Description of meson production in electron-positron annihilation and tau-lepton decays within the NJL model

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Outline

- Motivation, experiments
- Extended NJL model in brief
- Processes of e+e- annihilation into mesons
- Hadronic decays of tau leptons
- Discussion and future plans

The goals and motivation

Theretical description of several classes of processes with meson interactions at energies up to 2 GeV

- verification of the NJL models on a new class of tasks
- definition of its aplicability domain
- theoretical support of experimental programs
- construction of theoretical predictions
- "pinpointing" of exotic mesons with small masses

N.B. Approaches to chiral symmetry breaking, confinement etc.

Modern e+e- colliders

- Frascati: DAΦNE* (~1 GeV)
- Cornell: CESR (CLEO*) (3.5 — 12 GeV)
- Novosibirsk: VEPP-2000 (0.3 — 2.0 GeV), VEPP-4 (~4 GeV)
- Beijing: BEPC-II (3 — 5.6 GeV)
- B-factories: PEP-II (BaBar*), KEKB (Belle*, Belle-II)

  (the method of radiative return!)
- Super Charm-Tau factory - a mega-science project in Novosibirsk

R-ratio and muon g-2

- The R-ratio of e+e- annihlation cross sections into hadrons and muons is used for reconstruction of the hadronic contribution into vacuum polarization
- The alternative method exploits the hadronic decay width of tau leptons
- The hadronic contribution gives the dominant "theoretical" uncertainty in the muon g-2, and also crucially important for other high-precision tests of QED and the SM
- Measurements are not inclusive, so theoretical predictions are needed for each process separately

see the review [S.Actis, A.Arbuzov et al. "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", EPJC 2010]

The method of radiative return

- Measurement of e+e- annihilation processes at different centre-of-mass energies is very important for studies of strong interactions
- Beam energy scan is the best option but it is very expensive
- In ref. [A.A., E.Kuraev, N.Merenkov, L.Trentadue "Hadronic cross-sections in electron-positron annihilation with tagged photon" JHEP 1998] the application of the radiative return method at e+e- colliders was proposed for the first time

The Nambu-Jona-Lasinio model

Very simple four-fermion (current x current) interactions with **chiral symmetry**

$$\mathcal{L}_{\rm int}^{(4)} = \frac{G_1}{2}\int d^4x \sum_{j=1}^{9}\sum_{i=1}^{2}\left[J^j_{S,i}(x)J^j_{S,i}(x) + J^j_{P,i}(x)J^j_{P,i}(x)\right] -$$

$$-\int d^4x \sum_{j=1}^{9}\sum_{i=1}^{2}\left(\frac{G_2}{2}\, J^{j,\mu}_{V,i}(x)J^j_{V,i,\mu}(x) + \frac{G_3}{2}\, J^{j,\mu}_{A,i}(x)J^j_{A,i,\mu}(x)\right)$$

The model effectively describes **spontaneous breaking of the chiral symmetry**

[1] Y. Nambu, G. Jona-Lasinio, Phys. Rev. 1961

[2] T. Eguchi, PRD 1976; K. Kikkawa, Prog. Theor. Phys. 1976

[3] D. Ebert, M.K. Volkov, Z.Phys. C 1983; M.K.Volkov, Ann. Phys. 1984

NJL model derived from QCD:

[4] B.A. Arbuzov, M.K. Volkov, I.V. Zaitsev, IJMPA 2006; ibid. 2009

The extended NJL model

To include the first radial excited states of mesons, the NJL model is extended by introduction of simple polinomial form factors in currents, e.g. for the pseudoscalar one

$$F^{j}_{P,2}(\mathbf{k}^2) = \mathrm{i}\gamma_5 \tau^j c^j_P f_j(\mathbf{k}^2) \qquad f_j(\mathbf{k}^2) \equiv 1 + d_j \mathbf{k}^2$$

where c^j_P is a constant, d_j is the slope parameter, and $\mathbf{k}$ is the quark 3-momentum.

Parameters c^j_P are fixed from static observables, d_j - from unchanged quark condensate

The chiral symmetry is preserved and there are no any additional parameters for interaction of the excited mesons

[1] M. K. Volkov, C. Weiss, Phys. Rev. D 56, 221 (1997).
[2] M. K. Volkov, A.B. Arbuzov, Usp. Fiz. Nauk 187 (2017).

Quark-meson Lagrangian

- Interactions: $$L_{\text{int}} = L_{SM} + L_{NJL}$$

$$L_{NJL} = \bar{q}(i\gamma_5 \sum_{j=\pm} \lambda_j (a_K K^j + b_K K'^j) + \frac{1}{2}\gamma_\mu \lambda_V (a_V V_\mu + b_V V'_\mu))q$$

$$a_a = \frac{1}{\sin(2\theta_a^0)}\left[g_a \sin(\theta_a + \theta_a^0) + g_a' f_a(\vec{k}^2)\sin(\theta_a - \theta_a^0)\right]$$

$$b_a = \frac{-1}{\sin(2\theta_a^0)}\left[g_a \cos(\theta_a + \theta_a^0) + g_a' f_a(\vec{k}^2)\cos(\theta_a - \theta_a^0)\right]$$

*M.K. Volkov and A.B. Arbuzov, PEPAN **47**, 489 (2016)*

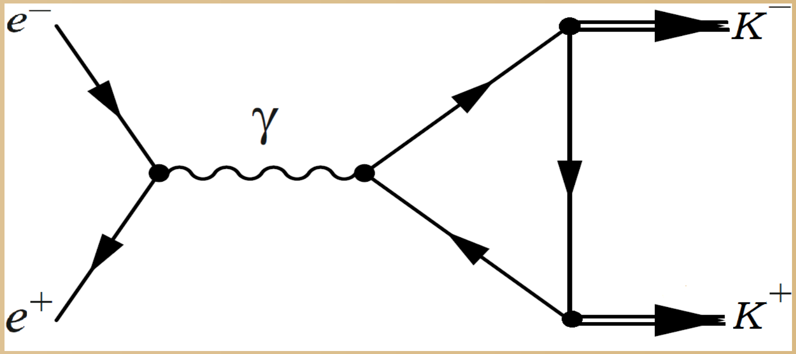
e+e- → K+ K-

The amplitude includes the contact Feynman diagram with direct interaction of virtual photon with quarks and diagrams with intermediate vector mesons:

$$T = \frac{16\pi\alpha_{em}}{s} l^{\mu}\left[B_{(\gamma)} + B_{(\rho+\rho')} + B_{(\omega+\omega')} + e^{i\pi}B_{(\varphi+\varphi')}\right]_{\mu\nu}\left(p_{K^+} - p_{K^-}\right)^{\nu}$$

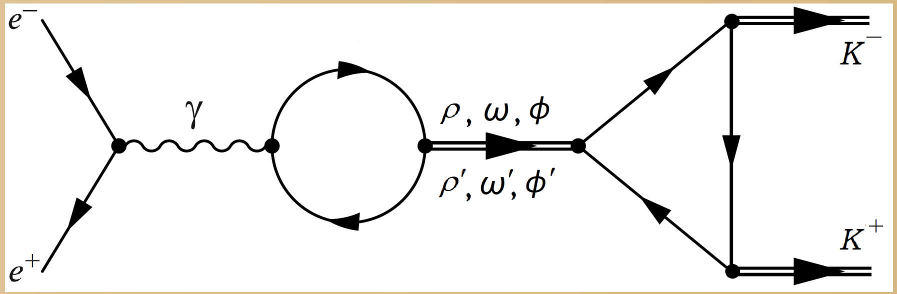
The contact diagram contribution reads

$$B_{(\gamma)\,\mu\nu} = g_{\mu\nu} I_2^{a_K a_K}\left(m_u, m_s\right)$$



e+e- → K+ K-

Contributions of intermediate vector mesons:



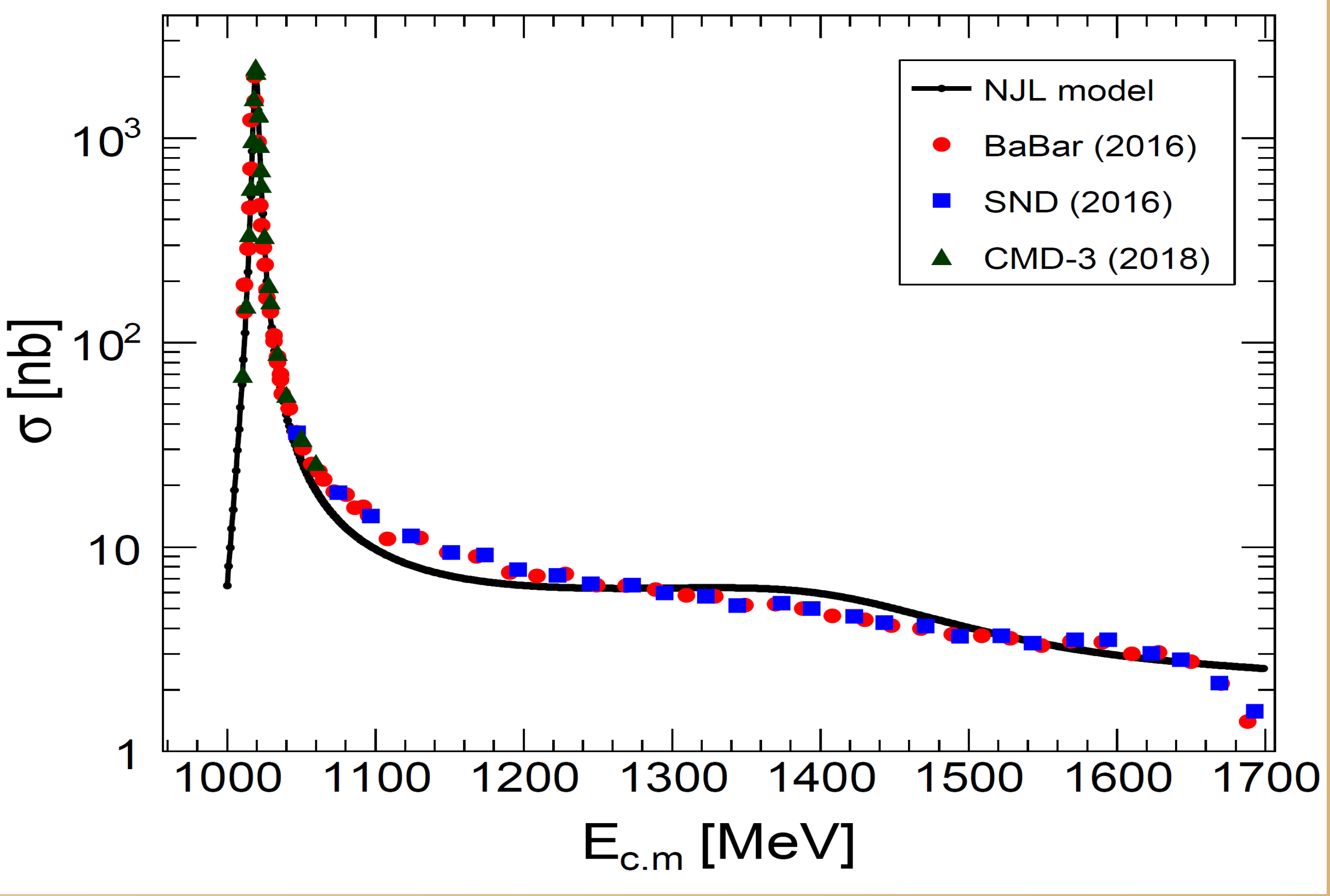
$$B_{(V+V')\mu\nu} = r_V \left[\frac{C_V}{g_V} \frac{g_{\mu\nu}s - p_\mu p_\nu}{M_V^2 - s - i\sqrt{s}\Gamma_V(s)} I_2^{a_V a_K a_K} + \frac{C_{V'}}{g_V} \frac{g_{\mu\nu}s - p_\mu p_\nu}{M_{V'}^2 - s - i\sqrt{s}\Gamma_{V'}(s)} I_2^{b_V a_K a_K} \right],$$

$$r_\rho = r_{\rho'} = 1/\ 2, r_\omega = r_{\omega'} = 1/\ 6,\ r_\varphi = r_{\varphi'} = 1/\ 3.$$

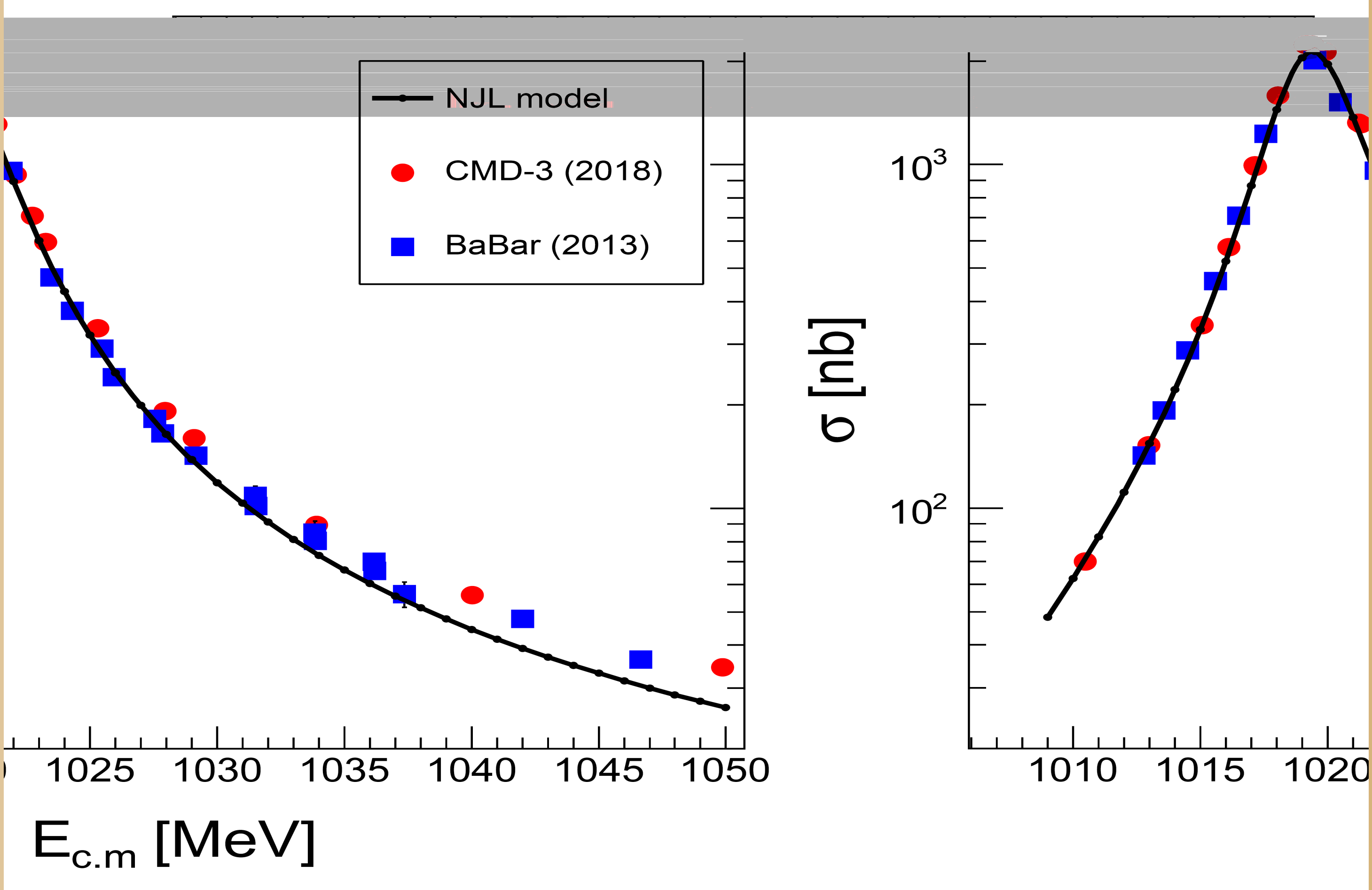
M.N. Achasov et al., Phys.Rev. D ***76****, 072012 (2007)*

J.P. Lees et al. [BaBar Collaboration], Phys.Rev. D ***88****, 032013 (213)*

e+e- → K+ K-


NJL model
BaBar (2016)
SND (2016)
CMD-3 (2018)
σ [nb]
10^3
10^2
10
1
1000
1100
1200
1300
1400
1500
1600
1700
E_c.m [MeV]

e+e- → K+ K-



NJL model
CMD-3 (2018)
BaBar (2013)
σ [nb]
10^3
10^2
1010
1015
1020
1025
1030
1035
1040
1045
1050
E c.m [MeV]

$e^+e^- \rightarrow K^+ K^-$

Comparisons of separate contributions with results of other models:

	our result	[4]	[14]
N_ρ	0.44	0.598	0.598
N_ω	0.147	0.171	0.199
N_ϕ	0.34	0.283	0.339
$N_{\rho'}$	0.066	0.056	0.056
$N_{\omega'}$	0.022	0.016	0.019
$N_{\phi'}$	0.0005	0.005	0.006

*[4] S.A. Ivashyn and A.Y. Korchin, Eur.Phys. J. C **49**, 697 (2007).*
*[14] C. Bruch, A. Khodjamirian and J.H. Kuhn, Eur.Phys.J. C **39**, 41 (2005).*

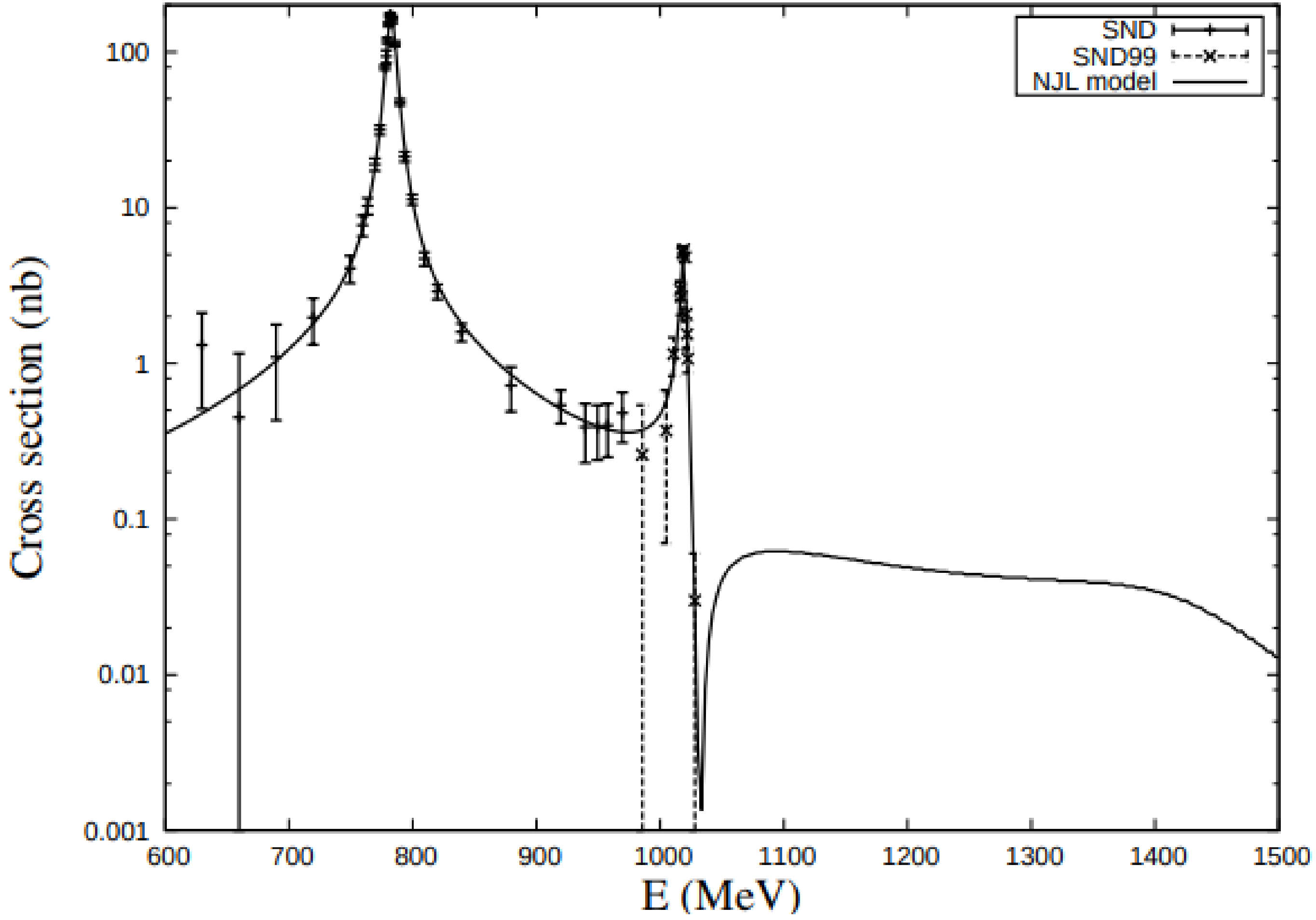


Figure 4. Comparison of experimental results for $e^+e^- \to \pi^0\gamma$ with the NJL model prediction.

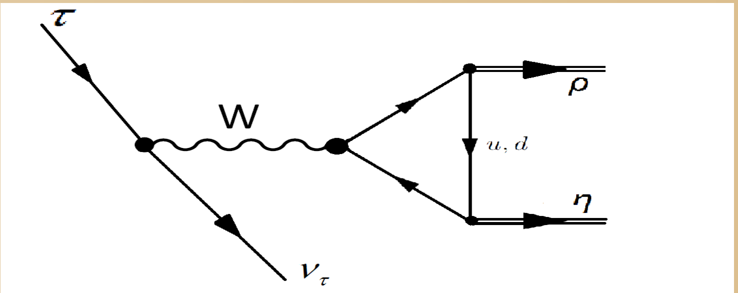
A good description of the following annihilation processes was achieved:

$e^+e^- \to [\pi, \pi(1300)]\gamma$	*A.B. Arbuzov, E.A. Kuraev, M.K. Volkov, EPJA 47 (2011) 103*
$e^+e^- \to [\eta, \eta'(958), \eta(1295), \eta(1475)]\gamma$	*A.I. Ahmadov, D.G. Kostunin, M.K. Volkov, PRC 87 (2013) 045203*
$e^+e^- \to [f_1(1285), a_1(1260)]\gamma$	*M.K. Volkov, A.A. Pivovarov and A.A. Osipov, IJMPA 32 (2017) 1750123*
$e^+e^- \to [\pi, \pi(1300)]\pi$	*M.K. Volkov and D.G. Kostunin, PRC 86 (2012) 025202*
$e^+e^- \to \omega(782)\pi^0$	*A.B. Arbuzov, E.A. Kuraev and M.K. Volkov, PRC 83 (2011) 048201*
$e^+e^- \to \rho(770)\eta$	*M.K. Volkov, K.Nurlan, A.A. Pivovarov, JETP Lett. 106 (2017) 771*
$e^+e^- \to K^{\pm}[K^{*\mp}(892), K^{*\mp}(1410)]$, $e^+e^- \to [\eta, \eta'(958)][\phi(1020), \phi(1680)]$	*M.K. Volkov, A.A. Pivovarov, IJMPA 31 (2016) 1650155*
$e^+e^- \to K^+K^-$	*M.K. Volkov, K. Nurlan, A.A. Pivovarov, PRC 98 (2018) 015206*
$e^+e^- \to [\eta, \eta'(958)]2\pi$	*M.K. Volkov, A.B. Arbuzov, D.G. Kostunin, PRC 89 (2014) 015202*

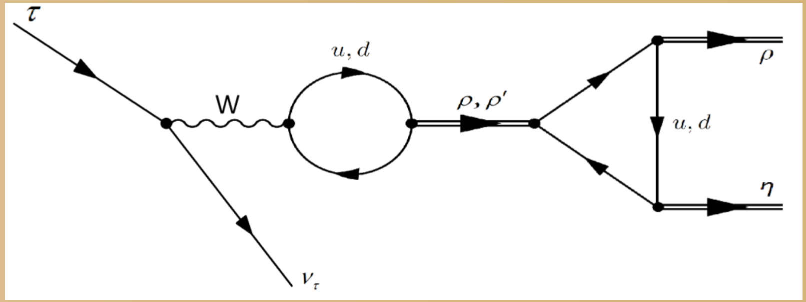
Predictions were obtained for:

$$e^+e^- \to \pi(1300)\gamma,$$
$$e^+e^- \to [\eta'(958), \eta(1295), \eta(1475)]\gamma,$$
$$e^+e^- \to [f_1(1285), a_1(1260)]\gamma,$$
$$e^+e^- \to \pi(1300)\pi,$$
$$e^+e^- \to K^\pm K^{*\mp}(1410),$$
$$e^+e^- \to \eta'(958)\phi(1020),$$
$$e^+e^- \to \eta, \phi(1680),$$
$$e^+e^- \to \eta'(958)2\pi$$

Three-body decay $\tau \to \rho\eta\nu_\tau$



Contact diagram



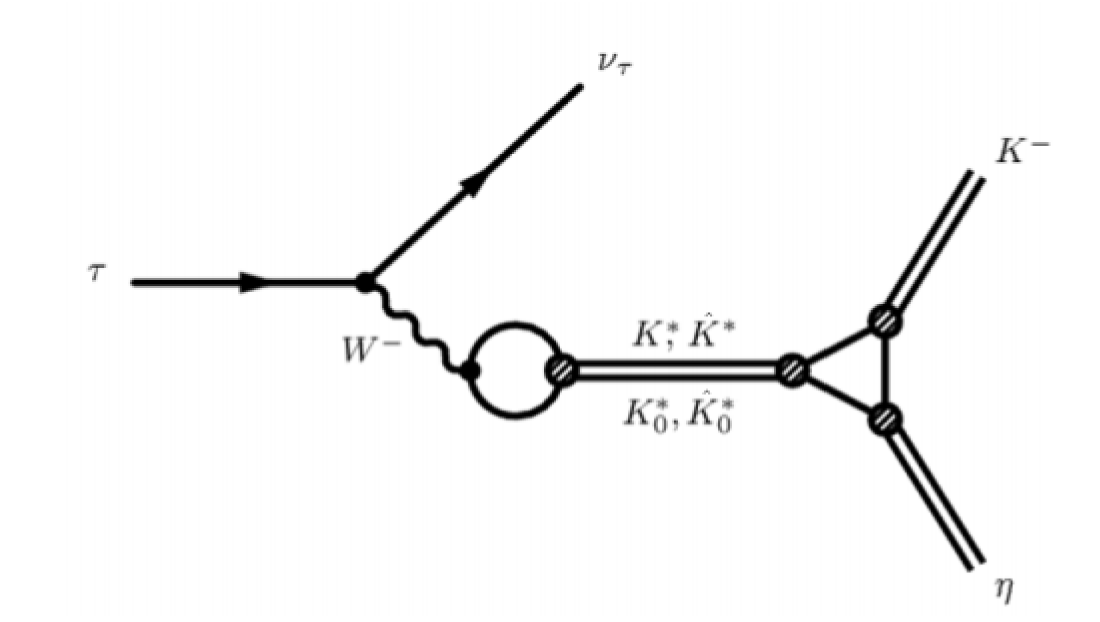
Diagrams with intermediate vector mesons

$$T = -i4m_u \frac{G_F}{\sqrt{2}} l_\mu V_{ud} \left\{ I_3^{\gamma\rho\eta} g_{\mu\nu} + \frac{C_\rho}{g_\rho} I_3^{\rho\rho\eta} \frac{g_{\mu\nu}s - p_\mu p_\nu}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho} + \frac{C_{\rho'}}{g_\rho} I_3^{\rho'\rho\eta} \frac{g_{\mu\nu}s - p_\mu p_\nu}{m_{\rho'}^2 - s - i\sqrt{s}\Gamma_{\rho'}} \right\}_{\mu\nu} \varepsilon_{\nu\lambda\delta\sigma} e_\lambda(p) p_\rho^\delta p_\eta^\sigma$$

$$Br(\tau \to \rho\eta\nu_\tau)_{NJL} = 1.44 \times 10^{-3}$$

$$Br(\tau \to 2\pi\eta\nu_\tau)_{exp} = (1.39 \pm 0.1) \times 10^{-3}$$

Decay \tau → \eta K \nu (I)



$\tau \to \eta K^- \nu_\tau$

The decay $\tau \to \eta K^- \nu_\tau$ with the intermediate vector $K^*(892)$, $K^*(1410)$ and scalar $K_0^*(800)$, $K_0^*(1430)$

Decay \tau → \eta K \nu (II)

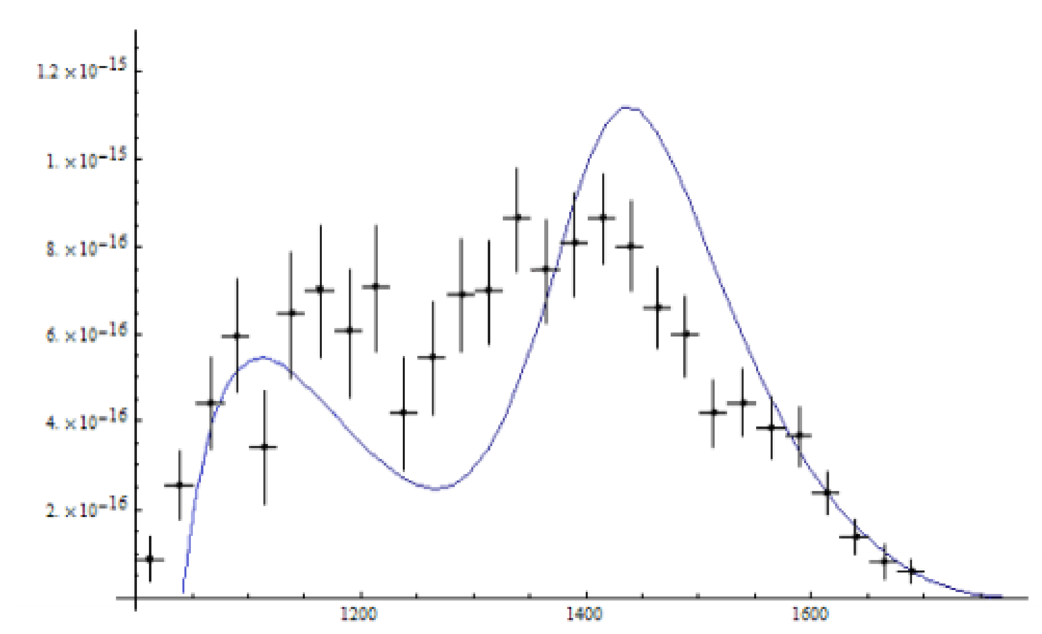
$$Br(\tau \to \eta K^- \nu_\tau) = 1.54 \cdot 10^{-4}$$

$$Br(\tau \to \eta K^- \nu_\tau)_{exp} = (1.58 \pm 0.14) \cdot 10^{-4}, \quad [30]$$

$$Br(\tau \to \eta K^- \nu_\tau)_{exp} = (1.42 \pm 0.18) \cdot 10^{-4}, \quad [31]$$

$$Br(\tau \to \eta K^- \nu_\tau)_{exp} = (1.52 \pm 0.08) \cdot 10^{-4}. \quad [34]$$

$$\tau \to \eta K^- \nu_\tau$$



$$Br(\tau \to \eta' K^- \nu_\tau) = 1.25 \cdot 10^{-6}$$

$$Br(\tau \to \eta' K^- \nu_\tau)_{exp} < 2.4 \cdot 10^{-6}$$

Process	NJL (Br)	Experiment (Br)	Publication
$\tau \to \pi \nu_\tau$ $\tau \to \pi(1300)\nu_\tau$	11.04% 9.8×10^{-5}	$(10.82 \pm 0.05)\%$ $(10 \div 19) \times 10^{-5}$	M.K.Volkov, A.B.Arbuzov, Phys. Usp. 60 (2017) 643
$\tau \to K^*(892)\nu_\tau$ $\tau \to K^*(1410)\nu_\tau$ $\tau \to K_1(1270)\nu_\tau$ $\tau \to K_1(1650)\nu_\tau$ $\tau \to a_1(1260)\nu_\tau$ $\tau \to a_1(1640)\nu_\tau$	1.15% 0.23% 0.4% 2.99×10^{-4} 14.1% 0.63%	$(1.2 \pm 0.07)\%$ $(0.15 + 1.4 - 1)$ $(0.47 \pm 0.11)\%$ - - -	M.K.Volkov, K.Nurlan, PEPAN Lett. 14 (2017) 677
$\tau \to \pi^- \pi^0 \nu_\tau$	24.76%	$(25.49 \pm 0.09)\%$	M.K.Volkov, D.G.Kostunin, PEPAN Lett. 10 (2013) 7
$\tau \to \pi\omega(782)\nu_\tau$	1.85%	$(1.95 \pm 0.06)\%$	M.K.Volkov, A.B.Arbuzov, D.G.Kostunin, PRD 86 (2012) 057301
$\tau \to \eta\pi^-\nu_\tau$ $\tau \to \eta'(958)\pi^-\nu_\tau$	4.72×10^{-6} 3.74×10^{-8}	$< 9.9 \times 10^{-5}$ $< 4 \times 10^{-6}$	M.K.Volkov, D.G.Kostunin, PRD 86 (2012) 013005
$\tau \to K^-\pi^0\nu_\tau$	4.13×10^{-3}	$(4.33 \pm 0.15) \times 10^{-3}$	M.K.Volkov, A.A.Pivovarov, MPLA 31 (2016) 1650043
$\tau \to \eta K^-\nu_\tau$ $\tau \to \eta'(958)K^-\nu_\tau$	1.45×10^{-4} 1.25×10^{-6}	$(1.55 \pm 0.08) \times 10^{-4}$ $< 2.4 \times 10^{-6}$	M.K.Volkov, A.A.Pivovarov, JETP Lett. 103 (2016) 613
$\tau \to K^0K^-\nu_\tau$	1.27×10^{-3}	$(1.48 \pm 0.05) \times 10^{-3}$	M.K.Volkov, A.A.Pivovarov, MPLA 31 (2016) 1650138
$\tau \to \rho(770)\eta\nu_\tau$	1.44×10^{-3}	-	M.K.Volkov, K.Nurlan, A.A.Pivovarov, JETP Lett. 106 (2017) 771
$\tau \to \bar{K}^{*0}(892)\pi^-\nu_\tau$	1.78×10^{-3}	$(2.2 \pm 0.5) \times 10^{-3}$	M.K.Volkov, A.A.Pivovarov, JETP Lett. 108, (2018) 369
$\tau \to f_1(1285)\pi^-\nu_\tau$	3.98×10^{-4}	$(3.9 \pm 0.5) \times 10^{-4}$	M.K.Volkov, A.A.Pivovarov, A.A.Osipov, EPJA 54 (2018) 61
$\tau \to \eta 2\pi\nu_\tau$ $\tau \to \eta'(958)2\pi\nu_\tau$	1.46×10^{-3} 9×10^{-7}	$(1.39 \pm 0.07) \times 10^{-3}$ $< 1.2 \times 10^{-5}$	M.K.Volkov, A.B.Arbuzov, D.G.Kostunin, PRC 89 (2014) 015202

The Vector Meson Dominance

- The vector meson dominance (VMD) (Sakurai '1960) automatically appears in the standard NJL model after summing up the contributions with intermediate photon and ρ meson in the ground state
- In tau lepton decays the VMD appears in transitions of W boson into axial-vactor mesons
- In the extended NJL model the VMD works only for the ground states of intermediate mesons, but fails for the first radial excited states

Discussion and future plans

- A series of 30+ works on the subject was published by M.K.Volkov & co. since 2011
- Whole classes of processes are systematically considered within the same model
- Theoretical predictions for future experiments were constructed
- The extended NJL model sucesfully works for e+e- annihilation and tau lepton decays at energies up to ~2 GeV
- A certain number of problems was revealed, they can be treated as indications of light exotic mesons: eta(1405), a0(980), a1(1410), f0(1500) etc.
- The nearest plans concern processes with axial-vector currents
- General lessons on effective QFT models: hints on symmetry breaking and energy scales
- Applications for hot dense matter...