Neutron star mergers
Lecture II

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Dubna International Advanced School of Theoretical Physics
Helmholtz International Summer School
Matter under Extreme Conditions in Heavy-Ion Collisions and Astrophysics

Plan of the lectures

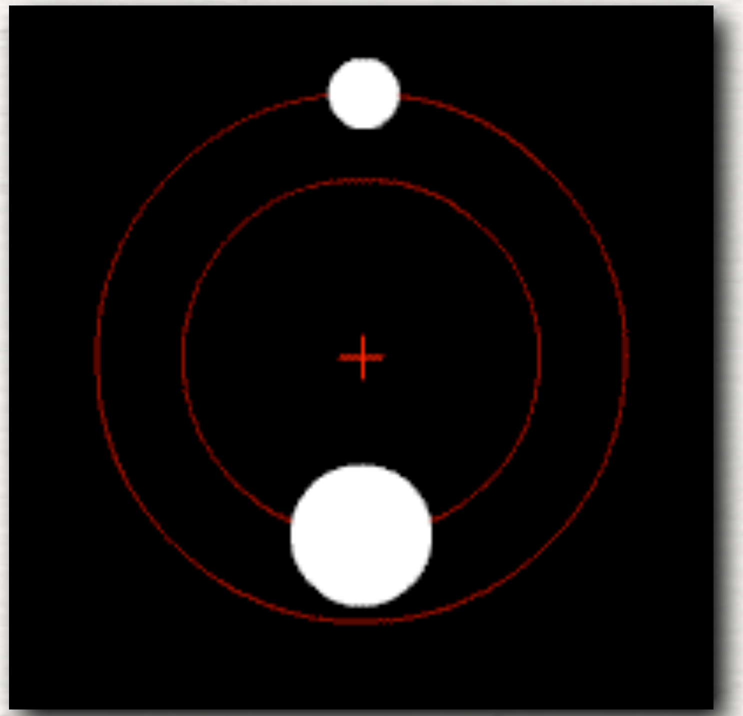
- ✴Lecture I: **brief** introduction to numerical relativity
- ✴Lecture II: **brief** review dynamics of merging binaries
- ✴Lecture III: **brief** overview of EOS constraints from mergers

✴**L. Baiotti and L. Rezzolla, Rep. Prog. Phys. 80, 096901, 2017**

✴V. Paschalidis, Classical Quantum Gravity 34, 084002 2017

✴Rezzolla and Zanotti, *"Relativistic Hydrodynamics"*, Oxford University Press, 2013

The two-body problem: Newton vs Einstein



Take two objects of mass m_1 and m_2 interacting only gravitationally

In **Newtonian gravity** solution is analytic: there exist closed orbits (circular/elliptic) with

$$\ddot{\boldsymbol{r}} = -\frac{GM}{d_{12}^3}\boldsymbol{r}$$

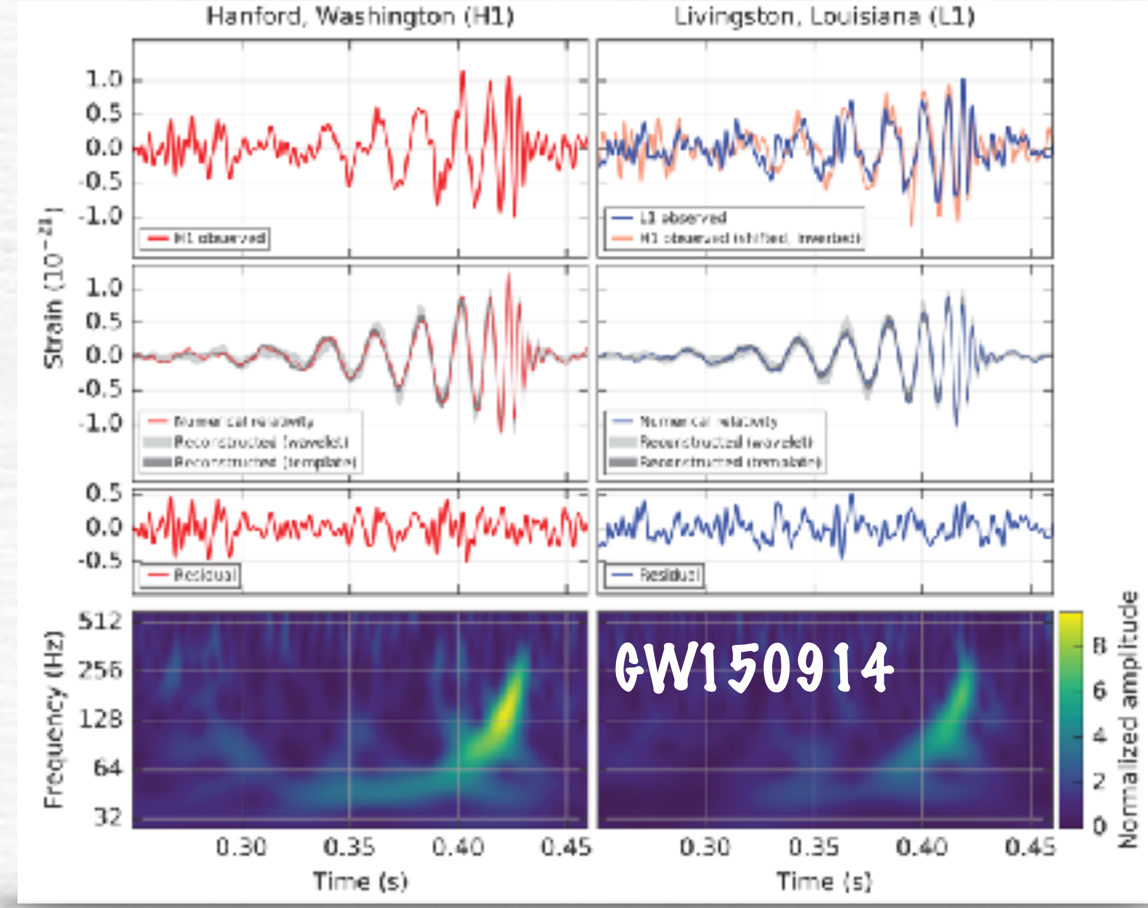
where $M \equiv m_1 + m_2\,, \boldsymbol{r} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2\,, d_{12} \equiv |\boldsymbol{r}_1 - \boldsymbol{r}_2|\,.$

In **Einstein's gravity** no analytic solution! No closed orbits: the system loses energy/angular momentum via gravitational waves.

The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH ⟶ BH + GWs



Abbott+ 2016

The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH ⟶ BH + GWs

- For NSs the question is more **subtle:** the merger leads to an hyper-massive neutron star (HMNS), ie a metastable equilibrium:

NS + NS ⟶ HMNS+**... ?** ⟶ BH+torus+**... ?** ⟶ BH + GWs

The two-body problem in GR

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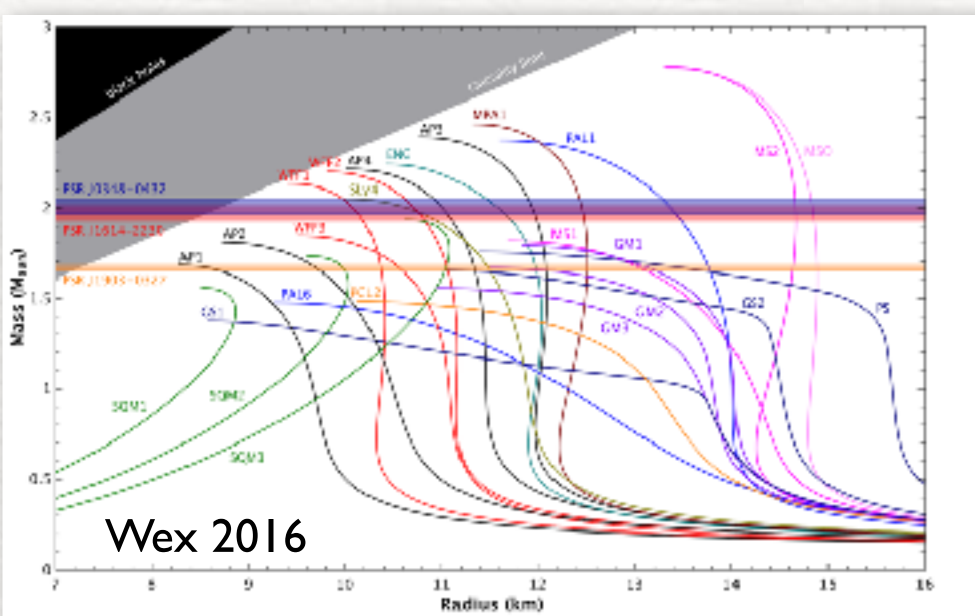
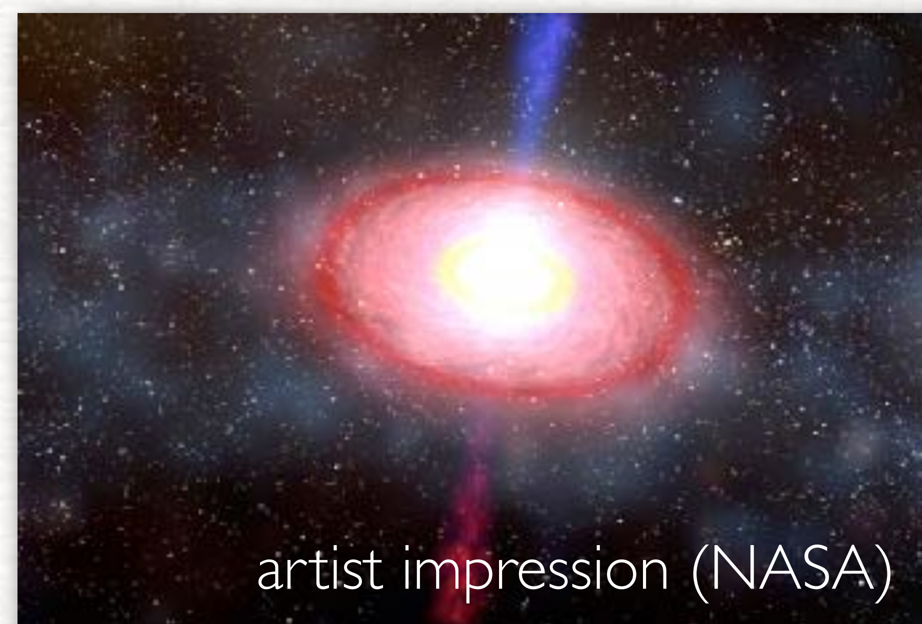
- **HMNS** phase can provide clear information on **EOS**





- **BH+torus** system may tell us on the central engine of **GRBs**

The two-body problem in GR

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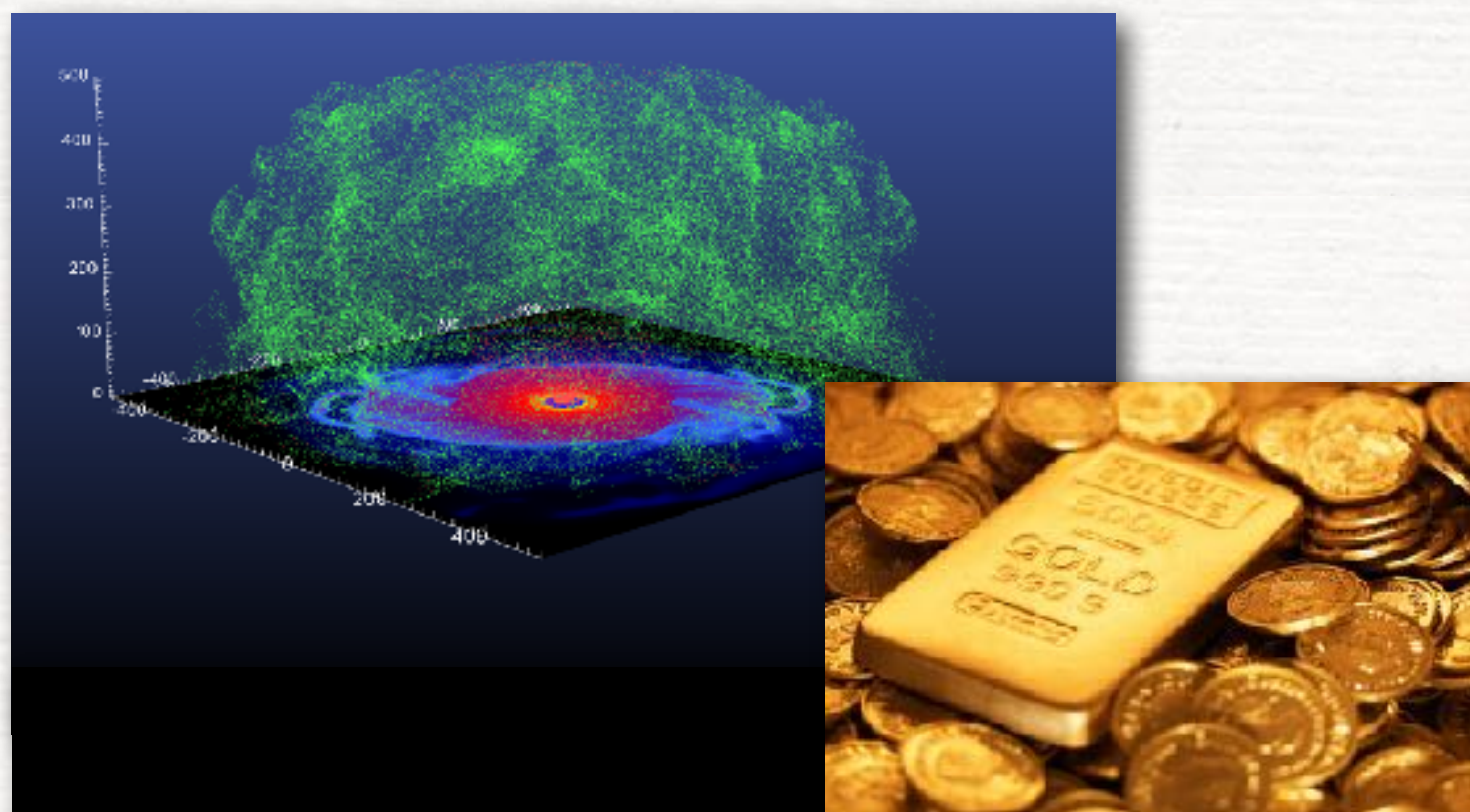
- **ejected matter** undergoes nucleosynthesis of heavy elements



The equations of numerical relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu} \,, \qquad \text{(field equations)}$$

$$\nabla_\mu T^{\mu\nu} = 0 \,, \qquad \text{(cons. energy/momentum)}$$

$$\nabla_\mu (\rho u^\mu) = 0 \,, \qquad \text{(cons. rest mass)}$$

$$p = p(\rho, \epsilon, Y_e, \ldots) \,, \qquad \text{(equation of state)}$$

$$\nabla_\nu F^{\mu\nu} = I^\mu \,, \quad \nabla_\nu^* F^{\mu\nu} = 0 \,, \qquad \text{(Maxwell equations)}$$

$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}} + \ldots \qquad \text{(energy} - \text{momentum tensor)}$$

In GR these equations do not possess an analytic solution in the regimes we are interested in

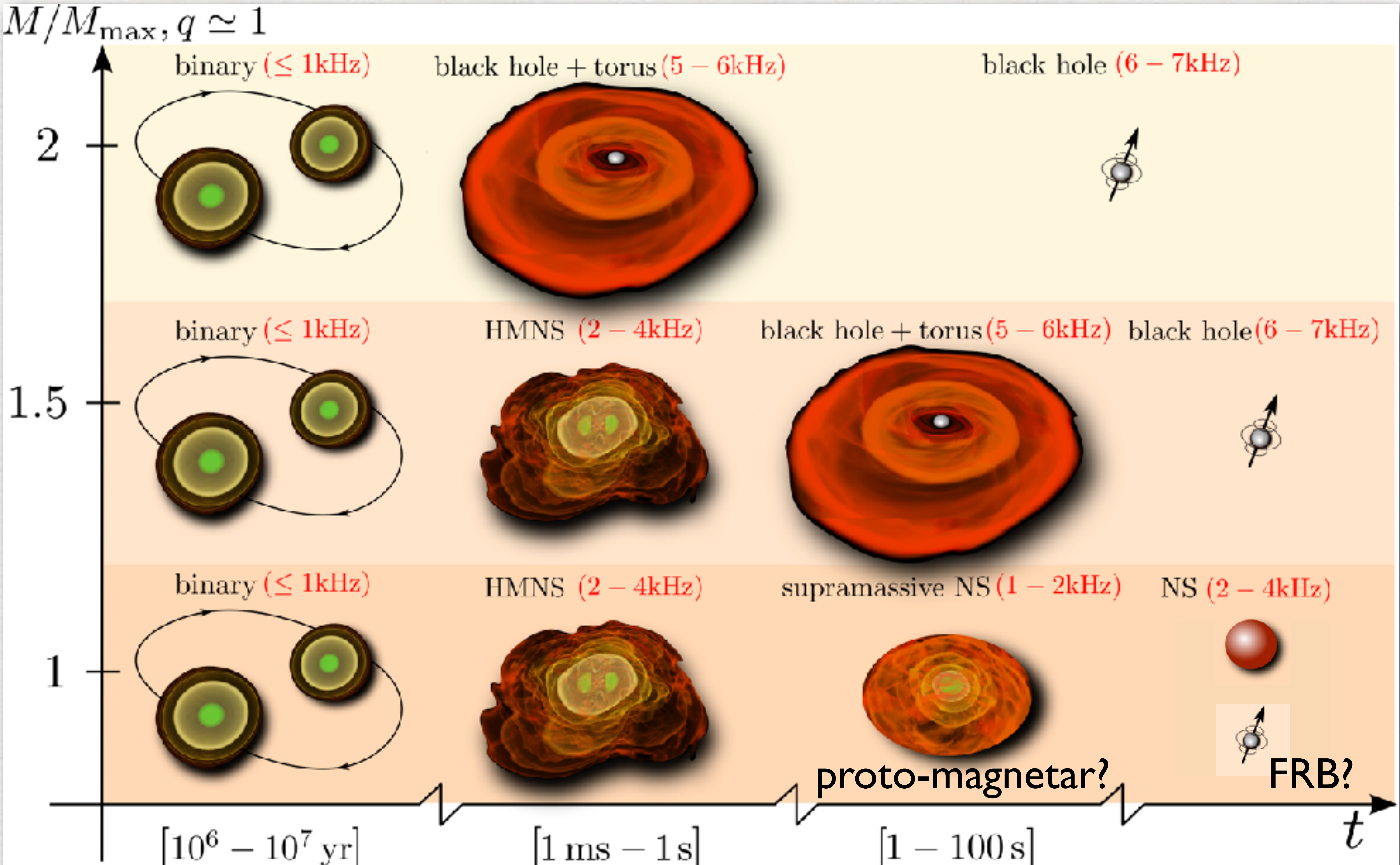
merger ⟶ HMNS ⟶ BH + torus

merger ⟶ HMNS ⟶ BH + torus

Quantitative differences are produced by:

- **total mass (prompt vs delayed collapse)**

Broadbrush picture



merger ⟶ HMNS ⟶ BH + torus

Quantitative differences are produced by:

- total mass (prompt vs delayed collapse)
- mass asymmetries (HMNS and torus)

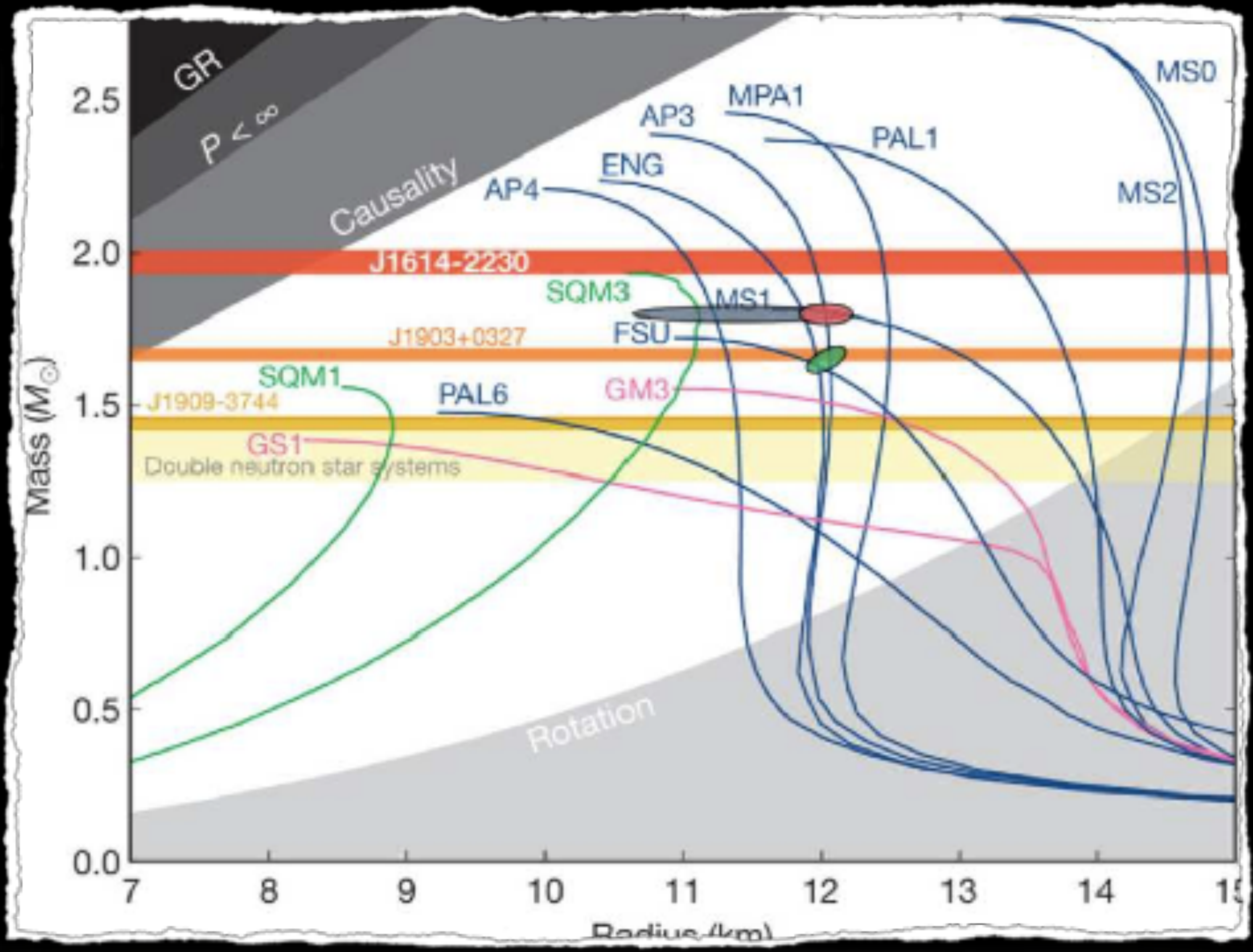
- the torii are generically more massive
- the torii are generically more extended
- the torii tend to stable quasi-Keplerian configurations
- overall unequal-mass systems have all the ingredients needed to create a GRB

merger ⟶ HMNS ⟶ BH + torus

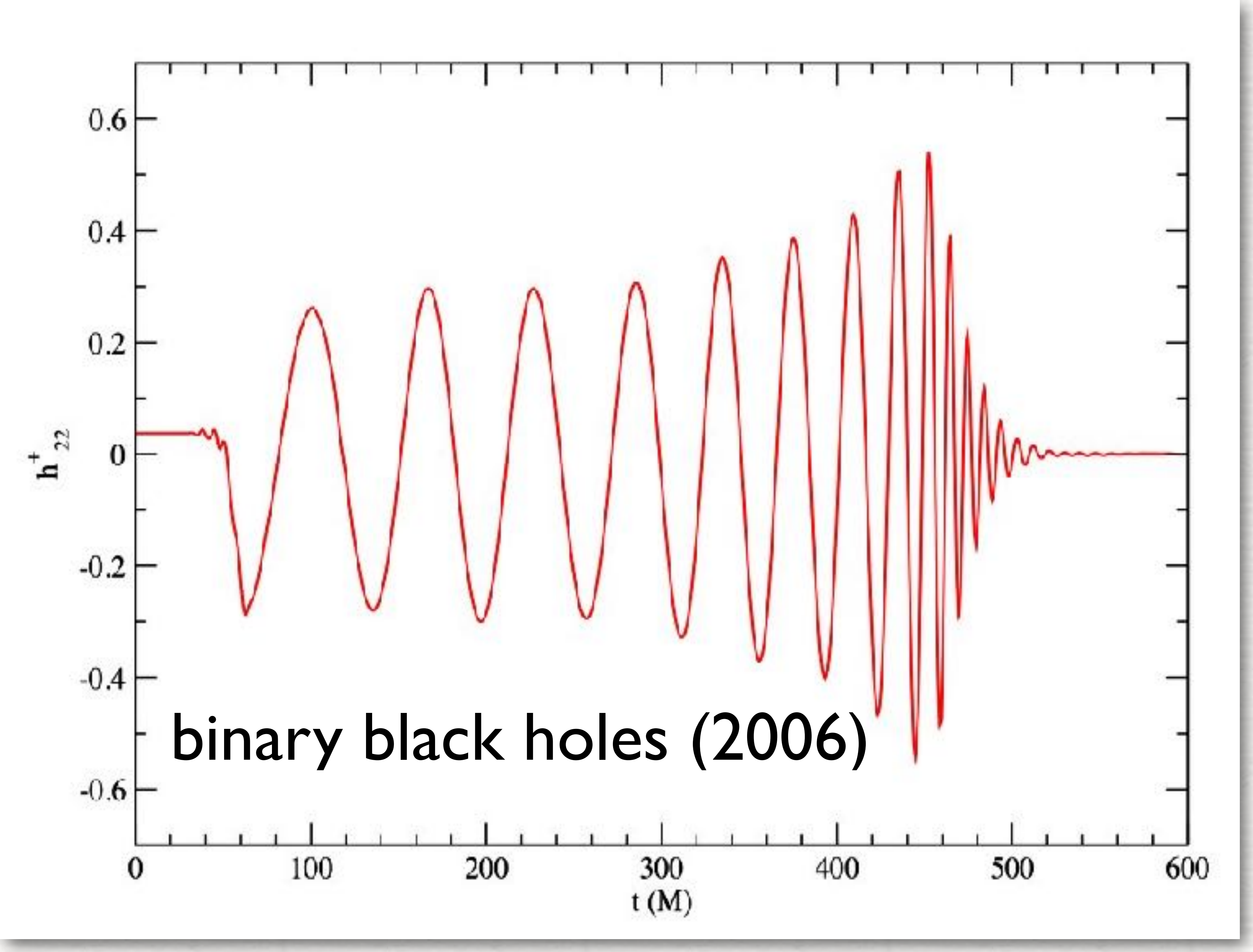
Quantitative differences are produced by:

- total mass (prompt vs delayed collapse)
- mass asymmetries (HMNS and torus)
- soft/stiff EOS (inspiral and post-merger)
- magnetic fields (equil. and EM emission)
- radiative losses (equil. and nucleosynthesis)

How to constrain the EOS from the GWs



Anatomy of the GW signal



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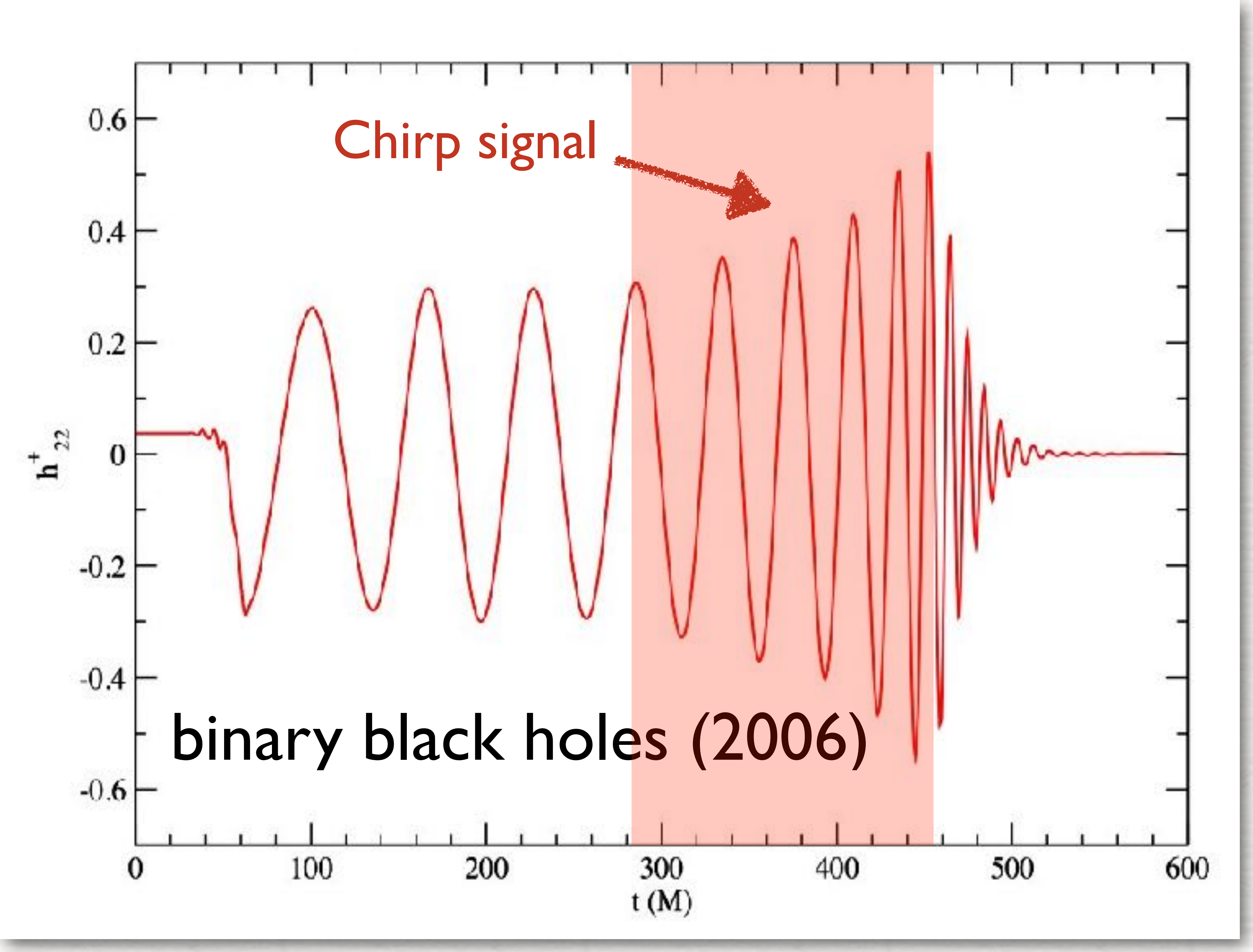

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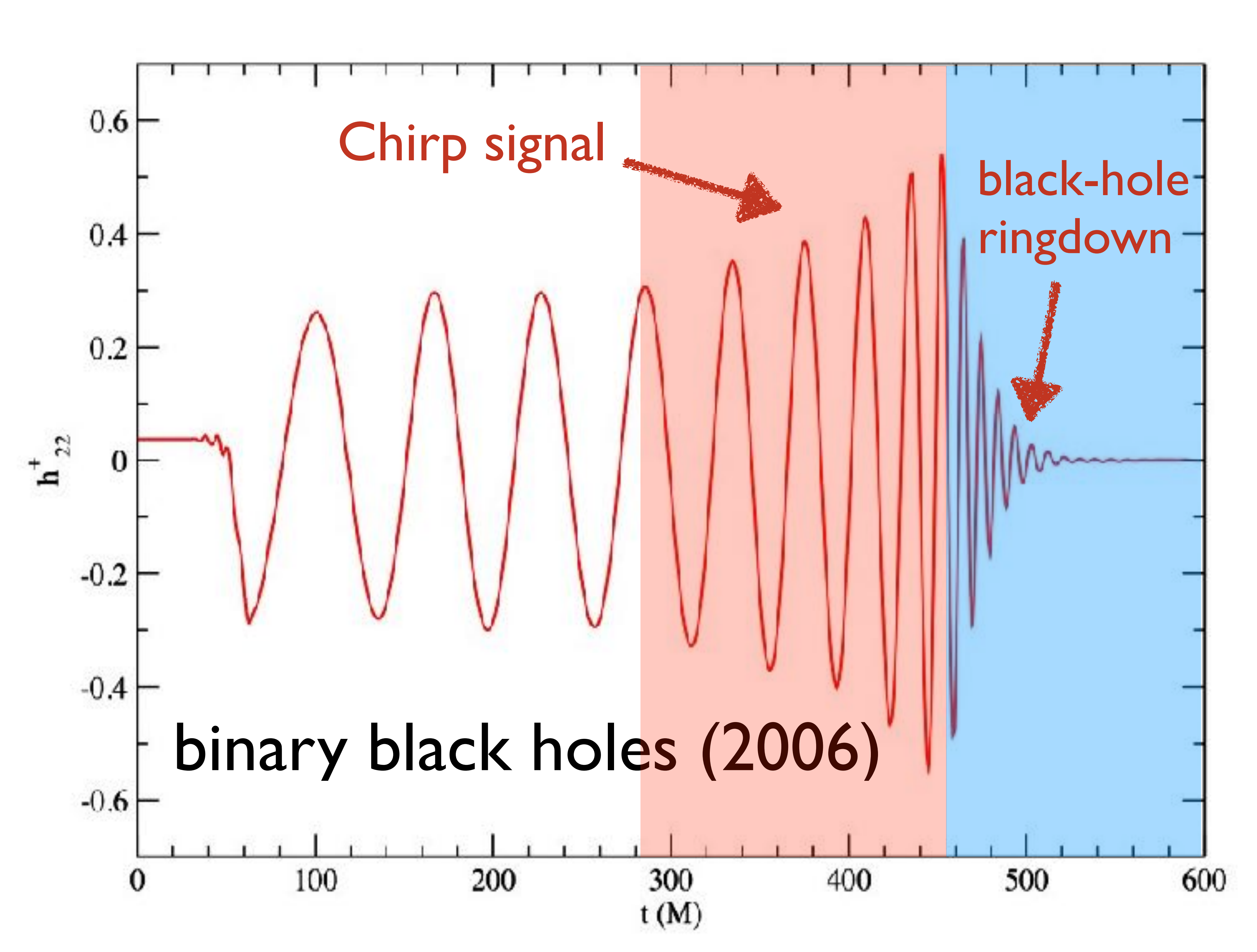
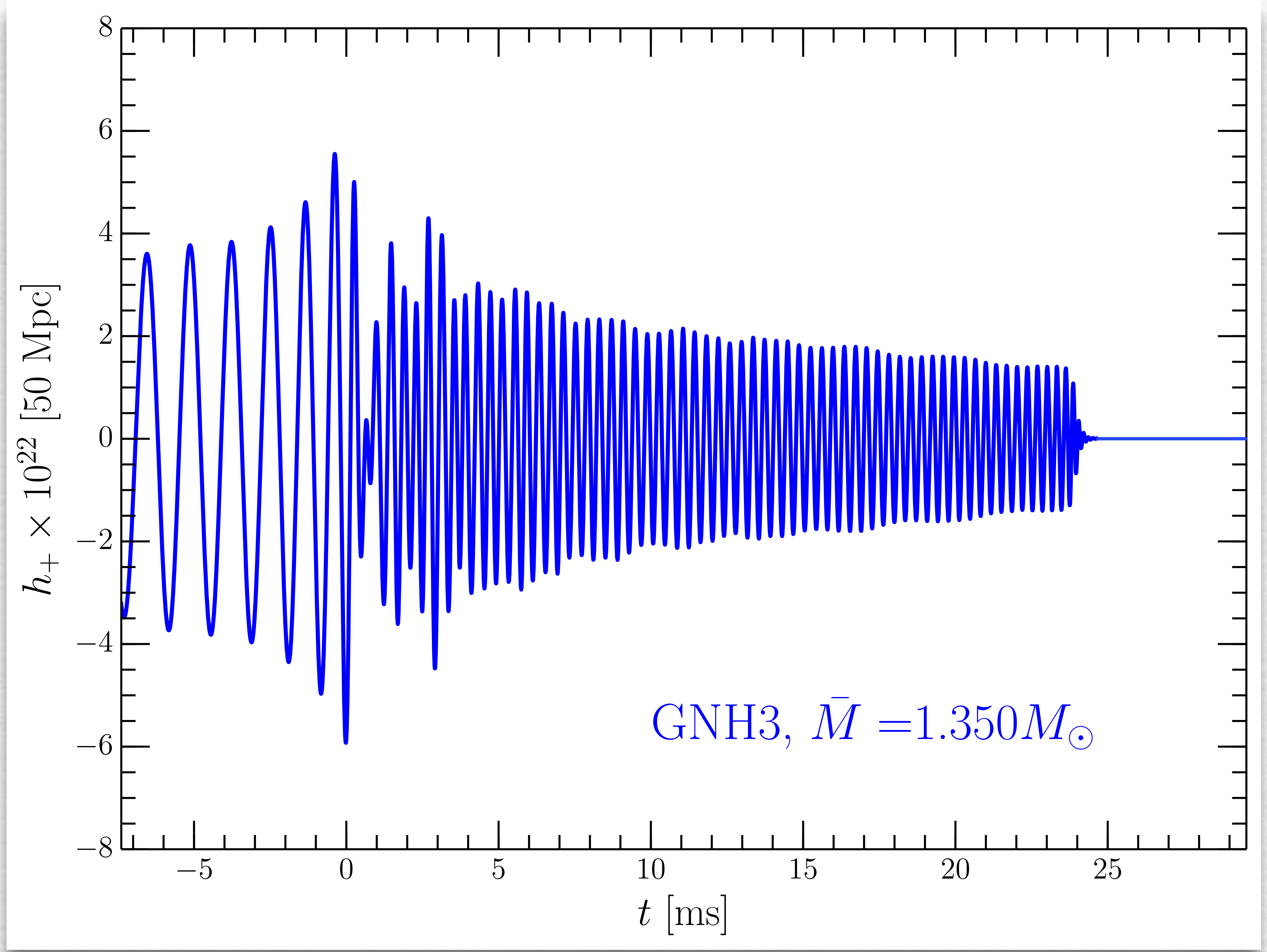
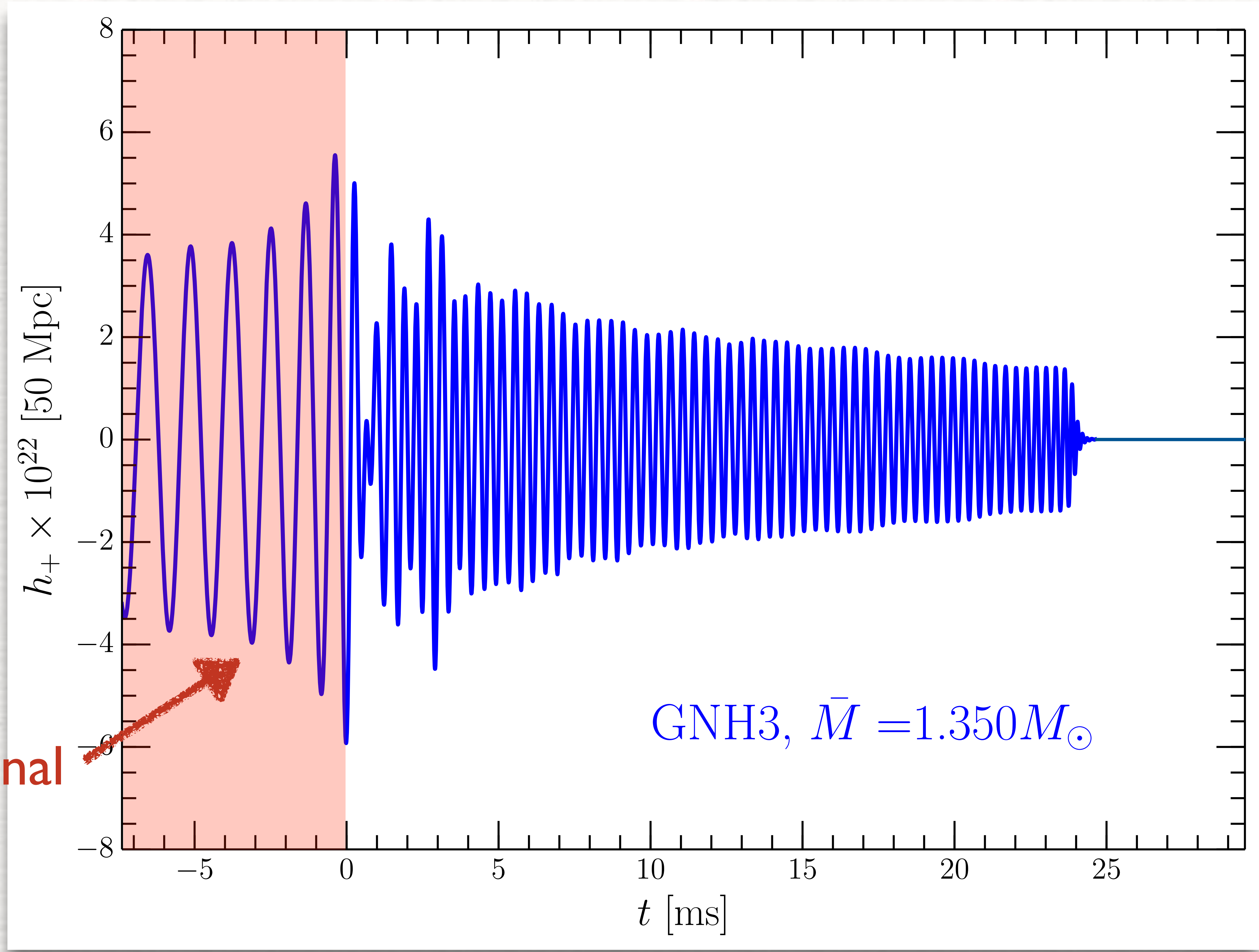


Anatomy of the GW signal



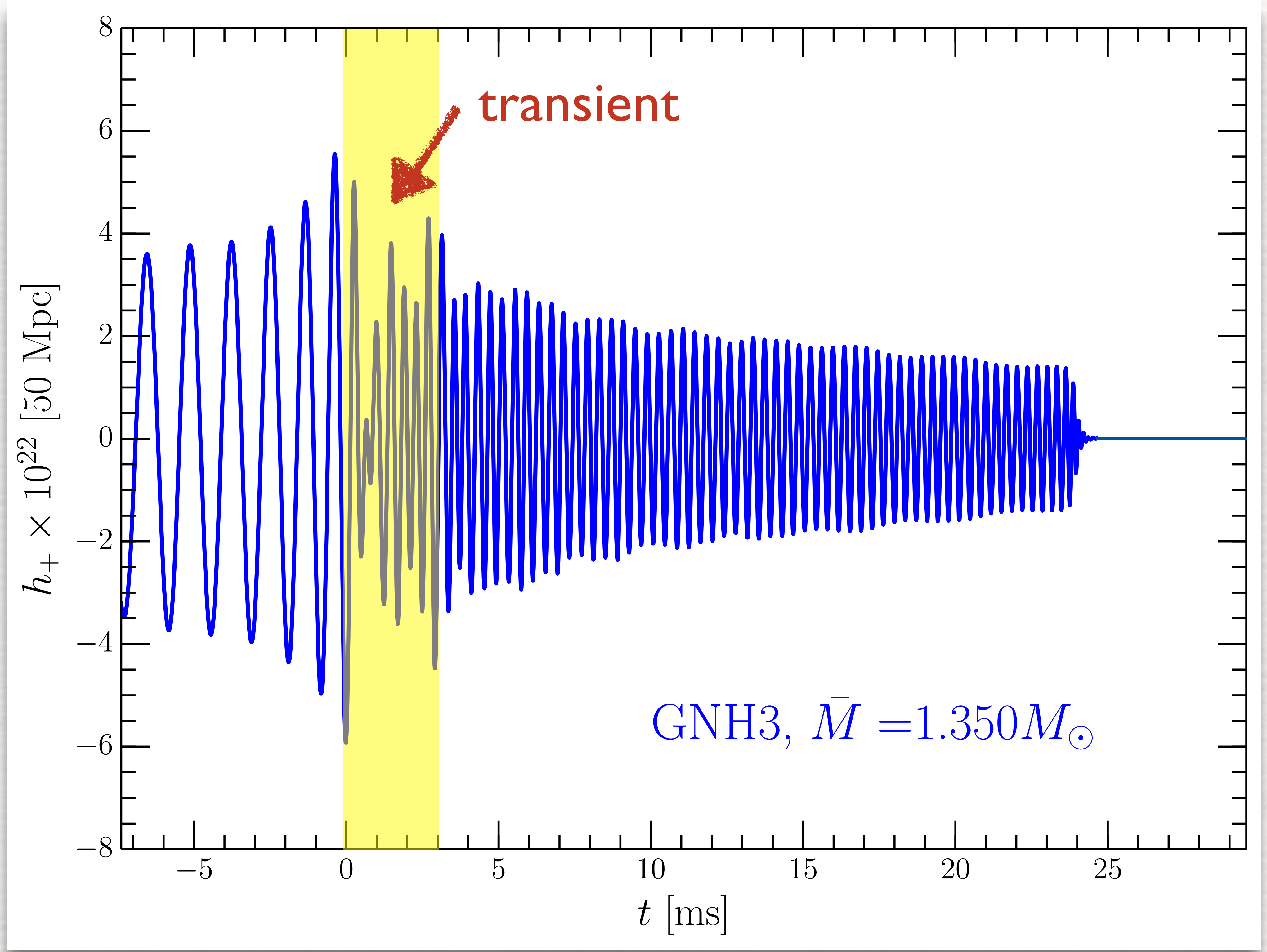
Anatomy of the GW signal

Chirp signal



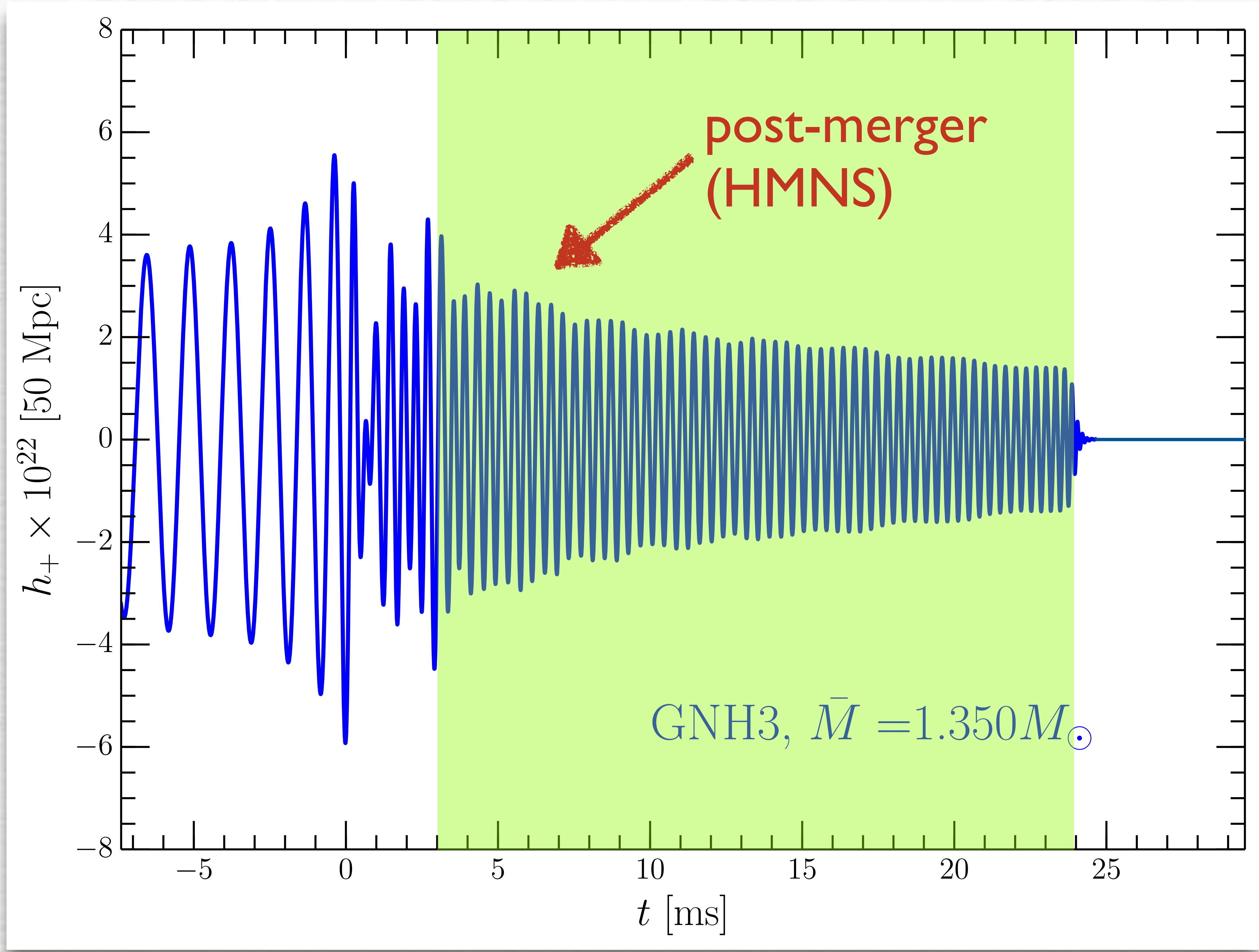
Inspiral: well approximated by PN/EOB; tidal effects important

Anatomy of the GW signal



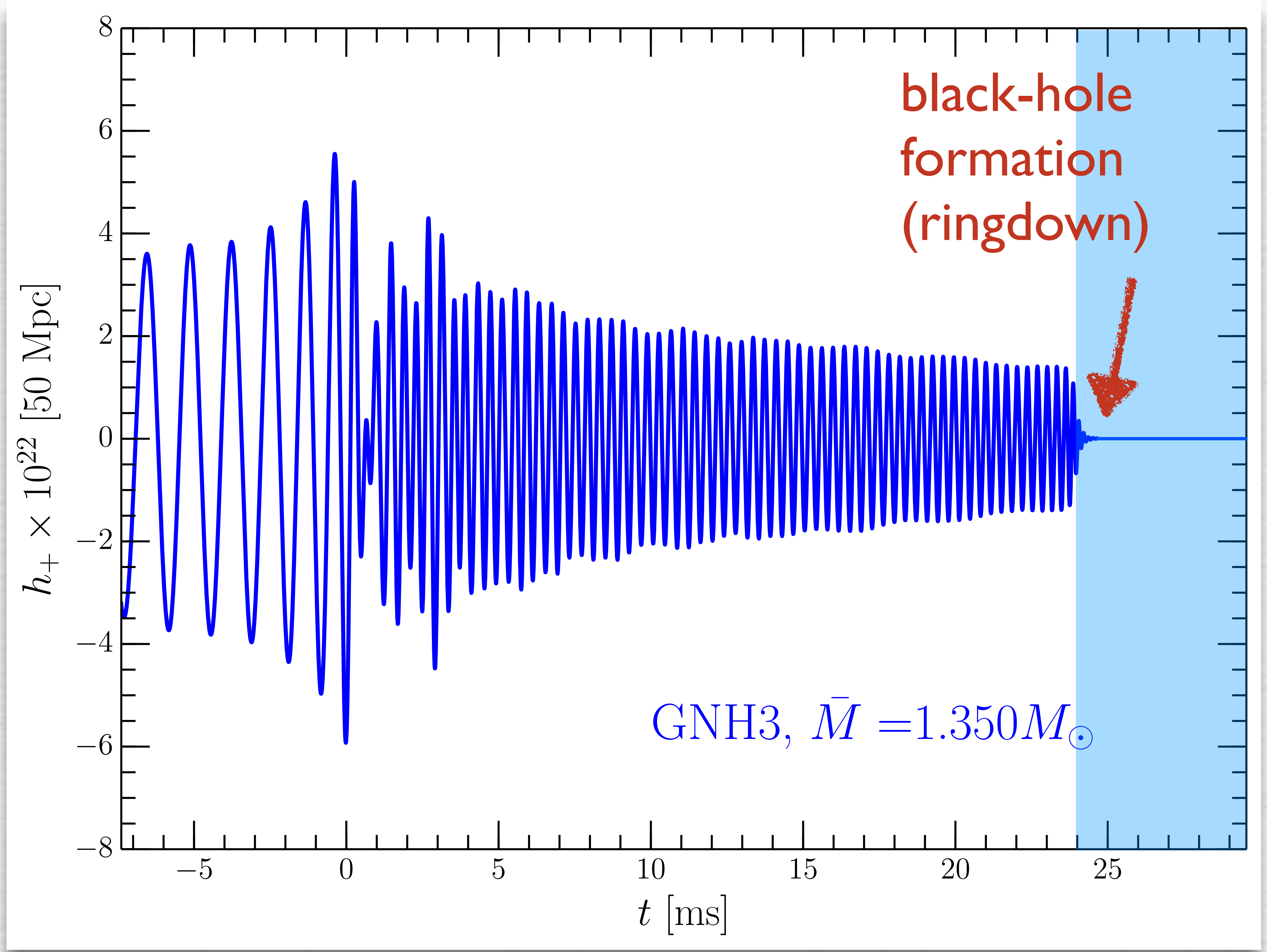
Merger: highly nonlinear but analytic description possible

Anatomy of the GW signal



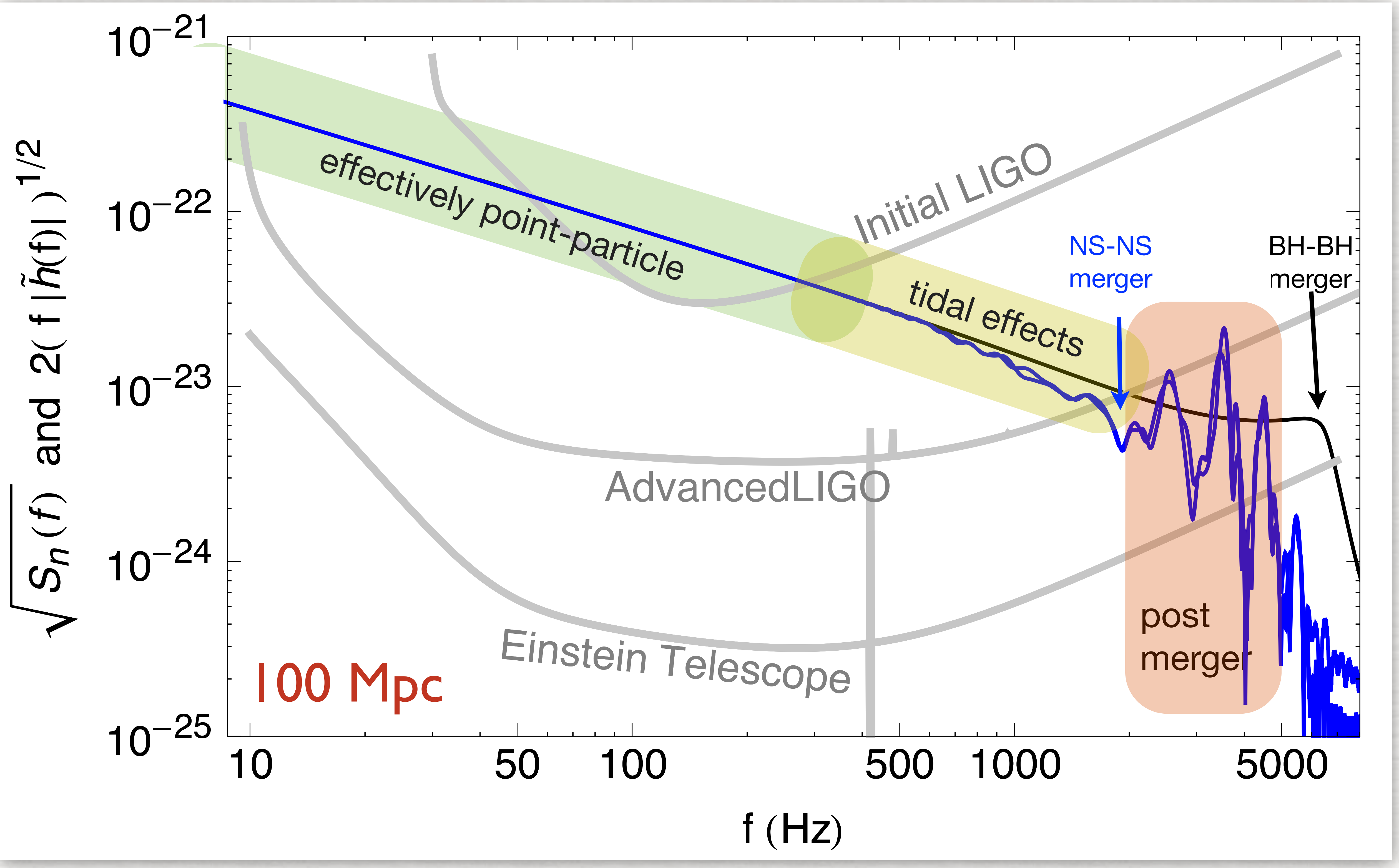
post-merger: quasi-periodic emission of bar-deformed HMNS

Anatomy of the GW signal



Collapse-ringdown: signal essentially shuts off.

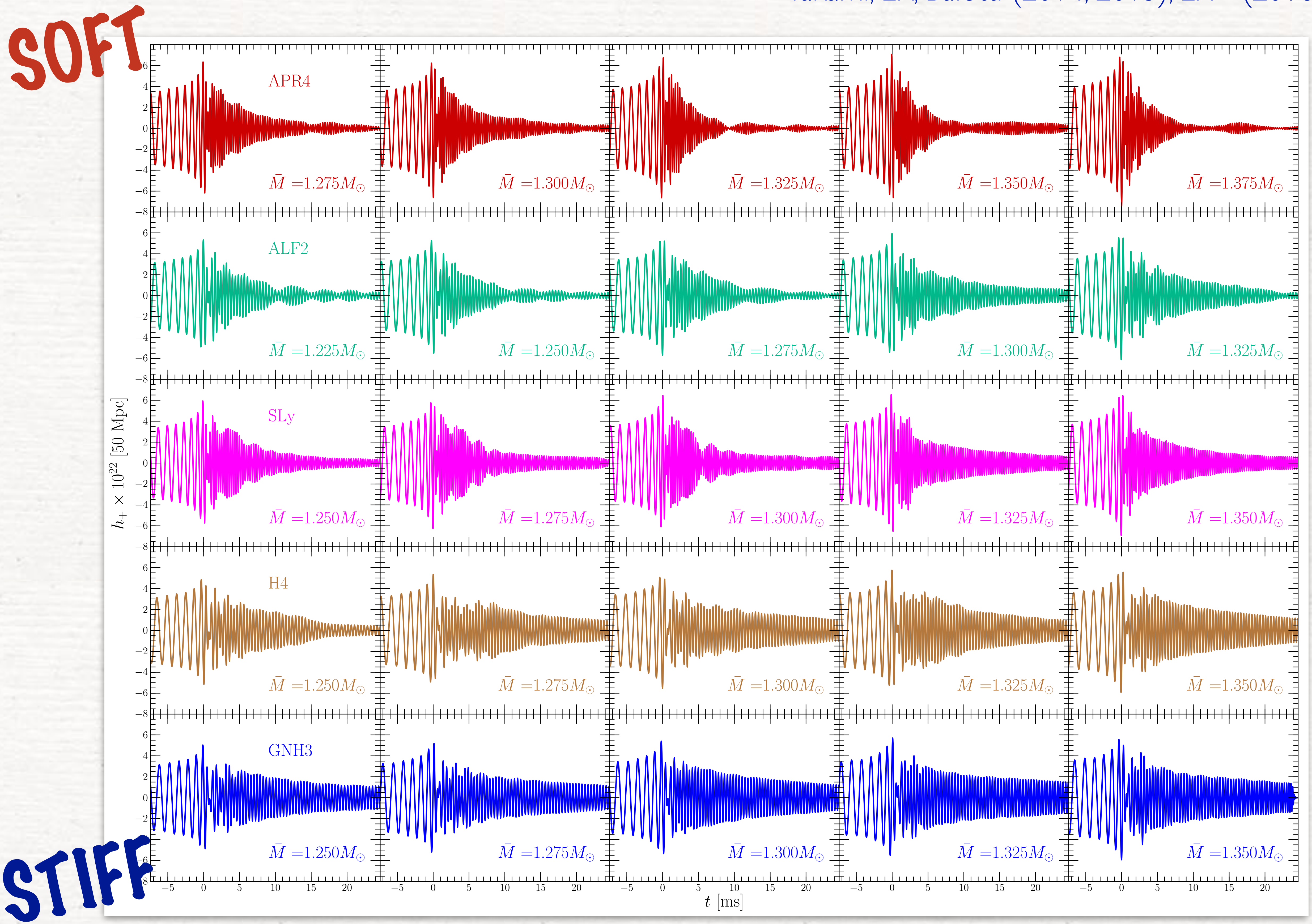
In frequency space



Read et al. (2013)

What we can do nowadays

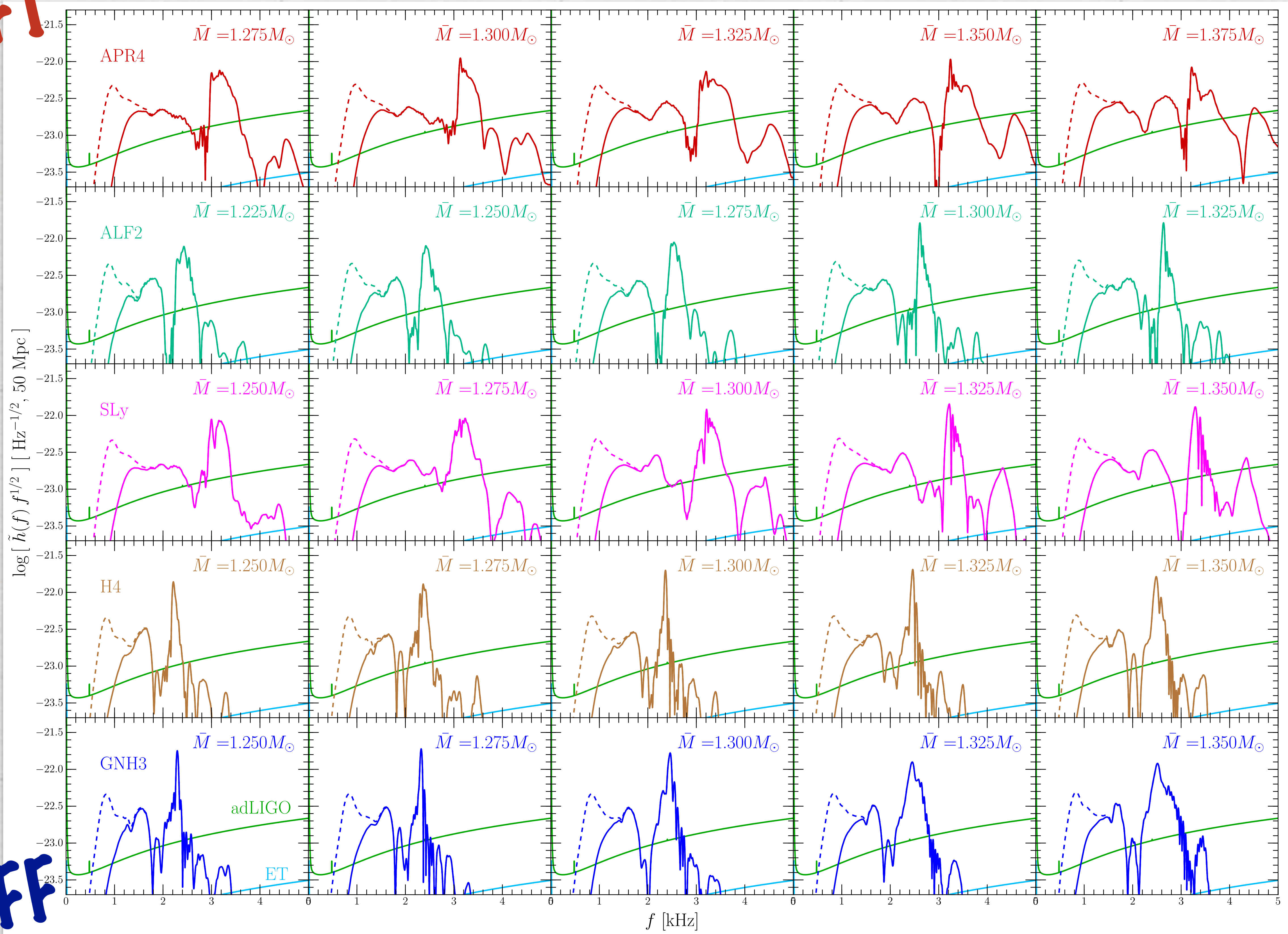
Takami, LR, Baiotti (2014, 2015), LR+ (2016)



Extracting information from the EOS

Takami, LR, Baiotti (2014, 2015), LR+ (2016)

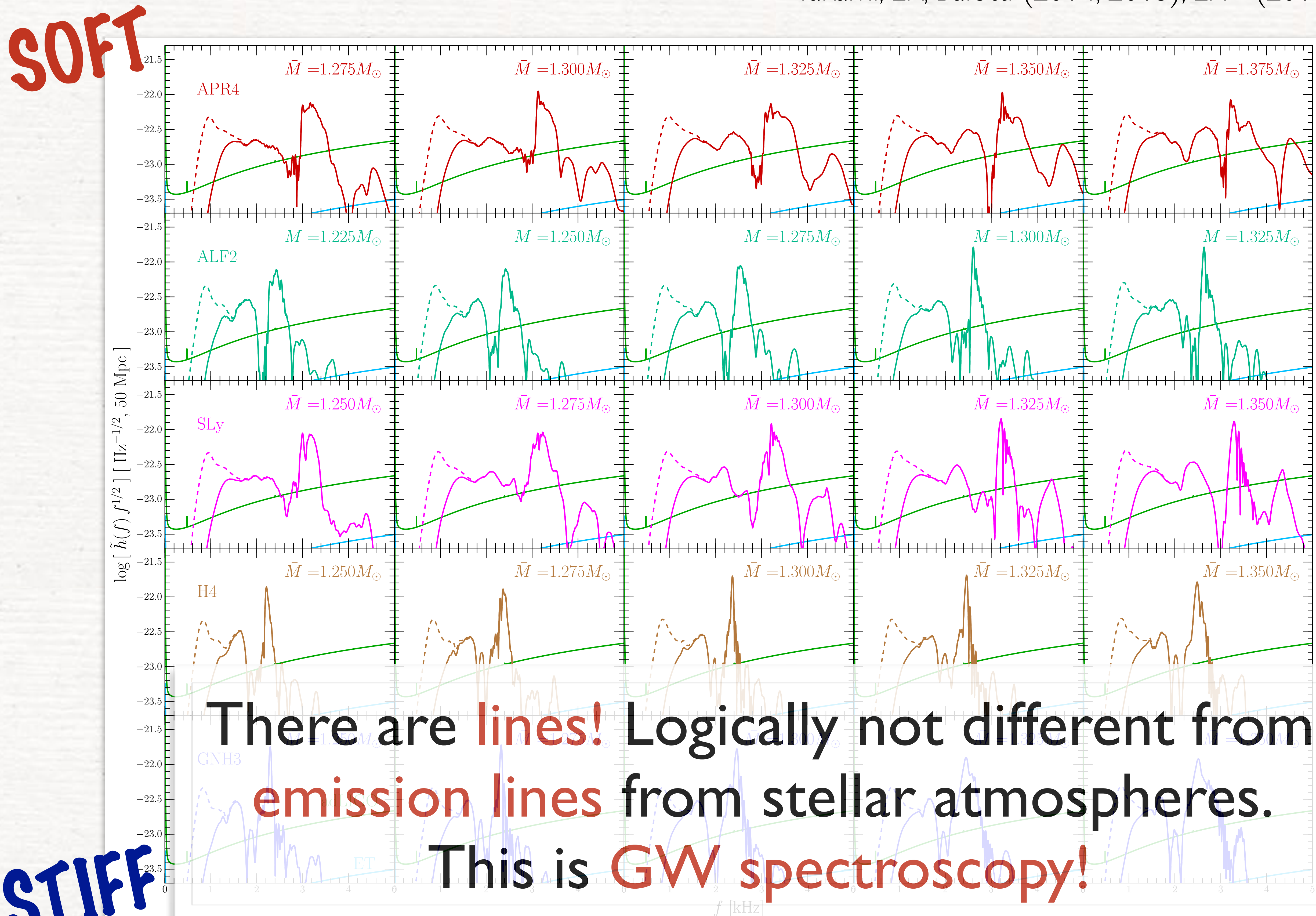
SOFT



STIFF

Extracting information from the EOS

Takami, LR, Baiotti (2014, 2015), LR+ (2016)



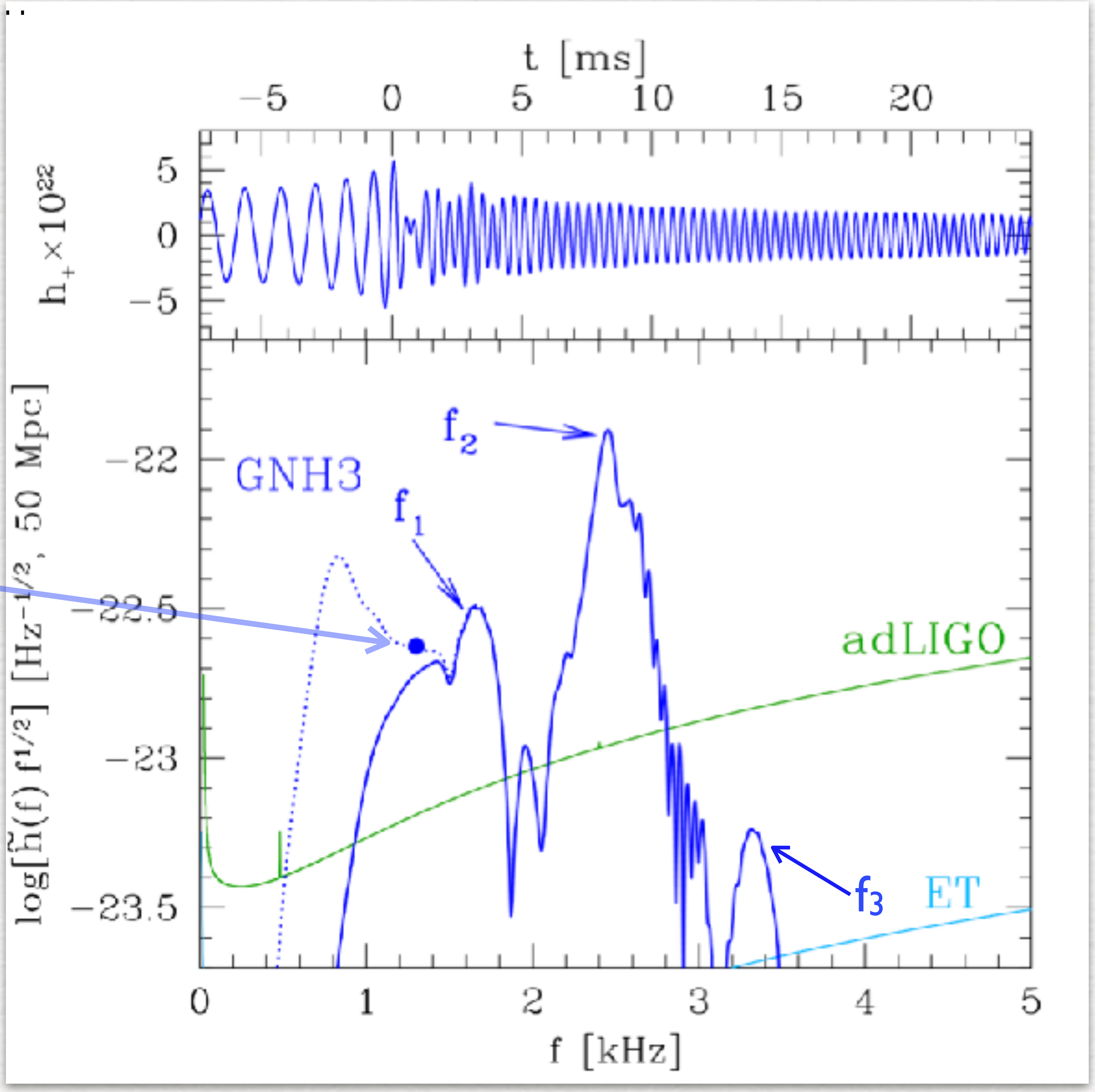
There are **lines!** Logically not different from **emission lines** from stellar atmospheres.

This is **GW spectroscopy!**

A new approach to constrain the EOS

Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, Clark+ 2016, LR+2016, de Pietri+ 2016, Feo+ 2017, Bose+ 2017 …

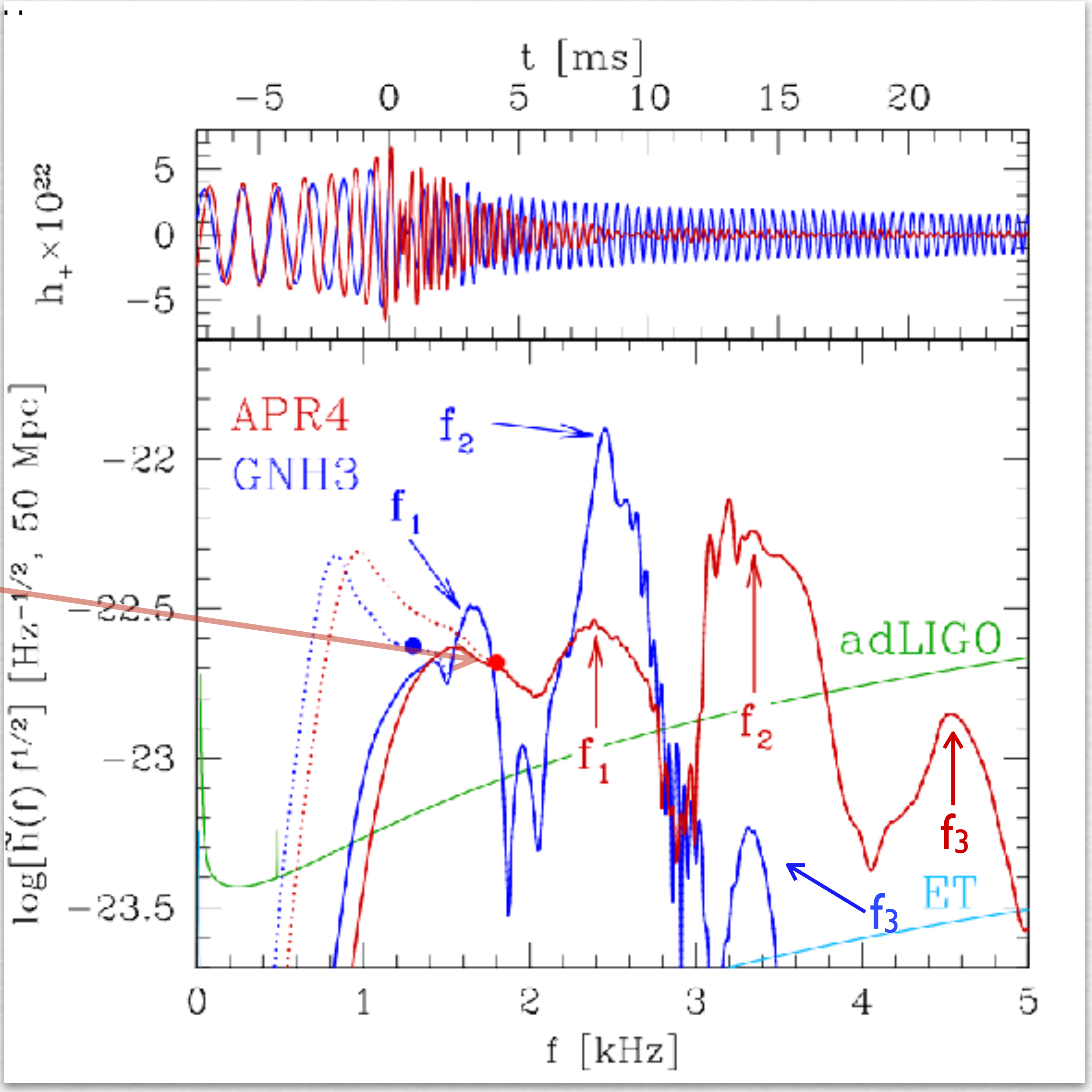
merger frequency



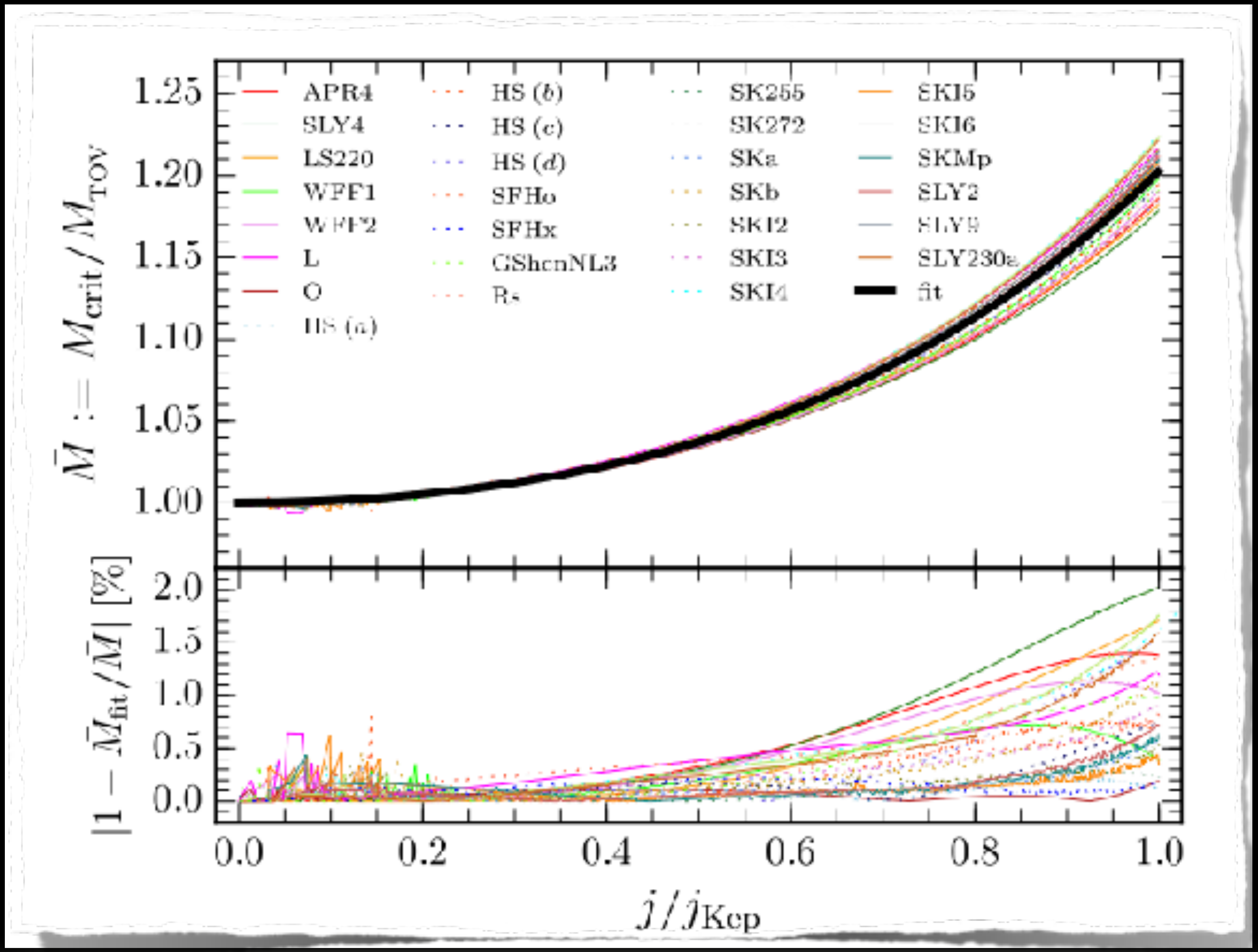
A spectroscopic approach to the EOS

Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, Clark+ 2016, LR+2016, de Pietri+ 2016, Feo+ 2017, Bose+ 2017 …

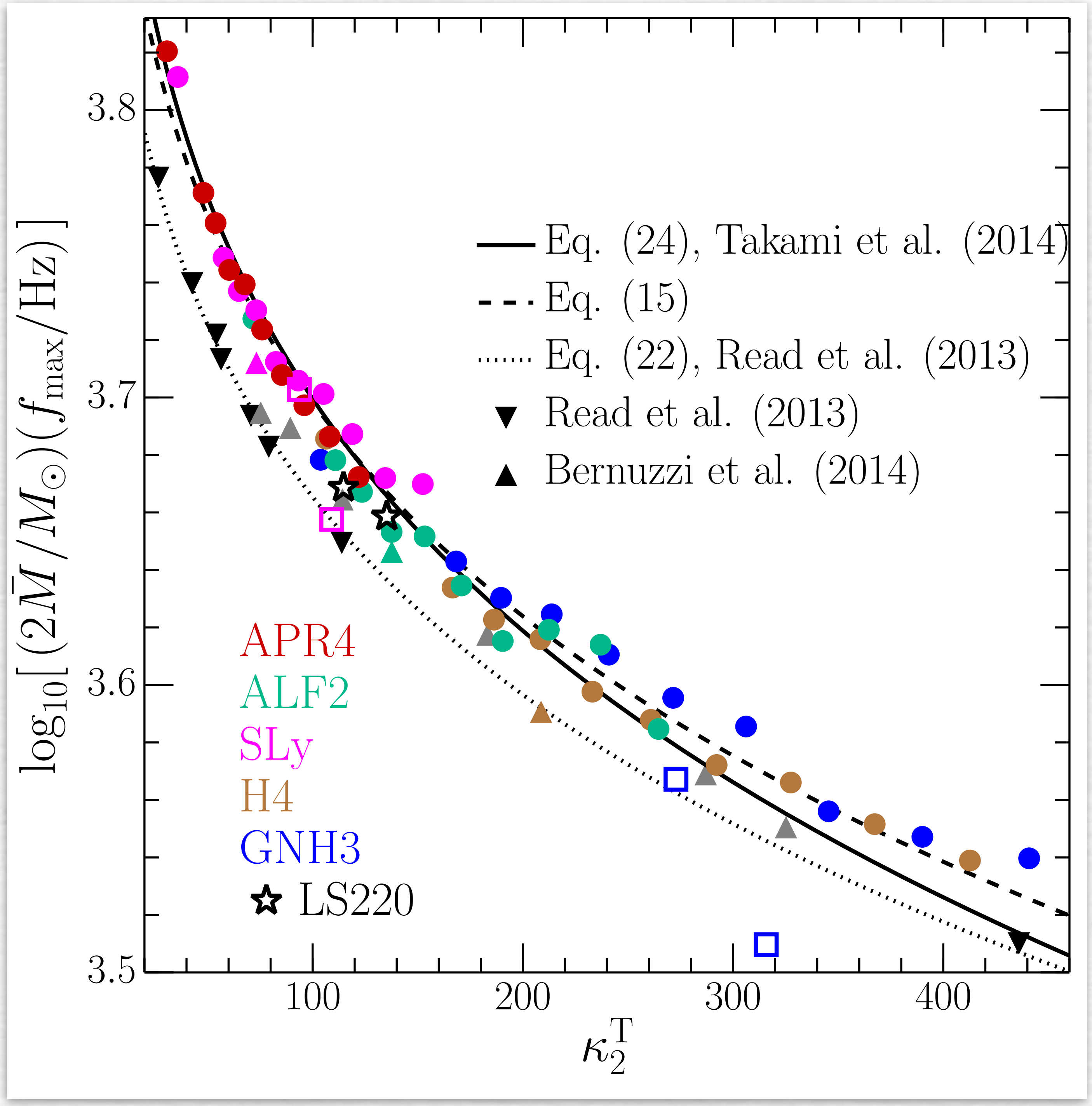
merger frequency



Quasi-universal behaviour



Quasi-universal behaviour: **inspiral**



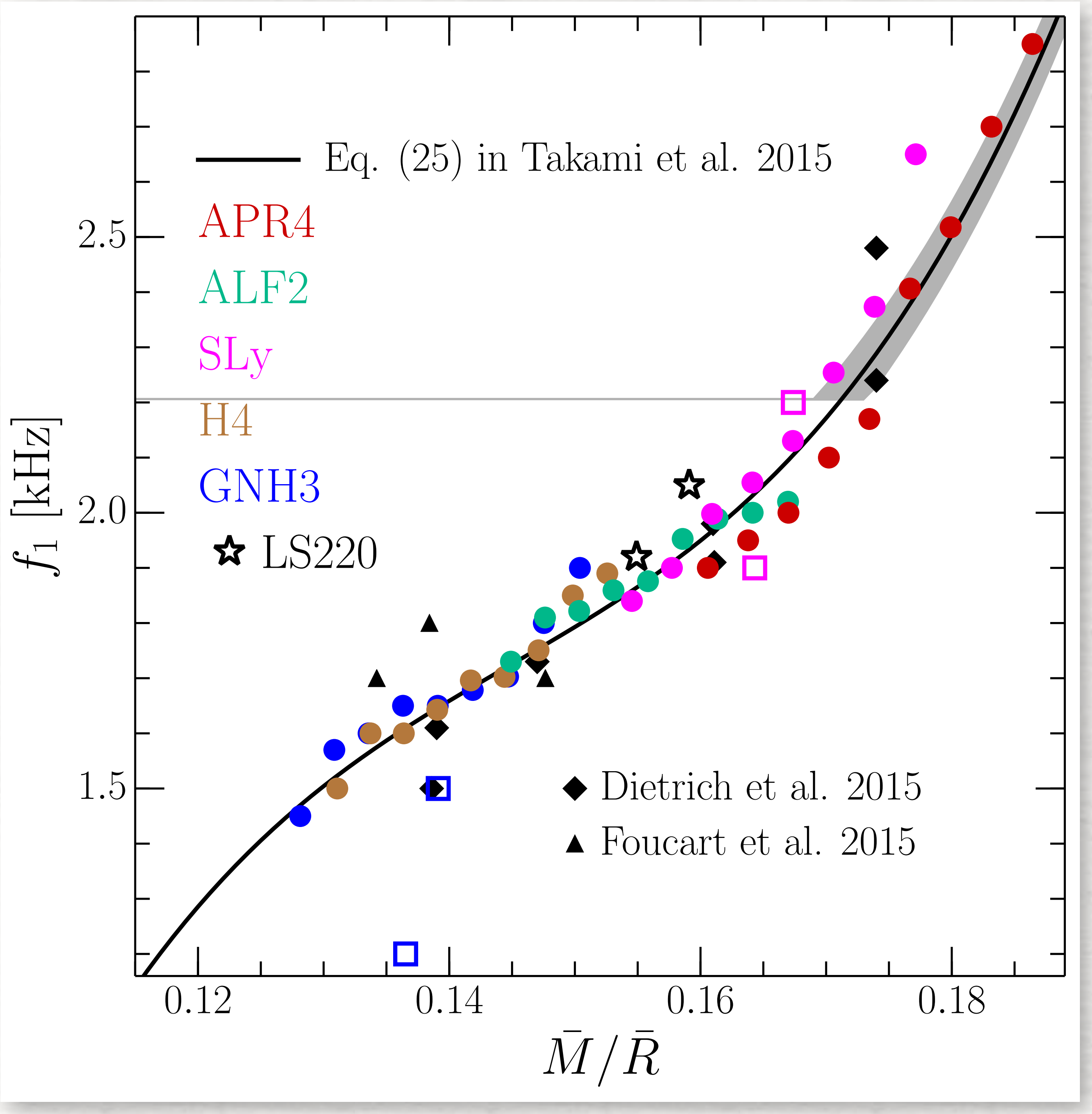
"surprising" result: quasi-universal behaviour of GW frequency at amplitude peak (Read+2013)

Many other simulations have confirmed this (Bernuzzi+ 2014, Takami+ 2015, LR+2016) .

Quasi-universal behaviour in the **inspiral** implies that once f_{max} is measured, so is tidal deformability, hence $I, \; Q, \; M/R$

$$\Lambda = \frac{\lambda}{\bar{M}^5} = \frac{16}{3}\kappa_2^T$$ tidal deformability or Love number

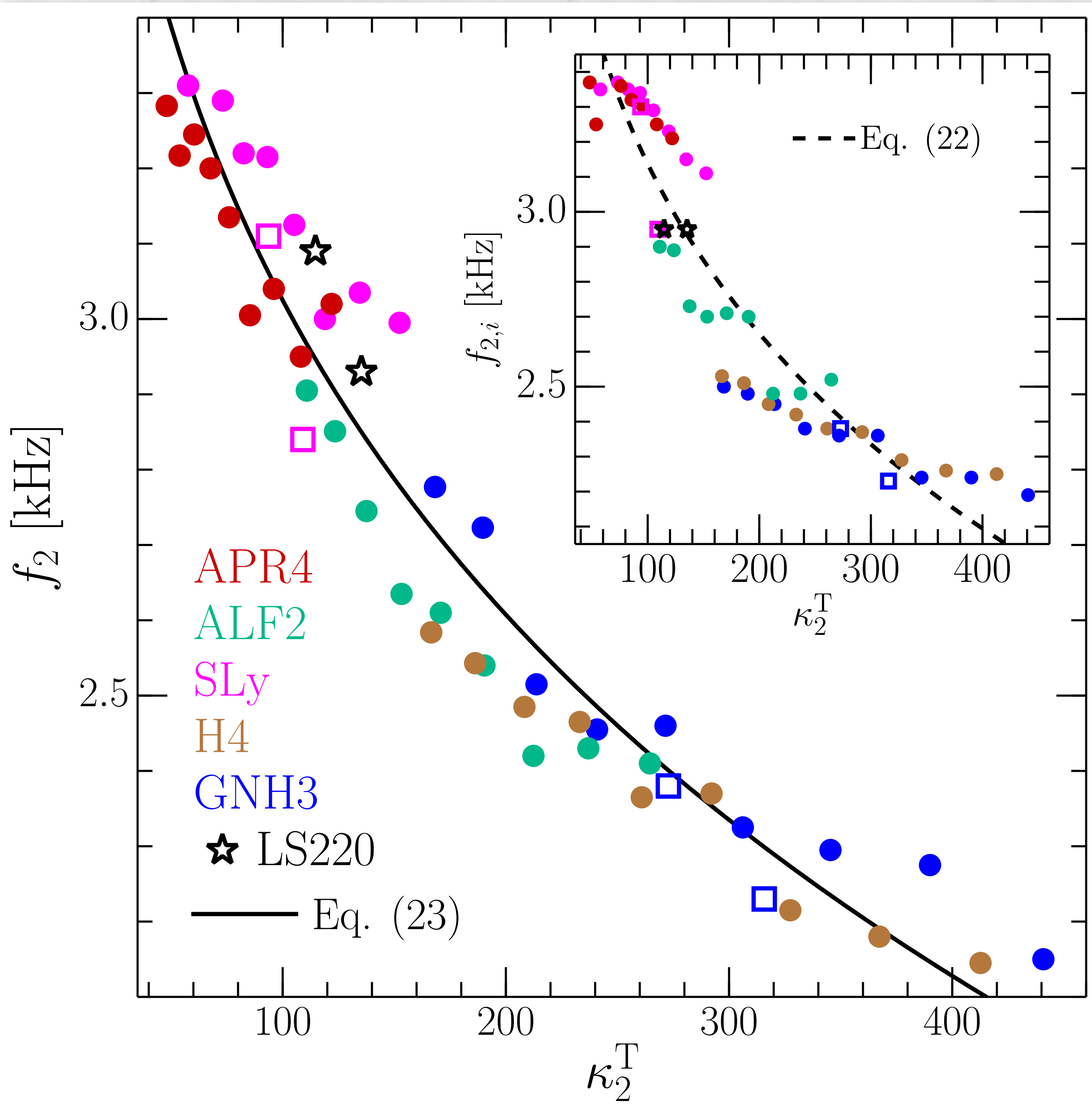
Quasi-universal behaviour: **post-merger**



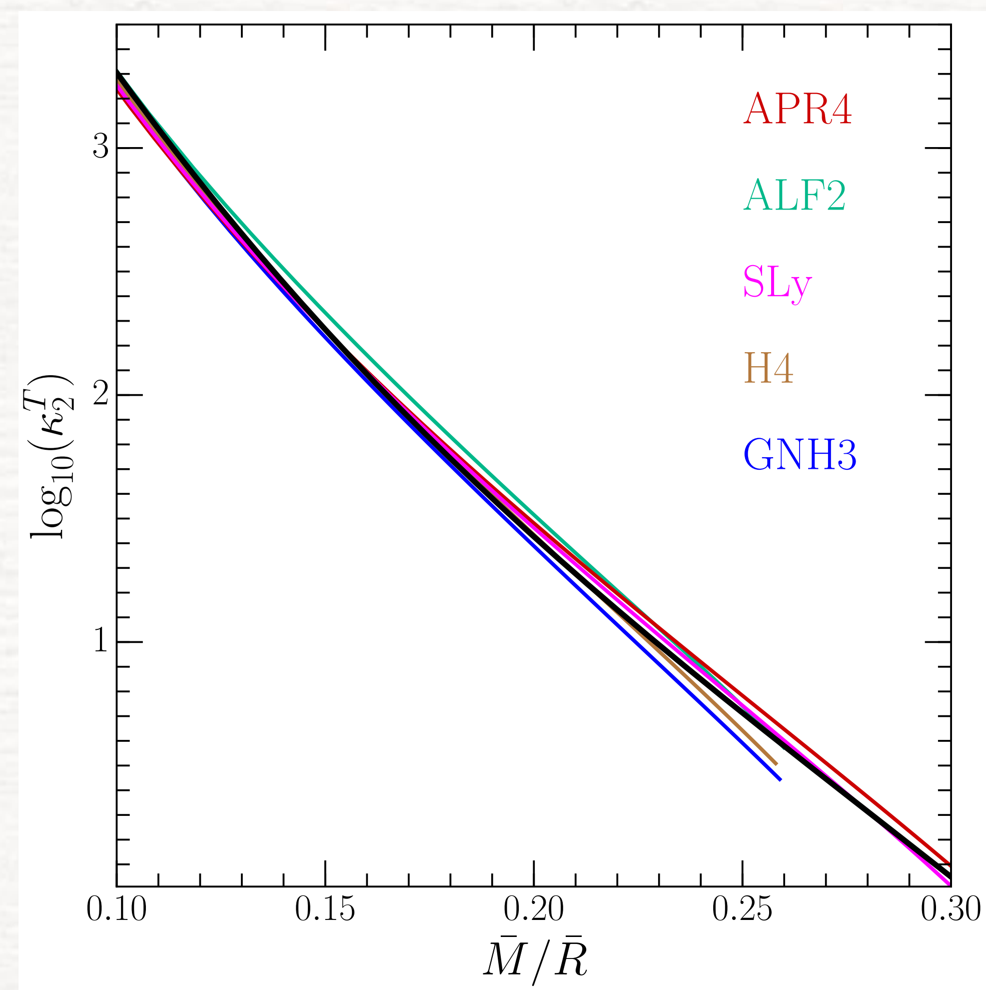
We have found **quasi-universal behaviour:** i.e., the properties of the spectra are only weakly dependent on the EOS.

This has profound implications for the analytical modelling of the GW emission: "what we do for one EOS can be extended to all EOSs."

Quasi-universal behaviour: **post-merger**



- Correlations with Love number found also for high frequency peak f_2.
- This and other correlations are **weaker** but equally useful.



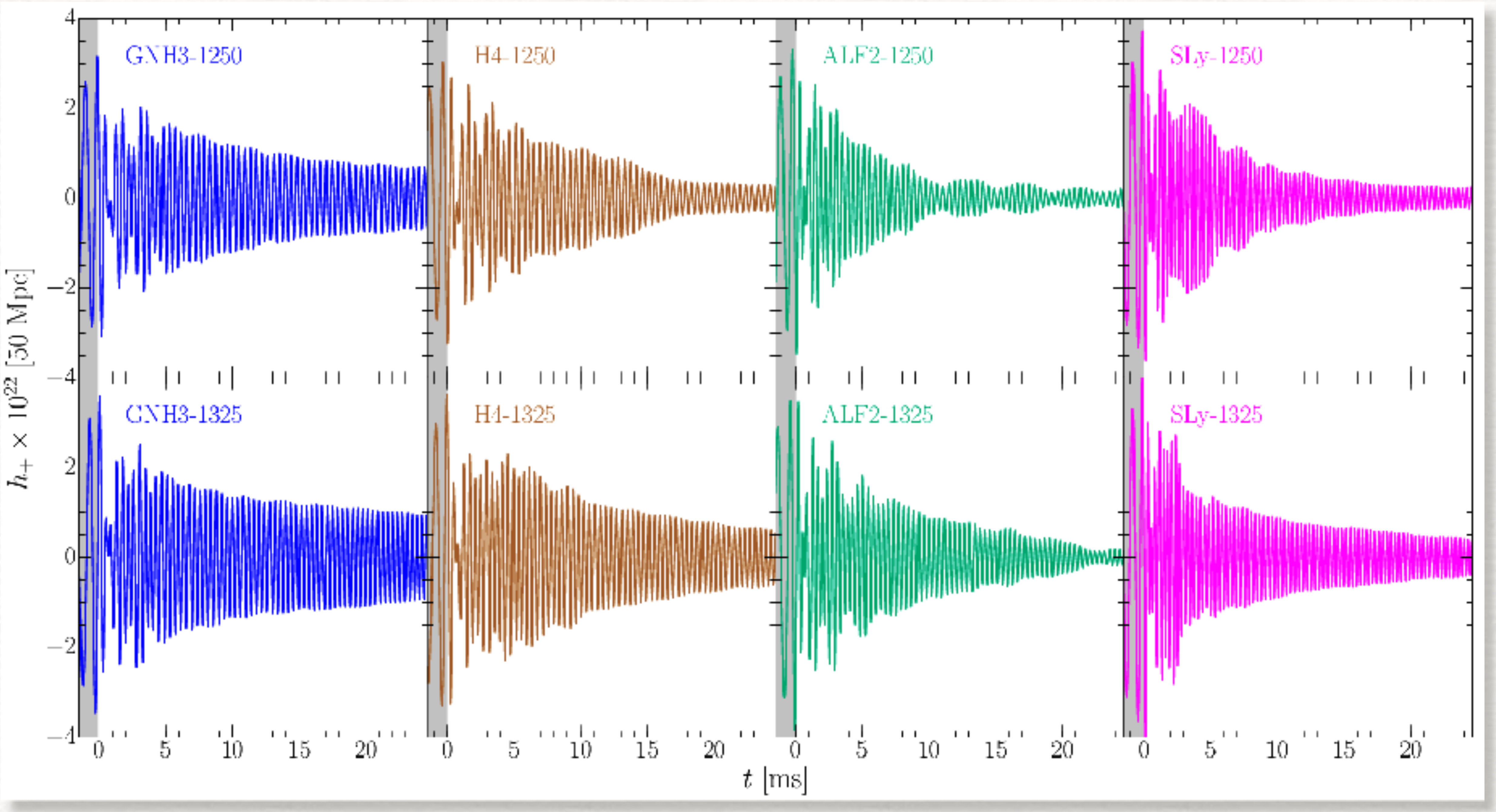
- Important correlation also between **compactness** and **deformability**

Radius estimate from binary population

Bose, Chakravarti, LR, Sathyaprakash, Takami (2017)

Analytical modelling of postmerger waveform

- **Postmerger** appears hopeless but isn't (Clark+14, 16; Bose+17)



Analytical modelling of postmerger waveform

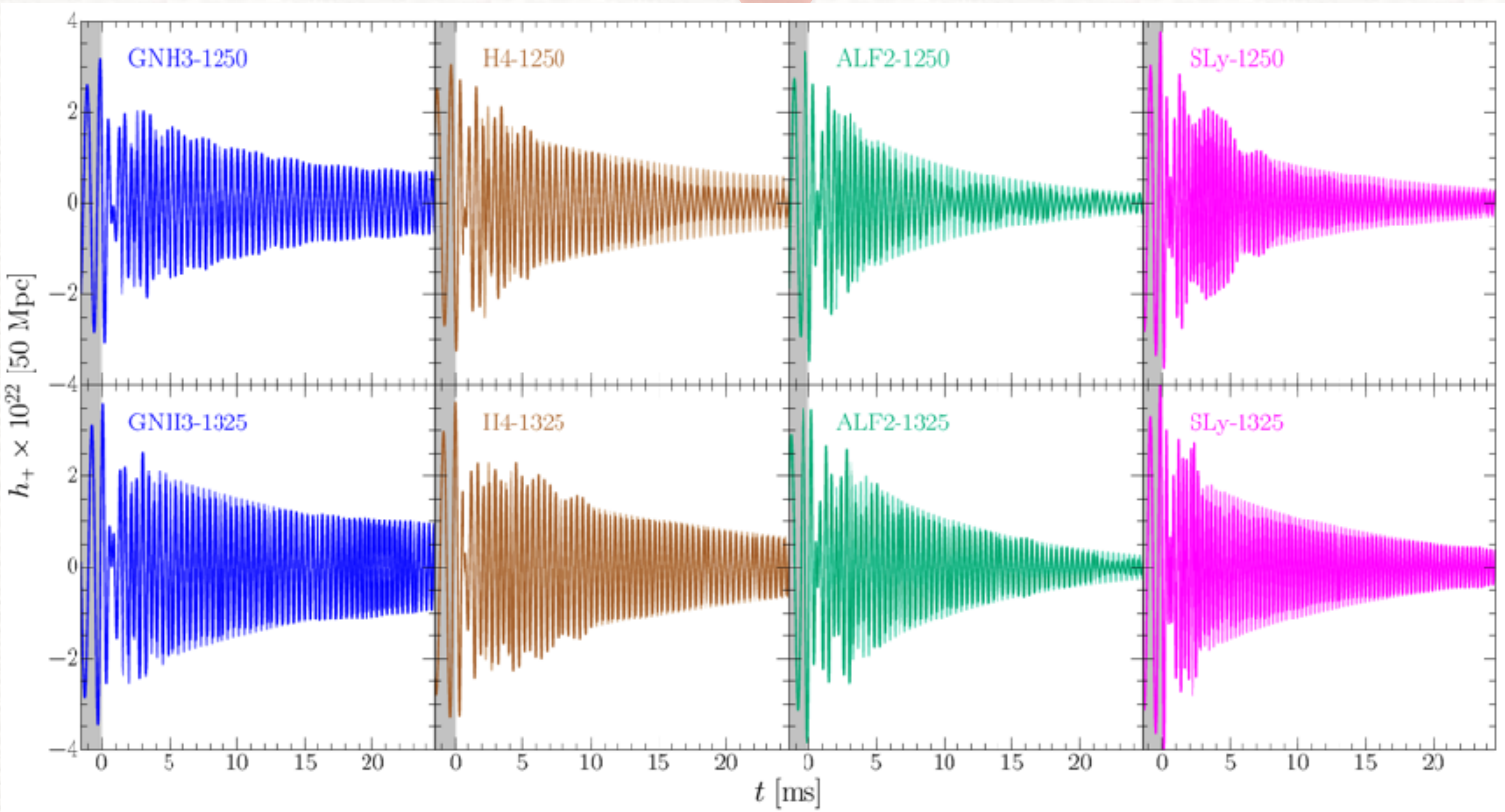
- Knowledge of spectral properties provides **analytic ansatz**

$$
\begin{aligned}
h(t) = \alpha \exp(-t/\tau_1)\big[&\sin(2\pi f_1 t) + \sin(2\pi(f_1 - f_{1\epsilon})t) + \\
&\sin(2\pi(f_1 + f_{1\epsilon})t)\big] + \\
\exp(-t/\tau_2)\,&\sin(2\pi f_2 t + 2\pi\gamma_2 t^2 + \pi\beta_2)\,.
\end{aligned}
$$

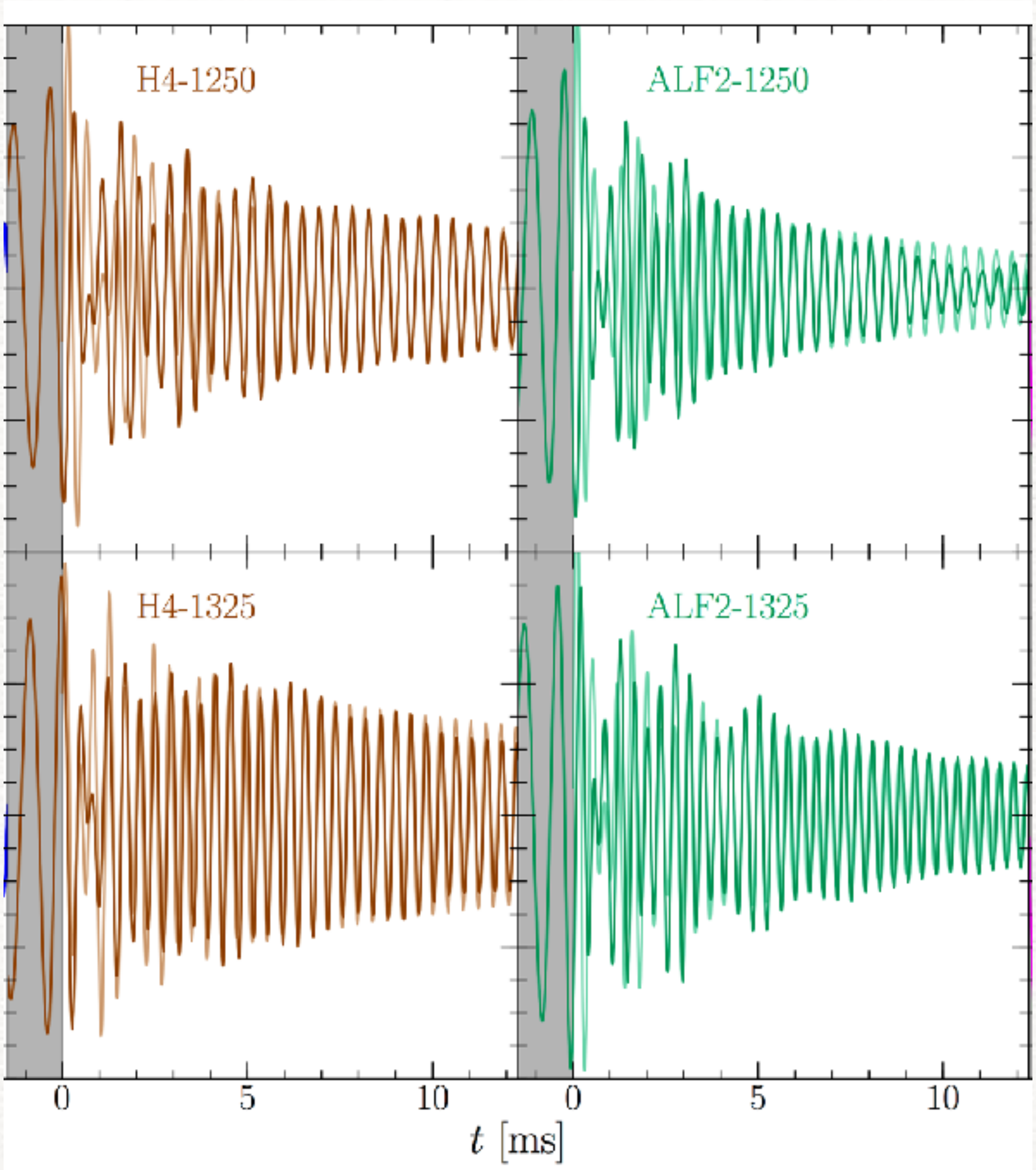
Analytical modelling of postmerger waveform

- Knowledge of spectral properties provides **analytic ansatz**

$$h(t) = \alpha \exp(-t/\tau_1)\left[\sin(2\pi f_1 t) + \sin(2\pi(f_1 - f_{1\epsilon})t) + \sin(2\pi(f_1 + f_{1\epsilon})t)\right] + \exp(-t/\tau_2)\sin(2\pi f_2 t + 2\pi\gamma_2 t^2 + \pi\beta_2)\,.$$

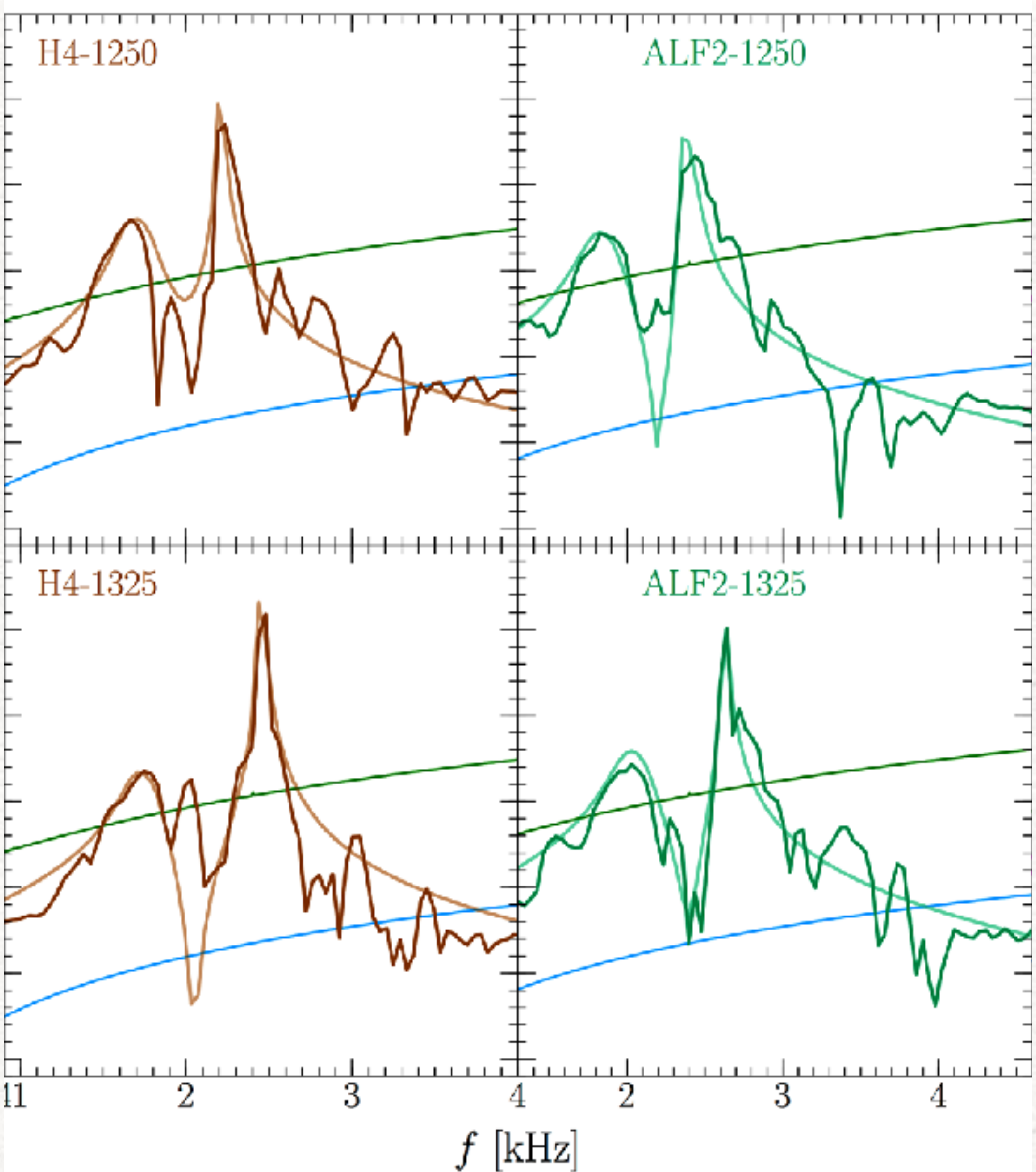


Analytical modelling of postmerger waveform



- Overall pretty decent fit in **phase**
- Fit in **amplitude** is less good but also less important

Analytical modelling of postmerger waveform



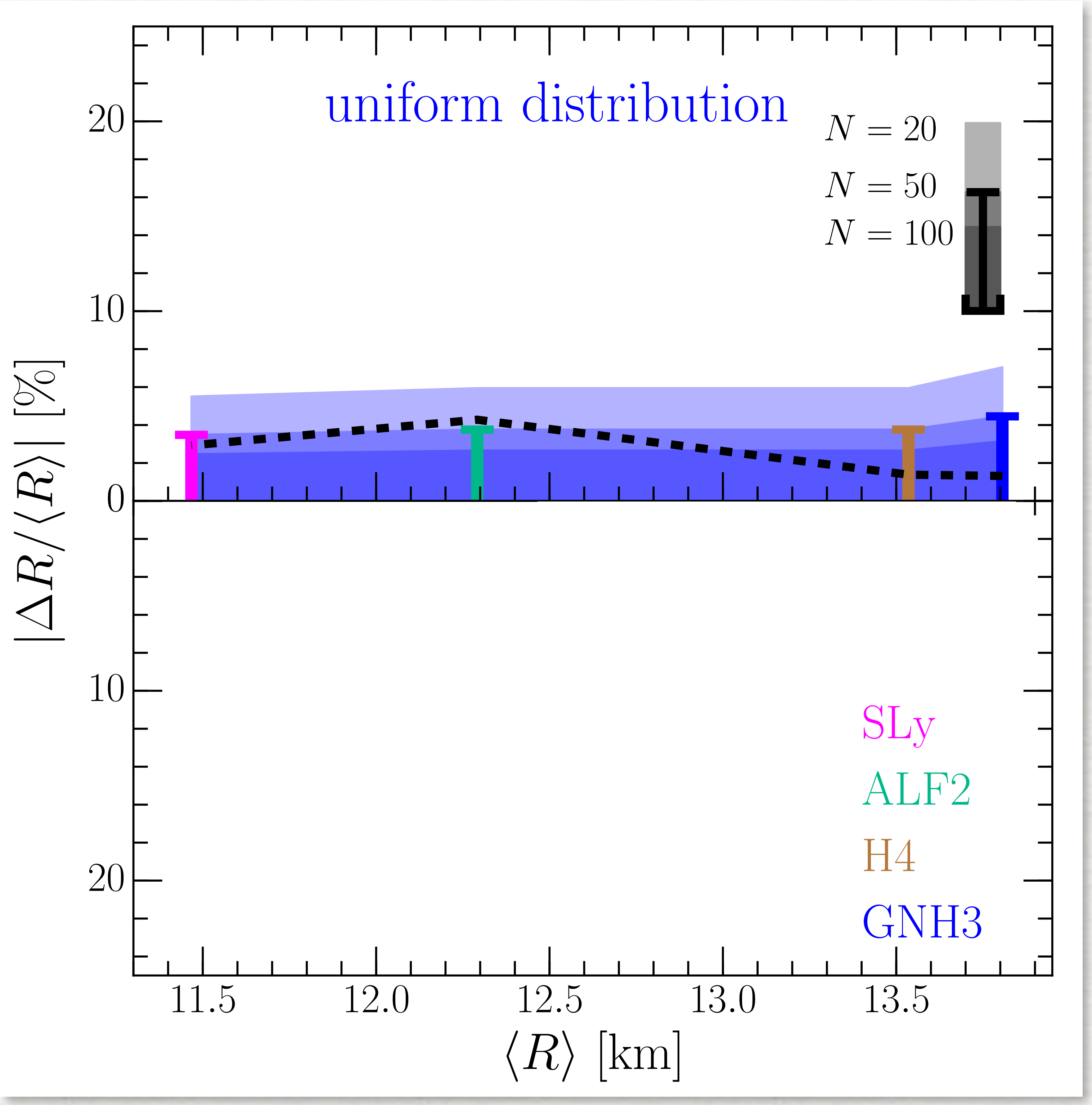
- Good match is clear also in **frequency space**

In summary: despite the complex signal, an **analytic** description of the **full GW signal** is now possible.

Even a small SNR counts

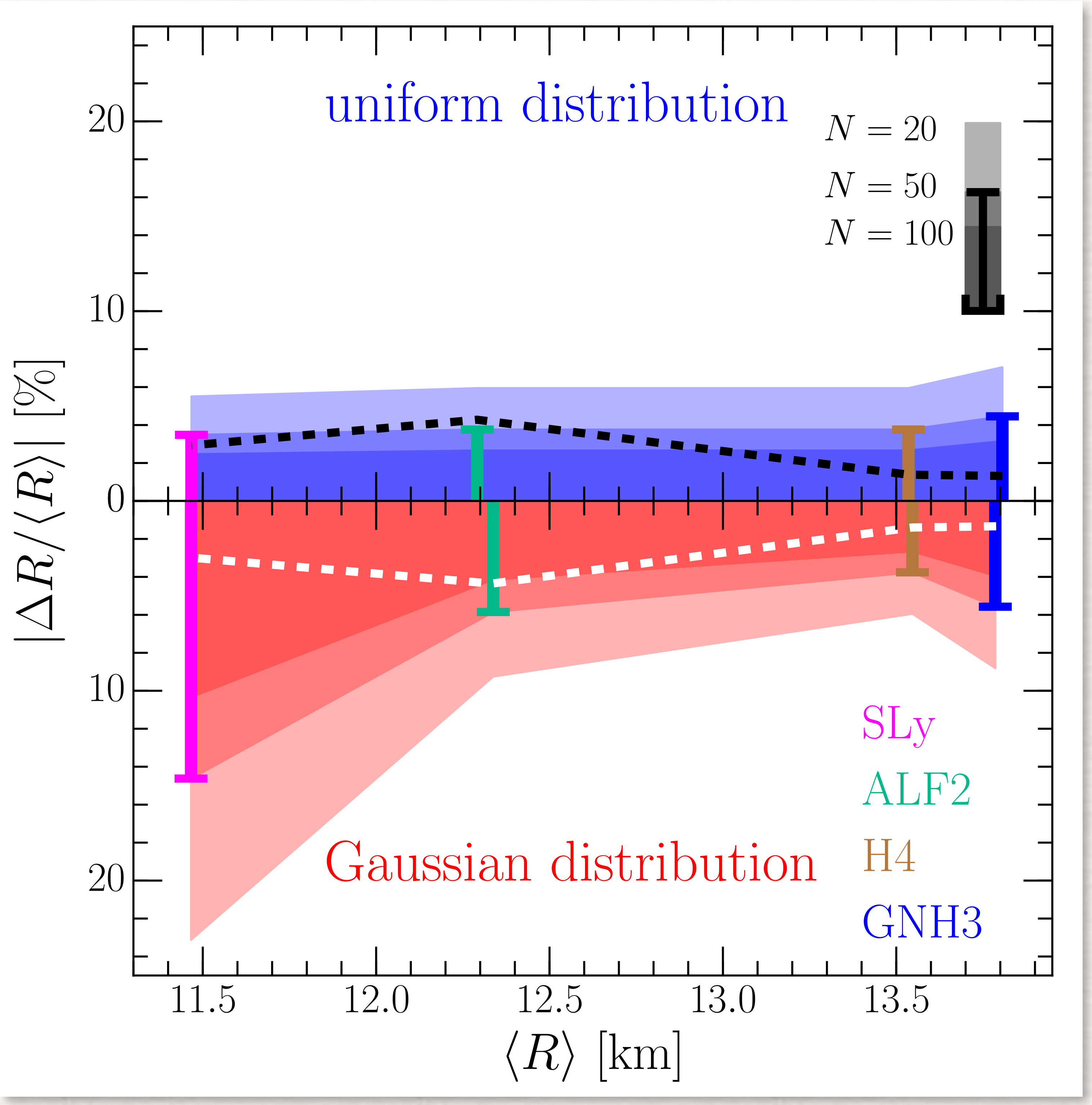
- Using **analytical modelling** performed **Fisher-matrix** analysis of GWs and **Monte-Carlo** simulation.
- Waveforms aligned at frequency, f_2^c. Standard frequency estimation yields value of f_2^c and statistical spread.
- **Quasi-universal relation** between f_2 and compactness, and error-propagation, to deduce the error in radius.
- Employed 100 BNS signals injected in 100 uncorrelated timeseries of Gaussian noise with aLIGO sensitivity.
- Used information on f_1 and chirp **mass** from **inspiral**.
- Repeated over 900 experiments to build statistics.

Constraining the radius: MonteCarlo vs Fisher



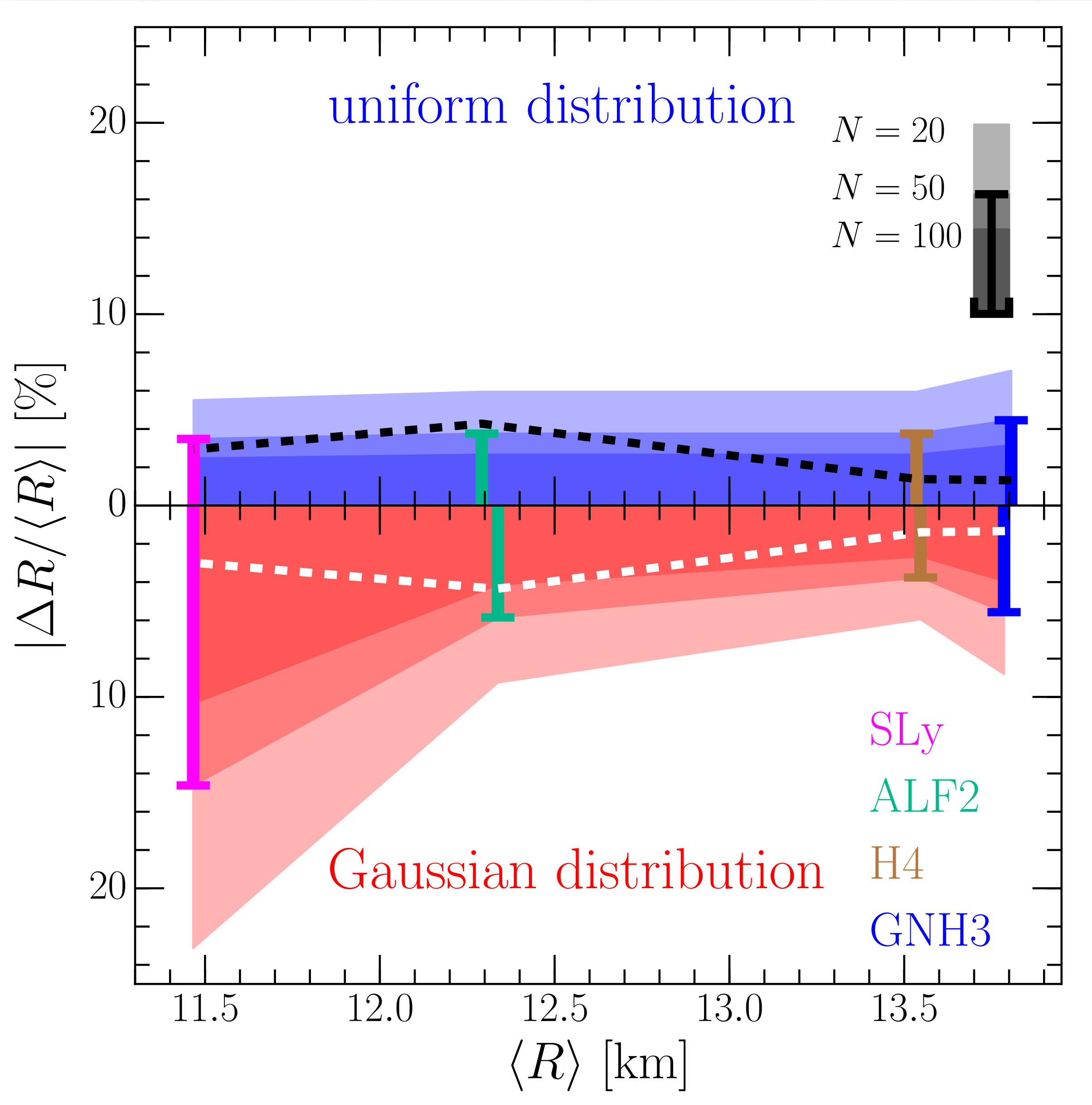
- **uniform** distribution in mass [1.21, 1.38] M☉ between 100 and 300 Mpc; isotropic distribution in space.
- dashed lines for results of Fisher-matrix analysis with N=50
- errors scale like $\sqrt{N}$

Constraining the radius: MonteCarlo vs Fisher



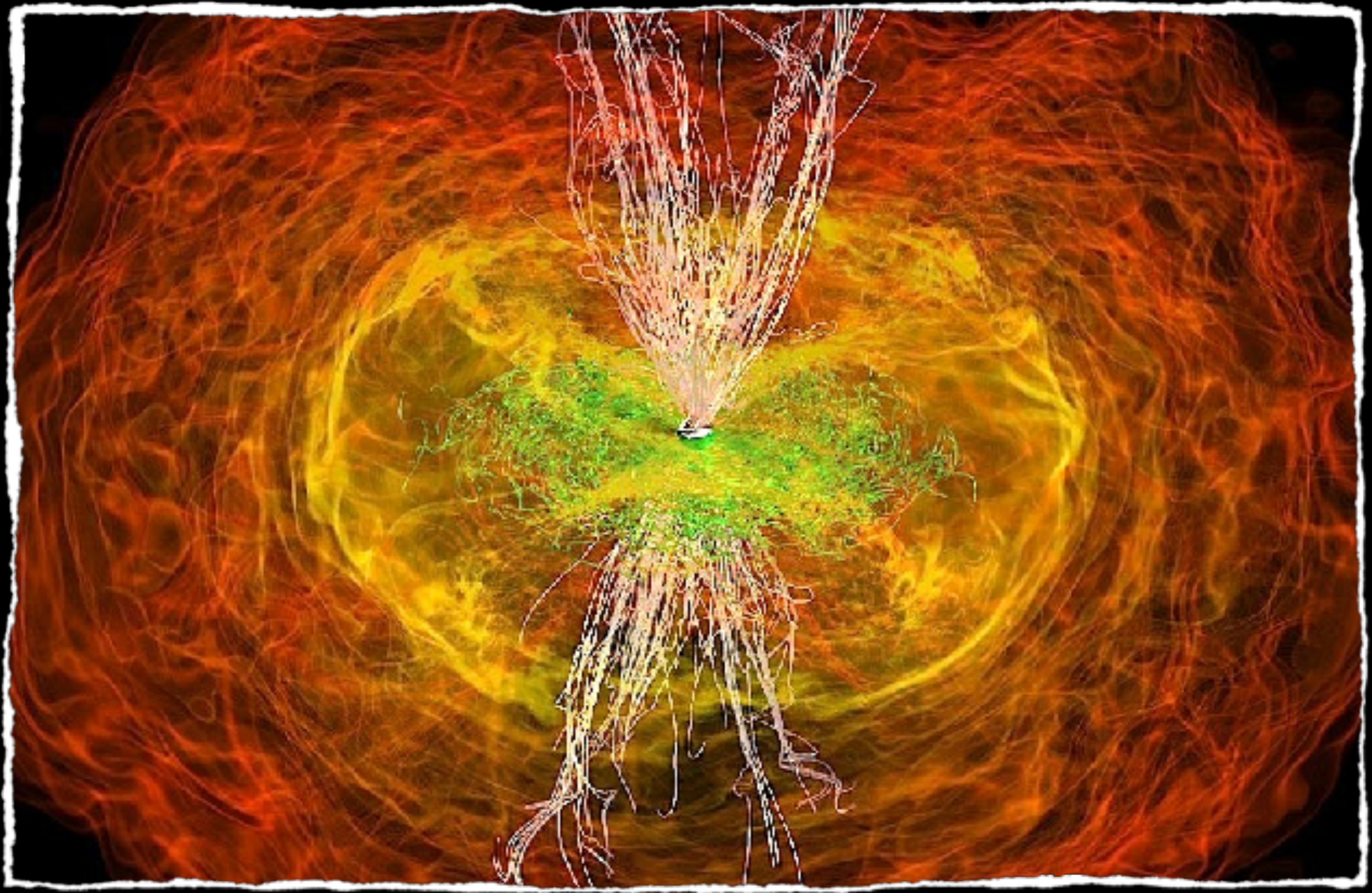
- **Gaussian** distribution in mass [1.21, 1.38] M⊙ centred at 1.35 M⊙ with variance 0.05 Binaries are between 100 and 300 Mpc; isotropic distribution in space.
- dashed lines for results of Fisher-matrix analysis with N=50
- errors scale like $\sqrt{N}$

All in all



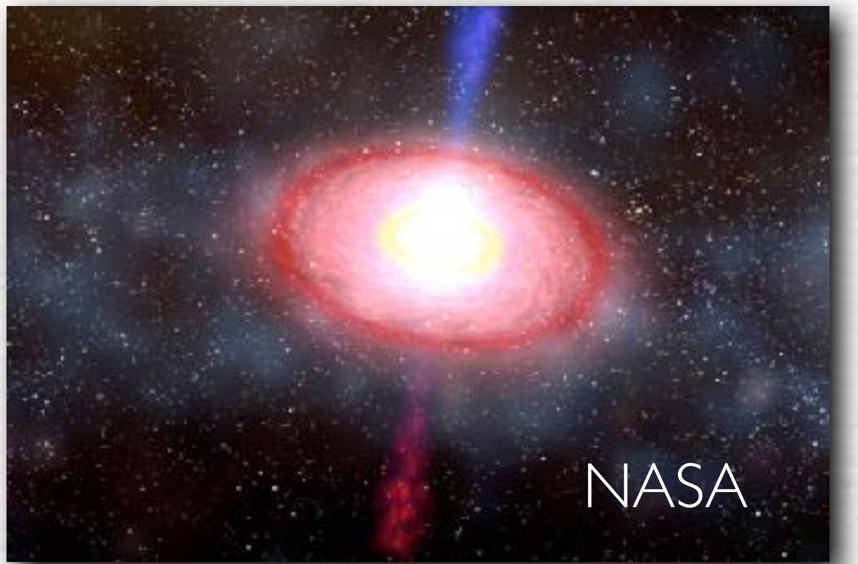
- stiff EOSs: $|\Delta R/\langle R \rangle| < 10\%$ for **N~20**
- soft EOSs: $|\Delta R/\langle R \rangle| \sim 10\%$ for **N~50**
- discriminating stiff/soft EOSs will possible even with moderate N
- discriminating two-stiff /two-soft EOSs will be harder
- very soft EOSs remain a challenge
- golden binary: **SNR ~ 6** at **30 Mpc** $|\Delta R/\langle R \rangle| \lesssim 2\%$ at 90% confidence

Electromagnetic counterparts



Electromagnetic counterparts

- Since 70's we have observed flashes of gamma rays with enormous energies 10^{50-53} erg: **gamma-ray bursts**.
- There are two families of bursts: "**long**" and "**short**".
- The first ones last **tens** or more of **seconds** and could to be due to the collapse of very massive stars.
- The second ones last **less** than a **second**.
- Merging neutron stars most reasonable explanation but how do you produce a **jet**?



Presence of a jet immediately implies presence of large-scale magnetic fields

What happens when magnetised stars collide?

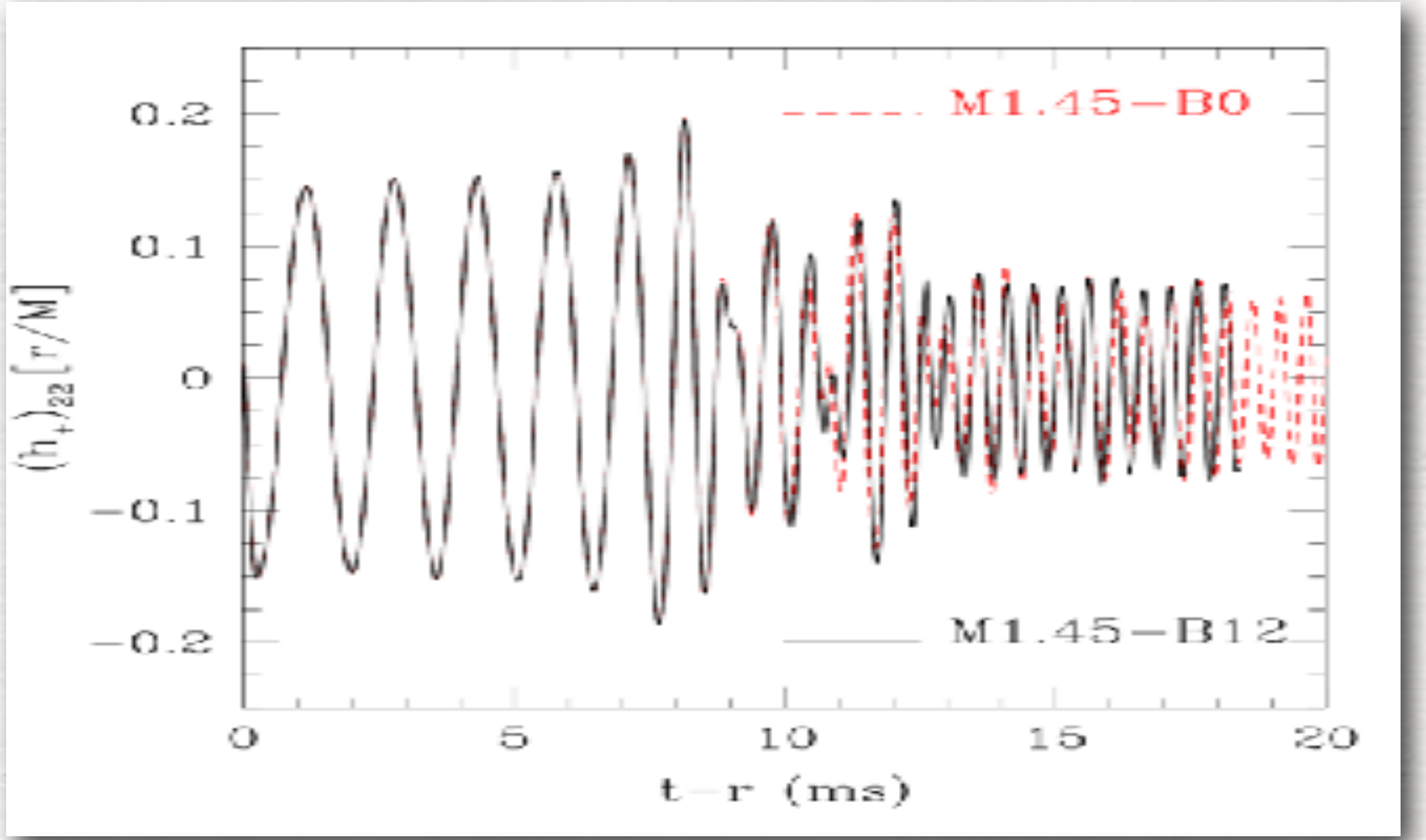
Need to solve equations of magnetohydrodynamics in addition to the Einstein equations

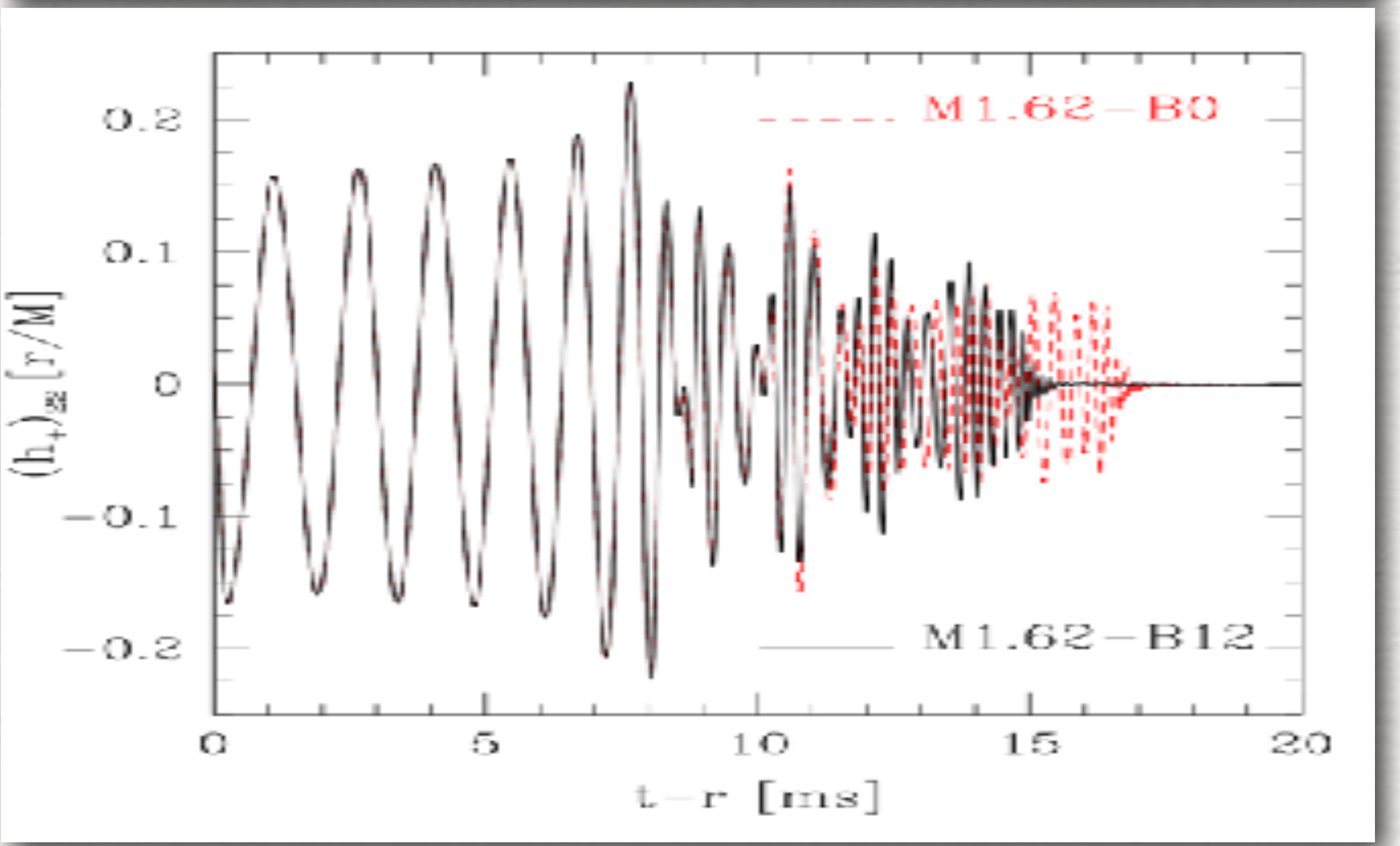
$$T_{\mu\nu} = (e+p)\, u_\mu u_\nu + p g_{\mu\nu} + F_\mu{}^\lambda F_{\nu\lambda} - \frac{1}{4} g_{\mu\nu}\, F^{\lambda\alpha} F_{\lambda\alpha},$$

$$\nabla^\nu T_{\mu\nu} = 0$$

$$\nabla_\nu (F^{\mu\nu} + g^{\mu\nu}\psi) = I^\mu - \kappa n^\mu \psi, \quad \nabla_\nu ({}^*F^{\mu\nu} + g^{\mu\nu}\phi) = -\kappa n^\mu \phi,$$

Can we detect B-fields in the inspiral?





Compare B/no-B field:

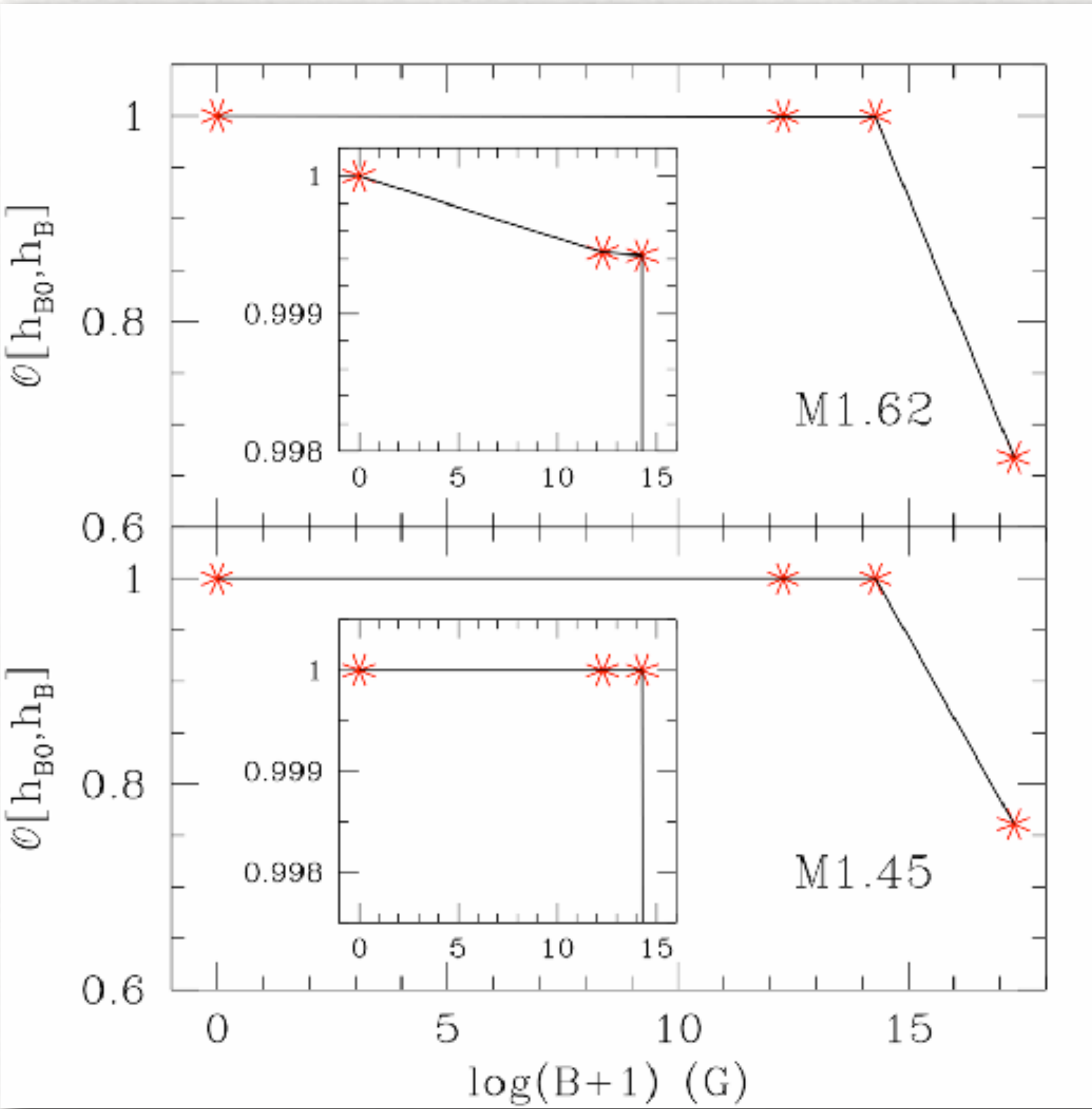
- **inspiral** waveform is different but for unrealistic B-fields (i.e. B~10^{17} G).

- **post-merger** waveform is different for all masses; strong B-fields delay the collapse to BH

Influence of B-fields on inspiral is **unlikely to be detected** for realistic fields

Can we detect B-fields in the inspiral?

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the overlap



$$\mathcal{O}[h_{B1}, h_{B2}] \equiv \frac{\langle h_{B1} | h_{B2} \rangle}{\sqrt{\langle h_{B1} | h_{B1} \rangle \langle h_{B2} | h_{B2} \rangle}}$$

where the scalar product is

$$\langle h_{B1} | h_{B2} \rangle \equiv 4\Re \int_0^\infty df \, \frac{\tilde{h}_{B1}(f) \tilde{h}^*_{B2}(f)}{S_h(f)}$$

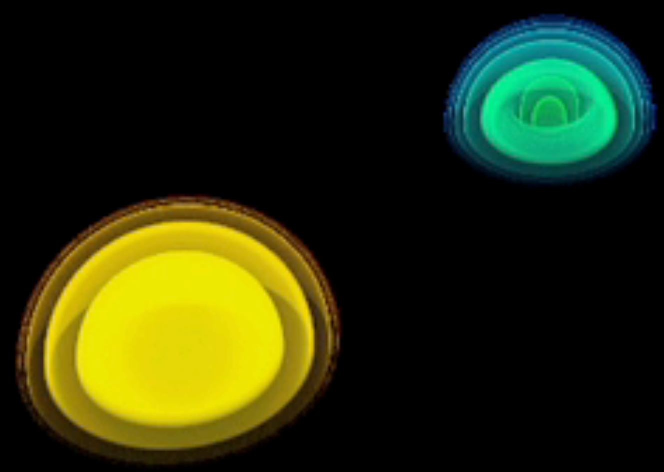
In essence, at these res:

$$\mathcal{O}[h_{B0}, h_B] \gtrsim 0.999$$

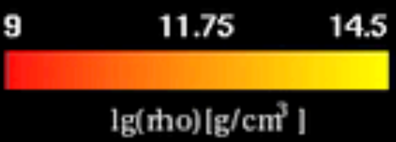
for $B \lesssim 10^{17}$ G

Influence of B-fields on inspiral is **unlikely to be detected**

If magnetic fields cannot be measured in the inspiral, what happens after merger?

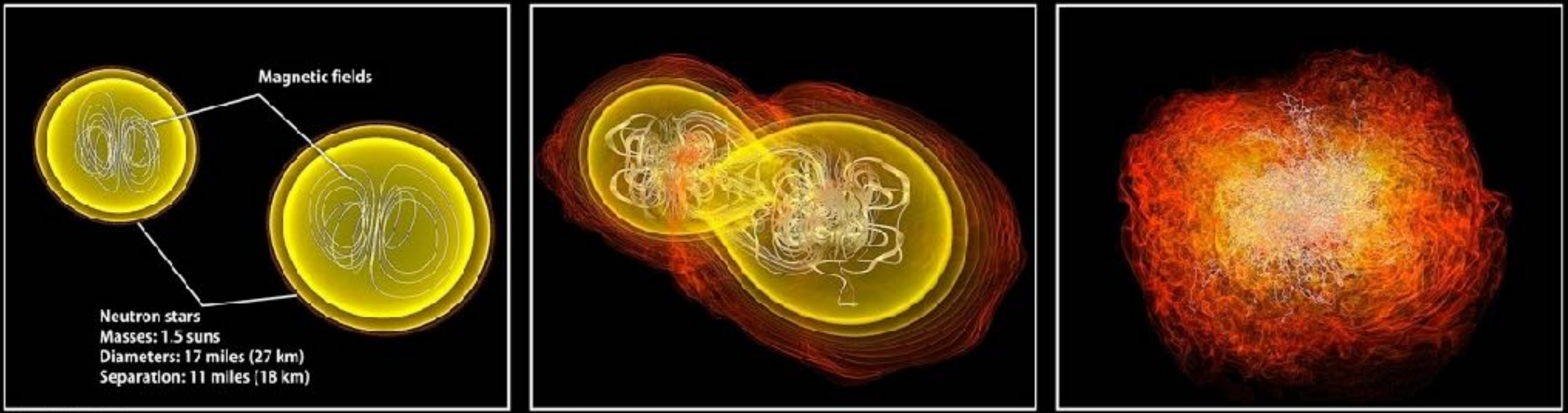


$$M = 1.5\,M_\odot, B_0 = 10^{12}\,\mathrm{G}$$



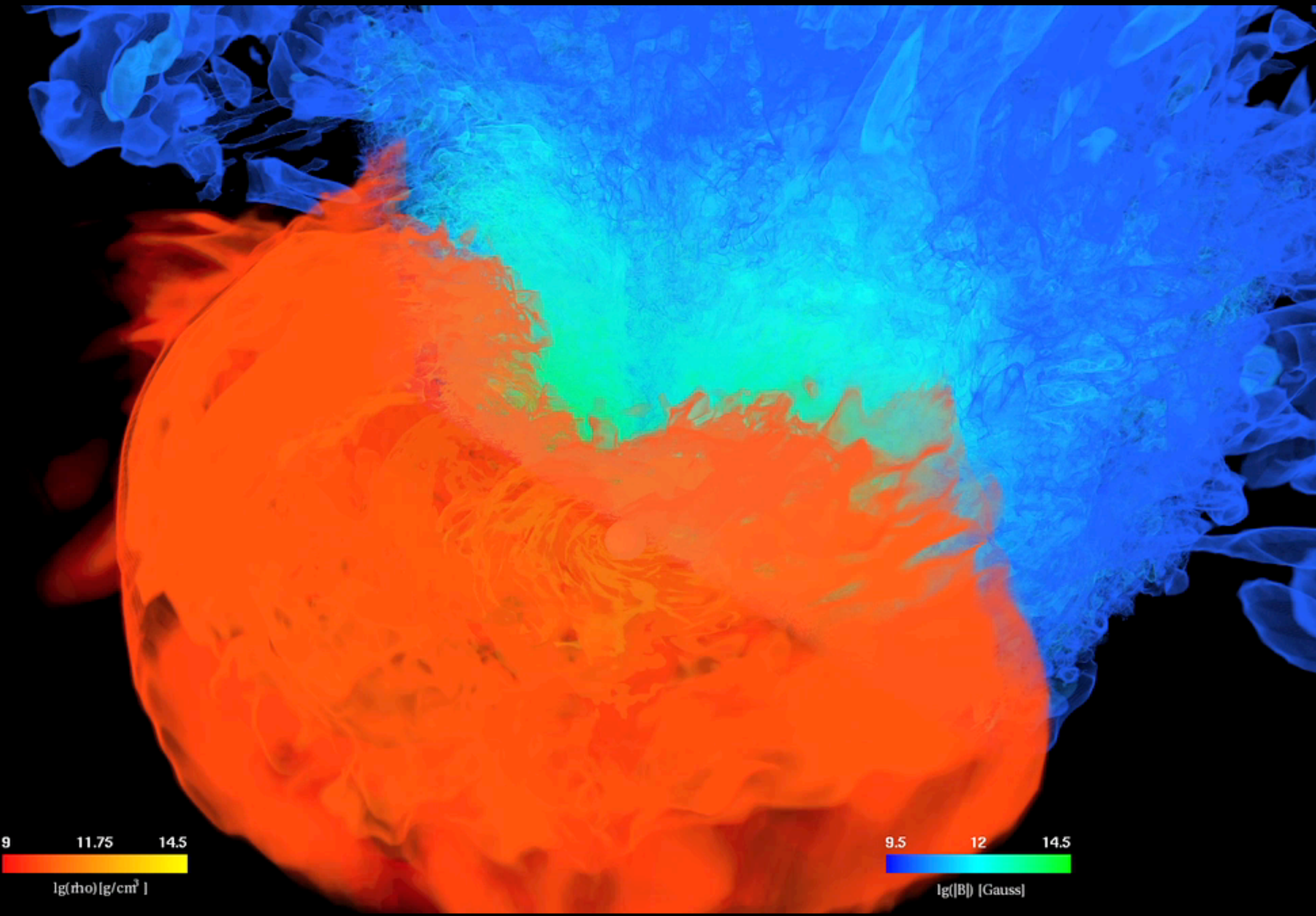
Animations:, LR, Koppitz

What happens when magnetised stars collide?

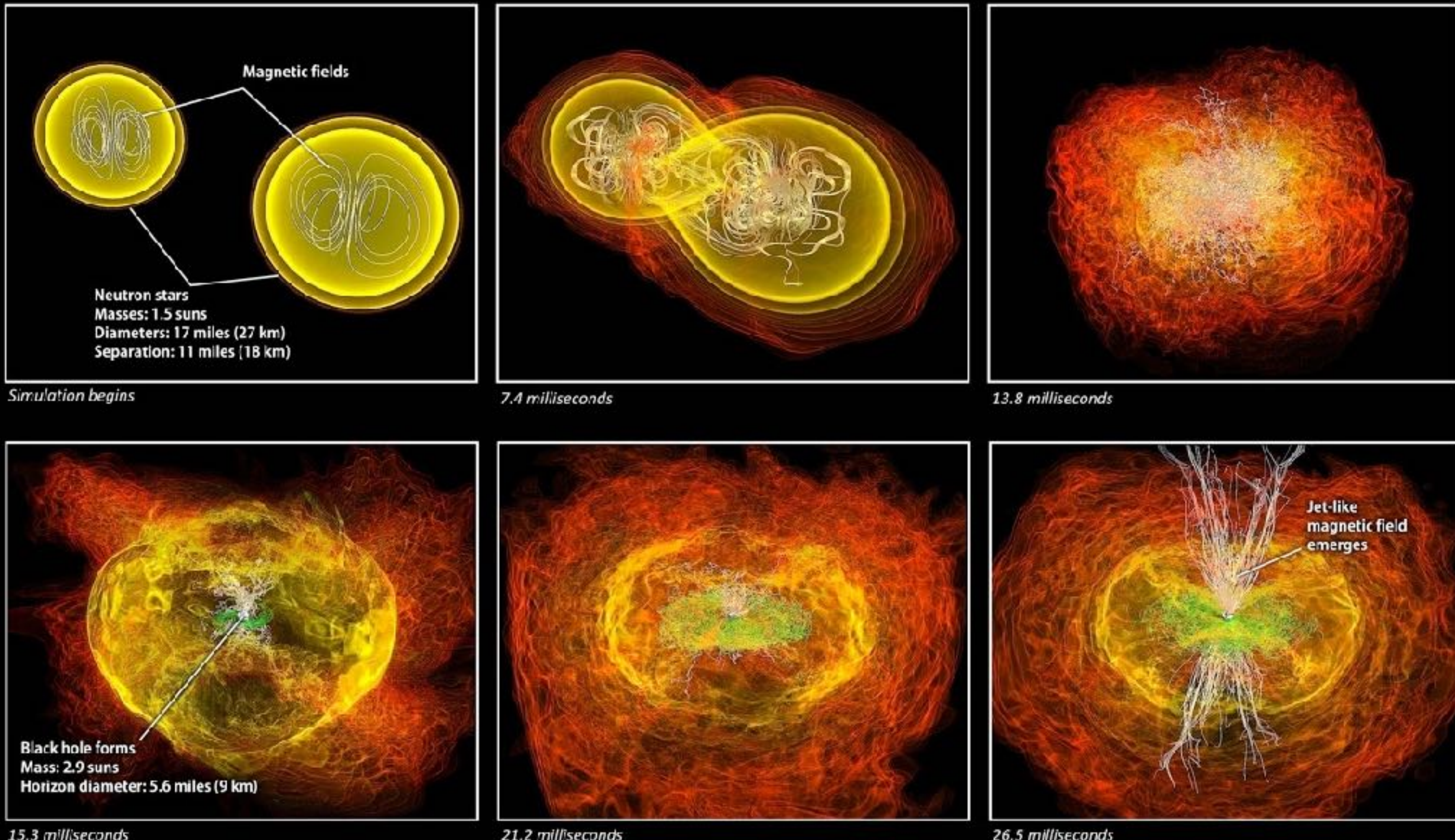


Simulation begins *7.4 milliseconds* *13.8 milliseconds*

Magnetic fields in the HMNS have complex topology: dipolar fields are destroyed.


9
11.75
14.5
lg(rho) [g/cm^3]
9.5
12
14.5
lg(|B|) [Gauss]

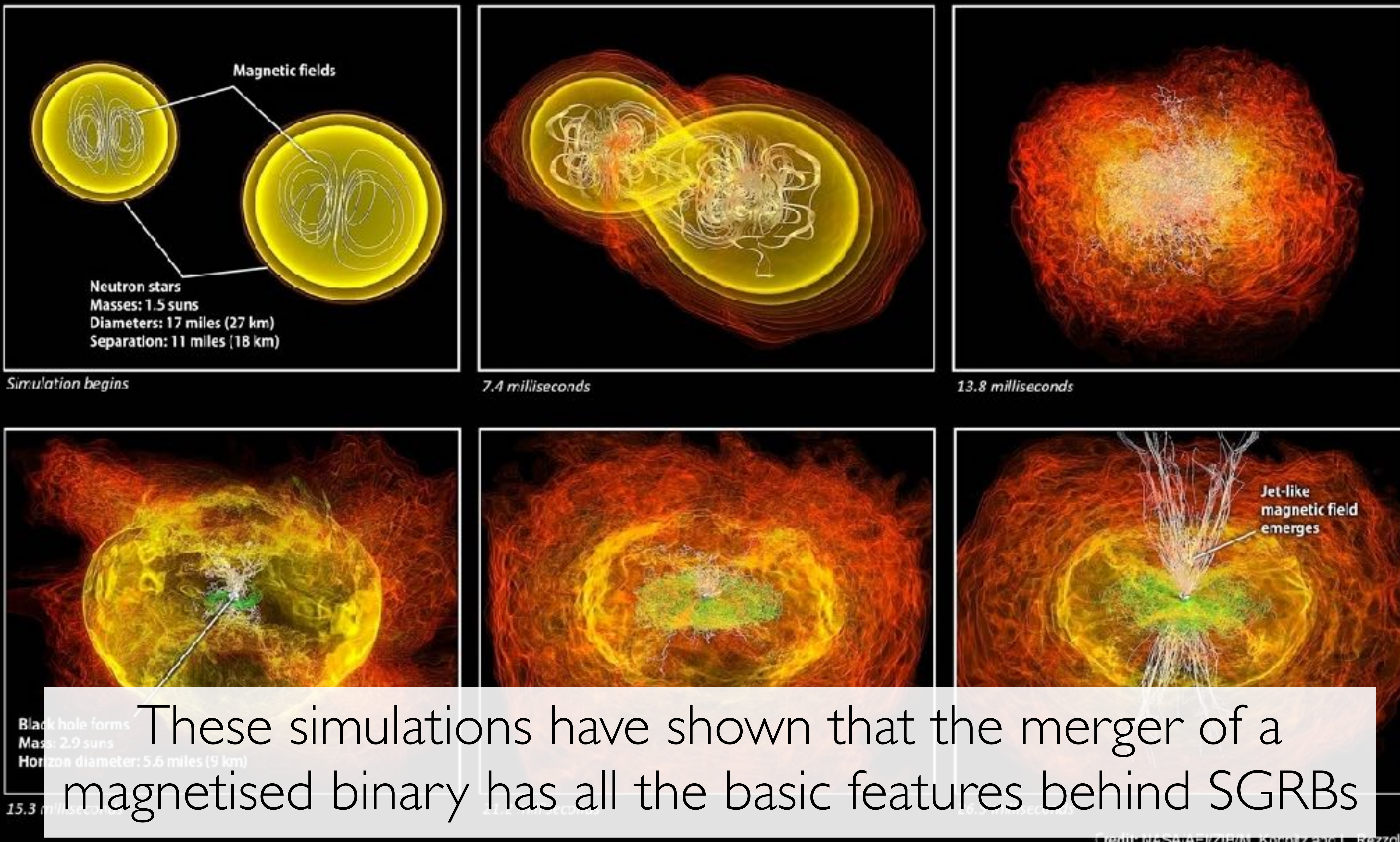
LR+ 2011



Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

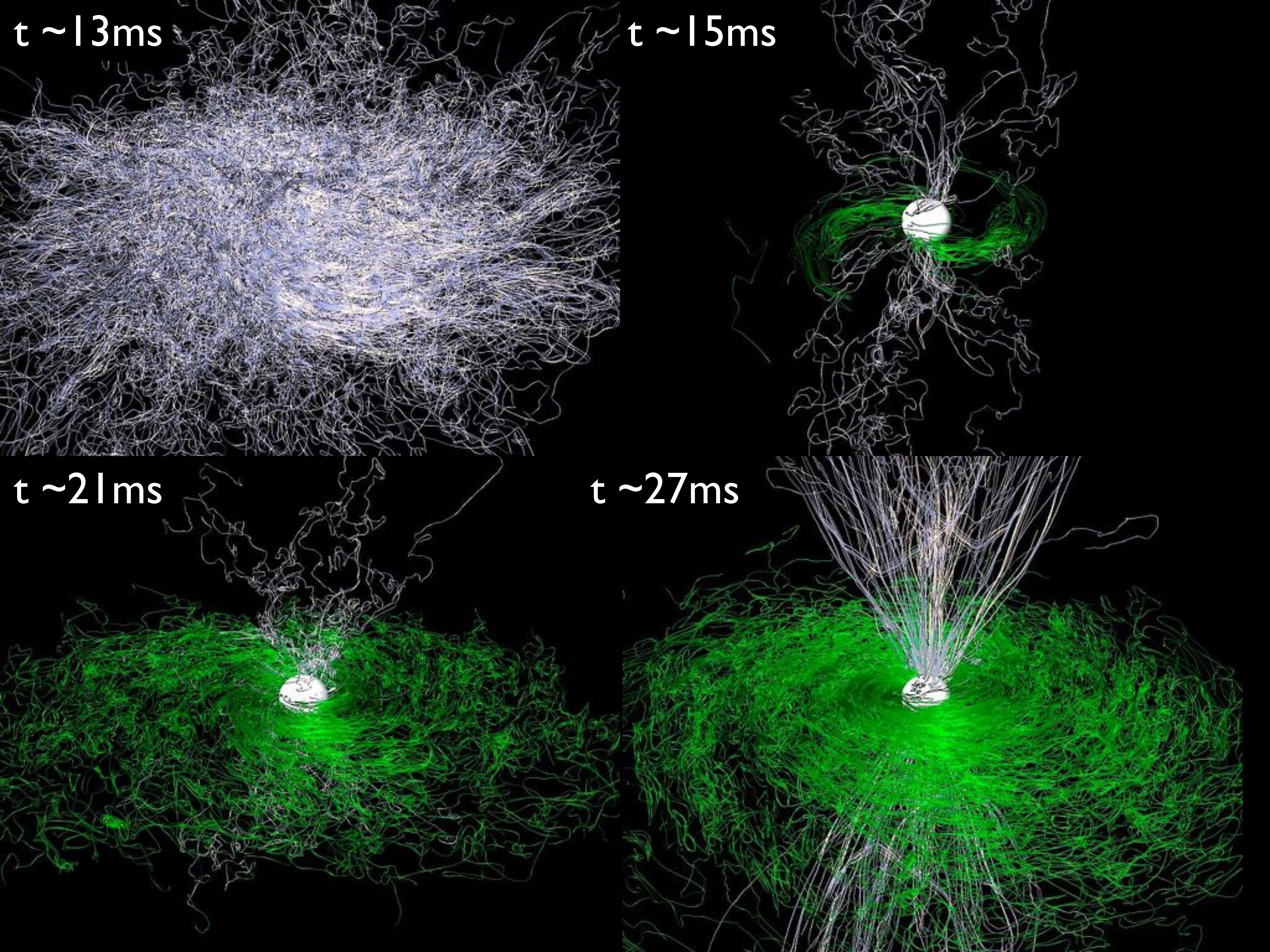
$J/M^2 = 0.83$ $\quad$ $\mathrm{M_{tor}} = 0.063 M_\odot$ $\quad$ $\mathrm{t_{accr}} \simeq M_{\mathrm{tor}}/\dot{M} \simeq 0.3\ \mathrm{s}$

LR+ 2011



These simulations have shown that the merger of a magnetised binary has all the basic features behind SGRBs

$J/M^2 = 0.83$ $\mathrm{M_{tor}} = 0.063 M_\odot$ $\mathrm{t_{accr}} \simeq M_{\mathrm{tor}}/\dot{M} \simeq 0.3\ \mathrm{s}$


t ~13ms
t ~15ms
t ~21ms
t ~27ms

Beyond IMHD: Resistive Magnetohydrodynamics

Dionysopoulou, Alic, LR (2015)

- Ideal MHD is a good approximation in the inspiral, but not after the merger; match to **electro-vacuum** not possible.
- Main difference in resistive regime is the current, which is dictated by Ohm's law but microphysics is **poorly** known.
- We know conductivity σ is a **tensor** but hardly know it as a scalar (prop. to density and inversely prop. to temperature).
- A simple prescription with scalar (isotropic) conductivity:

$$J^i = qv^i + W\sigma[E^i + \epsilon^{ijk}v_j B_k - (v_k E^k)v^i] \, ,$$

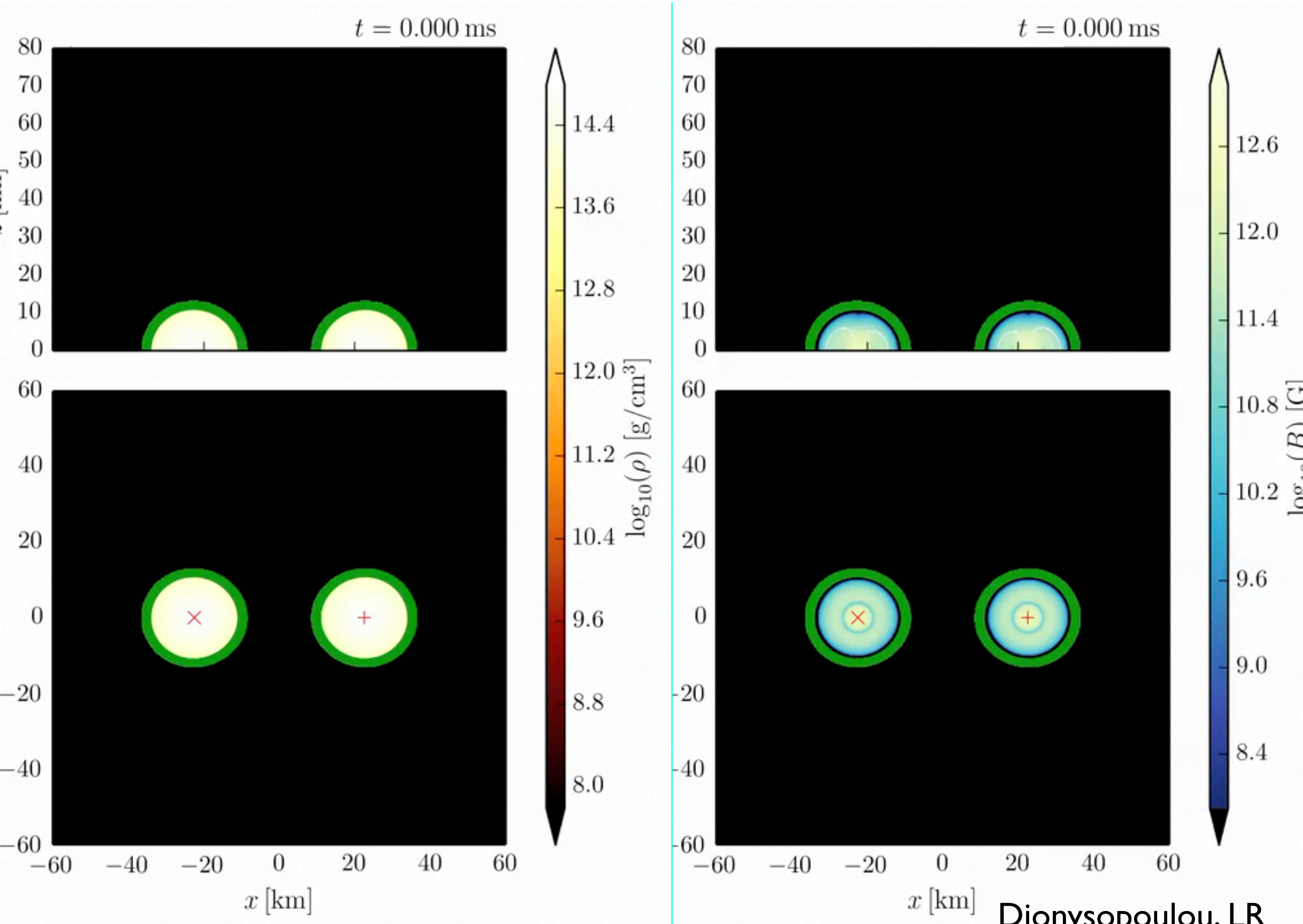
$\sigma \to \infty$ ideal-MHD (IMHD)

$\sigma \neq 0$ resistive-MHD (RMHD)

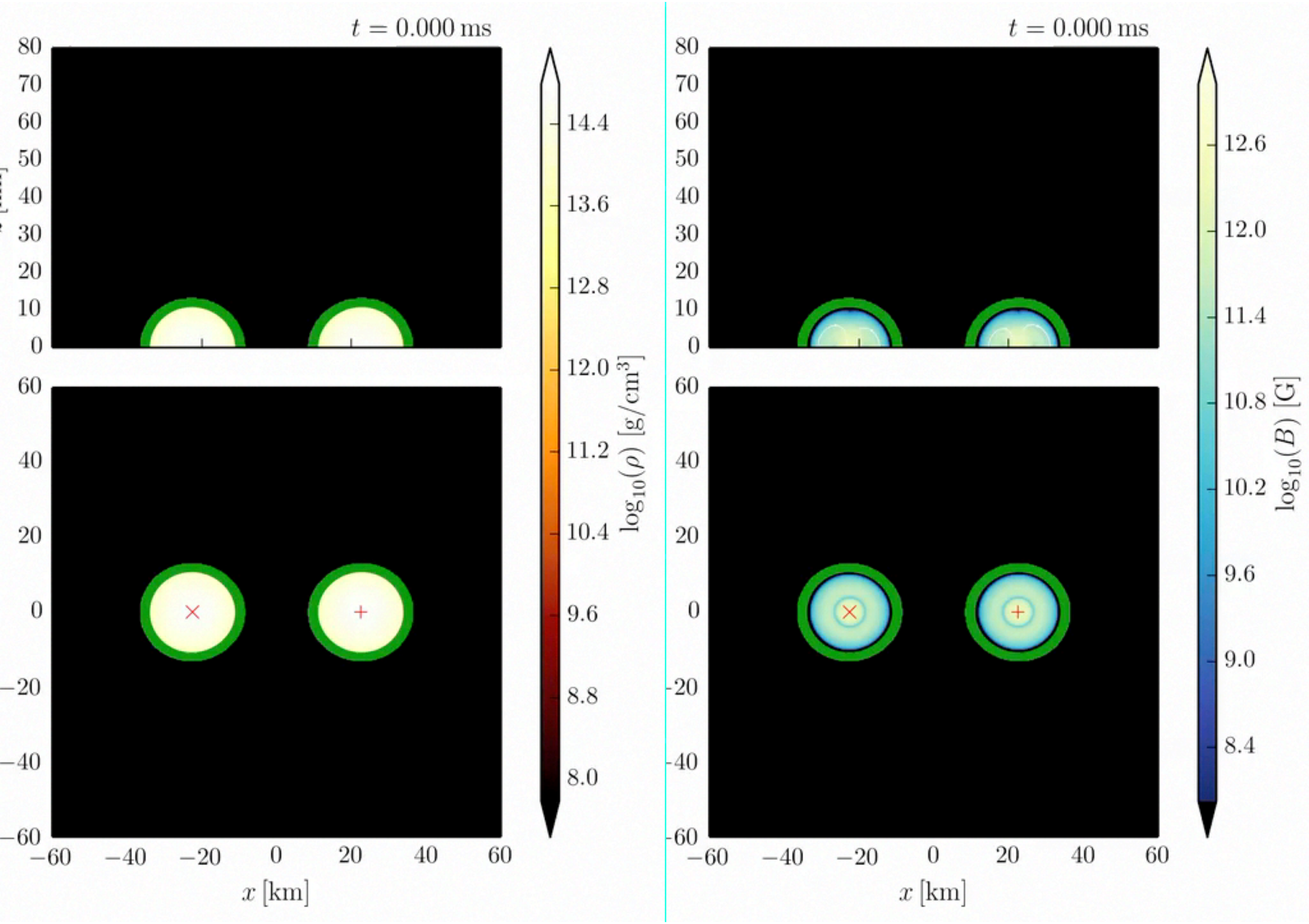
$\sigma \to 0$ electrovacuum

$$\sigma = f(\rho, \rho_{\rm min})$$

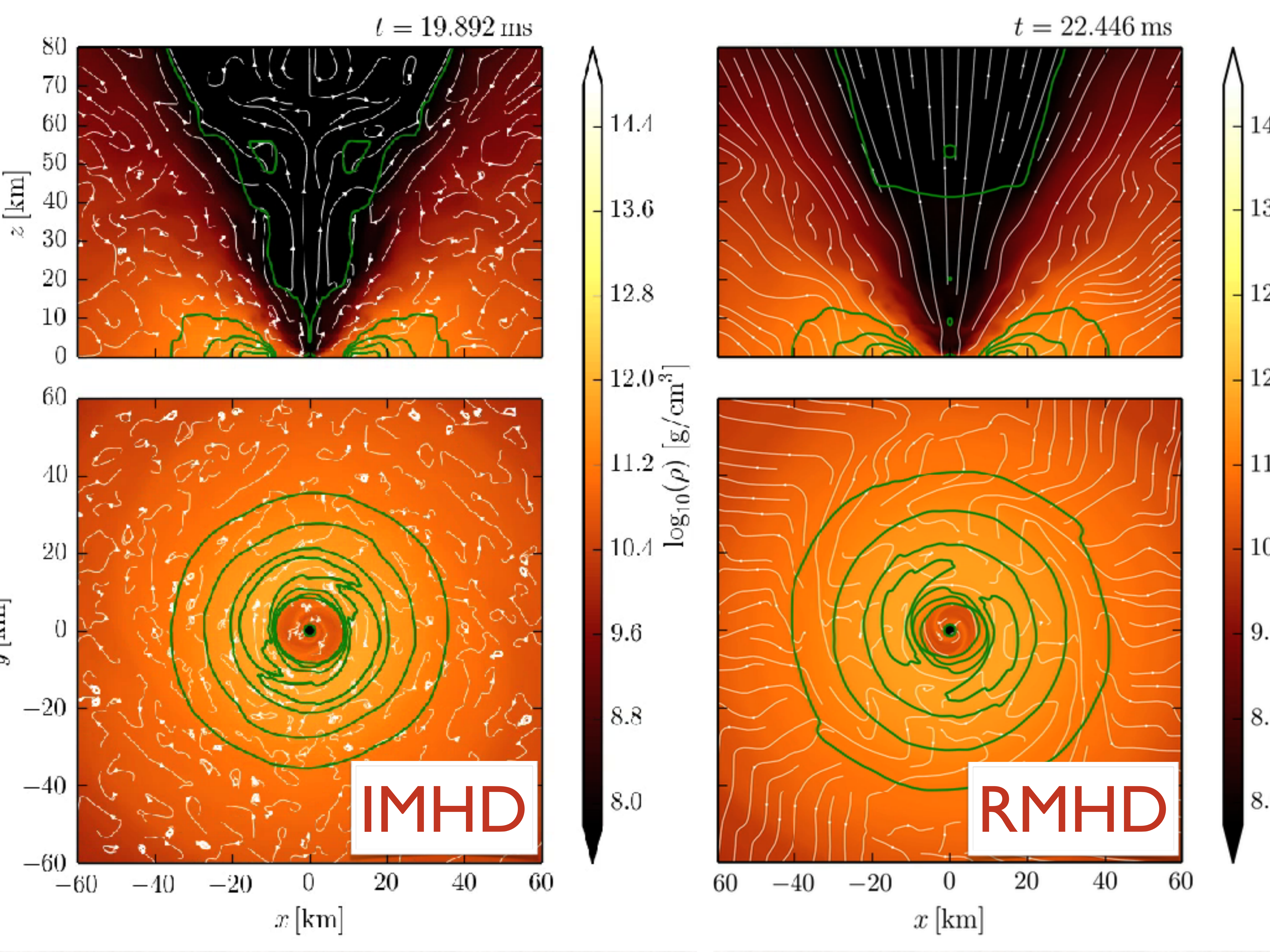
phenomenological prescription



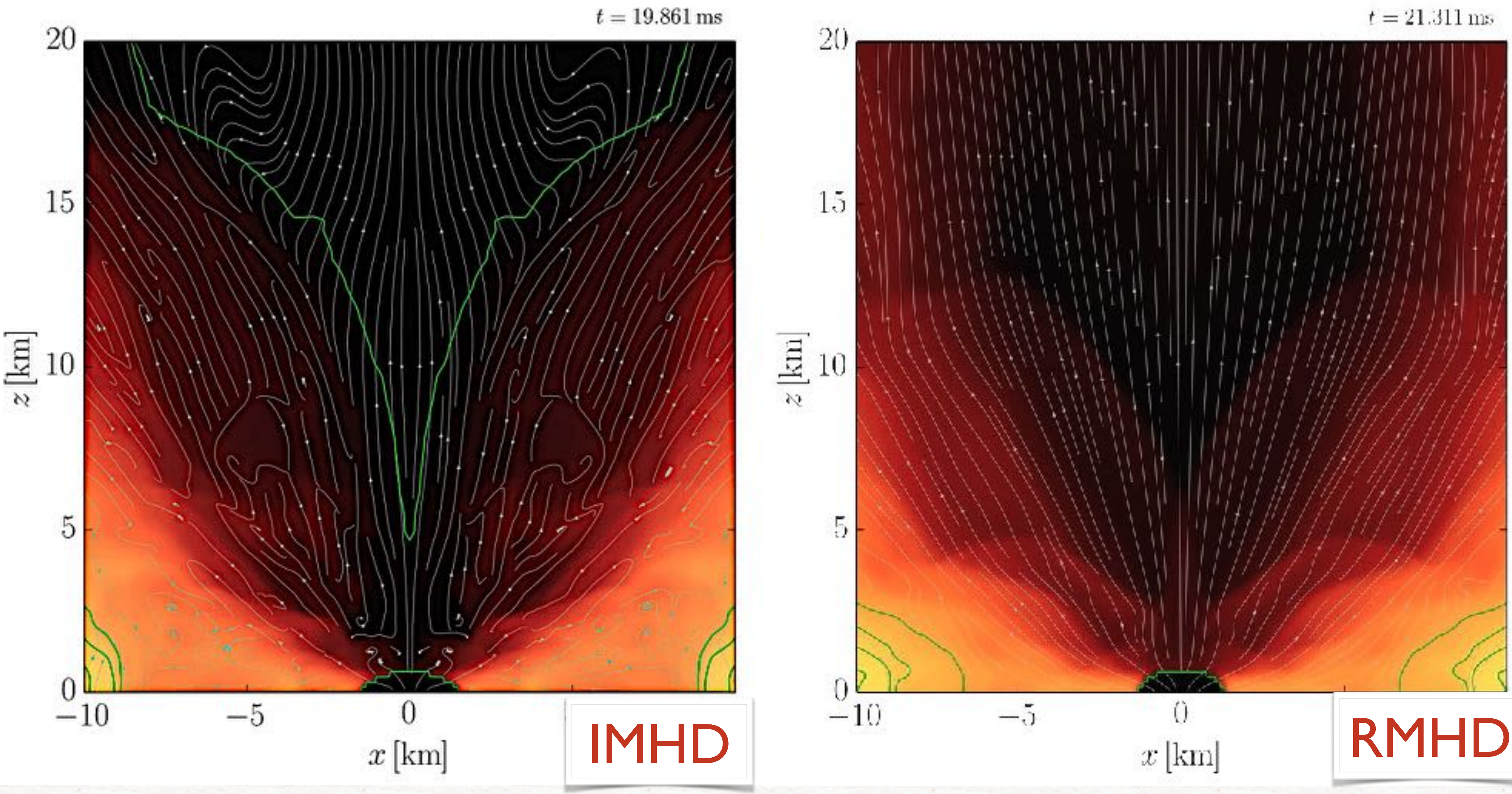
Dionysopoulou, LR



t = 0.000 ms
log10(ρ) [g/cm3]
14.4
13.6
12.8
12.0
11.2
10.4
9.6
8.8
8.0
x [km]
t = 0.000 ms
log10(B) [G]
12.6
12.0
11.4
10.8
10.2
9.6
9.0
8.4
x [km]



t = 19.892 ms
t = 22.446 ms
z [km]
y [km]
x [km]
x [km]
log10(ρ) [g/cm^3]
IMHD
RMHD

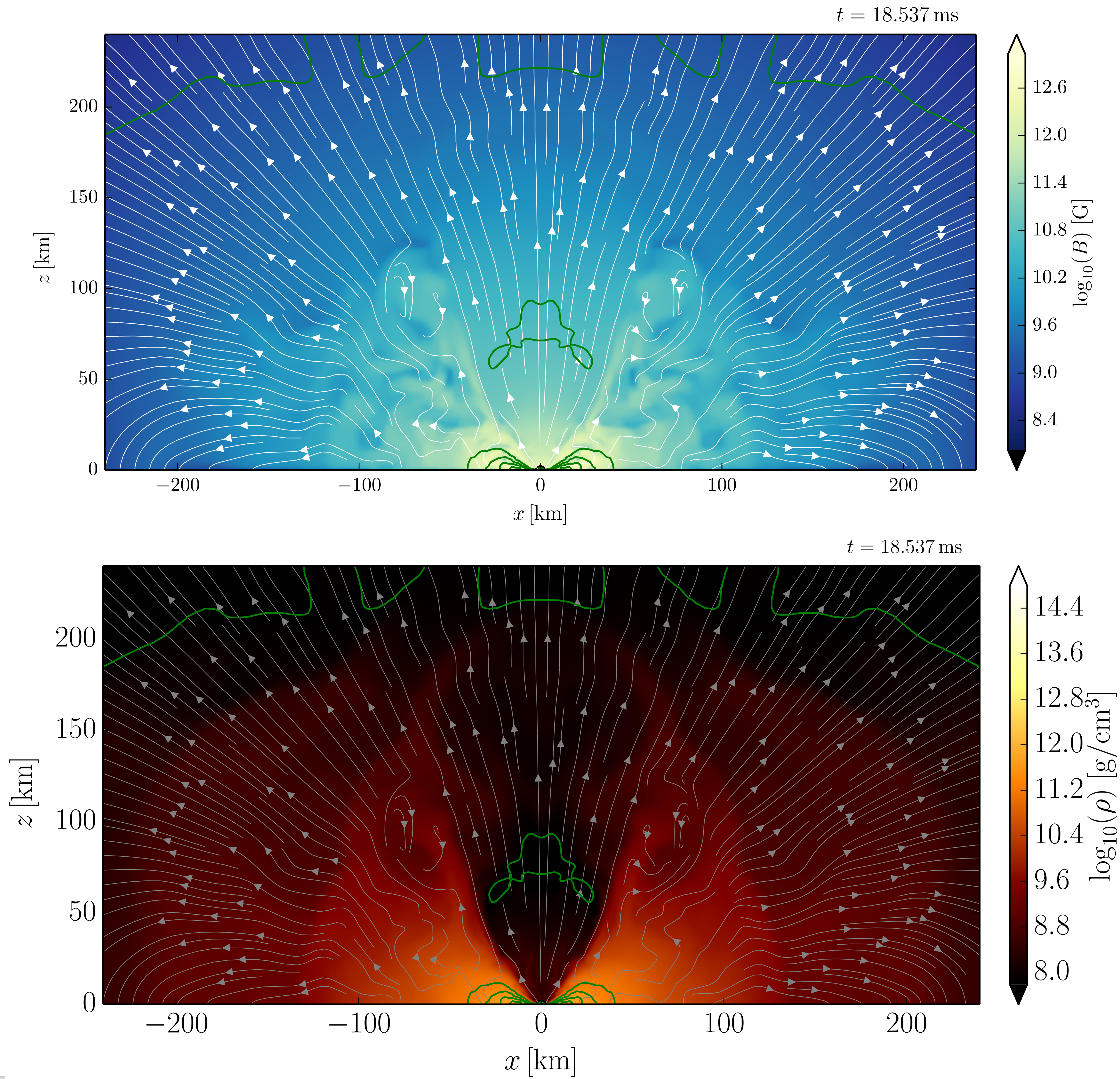


NOTE: the **magnetic jet structure** is **not** an **outflow**. It's a plasma-confining structure.

In **IMHD** the magnetic jet structure is present but less regular.

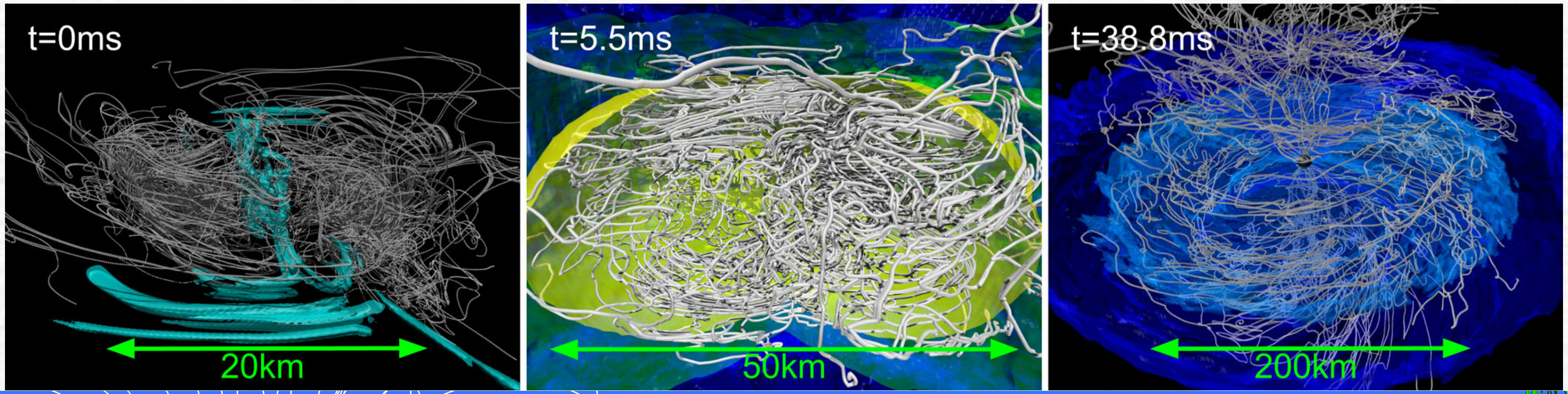
In **RMHD** it is more regular at all scales.

The magnetic jet structure maintains its coherence up to the largest scale of the system.

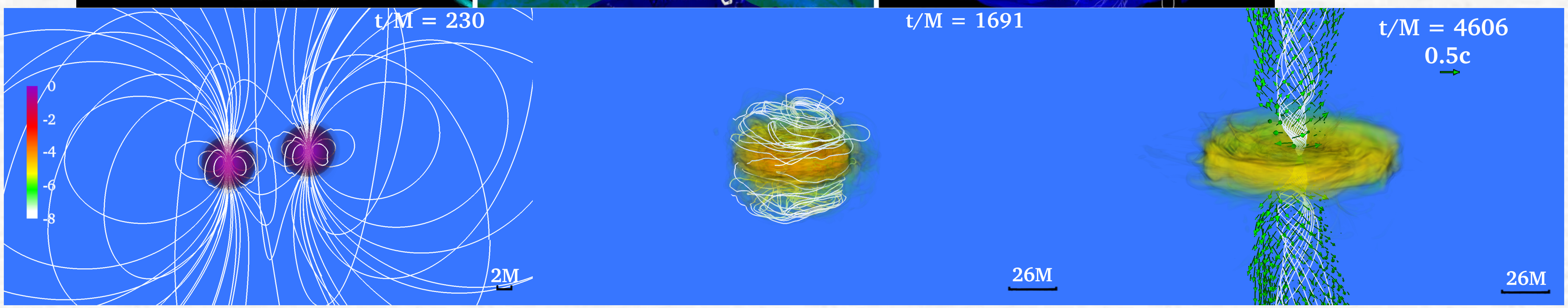


RMHD

With due differences, other groups confirm this picture



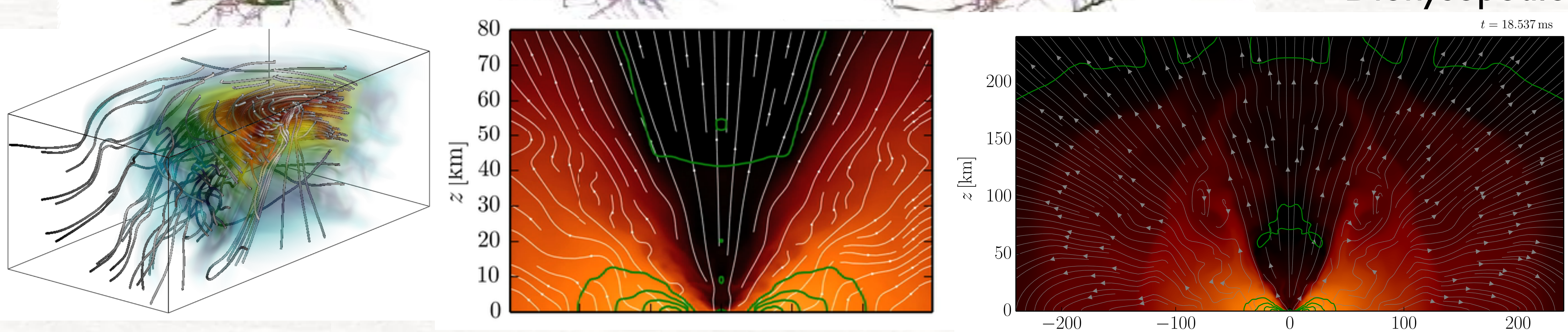
Kiuchi+ 2014



Ruiz+ 2016

Kawamura+2016

Dionysopoulou+ 2015



RMHD

Recap

- ☑ Spectra of post-merger shows clear **"quasi-universal"** peaks.
- ☑ Unless binary very close, peaks have **SNR ~ 1**. However, multiple signals can be stacked and **SNR** will **increase coherently**.
- ☑ Parallel Fisher-matrix and Monte-Carlo simulations can be performed combining information from inspiral and post-merger:
  - **stiff** EOSs: $|\Delta R/\langle R\rangle| < 10\%$ for ***N~20***
  - **soft** EOSs: $|\Delta R/\langle R\rangle| < 10\%$ for ***N~50***
  - **very soft** EOS will be a challenge for aLIGO-Virgo (ET?)
- ☑ **Electromagnetic counterparts** and a **jet** are **likely** to be produced but the details of this picture are still **far from clear**.