



Long-range rapidity correlations in the model with string fusion on transverse lattice

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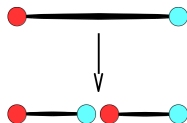
St.Petersburg State University

Helmholtz International Summer School "Matter under Extreme Conditions in Heavy-Ion Collisions and Astrophysics"
Dubna, August 20-31

The string model

First stage: colour quark-gluon strings (colour flux tubes) are formed

Second stage: hadronization of these strings produces the observed hadrons



A. Capella and A. Krzywicki, Phys.Rev.D18, 4120 (1978)

Event-by-event variance in the number of strings \Rightarrow

Long-Range FB Correlations (LRC) at large rapidity gap y_{gap}

The model with string fusion

$pp \rightarrow pA \rightarrow AA$ - the increase of the string density in transverse plain

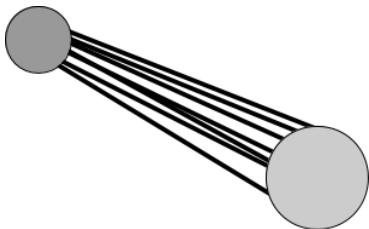
M.A. Braun, C. Pajares, Phys.Lett. B287, 154 (1992);

Nucl. Phys. B390, 542 (1993).

⇒ Reduction of multiplicity, increase of transverse momenta.

N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares,

Phys.Rev.Lett. 73, 2813 (1994).

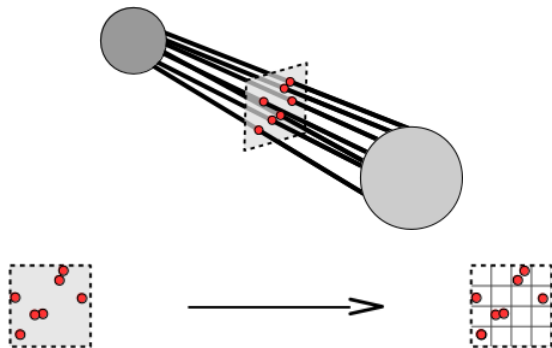


The version of SFM with the finite lattice in transverse plane

Vechernin V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136

Braun M.A., Kolevatov R.S., Pajares C., V.Vechernin Eur.Phys.J. C32 (2004) 535.

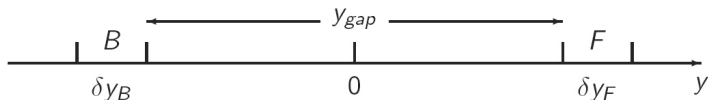
V.Vechernin, Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858.



Forward-Backward Rapidity Correlations

The definition of the rapidity:

$$y = \frac{1}{2} \ln \frac{p_0 + p_z}{p_0 - p_z}.$$



The correlation coefficient:

$$b_{FB} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{\text{cov}(F, B)}{D_F}, \quad b_{FB} = \left. \frac{d\langle B \rangle_F}{dF} \right|_{F=\langle F \rangle}$$

$\langle B \rangle_F = f(F)$ - the FB correlation function

$\langle B \rangle_F = a + b_{BF}F$ - the linear regression

The definitions are equivalent for linear regression function.

ALICE collaboration et al., J. Phys. G 32 1295 (2006), [Sect. 6.5.15]

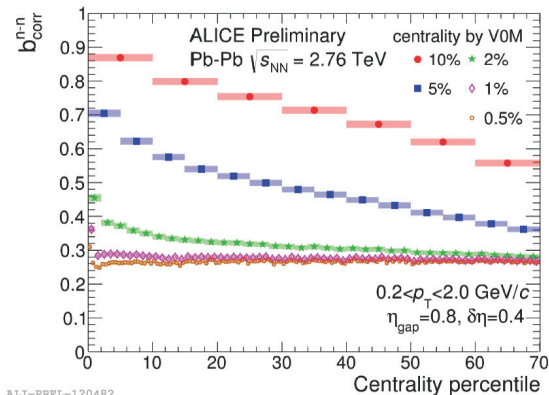
Three types of correlations:

- $n - n$ -correlations;
- $p_t - n$ -correlations;
- $p_t - p_t$ -correlations

$$p_{tB} \equiv \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}_{tB}^i| \quad p_{tF} \equiv \frac{1}{n_F} \sum_{i=1}^{n_F} |\mathbf{p}_{tF}^i|$$

$n - n$ correlations in ALICE

$B, F \Rightarrow n_B, n_F$ - the **extensive** variables $\Rightarrow b_{nn}$
 Strongly influenced by "volume" fluctuations.



I. Altsybeev for the ALICE Collaboration.
 Quark Matter 2017, 5-11 February 2017, Chicago, IL.

arXiv: 1711.04844v1

Intensive Observables

FB correlation between event-mean p_t of particles in $\delta\eta_F$ - $\delta\eta_B$ intervals:

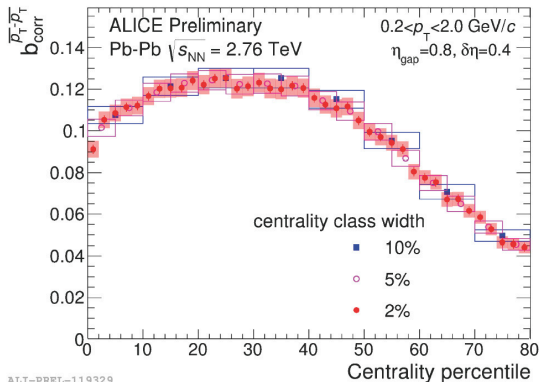
$$p_{tB} \equiv \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}_{tB}^i| \quad p_{tF} \equiv \frac{1}{n_F} \sum_{i=1}^{n_F} |\mathbf{p}_{tF}^i|$$

$B, F \Rightarrow p_{tB}, p_{tF}$ - the **intensive** variables $\Rightarrow b_{p_t p_t}$

LR event-mean p_t correlation is not sensitive to the fluctuation in the number of sources, but is sensitive to the fluctuation in the quality of sources.

$p_t - p_t$ correlations in ALICE

Resistant to "volume" fluctuations.



ALI-PREL-119329

I. Altsybeev for the ALICE Collaboration.
Quark Matter 2017, 5-11 February 2017, Chicago, IL.

arXiv: 1711.04844v1

Model

M is the number of cells.

Event is characterized by the set of numbers:

$$\begin{aligned}
 C &= \{C_\eta, C_n^B, C_n^F, C_p^B, C_p^F\}, \\
 C_\eta &= \{\eta_1, \dots, \eta_M\}, \\
 C_n^F &= \{n_1^F, \dots, n_M^F\}, \\
 C_p^F &= \{p_1^{1F}, \dots, p_1^{n_1^F}; \dots; p_M^{1F}, \dots, p_M^{n_M^F}\} \\
 C_n^B &= \{n_1^B, \dots, n_M^B\}, \\
 C_p^B &= \{p_1^{1B}, \dots, p_1^{n_1^B}; \dots; p_M^{1B}, \dots, p_M^{n_M^B}\}.
 \end{aligned}$$

$$n_F = \sum_{i=1}^M n_i^F, \quad n_B = \sum_{i=1}^M n_i^B$$

$$p_t^F = \frac{1}{n_F} \sum_{i=1}^M \sum_{j=1}^{n_i^F} p_i^{jF}, \quad p_t^B = \frac{1}{n_B} \sum_{i=1}^M \sum_{j=1}^{n_i^B} p_i^{jB}.$$

The Gaussian approximation

Fluctuations in the number of strings in the cell are assumed independent and Gaussian distributed with variance proportional to the average number of strings in the cell:

$$P(\eta_i) = \frac{1}{\sqrt{2\pi d_{\eta_i}}} e^{-\frac{(\eta_i - \bar{\eta}_i)^2}{2d_{\eta_i}}},$$

$$d_{\eta_i} = \omega_{\eta} \bar{\eta}_i,$$

and similarly for the number of particles in forward and backward windows:

$$P(n_i^F) = \frac{1}{\sqrt{2\pi d_{n_i^F}}} e^{-\frac{(n_i^F - \bar{n}_i^F)^2}{2d_{n_i^F}}}, \quad P(n_i^B) = \frac{1}{\sqrt{2\pi d_{n_i^B}}} e^{-\frac{(n_i^B - \bar{n}_i^B)^2}{2d_{n_i^B}}},$$

$$d_{n_i^F} = \omega_{\mu} \bar{n}_i^F, \quad d_{n_i^B} = \omega_{\mu} \bar{n}_i^B.$$

The average transverse momentum of the particles produced from the hadronization of the strings in the cells is assumed independent on the numbers of particles and depend on the numbers of strings in the cell. The variance of the transverse momentum of the one particle is assumed proportional to the mean transverse momentum squared.

$$d_{p_i}(\eta_i) = \overline{p^2}(\eta_i) - \bar{p}^2(\eta_i) = \gamma \bar{p}^2(\eta_i).$$

Model

The dependence of the average number of particles formed by hadronization of the string in the cell and the transverse momentum of these particles of the number of strings η_i in the cell:

$$\bar{n}(\eta_i) = \sqrt{\eta_i}, \quad \bar{p}(\eta_i) = p_0 \sqrt[4]{\eta_i}.$$

The numbers of particles formed from the hadronizations of the strings in i -th cell in forward rapidity window

$$\bar{n}_i^F = \mu_F \bar{n}(\eta_i).$$

And the same for the backward window

$$\bar{n}_i^B = \mu_B \bar{n}(\eta_i).$$

The small parameters:

$$\frac{1}{\bar{\eta}_i} \ll 1, \quad \frac{1}{M} \ll 1.$$

Definitions

$$b_{nn}^{mean} = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle (n_F)^2 \rangle - \langle n_F \rangle^2}, \quad b_{nn}^{corr f} = \left. \frac{d\langle n_B \rangle_{n_F}}{dn_F} \right|_{n_F=\langle n_F \rangle}.$$

$$b_{p_t n}^{mean} = \frac{\langle n_F p_t^B \rangle - \langle p_t^F \rangle \langle n_B \rangle}{\langle (n_F)^2 \rangle - \langle p_t^F \rangle^2}, \quad b_{p_t n}^{corr f} = \left. \frac{d\langle p_t^B \rangle_{n_F}}{dn_F} \right|_{n_F=\langle n_F \rangle}.$$

$$b_{p_t p_t}^{mean} = \frac{\langle p_t^F p_t^B \rangle - \langle p_t^F \rangle \langle p_t^B \rangle}{\langle (p_t^F)^2 \rangle - \langle p_t^F \rangle^2}, \quad b_{p_t p_t}^{corr f} = \left. \frac{d\langle p_B \rangle_{p_F}}{dp_F} \right|_{p_F=\langle p_F \rangle}$$

Calculation of the mean values

$$b_{BF}^{mean} \equiv \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2}.$$

$$\langle F \rangle = \left\langle \left\langle \langle F \rangle^{C_p^F} \right\rangle^{C_n^F} \right\rangle^{C_\eta},$$

where $\langle \dots \rangle^C$ is averaging over the configuration C ,

$$\langle F \rangle^{C_n^F} = \prod_{i=1}^M \int dn_i^F P(n_i^F) F.$$

$$\langle F \rangle^{C_\eta} = \prod_{i=1}^M \int d\eta_i P(\eta_i) F,$$

Calculating of the correlation function

$$b_{BF}^{corr f} \equiv \left. \frac{d\langle B \rangle_F}{dF} \right|_{F=\langle F \rangle}.$$

$$\langle B \rangle_F = \frac{\sum_C \langle B \rangle_C P(C) P_C(F)}{\sum_C P(C) P_C(F)}$$

In the case of great string density the configuration sum can be approximately rewritten as:

$$\sum_{C_\eta} \dots = \prod_{j=1}^M \sum_{\eta_j=0}^{\infty} \dots \longrightarrow \prod_{j=1}^M \int_0^{\infty} d\eta_j \dots,$$

$$\langle B \rangle_F = \frac{1}{P(F)} \prod_{j=1}^M \int_0^{\infty} d\eta_j P(C_{\eta_j}) \int_0^{\infty} dn_j^B P_{C_\eta}(C_n^B) \langle B \rangle_{C_\eta C_n^B} \int_0^{\infty} dn_j^F P_{C_\eta}(C_n^F) P_{C_\eta C_n^F}(F),$$

$$P(F) = \prod_{j=1}^M \int_0^{\infty} d\eta_j P(C_{\eta_j}) \int_0^{\infty} dn_j^B P_{C_\eta}(C_n^B) \int_0^{\infty} dn_j^F P_{C_\eta}(C_n^F) P_{C_\eta C_n^F}(F).$$

$$S_\nu = \sum_{i=1}^M \bar{\eta}_i^\nu$$

$$b_{nn} = \frac{\omega_\eta \mu_B M}{4\omega_\mu S_{1/2} + \mu_F \omega_\eta M},$$

$$b_{p_t n} = \frac{p_0 \omega_\eta \left(\frac{3}{2} \frac{S_{1/4}}{S_{1/2}} - \frac{MS_{3/4}}{(S_{1/2})^2} \right)}{4\omega_\mu S_{1/2} + M\mu_F \omega_\eta},$$

$$b_{p_t p_t} = \frac{\omega_\eta \mu_F \left(9S_{1/2}^3 - 12S_{1/4} S_{3/4} S_{1/2} + 4MS_{3/4}^2 \right)}{16\gamma S_1 S_{1/2}^2 + \omega_\eta \mu_F \left(9S_{1/2}^3 - 12S_{1/4} S_{3/4} S_{1/2} + 4MS_{3/4}^2 \right) + 16\omega_\mu S_{1/2} \left(S_1 S_{1/2} - S_{3/4}^2 \right)}.$$

Non-uniform string distribution in transverse plane:

$\bar{\eta}_+$	$\bar{\eta}_+$	$\bar{\eta}_0$	$\bar{\eta}_+$
$\bar{\eta}_+$	$\bar{\eta}_0$	$\bar{\eta}_+$	$\bar{\eta}_+$
$\bar{\eta}_0$	$\bar{\eta}_0$	$\bar{\eta}_0$	$\bar{\eta}_+$
$\bar{\eta}_0$	$\bar{\eta}_+$	$\bar{\eta}_0$	$\bar{\eta}_0$

$$\frac{M(\bar{\eta}_+ = a\bar{\eta}_0)}{M(\bar{\eta}_0)} = m$$

$$a > 1, 0 \leq m \leq 1$$

$$S_\nu = \sum_{i=1}^M \bar{\eta}_i^\nu = mM(a\bar{\eta}_0)^\nu + (1-m)M\bar{\eta}_0^\nu$$

$$b_{nn} = \frac{\omega_\eta \mu_B M}{4\omega_\mu S_{1/2} + \mu_F \omega_\eta M},$$

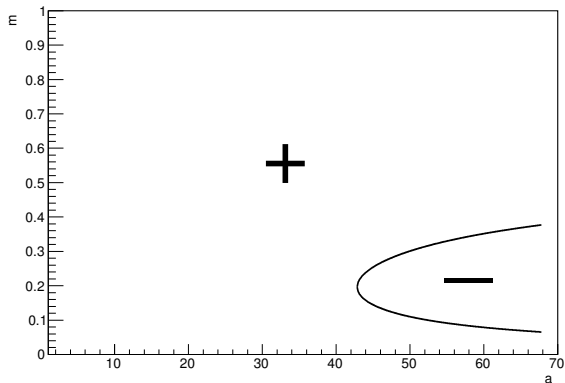
$$b_{p_t n} = \frac{p_0 \omega_\eta \left(\frac{3}{2} \frac{S_{1/4}}{S_{1/2}} - \frac{MS_{3/4}}{(S_{1/2})^2} \right)}{4\omega_\mu S_{1/2} + M\mu_F \omega_\eta} = b_{nn} \frac{p_0 S_{3/4}}{\mu_B (S_{1/2})^2} \left(\frac{3}{2} \frac{S_{1/4} S_{1/2}}{MS_{3/4}} - 1 \right)$$

$$\frac{3}{2} \frac{S_{1/4} S_{1/2}}{MS_{3/4}} - 1 < 0,$$

$$m^2(3a^{3/4} - 3a^{1/2} - 3a^{1/4} + 3) + m(-2a^{3/4} + 3a^{1/2} + 3a^{1/4} - 4) + 1 < 0$$

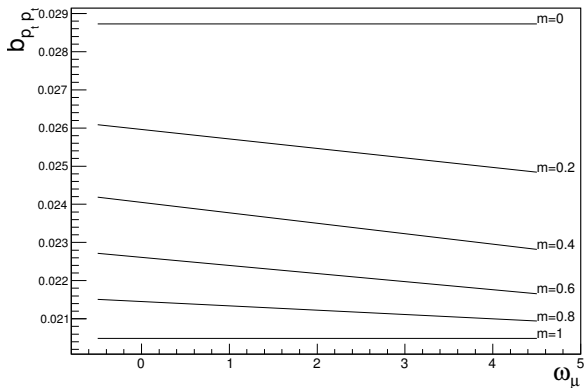
$$F(a, m) \equiv m^2(3a^{3/4} - 3a^{1/2} - 3a^{1/4} + 3) + m(-2a^{3/4} + 3a^{1/2} + 3a^{1/4} - 4) + 1.$$

$$F(a,m)=0$$



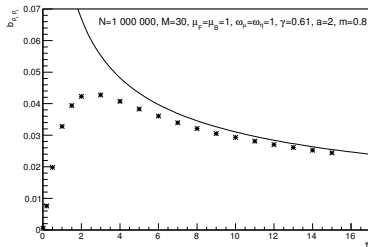
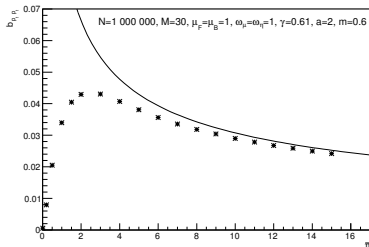
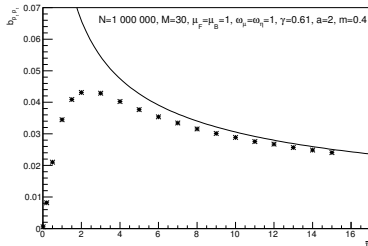
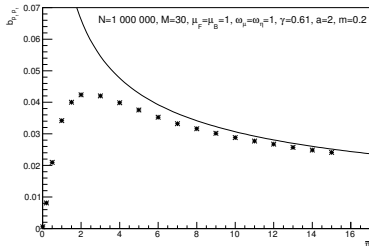
There are such non-uniform distributions of strings for which this coefficient becomes negative

Studing dependence of $b_{p_t p_t}$ on ω_μ



$$M = 450, a = 2, \bar{\eta}_0 = 12, \omega_\eta = 1, \mu_F = \mu_B = 1, \gamma = 0.61$$

MC numerical calculations of the coefficient $b_{p_t p_t}$



Conclusion

- $b_{nn}^{mean} = b_{nn}^{corr f}$;
- $b_{p_t n}^{mean} = b_{p_t n}^{corr f}$;
- $b_{p_t p_t}^{mean} = b_{p_t p_t}^{corr f}$;
- The examples with different cases of non-uniform string distribution in transverse plane were considered;
- It is shown that there are distributions of strings for which coefficient $b_{p_t n}$ becomes negative;
- The dependence of the correlation coefficient $b_{p_t p_t}$ on ω_μ disappears in the case of a homogeneous distribution of strings;
- The received asymptotes for the correlation coefficient between transverse momenta were compared with the results of the MC numerical calculations of this coefficient.