Stochastic Cooling for NICA

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Fermilab

Talk Objectives

- Why stochastic cooling?
- Stochastic cooling limitations
  - Type of longitudinal cooling
  - Band overlap
- Conceptual proposal for cooling system
  - Energy range where cooling is possible
  - Expected cooling rates and their comparison with IBS rates

Why do we need cooling?

- To prevent emittance growth due to IBS
    - Other emittance growth mechanisms make significantly smaller contributions

Why Stochastic Cooling?

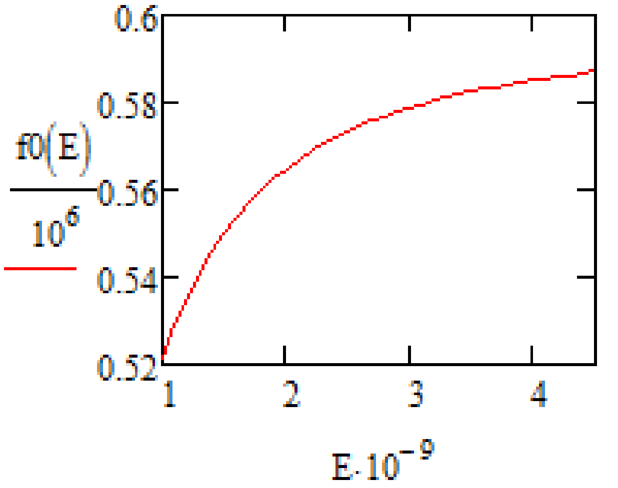
- Less expensive than electron cooling
    - More reliable
- Very good experience accumulated in Fermilab and other labs
    - Bunched beam cooling in BNL
- Good cooling rates for heavy ions due to small number of particles in the beam
- No recombination with electrons of the electron beam
    - Significantly longer beam lifetime => Longer luminosity lifetime
        - Larger integrated luminosity
    - Smaller load on injector

What is the major problem?

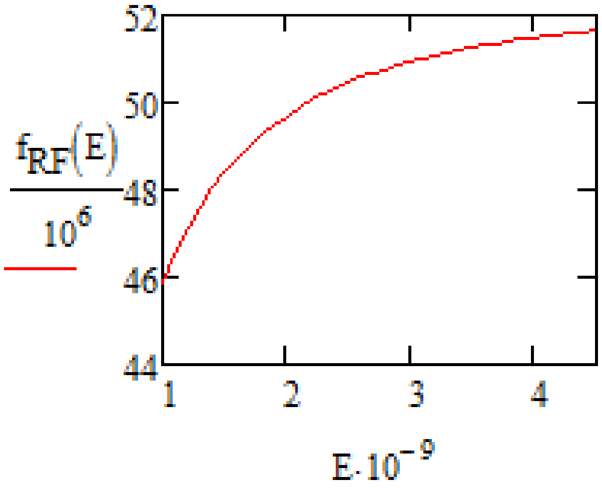
- Insufficient experience in JINR and BINP

NICA Main Parameters

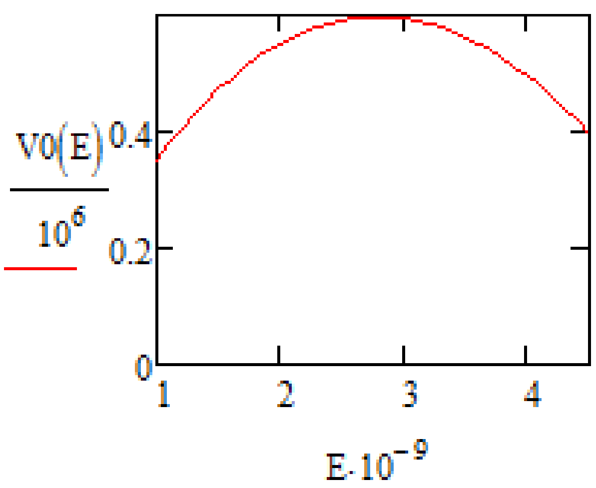
Revolution frequency, MHz



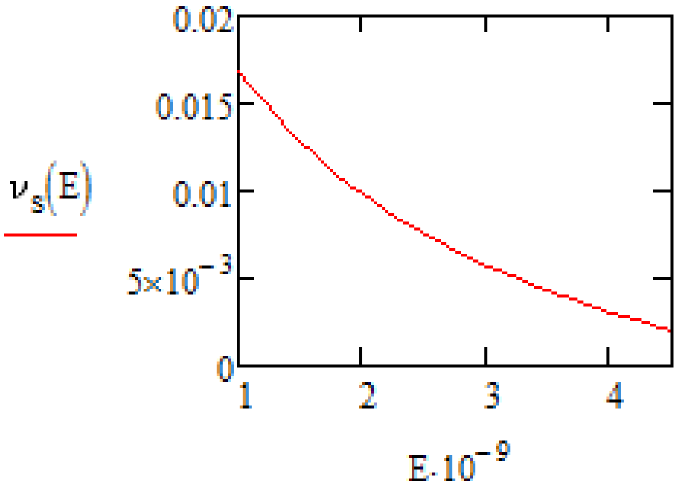
RF frequency, MHz



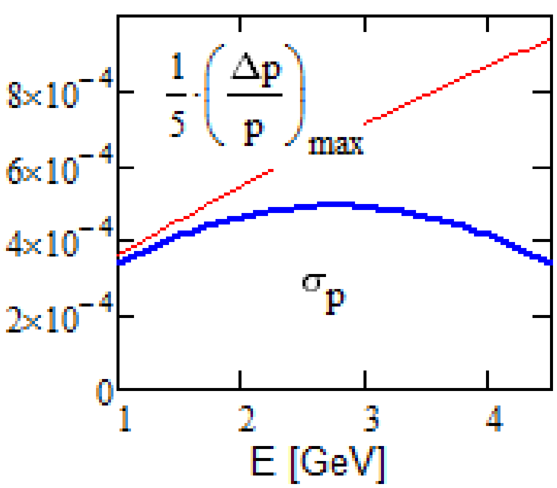
RF voltage, MV



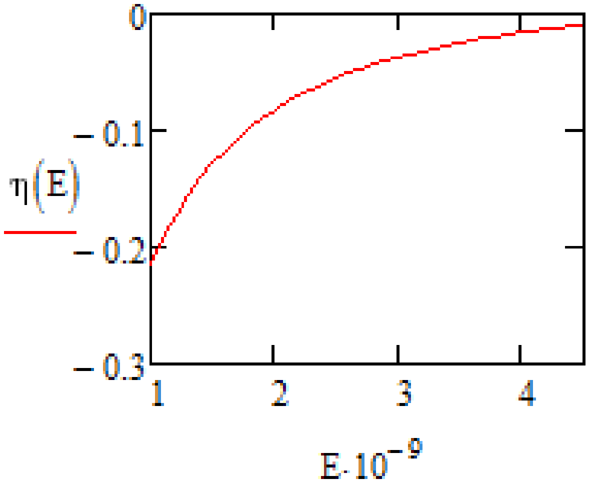
Synchrotron tune



Rms momentum spread



Slip-factor

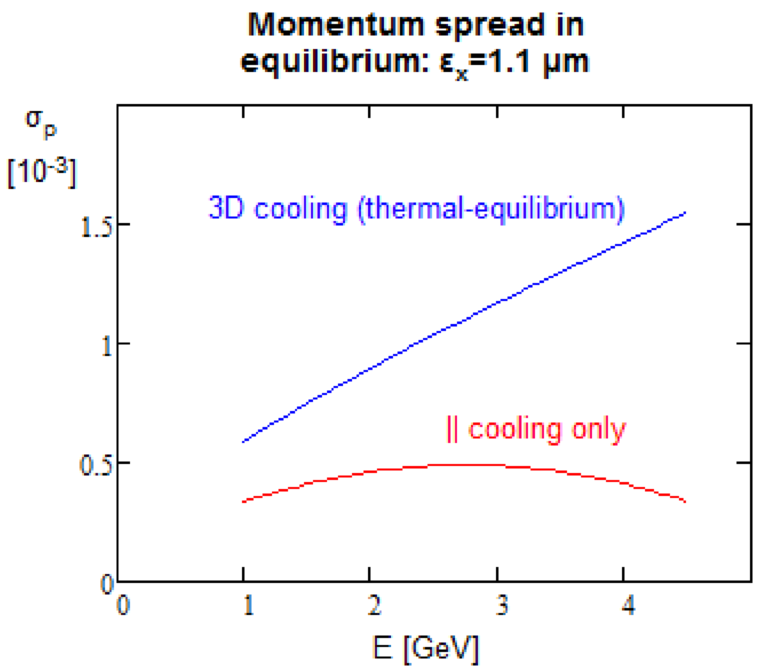


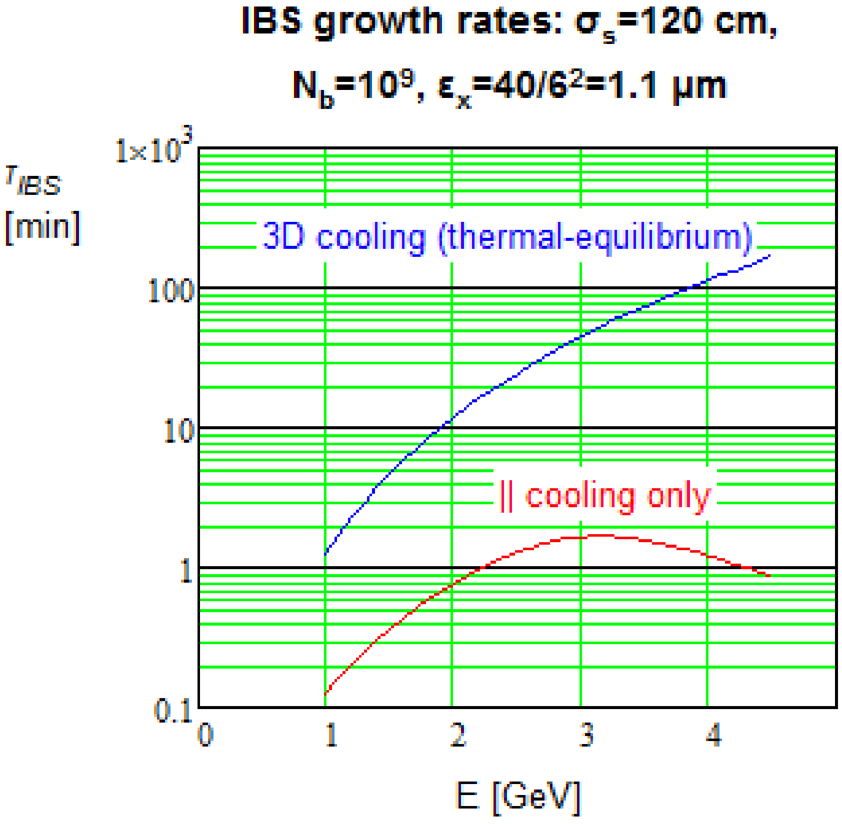
σ_s = 60 cm, $\varepsilon_x = 40/6^2$ μm, α = 0.2, *h* = 88, 22 bunches, $\Delta\nu_{max} = \Delta\nu_{SC} + \Delta\nu_{BB} =$ 0.05, Thermal equilibrium sets momentum spread and vertical emittance assuming that cooling rates in all planes are equal

IBS

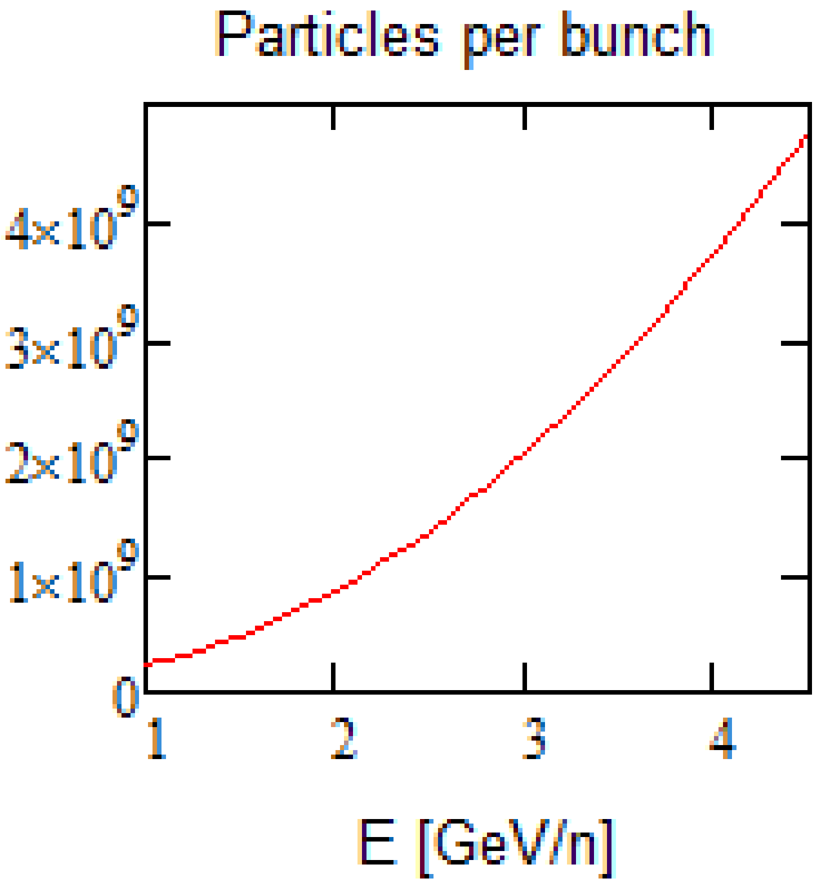
- Minimum IBS heating rates are achieved in a quasi-equilibrium – equal “temperatures” for all planes
- To keep the same horizontal emittance in sympathetic cooling the required momentum spread is significantly smaller

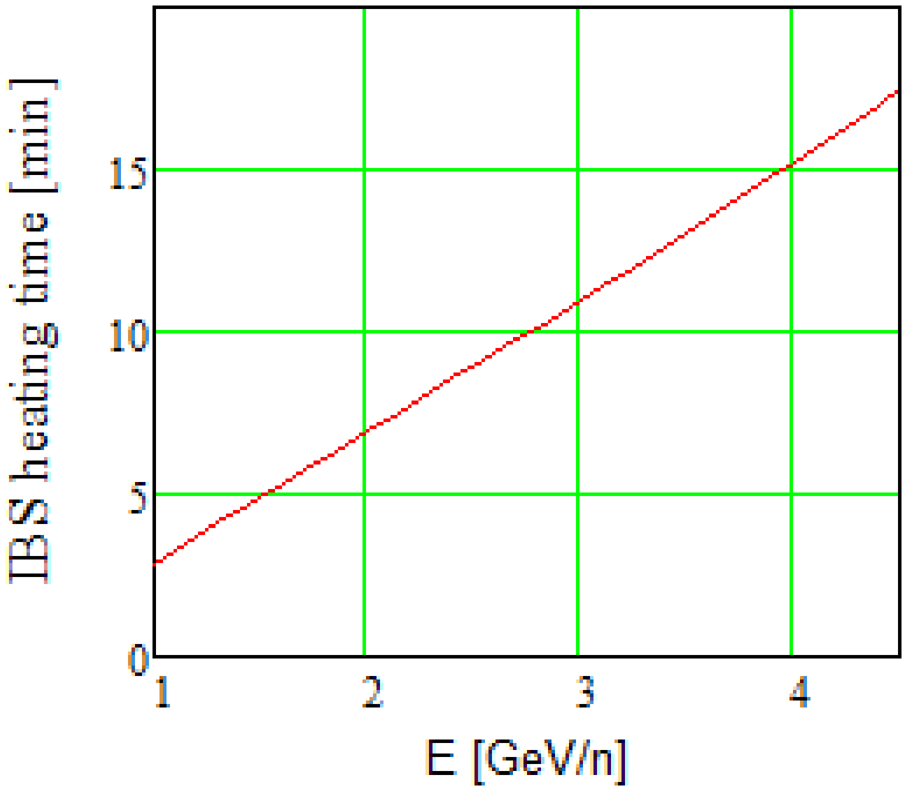
- Cooling in only longitudinal plane (sympathetic cooling in ⊥ planes) leads to the system being far from equilibrium
  - Very large increase of growth rates
- In the below estimates we assume that for 3D cooling the stochastic cooling rates in all planes are equal


Momentum spread in equilibrium: ε_x=1.1 μm


IBS growth rates: σ_s=120 cm, $N_b=10^9$, $\varepsilon_x=40/6^2$=1.1 μm

IBS (2)

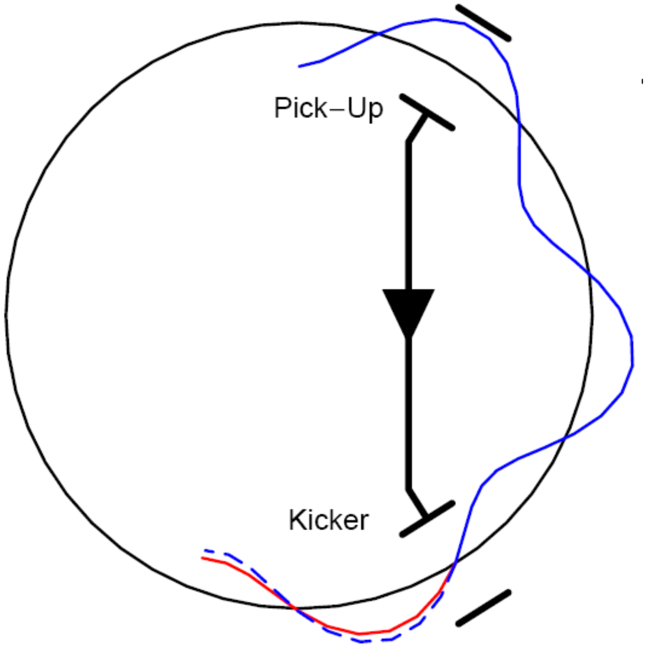

Particles per bunch
4×10^9
3×10^9
2×10^9
1×10^9
0
1
2
3
4
E [GeV/n]


IBS heating time [min]
15
10
5
0
1
2
3
4
E [GeV/n]

Stochastic Cooling Principles

- Suggested in 1969 by Simon van der Meer
- Used: CERN, FNAL & many other labs through the world

Transverse Microwave Stochastic Cooling



- Naïve model for transverse cooling
  - 90 deg. between pickup and kicker

$$\delta\theta = -g\theta$$

  - Averaging over betatron oscillations yields

$$\delta\overline{\theta^2} = -\frac{1}{2}2g\overline{\theta^2} \equiv -g\overline{\theta^2}$$

  - Adding noise of other particles yields

$$\delta\overline{\theta^2} = -g\overline{\theta^2} + N_{sample}g^2\overline{\theta^2} \equiv -\left(g - N_{sample}g^2\right)\overline{\theta^2}$$

That yields

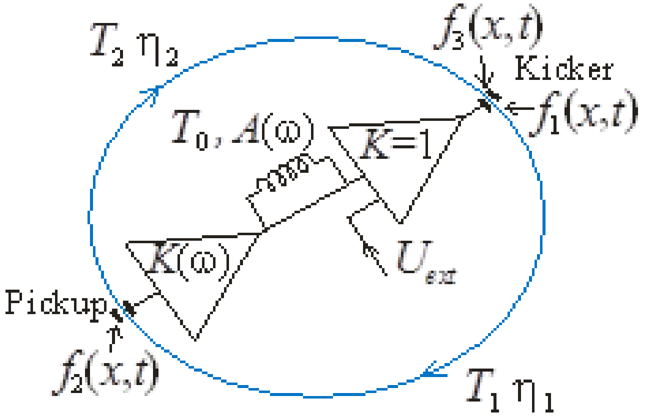
$$\delta\overline{\theta^2} = -\frac{1}{2}g_{opt}\overline{\theta^2} \quad , \quad g_{opt} = \frac{1}{2N_{sample}} \quad , \quad N_{sample} \approx N\frac{f_0}{W}$$

⇨ Cooling rate:

$$\boxed{\lambda_{opt} = \frac{1}{2}g_{opt}f_0 = \frac{W}{4N}}$$

- Accurate description is based on the Fokker-Plank equation(s)

SC Theory in One Slide



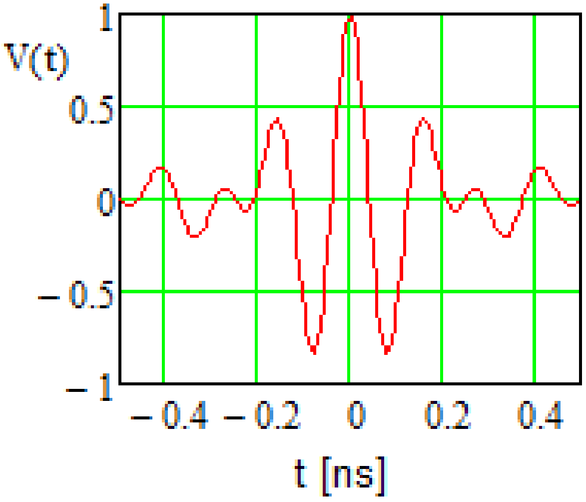
- Slip-factor and partial slip-factors

$$\begin{cases} T_1(x) = T_1 + T_0\eta_1 \Delta p / p \quad , \\ T_2(x) = T_2 + T_0\eta_2 \Delta p / p \quad , \\ T(x) = T_0\left(1 + \eta\Delta p / p\right) \quad , \end{cases} \Rightarrow \begin{cases} \eta_1 + \eta_2 = \eta \\ T_1 + T_2 = T_0 \end{cases}$$

- Longitudinal Cooling
  - Filter cooling (signal = difference Σ-signals of two consecutive turns)
  - Palmer Cooling (signal = Δ-signal at dispersive location)
- Transverse cooling
  - Kicker is 90° downstream of pickup
- Fokker-Planck equations:

$$\begin{cases} \dfrac{\partial\psi(x,t)}{\partial t} + \dfrac{\partial}{\partial x}\left(F(x)\psi(x,t)\right) = \dfrac{1}{2}\dfrac{\partial}{\partial x}\left(D(x)\dfrac{\partial\psi(x,t)}{\partial x}\right) \quad , \\ \dfrac{\partial\psi}{\partial t} + \lambda_\perp(x)\dfrac{\partial}{\partial I}\left(I\psi\right) = D_\perp(x)\dfrac{\partial}{\partial I}\left(I\dfrac{\partial\psi}{\partial I}\right), \end{cases}$$

$$x \equiv \frac{\Delta p}{p}$$



Voltage excited by a particle in 4-8 GHz system, G=const

- Cooling forces in the absence of band overlap

$$F(x) \equiv \frac{dx}{dt} = \frac{1}{T_0}\sum_{n=-\infty}^{\infty}\frac{G_1\left(x,\omega_n(x)\right)}{\varepsilon\left(\omega_n(x)\right)}\left(1 - A\left(n\omega_0(1-\eta x)\right)e^{-i\omega_n T_0}\right)e^{2\pi i n\eta_2 x} \ , \quad \omega_n = n\omega_0,$$

$$\lambda_\perp(x) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty}\mathrm{Re}\left(i\frac{G_{\perp 1}\left(\omega_{n\perp}\right)}{\varepsilon_\perp\left(\omega_{n\perp}\right)}e^{2\pi i n\eta_2 x}\right) , \quad \omega_{n\perp} = \omega_0\left(n - \nu\right) \quad .$$

Diffusion

- Longitudinal diffusion is proportional to the particle density distribution over momentum

$$D(x) = \frac{N}{T_0} \sum_{n=-\infty}^{\infty} \left|G(x,\omega_n)\right|^2 \left|\left(1 - A(\omega_n) e^{-i\omega_n T_0}\right)\right|^2 \begin{cases} \dfrac{\psi(x)}{|n\eta| \left|\varepsilon(\omega_n)\right|^2}, & \text{no band overlap} \\ 1, & \text{complete band overlap} \end{cases}$$

$$\omega_n = n\omega_0$$

- Transverse diffusion is proportional to the particle density distribution over momentum times average square amplitude (action) for given momentum

$$D_\perp(x) = \sum_{n=-\infty}^{\infty} \frac{\left|G_\perp(\omega_{n\perp}(x))\right|^2}{\left|\varepsilon_\perp(\omega_{n\perp}(x))\right|^2} \frac{\overline{I(x)} N \psi_{||}(x)}{2T_0 \left|\eta(x) n\right|}$$

$$\omega_{n\perp} = \omega_0 (n - \nu)$$

- For NICA thermal noise contribution can be neglected in all practical cases

Signal-to-noise Ratio

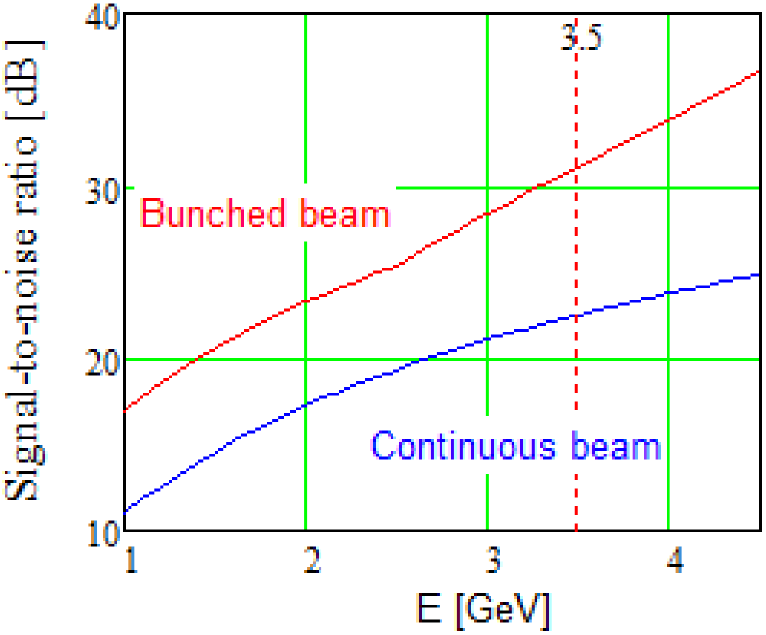
- Schottky noise for bunched beam (no band overlap)

$$S_n\Big|_{peak} = \frac{2Z^2e^2N_bqf_0}{\sqrt{2\pi}\sigma_p\,|\eta|}\frac{nZ_{pickup}{}^2}{n_\sigma{}^2}$$

- Thermal noise

$$S_U(f) = 4k_BTR$$

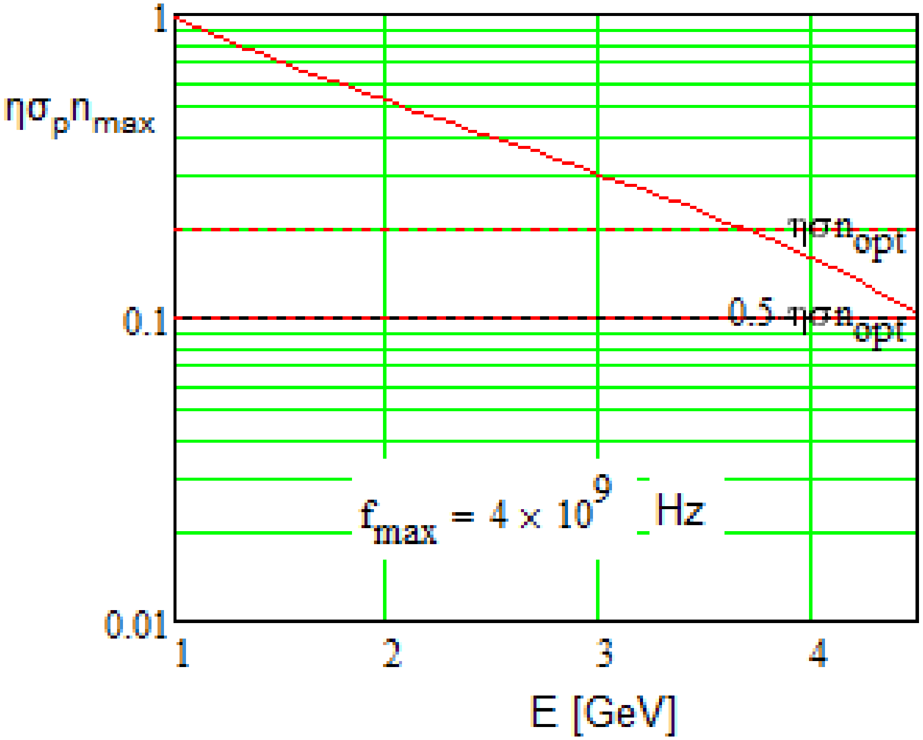
- There is very good signal-to-noise ratio due to large ion charge even if only one plate is used
  - ⇨Pickups can be very short ≤ 5 cm
- For FNAL systems
  - typical S/N ~15 dB
  - ♦ Debuncher
    256 loops * (300K/20K)=3840
  - ♦ NICA: Z^2=6241
    - Additionally NICA has much more particles ($2\cdot10^8$ -> [1 – 4]$\cdot10^9$)



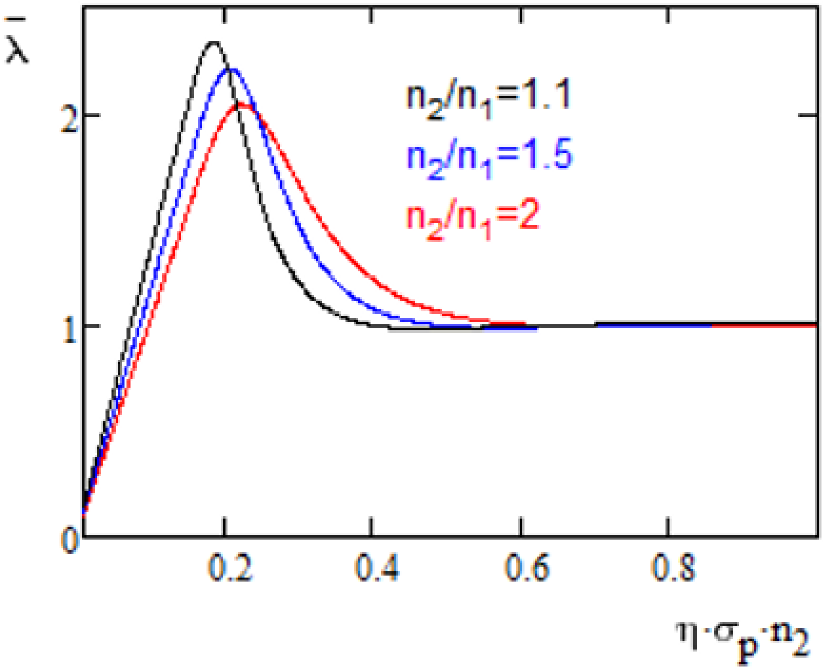
Z_{pickup}=25 Ω, T=300 K, R=50 Ω,
q=88, n_σ =7
σ_p is set by thermal equilibrium in the beam
N_b - set by space charge + beam-beam
n –harmonic number for f_{max}=4 GHz

Band Overlap

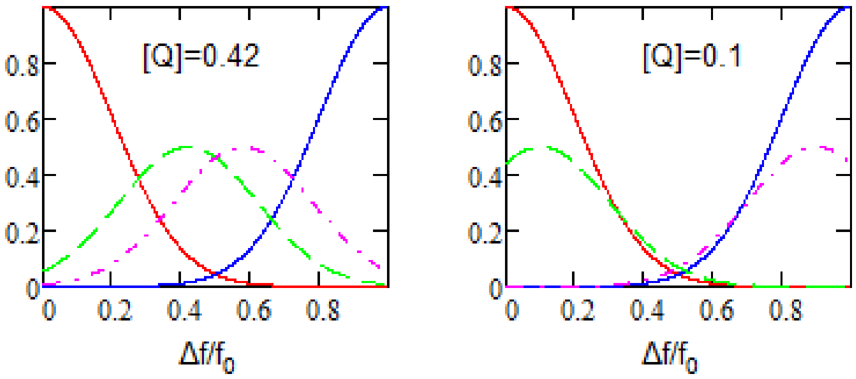
- Maximum damping is achieved at the onset of band overlap, $\eta\sigma_p n_2 = 0.2$. ($n_2 = f_{max}/f_0$)
  - That implies significant overlap with transverse bands



- No band overlap for E=4.5 GeV
- No. long. bands overlap for E>3.8 GeV

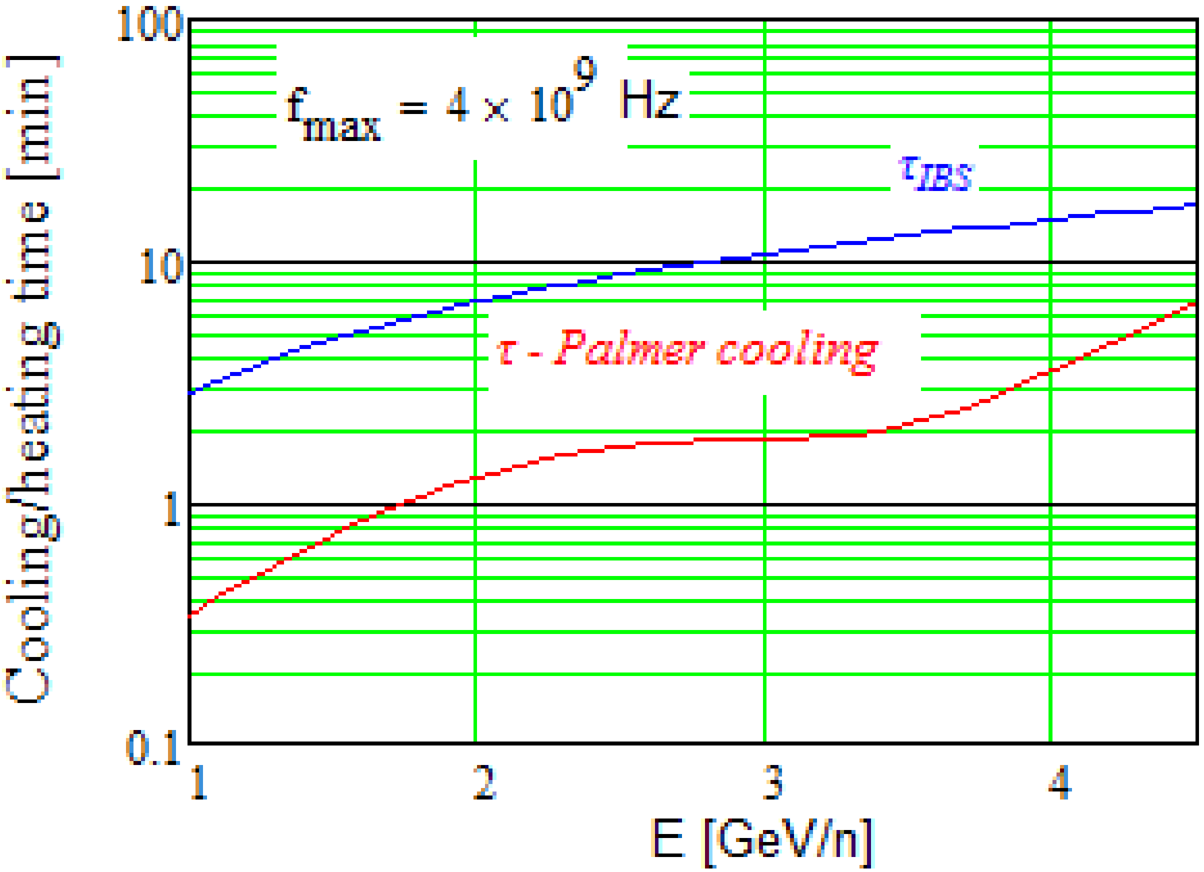


Dimensionless cooling rate for Palmer cooling at G_{opt} as function of band overlap at f_{max} for different system bandwidths.



Schottky spectra at optimal band overlap, $\eta\sigma_p n = 0.2$.

Cooling Rates versus IBS Heating Rates

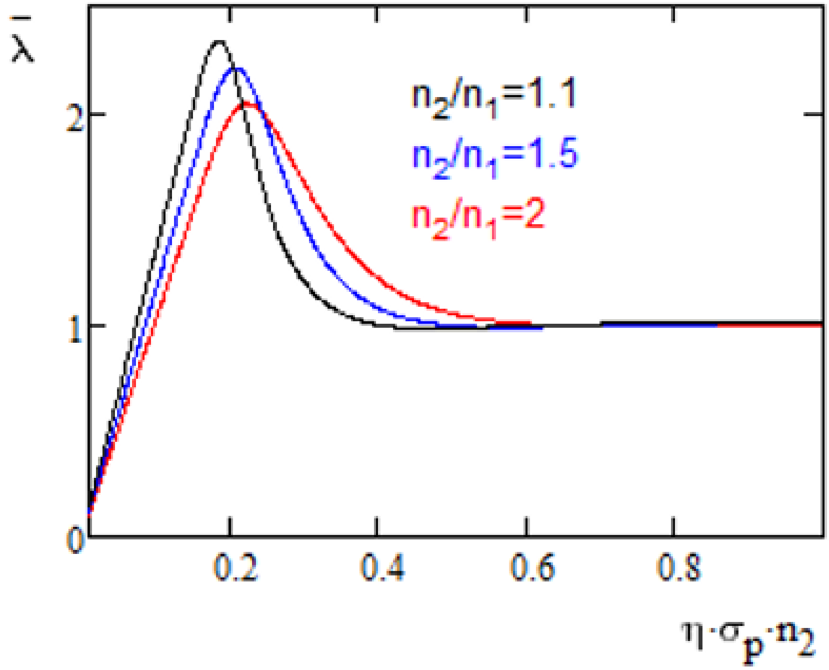


[2-4] GHz band, number of particles set by betatron tune shifts of 0.05.

- Cooling rates of Palmer cooling and transverse cooling exceed IBS heating rates in the entire energy range
  - An estimate accounts for band overlap but it assumes that ⊥ and || signals are separated (two-pickup choice)

Band Overlap Effects

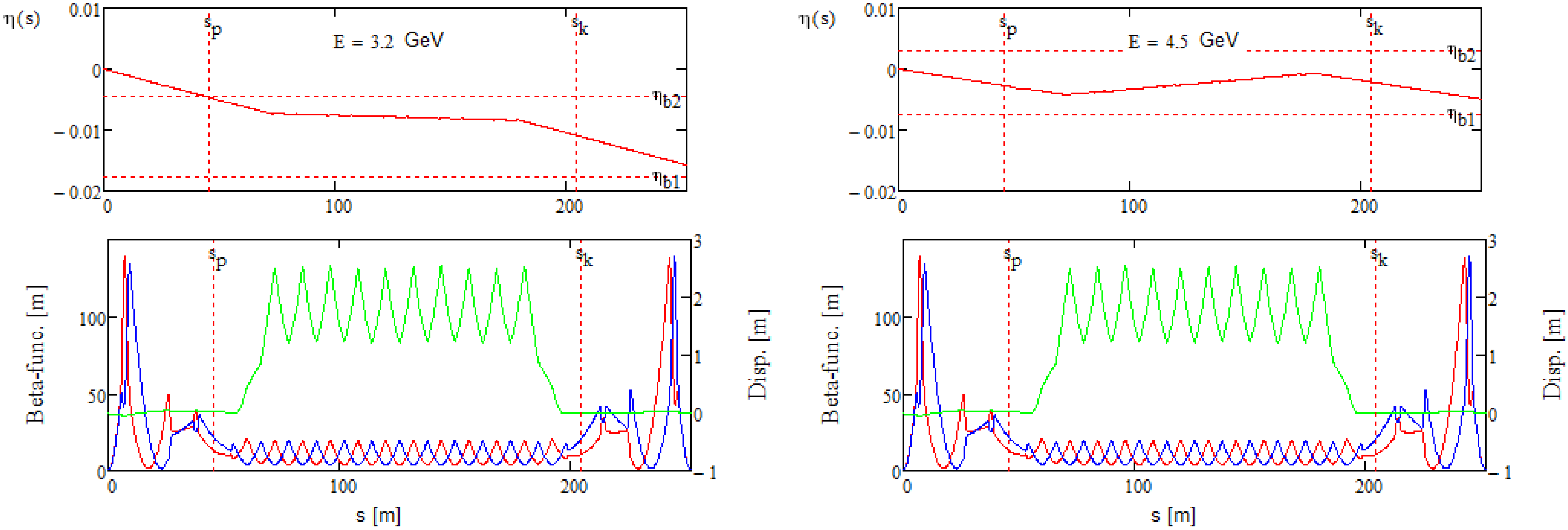
- Most operational SC systems operate without band overlap.
  - It simplifies their design and tuning but reduces their cooling rates
    - In NICA bands are overlapped for almost entire energy range
- Band overlap excludes a usage of filter cooling for high luminosity operation
  - Startup version has significantly smaller Δp/p => can use FC
- Band overlap may reduce cooling efficiency for horizontal and longitudinal cooling
  - For NICA parameters $D(\Delta p/p^2) \le \varepsilon\beta$ and, if not addressed it can seriously deteriorate || cooling
  - But it still delivers almost the same cooling rates as in the absence of band overlap
- Phasing of the system can be done for small intensity beam when the band overlap is absent



Boundaries for Partial Slip-factor

- For Palmer cooling the maximum partial slip-factor for the one-octave system is

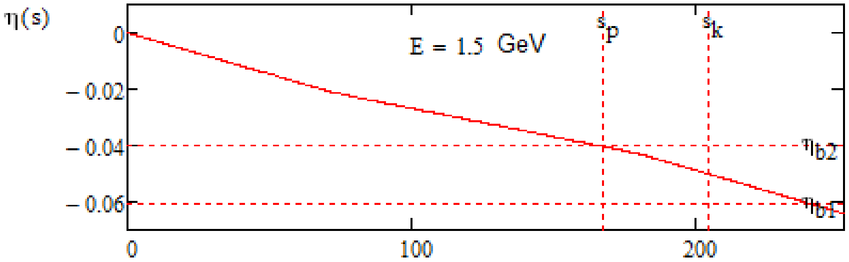
$$\eta_2\Big|_{\max} = \frac{1}{2\left(n_2 - n_1\right)\left(\Delta p / p\right)_{\max}} \xrightarrow[(\Delta p/p)_{\max}=n_\sigma\sigma_p]{n_1=n_2/2} \frac{1}{2n_2\sigma_p n_\sigma}, \quad n_\sigma = 6$$

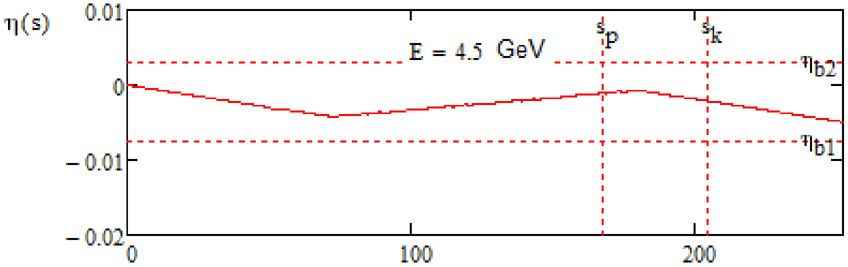


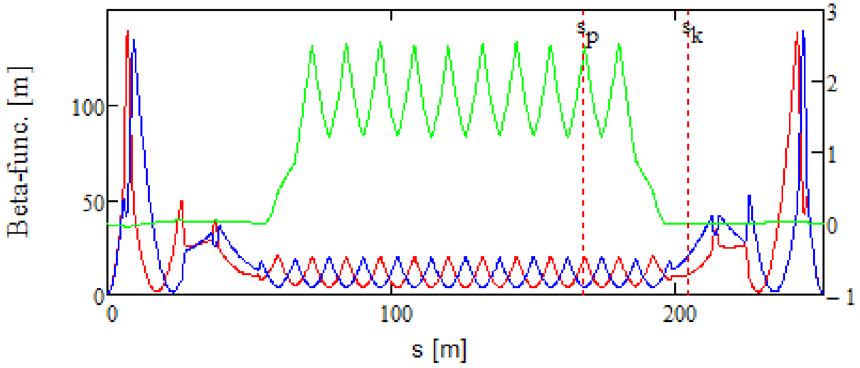
- If pickup is located in the straight line opposite to the kicker the energy range for ⊥ cooling where SC can be used is [3.2 – 4.5] GeV

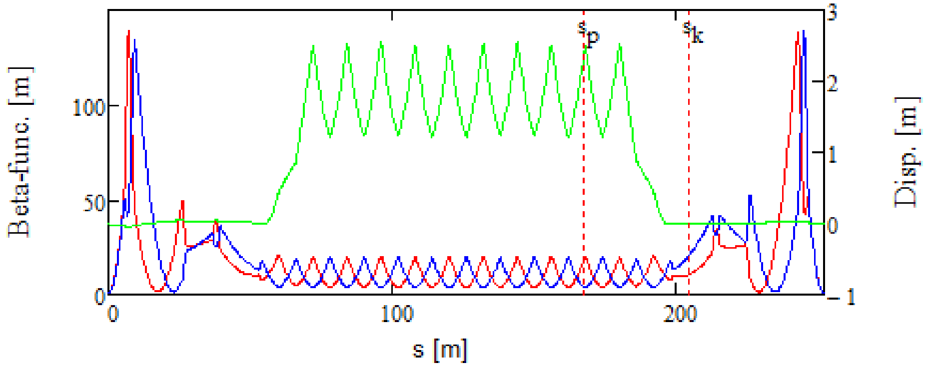
Partial Slip-factor (2)

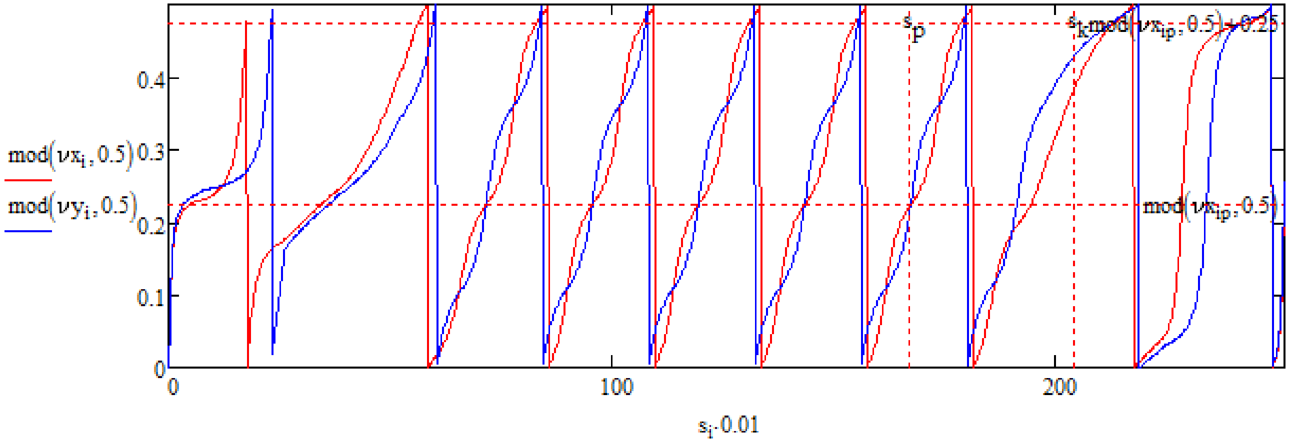
- Location of the pickup at the end of the arc preceding the kicker extends the energy range to [1.5 – 4.5] GeV
- Betatron phase advances suit well to transverse cooling







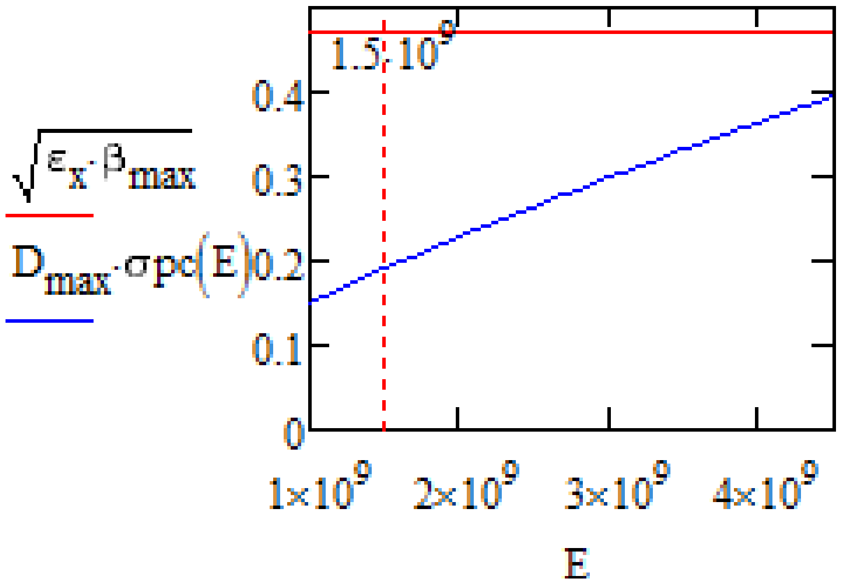


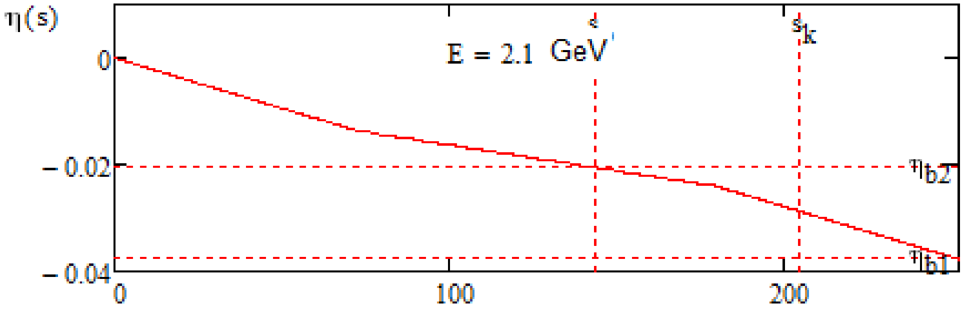


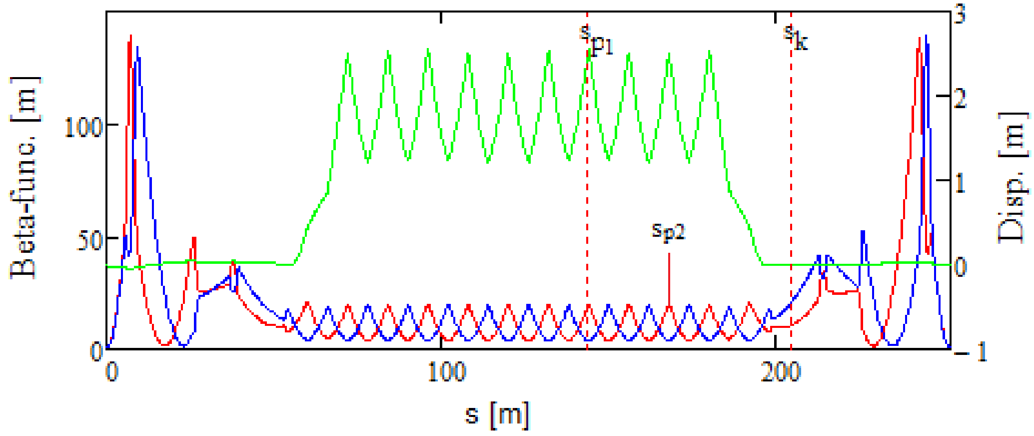
Coexistence of Palmer and Horizontal Cooling

- At low energy the synchrotron size in the pickup is significantly smaller than the betatron size.
  It compromises || cooling.
  - ♦ Too address it, one may use two pickups shifted by 180 deg. in betatron phase advance
    - Σ & Δ signals deliver signals for || & ⊥ cooling
    - That also improves S/N ratio, but increases the minimum energy where SC can be used.

    ⇨ The energy range is [2.1 – 4.5] GeV
- If required, minimum energy can be decreased by transition to lower frequencies [1-2] GHz







Conclusions

- Filter cooling cannot be used for the baseline NICA because of band overlap (too large $\Delta p/p$ & $\Delta\omega/\omega$ at f_{max})
- The energy range where SC can operate is 2.1 – 3.5 GeV for [2 -4] GHz system
  - It delivers sufficiently large cooling rates and does not need to be increased
- Longitudinal (Palmer) and horizontal cooling should originate from two pickups separated by 180 deg. phase advance and located at the end of arc preceding kickers
- Vertical cooling requires 1 pickup. If required two pickups can be used to adjust pickup-kicker betatron phase advance
- Pickups should include only one section longitudinally
  - It delivers sufficiently large S/N ratio and
  - enables an achievement of better quality (symmetry) of the pickups.
- The system has to be built to operate with pulsed and continuous beams with strong band overlap.
  - Betatron tunes near half integer are desirable but not necessary.