Real-time simulations:
Studying systems far from equilibrium

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

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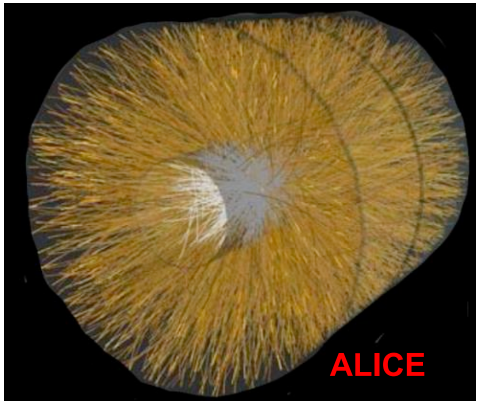
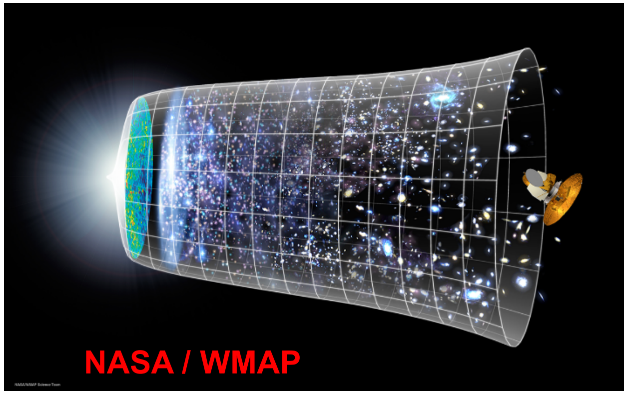
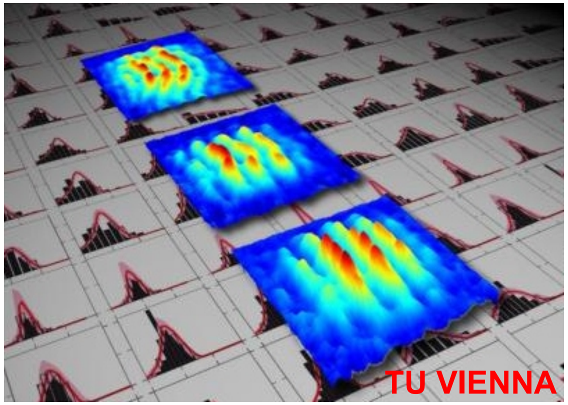
Motivation

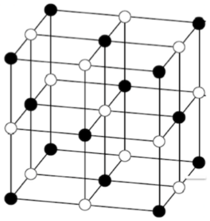
Far from equilibrium dynamics in

Heavy-ion collisions	*Inflationary cosmology*	*Ultracold atoms*
		
Longitudinally expanding non-Abelian plasmas	Relativistic scalar systems	Nonrelativistic scalar systems

- Couplings can be weak $g, \lambda \ll 1$ while occupation numbers large $f \sim 1/g^2 \gg 1$
- Then usual perturbation theory breaks down because propagators $\sim 1/g^2$
- Non-perturbative lattice technique needed $\Rightarrow$ classical-statistical simulations

Classical-statistical simulations

Principle

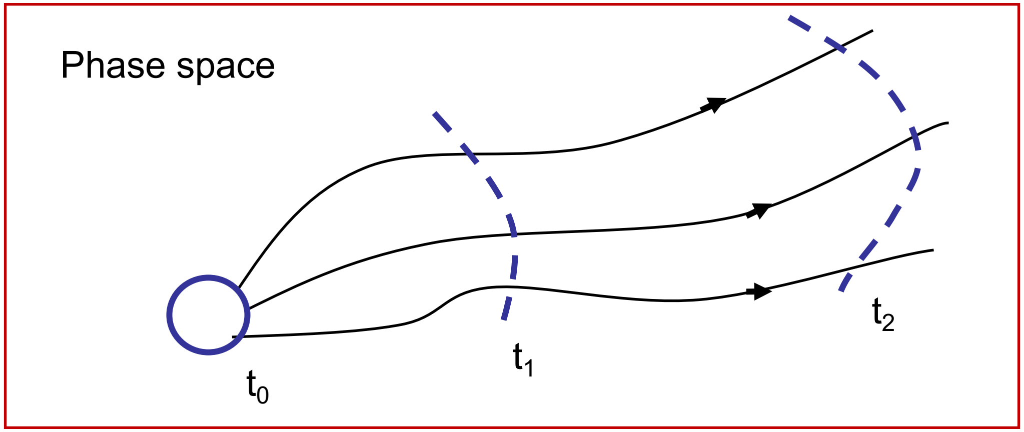


- Initial time t_0, quantum (Gaussian) initial conditions (IC):

  choose $\langle A \rangle$, $\langle AA \rangle$, $\langle E \rangle$, $\langle EE \rangle$ according to weight $W[A(t_0), E(t_0)]$

- Approximate quantum dynamics with classical field EOMs
- Evolve fields classically, obtain observables by averaging over IC at t

$$O(t) = \int DA(t_0)\, DE(t_0)\;\; W[A(t_0), E(t_0)]\, O_{\mathrm{cl}}[A, E](t)$$

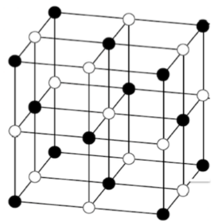


Trajectories of $O_{\mathrm{cl}}[A, E]$
classical observable

Examples:
Micha, Tkachev ; Smit, Tranberg; Nowak, Scholle, Sexty, Gasenzer ; Berges, KB, Schlichting, Venugopalan; Kurkela, Moore; …

Classical-statistical simulations

Example: non-Abelian gauge theory



- *SU*(N_c) *gauge theory* (often used $N_c = 2$) in temporal $A_0 = 0$ gauge
- Fields are link and chromo-electric fields U_i, E_i on 3D spatial lattice
- a_s: Lattice spacing, N_s: Lattice size in one direction
- Initialization: E.g., choose initial single-particle distribution $f(t_0, p)$

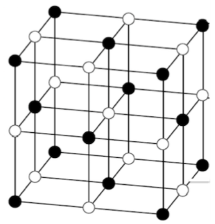
$$\langle |A(t_0,\boldsymbol{p})|^2 \rangle \sim \frac{f(t_0,p)}{p}, \quad \langle |E(t_0,\boldsymbol{p})|^2 \rangle \sim p\, f(t_0,p)$$

- $A_i(t_0,\boldsymbol{p}) \to U_i(t_0,\boldsymbol{x}) \simeq e^{i g a_s A_i(t_0,\boldsymbol{x})}$, $E_i(t_0,\boldsymbol{p}) \to E_i(t_0,\boldsymbol{x})$
- Restore Gauss law $D_j E^j = 0$, $j = 1,2,3$
- Evolve classical EOMs in time $D_\mu F^{\mu\nu} = 0$, but in U_i, E_i variables
- For gauge dependent observables: fix to Coulomb gauge $\partial_j A_j = 0$ before output

E.g., Berges, KB, Schlichting, Venugopalan; Kurkela, Moore ; KB, Kurkela, Lappi, Peuron; …

Classical-statistical simulations

Range of validity



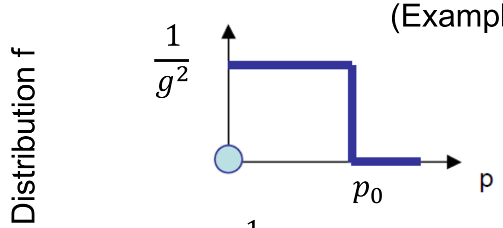
- Approximation only valid for large occupation numbers

Aarts, Berges (2002); Mueller, Son (2004); Jeon (2005); Berges, KB, Schlichting, Venugopalan (2014); ...

Classicality condition: $f(t,p) \simeq p\left\langle\left|A_j^a(t,\mathbf{p})\right|^2\right\rangle \gg 1$

- For typical hard momenta $p \lesssim \Lambda$, where $p^3 f(p)|_{p=\Lambda}$ maximal (dominating energy)
- Meaning: KMS relation $F \sim f\,\rho$, with $F = \langle AA\rangle$, $\rho \sim i[A,A]$, i.e., $F \gg \rho$
- However, *in thermal equilibrium:* $\Lambda \simeq T$, $f_{eq}(T) \sim 1 \;\Rightarrow\; F \sim \rho$
- Hence, classicality condition suitable for physics *far from equilibrium*
- In practice:

(Example)



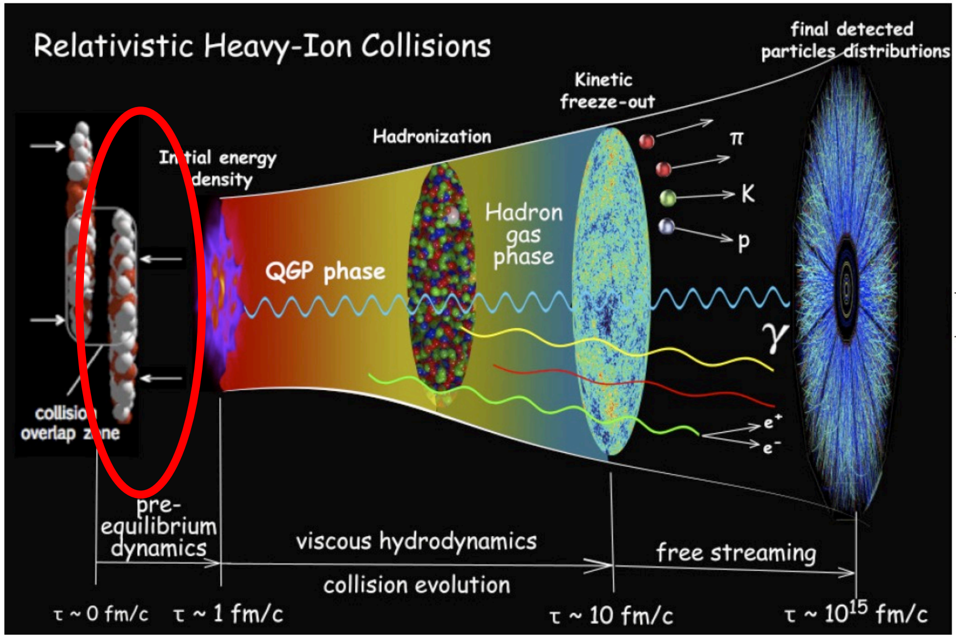
$$f(t_0,p) \sim \frac{1}{g^2}\,\Theta(p_0 - p), \quad \Lambda \sim p_0 \ll \frac{1}{a_s}$$

Hard scale Λ is physical scale, lattice spacing a_s should be small such that simulation not sensitive to a_s

Applications far from equilibrium

Examples in Heavy-ion collisions (HIC):
How does the created QCD matter evolve?

- CS simulations can be used to study initial stages in HIC
- Some phenomena:
  1) Saturation (Glasma)
  2) Instabilities
  3) Turbulence / NTFP
  4) …
- We will later concentrate on 3), nonthermal fixed points (NTFP), but first …



Little Bang by P. Sorensen and C. Shen

Applications far from equilibrium

Strong fields: Glasma and instabilities

Example: Heavy-ion collisions, after the collision (SU(N_c) theory, $g \ll 1$)

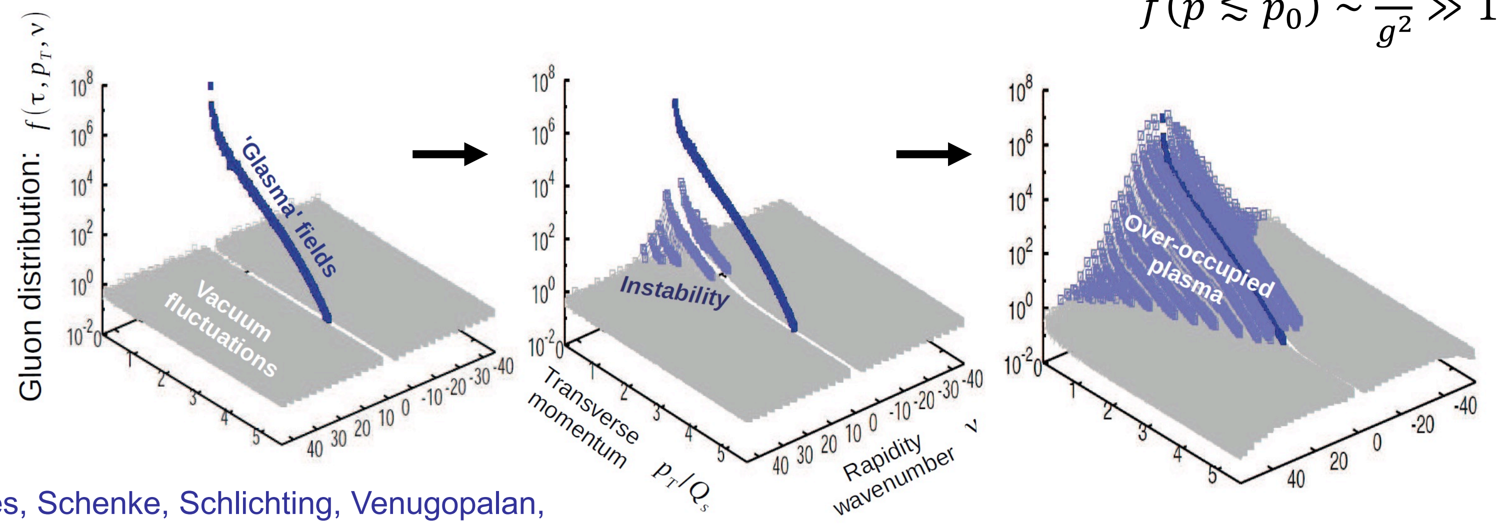
Large „Glasma" ($\nu = p_z t = 0$) + quantum vacuum

$A \sim \frac{1}{g}$

Plasma instabilities

Large gluon distribution

$f(p \lesssim p_0) \sim \frac{1}{g^2} \gg 1$

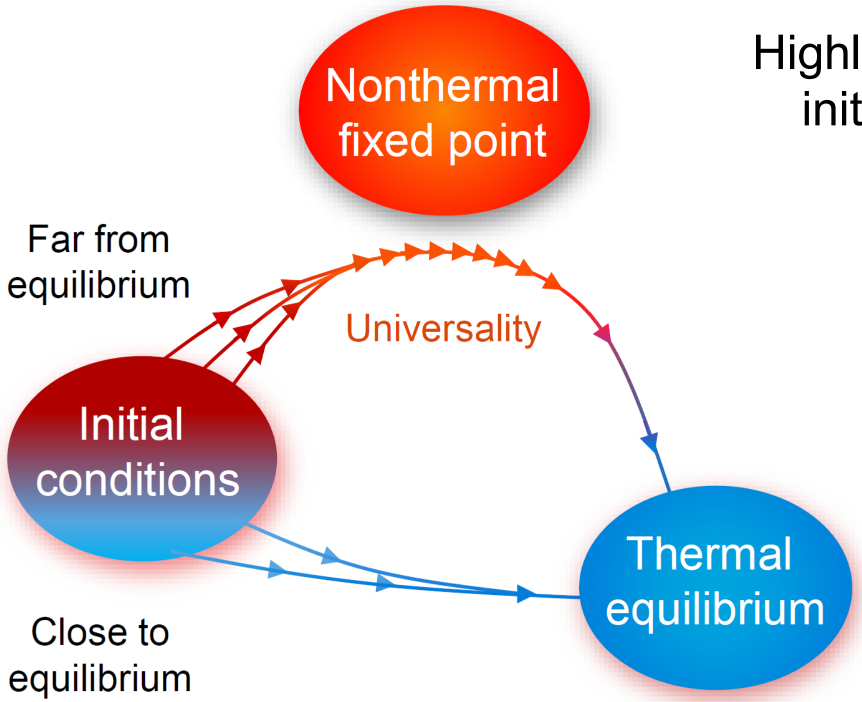


Berges, Schenke, Schlichting, Venugopalan, *Nucl. Phys. A 931, 348 (2014)*

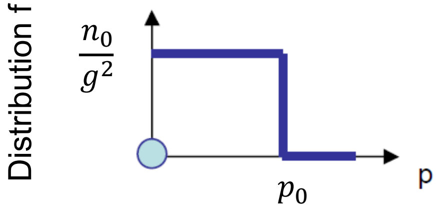
McLerran, Venugopalan (1999); Krasnitz, Venugopalan (1999, 2000, 2001); *Mrowczynski (1993); Arnold, Lenaghan, Moore (2003); Romatschke, Strickland (2003);* Lappi (2003, 2006, 2011); *Romatschke, Venugopalan (2006); Attems, Rebhan, Strickland (2012); Fukushima, Gelis (2012); Berges, Schlichting (2013); Epelbaum, Gelis (2013); ...*

Applications far from equilibrium

Nonthermal fixed points (NTFP)



Highly occupied initial state:



$$f(t_0, p) \sim \frac{n_0}{g^2}\,\Theta(p_0 - p)$$

Nonthermal fixed point (NTFP)

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

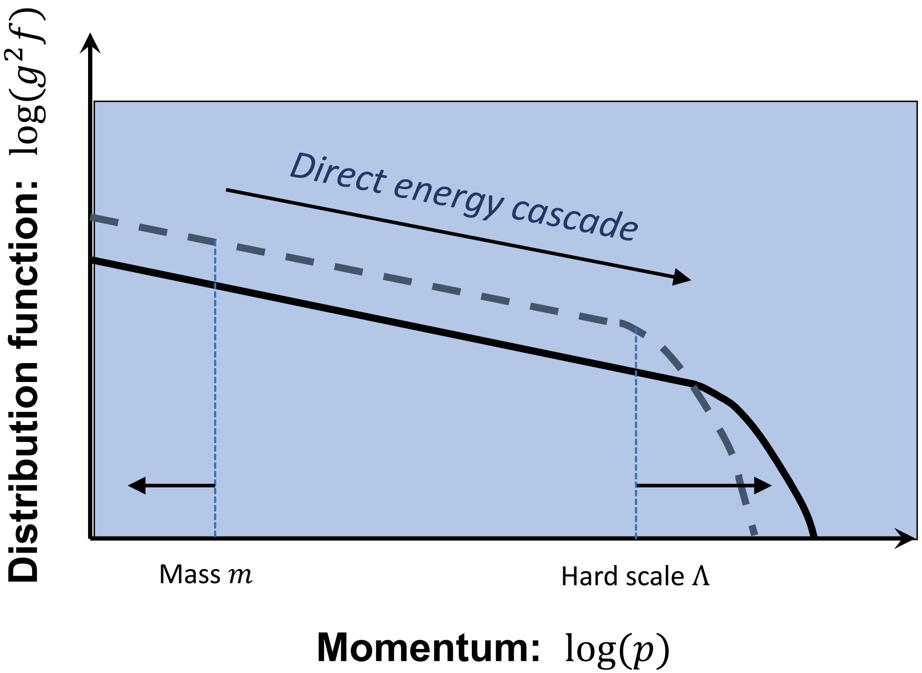
Self-similar distribution function: $f(t,p) = t^{\alpha}\, f_S(t^{\beta} p)$

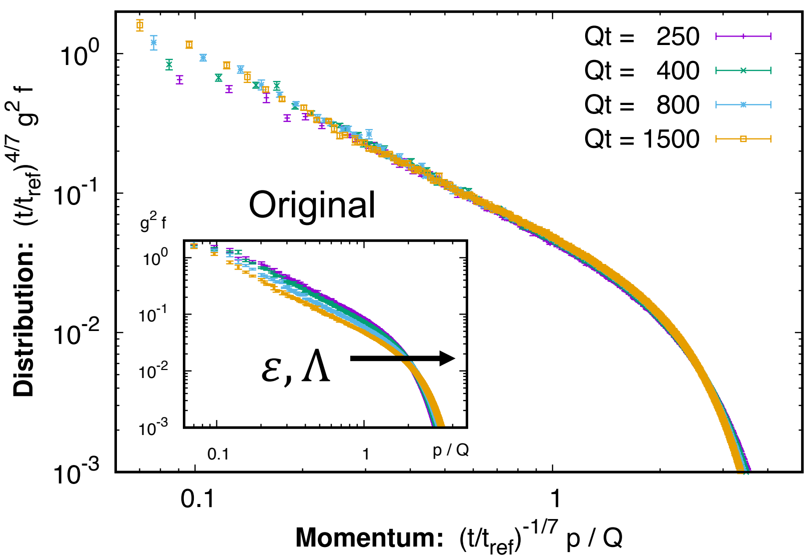
NTFPs in scalar theories relevant for *cosmology, ultra-cold atoms, …* ⇒ see Backup

Micha, Tkachev (2004); Berges, Rothkopf, Schmidt (2008); Piñeiro Orioli, KB, Berges (2015); Moore (2016); Karl, Gasenzer (2016); Walz, KB, Berges (2017); …

Applications far from equilibrium

Example: Yang-Mills theory (isotropic)





Q: constant scale from energy density $g^2\varepsilon \sim Q^4$

Self-similarity observed over extended time interval:

Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); York, Kurkela, Lu, Moore (2014); KB, Kurkela, Lappi, Peuron (2018)

Self-similar evolution

$$f(p,t) = t^{\alpha}\, f_S(t^{\beta} p)$$

Universal

$$\alpha = -4/7$$
$$\beta = -1/7$$

Applications far from equilibrium

Yang-Mills (anisotropic): NTFP in ultra-relativistic heavy-ion collisions

Berges, KB, Schlichting, Venugopalan, *PRD 89, 114007 (2014); PRD 89, 074011 (2014)*

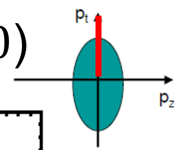
Reminder: *Self-similar evolution*

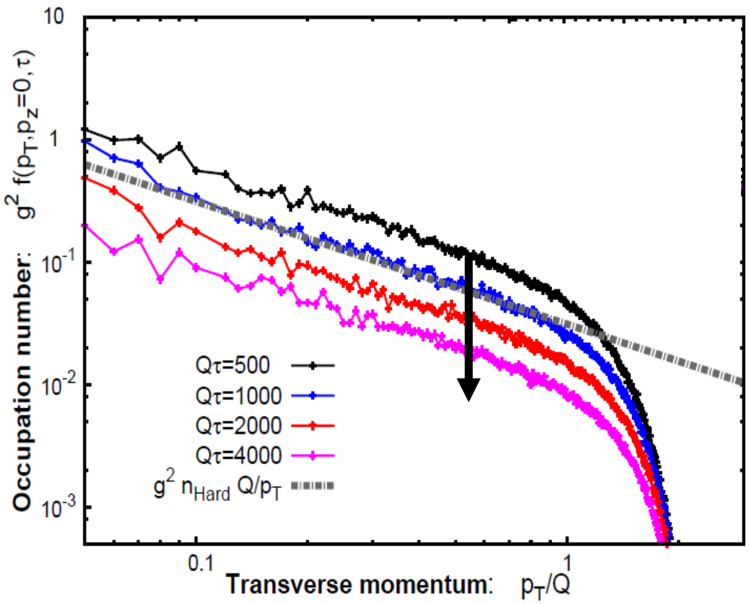
$$f(p_T, p_z, \tau) = \tau^{\alpha} f_s(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Universal scaling exponents

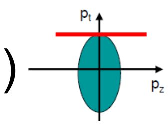
$$\alpha \simeq -2/3,\ \beta \simeq 0,\ \gamma \simeq 1/3$$

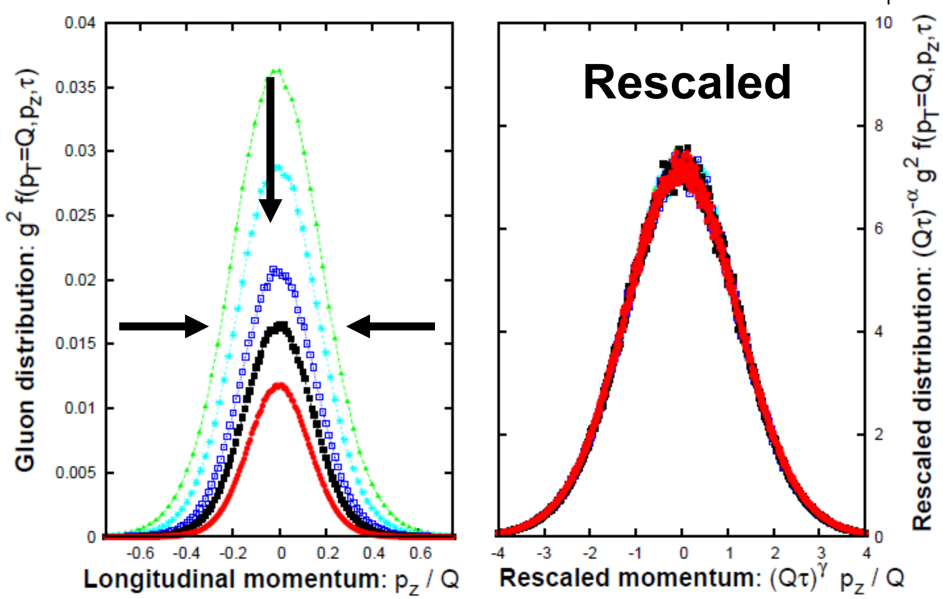
Transverse distribution $(p_z = 0)$





Longitudinal distribution $(p_T = Q)$





Applications far from equilibrium

Comparing to a scalar $\lambda\phi^4$ field theory ($O(N)$ symmetric)

Berges, KB, Schlichting, Venugopalan, *PRL 114, 061601 (2015) ; PRD 92, 096006 (2015)*

Reminder: *Self-similar evolution*

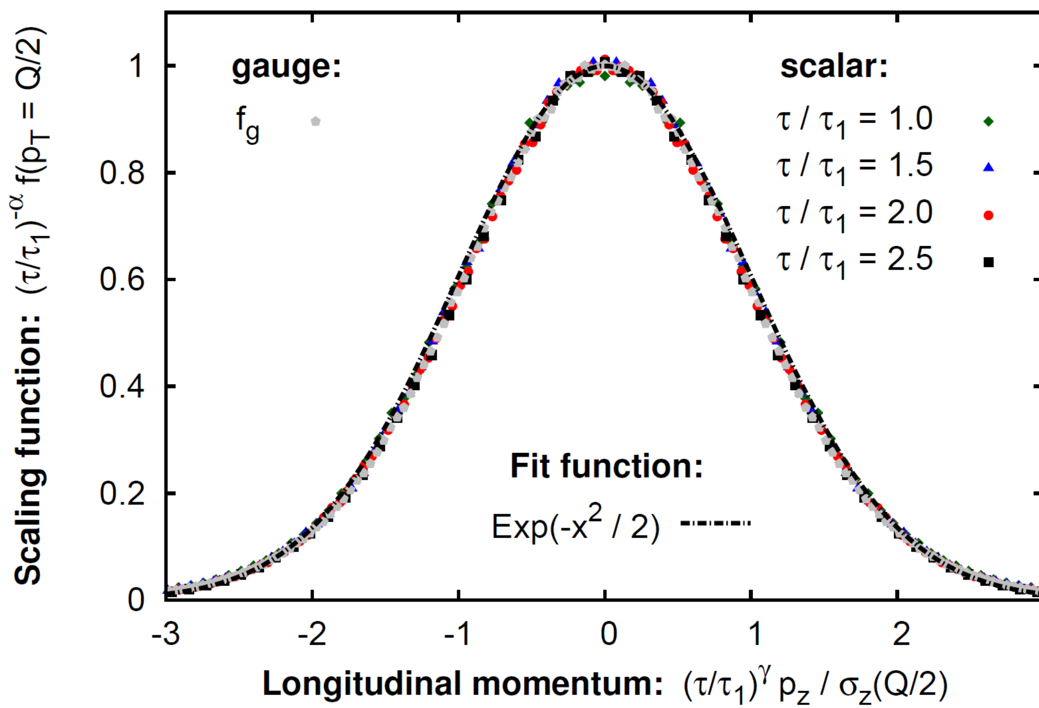
$$f(p_T, p_z, \tau) = \tau^\alpha f_s(\tau^\beta p_T, \tau^\gamma p_z)$$

Universality with gauge theory:

Same scaling exponents, same f_s

$$\begin{aligned} \alpha &\simeq -2/3 \\ \beta &\simeq 0 \\ \gamma &\simeq 1/3 \end{aligned}$$

Rescaled longitudinal distribution ($p_T = Q/2$)



Applications far from equilibrium

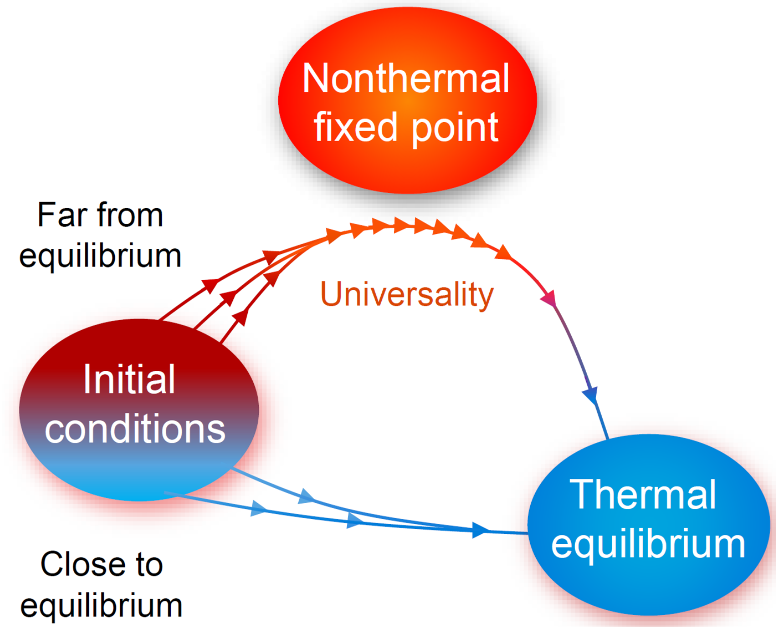
Summary: Nonthermal fixed points

Examples found for instance in

- Inflaton, dark-matter models, ultra-cold atom experiments (scalar theories, *Backup*)
- Heavy-ion collisions (Yang-Mills theory)

Common approach:

- Classical-statistical lattice simulations, or even experiment *, for *observation*
- *Understanding* with a kinetic or an effective theory (not discussed in this talk)
- **Often:** transport of a *conserved quantity* like energy density, particle number



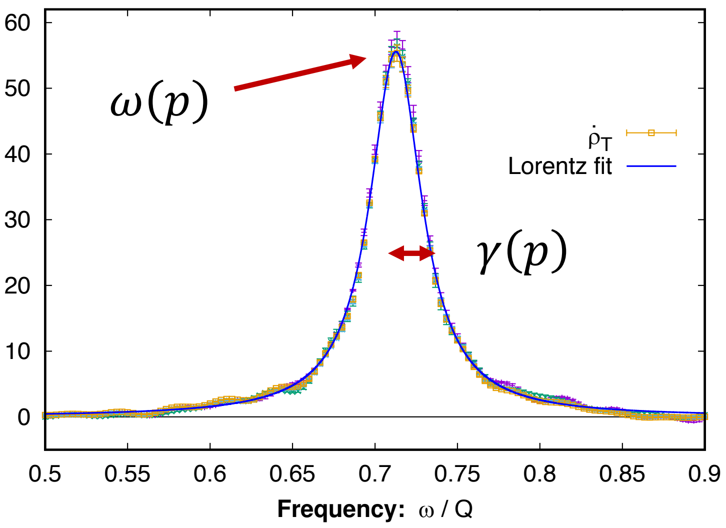
* *Exp. observation with ultra-cold atoms: M. Prüfer et al., arXiv:1805.11881*

Next step: extract spectral functions

Go beyond distribution functions

- To *better understand microscopic dynamics*
- To *test quasiparticle assumptions* underlying kinetic theories
- To extract *transport and diffusion* properties

A typical spectral function



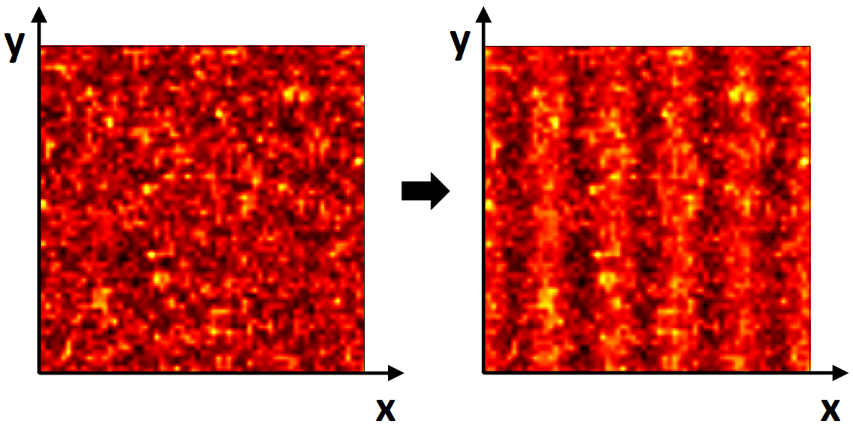
- $\rho(\omega, p)$ includes *all possible excitations*
- *Quasiparticles* emerge as Lorentz peaks
- *Dispersion* $\omega(p)$ is energy of "on-shell" particles
- *Damping rate* $\gamma(p)$ is inverse of their life time
- More complicated structures can also emerge (cuts, extra poles, etc.)

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

Next step: extract spectral functions

New method: Linear response theory on a classical-statistical lattice

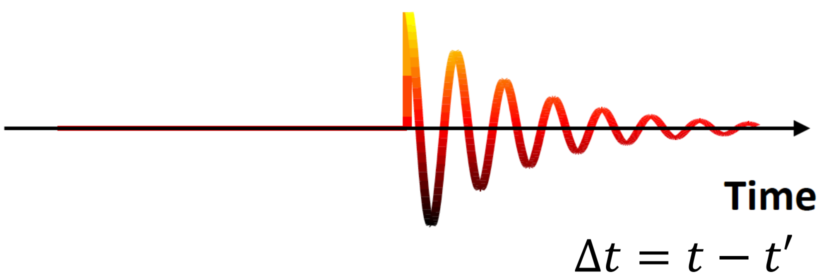
Perturbation



- Classical field simulations for background
- Source j at time t'
- Response in linear fluctuations a_j for $t > t'$

Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*

Response



- $\langle a_j(t,\boldsymbol{p})\rangle = \int dt' G_{R,jk}(t,t',\boldsymbol{p})\, j^k(t',\boldsymbol{p})$, obtain ret. propagator $G_{R,jk}$ from response
- Spectral function: $G_{R,jk} = \theta(t-t')\,\rho_{jk}$
- Distinguish polarizations

Self-similar evolution

$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

Next step: extract spectral functions

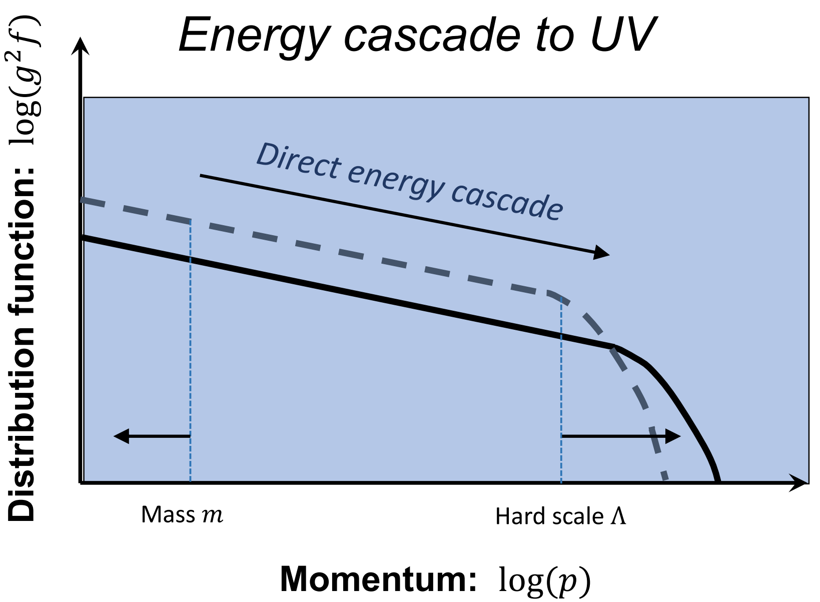
First application: isotropic Yang-Mills theory

Observations: *Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); …*

- Self-similar, cascade to UV
- *Scale separation grows* with time

$$m/\Lambda \sim (Qt)^{-2/7} \ll 1$$

<u>Asymptotic mass</u>: $m^2 \sim g^2 \int \mathrm{d}^3p \, \frac{f(t,p)}{p}$



Understanding: *Arnold, Moore, Yaffe (2003); York, Kurkela, Lu, Moore (2014)*

- *Effective kinetic theory* (AMY)

Scale separation allows usage of

- *Hard-thermal Loop* (HTL) *Braaten, Pisarski (1990); Blaizot, Iancu (2002)*

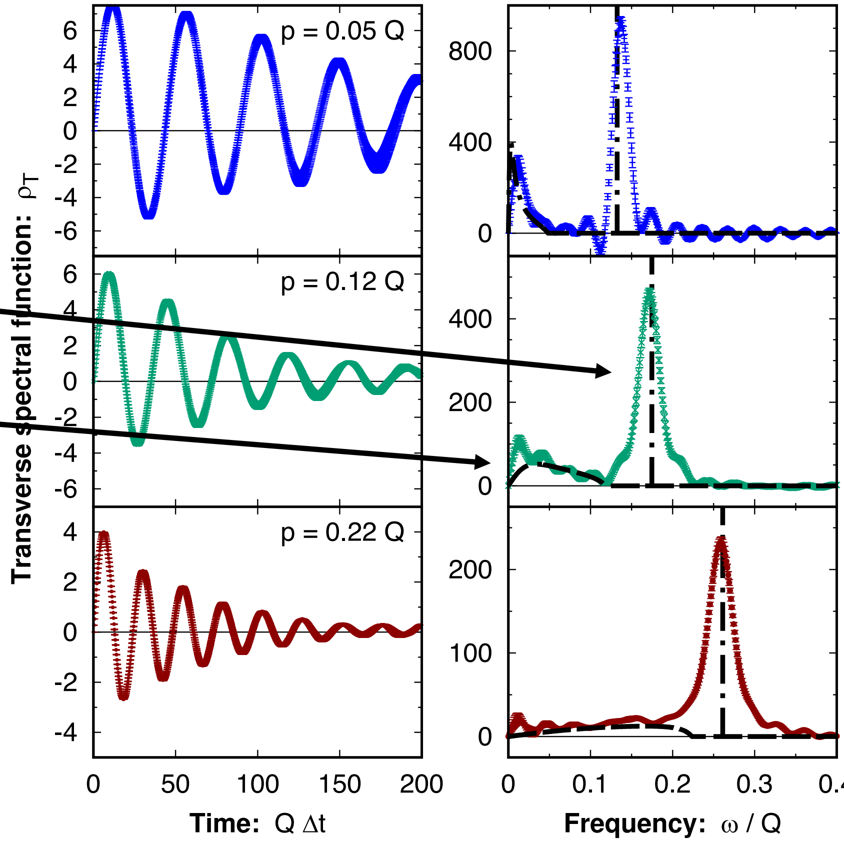
Next step: extract spectral functions

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Transverse spectral function ρ_T

ρ_T as function of $\Delta t = t - t'$ (left) or frequency ω (right) at late time $t, t' \gg \Delta t$

- Lorentzian peaks:
  existence of quasi-particles
- for $|\omega| \leq p$: *Landau cut*
- black dashed lines:
  Hard-thermal Loop (HTL) at LO



✓ Good agreement with HTL!

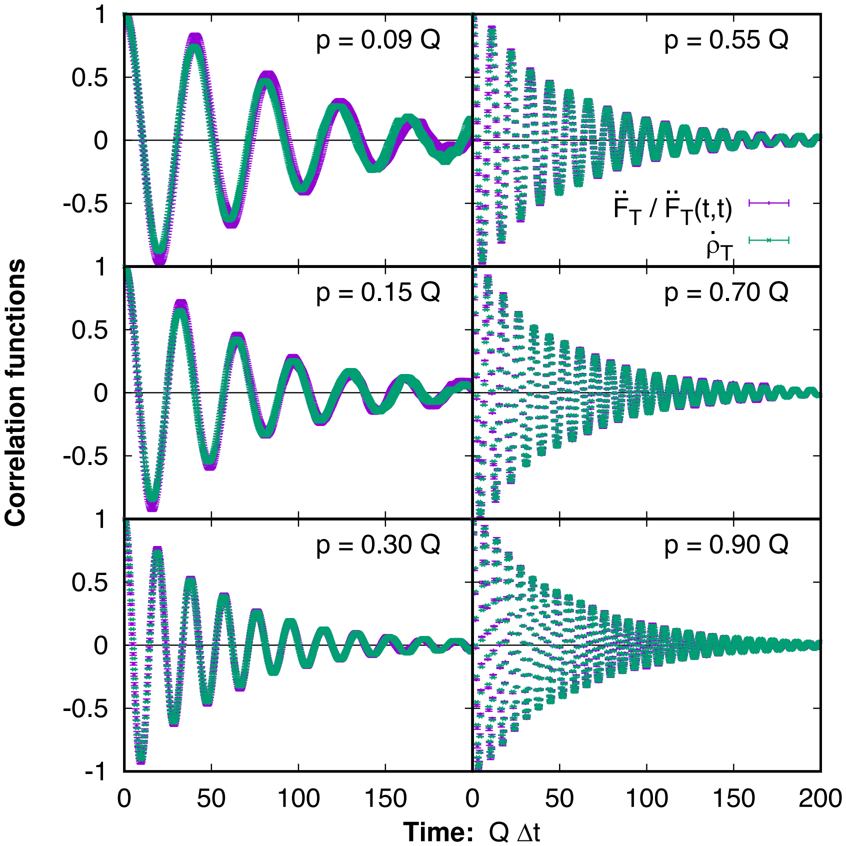
✓ System dominated by quasiparticles with relatively narrow width!

First determination of "width" $\gamma_{T,L}(p)$!

Next step: extract spectral functions

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

Statistical correlation function F



<u>(Classical) definition</u>:

$$\ddot{F}^{jk}(t,\Delta t,\boldsymbol{p}) = \langle E^j(t,\boldsymbol{p})\, E^{*,k}(t',\boldsymbol{p})\rangle$$

(As always: $\Delta t = t - t'$, Fourier transform to ω)

<u>(Thermal) fluctuation-dissipation relation</u>:

$$\ddot{F}_T(t,\omega,p)\;/\;T = \dot{\rho}_T(t,\omega,p)$$

Used to estimate thermal spectral function

Example: G. Aarts, *PLB 518, 315 (2001)*

We *extract $\ddot{F}$, $\dot{\rho}$ independently*, we observe a <u>generalized fluctuation-dissipation relation:</u>

$$\frac{\ddot{F}_T(t,\Delta t,p)}{\ddot{F}_T(t,\Delta t=0,p)} = \dot{\rho}_T(t,\Delta t,p)$$

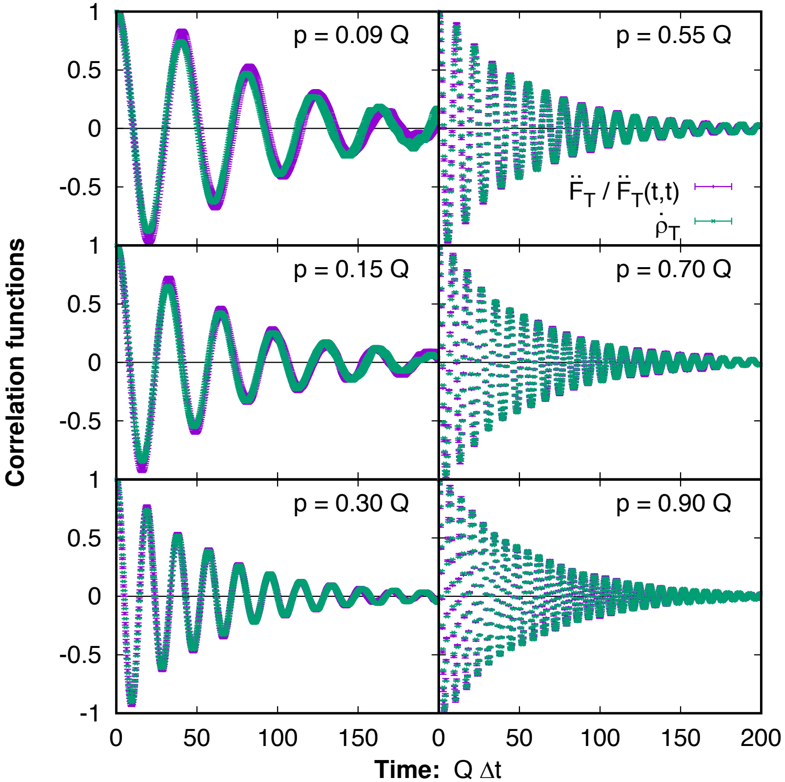
<u>Remarks</u>: $\dot{\rho}_T = \partial_t \rho_T$, $\dot{\rho}_T(t,\Delta t=0,p) = 1$

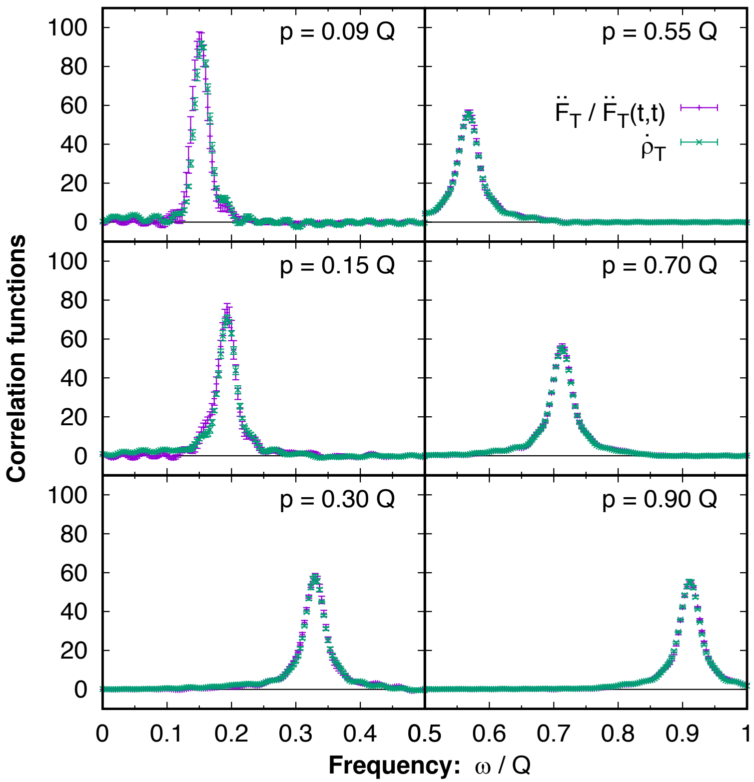
Next step: extract spectral functions

Observation: $\frac{\ddot{F}_T(t,\Delta t,p)}{\dot{\rho}_T(t,\Delta t,p)} = \frac{\ddot{F}_T(t,\omega,p)}{\dot{\rho}_T(t,\omega,p)} = \ddot{F}_T(t,\Delta t = 0,p)$

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

$\ddot{F}_T$ and $\dot{\rho}_T$ have same functional form in Δt or ω!





Remark: Similarly for longitudinal but with larger statistical error for $\ddot{F}_L$ in our simulations

Conclusion

a. Classical-statistical simulations: non-perturbative real-time lattice approach

b. Despite range if validity $f \gg 1$, can describe NTFPs, instabilities, …

c. Nonthermal fixed points (NTFP) commonly emerge far from equilibrium.
Examples: HICs (gluonic systems), inflaton (scalars), ultra-cold atoms, …

d. Non-perturbative numerical approach developed for spectral functions
$\Rightarrow$ more information on microscopic dynamics accessible

Outlook: We intend to use the new method to extract spectral functions for:

- *Transport* coefficients, jet quenching, diffusion
- *Anisotropy, quasiparticles, instabilities* for heavy-ion collisions

Thank you for your attention!

BACKUP SLIDES

Nonthermal fixed points

Scaling region (of a nonthermal fixed point)

Self-similar evolution of distribution function f

$$f(t,p) = t^{\alpha}\, f_S\left(t^{\beta} p\right)$$

with scaling behavior of typical scales $p_{\rm typ} \sim t^{-\beta}\ ,\ f(p_{\rm typ}) \sim t^{\alpha}$

Classification: universality classes far from equilibrium

Via scaling exponents α, β and the scaling function $f_s(x)$

NTFP	Close to 2nd order PT
Time scale t	Inverse reduced temp. T_c / $(T-T_c)$
Self-similar evolution	Critical slowing down, power laws
Scaling exponents & function	Critical exponents & surface

Applications far from equilibrium

Classic example: Turbulent thermalization for the inflaton (scalar $\lambda\phi^4$ theory)

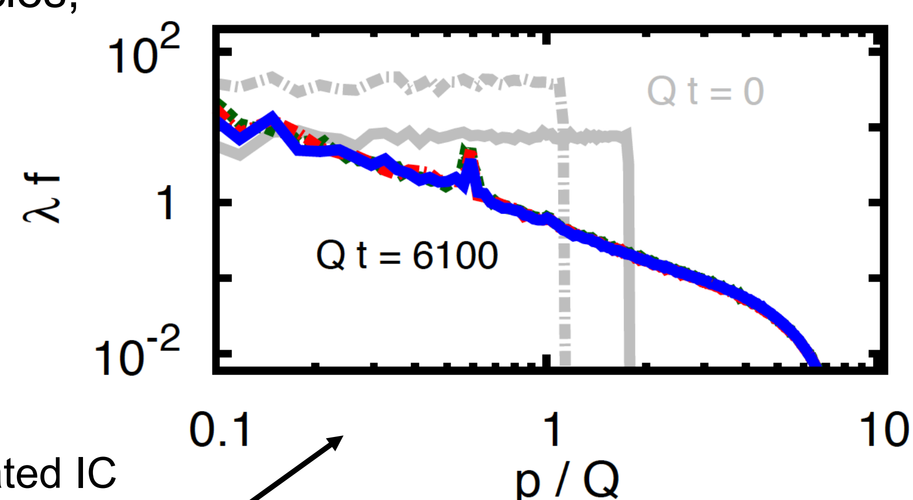
Micha, Tkachev,
PRL 90, 121301 (2003)
PRD 70, 043538 (2004)

- Reheating: inflaton decays into particles,
- They approach self-similar evolution

$$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$$

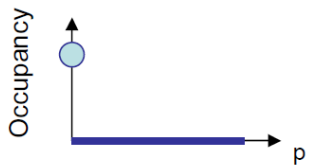
- With exponents $\alpha = -\frac{4}{5},\ \beta = -\frac{1}{5}$
- For different classes of ICs

Insensitive to details of IC



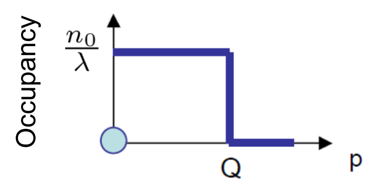
Berges, KB, Schlichting, Venugopalan,
JHEP 1405, 054 (2014)

Condensate driven IC



$\phi_0 \sim 1/\sqrt{\lambda}$ $\qquad f = 0$

Fluctuation dominated IC



$\phi_0 = 0$ $\qquad f(p \leq Q_0) = n_0/\lambda$

Self-similar evolution

$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$

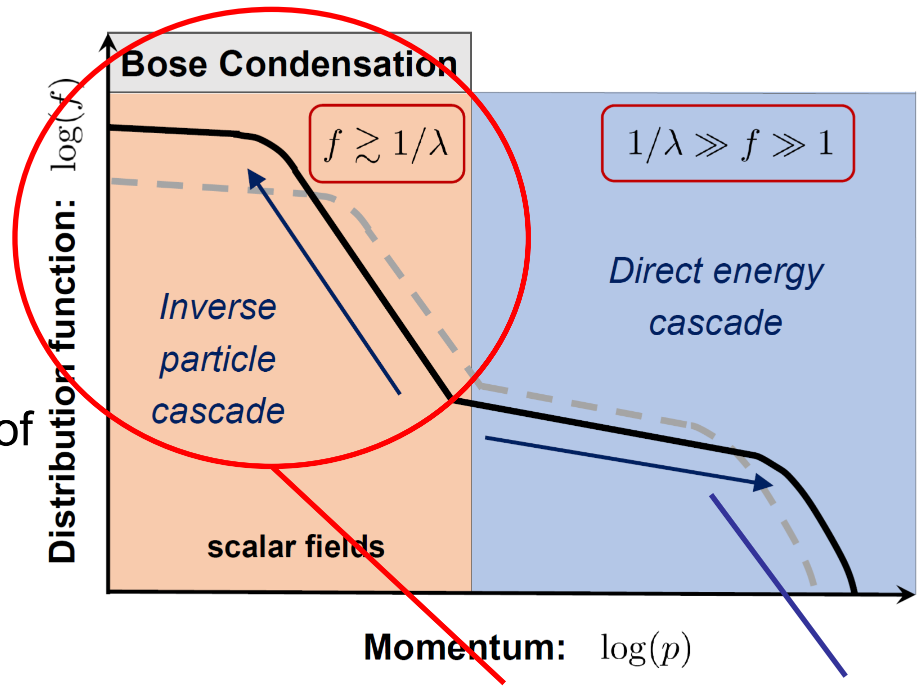
Applications far from equilibrium

In scalar systems: Universality in IR

Two scaling regions: *Inverse N-cascade to IR*, *direct E-cascade to UV*

- **Relativistic** scalars ($\lambda\phi^{2n}$, $O(N)$) *(inflaton, axions, dark matter, …)*
- **Nonrelativistic** scalars *(ultra-cold atom experiments, …)*
- Each region self-similar, own set of $\alpha, \beta, f_S(x)$
- **IR is universal:** same in both

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*



Regions Discovered by: Berges, Rothkopf, Schmidt, *PRL 101, 041603 (2008)*; Micha, Tkachev, *PRL 90*, 121301 (2003)

Nonthermal fixed points

Scalars: Inverse particle cascade to IR

Self-similar evolution

$$f(p,t) = t^{\alpha}\, f_S(t^{\beta} p)$$

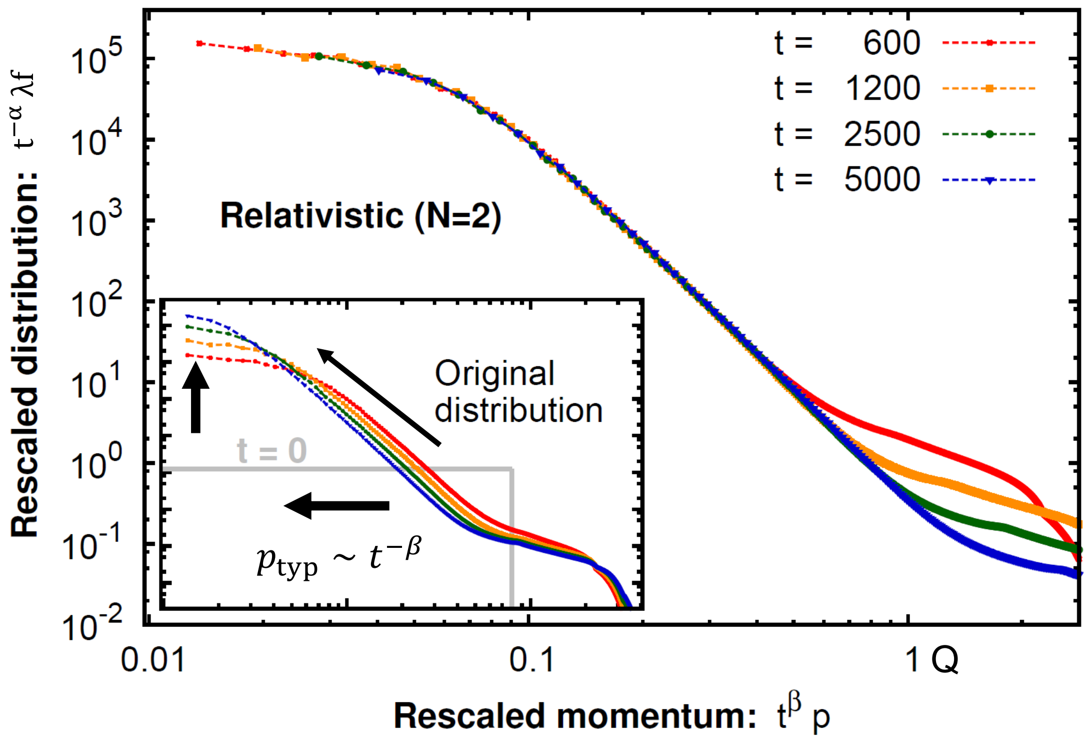
Scaling exponents

$$\alpha \approx \frac{3}{2}, \qquad \beta \approx \frac{1}{2}$$

Particle conservation

$$n = \int_{IR} \frac{d^3p}{(2\pi)^3}\, f(p) \approx const$$

Piñeiro Orioli, KB, Berges,
PRD 92, 025041 (2015)



Berges, Rothkopf, Schmidt (2008); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Karl, Gasenzer (2016); Berges, KB, Chatrchyan, Jäckel (2017); Walz, KB, Berges (2017); Schmied, Mikheev, Gasenzer (2018) ...

Nonthermal fixed point in a spin gas (ultracold atoms)

First experimental observation of a NTFP

Prüfer, Kunkel, Strobel, Lanning, Linnemann, Schmied, Berges, Gasenzer, Oberthaler, *arXiv:1805.11881*

Self-similar evolution

$$f(p,t) = t^{\alpha}\, f_S(t^{\beta} p)$$

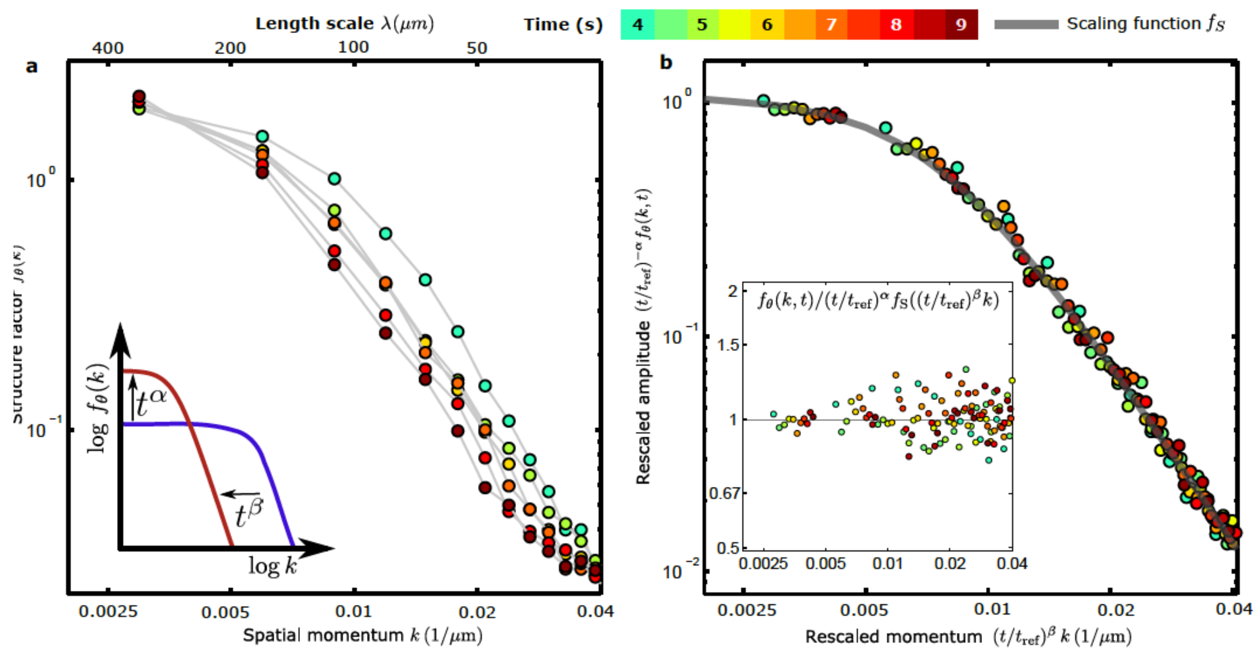
Observation also

$$\alpha \approx \frac{d}{2}, \qquad \beta \approx \frac{1}{2}$$

Same as for IR in scalar systems!

Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*

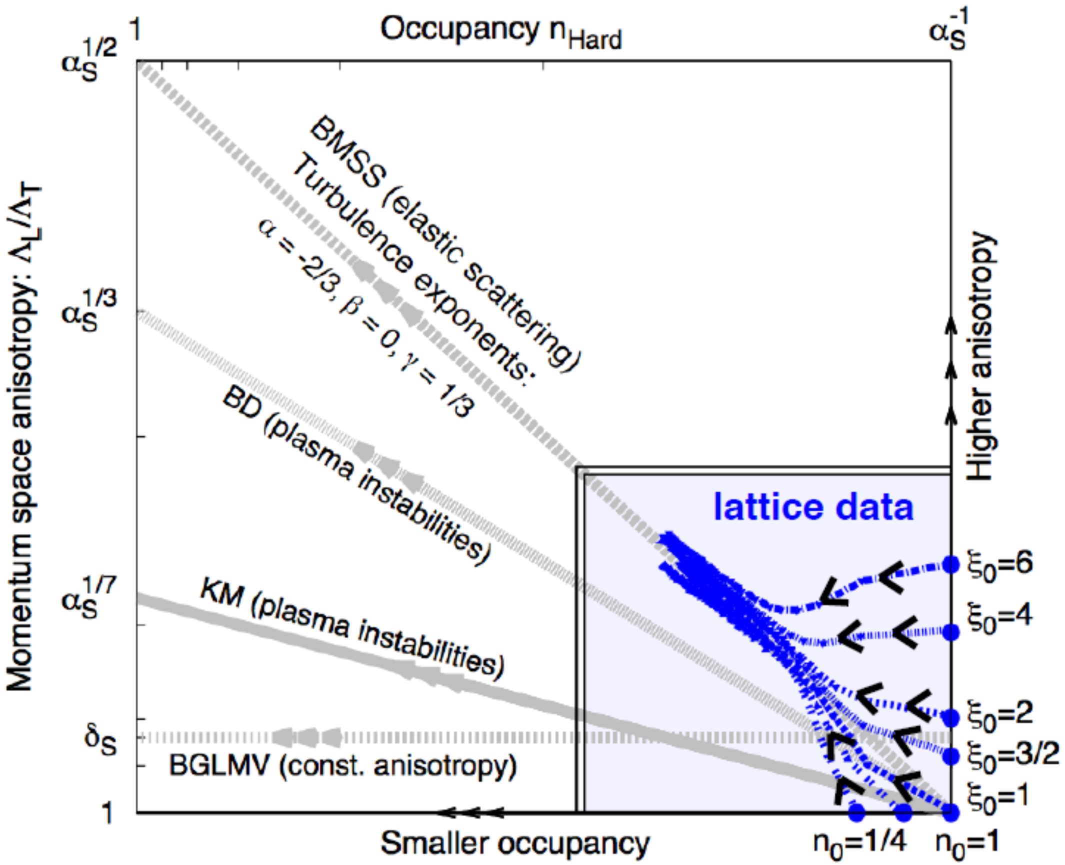
Original distribution **Rescaled** distribution



In $d = 1$, system prepared highly occupied

NTFP in HICs: picked correct kinetic scenario

Real-time lattice simulations



Berges, KB, Schlichting, Venugopalan, *PRD 89, 074011 (2014)*

Thermalization scenarios

- Baier, Mueller, Schiff, Son (BMSS), (2001)
- Bodeker (BD), (2005)
- Kurkela, Moore (KM), (2011)
- Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

Well described by “Bottom-up” (BMSS)! Baier, Mueller, Schiff, Son, *PLB 502, 51 (2001)*

Scenario consists of 3 stages;
Self-similar evolution is 1. stage

Further studies of different stages:

Blaizot, Iancu, Mehtar-Tani, *PRL 111, 052001 (2013)*
Kurkela, Lu, *PRL 113, 182301 (2014)*
Kurkela, Zhu, *PRL 115, 182301 (2015)*

Computational method

What is gauge invariant?

- *Equations of motion* of background (BG) field and linearized fluctuations
- *Initial conditions for the BG* can, in principle, be set gauge-invariantly (starting with classical thermal system $T \gg 1/a_s$, gauge cool until $\Lambda \ll 1/a_s$)
- *Correlation function* $\langle \langle a_k^a(t,\boldsymbol{x})\rangle \, U_0^{ab}(t,t_{\text{pert}},\boldsymbol{x}) \, j_{0,l}^b(\boldsymbol{x})\rangle_j$, where $U_0(t,t_{\text{pert}},\boldsymbol{x})$ is a Wilson line, i.e., product of U_0 links. In our framework:
  - $A_0 = 0 \;\Rightarrow\; U_0^{ab} = \delta_{ab}$
  - Use source only for one momentum $\boldsymbol{p}$

$\Rightarrow$ $\boxed{\langle \langle a_k^a(t,\boldsymbol{x})\rangle \, U_0^{ab}(t,t_{\text{pert}},\boldsymbol{x}) \, j_{0,l}^b(\boldsymbol{x})\rangle_j \propto G_{R,kl}\big(t,t_{\text{pert}},\boldsymbol{p}\big)}$, a *gauge-inv. observ.*!

<u>Remark</u>: But it corresponds to G_R only in temporal gauge

- <u>*Gauge-dependent*</u>: The choice of the source $j_{0,l}^b(\boldsymbol{x}) \propto \sqrt{V}\, v_l(\boldsymbol{p}) \cos \boldsymbol{p}\boldsymbol{x}$

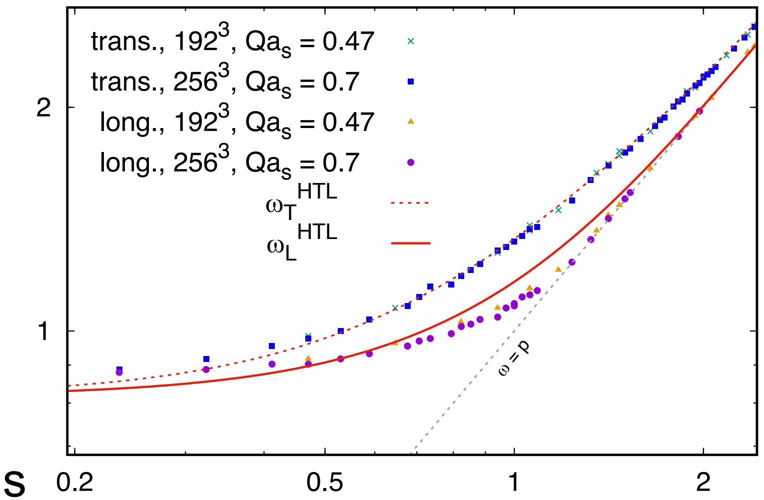
Extracted spectral function vs. HTL predictions

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Extracted dispersion relations $\omega_{T,L}(p)$

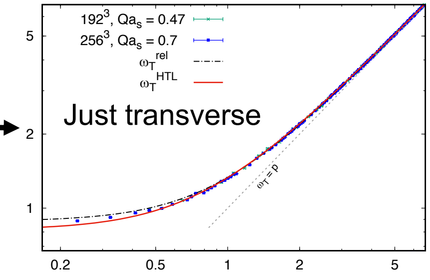
- Extracted from peak position (for ω_L after subtracting HTL Landau cut)
- *Similar to HTL* predictions: $\omega_{T,L}^{\mathrm{HTL}}(p)$
- Deviations at small p, for finite m/Λ?
- "$\omega_L(p)$" deviates at $p \sim m$ because peak is smaller than Landau cut, harder to measure

$\omega_{T,L} \,/\, m_{\mathrm{HTL}}$



Momentum: $p \,/\, m_{\mathrm{HTL}}$

Remark: $\omega_T(p)$ also compatible with $\omega_T^{\mathrm{rel}} = \sqrt{m_\infty^2 + p^2}$ →

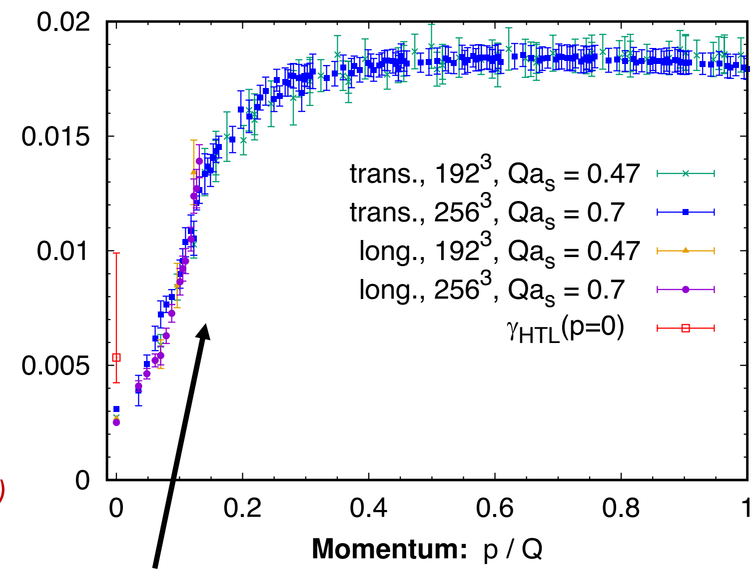


Extracted spectral function vs. HTL predictions

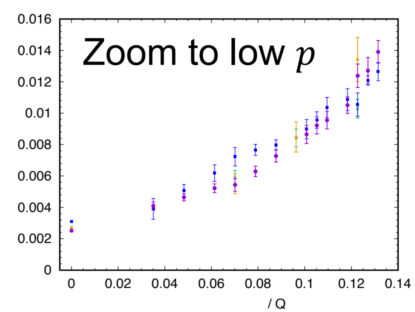
KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Extracted damping rates $\gamma_{T,L}(p)$

$\gamma_{T,L}(p)\,/\,Q$



- $\gamma_{T,L}(p)$ is $\mathcal{O}(g^2 Q)$ and *beyond HTL at LO*, it may contain non-perturbative contributions (*magnetic scale*)
- Here *first determination* of $\gamma_{T,L}(p)$!
- Extracted by fitting to a damped oscillator
- HTL prediction: $\gamma_{\mathrm{HTL}}(p=0)$ — Braaten, Pisarski, *PRD 42, 2156 (1990)*
- "Isotropic" $\gamma_T \approx \gamma_L$ for $p \lesssim m$



Extracted spectral function vs. HTL predictions

Check: Is damping rate indeed subleading?

- Observe: (indication, but low statistics)

  $\gamma(p) \sim Q(Qt)^{-3/7}$

  as function of p/m_{HTL}

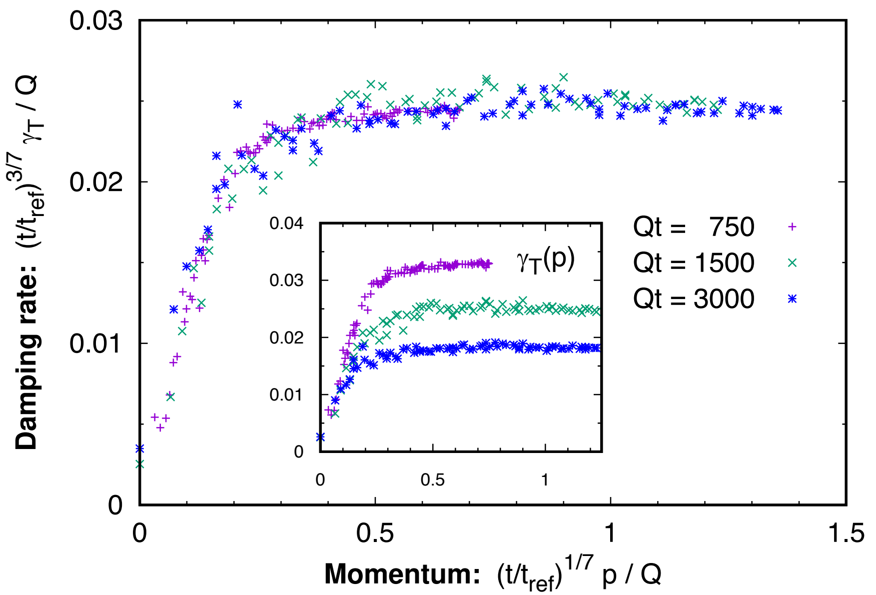
- Reminder:

  $m/\Lambda \sim (Qt)^{-2/7}$

- Conclusion:

  $\gamma(p)/\Lambda \sim (m/\Lambda)^2$

- This shows $\gamma(p)/m \xrightarrow{t\to\infty} 0$, quasi-particle peaks become Delta functions



- *Late-time limit* corresponds to *LO HTL*, damping rates are effects beyond LO

Extracted spectral function vs. HTL predictions

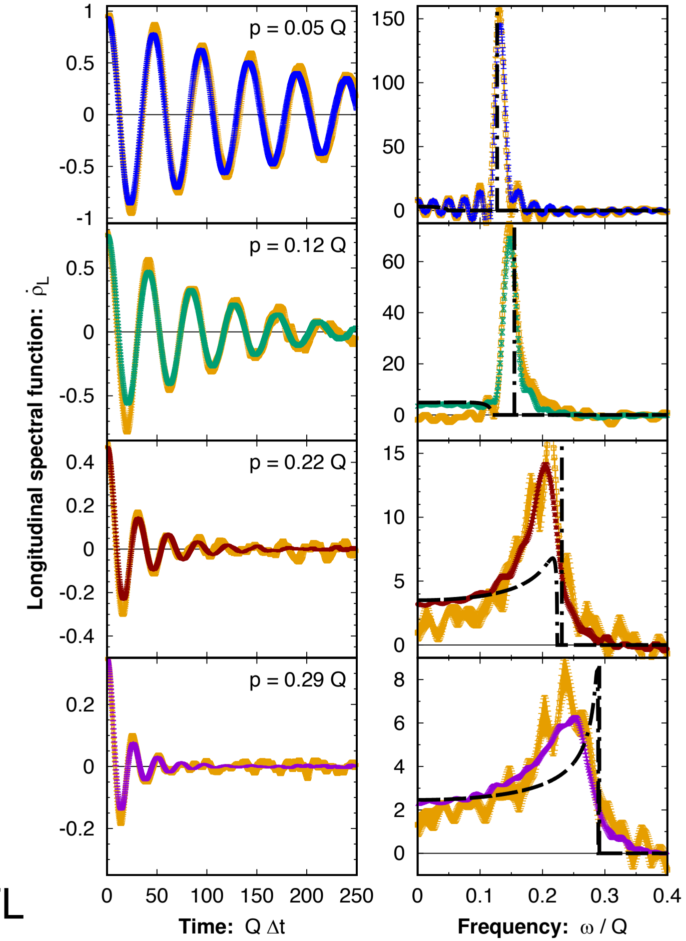
Longitudinal statistical function $\ddot{F}_L$

Orange curves are normalized $\ddot{F}_L (\simeq \omega^2 F_L)$ as functions of $\Delta t = t - t'$ or ω, also $\dot{\rho}_L$

- *Generalized fluctuation-dissipation relation:*

$$\frac{\ddot{F}_L(t,\Delta t,p)}{\dot{\rho}_L(t,\Delta t,p)} = \frac{\ddot{F}_L(t,\omega,p)}{\dot{\rho}_L(t,\omega,p)} = \frac{\ddot{F}_L(t,\Delta t = 0,p)}{\dot{\rho}_L(t,\Delta t = 0,p)}$$

- Higher statistical uncertainty because $\ddot{F}^{jk}(t,t',\boldsymbol{p})$ has both polarizations, the transverse one leads to additional noise



Remark: sum rule $\dot{\rho}_L(t,\Delta t = 0,p) = \frac{2m^2}{2m^2 + p^2}$ from HTL

Extracted spectral function vs. HTL predictions

In detail: deviations from HTL

- *HTL expectation* for $p \ll \Lambda$ in gray, e.g.

$$\ddot{F}_T(t,\omega,p) \,/\, T_* = \dot{\rho}_T(t,\omega,p)$$

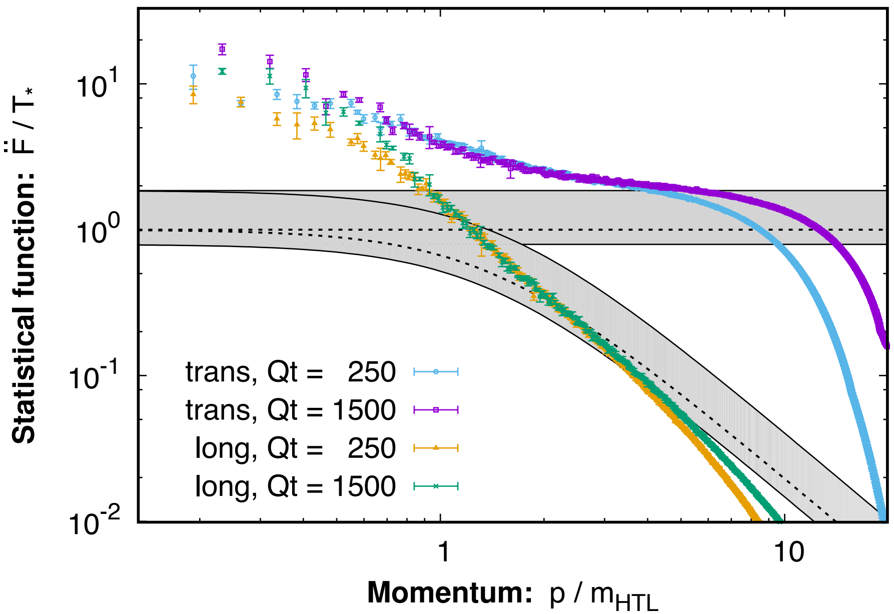
$$\Rightarrow \quad \ddot{F}_T(t,\Delta t = 0,p) = T_*$$

Arnold, Moore, Yaffe, *JHEP 01, 030 (2001)*

- With effective temperature

$$g^2 T_* \approx \frac{\int \mathrm{d}^3p \, (g^2 f)^2}{2\int \mathrm{d}^3p \; (g^2 f)/p} \sim \gamma_{\mathrm{HTL}}\,(p=0)$$

Statistical function: $\ddot{F}_{T,L}(t,\Delta t = 0,p) \,/\, T_*$



Deviations:

- p-dependence even for $p \ll \Lambda$
- *Enhancement* of $\ddot{F}_{T,L}$ for $p \lesssim m$ visible

$\Rightarrow$ *Scale separation* not good enough? Effect of *magnetic scale*?