Constraints on the Intrinsic Charm Content of the Proton from Recent ATLAS Data

V.A. Bednyakov,[1] S.J. Brodsky,[2] A.V. Lipatov,[1,3] G.I. Lykasov,[1,*] M.A. Malyshev,[3] J. Smiesko,[4] and S. Tokar[4]

[1] *Joint Institute for Nuclear Research, Dubna 141980, Moscow region, Russia*
[2] *SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94025, United States*
[3] *Skobeltsyn Institute of Nuclear Physics, Moscow State University, 119991 Moscow, Russia*
[4] *Comenius University in Bratislava, Faculty of Mathematics, Physics and Informatics, Mlynska Dolina, 842 48 Bratislava, Slovakia*

Constraints on the intrinsic charm probability $w_{c\bar{c}} = P_{c\bar{c}/p}$ in the proton are obtained for the first time from LHC measurements. The ATLAS Collaboration data for the production of prompt photons, accompanied by a charm-quark jet in pp collisions at $\sqrt{s} = 8$ TeV, are used. The upper limit $w_{c\bar{c}} < 1.93$ % is obtained at the 68 % confidence level. This constraint is primarily determined from the theoretical scale and systematical experimental uncertainties. Suggestions for reducing these uncertainties are discussed. The implications of intrinsic heavy quarks in the proton for future studies at the LHC are also discussed.

PACS numbers: 12.15.Ji, 12.38.Bx, 13.85.Qk
Keywords: Quarks, gluons, charm, QCD, PDF

One of the fundamental predictions of quantum chromodynamics is the existence of Fock states containing heavy quarks at large light-front (LF) momentum fraction x in the LF wavefunctions of hadrons [1, 2]. A key example is the $|uudc\bar{c}\rangle$ intrinsic charm Fock state of the proton's QCD eigenstate generated by $c\bar{c}$-pairs which are multiply connected to the valence quarks. The resulting intrinsic charm (IC) distribution $c(x, Q^2)$ is maximal at minimal off-shellness; i.e., when all of the quarks in the $|uudc\bar{c}\rangle$ LF Fock state have equal rapidity. Equal rapidity implies that the constituents in the five-quark light-front Fock state have momentum fractions $x_i = \frac{k_i^+}{P^+} \propto \sqrt{m_i^2 + k_{\mathrm{T}i}^2}$, so that the heavy quarks carry the largest momenta.

The study of the intrinsic heavy quark structure of hadrons provides insight into fundamental aspects of QCD, especially its nonperturbative aspects. The operator product expansion (OPE) predicts that the probability for intrinsic heavy Q-quarks in a light hadron scales as $\kappa^2/M_{\mathrm{Q}}^2$ due to the twist-6 $G_{\mu\nu}^3$ non-Abelian couplings of QCD [2, 3]. Here κ is the characteristic mass scale of QCD. In the case of the BHPS model [1] within the MIT bag approach [4], the probability to find a five-quark component $|uudc\bar{c}\rangle$ bound in the nucleon eigenstate is estimated to be in the range 1–2%. Although there are many phenomenological signals for heavy quarks at high x, the precise value for the intrinsic charm probability $w_{c\bar{c}} = P_{c\bar{c}/p}$ in the proton has not as yet been definitively determined.

The first evidence for intrinsic charm (IC) in the proton originated from the EMC measurements of the charm structure function $c(x, Q^2)$ in deep inelastic muon-proton scattering [5]. The charm distribution measured by the EMC experiment at $x_{\mathrm{bj}} = 0.42$ and $Q^2 = 75$ GeV was found to be approximately 30 times that expected from the conventional gluon splitting mechanism $g \to c\bar{c}$. However, as discussed in ref. [6], this signal for (IC) is not conclusive because of the large statistical and systematic uncertainties of the measurement.

A series of experiments at the Intersection Storage Ring (ISR) at CERN, as well as the fixed-target SELEX experiment at Fermilab, measured the production of heavy baryons Λ_c and Λ_b at high x_F in pp, $\pi^- p$, $\Sigma^- p$ collisions [7–9]. For example, the Λ_c will be produced at high x_F from the excitation of the $|uudc\bar{c}\rangle$ Fock state of the proton and the comoving u, d, and c quarks coalesce. The x_F of the produced forward heavy hadron in the $pp \to \Lambda_c$ reaction is equal to the sum $x_F = x_u + x_d + x_c$ of the light-front momentum fractions of the three quarks in the five-quark Fock state. However, the normalization of the production cross section has sizable uncertainties, and thus one cannot obtain precise quantitative information on the intrinsic charm contribution to the proton charm parton distribution function (PDF) from the ISR and SELEX measurements. Another important way to identify IC utilizes the single and double J/Ψ hadroproduction at high x_F as measured by the NA3 fixed target experiment [10–12].

The first indication for intrinsic charm at a high energy collider was observed in the $\bar{p}p \to c\gamma X$ reaction at the Tevatron [13–17]. An explanation for the large rate observed for events at high E_T based on the intrinsic charm contribution is given in refs. [18, 19]. A comprehensive review of the experimental results and global analysis of PDFs with intrinsic charm was presented in [20, 21].

Intrinsic heavy quarks also leads to the production of the Higgs boson at high x_F the LHC energies [22]. It also implies the production of high energy neutrinos from the interactions high energy cosmic rays in the atmosphere, a reaction which can be measured by the IceCube detector [23].

In our previous publications [24–28], we showed that the IC signal can be visible in the production of prompt photons and vector bosons Z/W in pp collisions, accompanied by heavy-flavor c/b-jets at large transverse momenta and the forward rapidity region ($|y| > 1.5$), kinematics within the acceptance of the ATLAS and CMS experiments at the LHC.

The main goal of this paper is to test the intrinsic charm hypothesis utilizing recent ATLAS data on prompt photon production accompanied by a c-jet in pp collisions at $\sqrt{s} = 8$ TeV. We perform this analysis using the analytical QCD calculation and the MC generator Sherpa [29].

We will first present the scheme of our QCD analysis for such processes. The systematic uncertainties due to hadronic structure are evaluated using a combined QCD approach, based on the $k_{\rm T}$-factorization formalism [30, 31] in the small-x domain and the assumption of conventional (collinear) QCD factorization at large x. Within this approach, we have employed the $k_{\rm T}$-factorization formalism to calculate the leading contributions from the $\mathcal{O}(\alpha\alpha_s^2)$ off-shell gluon-gluon fusion $g^*g^* \to \gamma c\bar{c}$. In this way one takes into account the conventional perturbative charm contribution to associated γc production. In addition there are backgrounds from jet fragmentation.

The IC contribution is computed using the $\mathcal{O}(\alpha\alpha_s)$ QCD Compton scattering $cg^* \to \gamma c$ amplitude, where the gluons are kept off-shell and incoming quarks are treated as on-shell partons. This is justified by the fact that the IC contribution begins to be visible at the domain of large $x \geq 0.1$, where its transverse momentum can be safely neglected. The $k_{\rm T}$-factorization approach has technical advantages, since one can include higher-order radiative corrections by adopting a form for the transverse momentum dependent (TMD) parton distribution of the proton (see reviews [32] for more information). In addition, we take into account several standard pQCD subprocesses involving quarks in the initial state. These are the flavor excitation $cq \to \gamma cq$, quark-antiquark annihilation $q\bar{q} \to \gamma c\bar{c}$ and quark-gluon scattering subprocess $qg \to \gamma qc\bar{c}$. These processes become important at large transverse momenta p_T (or, respectively, at large parton longitudinal momentum fraction x, which is the kinematics needed to produce high p_T events); it is the domain where the quarks are less suppressed or can even dominate over the gluon density. We rely on the conventional (DGLAP) factorization scheme, which should be reliable in the large-x region. Thus, we apply a combination of two techniques (referred as a "combined QCD approach") employing each of them in the kinematic regime where it is most suitable. More details can be found in [33] (see also references therein).

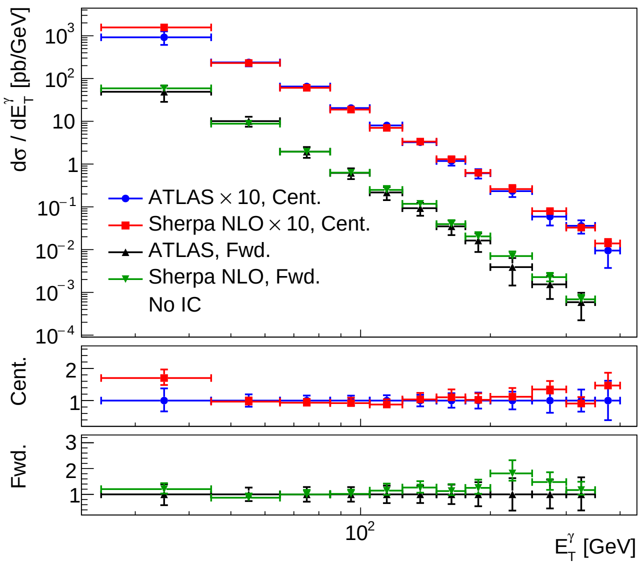
According to the BHPS model [1, 34], the total charm distribution in a proton is the sum of the *extrinsic* and *intrinsic* charm contributions.

$$xc(x,\mu_0^2) = xc_{\rm ext}(x,\mu_0^2) + xc_{\rm int}(x,\mu_0^2). \tag{1}$$

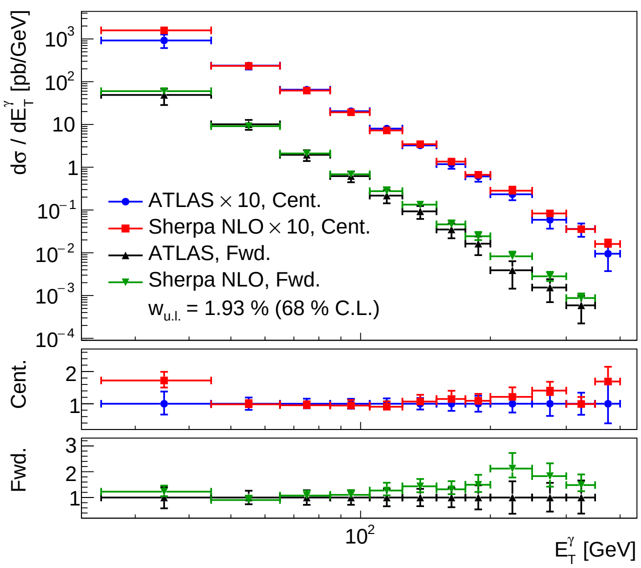
The *extrinsic* quarks and gluons are generated by perturbative QCD on a short-time scale associated within the large-transverse-momentum processes. Their distribution functions satisfy the standard QCD evolution equations. In contrast, the *intrinsic* quarks and gluons are associated with a bound-state hadron dynamics and thus have a non-perturbative origin. In Eq. 1 the IC weight is included in $xc_{\rm int}(x,\mu_0^2)$ and the total distribution $xc(x,\mu_0^2)$ satisfies the QCD sum rule, which determines its normalization [35] and see Eq. 6 in [27]. We define the IC probability as the $n = 0$ moment of the charm PDF at the scale $\mu_0 = m_{\rm c}$, where $m_{\rm c} = 1.29$ GeV is the c-quark mass.

As shown in [26, 27], the interference between the two contributions to Eq. 1 can be neglected, since the IC term $xc_{\rm int}(x,\mu^2)$ is much smaller than the extrinsic contribution generated at $x < 0.1$ by DGLAP evolution [36–38] from gluon splitting. Therefore, since the IC probability $w_{c\bar{c}}$ enters into Eq. 1 as a constant in front of the function dependent on x and μ^2, one can adopt a simple linear relation for any $w_{\bar{c}c} \leq w_{\bar{c}c}^{\rm max}$ [26, 27], which provides an interpolation between two charm densities at the scale μ^2, obtained at $w_{\bar{c}c} = w_{\bar{c}c}^{\rm max}$ and $w_{\bar{c}c} = 0$. We have performed a three-point interpolation of the all parton (quark and gluon) distributions for $w_{\bar{c}c} = 0$, 1 and 3.5 %, which correspond to the CTEQ66M, CTEQ66c0 and CTEQ66c1 sets, respectively [39]. For the interpolation function we used the linear and the quadratic $w_{c\bar{c}}$ interpolation. The difference between linear and quadratic interpolation functions in the interval $0 < w_{\bar{c}c} \leq 3.5$ % is no greater than 0.5 %, thus giving confidence in our starting point [26]. Given that, we used the quadratic interpolation for the all parton flavors at μ_0 and $w_{\bar{c}c} < w_{\bar{c}c}^{\rm max}$ to satisfy the quark and gluon sum rules, see [40, 41]. Let us stress that at $w_{\bar{c}c} = w_{\bar{c}c}^{\rm max}$ the quark sum rule is satisfied automatically in the used PDF because the intrinsic light $q\bar{q}$ contributions are included [39]. Note that $w_{c\bar{c}}$ is treated in $xc_{\rm int}(x,\mu_0^2)$ of Eq. 1 as a parameter which does not depend on μ^2. Therefore, its value can be determined from the fit to the data.

For the second approach, we use the MC generator Sherpa [29, 43, 44] with next-to-leading order (NLO) matrix elements (version 2.2.4) to generate samples for the extraction of the $w_{c\bar{c}}$ from the ATLAS data. The recent versions of Sherpa generator can provide additional weights, which are used to reweight our spectra to PDFs with different IC contribution [44]. In order to obtain weights corresponding to any w we quadratically interpolate three PDF weights: W_0 (0 % IC), W_1 corresponding to the BHPS1 (the mean value of the $c\bar{c}$ fraction is $\langle x_{c\bar{c}}\rangle \simeq 0.6$ %, which corresponds to the IC probability $w_{\bar{c}c} = 1.14$ %) and W_2 corresponding to the BHPS2 ($\langle x_{c\bar{c}}\rangle \simeq 2.1$ %, $w_{\bar{c}c} = 3.54$ %). We used CT14nnloIC [45] PDF included with the help of LHAPDF6 [46]. The process $p + p \to \gamma +$ any jet (up to 3 additional jets) is simulated with the requirement $E_{\rm T}^{\gamma} > 20$ GeV and $\eta^{\gamma} < 2.7$. Additional cuts are applied to match the AT-

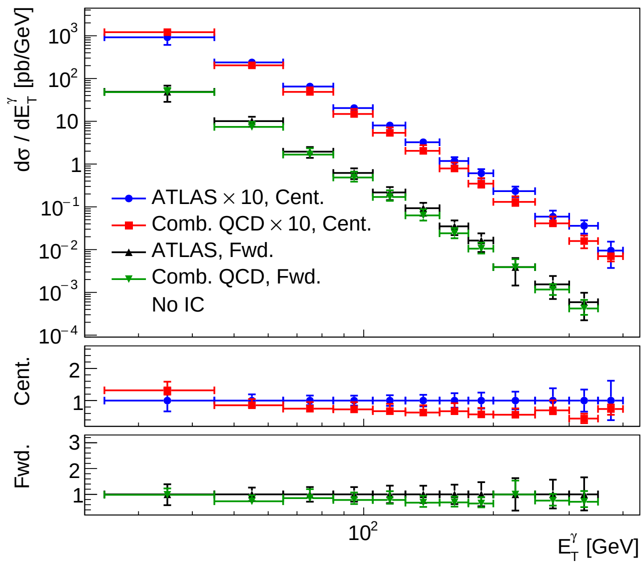


(a) $w_{\rm c} = 0$ %

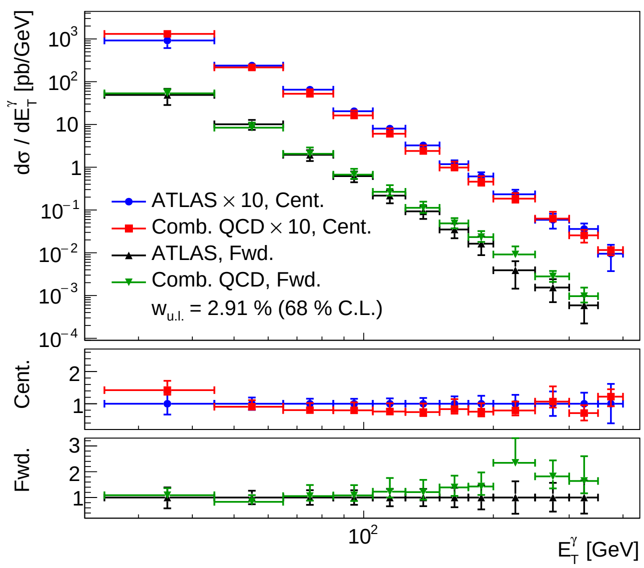


(b) $w_{\rm u.\,l.} = 1.93$ %

FIG. 1: The $E_{\rm T}^{\gamma}$-spectrum calculated with MC generator SHERPA, NLO compared with the ATLAS data [42].
(a) top: the spectrum at the central rapidity region $|\eta^{\gamma}| < 1.37$ and forward $1.56 \leq |\eta^{\gamma}| < 2.37$ region without the IC contribution;
(a) middle: the ratio of the MC calculation to the data for the central rapidity region ($w_{\rm c} = 0$ %);
(a) bottom: the ratio of the MC calculation to the data for the for forward rapidity regions ($w_{\rm c} = 0$ %).
(b): the same spectra, as in (a), but with the upper limit of IC contribution $w_{\rm u.\,l.} = 1.93$ %.



(a) $w_{\rm c} = 0$ %



(b) $w_{\rm u.\,l.} = 2.91$ %

FIG. 2: The spectrum of prompt photons as a function of its transverse energy $E_{\rm T}^{\gamma}$ calculated with the combined QCD analysis, compared with ATLAS data [42].
(a) top: the spectrum in the central rapidity region $|\eta^{\gamma}| < 1.37$ and forward $1.56 \leq |\eta^{\gamma}| < 2.37$ region without the IC contribution;
(a) middle: the ratio of the MC calculation to the data for the central rapidity region ($w_{\rm c} = 0\%$);
(a) bottom: the ratio of the MC calculation to the data for the forward rapidity regions ($w_c = 0\%$).
(b): the same spectra, as in (a), but with upper limit of IC contribution $w_{\rm u.\,l.} = 2.91$ % corresponding to the best fit of the data.

LAS event selection [42]. In order to extract the w-value from the data we first calculate the $E_{\rm T}^{\gamma}$-spectrum using the SHERPA MC generator in the central rapidity region

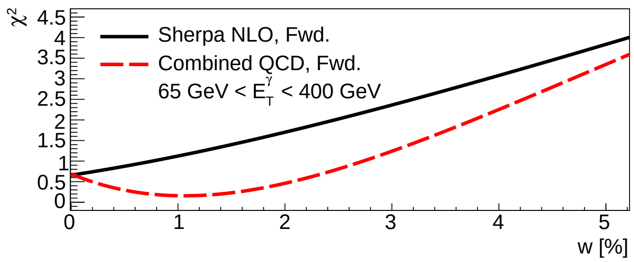


FIG. 3: Solid line: χ^2 as a function of w in the forward rapidity region in SHERPA NLO. Dashed line: Same but χ^2 obtained within the combined QCD calculation.

($|\eta^\gamma| < 1.37$) and compare it with the $\gamma + c$-jet ATLAS spectra. Next, we show that one can obtain a satisfactory description of the ATLAS data in the central rapidity region using the SHERPA (NLO) calculation without IC.

These results using the PDF CT14nnloIC without the IC contribution are presented in Fig. 1 (top). One can see that the difference between these experimental E_T^γ data and the MC calculation is less than the total uncertainties. Therefore, we are unable to determine a precise value of the IC probability from recent ATLAS data. However, one can extract an upper limit of the IC contribution from the data. Therefore, the SHERPA NLO calculation at the upper limit of the IC contribution $w_{\mathrm{u.l.}} = 1.93$ % is presented in Fig. 1 (bottom). This value of $w_{\mathrm{u.l.}}$ corresponds to the χ^2 at minimum plus one, see the solid line in Fig. 3.

The $w_{c\bar{c}}$ extraction method was repeated using the above mentioned combined QCD scheme instead of SHERPA (NLO). The CTEQ66c PDF, which includes the IC fraction in the proton, was used to calculate the E_T^γ-spectrum in the forward rapidity region E_T^γ spectra and χ^2 as a function of w obtained within these approach are presented in Fig. 2 and Fig. 3 (dashed red line) respectively. The upper limit of the IC contribution obtained within the cobmbined QCD is about $w_{\mathrm{u.l.}} = 2.91$ %. As was shown in [28], the combined QCD does not include parton showers and hadronization, a contribution which is sizable at E_T^γ>100 GeV, where the IC signal could be visible. Therefore, the results obtained SHERPA (NLO), which include these effects, are more realistic.

The w-dependence of χ^2-functions obtained with both SHERPA and the combined QCD approach in the forward region are presented in Fig. 3. By definition, the minimum of the χ^2-function is reached at a central value w_c which corresponds to the best description of the ATLAS data. The application of SHERPA results in $w_c = 0.00$ %, and the combined QCD gives us $w_c = 1.00$ %.

Figure 3 shows a rather weak χ^2-sensitivity to the w-value. It is due to the large experimental and theoretical QCD scale uncertainties especially at $E_T^\gamma > 100$ GeV (Figs. 1 and 2). Therefore, it is not possible to extract the w_c-value with a requested accuracy (3–5 σ), instead, we present relevant upper limit at the 68 % confidence level (C.L.).

As a first estimate of the scale uncertainty we have used the conventional procedure, used in a literature, varying the values of the QCD renormalization scale μ_R and the factorization one μ_F in the interval from $0.5E_T^\gamma$ to $2E_T^\gamma$. In fact, there are several methods to check the sensitivity of observables to the scale uncertainty, see, for example, [47] and references there in. The renormalization scale uncertainty of the E_T^γ-spectra poses a serious theoretical problem for obtaining more precise estimate of the IC probability from the LHC data.

The precision is limited by the experimental systematic uncertainties — mainly, by the c-tagging uncertainty which is predominantly connected with the light jet scaling factors [42]. It is also limited by theoretical QCD scale uncertainties. The PDF uncertainties are included in the predictions using the SHERPA (NLO). In contrast to these uncertainties, the statistical uncertainty does not play a large role. All this can be seen in Fig. 4, where the dependence of the allowed IC upper limit $w_{\mathrm{u.l.}}$ on different components of uncertainty is shown. The allowed upper limit is presented four times, every time the component of uncertainty in question is reduced from its actual value (100 %). This asssumes that the central values of the experiment does not change. In order to obtain more reliable information on the IC probability in the proton from future LHC data at $\sqrt{s} = 13$ TeV it is needed to have a more realistic estimate of the theoretical scale uncertainties and reduce the systematic uncertainties.

This problem can be, in fact, eliminated by employing the “principle of maximum conformality” (PMC) [48] which sets renormalization scales by shifting the β terms in the pQCD series into the running coupling. The PMC predictions are independent of the choice of renormalization scheme — a key requirement of the renormalization group. Its utilization will be the next step of our study.

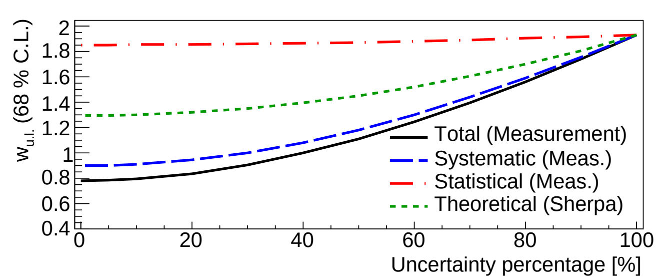


FIG. 4: The dependence of the IC upper limit $w_{\mathrm{u.l.}}$ at 68% C.L. on the uncertainty percentage of the particular uncertainty component.

In summary:
A first estimate of the intrinsic charm probability in the

proton has been carried out utilizing recent ATLAS data on the prompt photon production accompanied by the c-jet at $\sqrt{s} = 8$ TeV [42]. We estimate the upper limit of the IC probability in proton about 1.93%. In order to obtain more precise results on the intrinsic charm contribution one needs additional data and at the same time reduced systematic uncertainties which come primarily from c-jet tagging. In particular, measurements of cross sections of $\gamma + c$ and $\gamma + b$ production in pp-collisions at $\sqrt{s} = 13$ TeV at high transverse momentum with high statistics [26] will be very useful since the ratio of photon + charm to photon + bottom cross sections is very sensitive to the IC signal [26, 27]. The ratio, when E_T^γ grows, decreases in the absence of the IC contribution and stays flat or increases when the IC contribution is included. Furthermore, measurements of $Z/W + c/b$ production in pp collision at 13 TeV could also give additional significant information on the intrinsic charm contribution [25–28]. Our study shows that the most important source of theoretical uncertainty on $w_{c\bar{c}}$, from the theory point of view, is the dependence on the renormalization and factorization scales. This can be reduced by the application of the Principle of Conformality (PMC), which produces scheme-independent results, as well the calculation of the NNLO pQCD contributions. Data at different energies at the LHC which checks scaling predictions and future improvements in the accuracy of flavor tagging will be important. These advances, together with a larger data sample (more than 100 fb^{-1}) at 13 TeV, should provide definitive information from the LHC on the contribution of the non-perturbative intrinsic heavy quark contributions to the fundamental structure of the proton.

Acknowledgements

We thank A.A. Glasov, R. Keys, E.V. Khramov, S. Prince and S.M. Turchikhin for very helpful discussions. The authors are also grateful to S.P. Baranov, P.-H. Beauchemin, I.R. Boyko, Z. Hubacek, H. Jung, F. Hautmann, B. Nachman, P.M. Nadolsky, H. Tores and L. Rotali, N.A. Rusakovich for useful discussions and comments. The SHERPA calculations by J.S. were done at SIVVP, ITMS 26230120002 supported by the Research & Development Operational Programme funded by the ERDF. J.S. is thankful for the opportunity of using this cluster. The work of A.V.L and M.A.M was supported in part by the grant of the President of Russian Federation NS-7989.2016.2. A.V.L. and M.A.M. are also grateful to DESY Directorate for the support in the framework of Moscow — DESY project on Monte-Carlo implementation for HERA — LHC. M.A.M. was supported by a grant of the Foundation for the Advancement of Theoretical physics “Basis” 17–14–455–1 and RFBR grant 16–32–00176-mol-a. SJB is supported by the U.S. Department of Energy, contract DE–AC02–76SF00515, the SLAC Pub number is SLAC-PUB-17198 with title: Constraints on the Intrinsic Charm Content of the Proton from Recent ATLAS Data.

* gennady.lykasov@cern.ch

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