$\mathcal{N} = 1$ Lagrangians for Argyres-Douglas theories

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- Based on

1. **N=1 Deformations and RG Flows of N=2 SCFTs - J. Song, K. Maruyoshi (arXiv: 1607.04281)**

2. **N=1 Deformations and RG Flows of N=2 SCFTs, Part II: Non-principal deformations – P. A. , J. Song, K. Maruyoshi (arXiv:1610.05311)**

3. **N=1 Lagrangians for generalized Argyres-Douglas theories – P. A. , J. Song, A. Sciarappa , (arXiv:1707.04751)**

Plan of the talk

- Brief review of Argyres-Douglas theories
- An $\mathcal{N}=1$ Lagrangian for the H_0 theory
- Generalization of the Lagrangian for H_0 theory
- Superconformal Index
- Summary

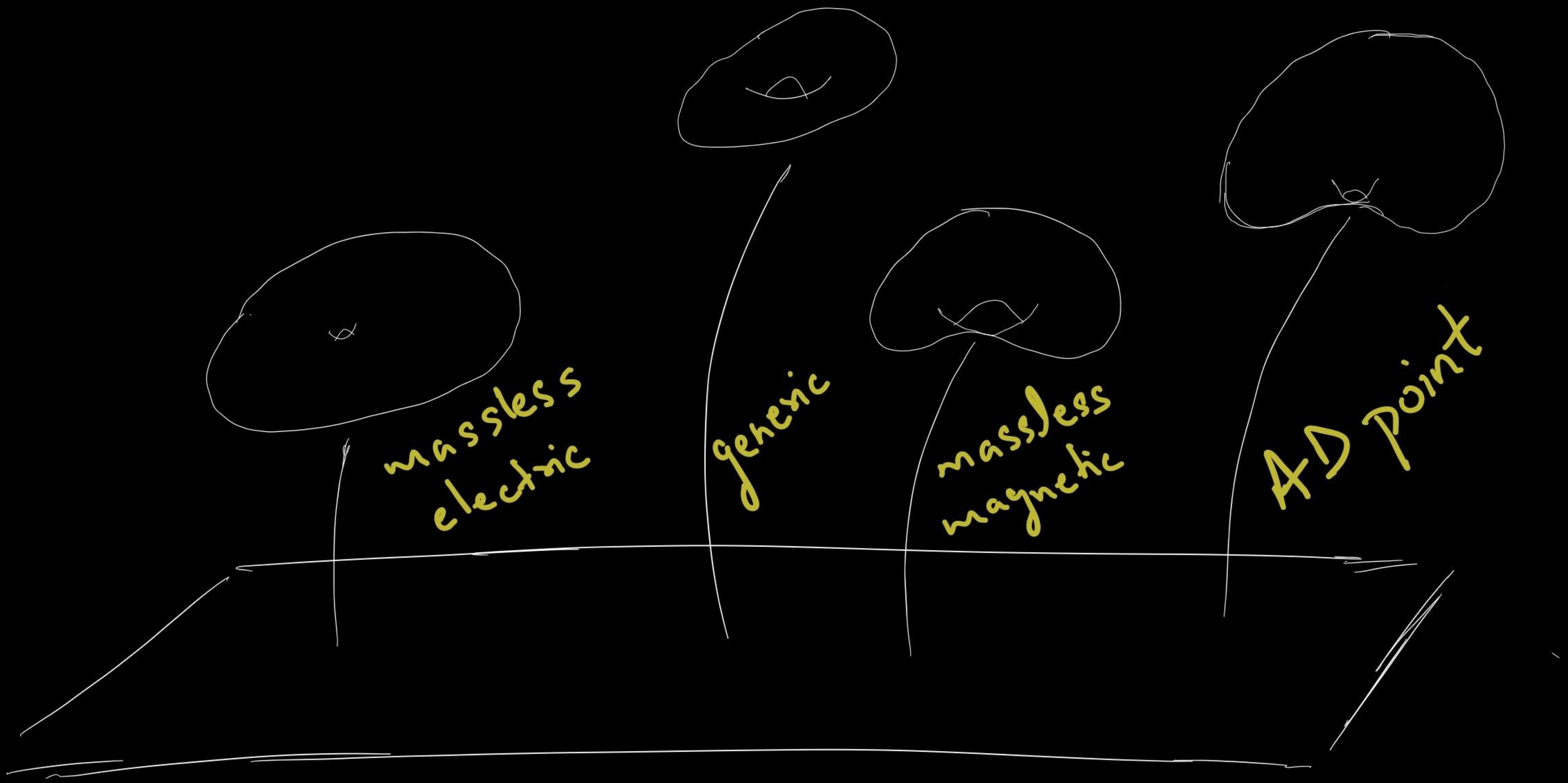
Review of Argyres-Douglas theories

Review of Argyres-Douglas theories

- $\mathcal{N} = 2$ Superconformal theories (SCFTs)

- Describe the low energy theory at special loci on the Coulomb branch of generic $\mathcal{N} = 2$ theories

- At these special loci, monopoles and electrically charged particles simultaneously become massless


massless
electric
generic
massless
magnetic
AD point

AD theories are Non-Lagrangian

- Impossible to write a manifestly Lorentz invariant Lagrangian with electrons as well as monopoles as elementary degrees of freedom
- This implies that the AD theories are inherently non-perturbative.
- This poses a challenge in studying AD theories
- Most of the progress in understanding them has only happened in the past decade or so

Simplest AD theory

- Supersymmetric U(1) gauge theory + electron + monopole/dyon
- AD point on the Coulomb branch of $\mathcal{N}=2$ SU(2) gauge theory with 1 doublet hyper
- Often called as the H_0 theory
- Discovered by Argyres and Douglas in 1995

- H_0 has a single Coulomb branch operator with scaling dimension

$$\Delta_{\mathcal{O}} = \frac{6}{5}$$

- central charges were computed by Shapere and Tachikawa in 2008

$$a = \frac{43}{120}, \;\; c = \frac{11}{30}$$

Minimal 4d theory with $\mathcal{N}=2$ SUSY

- H_0 is believed to be the minimal 4d theory with 8 supercharges
- All 4d $\mathcal{N}=2$ SCFTs obey an analytic lower bound on their central charge c

Liendo, Ramirez, Seo (**arXiv:1509.00033**)

- H_0 theory saturates this bound
- Despite being the simplest of all $\mathcal{N}=2$ SCFTs, very little is known about it: Conventional methods to compute partition functions on $S^4,\ S^3\times S^1$ etc do not apply

$\mathcal{N} = 1$ Lagrangian for H_0 theory

- Discovered by Jaewon Song and Kazunobu Maruyoshi in 2016

- $\mathcal{N}=1$ SU(2) gauge theory coupled to one adjoint chiral (Φ) , two chiral doublets (q, q') and two SU(2) singlets (M, β)

$$W = \mathrm{tr} q\Phi q + \beta \mathrm{tr}\Phi^2 + M\mathrm{tr}\Phi q' q'$$

- The term in red is irrelevant in the UV
- In reality it is dangerously irrelevant and therefore can not be ignored

- There is an axial U(1) flavor symmetry under which the various fields have charges given by

$$q:\frac{1}{2},\ q':\frac{7}{2},\ \Phi:-1,\ \beta:2,\ M:-6$$

- In $\mathcal{N}=1$ theories axial U(1) symmetries generically mix with the R-symmetry along the RG flow.

$$R_{IR}=R_{UV}+\sum\alpha_i A_i$$

- The mixing coefficients α_i are such that they maximize the central charge a
  - Intriligator, Wecht (**arXiv:hep-th/0304128v3**)

- Central charges of supersymmetric theories are exact functions of the R-charge – Anselmi, Freedman, Grisaru, Johansen (**arXiv:hep-th/9708042**)

$$a = \frac{9}{32}\mathrm{tr}R^3 - \frac{3}{32}\mathrm{tr}R$$

$$c = \frac{9}{32}\mathrm{tr}R^3 - \frac{5}{32}\mathrm{tr}R$$

- For the above Lagrangian a gets maximized at a point where R-charge of M is given by $R_M = \frac{4}{5}$

- Charges of other fields can be fixed by requiring IR R-symmetry to be non-anomalous and that each term in the superpotential should have R-charge 2

- The central charges at the fixed point of the above Lagrangian we therefore find that

$$a = \frac{43}{120}, c = \frac{11}{30}$$

- The dimension of various gauge invariant chiral operators can be obtained from their R-charges

$$\Delta_{\mathcal{O}} = \frac{3}{2} R_{\mathcal{O}}$$

- In particular, we find that $\Delta_M = \frac{3}{2} \times \frac{4}{5} = \frac{6}{5}$

- Recall, that for H_0

$$\Delta_{\mathcal{O}} = \frac{6}{5},\ a = \frac{43}{120},\ c = \frac{11}{30}$$

- We see, that the central charges at the fixed point of our Lagrangian match that of the H_0 theory with M playing the role of the Coulomb branch operator

- Claim: The above Lagrangian experiences SUSY enhancement and flows to the H_0 theory in the IR

- We can use the above Lagrangian to compute the full $\mathcal{N}=2$ superconformal index of H_0

- This was an open problem until now

- The Schur and Macdonald limits of the superconformal index were previously obtained by using 4d/2d correspondence

M. Buican, T. Nishinaka (**arXiv:1505.05884v2, 1509.05402**), J. Song (**arXiv:1509.06730**)

- These corresponding limits of the superconformal index computed from the Lagrangian match the above results

Lagrangians for generalized AD theories

- Let us take a quick look at how the above lagrangian was obtained

- $\mathcal{N}=1$ deformation of the $\mathcal{N}=2$ SCFT given by an SU(2) gauge theory with 8 fundamental half-hypers (Q)

- The $\mathcal{N}=2$ SCFT has an SO(8) flavor symmetry

- Deform this by introducing gauge singlet chiral multiplets (M) in the adjoint representation of SO(8).

- Coupling is through a superpotential $W = \mathrm{tr} MQQ$

- Consider the following vev for M, $\langle M \rangle = \rho(\sigma^+)$

- ρ is the choice an $SU(2) \subset SO(8)$, such that

$$(\mathbf{8})_{SO(8)} \to (\mathbf{7} \oplus \mathbf{1})_{SU(2)}$$

- This is called the principle embedding in math literature

- Integrate out the quarks that get masses as a result of the above vev

- Decouple the multiplets containing Goldstone bosons

- There is a straight forward generalization of the above Lagrangian

- Consider $\mathcal{N}=1$ deformations of the $\mathcal{N}=2$ SCFT based on SU(N) gauge theory with 2N fundamental hypers

- This $\mathcal{N}=2$ SCFT has an SU(2N) flavor symmetry

- The $\mathcal{N}=1$ deformation consists of introducing a gauge singlet chiral multiplet M transforming in the adjoint representation of the flavor symmetry

- We are interested in effective theories obtained after giving the following vev to M

$$\langle M \rangle = \rho(\sigma^+), \;\; \rho : SU(2) \hookrightarrow SU(2N)$$

- The SU(2) embeddings into Lie Algebras were classified by Dynkin

- For SU(2N), the SU(2) embeddings are in one-one correspondence with integer partitions of 2N

- The choice of integer partition tells us how to decompose the fundamental representation of SU(2N) into irreps of SU(2)

- Principle embedding : $(\mathbf{2N})_{SU(2N)} \to (\mathbf{2N})_{SU(2)}$

- This was studied by Song and Maruyoshi in **arXiv:1607.04281**

- They found that the resulting theory flows to an IR fixed point that describes the so called (A_1, A_{2N-1}) type AD theories

- The deformation corresponding to all possible SU(2) embeddings was studied in **arXiv:1610.05311**

- It was found that other than sporadic occurrences, only one other SU(2) embedding gives AD theory at the fixed point. This is

$$(\mathbf{2N})_{SU(2N)} \to (\mathbf{2N-1} \oplus \mathbf{1})_{SU(2)}$$

- This flows to the (A_1, D_{2N}) type AD theory

- We can also consider similar deformation of the $\mathcal{N}=2$ SCFT based on Sp(N) gauge theory with (4N+4) half-hypers

- This has an SO(4N+4) flavor symmetry

- The deformations to be studied are therefore labelled by SU(2) embeddings of SO(4N+4)

- The principal embedding : $(\mathbf{4N+4})_{SO(4N+4)} \rightarrow (\mathbf{4N+3 \oplus 1})_{SU(2)}$

- The principal embedding gives an $\mathcal{N}=1$ theory that flows to (A_1, A_{2N}) type AD theories

- When $\rho : (\mathbf{4N+4})_{SO(4N+4)} \rightarrow (\mathbf{4N+1 \oplus 3})_{SU(2)}$

- The resulting Lagrangian describes (A_1, D_{2N+1}) AD theories at it's fixed point

- Other embeddings do not give anything interesting other than sporadically

- The deformations of $\mathcal{N}=2$ gauge theories based on SU(N) and Sp(N) gauge groups, together give all the AD theories of type (A_1, A_n) and (A_1, D_n)

- However, the AD theories have a much richer classification

- To begin with there are AD theories of type (A_k, A_n) and (A_k, D_n)

- Therefore an immediate question is to look for Lagrangians for these more general classes of AD theories

- A partial solution to this question was reported in **arXiv:1707.04751** (written in collaboration with A. Sciarappa and J. Song)

- We have been able to establish that (A_{m-1}, A_{Nm-1}) type AD theories can be obtained from considering $\mathcal{N}=1$ preserving "principal nilpotent" deformations of the following $\mathcal{N}=2$ quivers

$$(N) - (2N) - \cdots - (mN - N) - [mN]$$

- Deformations corresponding to other SU(2) embeddings of SU(mN) do not give anything interesting

- (A_{2m-1}, D_{2Nm+1}) type AD theories can be obtained from principal deformation of

$$[SO(2)]-Sp(N)-SO(4N+2)-Sp(3N)-\cdots-Sp(2mN-N)-[SO(4mN+2)]$$

- $(A_{2m}, D_{m(N-2)+\frac{N}{2}})$ type AD theory can be obtained from principal deformations of

$$[SO(N)]-Sp(N-2)-SO(3N-4)-Sp(2N-4)- \\ \cdots - Sp(mN-2m)-[SO(2mN+N-4m)]$$

- In addition to the above quivers, we also found that

$$\boxed{1} - (k+1) - (2k+1) - \ldots - (mk-k+1) - \boxed{mk+1} \rightsquigarrow (I_{m,mk}, S)$$

and

$$Sp(N) - SO(4N+4) - Sp(3N+2) - SO(8N+8) - \\ \ldots - Sp((m-1)(2N+2)+N) - \boxed{SO(4m(N+1))} \rightsquigarrow D_{m(2N+2)}^{m(2N+2)}[m]$$

Superconformal Index

- The $\mathcal{N}=1$ superconformal index is defined as

$$\mathcal{I}_{\mathcal{N}=1} = \mathrm{tr}(-1)^F p^{j_1+j_2+\frac{R}{2}} q^{j_2-j_1+\frac{R}{2}} \prod_i a_i^{f_i}$$

- generically, a function of two fugacities p and q

- The $\mathcal{N}=2$ superconformal index is a function of 3 fugacities p, q and t

$$\mathcal{I}_{\mathcal{N}=2} = \mathrm{tr}(-1)^F p^{j_1+j_2+\frac{r}{2}} q^{j_2-j_1+\frac{r}{2}} t^{R-\frac{r}{2}} \prod_i a_i^{f_i}$$

- Recall, that all our Lagrangians necessarily have a U(1) axial symmetry

- A linear combination of this with the $\mathcal{N}=1$ R-symmetry becomes the cartan of the SU(2) R-symmetry of the $\mathcal{N}=2$ algebra

- A second independent linear combination becomes the $U(1)_r$

- Call the fugacity for axial U(1) as ξ

- $\mathcal{N}=1$ superconformal index can be transformed into $\mathcal{N}=2$ if

$$\xi = (t(pq)^{-\frac{2}{3}})^{\beta}$$

- β can be fixed by comparing the axial charge of the gauge singlets in the Lagrangian, to the $U(1)_r$ charge of the corresponding Coulomb branch operator in the AD theory

$$\mathcal{F}_{axial} = \frac{1}{\beta}(R - \frac{r}{2})$$

Summary

- Non- Lagrangianity of AD theories poses a major hurdle in our understanding of them
- We have been successful in constructing $\mathcal{N} = 1$ Lagrangians whose IR fixed points describe AD theories
- Can use these to compute RG protected quantities such as SCI

- Closed form formulae for the Schur and Macdonald limit of the SCI for AD theories have been given in arXiv: 1706.01607. These match order by order with the result obtained using our Lagrangians. It will be nice to have an analytic proof of these equalities.

For e.g. the Schur limit of the SCI of the H_0 theory is given by

$$I_{Schur,H_0} = \frac{1}{(q^2;q^5)_\infty (q^3;q^5)_\infty}$$

Our Lagrangians suggest that the Schur index should be

$$I_{Schur,H_0} = \lim_{t\to q} \kappa \frac{\Gamma((\frac{pq}{t})^{\frac{6}{5}})}{\Gamma((\frac{pq}{t})^{\frac{2}{5}})} \oint_C \frac{dz}{2\pi i z} \frac{\Gamma(z^{\pm}(pq)^{\frac{2}{5}} t^{\frac{1}{10}})\Gamma(z^{\pm}(pq)^{-\frac{1}{5}} t^{\frac{7}{10}})\Gamma(z^{\pm 2,0}(\frac{pq}{t})^{\frac{1}{5}})}{2\Gamma(z^{\pm 2})}$$

- These lagrangians are interesting in their own regard.
- Rare examples of 4d QFTs with accidental SUSY
- The mechanism of SUSY enhancement is still not understood. This will be an interesting direction to pursue
- It will also be interesting to find string theory realization of the above lagrangians and thereby understand the geometric settings that lead to theories with accidental SUSY

THANK YOU!