CHARGED-CURRENT NEUTRINO-NUCLEON REACTIONS IN THE SUPERNOVA NEUTRINO-SPHERE

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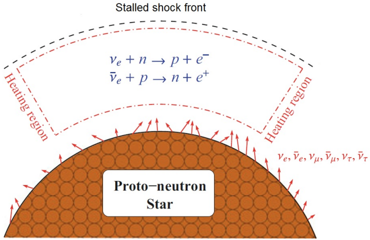
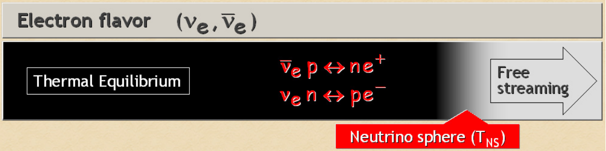


The (anti)neutrino heating rate

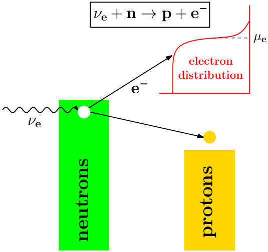
$$Q_{\nu,\bar{\nu}} \sim L_{\nu,\bar{\nu}} \langle E^2_{\nu,\bar{\nu}} \rangle Y_{n,p},$$

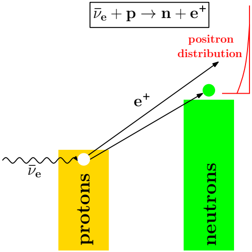
where $L_{\nu,\bar{\nu}}$ ($E_{\nu,\bar{\nu}}$) is the neutrino luminosity (energy), and $Y_{n,p}$ is the number fractions of free protons and neutrons.

The success of the delayed supernova explosion depends critically on $L_{\nu,\bar{\nu}}$ and $\langle E^2_{\nu,\bar{\nu}} \rangle$.



The deeper and the hotter is the neutrino-sphere, the larger is a fraction of hight-energy (anti) neutrinos and the larger is the luminosity.





Neutrino-sphere – **neutron-rich** gas of nucleons:

$$\rho \approx (0.01 - 0.1)\rho_0, \quad T = 2 - 10 \text{ MeV}.$$

Due to neutron-rich conditions $\Delta U_{np} = U_n - U_p \gg 0$. In elastic approximation ($q = 0$):

$$E_{e^-} = E_{\nu_e} + \Delta U_{np}, \quad \text{while} \quad E_{e^+} = E_{\bar\nu_e} - \Delta U_{np}.$$

Mean-field effect favors ν_e-absorption, but suppresses $\bar\nu_e$-absorption.

Nucleon-nucleon correlations may be important because:

- Neutrino wavelength: $\Lambda_\nu = \frac{2\pi\hbar c}{E_\nu} \approx 100\,\text{fm}$.

  Inter-nucleon distance: $a = \left(\frac{3}{4\pi\rho}\right)^{1/3} \approx 10\,\text{fm}$.

- We have $E_\nu \sim \Gamma$ for $E_\nu \approx 3T$ and $\Gamma \sim \langle \sigma_{NN} v \rangle \rho \approx 30\,\text{MeV}$.

Neutrino and antineutrino opacity: $\chi(\text{or } \lambda^{-1}) = \rho \times \sigma_{\text{abs}}$**.** The smaller **opacity** leads to larger $\langle E_{\nu,\bar{\nu}} \rangle$ and $L_{\nu,\bar{\nu}}$.

The differential cross section for ν_e- and $\bar{\nu}_e$-absorption:

$$\frac{1}{V}\frac{d^2\sigma(E_{\nu,\bar{\nu}})}{d cos\theta d\omega} = \frac{G_F^2 cos^2\theta_c}{4\pi^2} E_e p_e (1 - f(E_e)) \Big\{ g_V^2 \left[1+cos\theta\right] S_\tau^{(\pm)}(q,\omega,T) + g_A^2 \left[3-cos\theta\right] S_{\sigma\tau}^{(\pm)}(q,\omega,T) \Big\}.$$

Fermi and Gamow-Teller strength functions :

$$S_\alpha^{(\pm)}(q,\omega) = Z^{-1} \sum_{ij} e^{-E_i/T} |\langle f | \boldsymbol{Q}_\alpha^{(\pm)} | i \rangle|^2 \delta(E_f - E_i - \omega), \quad \begin{cases} (+) \; for \; n \to p \\ (-) \; for \; p \to n \end{cases}$$

where $\boldsymbol{Q}_\tau^{(\pm)} \sim e^{iqr}\tau_\pm$ (Fermi), $\boldsymbol{Q}_{\sigma\tau}^{(\pm)} \sim e^{iqr}\sigma\tau_\pm$ (Gamow-Teller), and $\tau_+(\tau_-)$ for ν_e $(\bar{\nu}_e)$-absorption.

The detailed balance principle:

$$S_{\tau,\sigma\tau}^{(\mp)}(q,\omega,T) = e^{(\omega-\mu_p+\mu_n)/T} S_{\tau,\sigma\tau}^{(\pm)}(q,-\omega,T).$$

Fluctuation-dissipation theorem:

$$S^{(\pm)}_{\tau,\sigma\tau}(q,\omega,T) = -\frac{1}{\pi}\frac{\mathrm{Im}\left[\Pi^{(\pm)}_{\tau,\sigma\tau}(q,\omega,T)\right]}{1-e^{-(\omega\mp\Delta\mu_{np})}}.$$

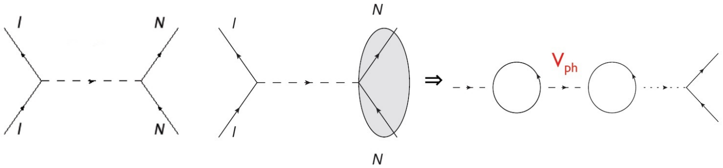
The response function and the particle-hole Green's function

$$\Pi(q,\omega,T) = 2\int\frac{d^3\mathbf{k}}{(2\pi)^3}G(q,\omega,\mathbf{k},T).$$

Within the RPA the Green's function is the solution of the Bethe-Salpeter equation:

$$G_{RPA}(q,\omega,\mathbf{k}_1) = G_{HF}(q,\omega,\mathbf{k}_1)+G_{HF}(q,\omega,\mathbf{k}_1)\int\frac{d^3\mathbf{k}_2}{(2\pi)^3}V_{ph}(q,\mathbf{k}_1,\mathbf{k}_2)G_{RPA}(q,\omega,\mathbf{k}_2).$$

where $G_{HF}(q,\omega,\mathbf{k}_1)$ is the free Hartree-Fock propagator.



Free space In medium

The Skyrme effective interaction:

$$V(\mathbf{r}_1, \mathbf{r}_2) = t_0(1 + x_0 P_\sigma)\delta(\boldsymbol{r}) + \frac{1}{2}t_1(1 + x_1 P_\sigma)(\mathbf{k}^2\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{k}'^2)$$
$$+ t_2(1 + x_2 P_\sigma)\mathbf{k}'\delta(\mathbf{r})\mathbf{k} + \frac{1}{6}t_3\rho^\alpha(\mathbf{R})(1 + x_3 P_\sigma)\delta(\mathbf{r})$$
$$+ iW_{SO}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)[\mathbf{k}' \times \delta(\mathbf{r})\mathbf{k}],$$

The Skyrme energy-density functional:

$$\mathcal{E}[\rho] = \mathcal{E}_{kin} + \mathcal{E}_{Skyrme}(x_i, t_i, W_{SO}, \rho, \tau, \ldots)$$

The Hartree-Fock mean-field:

$$U_\tau(k) = \frac{\delta\mathcal{E}[\rho]}{\delta\rho_{(kk)}} \;\; and \;\; U_n \gg U_p \;\; for \;\; \rho_n \gg \rho_p.$$

The particle-hole interaction:

$$\langle k_1', k_2'|V_{ph}|k_1, k_2\rangle = \frac{\delta^2\mathcal{E}[\rho]}{\delta\rho_{(k_1',k_2')}\delta\rho_{(k_1,k_2)}}$$

With the Skyrme interaction the Bethe-Salpeter equation reduces to a system of 23 linear equations.

"Consistent" Skyrme parametrizations

PHYSICAL REVIEW C **85**, 035201 (2012)

Skyrme interaction and nuclear matter constraints

M. DUTRA *et al.* PHYSICAL REVIEW C **85**, 035201 (2012)

TABLE I. List of macroscopic constraints and the range of their experimental (exp) and empirical (emp) values, density region in which they are valid, and the corresponding range as found using successful Skyrme parametrizations (CSkP). For more explanation see text.

Constraint	Quantity	Eq.	Density region	Range of constraint (exp and emp)	Range of constraint from CSkP	Ref.
SM1	K_o	(7),(15)	ρ_o (fm^{-3})	200–260 MeV	202.0–240.3 MeV	[64]
SM2	$K' = -Q_o$	(8),(16)	ρ_o (fm^{-3})	200–1200 MeV	362.5–425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_o} < 2.2$	Band region	see Fig. 2	[80]
PNM1	$\frac{E_{\rm PNM}}{E^o_{\rm PNM}}$	(31)	$0.014 < \frac{\rho}{\rho_o} < 0.106$	Band region	see Fig. 3	[39,40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band region	see Fig. 5	[78]
MIX1	J	(9)	ρ_o (fm^{-3})	30–35 MeV	30.0–35.5 MeV	[44]
MIX2	L	(10)	ρ_o (fm^{-3})	40–76 MeV	48.6–67.1 MeV	[101]
MIX3	$K_{\tau,v}$	(21)	ρ_o (fm^{-3})	−760 to −372 MeV	−407.1 to −360.1 MeV	[107]
MIX4	$\frac{S(\rho_o/2)}{J}$	—	ρ_o (fm^{-3})	0.57–0.86	0.61–0.67	[110]
MIX5	$\frac{3P_{\rm PNM}}{L\rho_o}$	(41)	ρ_o (fm^{-3})	0.90–1.10	1.02–1.10	[112]

240 Skyrme interaction parameter sets were tested and only **16** satisfy all criteria.

"Best fit" Skyrme parametrizations

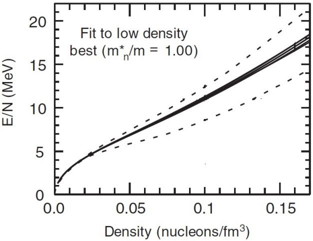
PHYSICAL REVIEW C **89**, 011307(R) (2014)

Constraints on Skyrme equations of state from properties of doubly magic nuclei and *ab initio* calculations of low-density neutron matter

B. Alex Brown[1] and A. Schwenk[2,3]

TABLE I. Properties of the fitted Skyrme functionals. The effective mass m_n^*/m in neutron matter at $\rho_n = 0.10$ fm^{-3} is constrained to be 0.9 in the first part of the table and 1.0 in the second part. The symmetry energy J, its density derivative L, the symmetry-energy incompressibility K_s, the symmetric-nuclear-matter incompressibility K_m, and effective mass m^*/m are evaluated at $\rho = 0.16$ fm^{-3}. The mean value is from the entire set, and the best value (b) is for the six cases that give the best fit to the data.

Name		σ	m_n^*/m	χ^2	K_m (MeV)	m^*/m	a_n (MeV fm^3)	b_n (MeV fm$^{3\gamma}$)	d_n (MeV fm^5)	J (MeV)	L (MeV)	K_s (MeV)	R_{np} (fm) ^{208}Pb	R_{np} (fm) ^{48}Ca
Ska25	$s7$	0.25	1.00	0.85 (b)	218	0.99	−386	424	2	32.6	48	−165	0.173	0.168
Ska35	$s8$	0.35	1.00	0.73 (b)	244	1.00	−333	419	−3	33.1	53	−165	0.179	0.171
SkT1	$s10$	1/3	0.99	0.76 (b)	241	0.97	−341	423	0	33.4	53	−163	0.181	0.170
SkT2	$s11$	1/3	1.00	0.76 (b)	230	0.97	−338	413	4	33.1	52	−164	0.179	0.170
SkT3	$s12$	1/3	1.00	0.73 (b)	241	0.98	−337	408	3	32.8	50	−166	0.176	0.170
SV-sym32	$s16$	0.30	1.02	1.04 (b)	242	0.91	−364	450	−21	33.4	51	−176	0.178	0.173

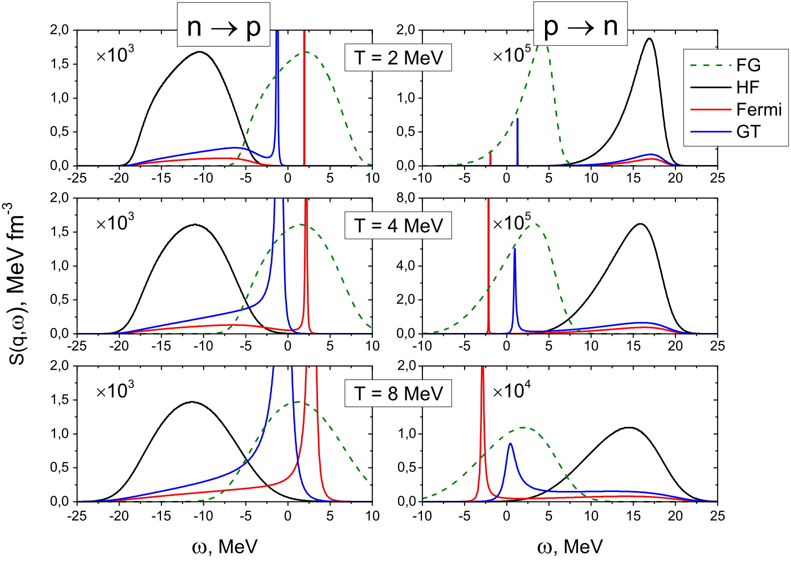


Calculations for PNM based on the chiral EFT and RG predict that $m^*/m \approx 1.0$ for densities $\rho < 0.1\,\mathrm{fm}^{-3}$.

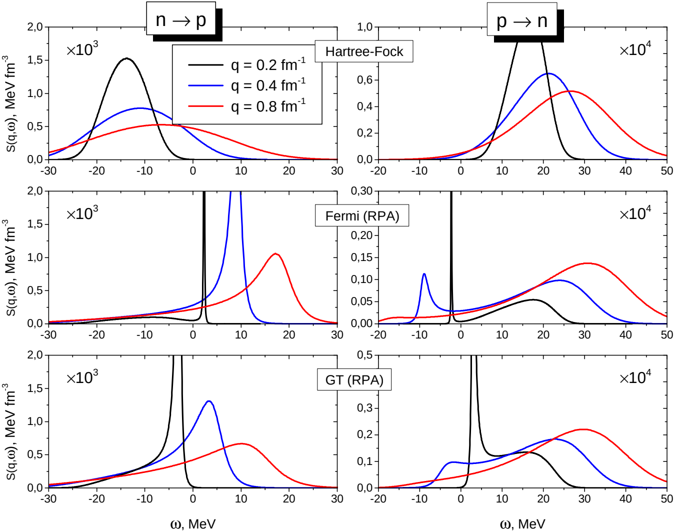
$\Longleftarrow$ EOS for PNM obtained by N^{3}LO calculations based on chiral EFT

Strength functions with Ska35

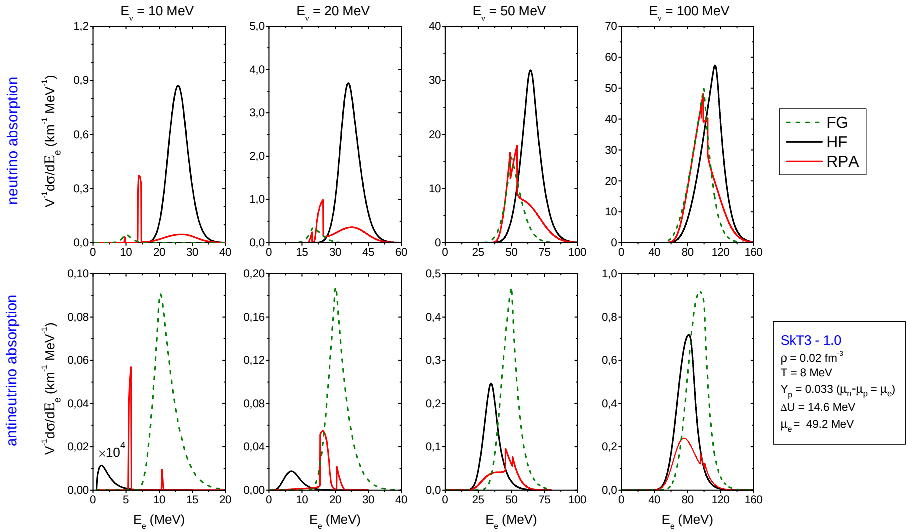
$\rho = 0.02\,\mathrm{fm}^{-3},\ Y_p = 0.1,\ q = 0.2\,\mathrm{fm}^{-1}\ (U_n - U_p \approx 13$ MeV)



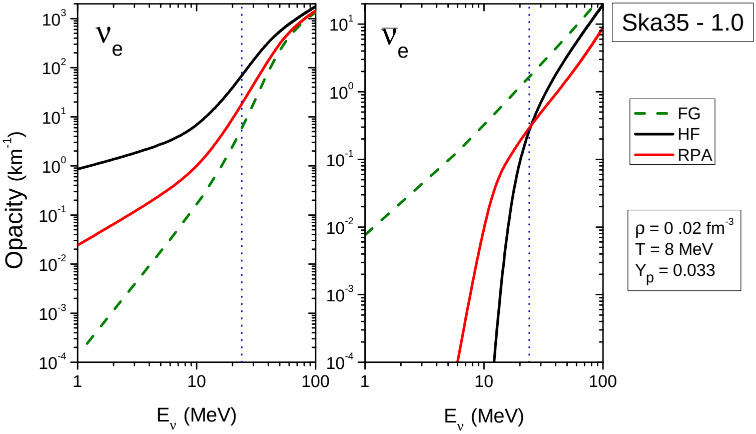
$\rho = 0.02\,\mathrm{fm}^{-3},\ Y_p = 0.1,\ T = 8\,\mathrm{MeV}$

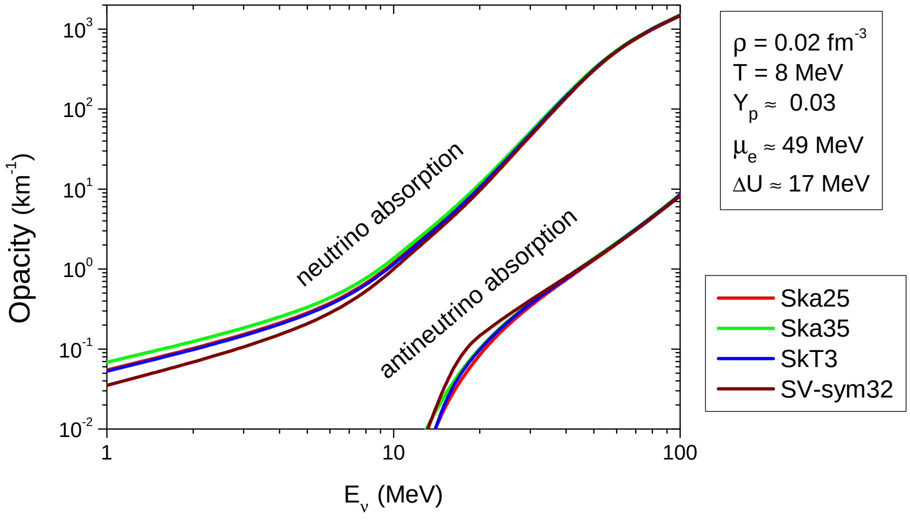


Differential cross sections



Neutrino and antineutrino opacities





ρ = 0.02 fm^-3
T = 8 MeV
Y_p ≈ 0.03
μ_e ≈ 49 MeV
ΔU ≈ 17 MeV
Ska25
Ska35
SkT3
SV-sym32
neutrino absorption
antineutrino absorption
Opacity (km^-1)
E_ν (MeV)
10^3
10^2
10^1
10^0
10^-1
10^-2
1
10
100

Conclusion

- The neutrino and antineutrino opacities due to charged-current reactions near neutrino-sphere have been studied by applying the Skyrme-RPA.
- The opacities are found to be quite sensitive to the difference between neutron and proton mean-field potentials and RPA correlations.
- It is found that Skyrme parametrizations from the "best fit set" give very close results.
- One expect that RPA particle-hole correlations increase the efficiency of neutrino heating and favor the supernova explosion.