojo+2014, ...

soft-stiff EoS & causality → **crossover** or **weak** 1st order

McLerran-Pisarski’s picture

[McLerran-Pisarski ’07]



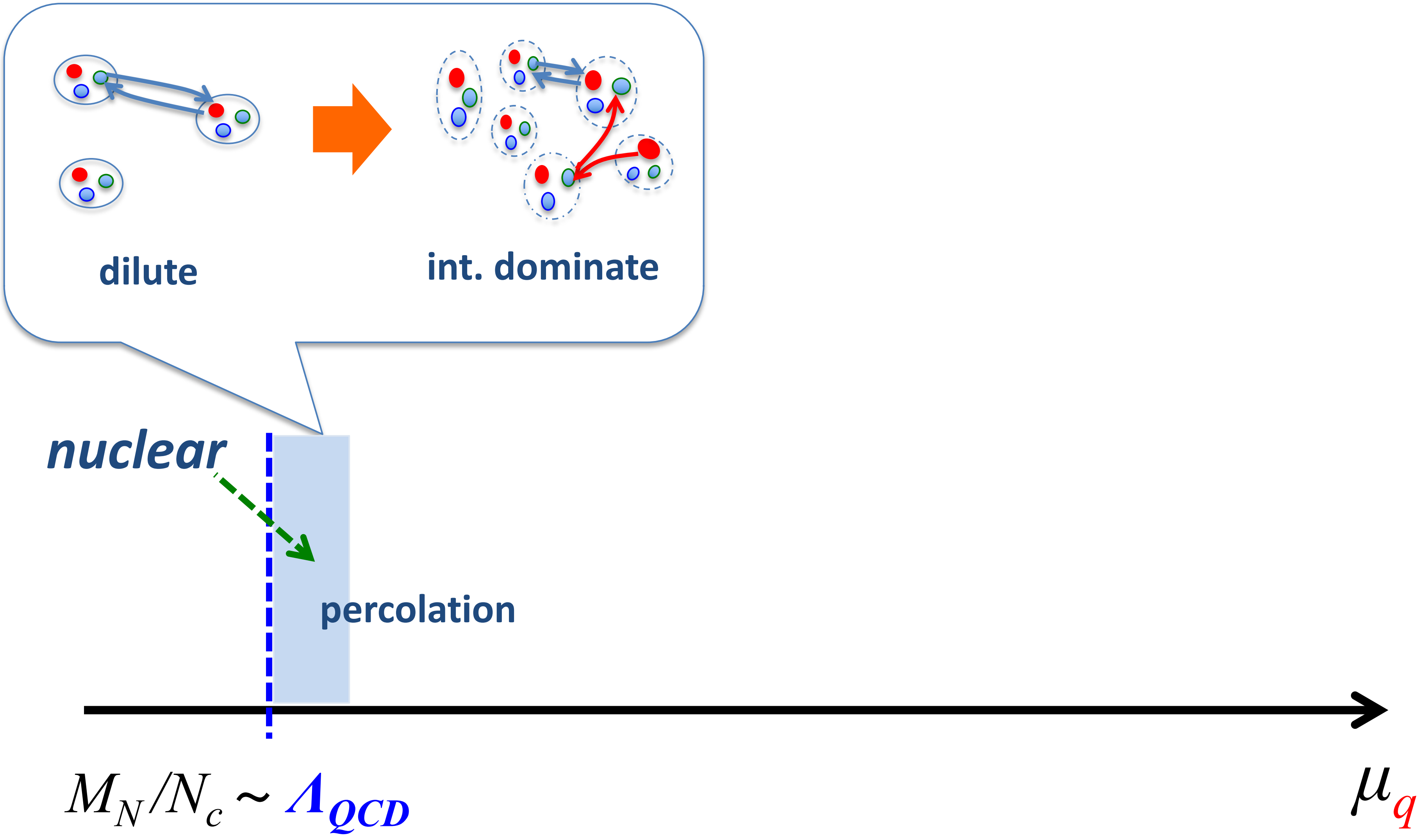
nuclear

$M_N/N_c \sim \Lambda_{QCD}$

μ_q

McLerran-Pisarski's picture

[McLerran-Pisarski '07]



dilute

int. dominate

nuclear

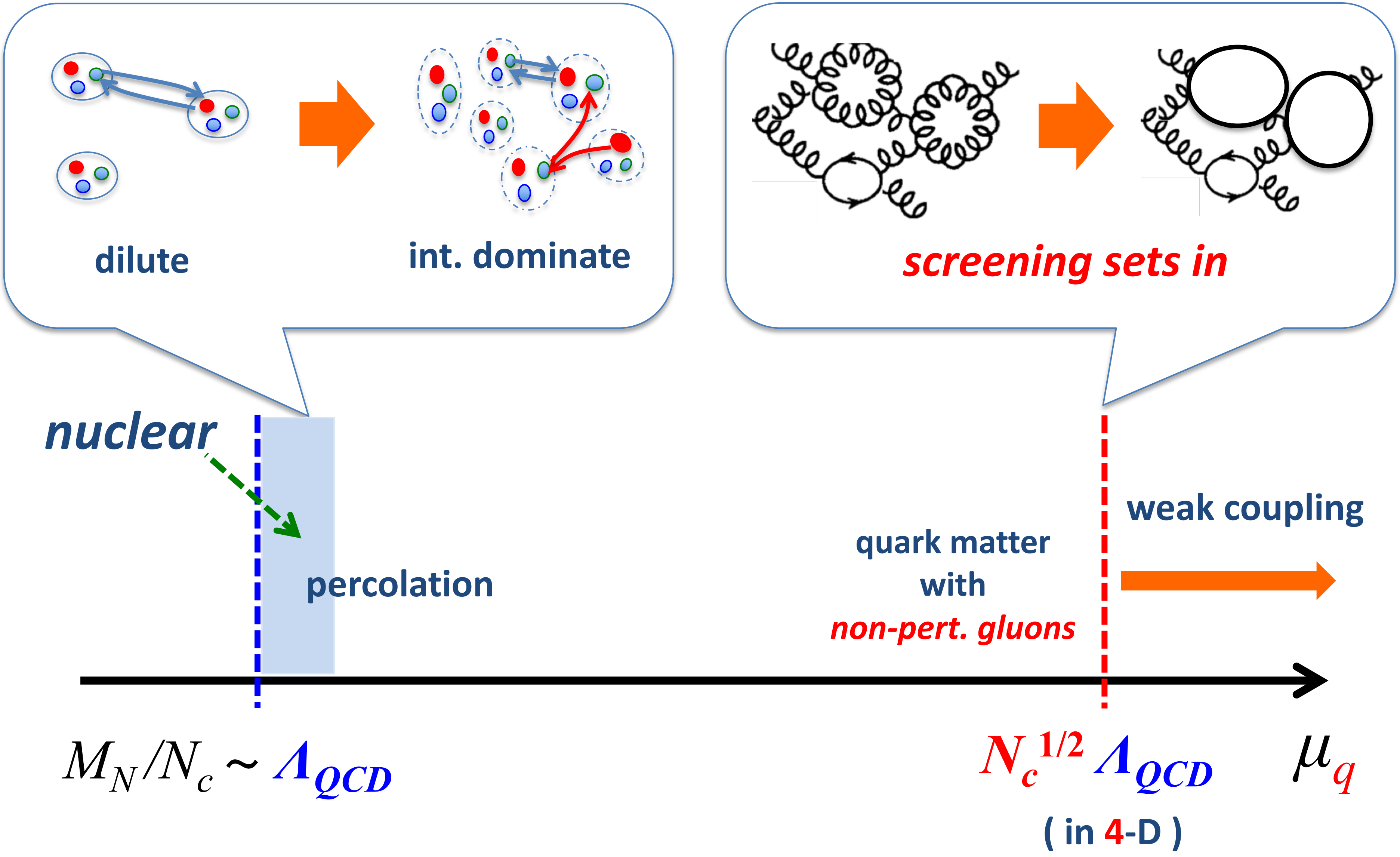
percolation

$M_N/N_c \sim \Lambda_{QCD}$

μ_q

McLerran-Pisarski's picture

[McLerran-Pisarski '07]



dilute

int. dominate

screening sets in

nuclear

percolation

quark matter
with
non-pert. gluons

weak coupling

$M_N/N_c \sim \Lambda_{QCD}$

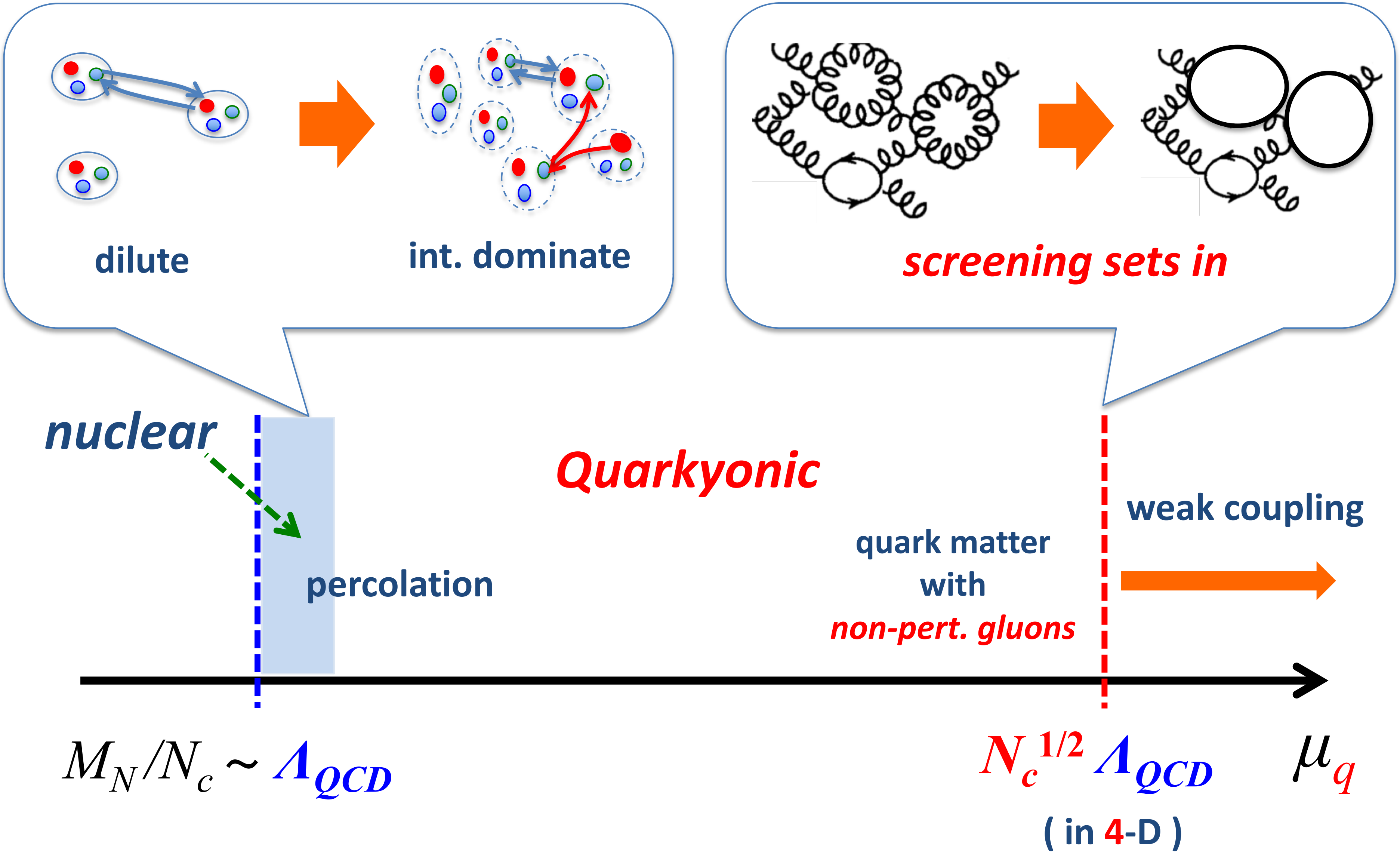
$N_c^{1/2}\,\Lambda_{QCD}$

(in 4-D)

μ_q

McLerran-Pisarski's picture

[McLerran-Pisarski '07]



dilute

int. dominate

screening sets in

nuclear

Quarkyonic

percolation

quark matter
with
non-pert. gluons

weak coupling

$M_N/N_c \sim \Lambda_{QCD}$

$N_c^{1/2}\,\Lambda_{QCD}$

(in 4-D)

μ_q

Several ***branches***

- **Confined, *but chiral symmetric* matter** (many papers ...)
  - **have been *challenged* by many model calculations** [Glozman et al. 2007, ...]

- **Confined, *inhomogeneous* chiral *SSB*** (still ongoing ...)

  (*chiral sym. broken only locally*)
  - **Skyrme crystals, ...**
  - **Chiral density wave (1-D periodic structure)** [Carignano-Nickel-Bubbala]
  - **Quarkyonic Chiral Spirals**
  - **Interweaving Chiral Spirals**

  } [TK-Hidaka-Fukushima -McLerran-Pisarski-Tsvelik 09-11]

- **Reinterpretation of *Hadron-Quark Continuity***
  - **Original proposal : Schafer-Wilczek**
  - **CSC in quarkyonic matter & NS context** [Fukushima-TK '15]

Plan of lectures

1, Lessons from hot QCD: how 3-window works

2, Theoretical orientation: high & low density limits (T=0)

3, NS constraints on EoS : hints for **soft-stiff** EoS

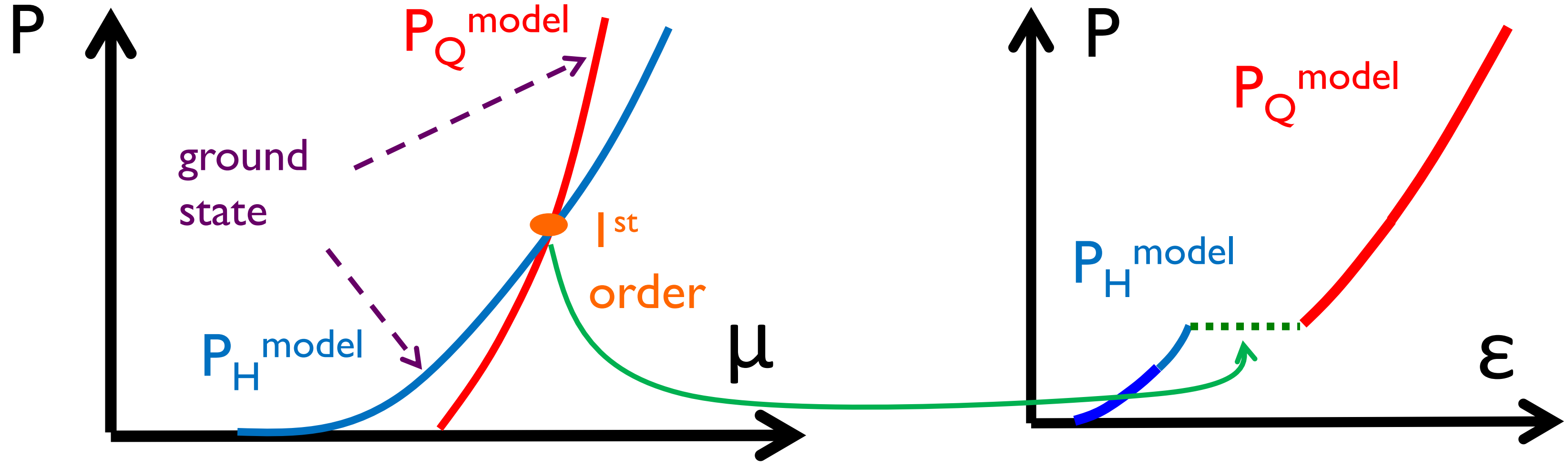
4, Crossover scenario: chiral restorations, etc.

5, A quark model : delineating the properties of matter

6, The astrophysical results from EoS QHC18

7-, Other topics: warm EoS, beyond-MF, etc.

Traditional hybrid construction



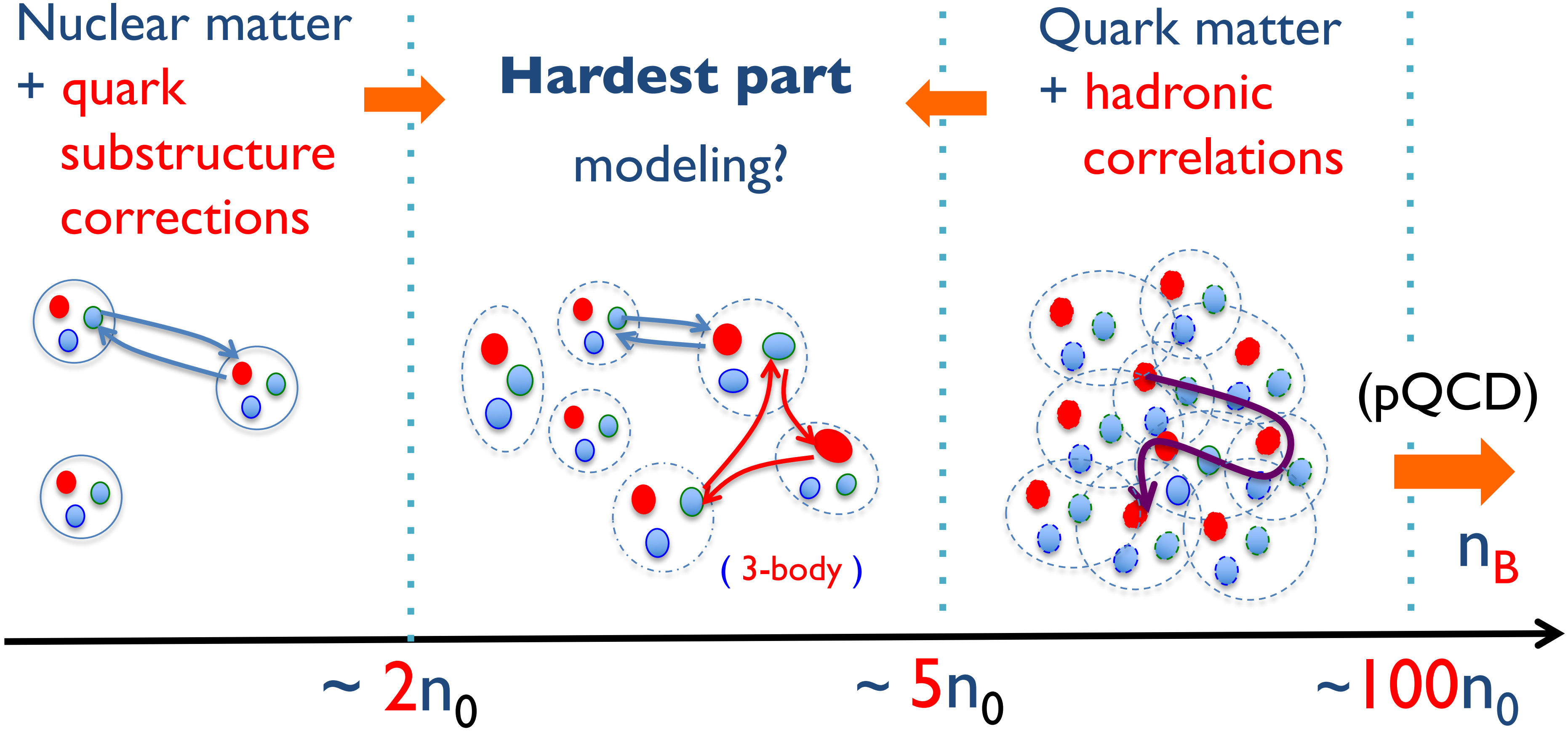
- Key (implicit) **assumptions** :
  - 1) Hadronic & quark phases are distinct (e.g. by order parameters)
  - 2) Both P_H and P_Q are reliable in the overlap region

→ by construction, Q-EoS must be much softer than H-EoS

(unless fine tuning worked out)

0, quark matter can be stiff
1, chiral restoration, color-super
2, expand quark-hadron continuity picture,
  percolation model, quarkyonic matter,
  Wilczek-Schafer, interplay b.t.w chiral & diquark,
  phases separated by symmetry
3, P vs mu : graphical rep.
4, NN, NY interaction: universal repulsion & strangeness?
5, a schematic quark model
6, astrophysical outputs

To Do (work in progress...)



Nuclear matter
+ quark
substructure
corrections

Hardest part

modeling?

(3-body)

Quark matter
+ hadronic
correlations

(pQCD)

n_B

$\sim 2n_0$ $\sim 5n_0$ $\sim 100n_0$

Then the matter should be heated up → predictions for HMNS

excitation modes ⟷ **the phase structure**

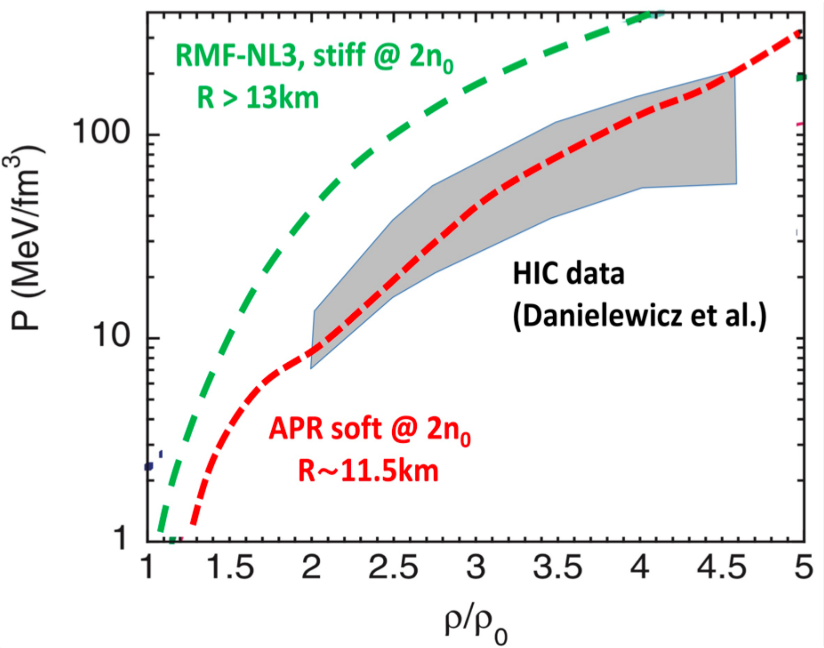
Small $R_{1.4}$ & soft EoS @ 1-2 n_0 ?

▪ Thermal X-rays analyses for NS radii :

- ▪ Suleimanov et al (2011) : > 13.9 km
- ▪ Guillot et al. (2011) : $9.1^{+1.3}_{-1.5}$ km
- ▪ Ozel & Freire (2015) : 10.6 ± 0.6 km
- ▪ Steiner et al (2015) : 12.0 ± 1.0 km

systematic uncertainties : distance to NS, atmosphere of NS, uniform T distributions,…

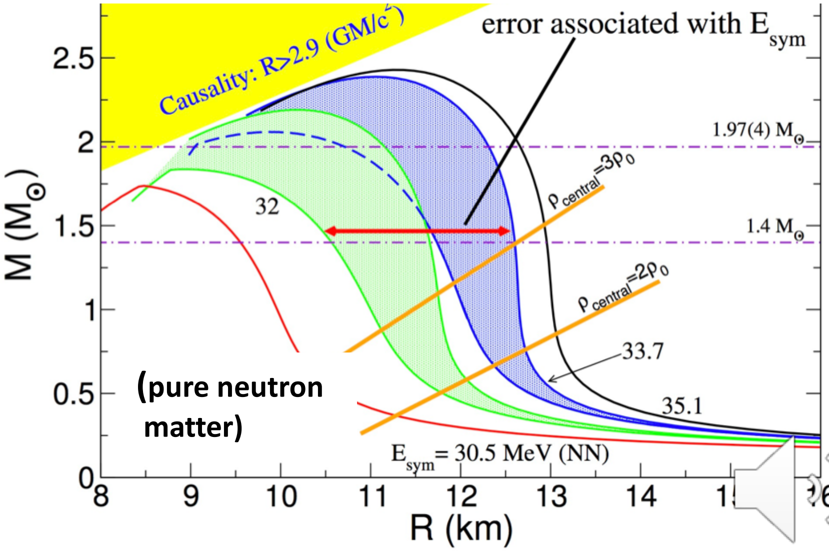
▪ HIC : (Danielewicz et al. 2002)



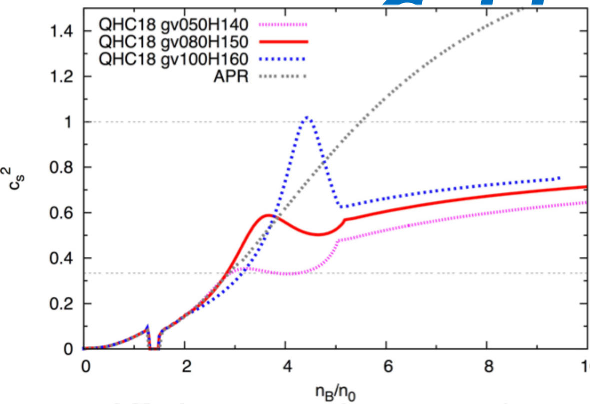
▪ nuclear EoS extrapolation :

(Gandolfi et al. 2015)

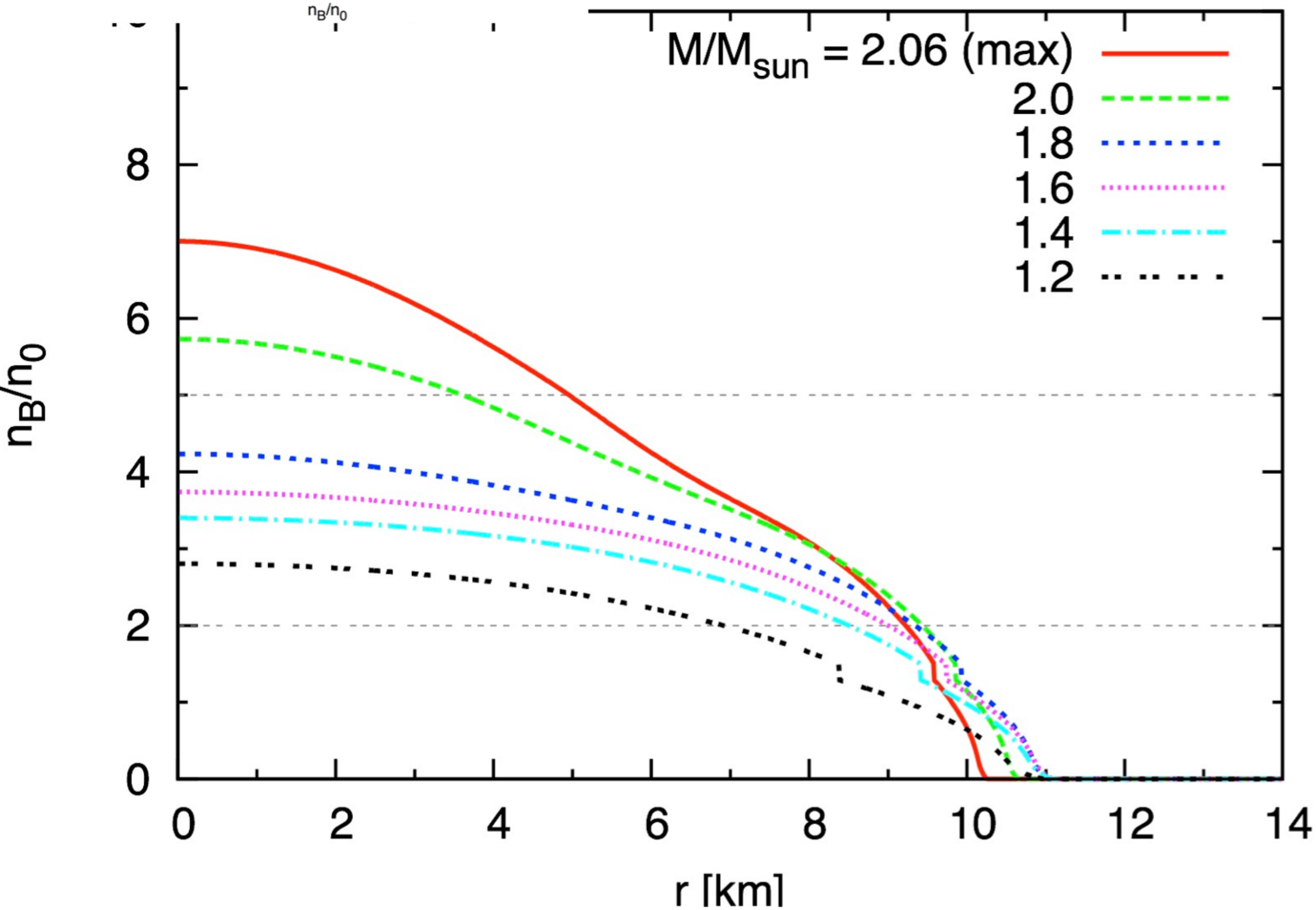
(*sophisticated* potentials & Monte-Carlo)

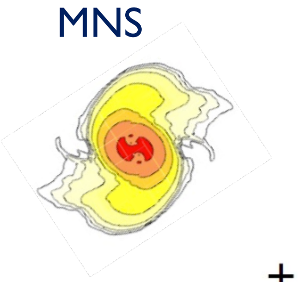


dense EoS : Low density



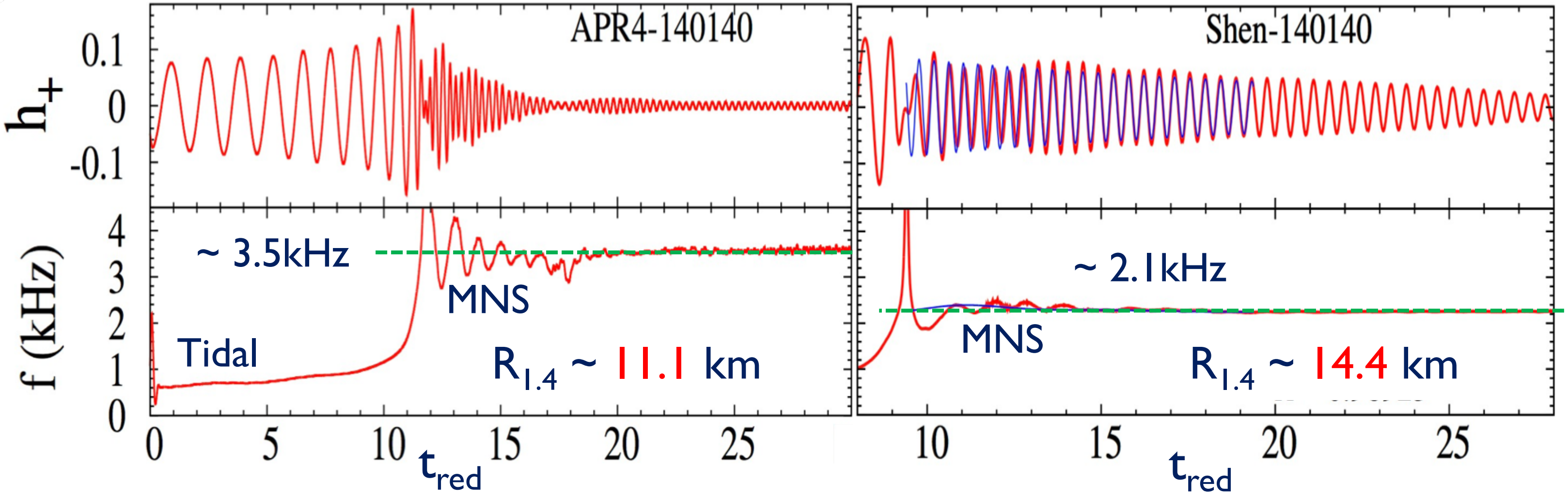
$$R_1 = 11.9^{+1.4}_{-1.4}\ \mathrm{km}$$





Merger & HMNS: $f_{GW} \rightarrow R_{NS}$

Figs from Hotokezaka+ 2013



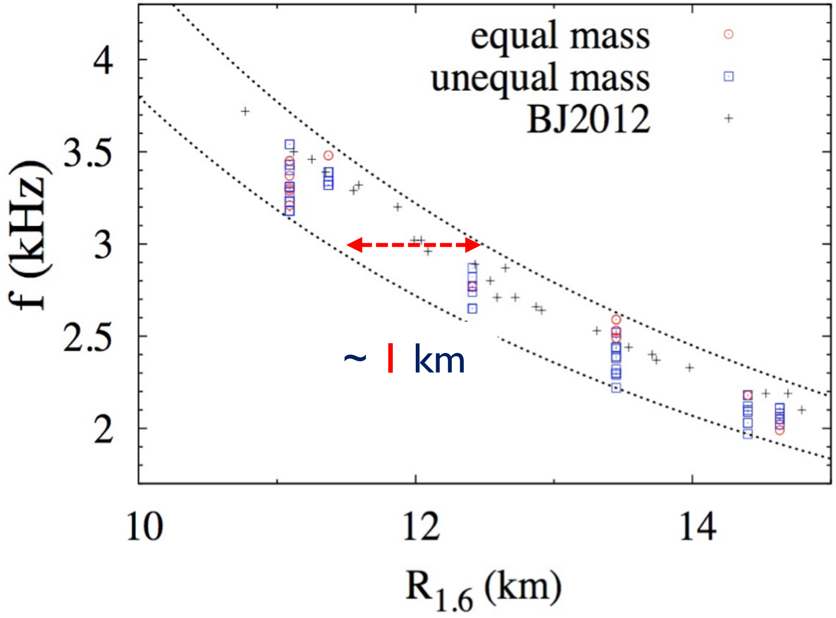
compact stars → high frequency GW

smaller $R_{NS} \rightarrow$ larger f_{GW}

(Bauswein and Janka 2012)

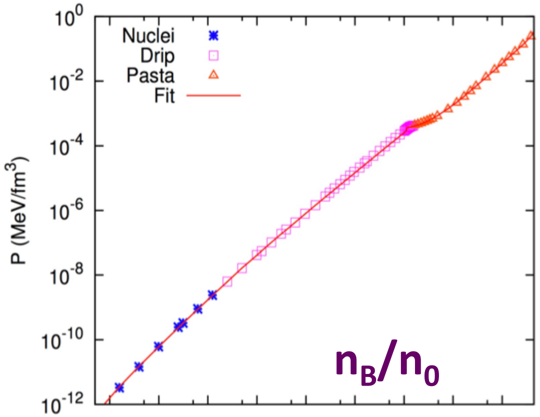
For **GW170817** :

f_{GW} is **NOT measured yet**;
high frequency region → smaller S/N



EoS & M-R relation

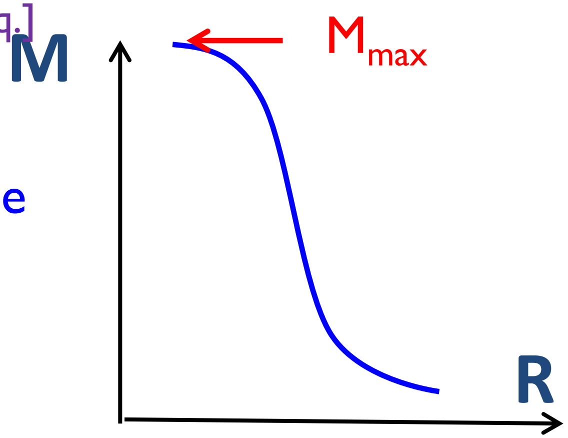
Einstein eq. : $$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$ QCD EoS



[for spherical NS : TOV eq.]

1-to-1 correspondence

Lindblom (1992)



1) non-rotating, spherical NS : TOV equation $\quad M_{TOV} > 2M_{sun}$

2) uniformly rotating NS : e.g. Hartle-Thorne $\quad M_{uni} \sim 1.2\, M_{TOV}$

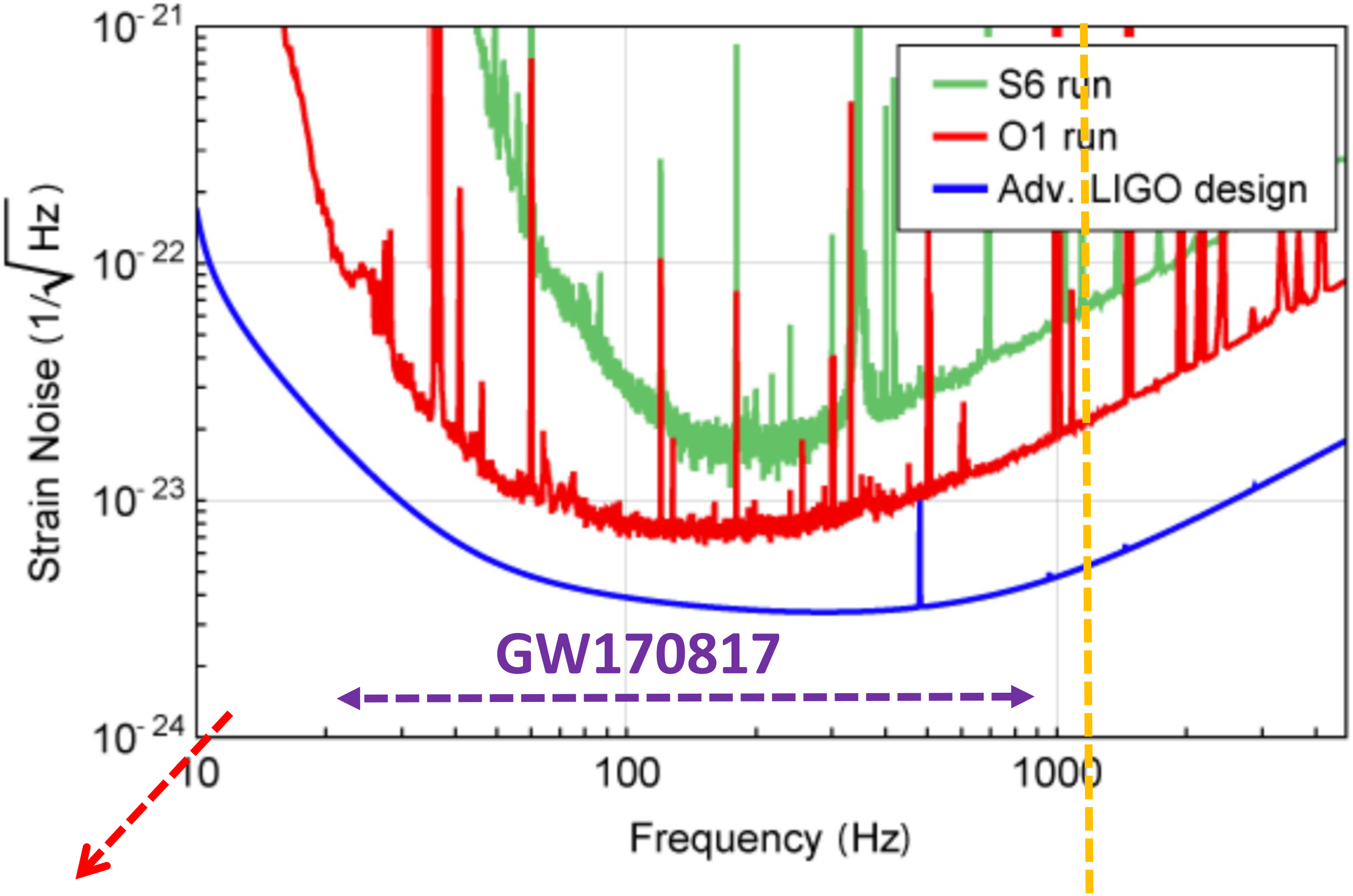
(stable if rotation is slow enough)

3) differentially rotating NS : Numerical GR $\quad M_{diff} \sim 1.5\, M_{TOV}$

(short-live; dissipation and magnetic braking → collapse)

Baryon number density

Design sensitivity



inspiral

(noise: seismology)

tidally deformed phase

(noise: mirror)

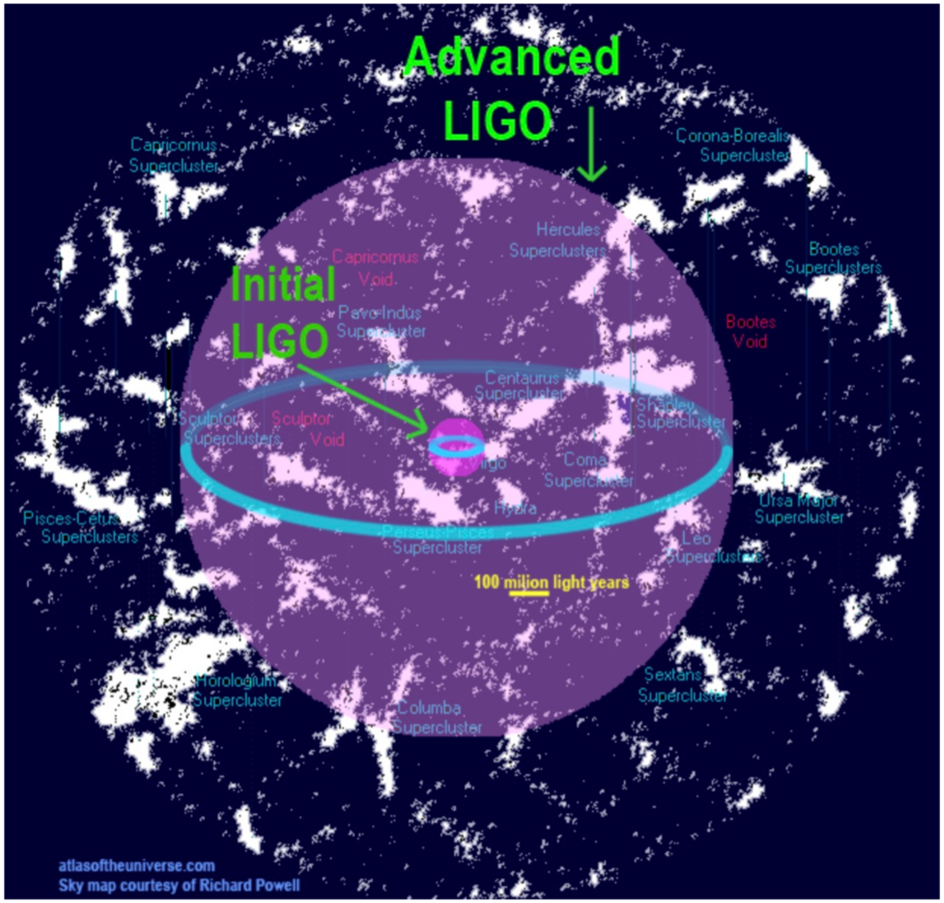
~ post-merger

HMNS or BH

(quantum noise: laser)

To detect rare events

1pc = 3.26 lyr



atlasoftheuniverse.com
Sky map courtesy of Richard Powell

- **our galaxy (milky-way) ~ 31-55 kpc**
- **to the edge of universe ~ 14 Gpc**
- ***detector horizon***
  - **aLIGO**

    Livingston ~ 218 Mpc

    Hanford ~ 107 Mpc
  - **Virgo ~ 58 Mpc**
- ***expected detection rate***

  0.1 – 100 events/year

- **GW170817** **happened at** 40^{+8}_{-14} Mpc

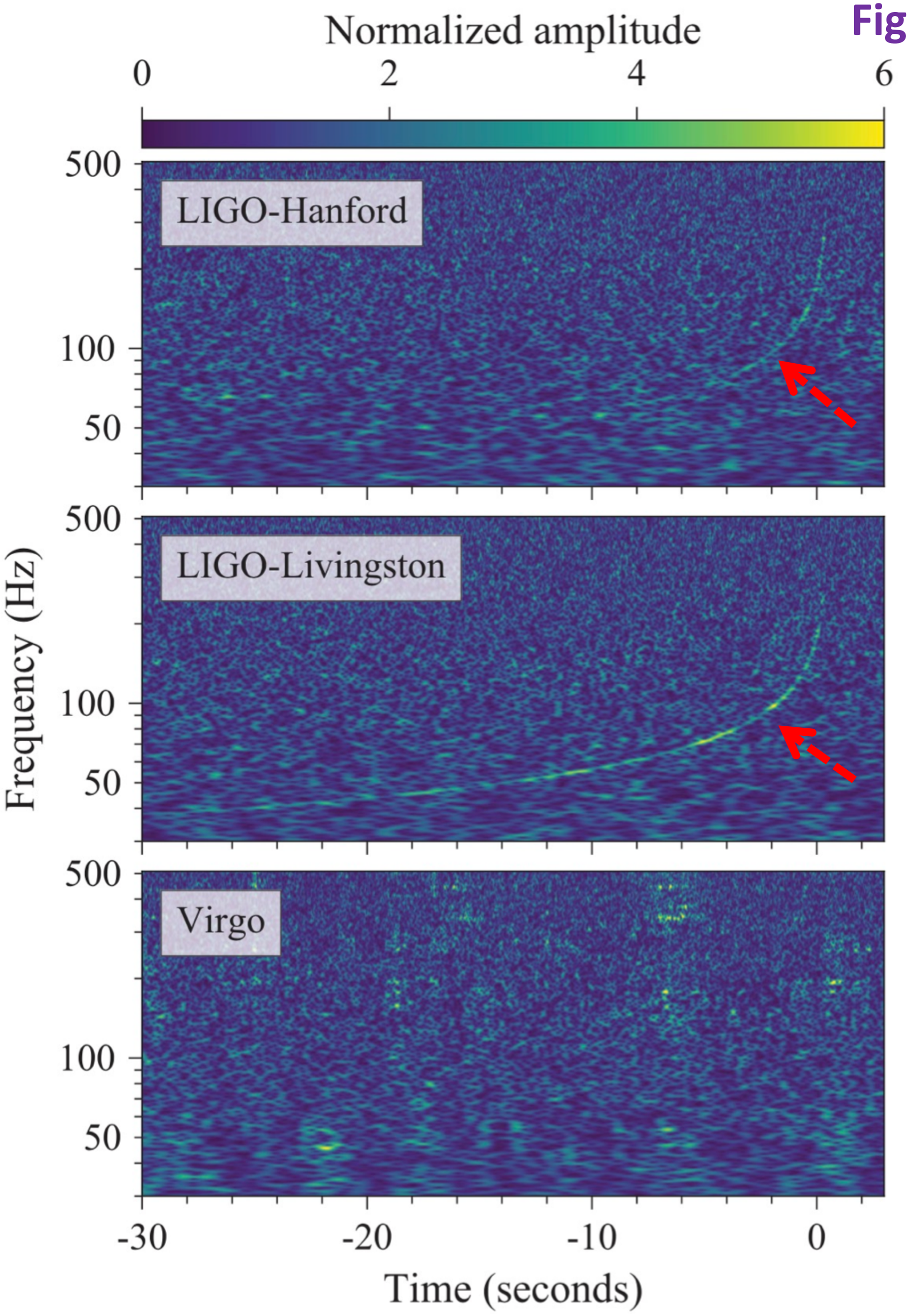


Fig. from PRL 119, 161101 (2017)

- aLIGO: signal-to-noise = 32.4 !

  (largest GW signal ever)

- Virgo did not find it

  GWs from the blind spot of Virgo

  → strongly constrain the location

  → trigger follow-up EM studies

- clear signal 20 Hz - 1kHz

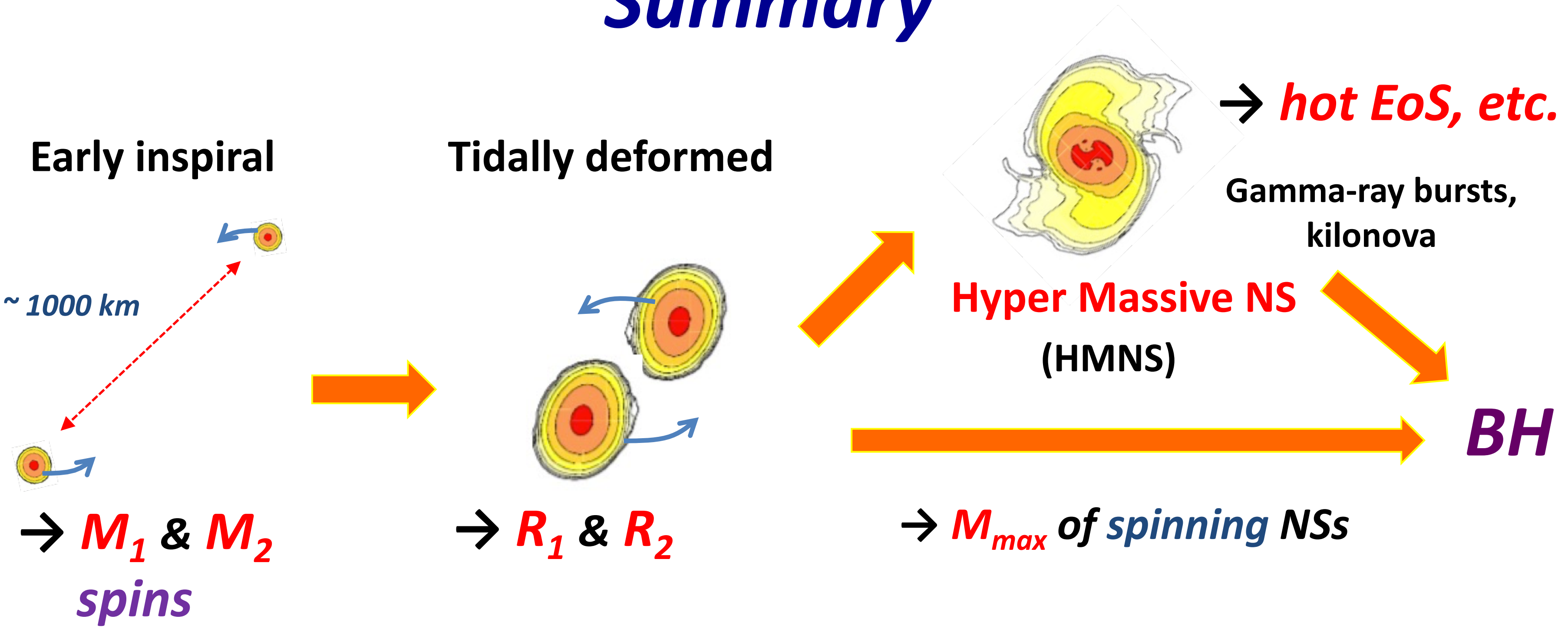
  inspiral – *tidal deformed* phases

  BH ring-down not measured

  (larger noise at higher frequency)

- EM signals from objects just after merger

Summary



Early inspiral

~ 1000 km

→ M_1 & M_2

spins

Tidally deformed

→ R_1 & R_2

→ hot EoS, etc.

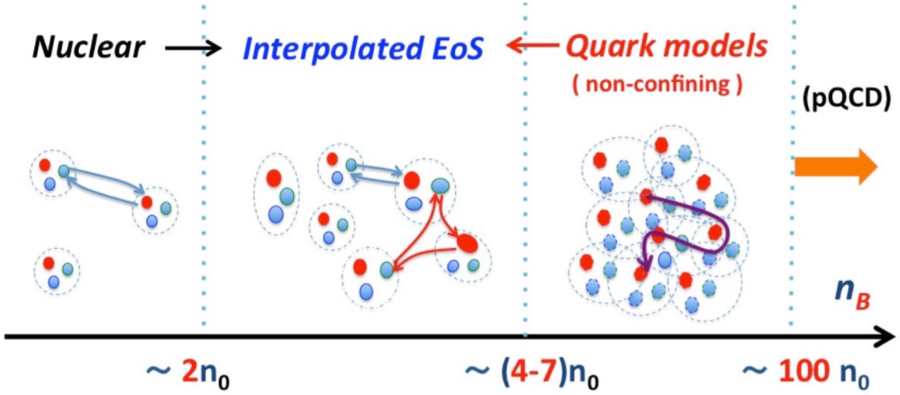
Gamma-ray bursts, kilonova

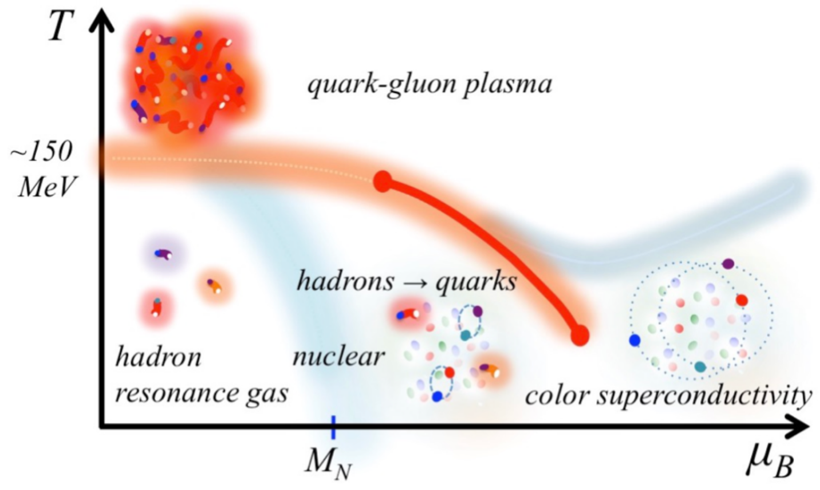
Hyper Massive NS

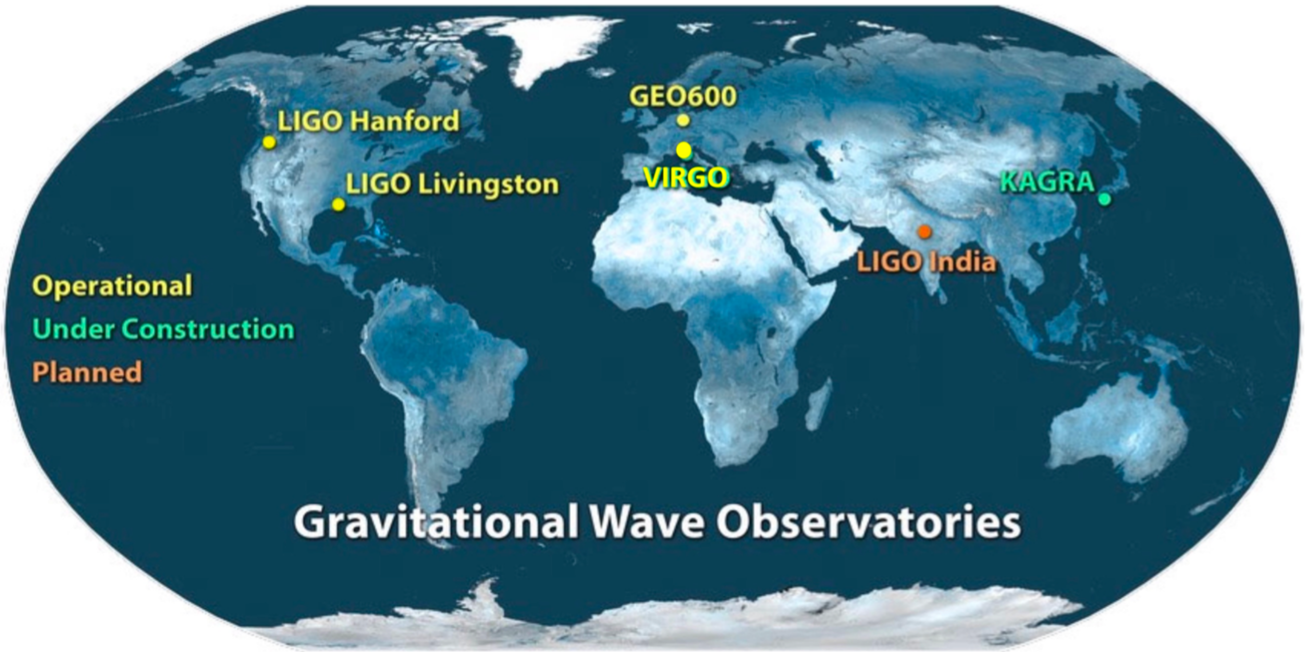
(HMNS)

BH

→ M_{max} of spinning NSs







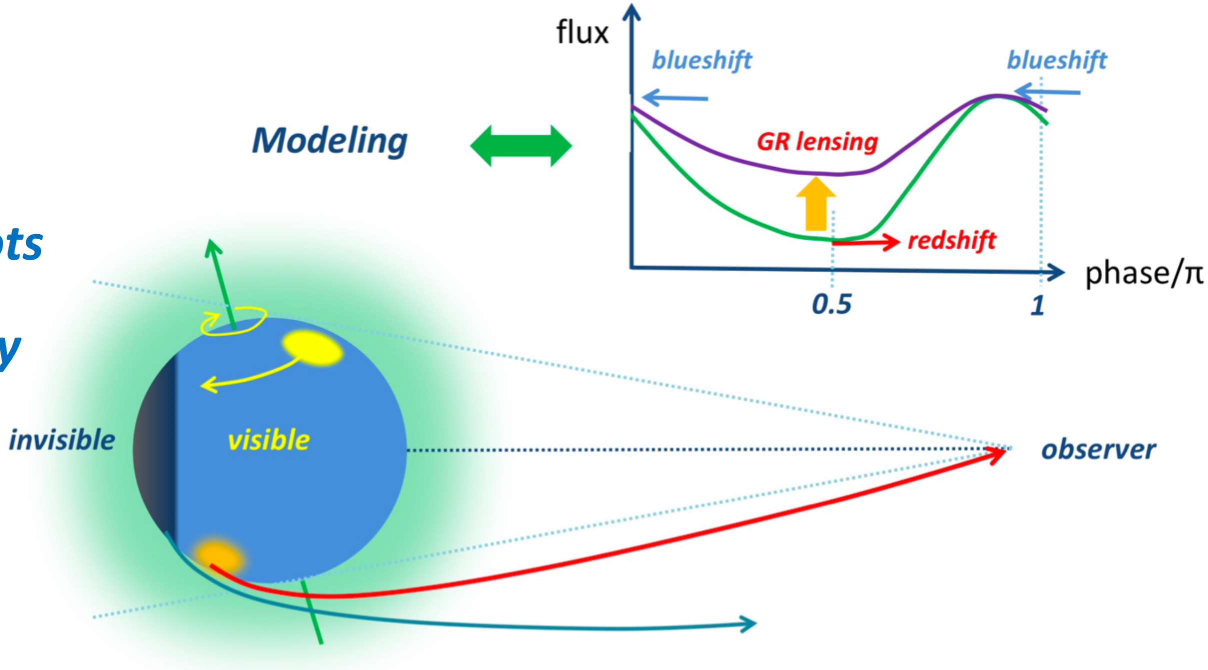
- *GW detectors :*

  aLIGO (O3)
  VIRGO
  KAGRA
  LIGO India, …

- *NICER (2017~) :*

  timing analyses of hot spots

  R & M/R → 5-10 % accuracy

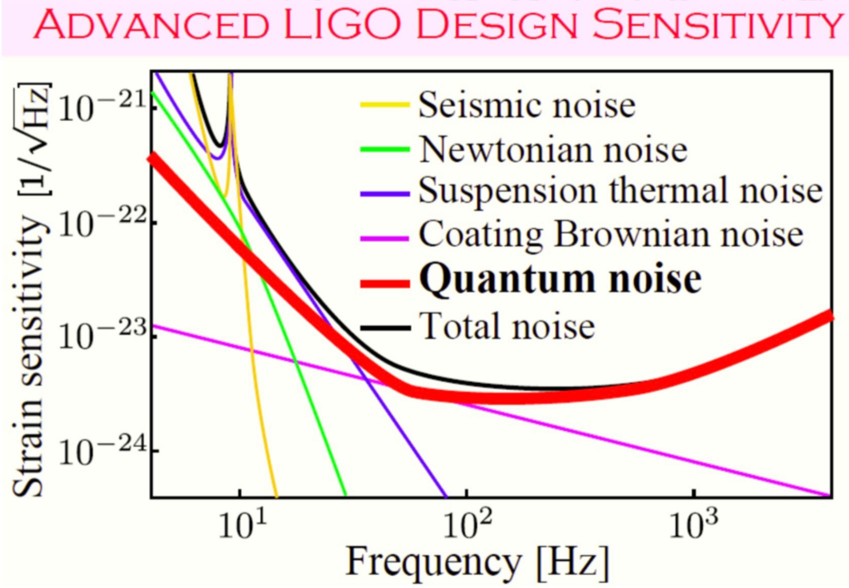


Template 1: post-Newtonian for f < ~1kHz

Cutler et al., PRL70, 2984 (1993)

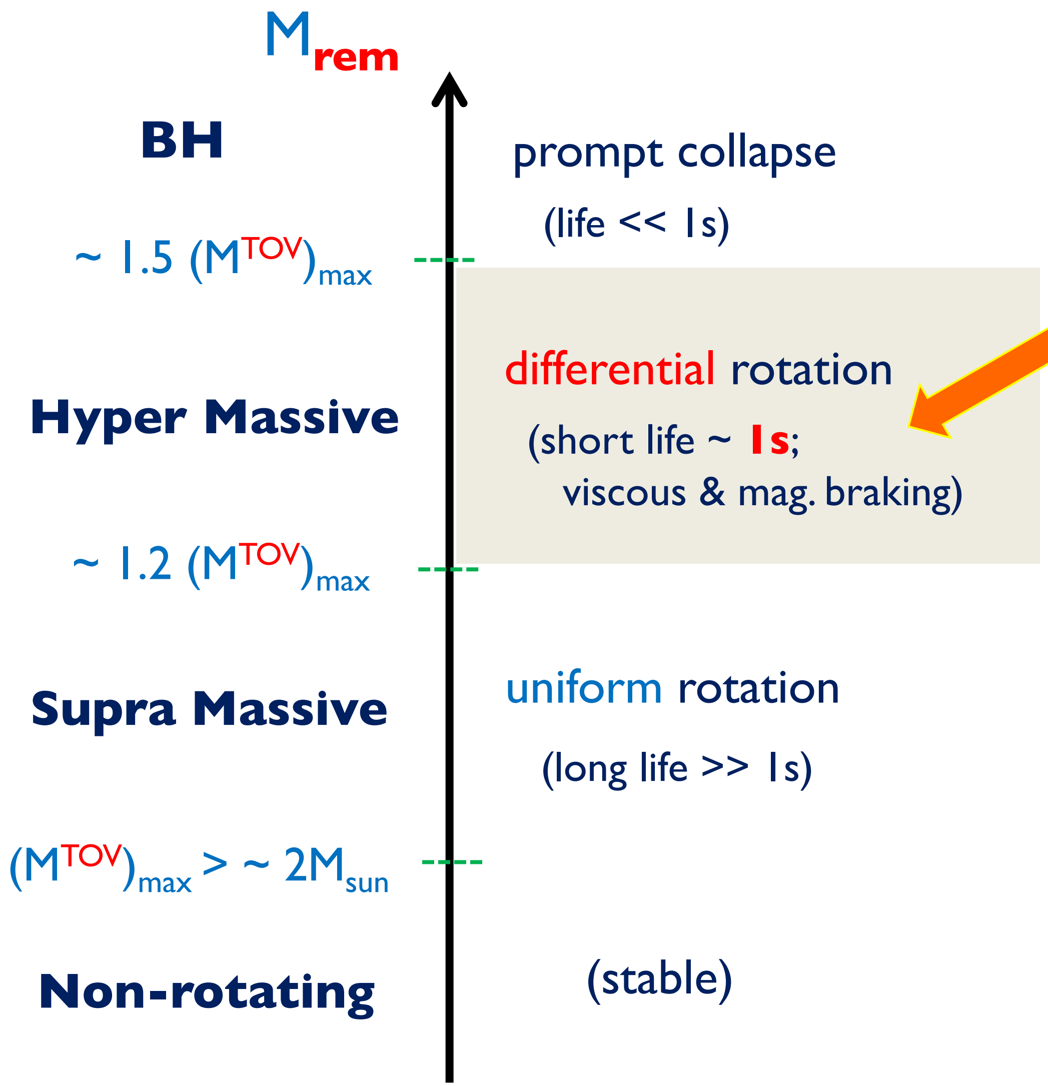
$$\frac{d\mathcal{N}_{\text{cyc}}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1 + \left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)x\right.$$

$$\left.\text{S.}]x^2 + O(x^{2.5})\right\}.$$


ADVANCED LIGO DESIGN SENSITIVITY

Delayed vs prompt collapse → $(M^{TOV})_{max}$

Lattimer, talk at INT, 2018



M_{rem}

BH

prompt collapse

(life << 1s)

~ 1.5 $(M^{TOV})_{max}$

Hyper Massive

differential rotation

(short life ~ **1s**;
viscous & mag. braking)

~ 1.2 $(M^{TOV})_{max}$

Supra Massive

uniform rotation

(long life >> 1s)

$(M^{TOV})_{max}$ > ~ $2M_{sun}$

Non-rotating

(stable)

For **GW170817** :

- collapse to BH after ~ **1s**
- ~2.28 < M_{rem}/M_{sun} < ~2.53
  (estimated)

$1.2\,(M^{TOV})_{max} < 2.53\,M_{sun}$

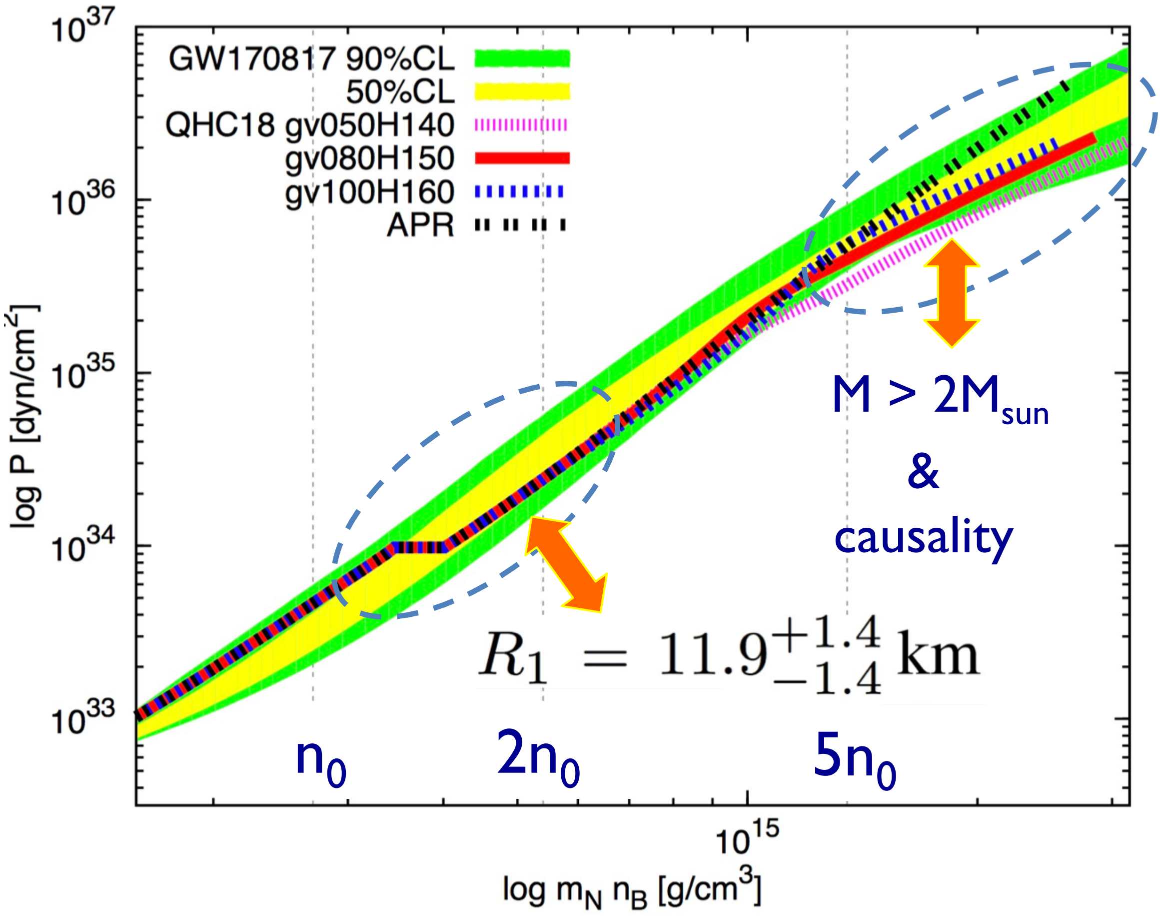
➡ $(M^{TOV})_{max} < 2.11\,M_{sun}$

- If thermal effects are included,
  the constraint may be even stronger

EoS from aLIGO vs QHC18

aLIGO & Virgo new analyses for GW170817

arXiv: 1805.11581 [gr-qc]



$$R_1 = 11.9^{+1.4}_{-1.4}\ \text{km}$$

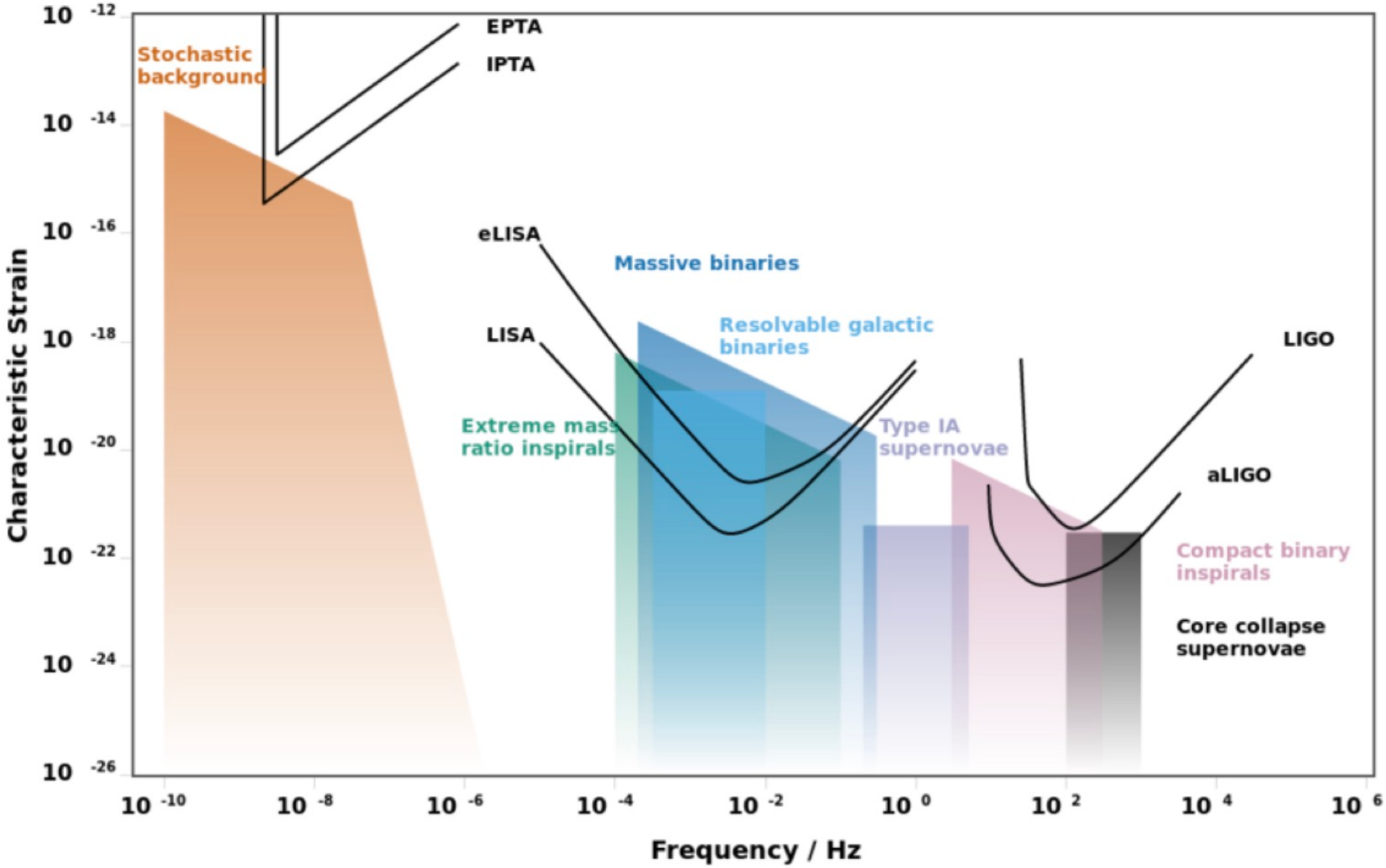
$M > 2M_{sun}$

&

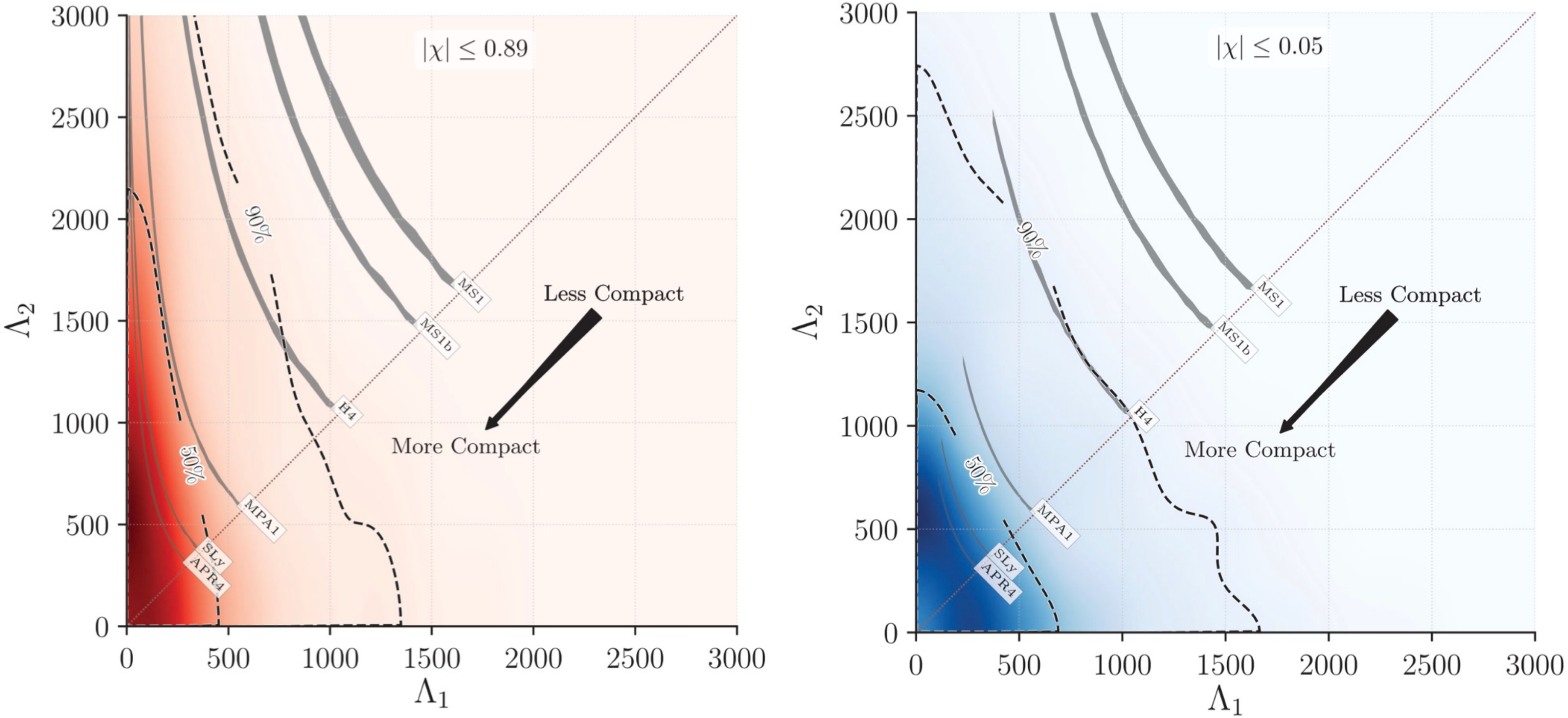
causality

EoS constraints with

- $M > 2M_{sun}$
- tidal deformability
- causality


EPTA
IPTA
Stochastic background
eLISA
LISA
Massive binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Type IA supernovae
LIGO
aLIGO
Compact binary inspirals
Core collapse supernovae
Characteristic Strain
10 -12
10 -14
10 -16
10 -18
10 -20
10 -22
10 -24
10 -26
10 -10
10 -8
10 -6
10 -4
10 -2
10 0
10 2
10 4
10 6
Frequency / Hz

APR~11.1km, H4~13.6km, MS1~14.5km



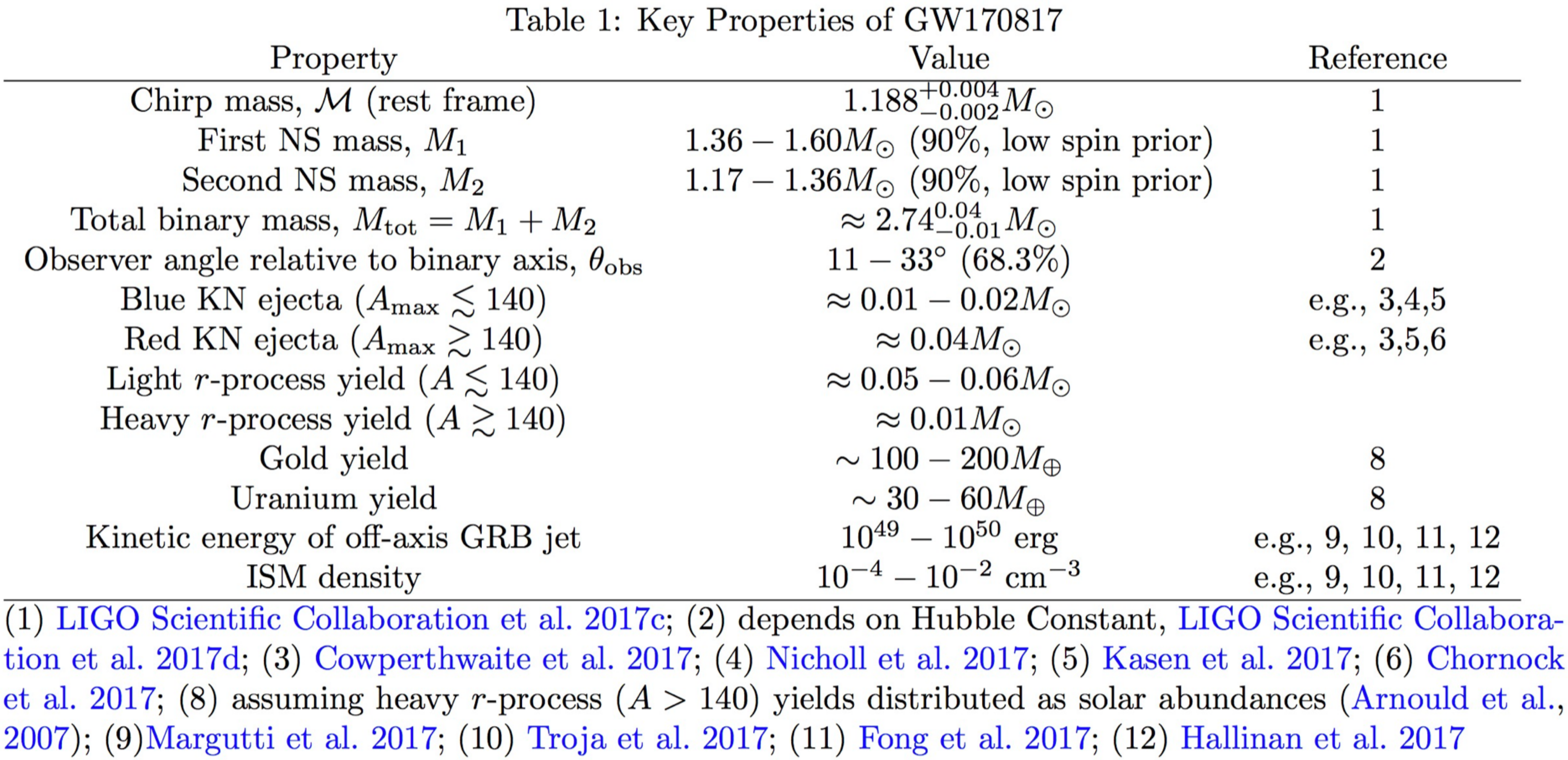


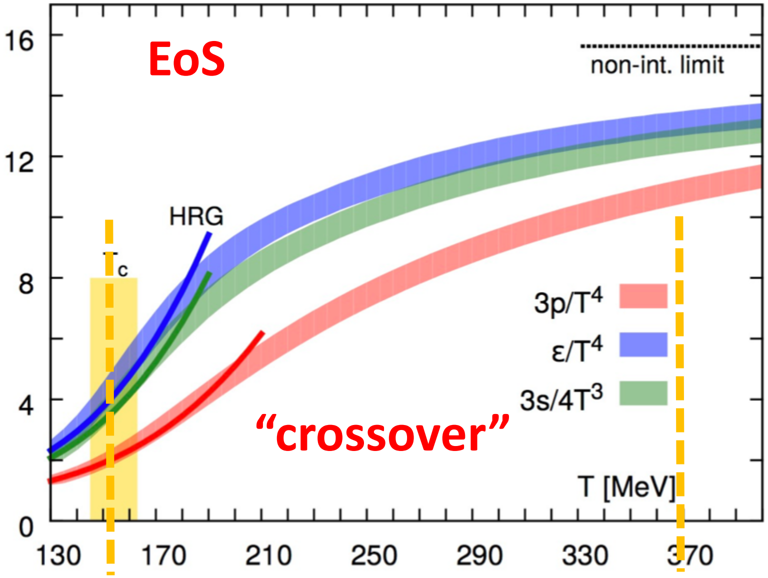
Table 1: Key Properties of GW170817

Property	Value	Reference
Chirp mass, $\mathcal{M}$ (rest frame)	$1.188^{+0.004}_{-0.002} M_\odot$	1
First NS mass, M_1	$1.36 - 1.60 M_\odot$ (90%, low spin prior)	1
Second NS mass, M_2	$1.17 - 1.36 M_\odot$ (90%, low spin prior)	1
Total binary mass, $M_{\rm tot} = M_1 + M_2$	$\approx 2.74^{0.04}_{-0.01} M_\odot$	1
Observer angle relative to binary axis, $\theta_{\rm obs}$	$11 - 33°$ (68.3%)	2
Blue KN ejecta ($A_{\rm max} \lesssim 140$)	$\approx 0.01 - 0.02 M_\odot$	e.g., 3,4,5
Red KN ejecta ($A_{\rm max} \gtrsim 140$)	$\approx 0.04 M_\odot$	e.g., 3,5,6
Light r-process yield ($A \lesssim 140$)	$\approx 0.05 - 0.06 M_\odot$	
Heavy r-process yield ($A \gtrsim 140$)	$\approx 0.01 M_\odot$	
Gold yield	$\sim 100 - 200 M_\oplus$	8
Uranium yield	$\sim 30 - 60 M_\oplus$	8
Kinetic energy of off-axis GRB jet	$10^{49} - 10^{50}$ erg	e.g., 9, 10, 11, 12
ISM density	$10^{-4} - 10^{-2}$ cm^{-3}	e.g., 9, 10, 11, 12

(1) LIGO Scientific Collaboration et al. 2017c; (2) depends on Hubble Constant, LIGO Scientific Collaboration et al. 2017d; (3) Cowperthwaite et al. 2017; (4) Nicholl et al. 2017; (5) Kasen et al. 2017; (6) Chornock et al. 2017; (8) assuming heavy r-process ($A > 140$) yields distributed as solar abundances (Arnould et al., 2007); (9)Margutti et al. 2017; (10) Troja et al. 2017; (11) Fong et al. 2017; (12) Hallinan et al. 2017

Delineating QCD matter from HOT EoS

lattice calculations



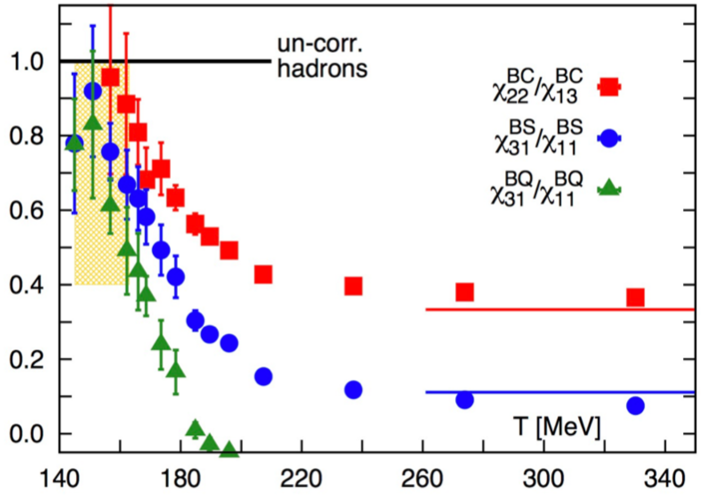
Hadron resonance gas

semi-QGP

pQGP

(Ding-Karsch-Makherjee, review 2015)

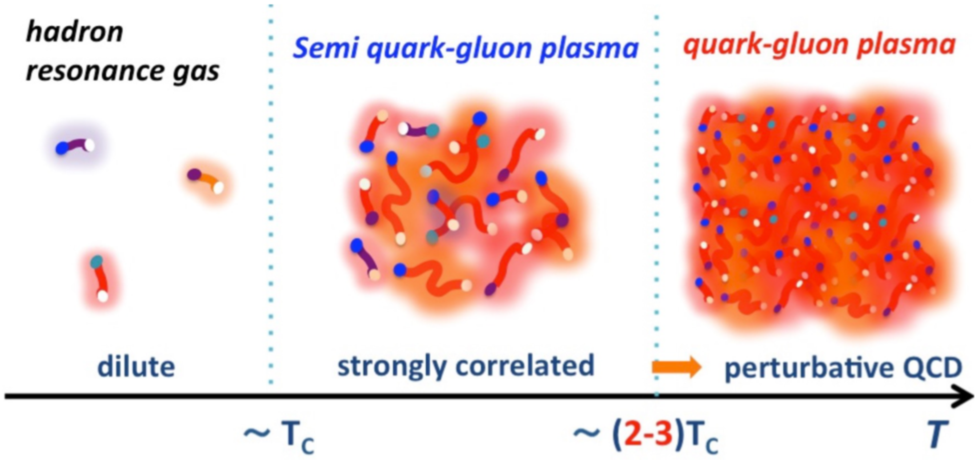
derivatives of EoS



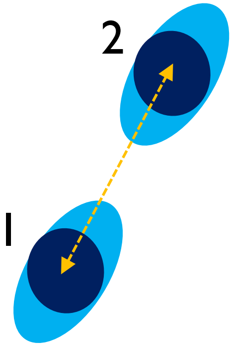
→ T_c: **universal for different flavors**

plausible picture

Fig. from Baym et al. 2018



Dimensionless tidal deformability → R_{NS}



more common to use

$$\Lambda(M) = 32\frac{\lambda G}{R^5} = \frac{2}{3}k_2\left(\frac{R}{GM}\right)^5$$

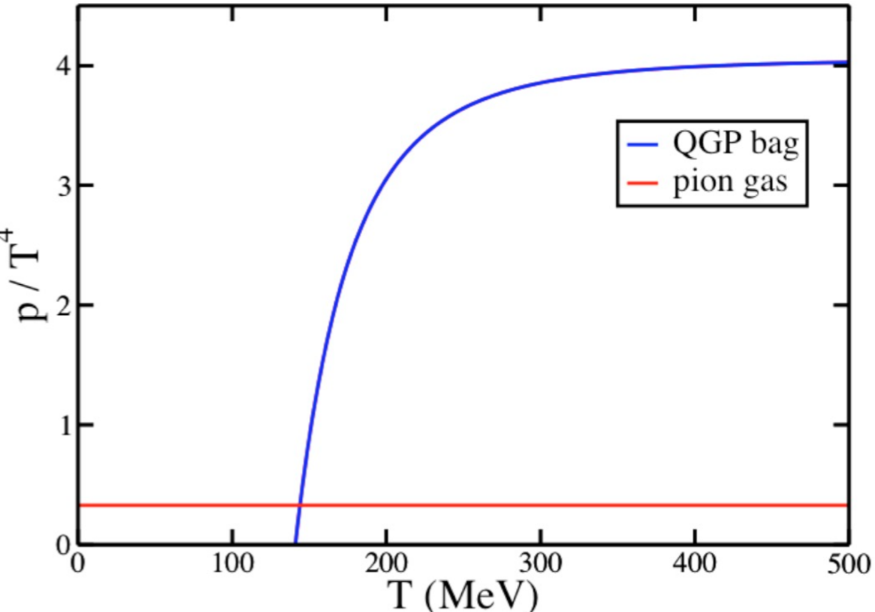
(k_2 : Love number)

What GW analyses measure: combination of Λ for star 1 & 2 :

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_1+12M_2)M_1^4\Lambda_1 + (M_2+12M_1)M_2^4\Lambda_2}{(M_1+M_2)^5}$$

(measured)

2-parameters: M_1 & M_2



$$= 32\frac{\lambda G}{R^5}$$