

Centrality determination in MPD using MC Glauber model

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Outline

- Introduction
- Fitting procedure using MC Glauber Centrality framework
- Comparison of the initial geometry parameters between models (UrQMD, SMASH, PHSD and MC Glauber)
- Comparison of the impact parameter distribution between MC Glauber used in Centrality framework and the new method proposed by R. Rogly and G. Giacalone

Motivation

Evolution of matter produced in heavy-ion collisions depend on its initial geometry

Goal: map collision geometry to the measurable quantities

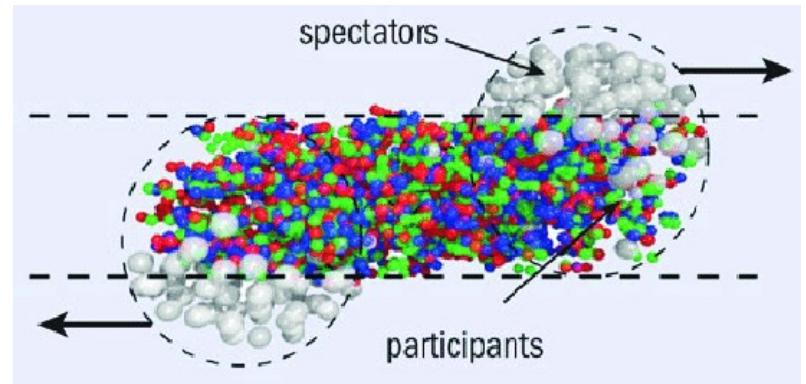
Comparison with existing data (RHIC BES, NA49/NA61 scans)

- **Collision geometry:**

impact parameter, number of participating nucleons, number of binary NN collisions, etc.

- **Measurable quantities:**

multiplicity of the produced charged particles, energy of the spectators



Application of the MC Glauber model in centrality determination in MPD experiment

STAR BES-II program

Beam Energy (GeV/nucleon)	$\sqrt{s_{NN}}$ (GeV)	Run Time	Species	Number Events
9.8	19.6	4.5 weeks	Au+Au	400M MB
7.3	14.5	5.5 weeks	Au+Au	300M MB
5.75	11.5	5 weeks	Au+Au	230M MB
4.6	9.1 ¹	4 weeks	Au+Au	160M MB
9.8	4.5 (FXT)	2 days	Au+Au	100M MB
7.3	3.9 (FXT)	2 days	Au+Au	100M MB
5.75	3.5 (FXT)	2 days	Au+Au	100M MB
31.2	7.7 (FXT)	2 days	Au+Au	100M MB
19.5	6.2 (FXT)	2 days	Au+Au	100M MB
13.5	5.2 (FXT)	2 days	Au+Au	100M MB

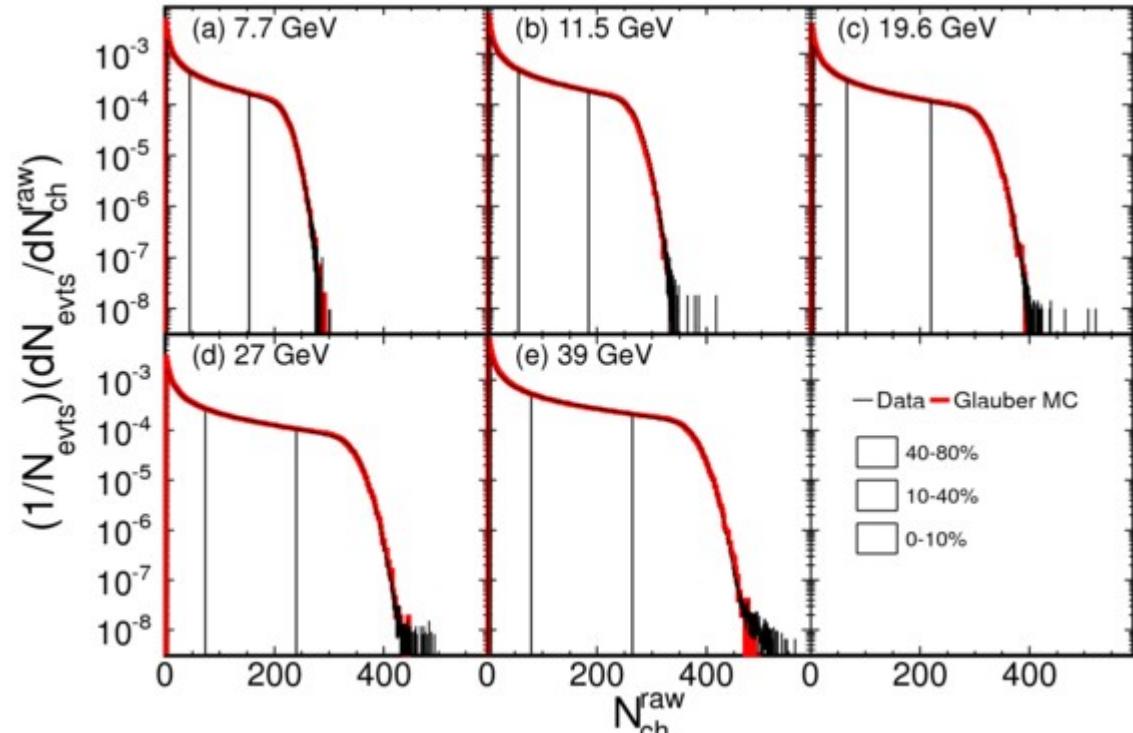
Many measurements at NICA energy range will be done during STAR BES-II
Will require comparison of the future MPD measurements with the RHIC/SPS₅

Centrality in STAR

- Uncorrected charged particle multiplicity distribution in TPC ($|\eta|<0.5$)
- Comparison with MC Glauber simulations
- Fitted using two-component model:

$$\frac{dN_{ch}}{d\eta} \Bigg|_{\eta=0} = n_{pp} \left[(1-x) N_{part}/2 + x N_{coll} \right]$$

Similar centrality estimator is needed for comparisons with STAR

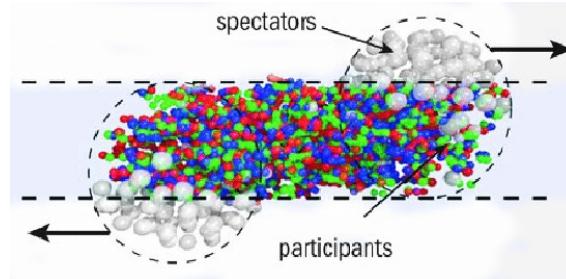


Phys. Rev. C 86 (2012) 54908

Centrality determination in MPD (NICA)

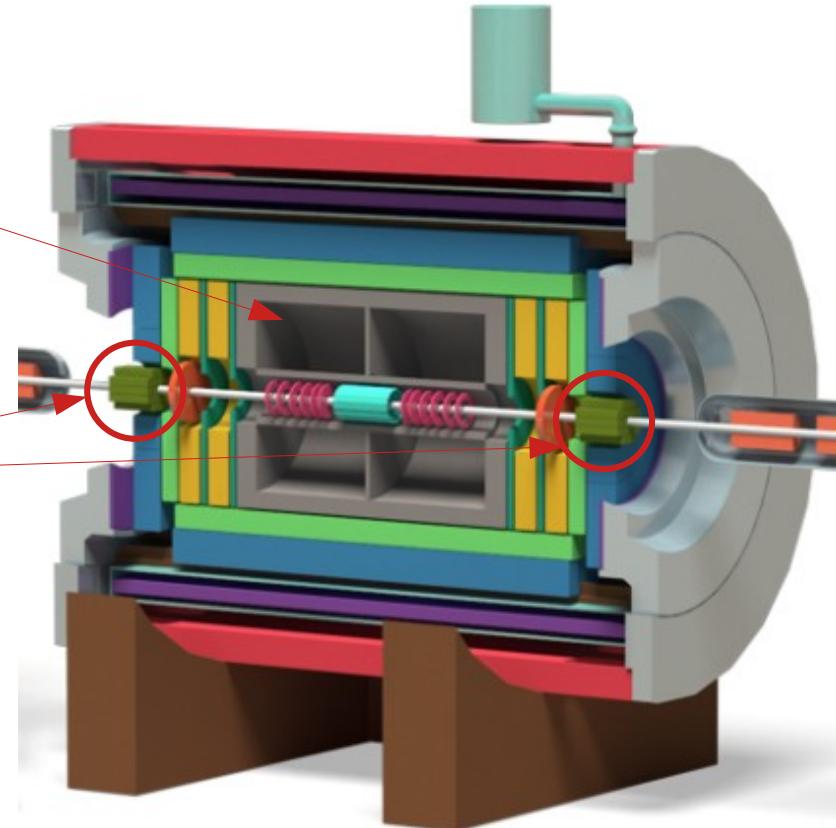
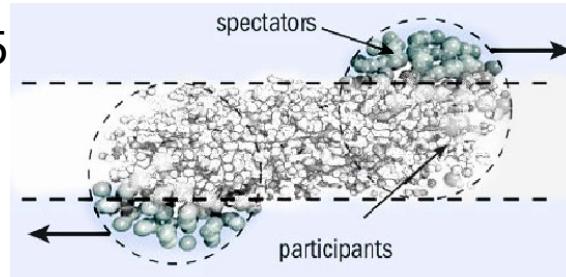
- Time Projection Chamber (TPC)

$$|\eta| < 1.5$$

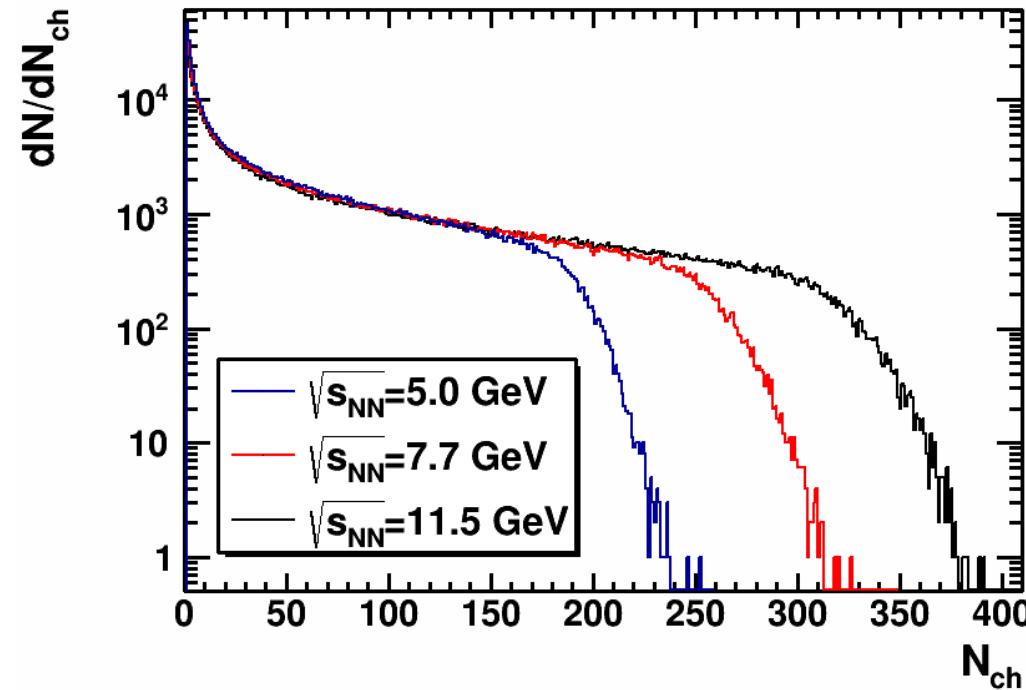


- Forward Hadron Calorimeter (FHCAL)

$$2 < |\eta| < 5$$



Charged particle multiplicity in MPD



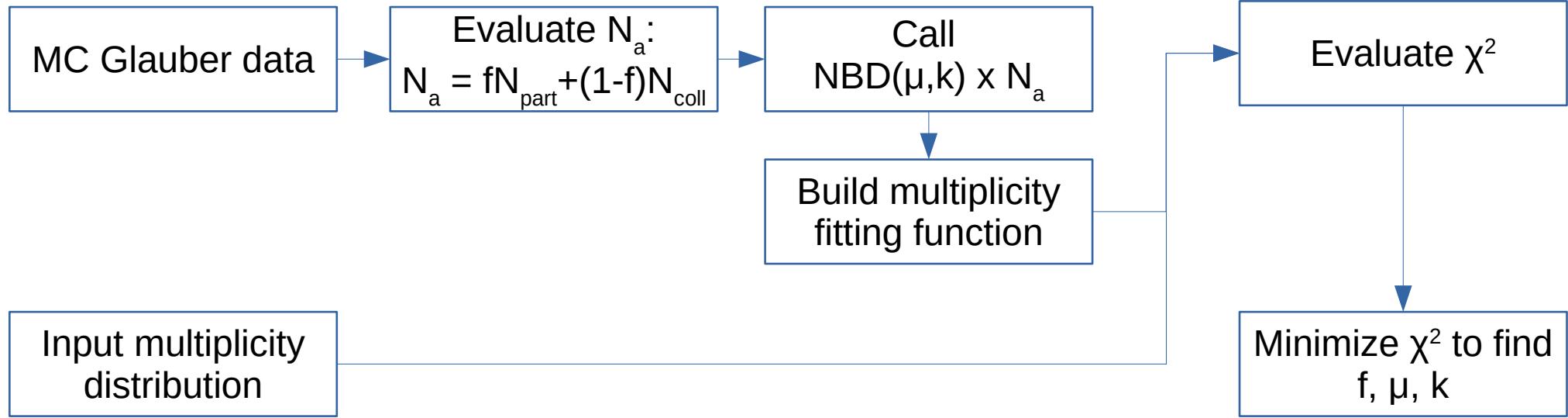
Reconstructed data:

- UrQMD 3.4 simulation
 - Au+Au, $N_{ev}=500k$, $\sqrt{s_{NN}}=5, 7.7, 11.5 \text{ GeV}$
- GEANT4 MPD detector simulation
- Reconstruction procedure:
 - Realistic tracking in TPC (Cluster Finder)

Used particle selection:

- $|\eta| < 0.5$
- $p_T > 0.15 \text{ GeV}/c$

Integrating the CBM Centrality framework



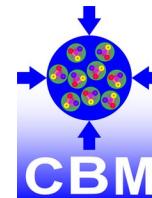
This centrality procedure was used in CBM, NA49, and NA61/SHINE:

Acta Phys. Polon. Supp. 10 (2017) 919

EPJ Web Conf. 182 (2018) 02132

Implementation in MPD: <https://github.com/IlyaSegal/NICA>

Lubynets O., Selyuzhenkov I., Klochkov V. 33-rd CBM CM



Glauber Model configuration

C. Loizides, J. Nagle and P. Steinberg, SoftwareX 1-2 (2015) 13-18
Used TGlauberMC-3.2 version from tgleubermc.hepforge.org

Input to the model

- Inelastic NN cross section
 - $\sigma_{\text{NN}}=29.3 \text{ mb}$ for $\sqrt{s}_{\text{NN}}=5.0 \text{ GeV}$
 - $\sigma_{\text{NN}}=29.7 \text{ mb}$ for $\sqrt{s}_{\text{NN}}=7.7 \text{ GeV}$
 - $\sigma_{\text{NN}}=31.2 \text{ mb}$ for $\sqrt{s}_{\text{NN}}=11.5 \text{ GeV}$
- Colliding nuclei
 - “Au(197,79)”+”Au(197,79)”

Output from the model

- TNtuple with model parameters:
 - Impact parameter b
 - Number of participating in the collision nucleons N_{part}
 - Number of NN collisions N_{coll}
 - Participant eccentricity ϵ_n
 - etc.

In progress: comparison MC Glauber with GLISSANDO arXiv:1901.04484 [nucl-th]

Centrality framework configuration

NBD Equation:

$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \cdot \left(\frac{\mu}{k}\right)^n \cdot \left(\frac{\mu}{k} + 1\right)^{n+k}$$

Parameter range:

$$f = (0-1), f_{step} = 0.01$$
$$k = (0-50), k_{step} = 1$$

Fitting function for charged particle multiplicity:

$$N_{ch}(f, \mu, k) = P_{\mu,k}(n) \cdot [f N_{part} + (1-f) N_{coll}]$$

Normalization of the total number of events:

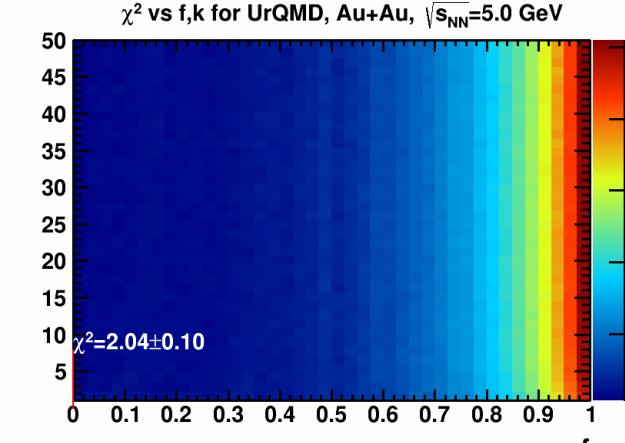
$$\frac{N_{ev}^{reco}}{N_{ev}^{MC Glauber}} = \frac{1}{10}$$

Fitting region:

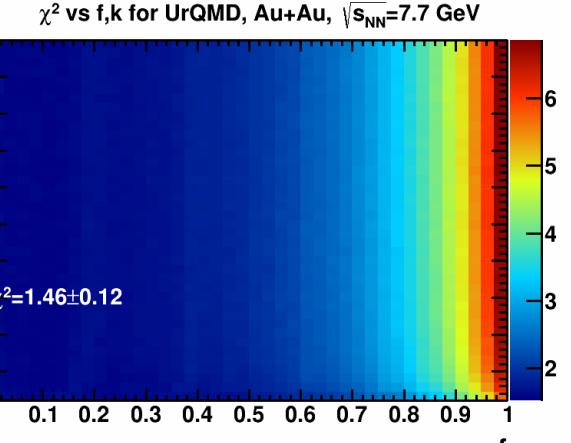
$$N_{ch} = \begin{cases} (10-235), \sqrt{s_{NN}} = 5. GeV \\ (20-310), \sqrt{s_{NN}} = 7.7 GeV \\ (15-380), \sqrt{s_{NN}} = 11.5 GeV \end{cases}$$

Fit parameters f,k vs χ^2

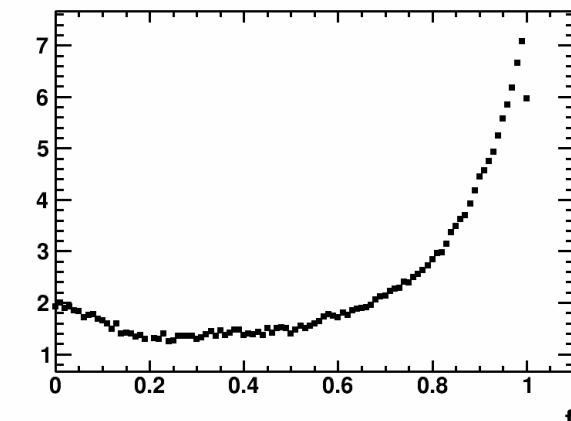
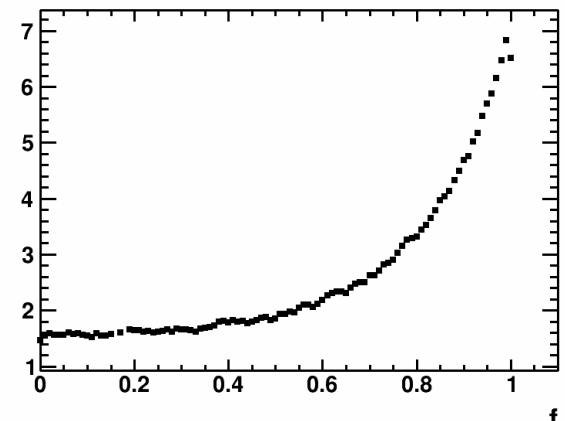
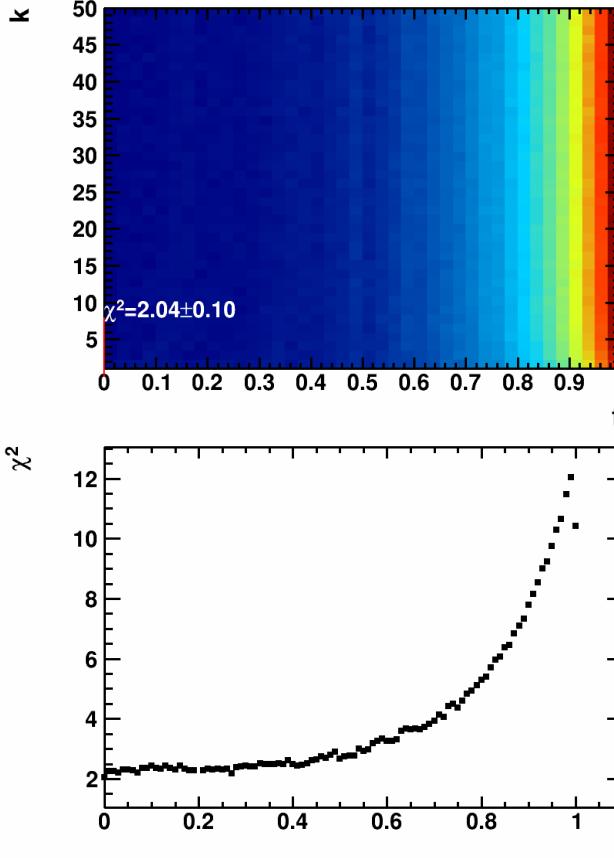
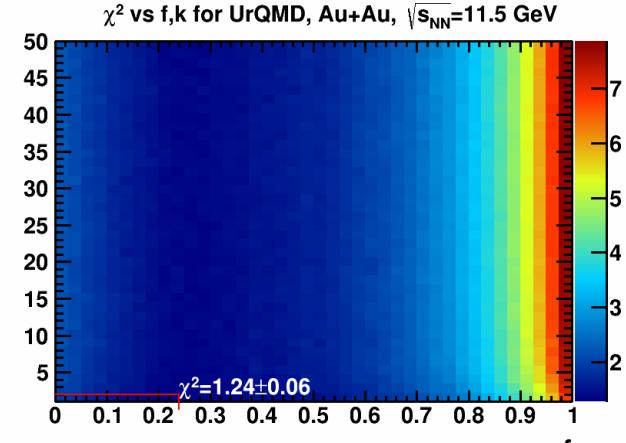
$f=0, k=8, \mu=0.23,$
 $\chi^2=2.04\pm0.10, M=(10,235)$



$f=0, k=14, \mu=0.31,$
 $\chi^2=1.46\pm0.12, M=(20,310)$

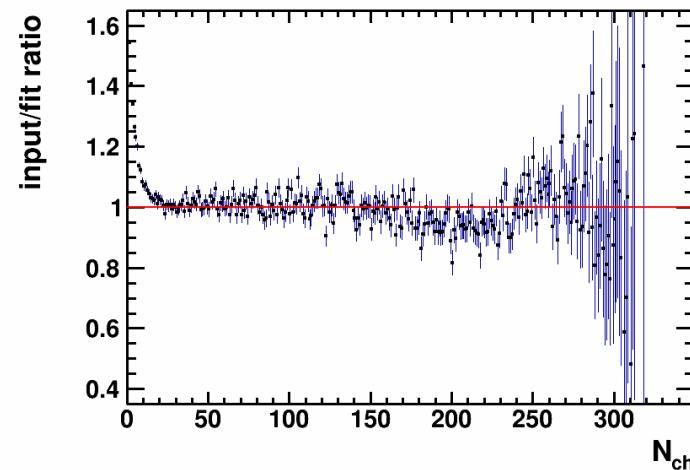
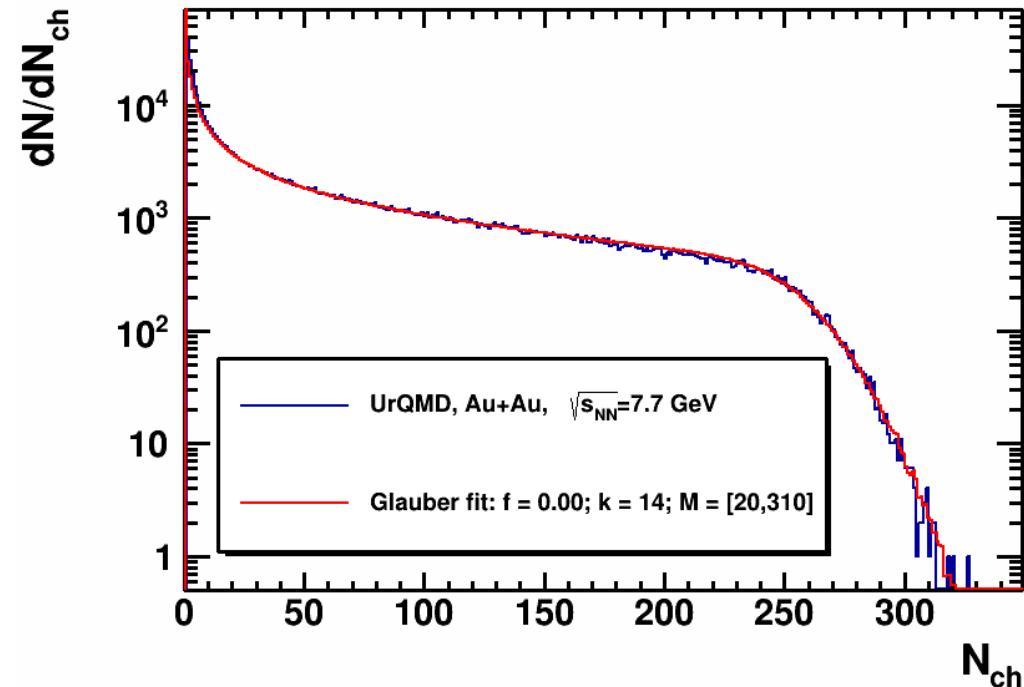


$f=0.24, k=2, \mu=0.71,$
 $\chi^2=1.24\pm0.06, M=(15,380)$



MC Glauber fit: h^\pm multiplicity

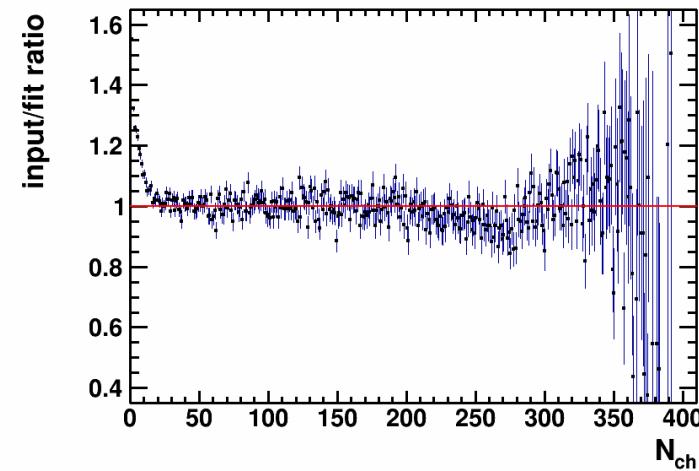
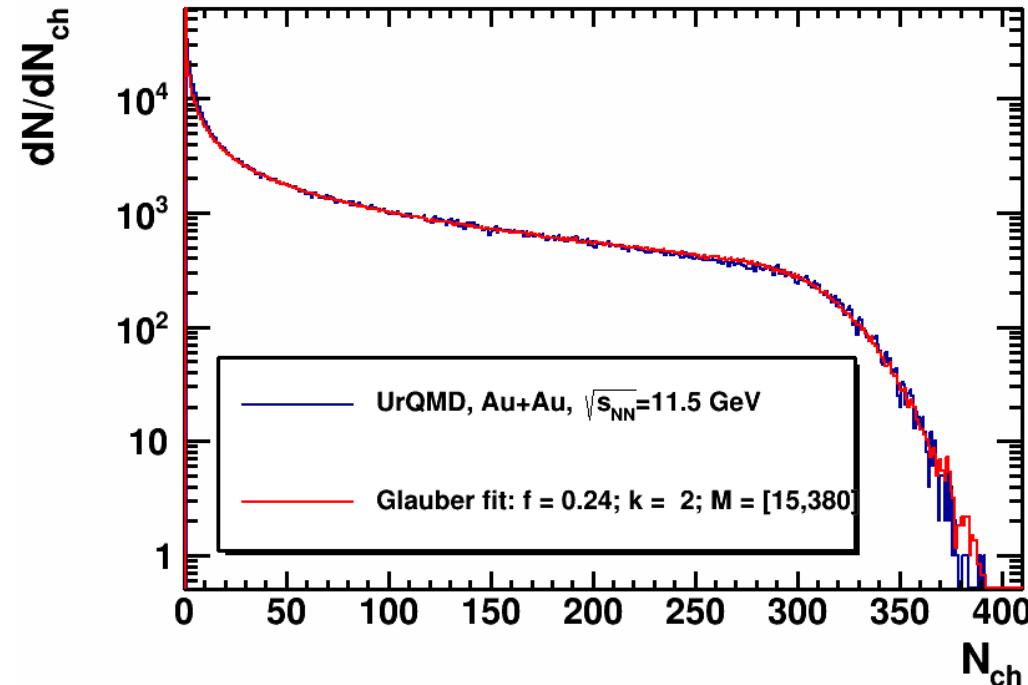
$f=0, k=14, \mu=0.31, \chi^2=1.46\pm0.12, M=(20,310)$



MC Glauber fit is in the good agreement with simulated input for the large multiplicity region

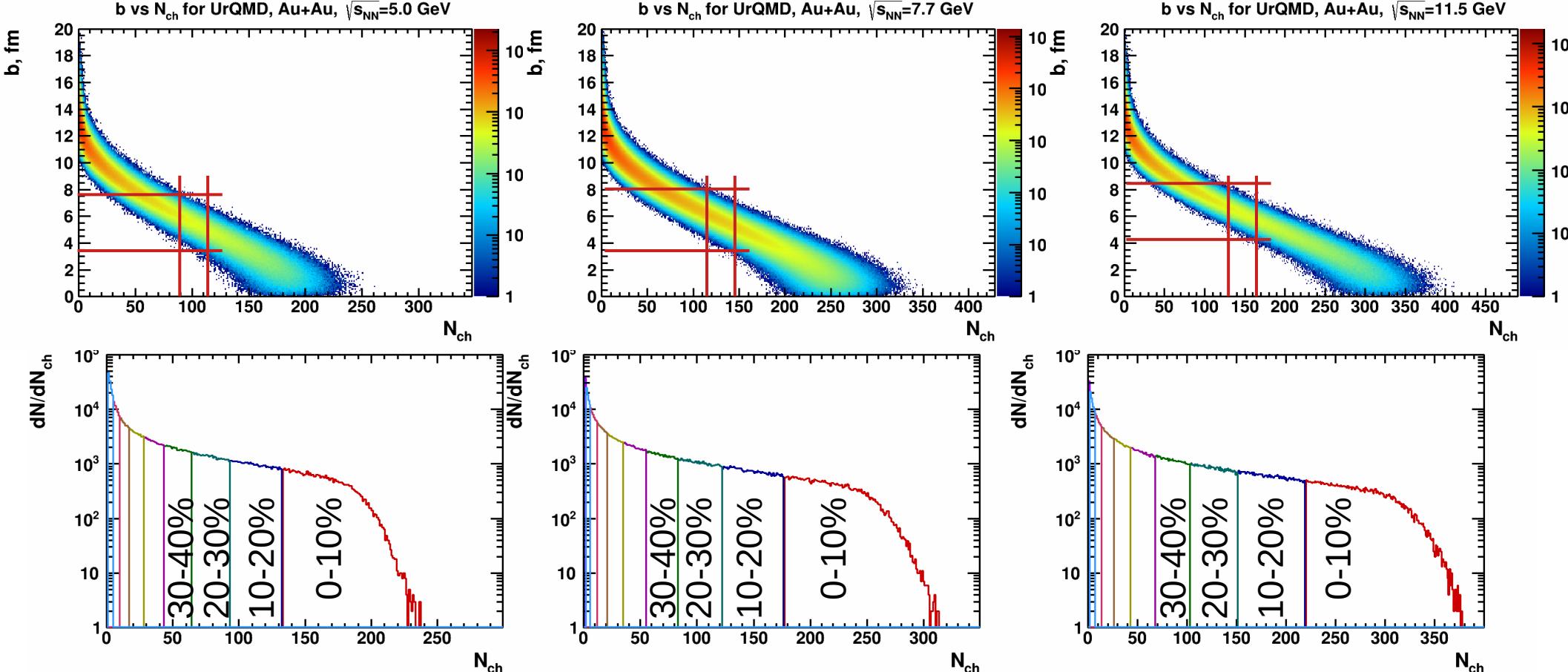
MC Glauber fit: h^\pm multiplicity

$f=0.24, k=2, \mu=0.71, \chi^2=1.24\pm0.06, M=(15,380)$



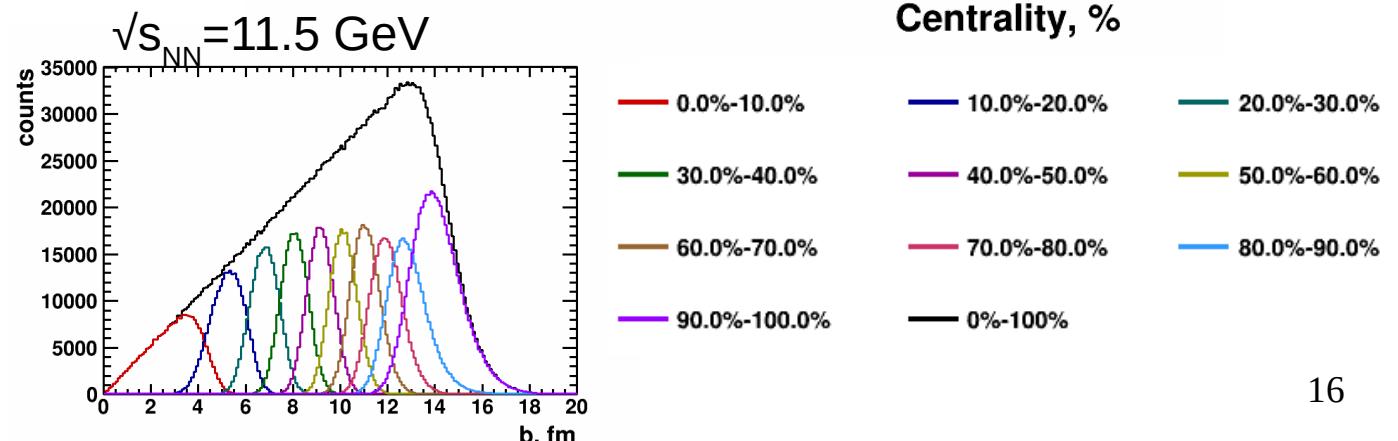
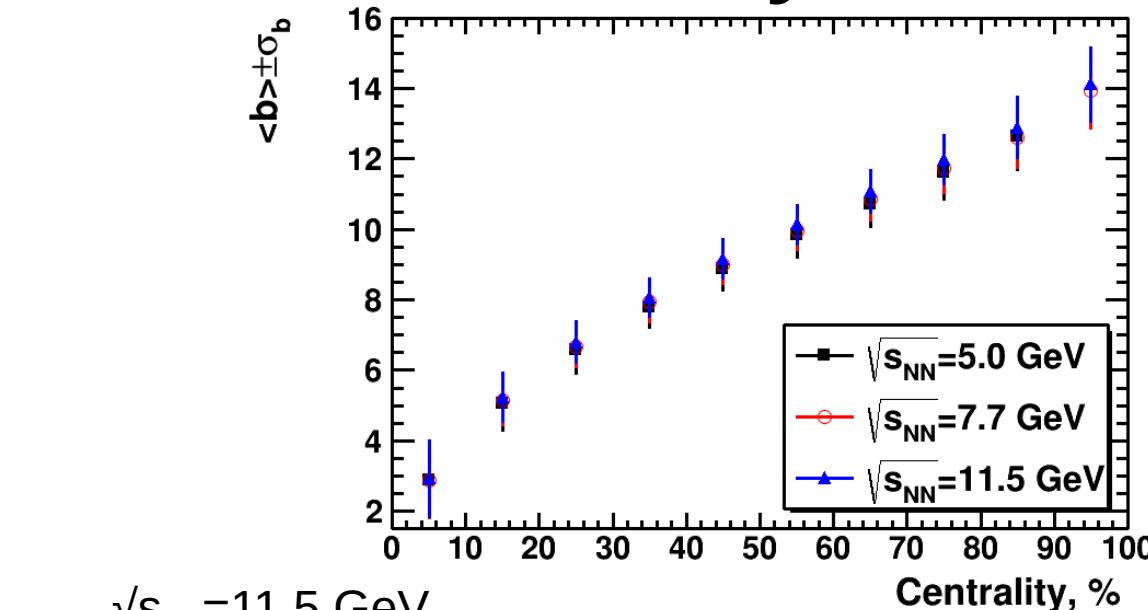
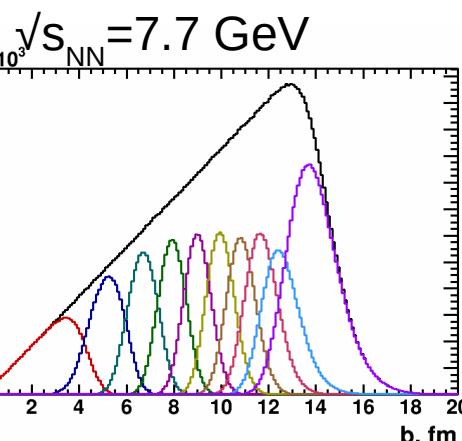
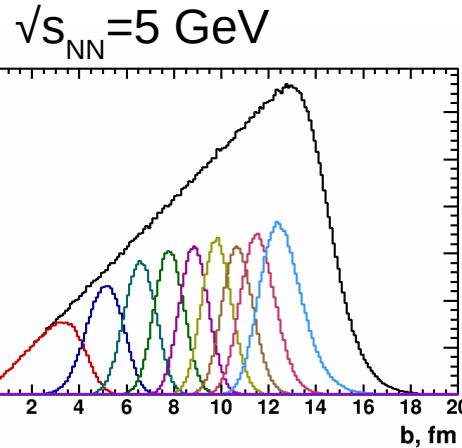
MC Glauber fit is in the good agreement with simulated input for the large multiplicity region

B vs N_{ch} & N_{ch} centrality classes

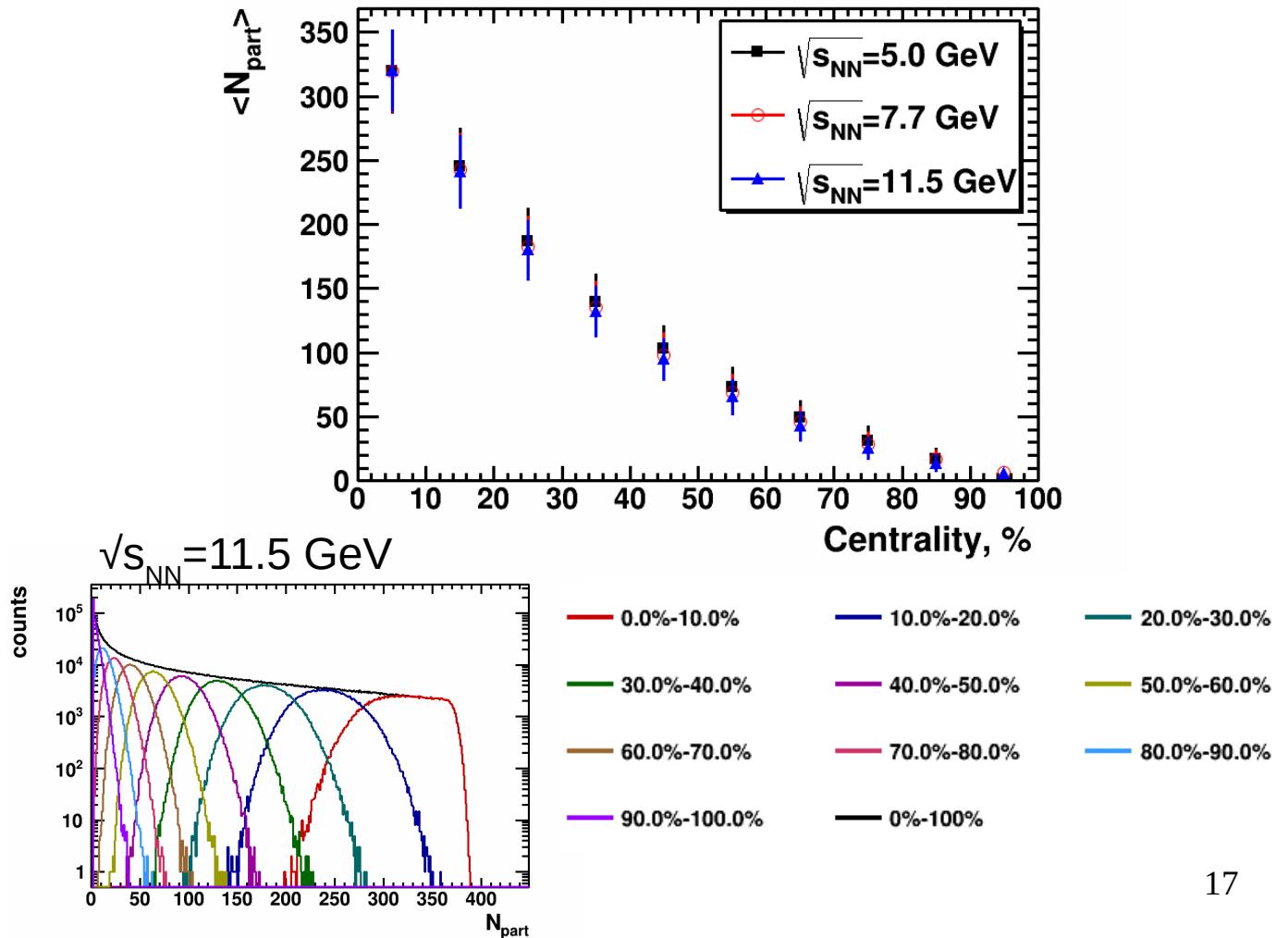
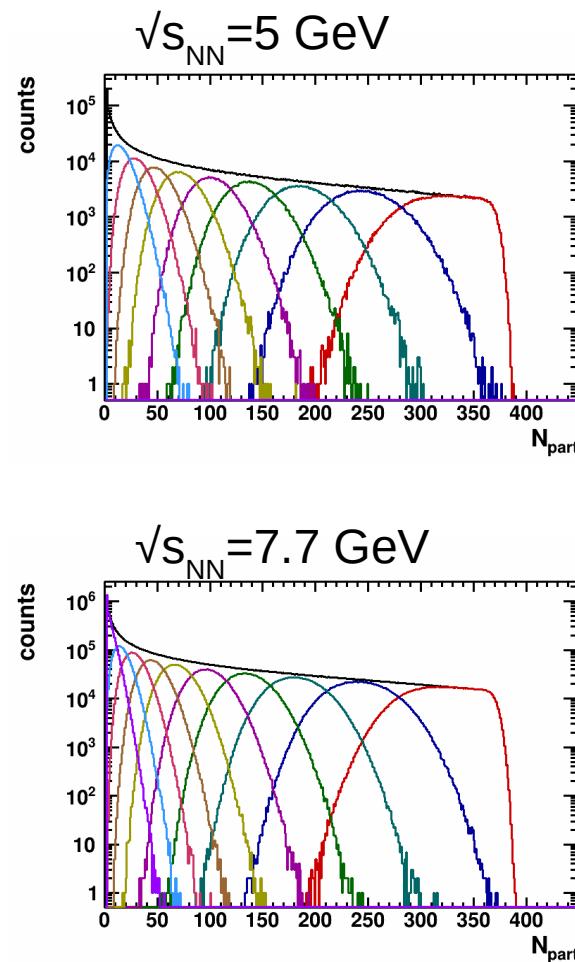


Events in multiplicities $M \pm \Delta M$ have impact parameter in range $b \pm \sigma_b$

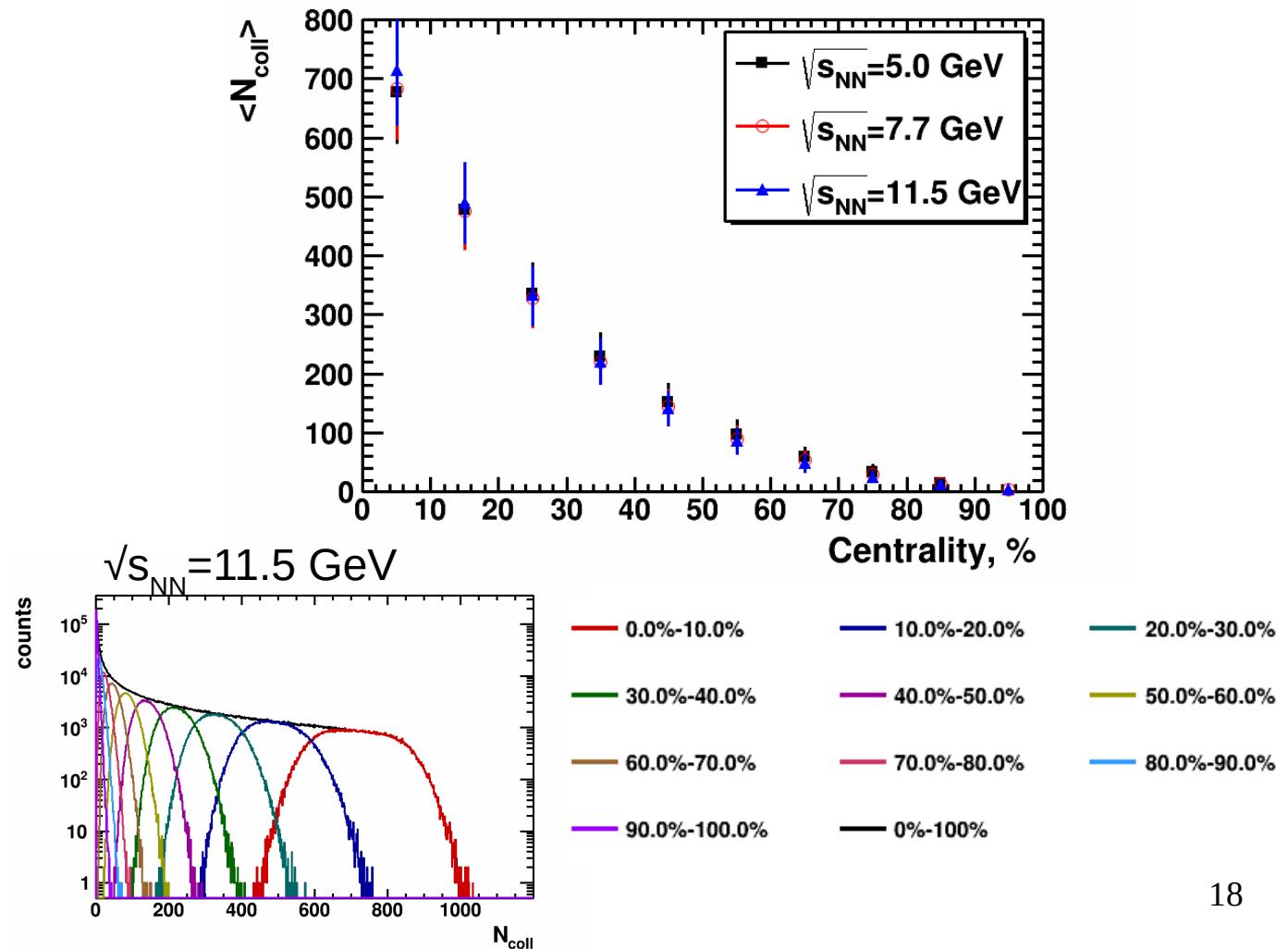
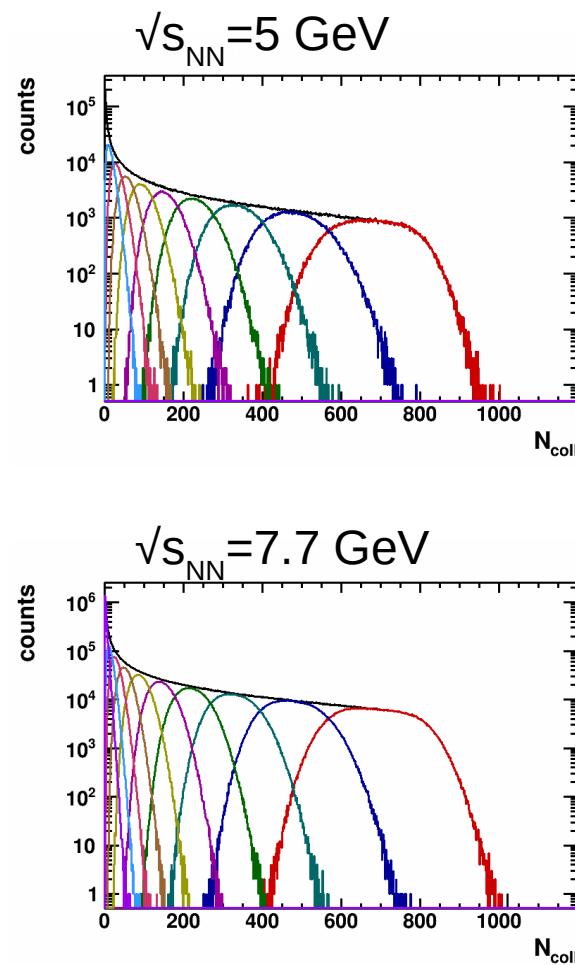
b distribution in centrality classes



N_{part} distribution in centrality classes

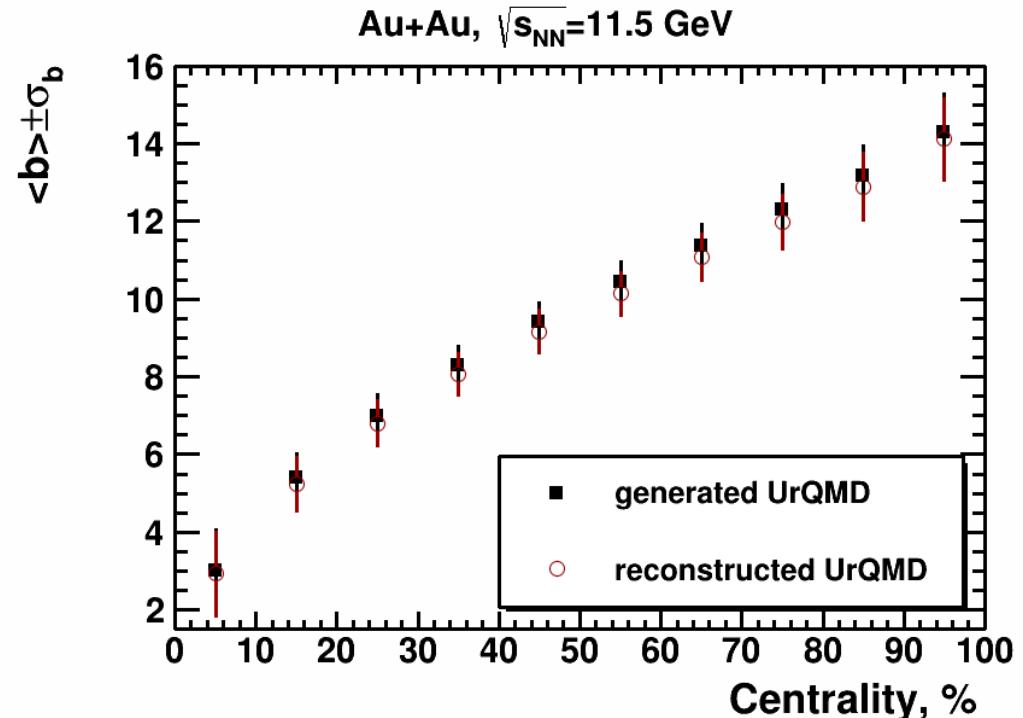
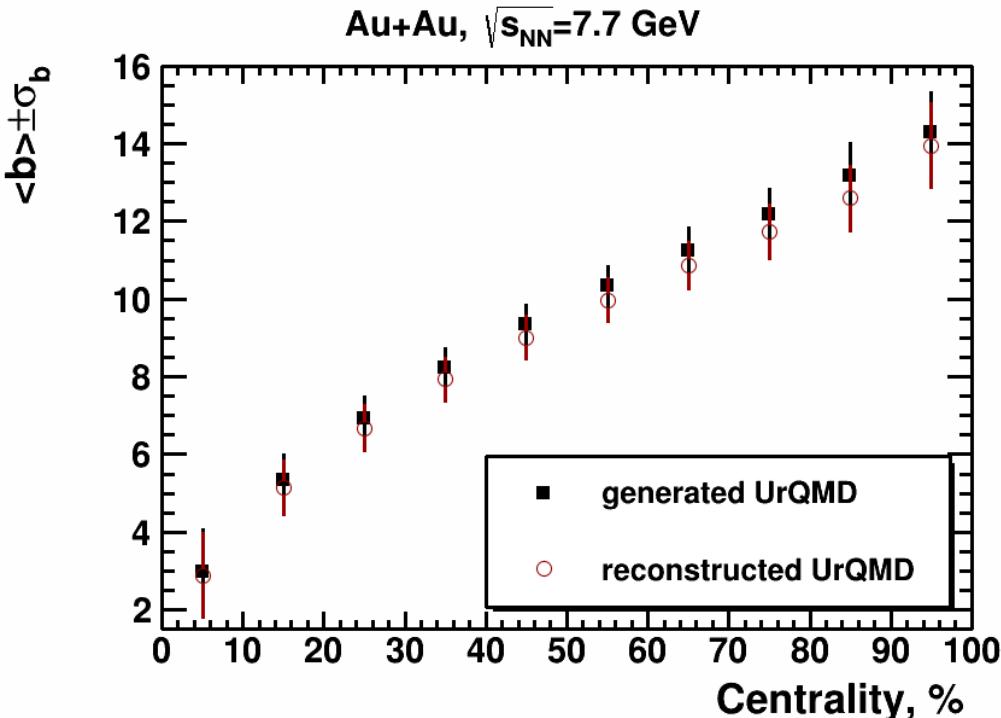


N_{coll} distribution in centrality classes



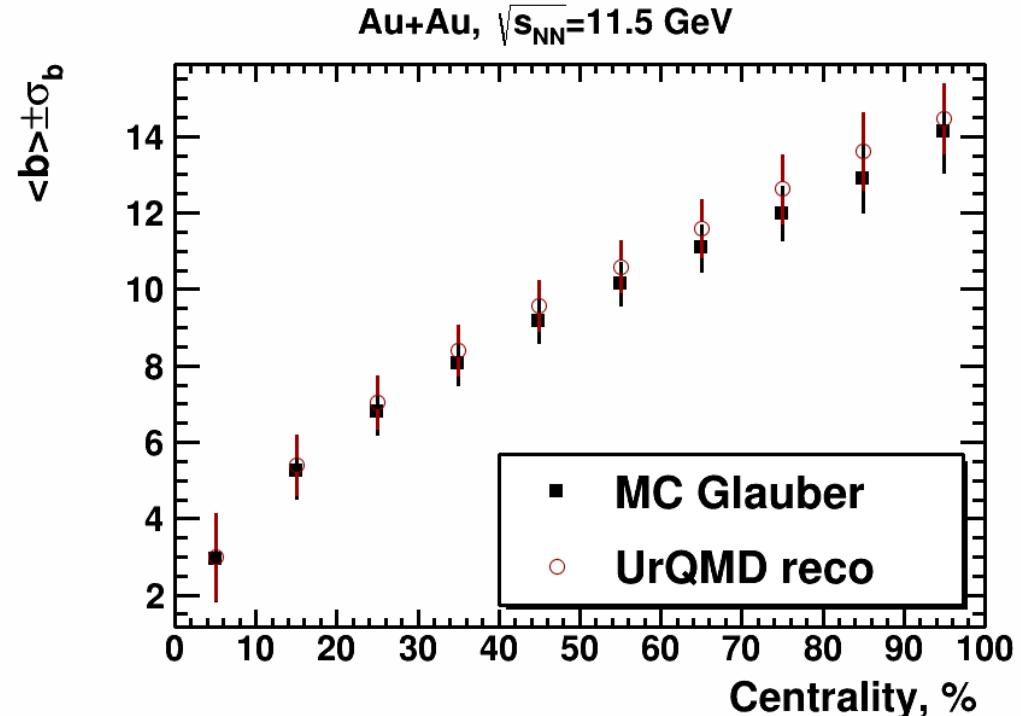
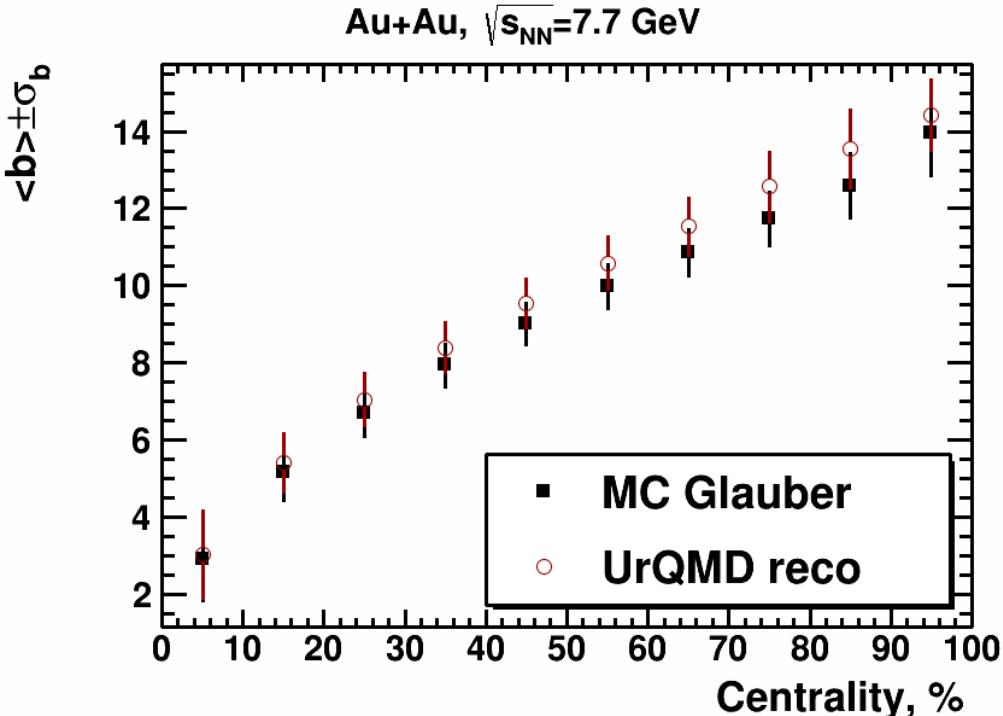
Comparison of the UrQMD, PHSD, SMASH and MC Glauber parameters

Generated vs reconstructed UrQMD



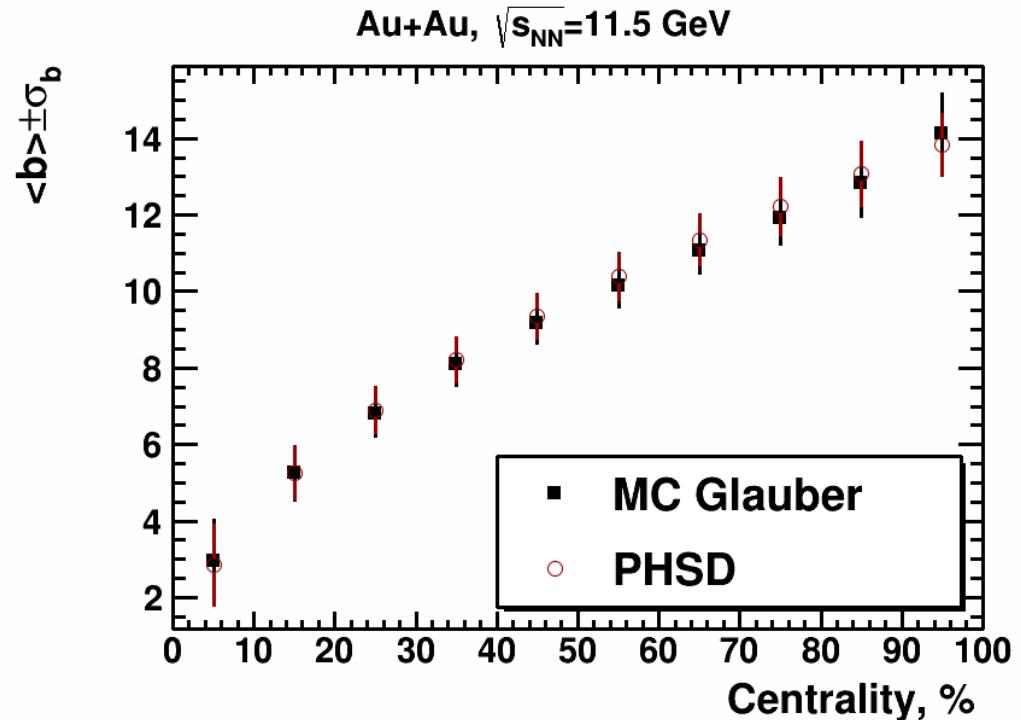
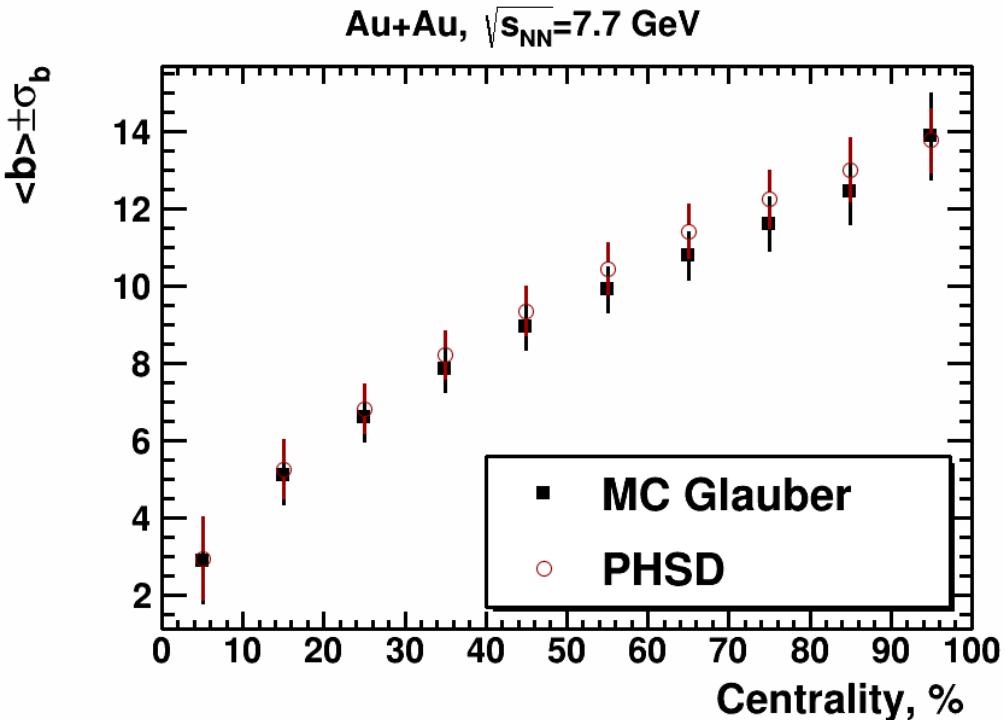
Reasonable agreement between reconstructed data and pure model

b vs centrality: MC Glauber vs UrQMD

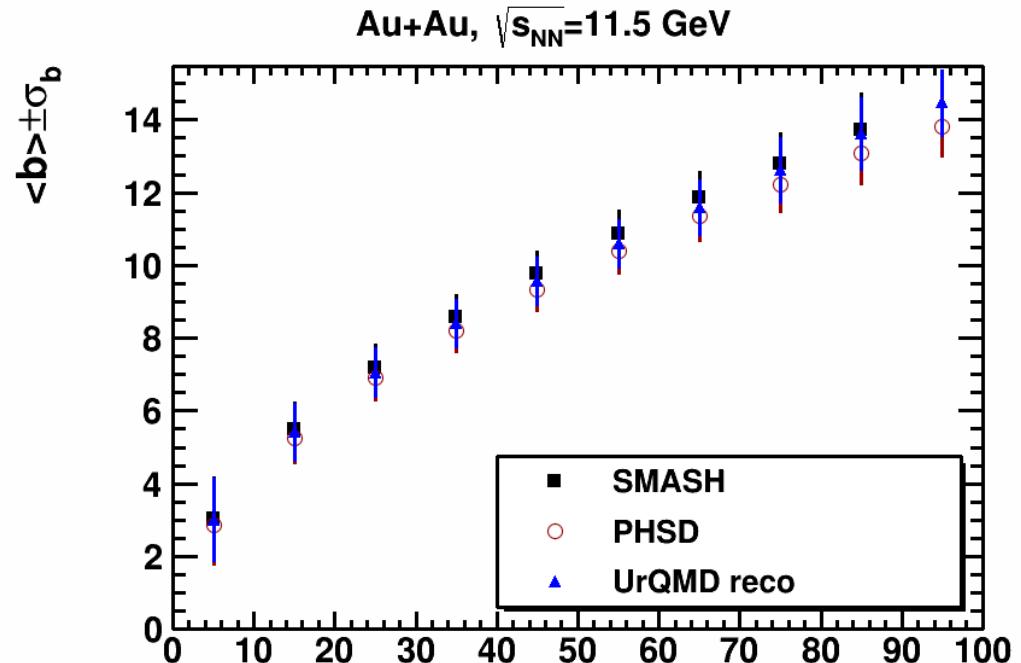
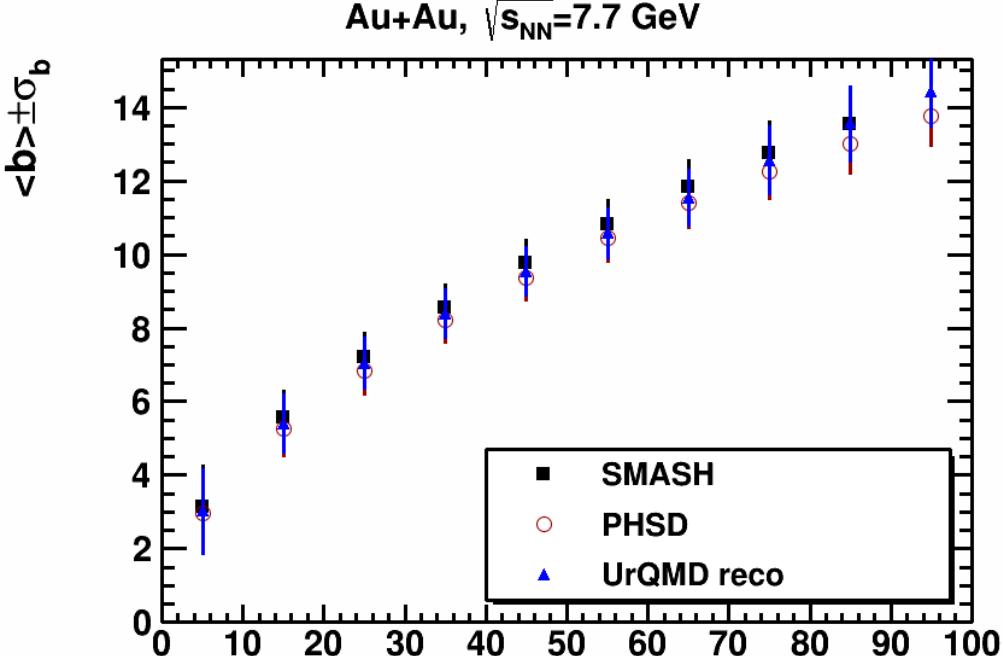


Reasonable agreement between MC Glauber and UrQMD

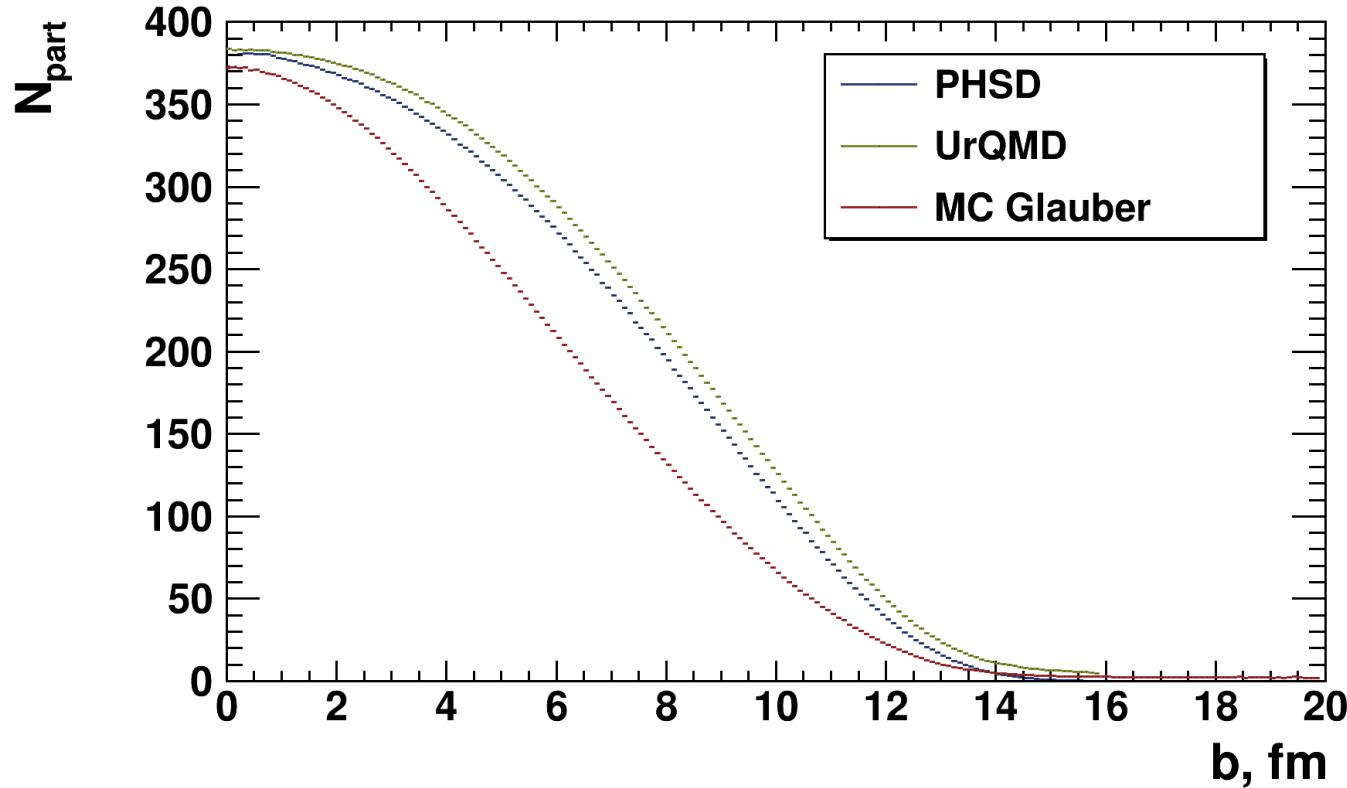
b vs centrality: Glauber vs PHSD



b vs centrality: all models



N_{part} vs b : all models



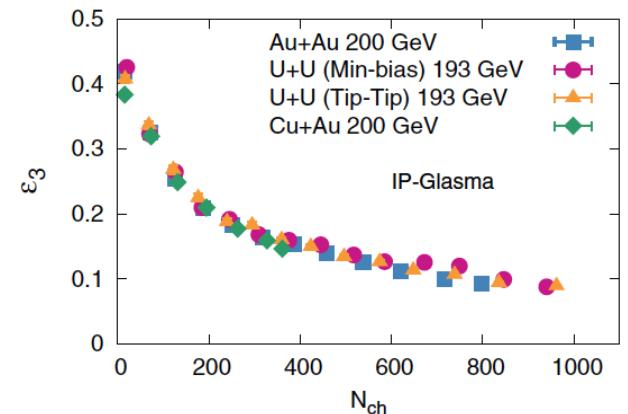
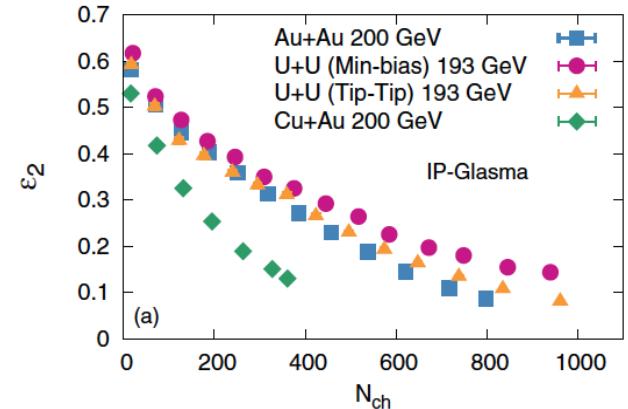
Eccentricity ε_n

- Eccentricity characterizes initial-state spatial anisotropy
- In MC Glauber, ε_n defined as a $\varepsilon_{\text{part}}$ in the center-of-mass system of the participant nuclei (Phys.Rev. C81 (2010) 054905):

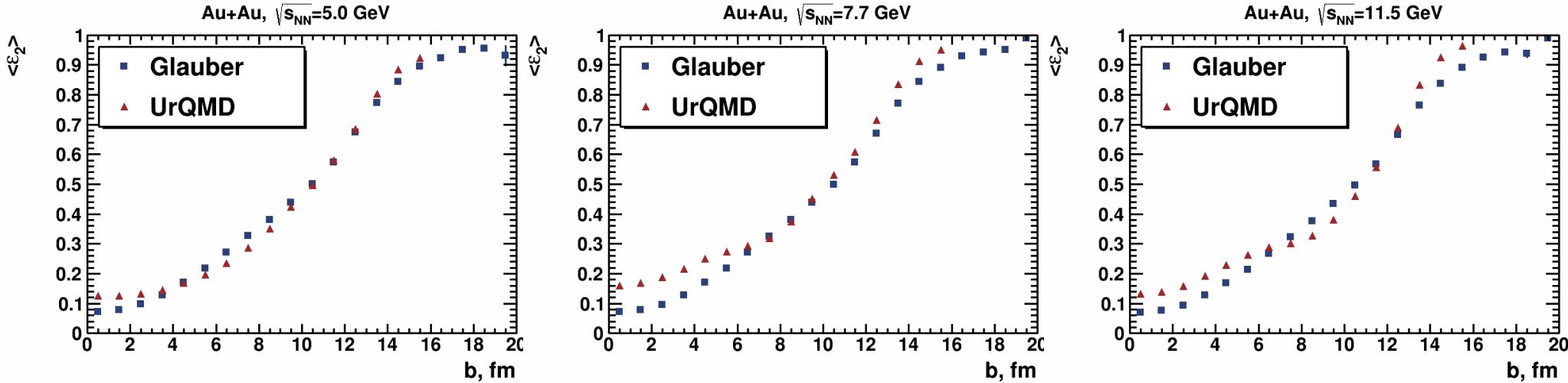
$$\varepsilon_n = \frac{\sqrt{\langle r^2 \cos(n\varphi) \rangle^2 + \langle r^2 \sin(n\varphi) \rangle^2}}{\langle r^2 \rangle}$$

- ε_2 is system dependent
- ε_3 is system independent

B. Schenke, et al.
PRC 89, 064908 (2014)

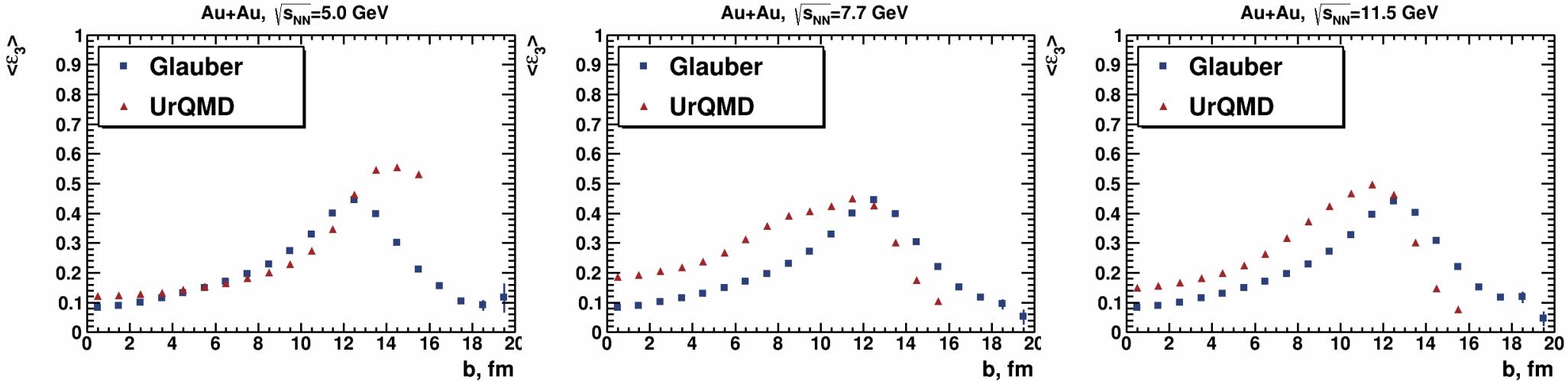


Eccentricity: Comparison w/ UrQMD



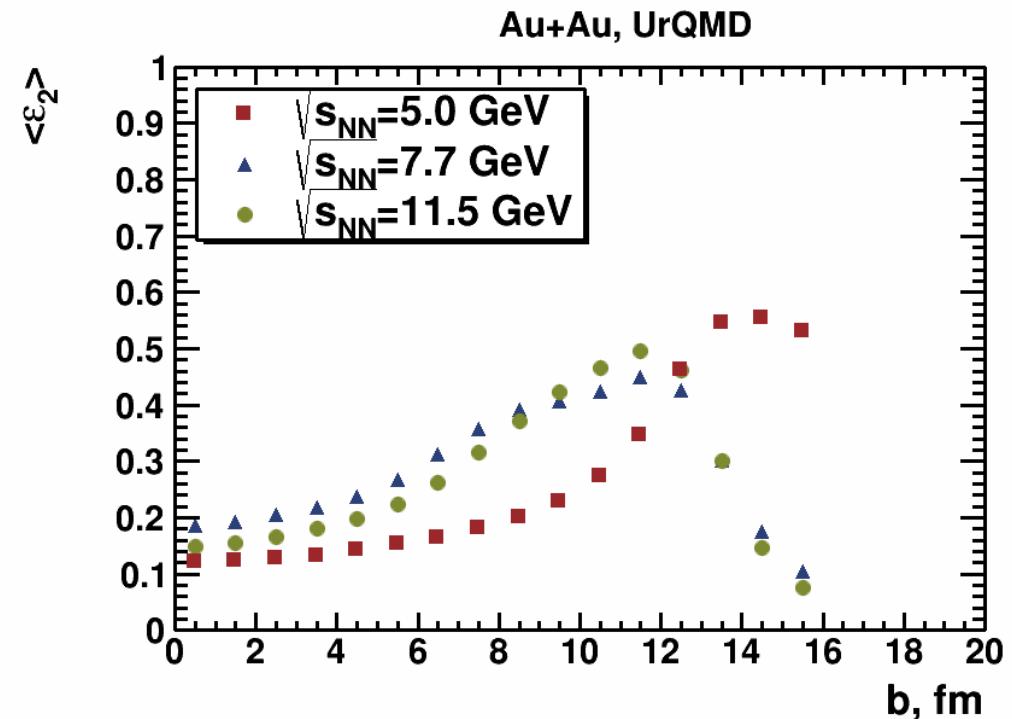
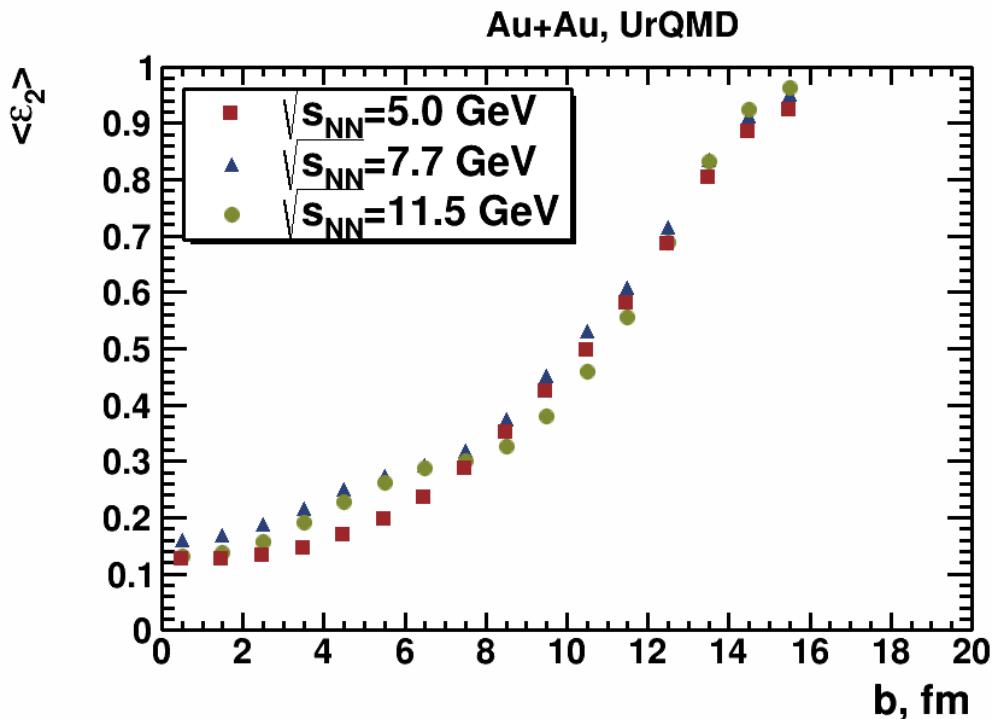
Notable difference between MC Glauber and UrQMD eccentricities

ε_3 : Comparison w/ UrQMD



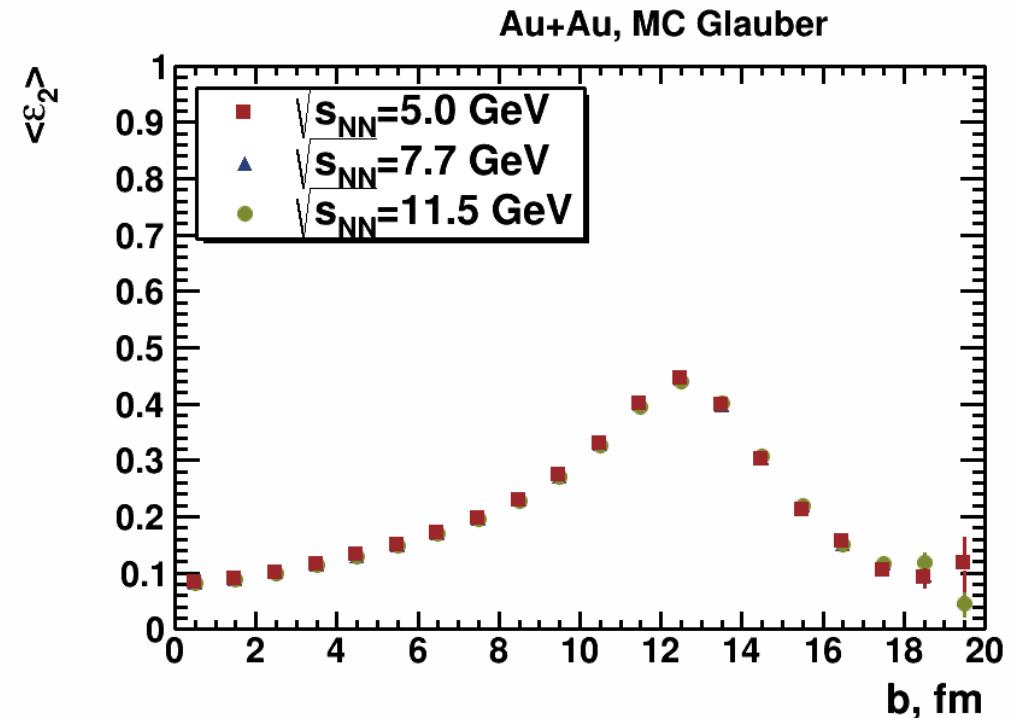
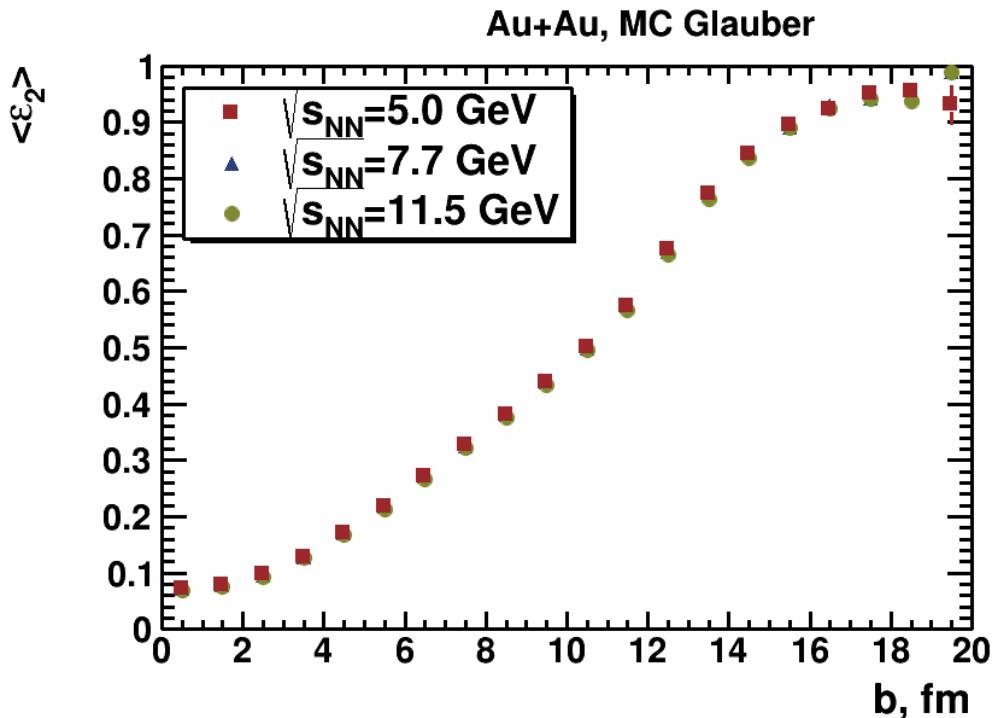
Notable difference between MC Glauber and UrQMD

ε_n : UrQMD at different \sqrt{s}_{NN}



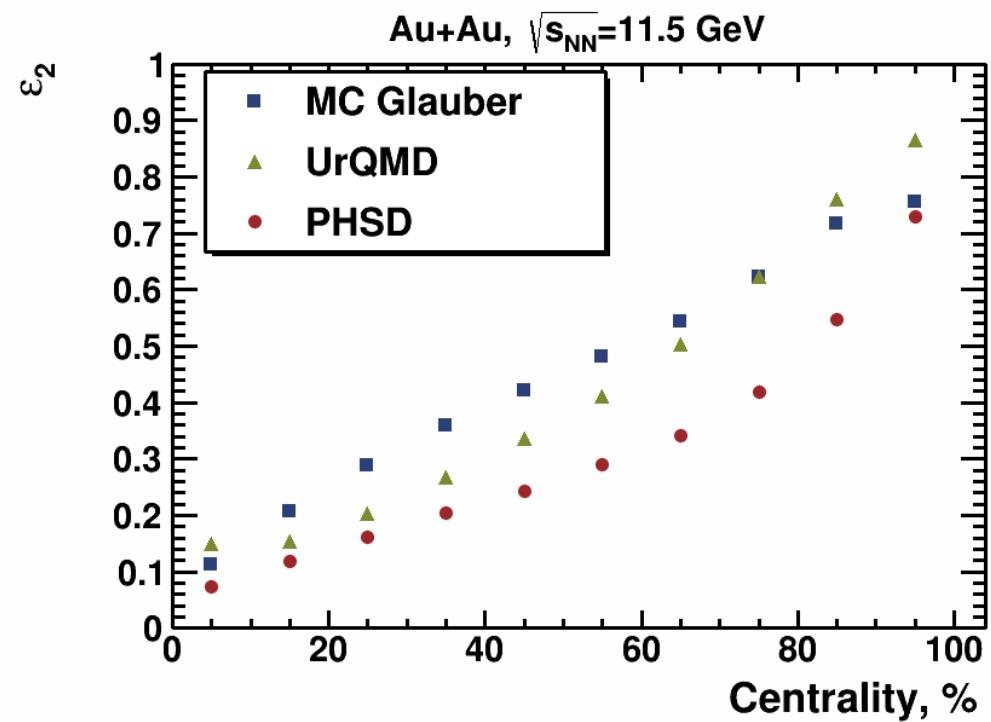
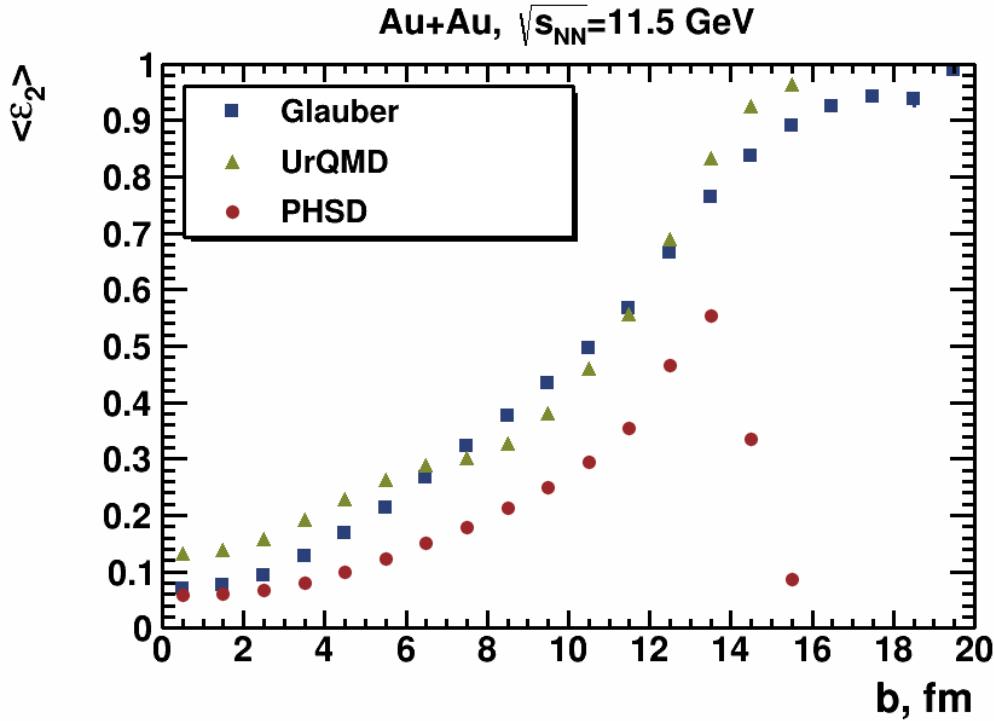
Notable difference for ε_3 at $\sqrt{s}_{NN} = 5$ GeV for UrQMD

ε_n : MC Glauber at different \sqrt{s}_{NN}

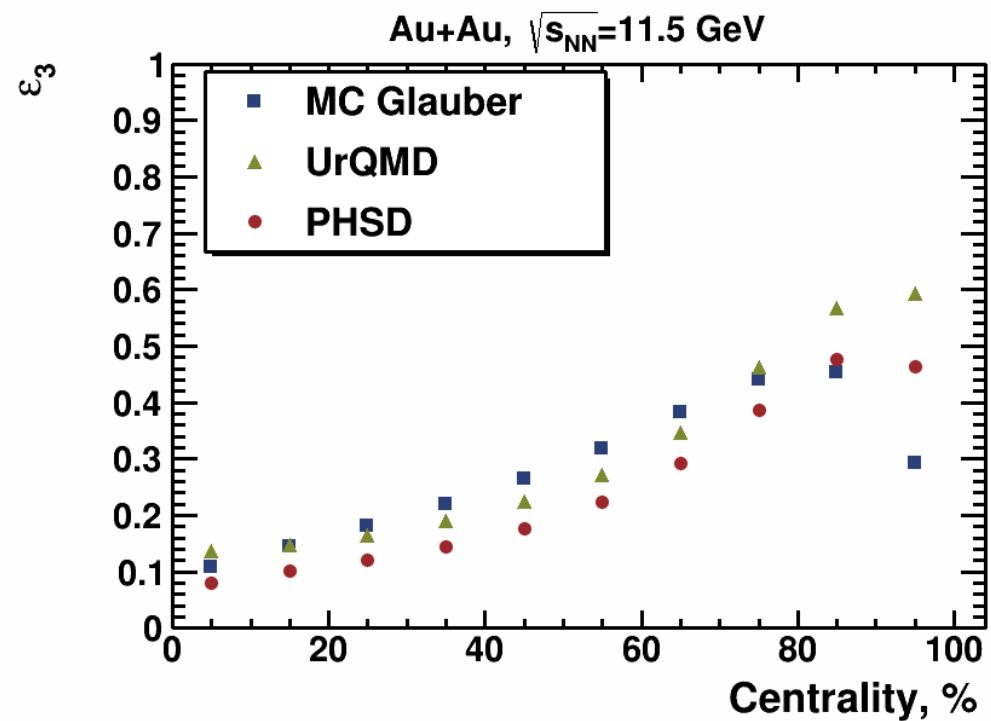
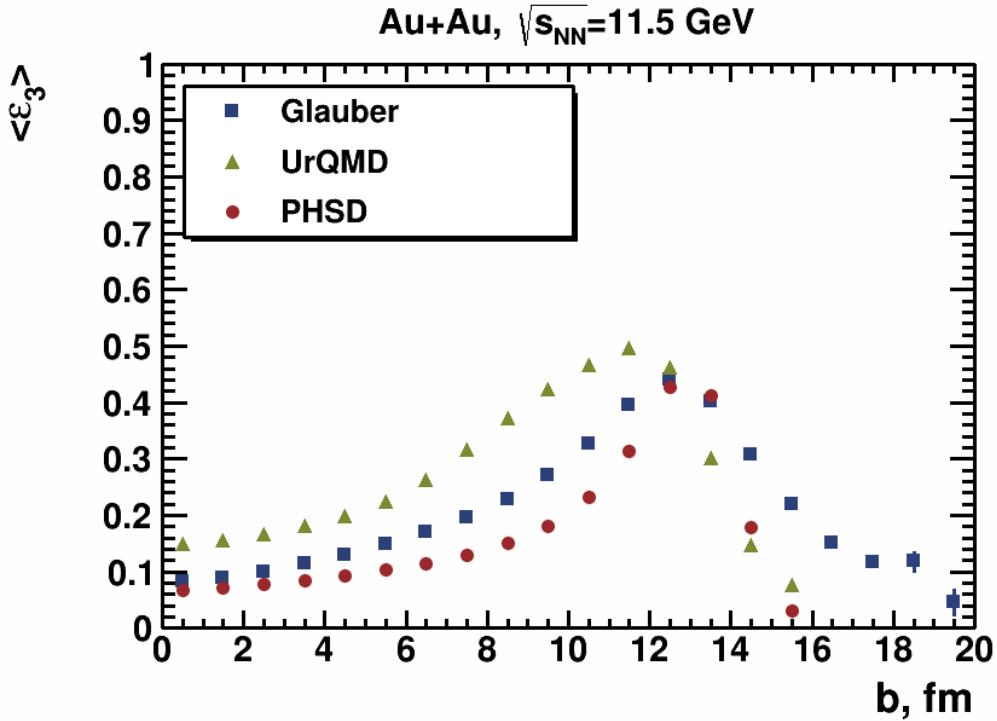


No notable difference for ε_3 for MC Glauber

Eccentricity comparison: ε_2



Eccentricity comparison: ε_3



Reconstructing the impact parameter of nucleus-nucleus collisions

based on the work of R. Rogly and G. Giacalone

Phys. Rev. C97 (2018) no.1, 014905
Phys. Rev. C98 (2018) no.2, 024902

N_{ch} to b correlation

Probability of N_{ch} for fixed b :

-Gaussian approach

(Phys.Rev. C97 (2018) no.1, 014905):

$$P(N_{ch}|c_b) = \frac{1}{\sigma(c_b)} \exp\left(\frac{-(N_{ch} - \bar{N}_{ch})^2}{2 \sigma(c_b)^2}\right)$$

-Approach using gamma distribution

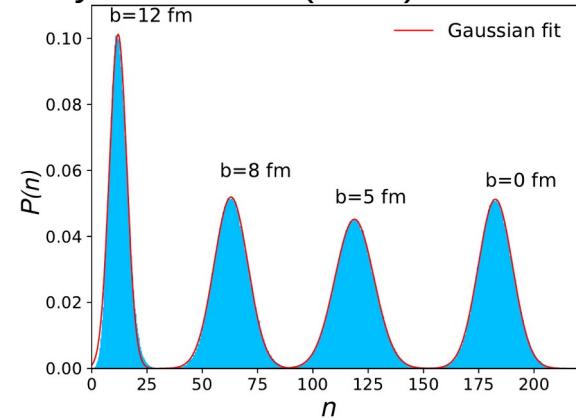
(Phys.Rev. C98 (2018) no.2, 024902):

$$P(N_{ch}|c_b) = \frac{1}{\Gamma(k(c_b)) \theta^k} N_{ch}^{k(c_b)-1} e^{-n/\theta}$$

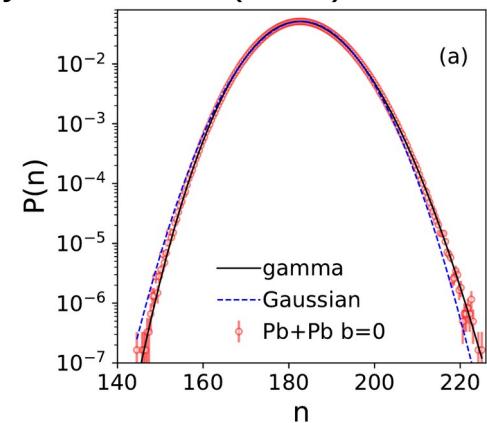
where c_b – centrality defined from impact parameter:

$$c_b = \frac{1}{\sigma_{inel}} \int_0^b P_{inel}(b') 2\pi b' db' \simeq \frac{\pi b^2}{\sigma_{inel}}$$

Phys.Rev. C97 (2018) no.1, 014905



Phys.Rev. C98 (2018) no.2, 024902



Fit function

Phys.Rev. C97 (2018) no.1, 014905

Probability function for multiplicity distribution:

$$P(N_{ch}) = \int_0^1 P(N_{ch} | c_b) d c_b$$

Parameters for Gaussian approach:

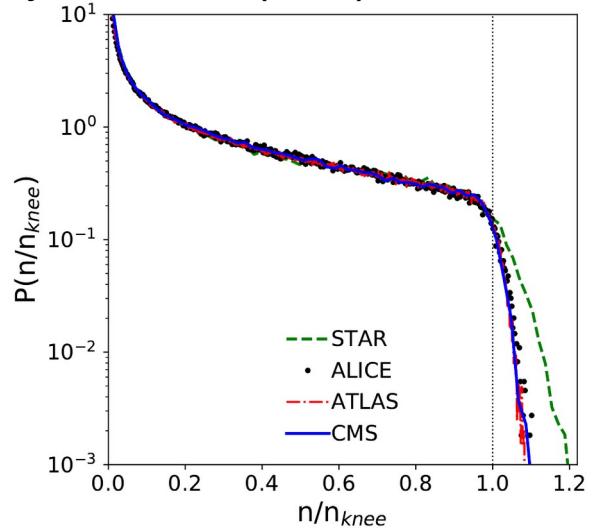
$$\overline{N}_{ch}(c_b) = N_{knee} \cdot \exp\left(-\sum_{i=1}^3 a_i c_b^i\right), \quad \sigma(c_b) = \sigma(0) \sqrt{\frac{\overline{N}_{ch}(c_b)}{\overline{N}_{ch}(0)}}$$

Free parameters: $N_{knee}, \sigma(0), a_i$.

Parameters for gamma-function approach:

$$k(c_b) = k_{max} \cdot \exp\left(-\sum_{i=1}^4 a_i c_b^i\right), \quad \theta = const \quad (k(c_b) \theta \equiv \overline{N}_{ch}(c_b), \quad \sqrt{k(0)} \theta \equiv \sigma(0))$$

Free parameters: k_{max}, θ, a_i .



Reconstruction of b

- Find probability of b for fixed N_{ch} using Bayes' theorem:

$$P(b|N_{ch}) = \frac{P(N_{ch}|b)P(b)}{P(n)} = \frac{P(N_{ch}|c_b) \frac{2\pi b}{\sigma_{inel}} P_{inel}(b)}{P(n)} = \frac{2\pi b}{\sigma_{inel} P(n)} P(N_{ch}|b),$$

where $P_{inel}(b) \approx 1$, $\sigma_{inel} = 685 \text{ fm}^2$, $c_b \approx \frac{\pi b^2}{\sigma_{inel}}$

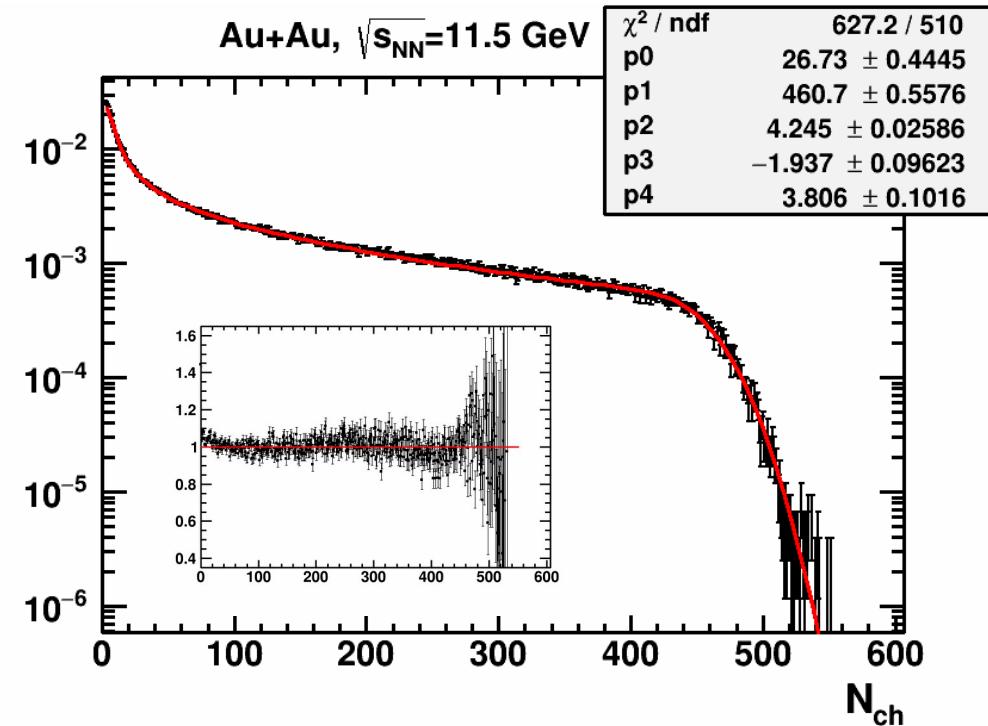
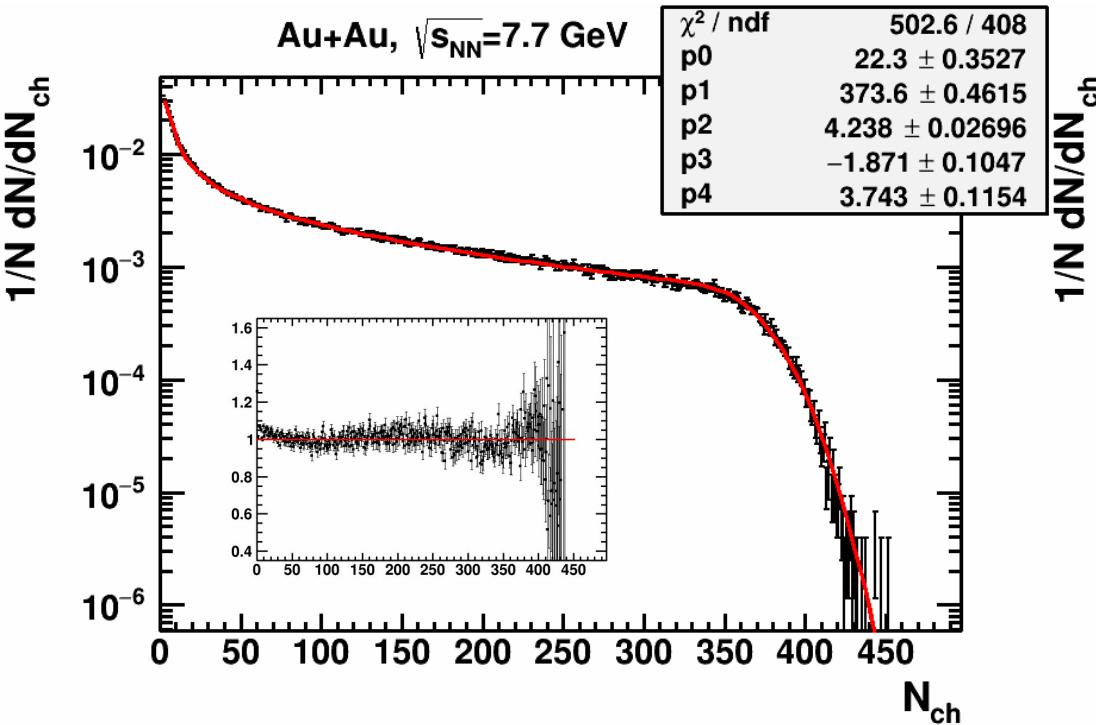
Experimental centrality:

$$c(N_{ch}) = \int_{N_{ch}}^{\infty} P(N'_{ch}) dN'_{ch}$$

Main steps to reconstruct b from N_{ch} :

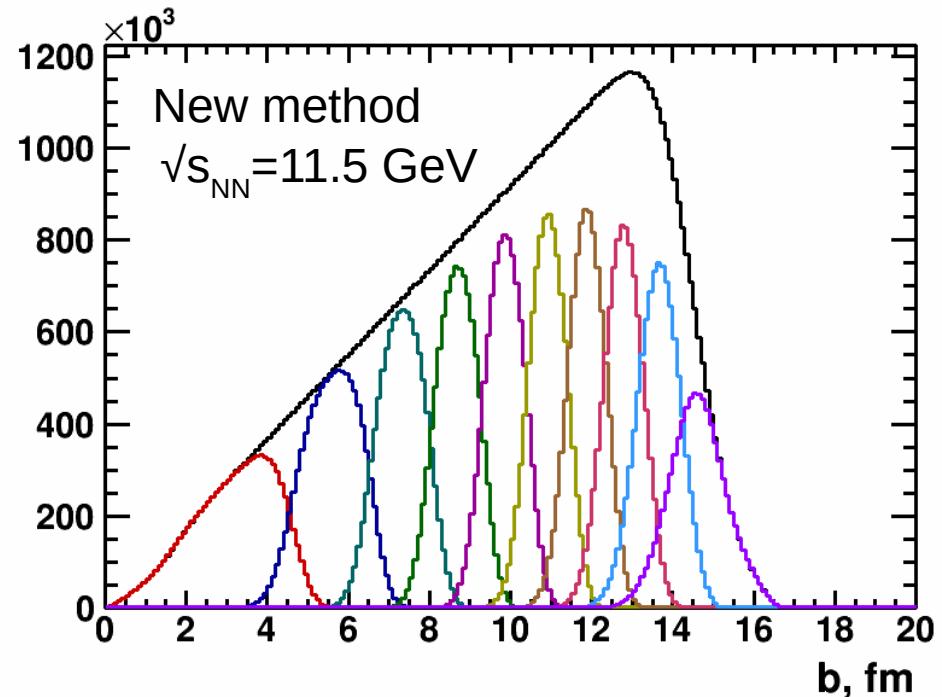
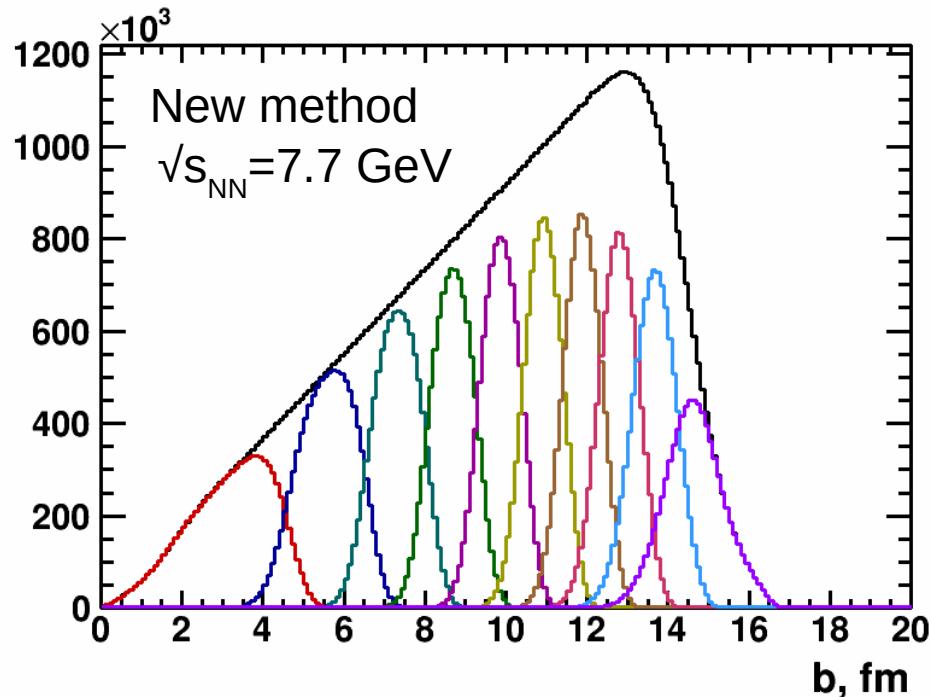
- Fit normalized multiplicity distribution with $P(N_{ch})$
- Define centrality $c(N_{ch})$
- Construct $P(b|N_{ch})$ using Bayes' theorem using parameters from the fit

Gaussian approach: fitting procedure

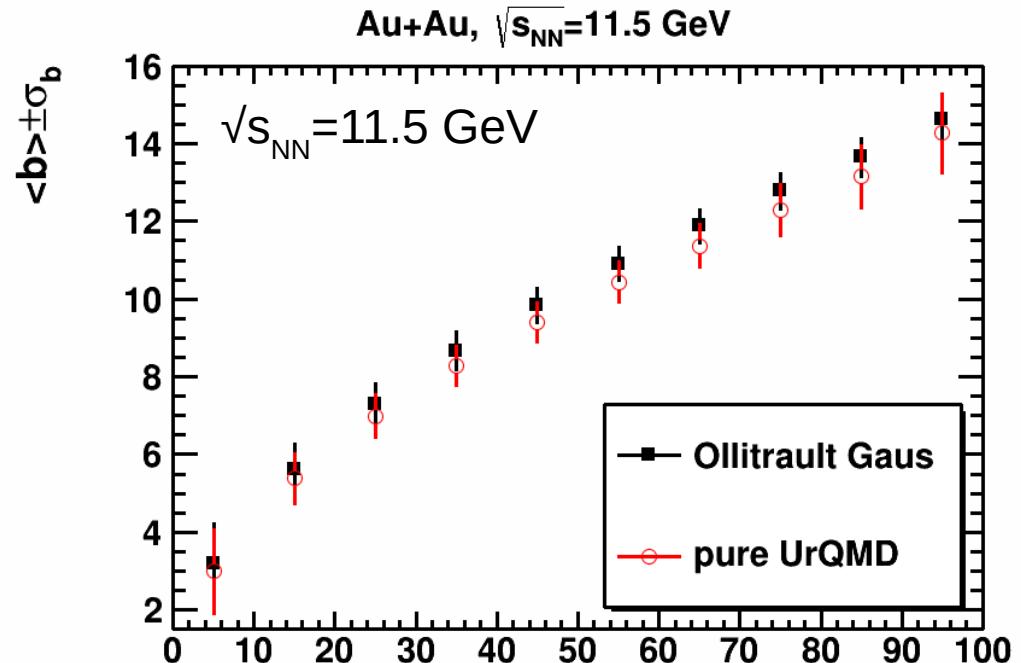
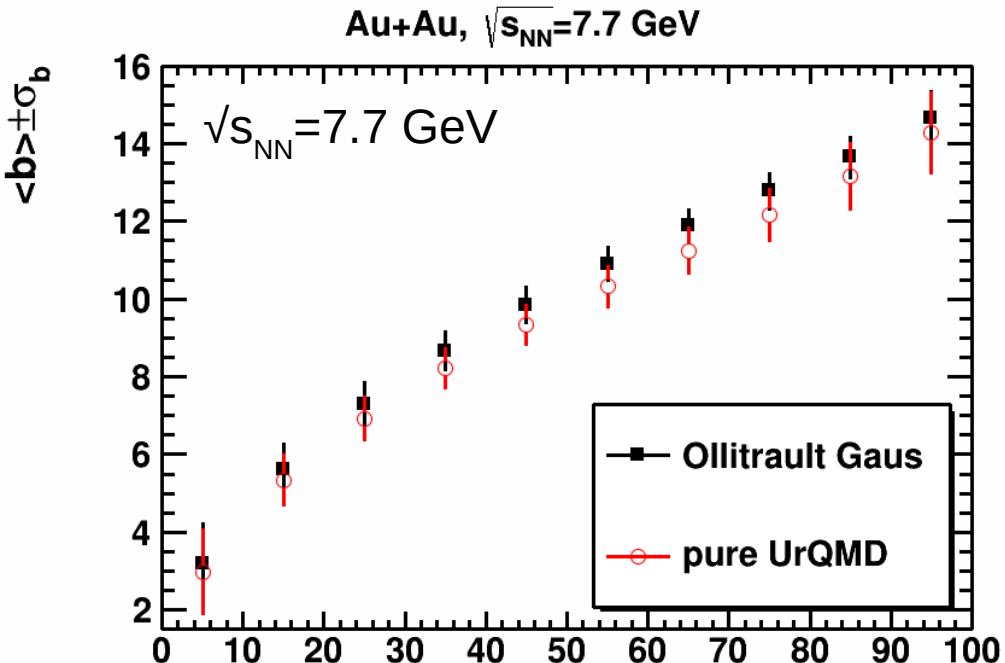


Fit reproduces charge particle multiplicity from pure UrQMD

Gaussian approach: b distribution

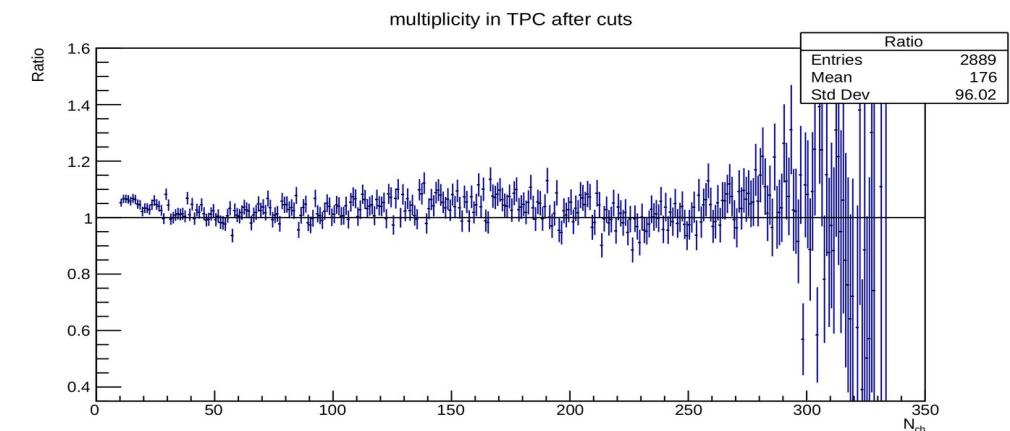
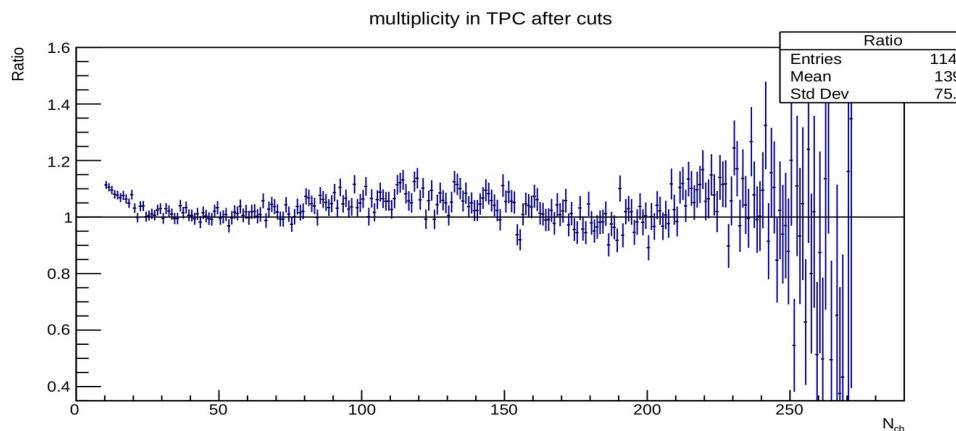
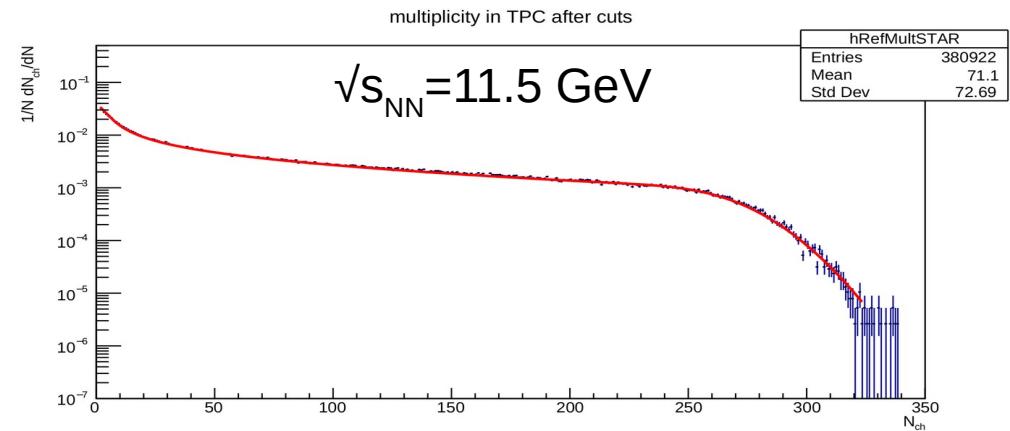
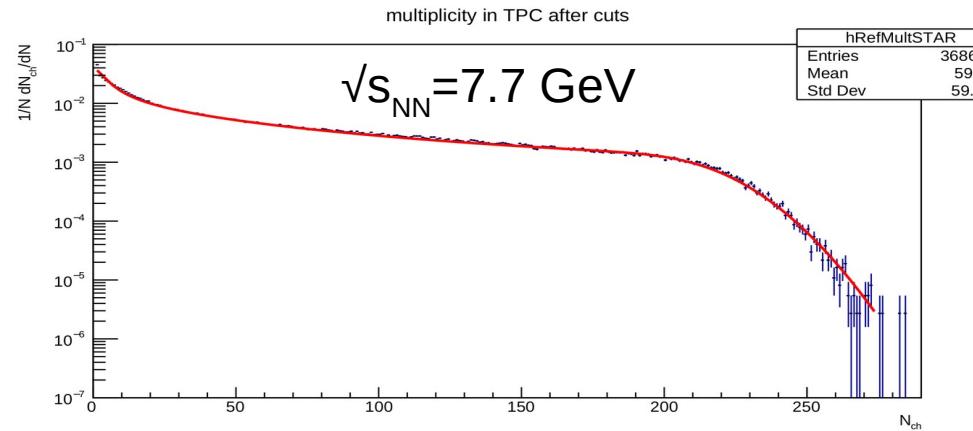


b vs centrality: method comparison



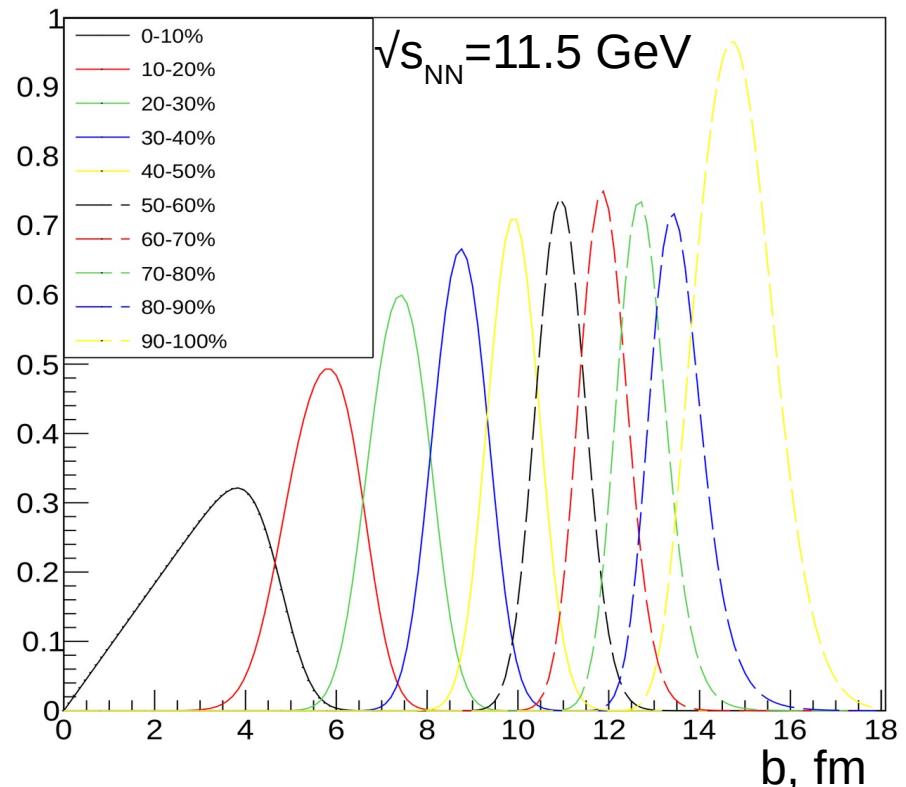
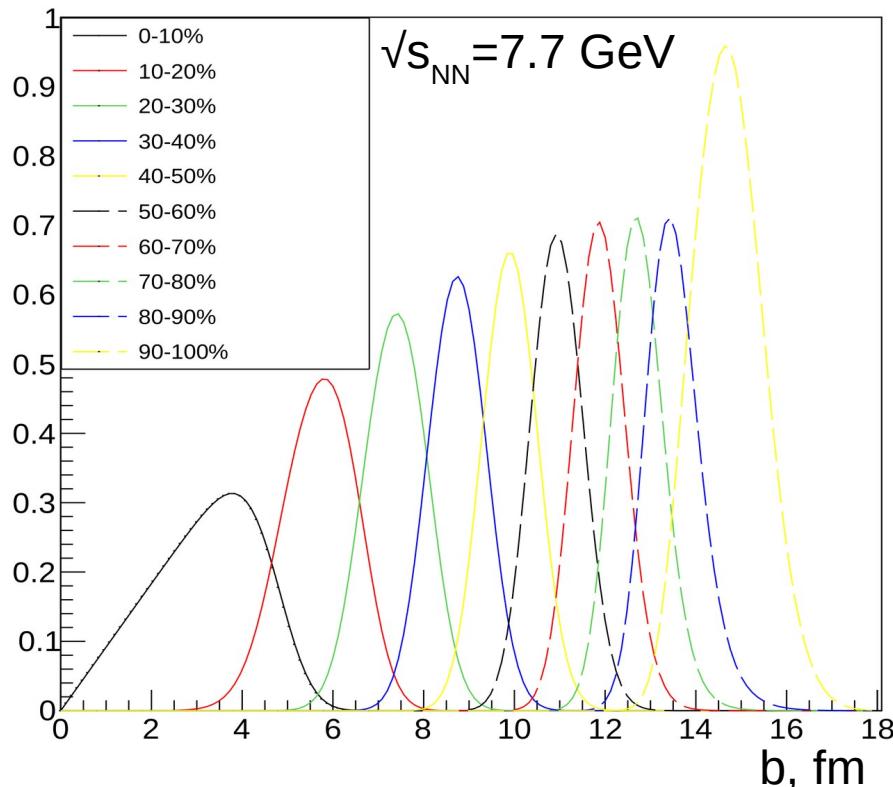
Reasonable agreement between two methods

Γ -function approach: fitting procedure



Fit reproduces charge particle multiplicity from reco UrQMD

Γ -function approach: b distribution



Comparison with method based on MC Glauber is in progress

Summary and next steps

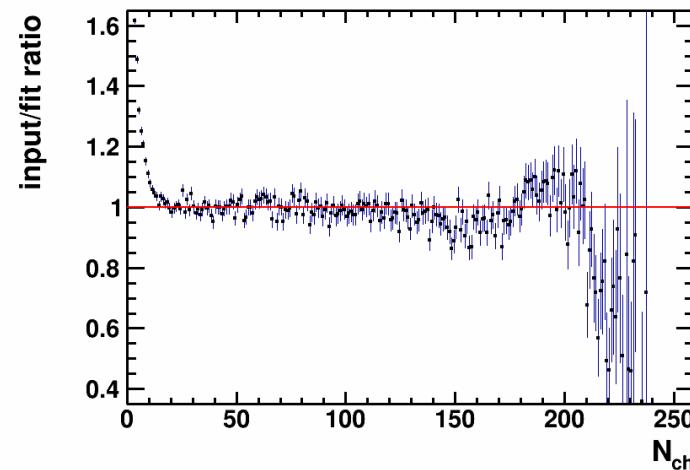
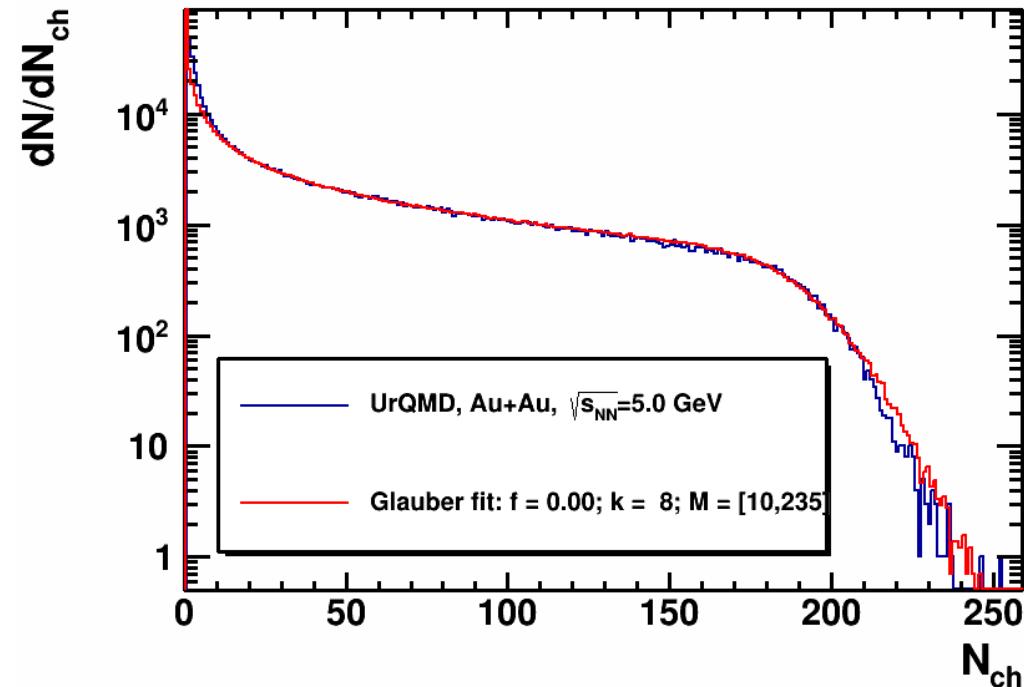
- MC-Glauber based procedure for centrality determination is established
 - UrQMD at 3 energies ($\sqrt{s_{NN}}=5, 7.7, 11.5$ GeV) are under study
- Fit reproduces charged particle multiplicity with chosen parameters
- Extracted relation between model parameters ($b, N_{\text{part}}, N_{\text{coll}}$) and multiplicity centrality classes
 - Impact parameter from MC Glauber and UrQMD in given centrality classes are in reasonable agreement
- Comparison of the ε_n between MC Glauber and UrQMD and for different $\sqrt{s_{NN}}$ shows notable difference
- First comparison with the new method proposed by J. Ollitrault shows reasonable agreement with standard method based on MC-Glauber
 - Comparison with the new improved method based on Gamma function is in progress
- Systematic study and analysis note are under preparation

Thank you for your attention!

Backup

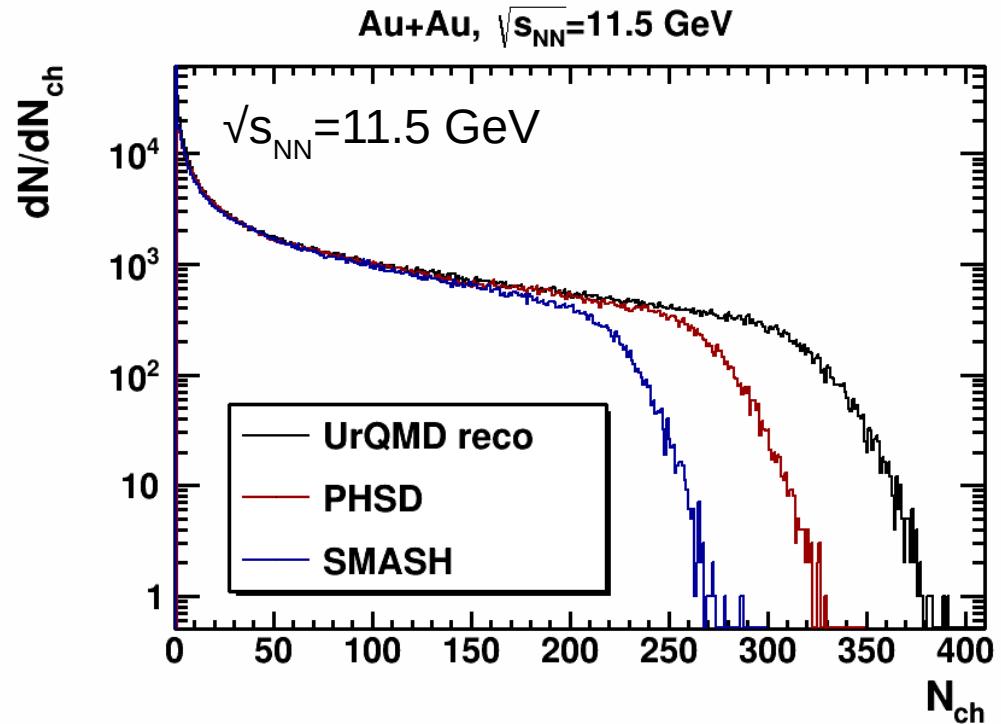
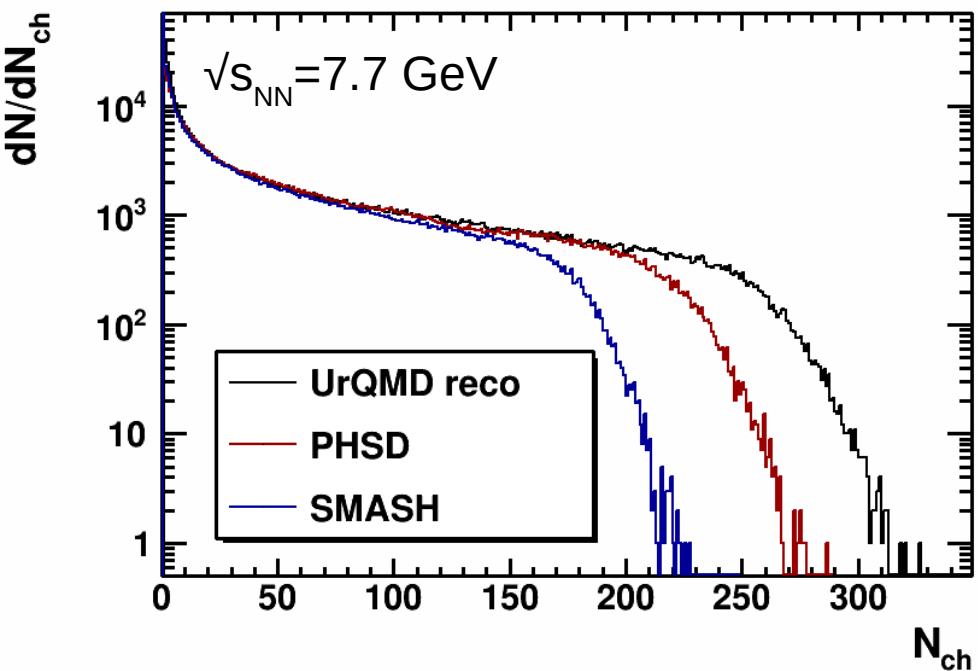
MC Glauber fit: h^\pm multiplicity

$f=0, k=8, \mu=0.23, \chi^2=2.04\pm0.10, M=(10,235)$



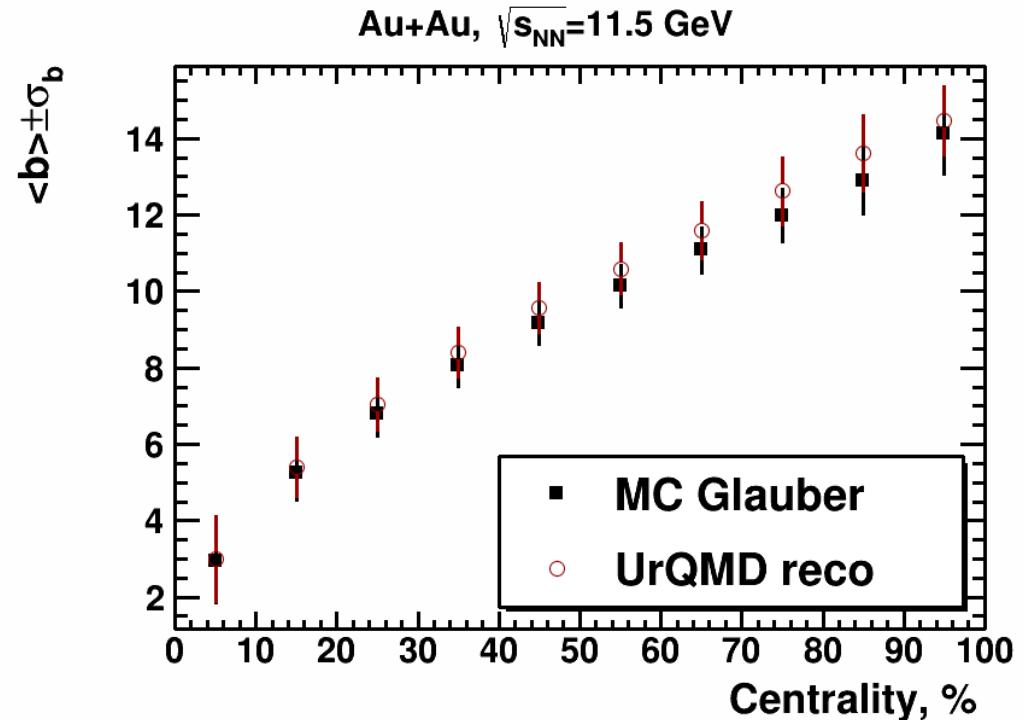
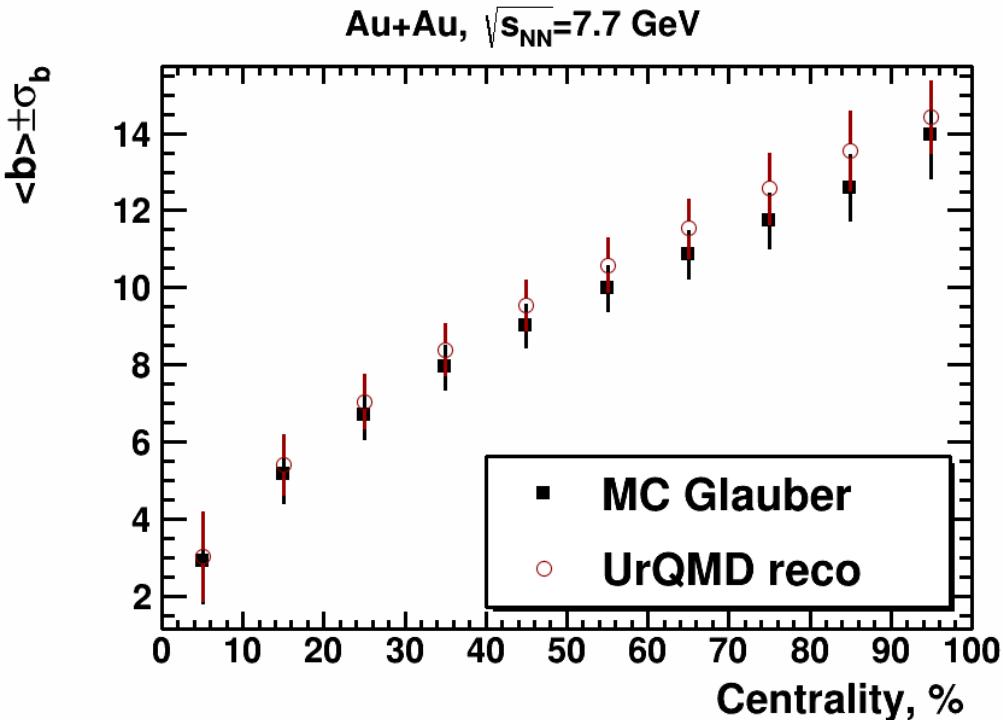
MC Glauber fit deviates from UrQMD data for large multiplicity region

h^\pm multiplicity

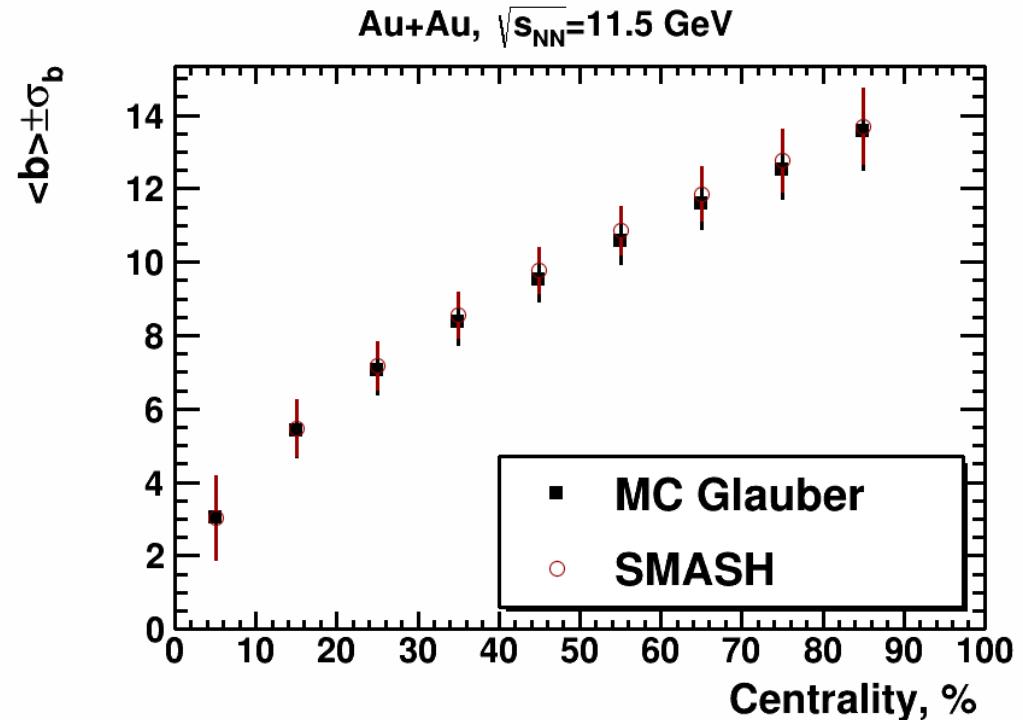
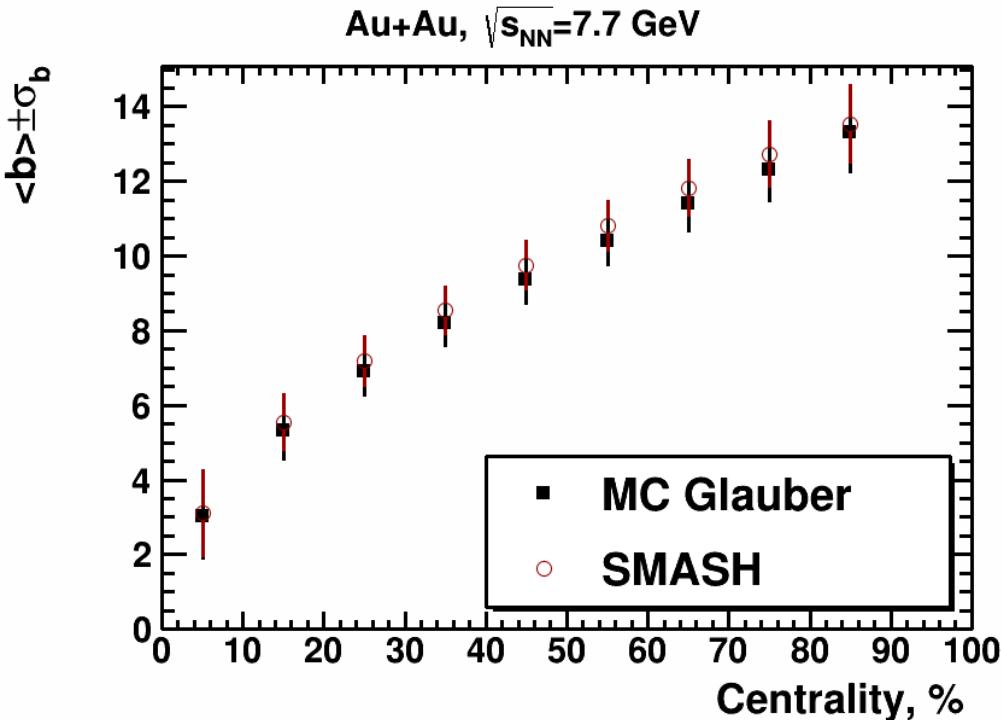


$\langle b \rangle$ vs centrality: comparison
between models

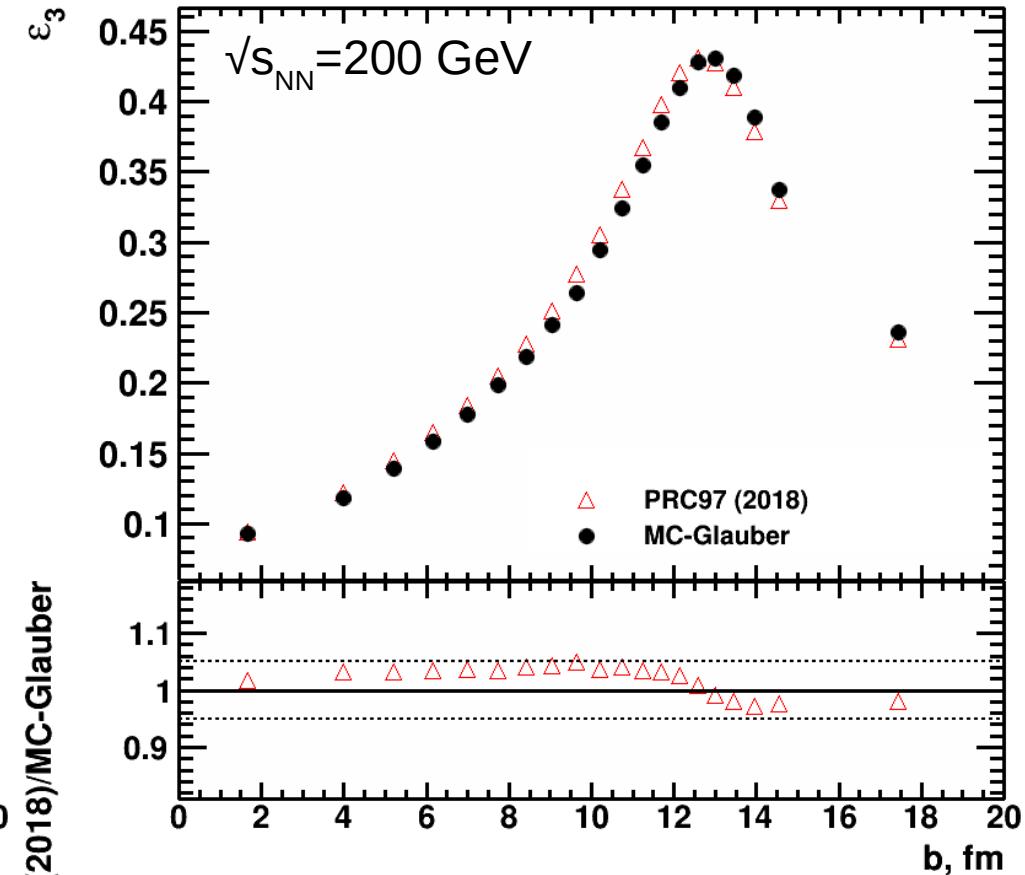
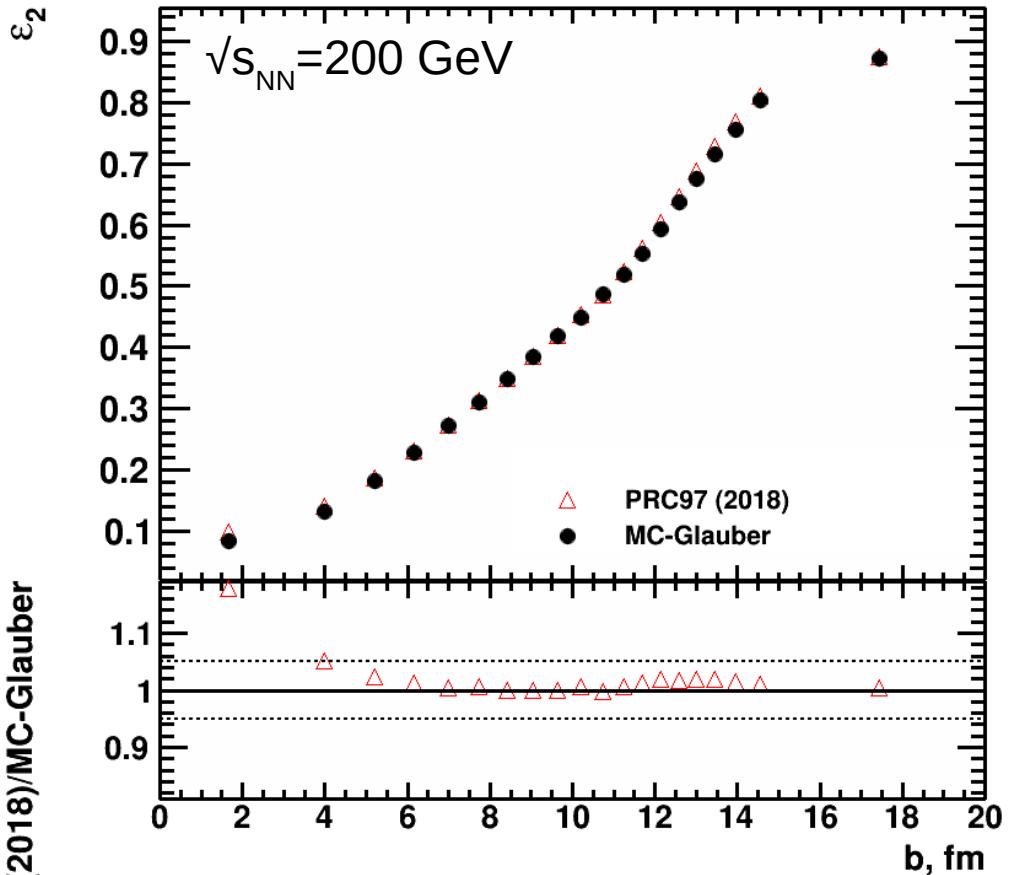
b vs centrality: Glauber vs UrQMD



b vs centrality: Glauber vs SMASH



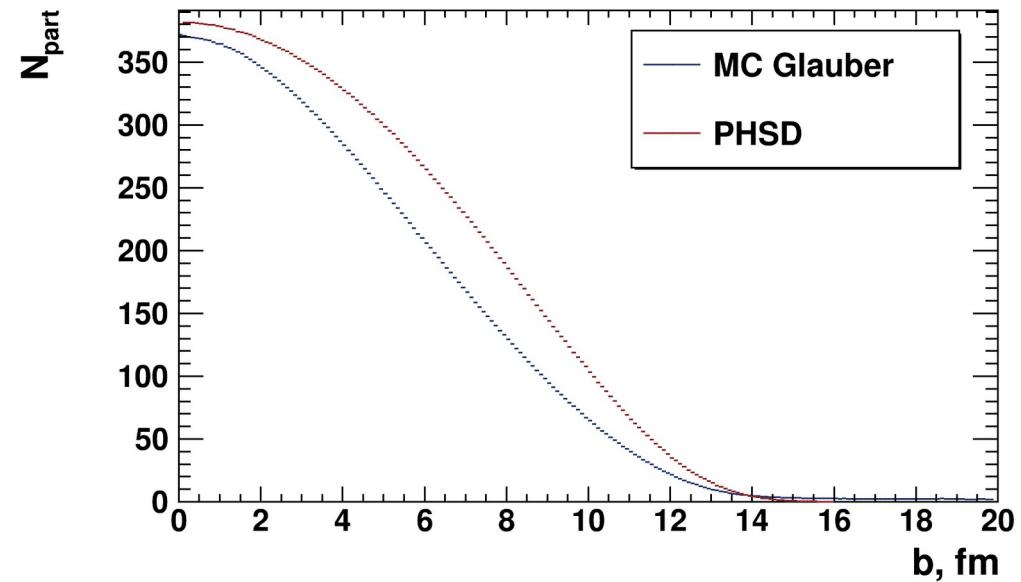
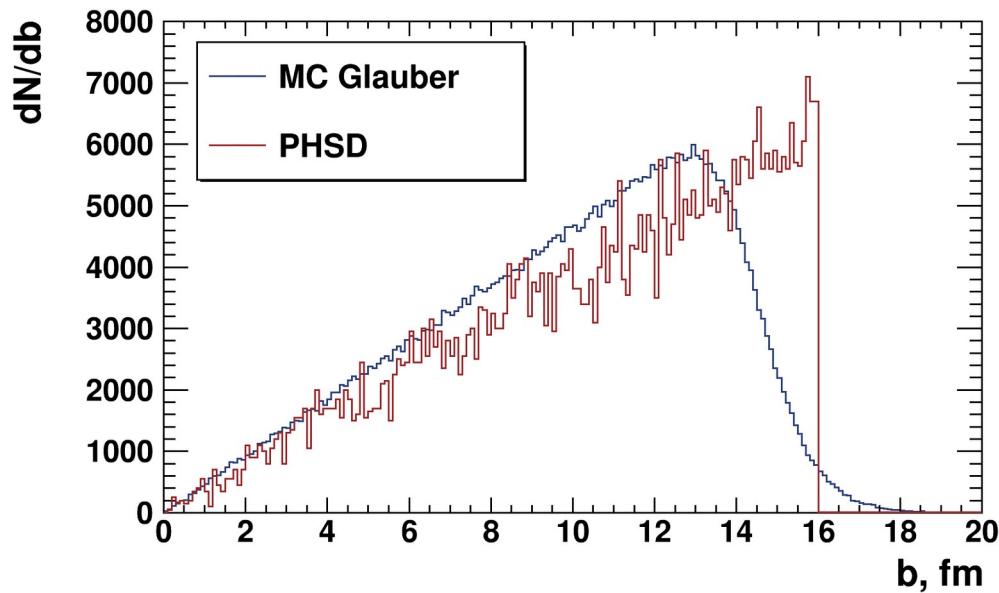
Eccentricity: comparison with STAR



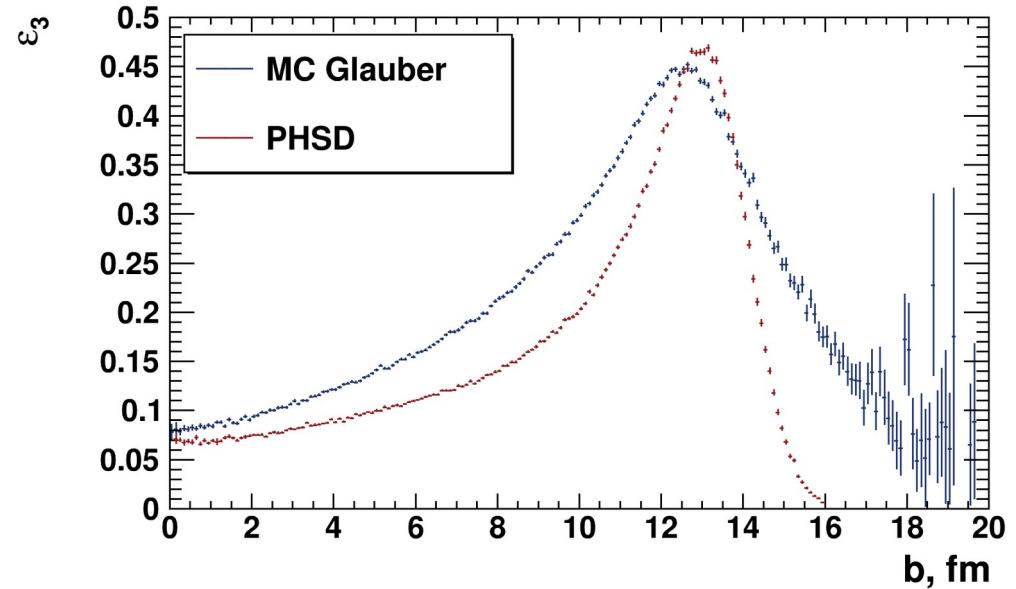
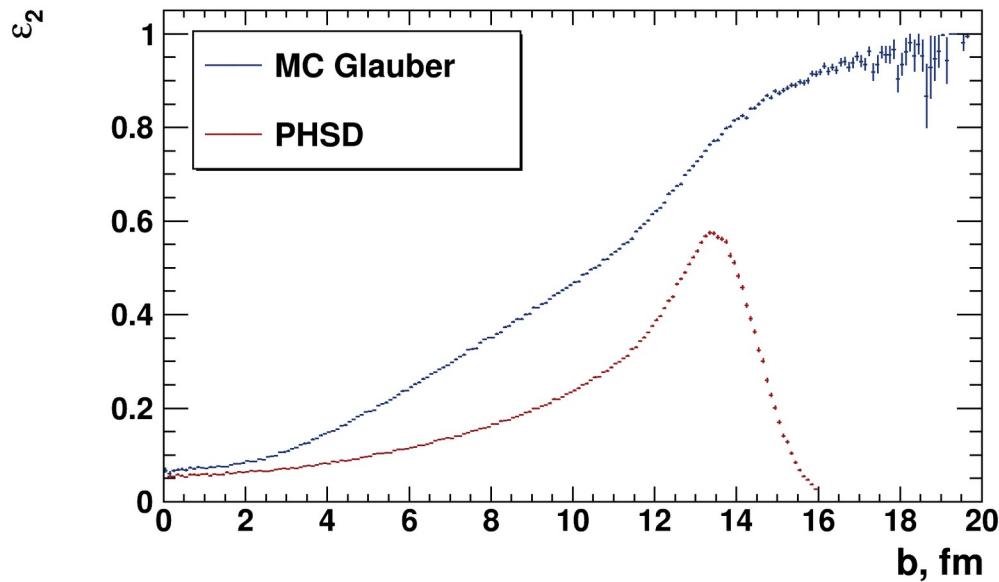
Good agreement with the published data

Initial state comparison:
 $\sqrt{s}_{\text{NN}} = 7.7 \text{ GeV}$

MC Glauber vs PHSD: b , N_{part}

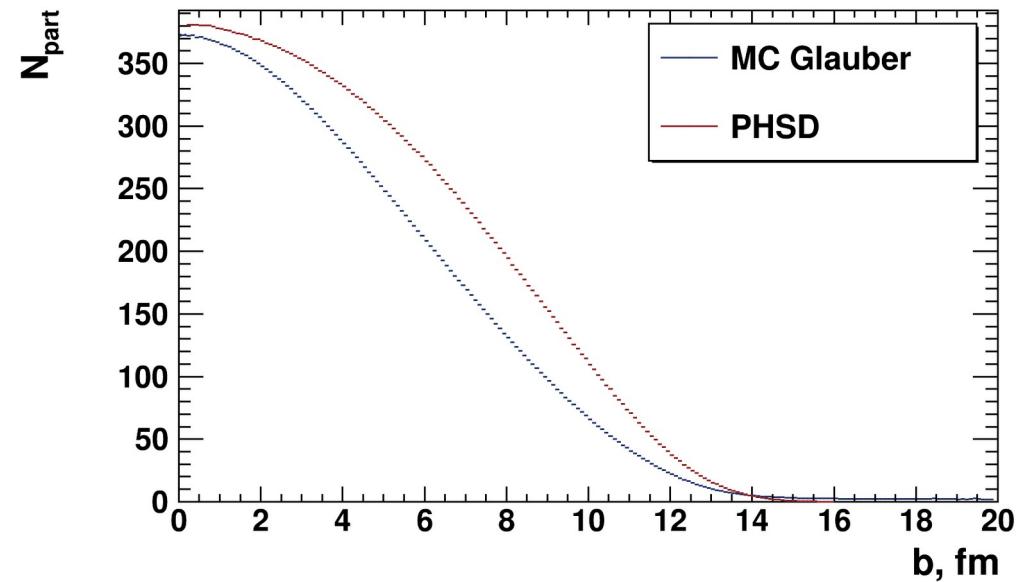
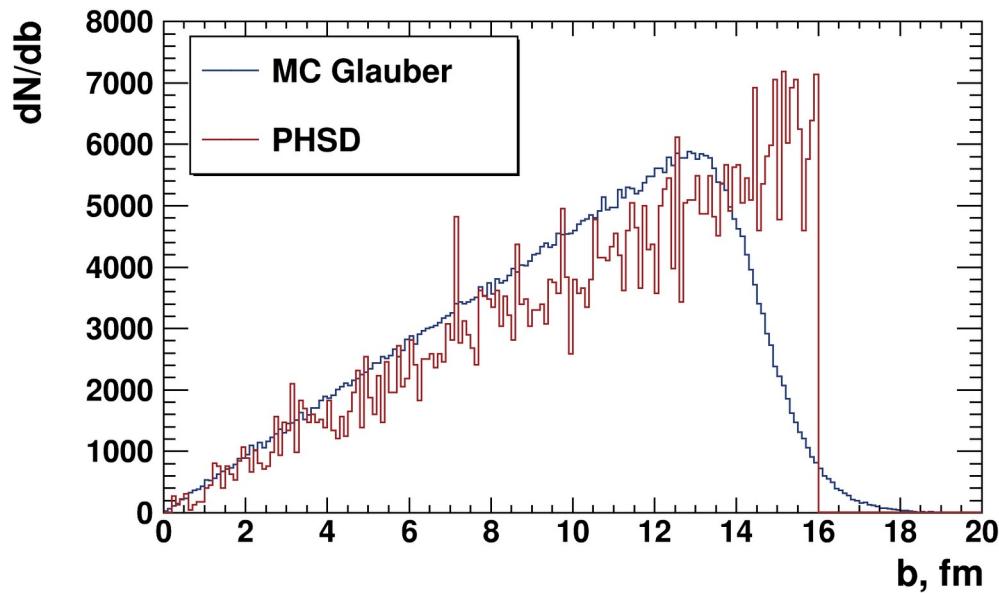


MC Glauber vs PHSD: ε_n

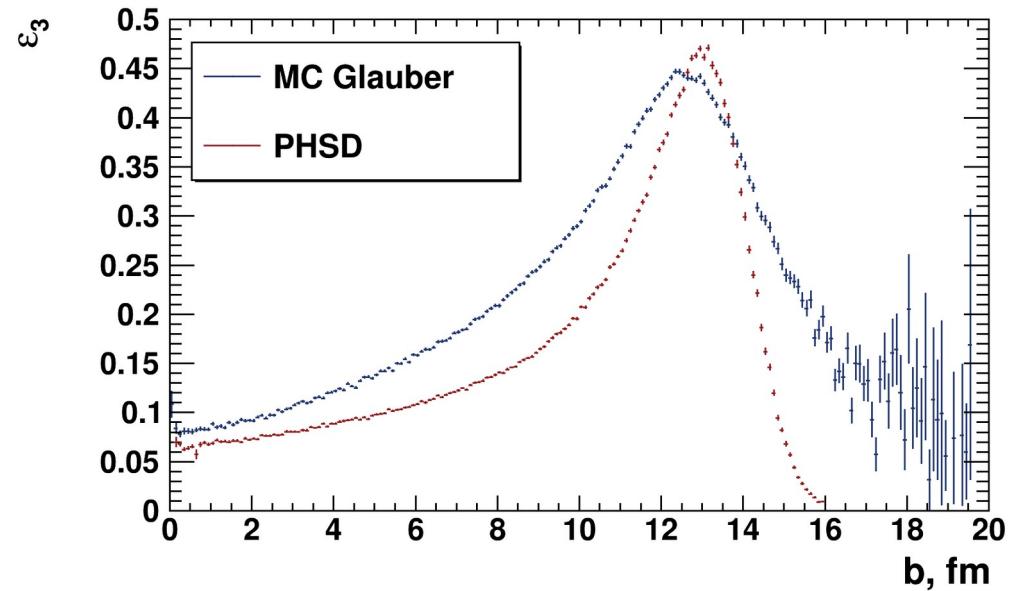
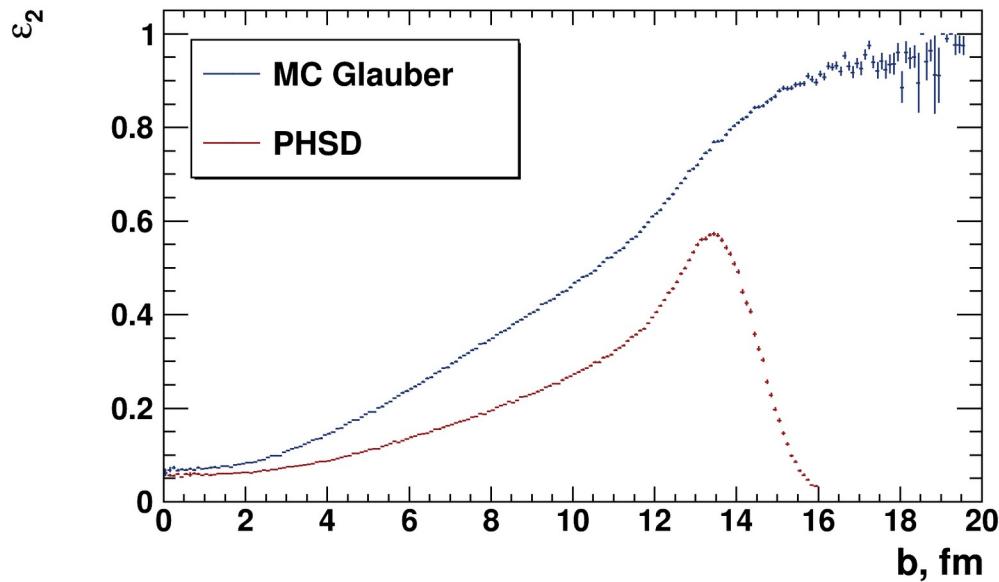


Initial state comparison:
 $\sqrt{s}_{\text{NN}} = 11.5 \text{ GeV}$

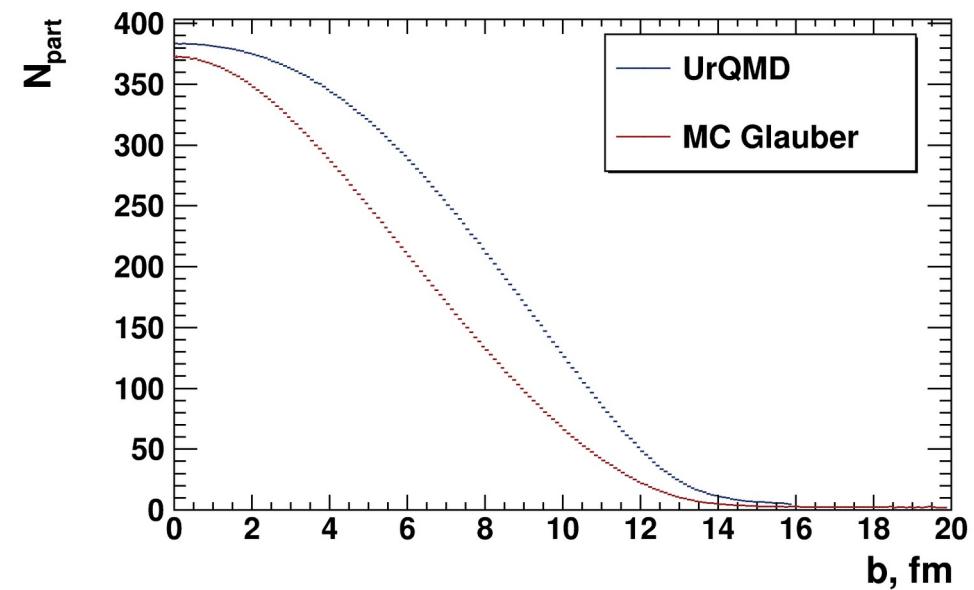
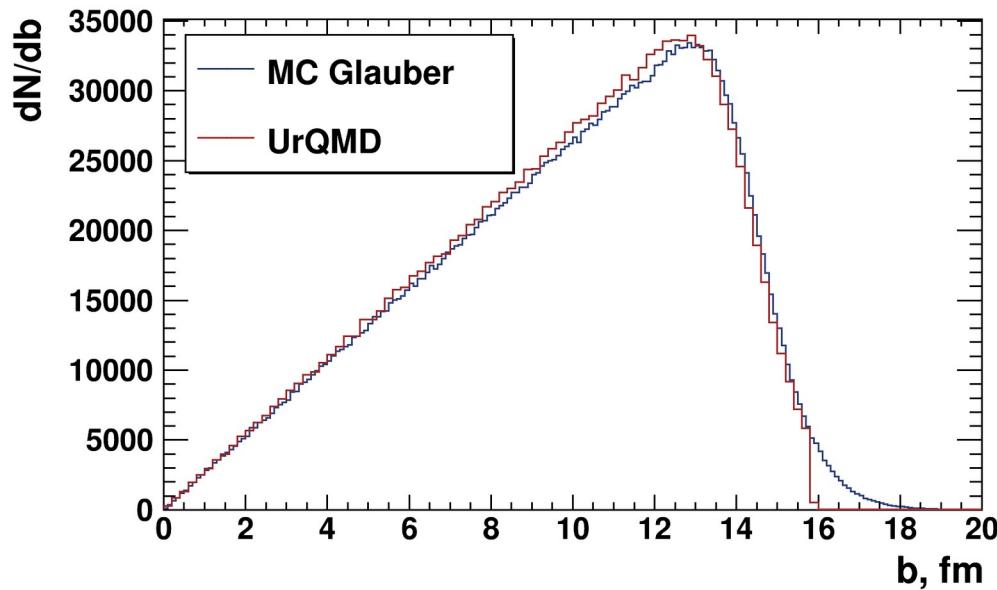
MC Glauber vs PHSD: b , N_{part}



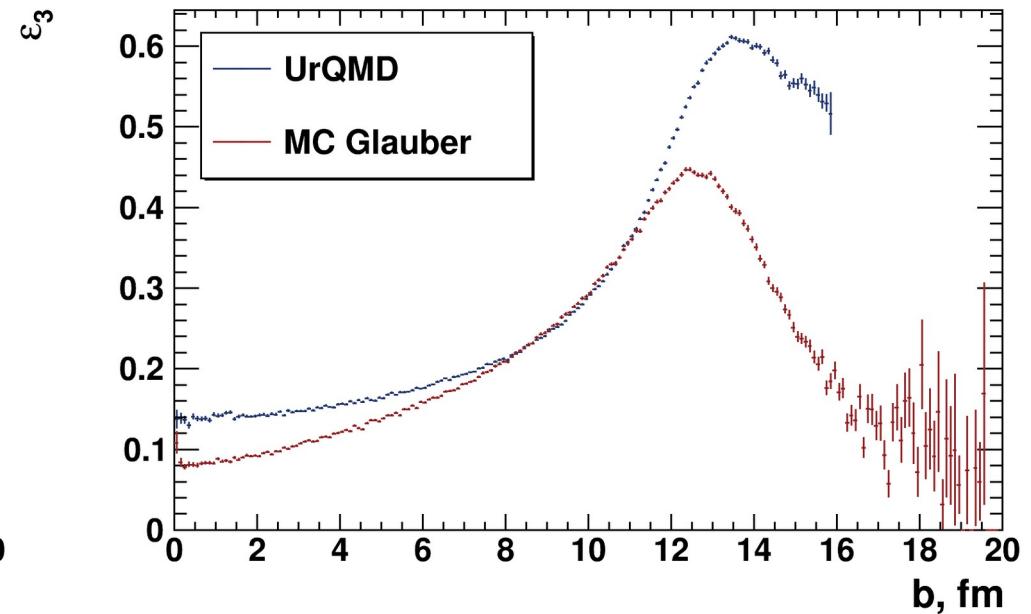
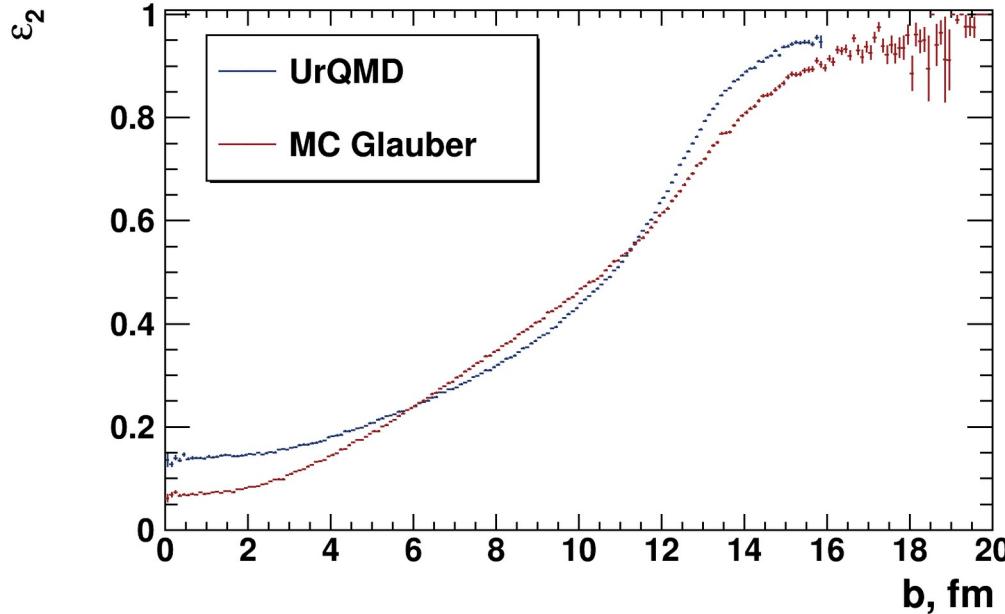
MC Glauber vs PHSD: ε_n



MC Glauber vs UrQMD: b, N_{part}

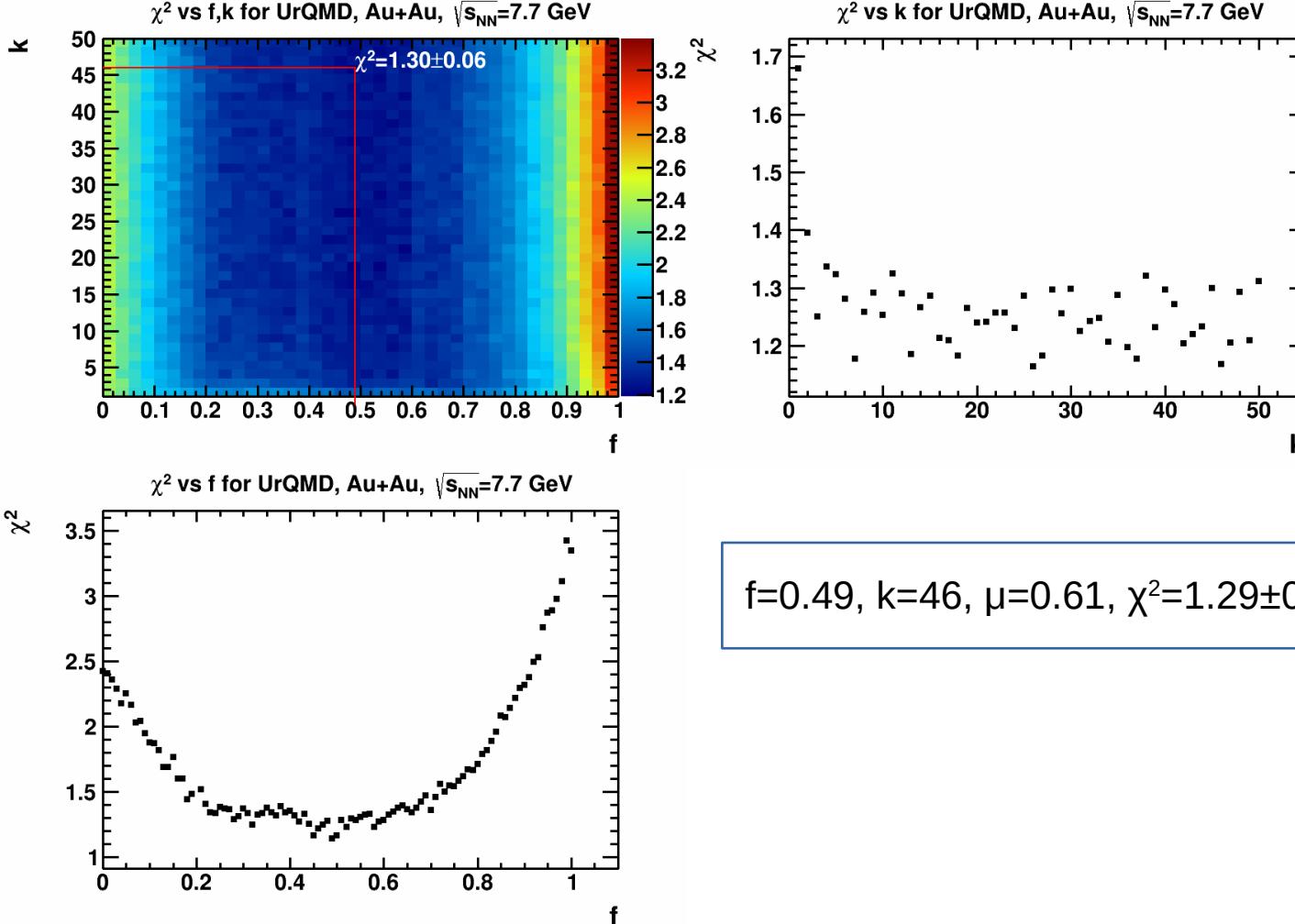


MC Glauber vs UrQMD: ϵ_n

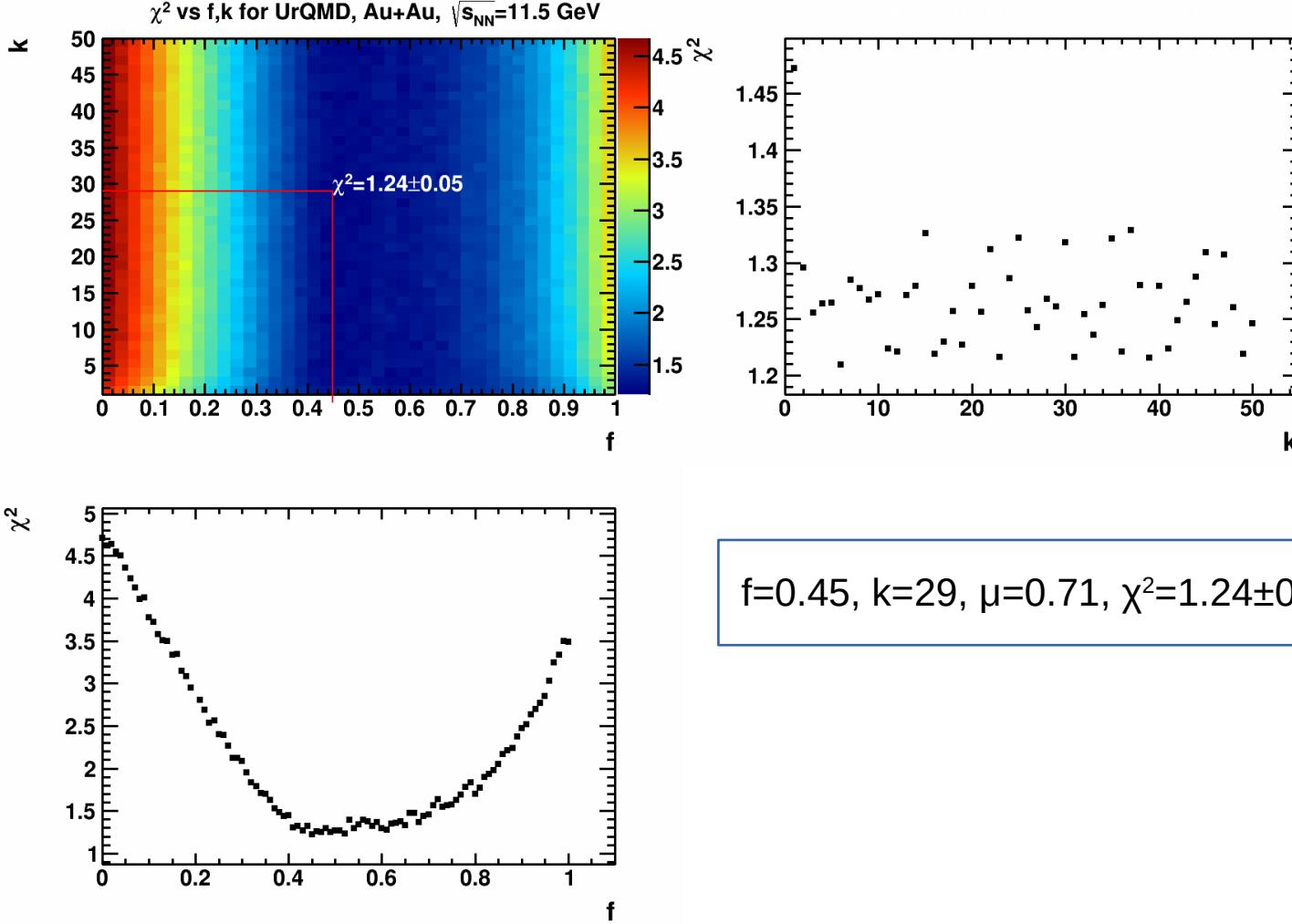


MC Glauber vs pure UrQMD

Fit parameters f,k vs χ^2

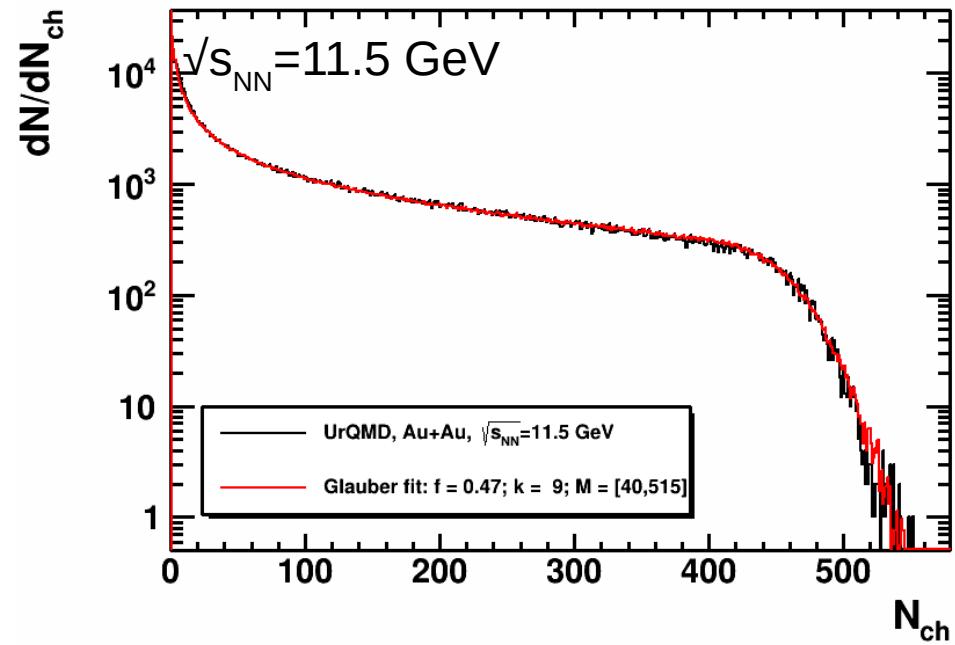
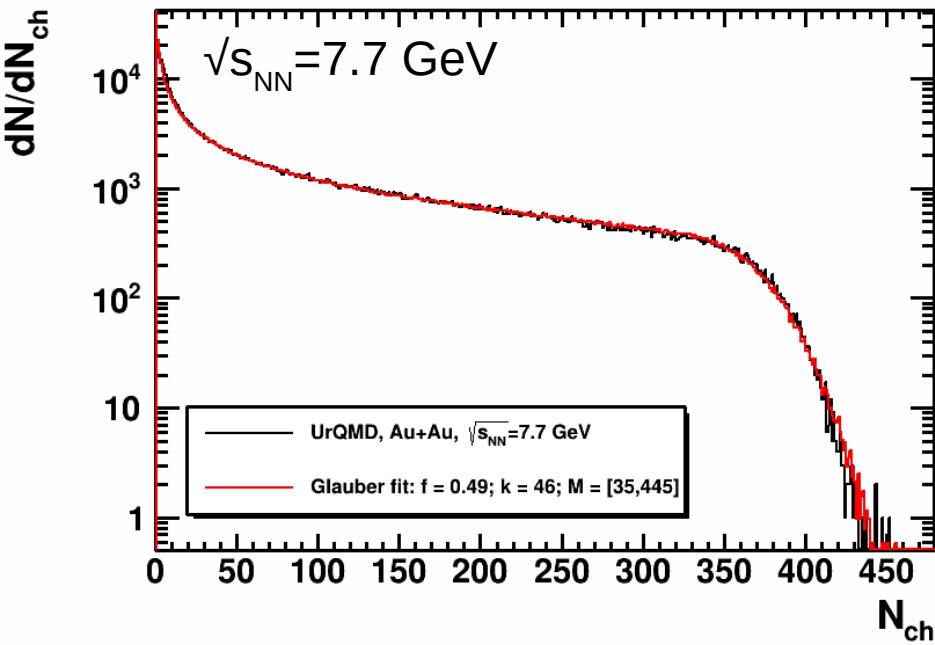


Fit parameters f,k vs χ^2

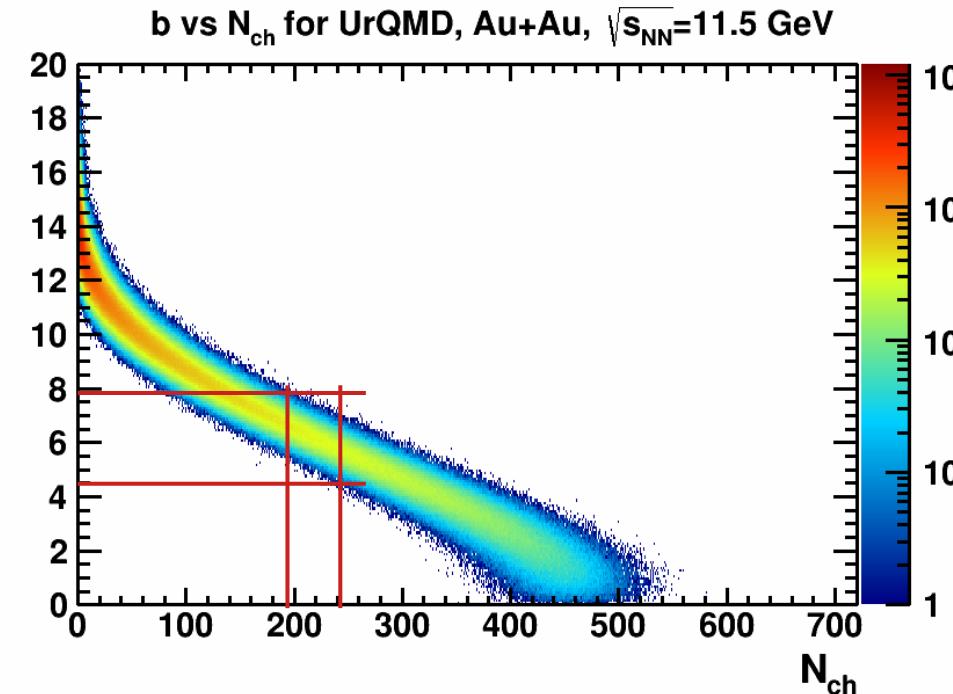
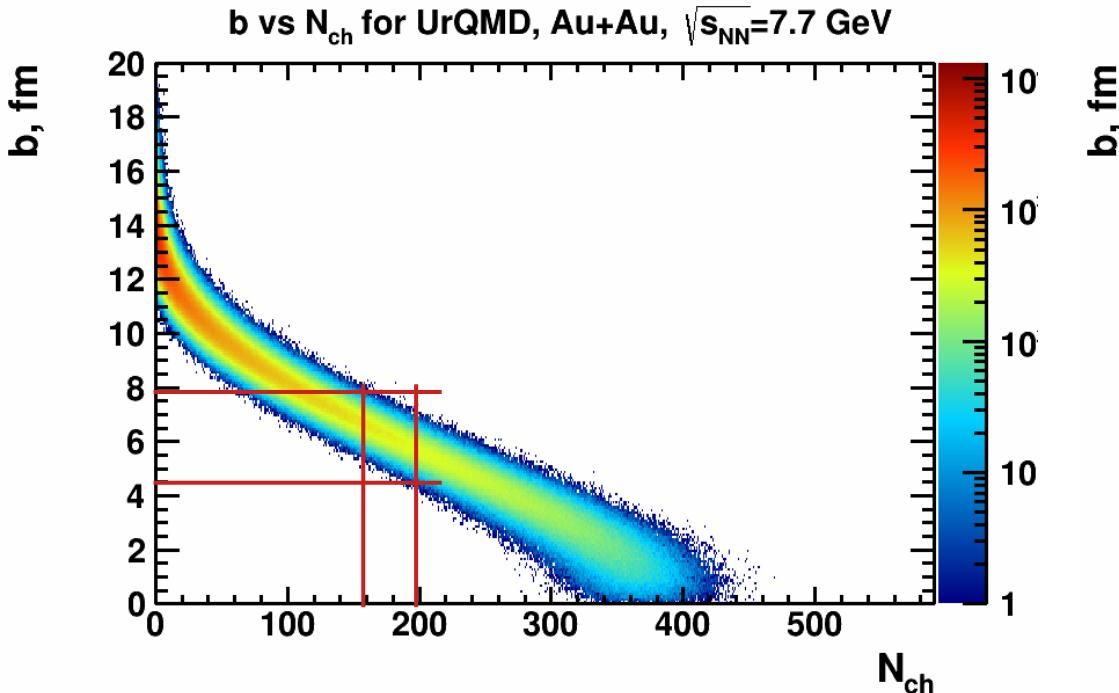


f=0.45, k=29, $\mu=0.71$, $\chi^2=1.24\pm0.05$, M=(40,540)

MC Glauber fit: h^\pm multiplicity

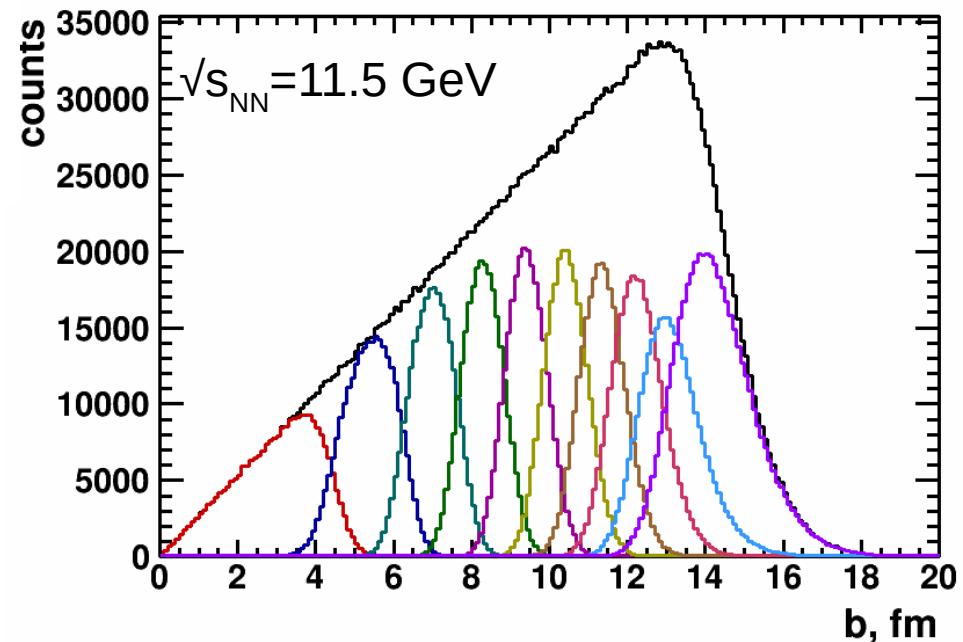
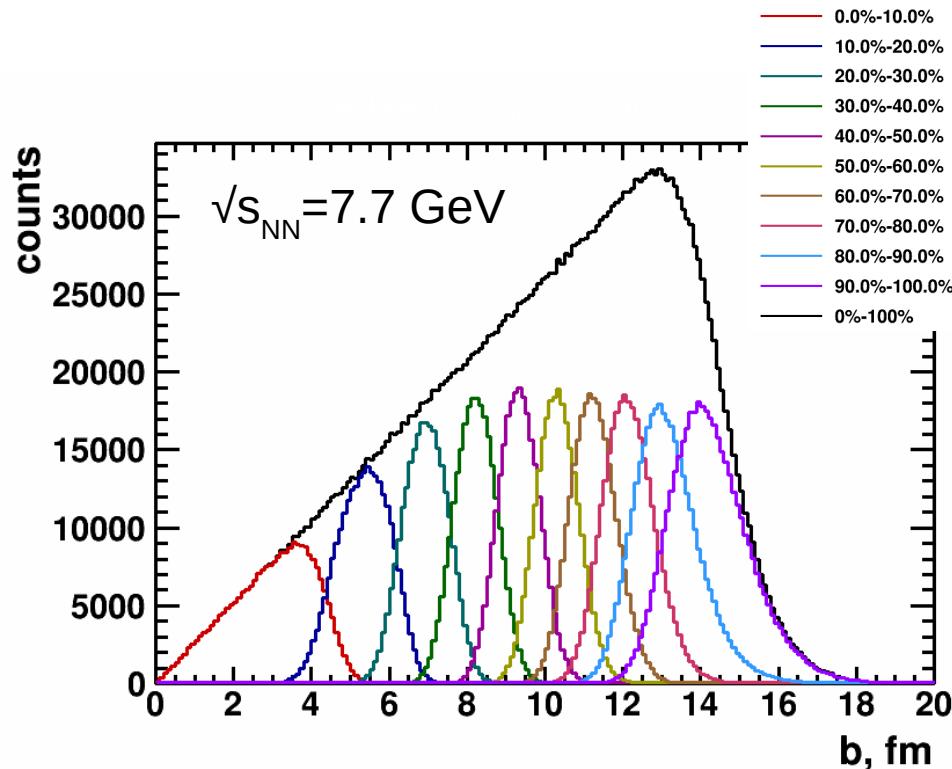


b-multiplicity correlation

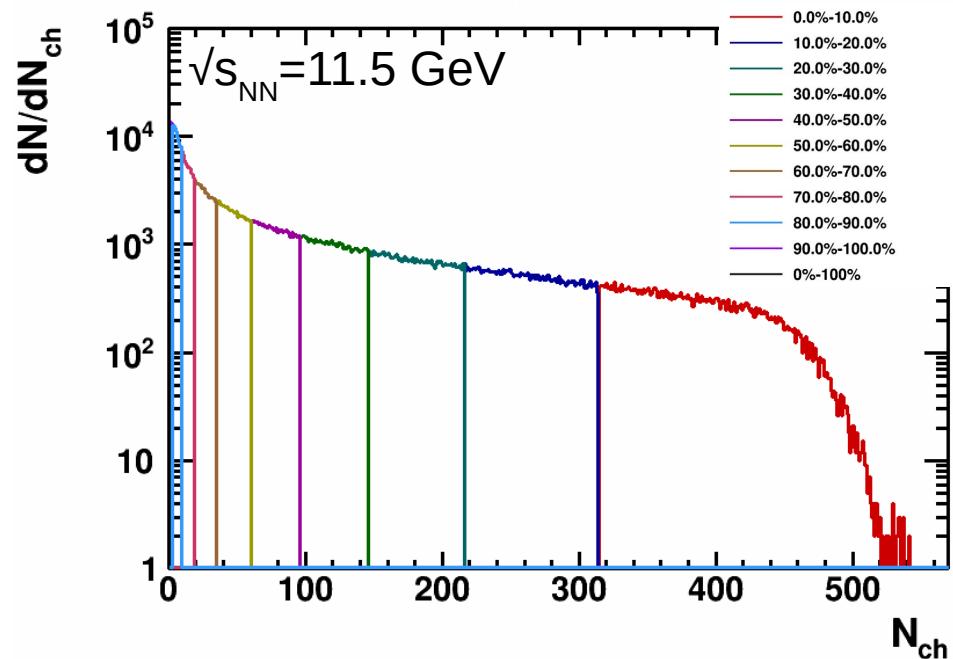
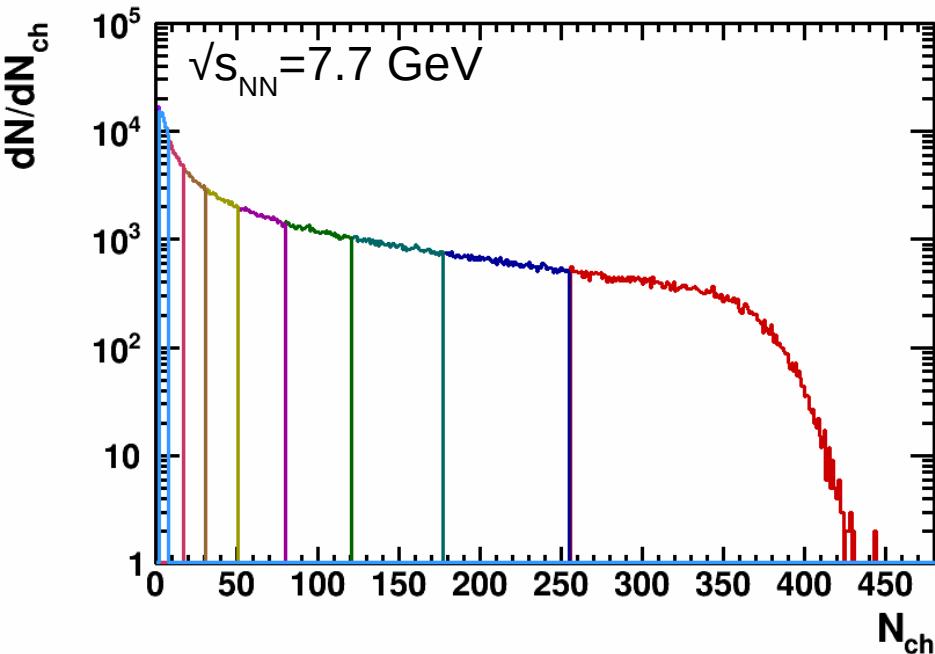


Events in multiplicities $M \pm \Delta M$ have impact parameter in range $b \pm \sigma_b$

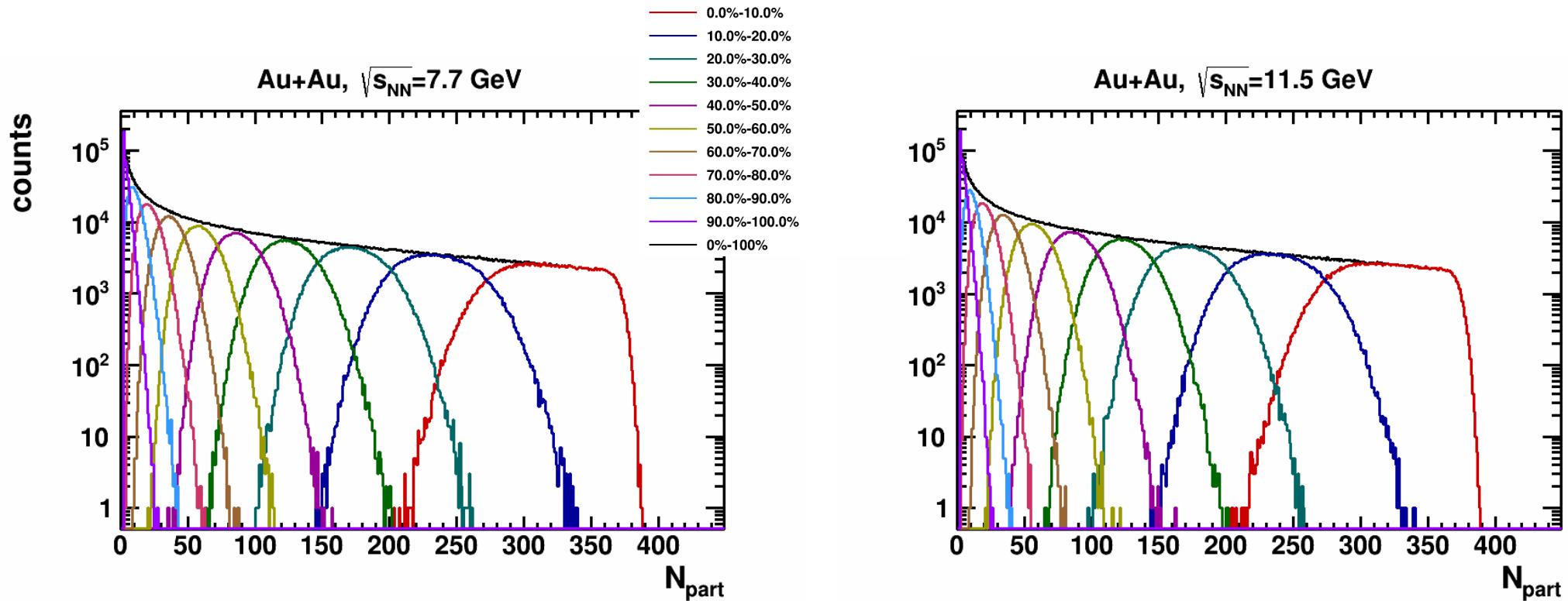
b distribution in centrality classes



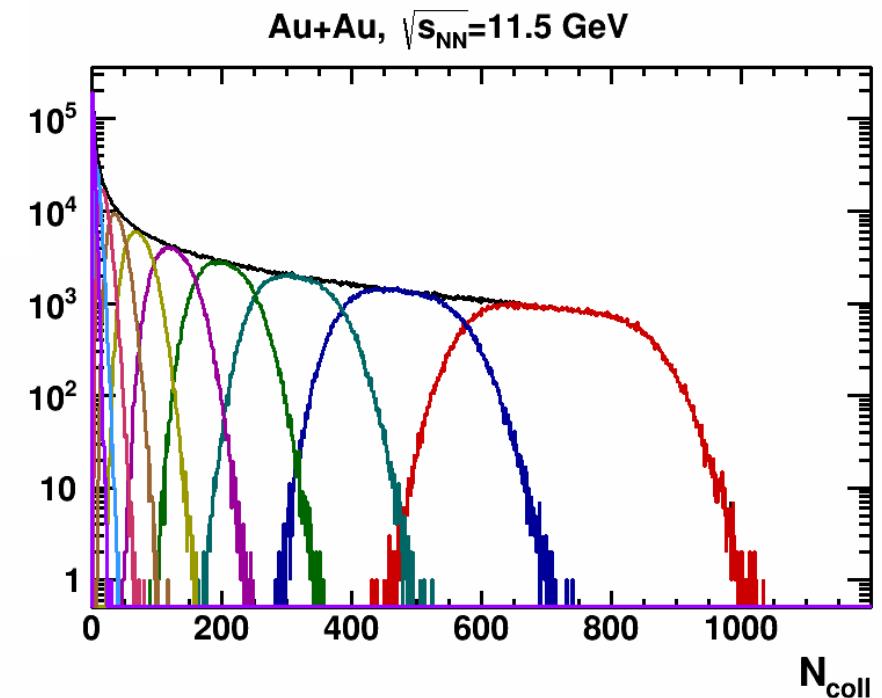
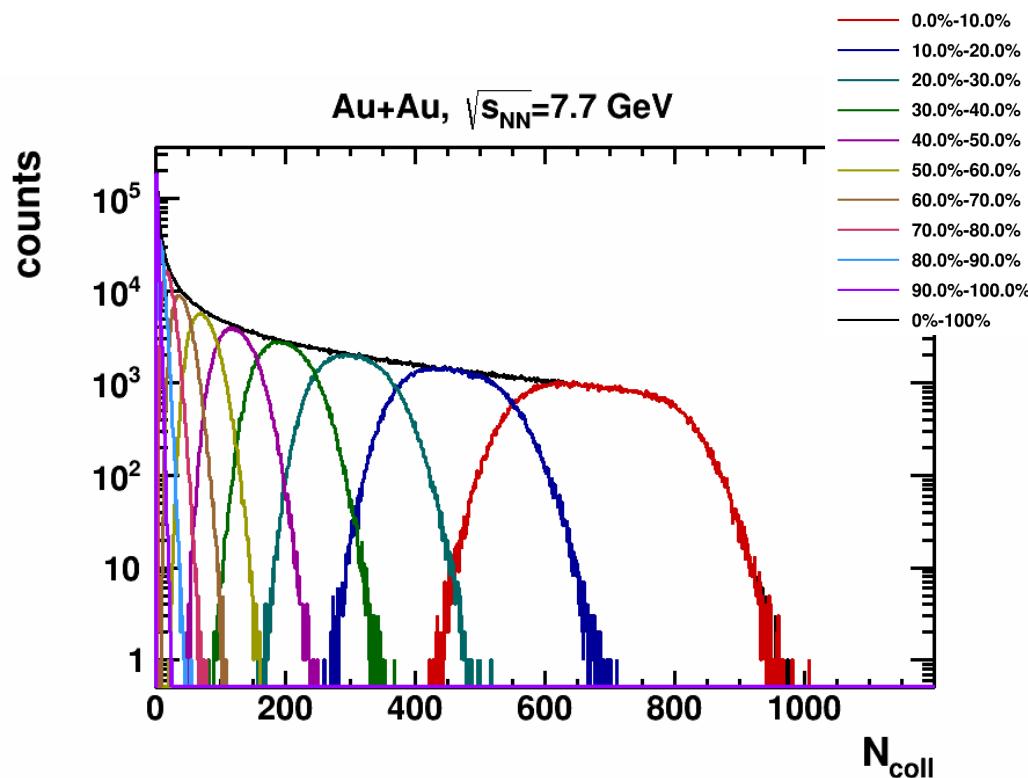
Multiplicity distribution in centrality classes



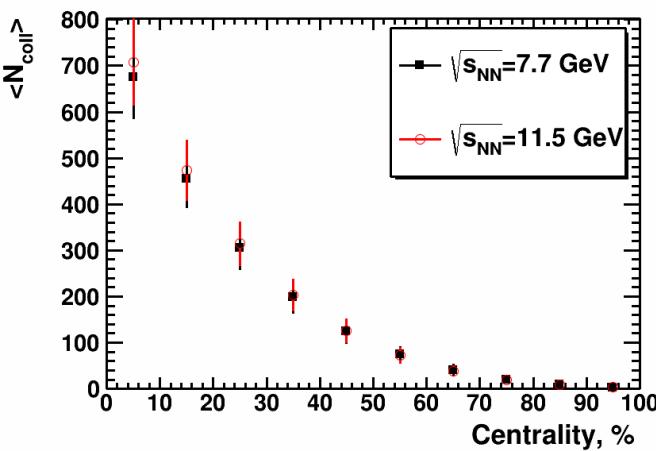
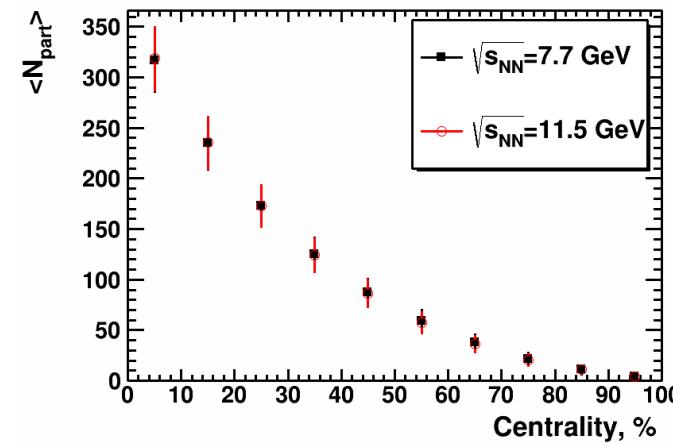
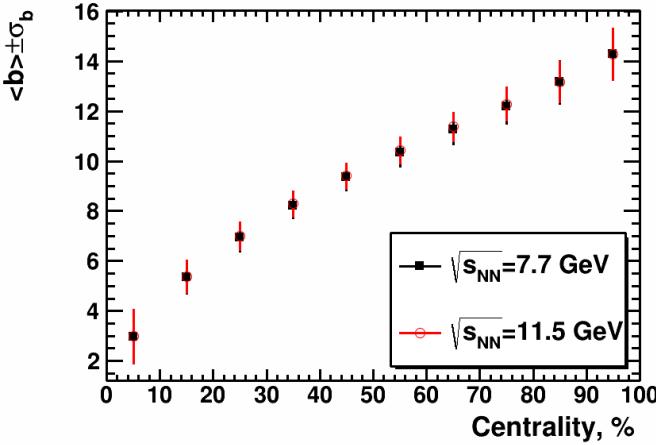
Centrality classes: Npart



Centrality classes: Ncoll

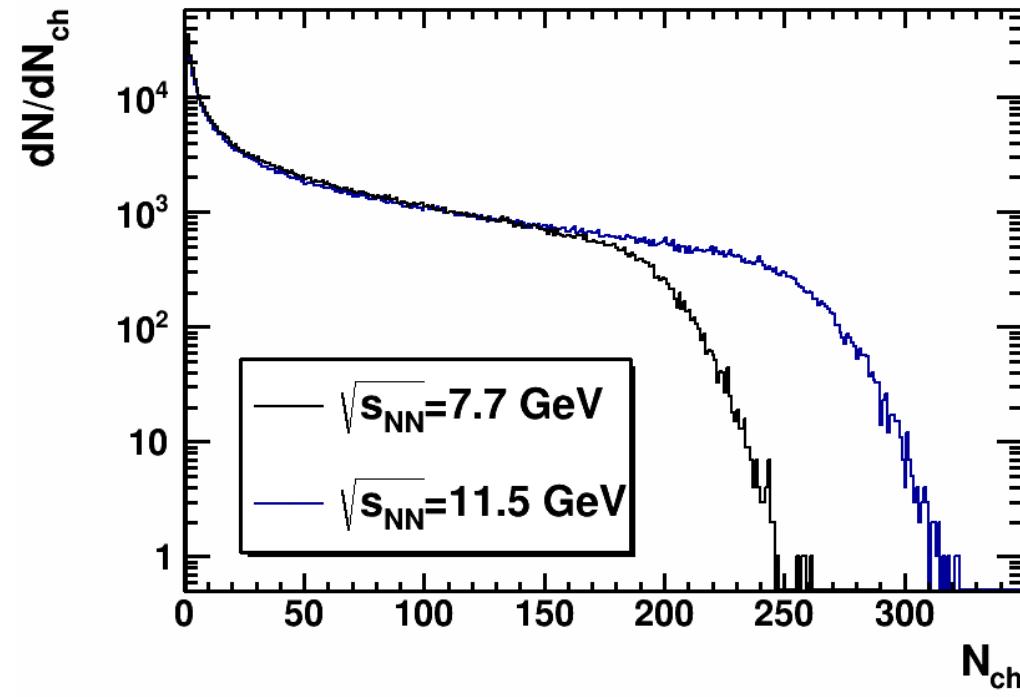


Centrality classes: Ncoll



Centrality framework results for UrQMD reco with pion multiplicity

Charged particle multiplicity in MPD



Reconstructed data:

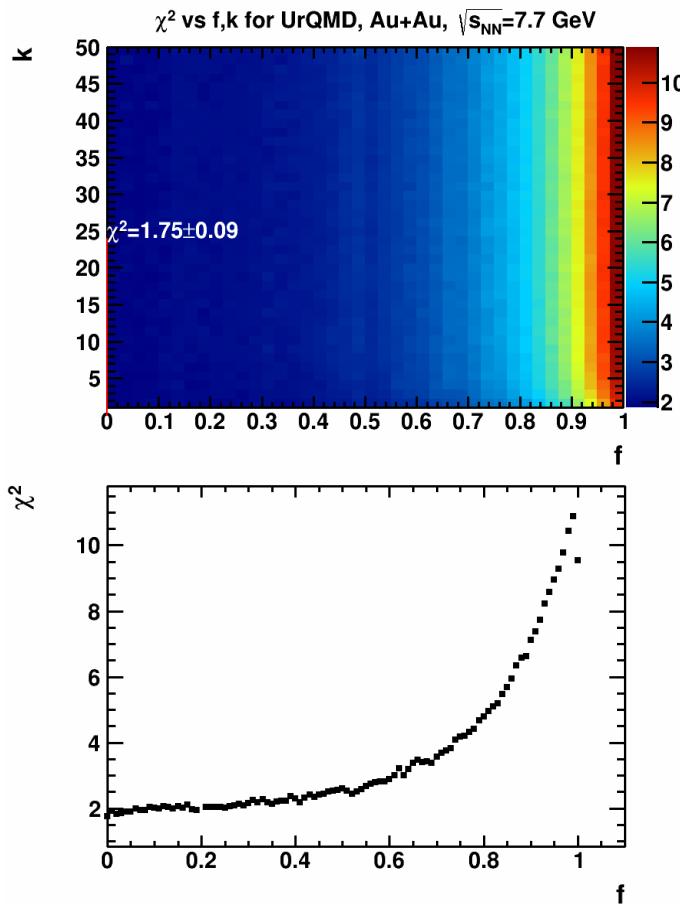
- UrQMD 3.4 simulation
 - Au+Au, $N_{ev}=500k$, $\sqrt{s_{NN}}=7.7, 11.5 \text{ GeV}$
- GEANT4 MPD detector simulation
- Reconstruction procedure:
 - Realistic tracking in TPC (Cluster Finder)

Used particle selection:

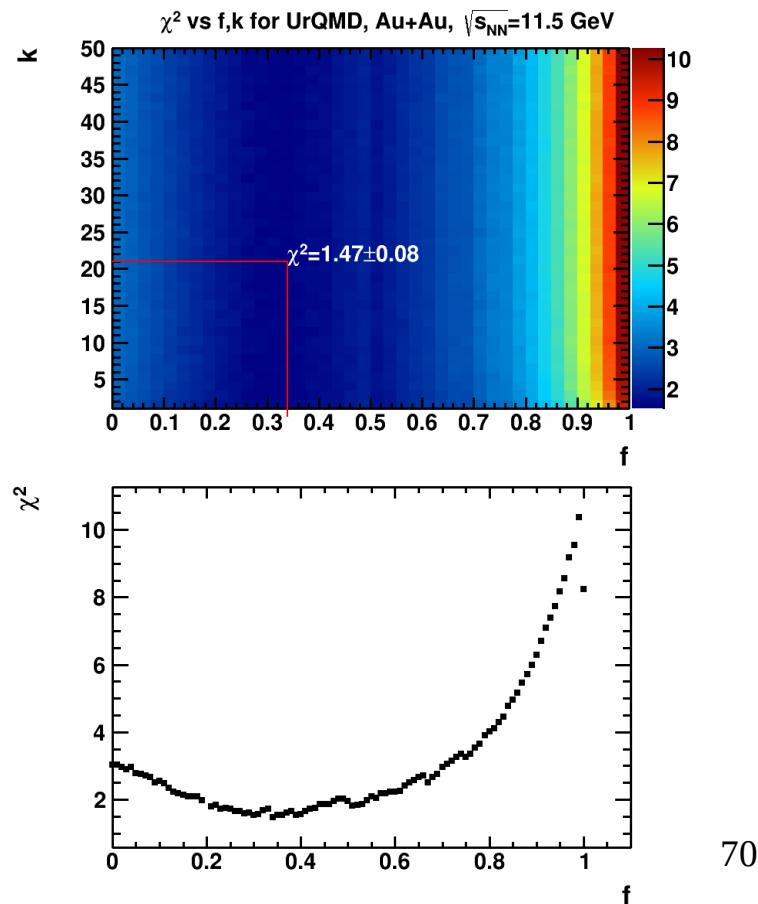
- Only charged pions
- $|\eta| < 0.5$
- $p_T > 0.15 \text{ GeV}/c$

Fit parameters f,k vs χ^2

$f=0, k=42, \mu=0.24, \chi^2=1.39\pm0.1, M=(10,240)$

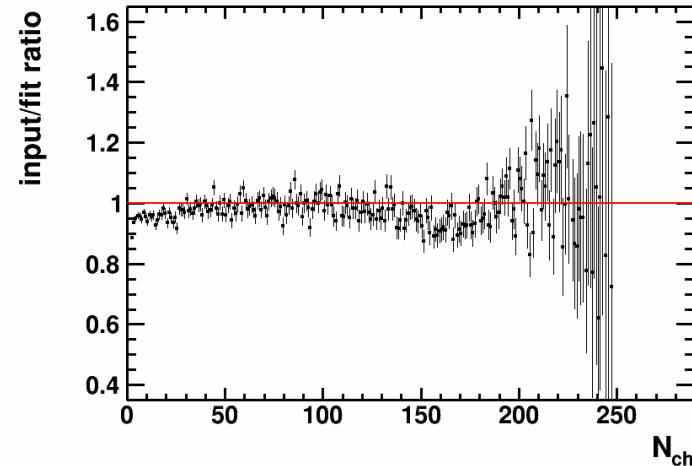
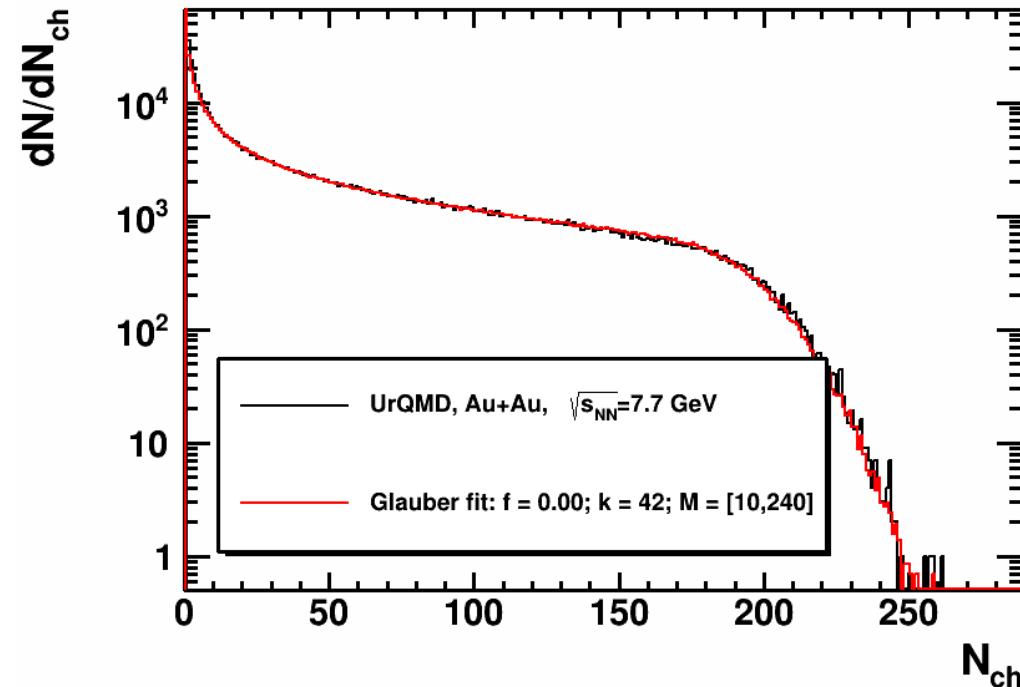


$f=0.01, k=43, \mu=0.3, \chi^2=1.17\pm0.07, M=(10,320)$



MC Glauber fit: π^\pm multiplicity

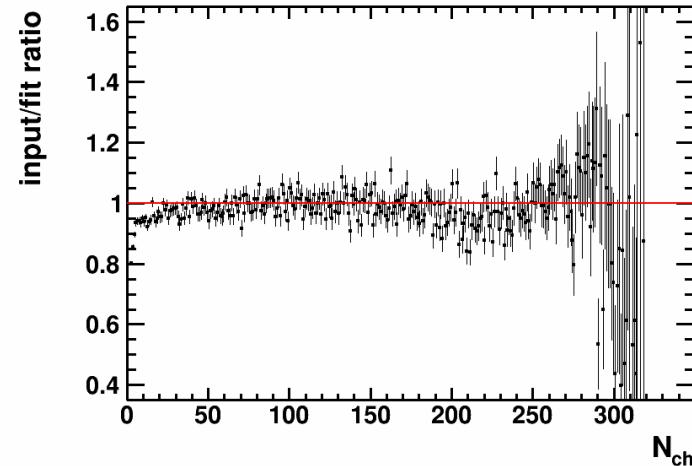
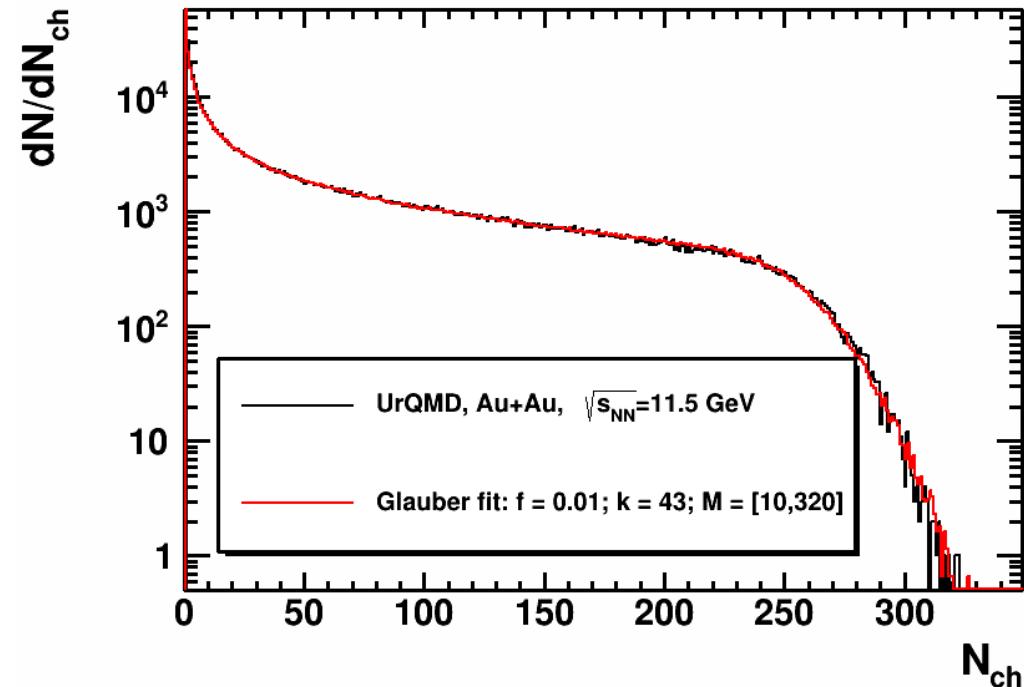
$f=0, k=42, \mu=0.24, \chi^2=1.39\pm0.1, M=(10,240)$



MC Glauber fit is in the good agreement with simulated input for the large multiplicity region

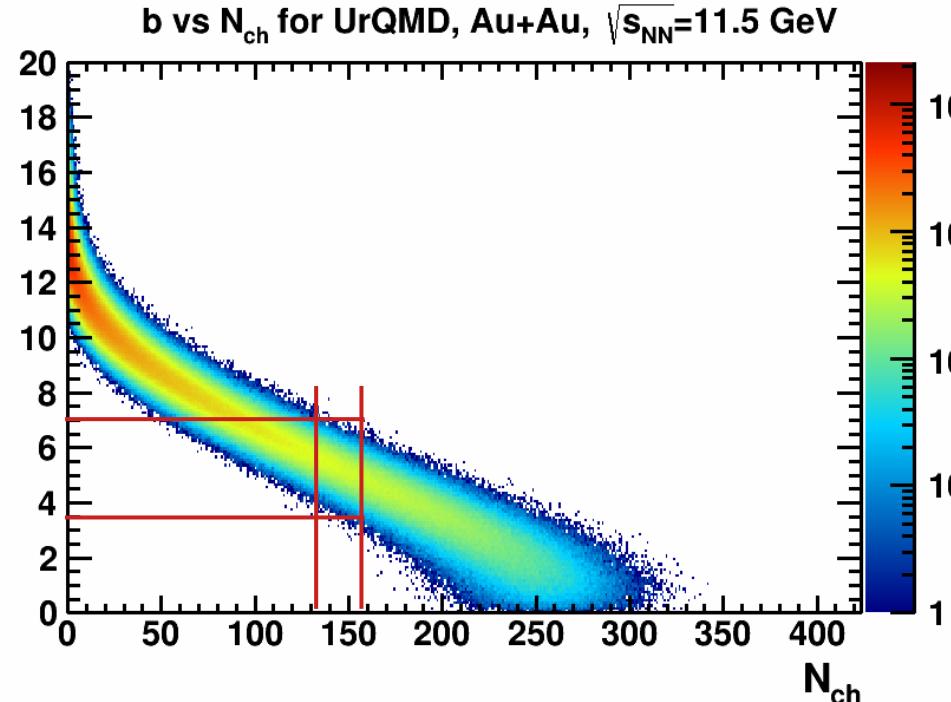
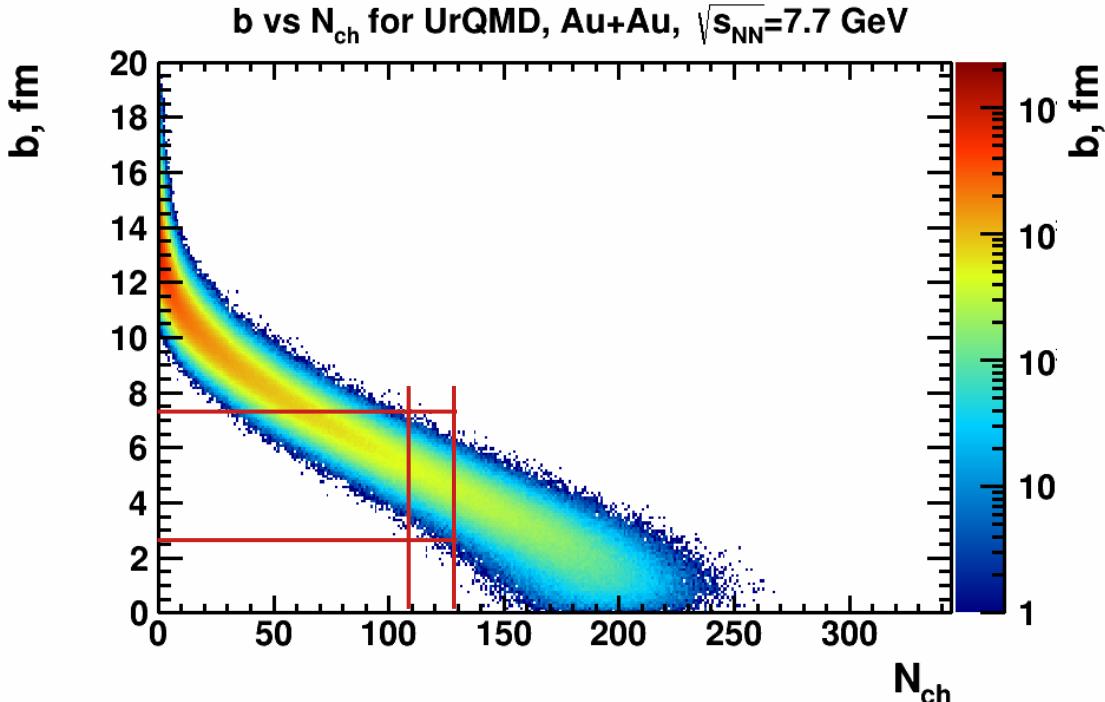
MC Glauber fit: π^\pm multiplicity

$f=0.01, k=43, \mu=0.3, \chi^2=1.17\pm0.07, M=(10,320)$



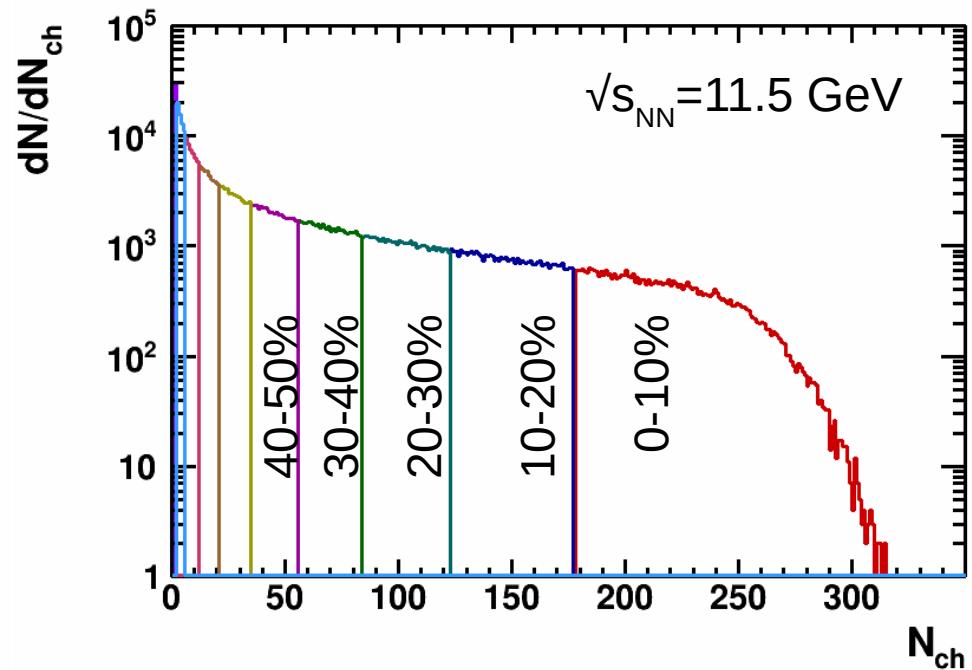
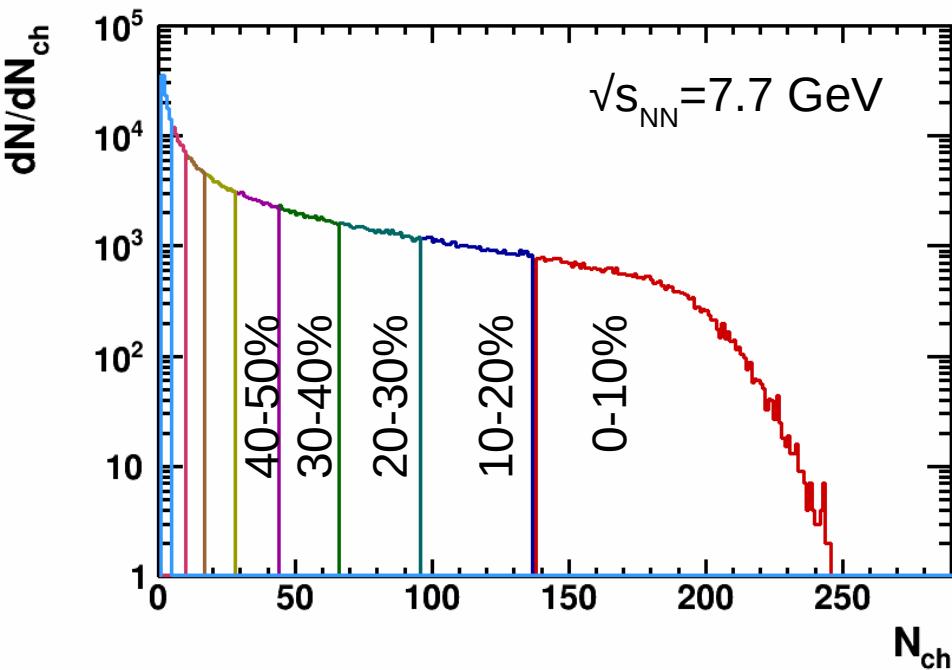
MC Glauber fit is in the good agreement with simulated input for the large multiplicity region

b vs. multiplicity correlation

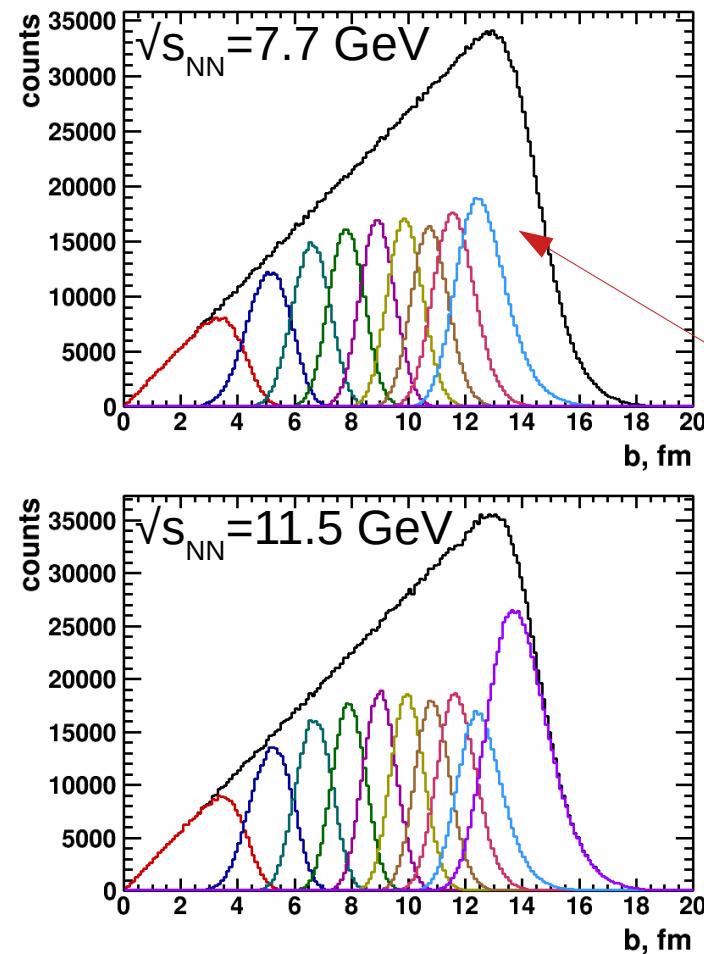


Events in multiplicities $M \pm \Delta M$ have impact parameter in range $b \pm \sigma_b$

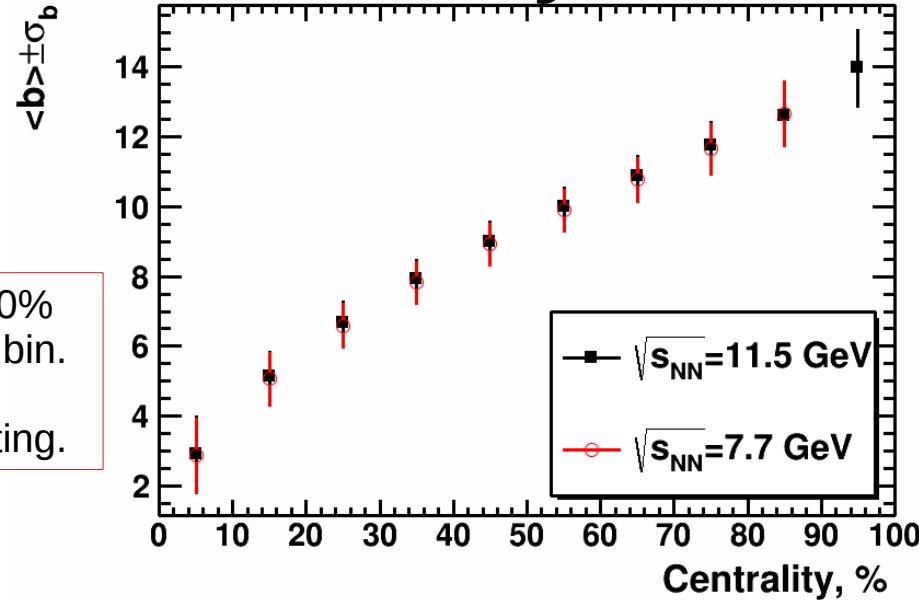
N_{ch} distribution in centrality classes



b distribution in centrality classes

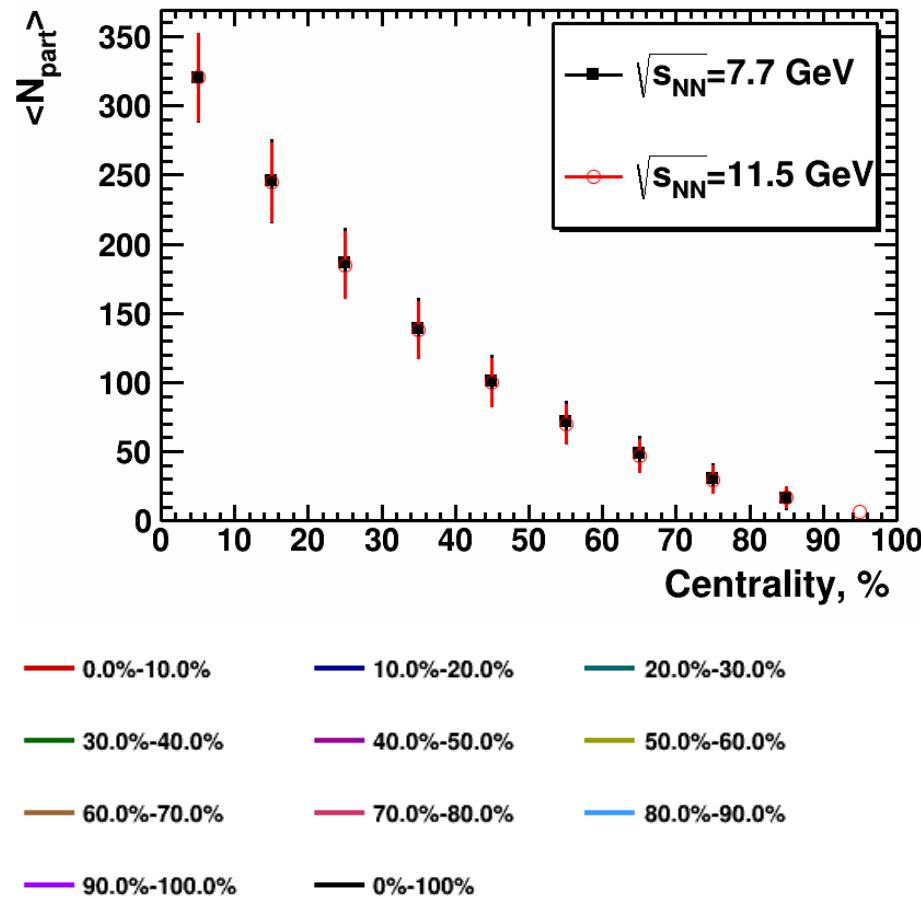
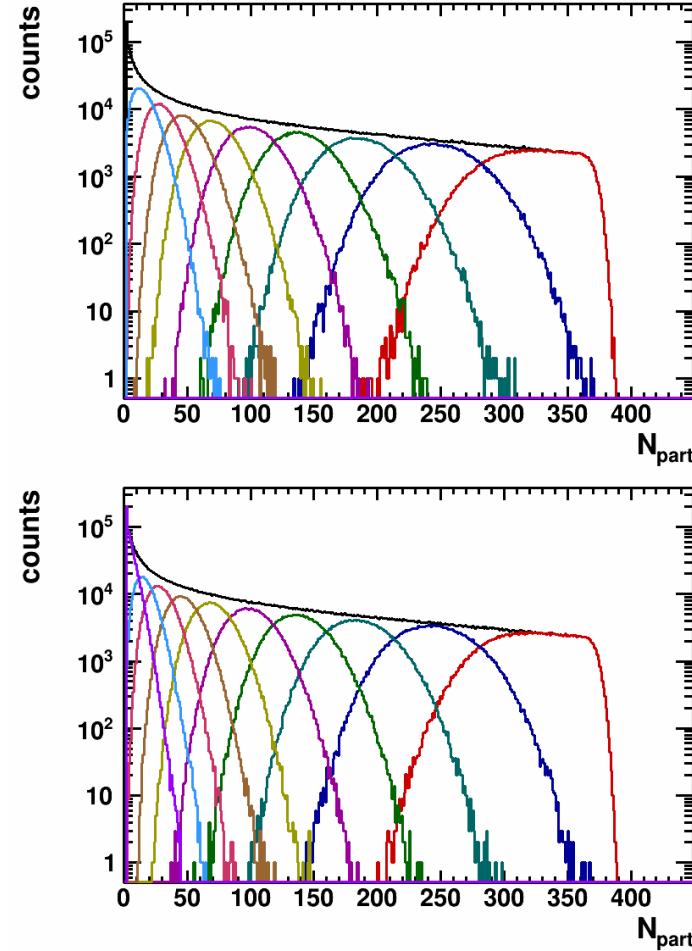


No 90-100% centrality bin.
Investigating.

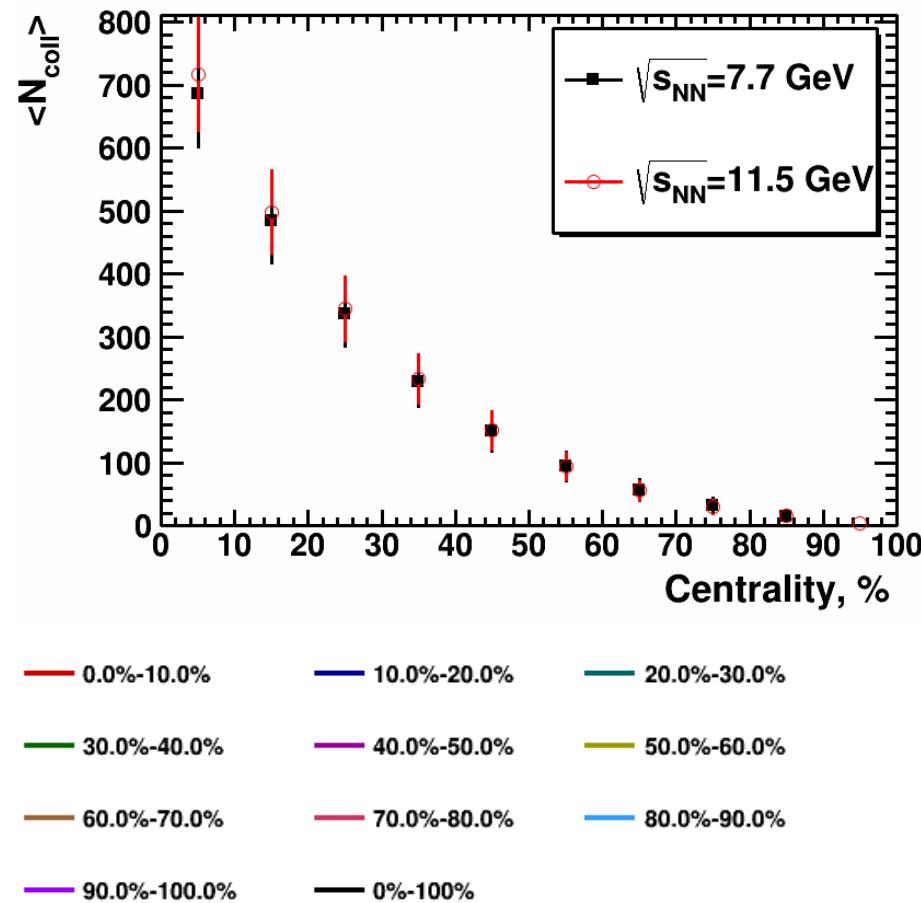
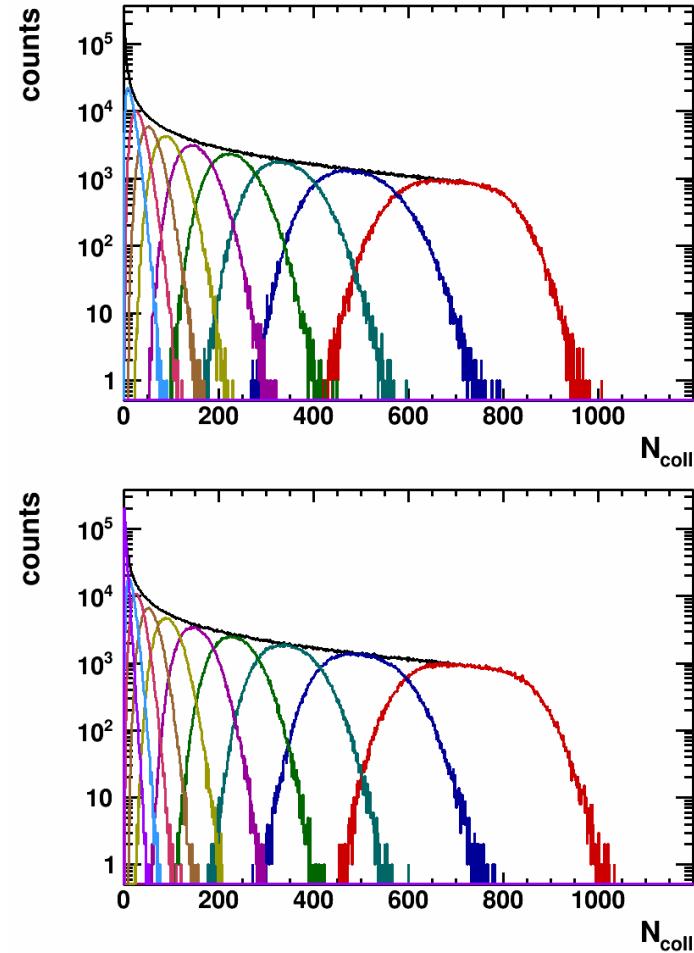


- | Centrality Class | Color |
|------------------|--------------|
| 0.0%-10.0% | Red |
| 10.0%-20.0% | Blue |
| 20.0%-30.0% | Teal |
| 30.0%-40.0% | Green |
| 40.0%-50.0% | Magenta |
| 50.0%-60.0% | Yellow-Green |
| 60.0%-70.0% | Brown |
| 70.0%-80.0% | Pink |
| 80.0%-90.0% | Cyan |
| 90.0%-100.0% | Purple |
| 0%-100% | Black |

N_{part} distribution in centrality classes

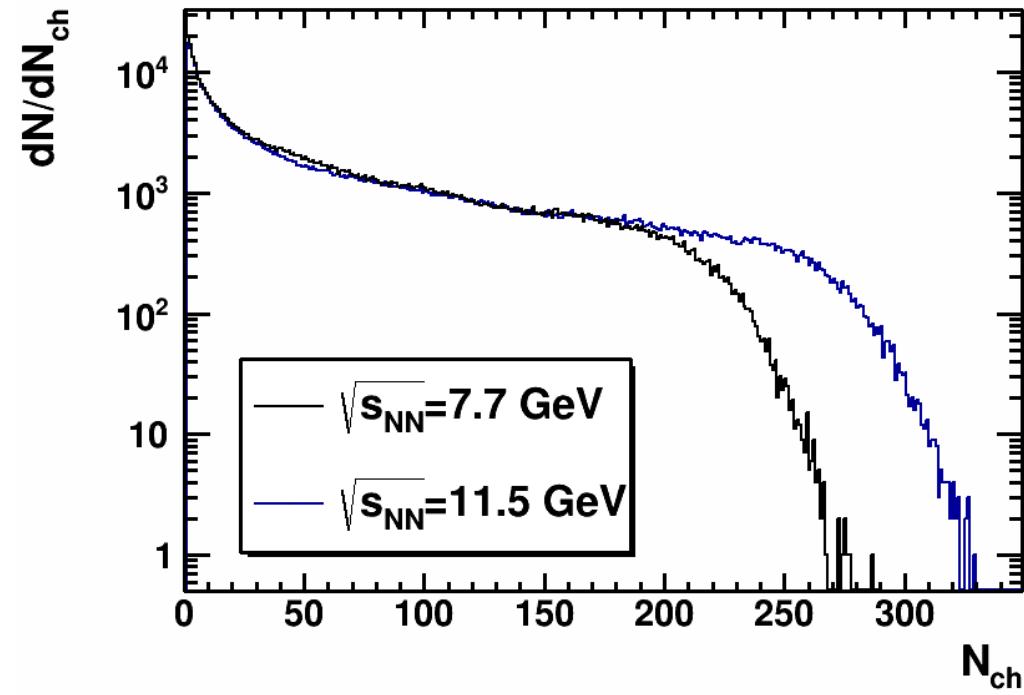


N_{coll} distribution in centrality classes



Centrality framework results for PHSD

Charged particle multiplicity in PHSD



Generated data:

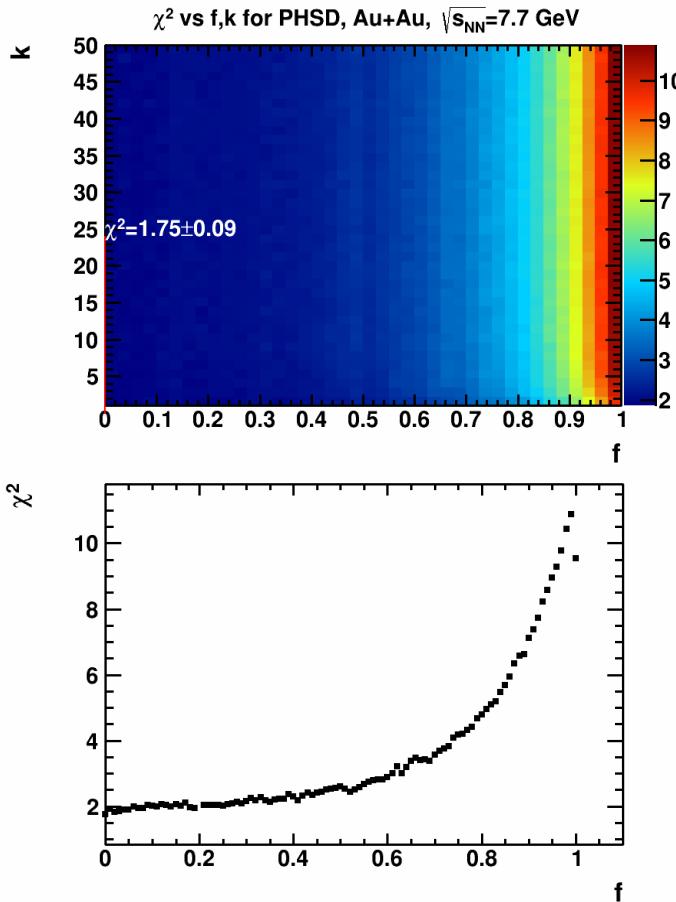
- PHSD v4.0 simulation
 - Au+Au, $N_{ev}=500k$,
 $\sqrt{s_{NN}}=7.7, 11.5 \text{ GeV}$

Used particle selection:

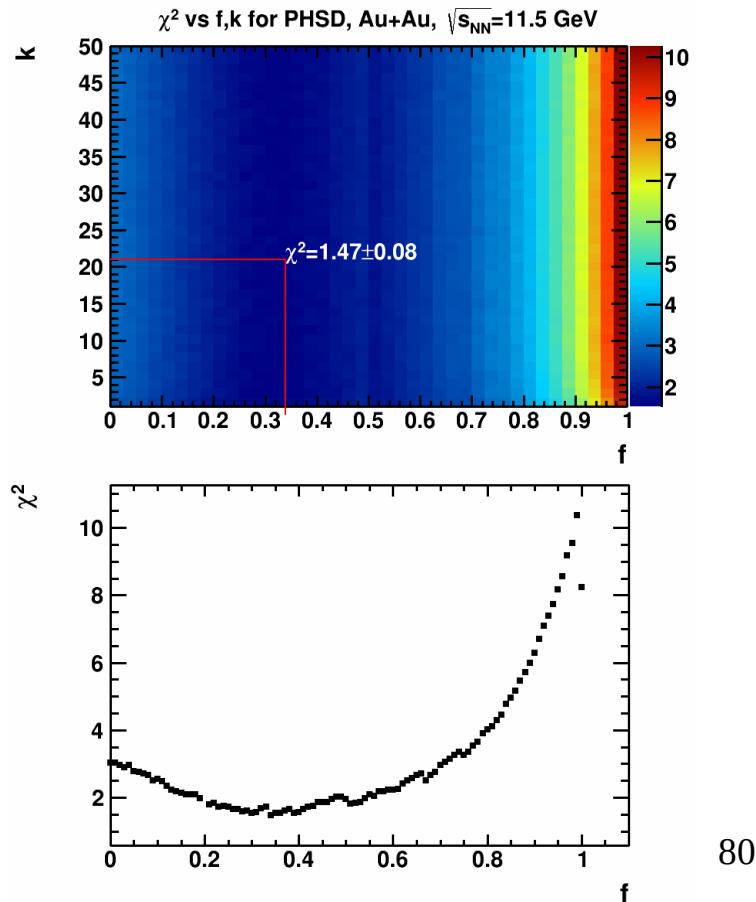
- $|\eta| < 0.5$
- $p_T > 0.15 \text{ GeV}/c$

Fit parameters f,k vs χ^2

$f=0, k=24, \mu=0.27, \chi^2=1.75\pm0.09, M=(10,265)$

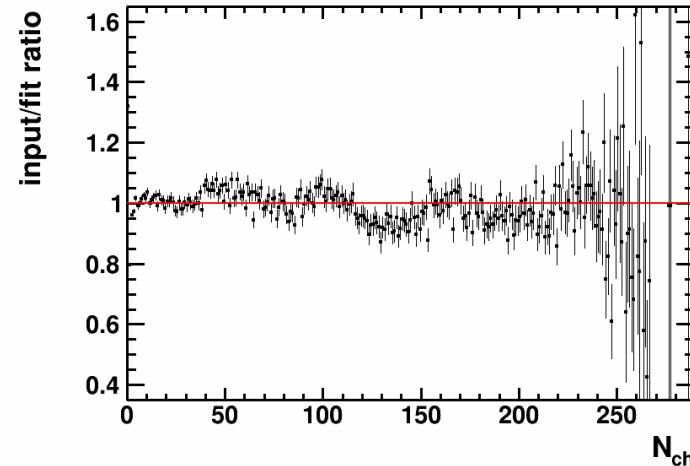
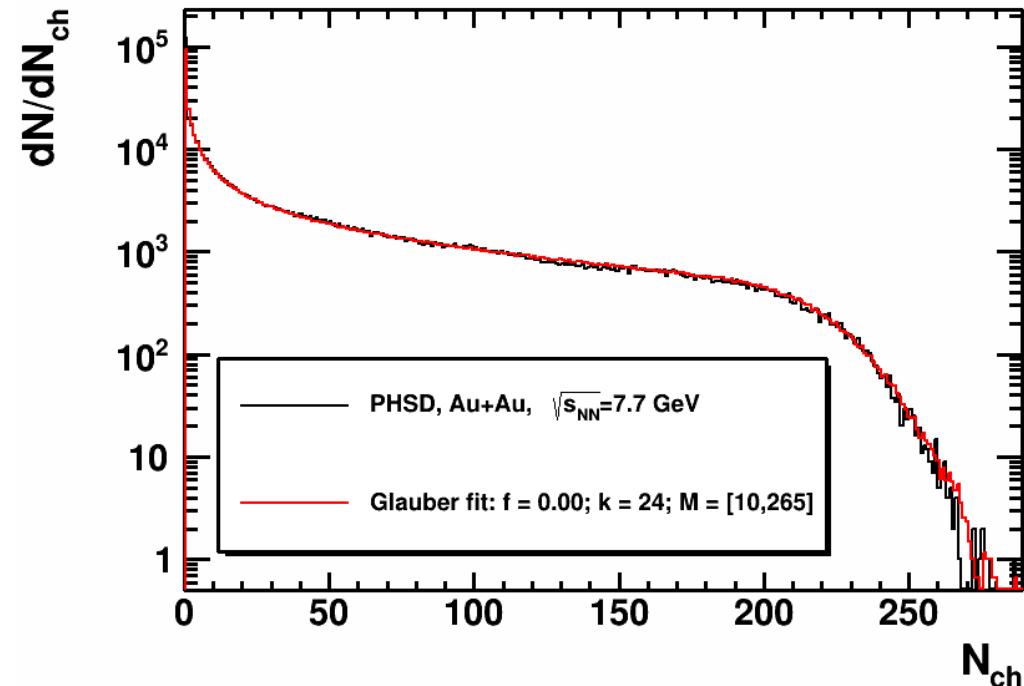


$f=0.34, k=21, \mu=0.39, \chi^2=1.47\pm0.08, M=(10,320)$



MC Glauber fit: h^\pm multiplicity

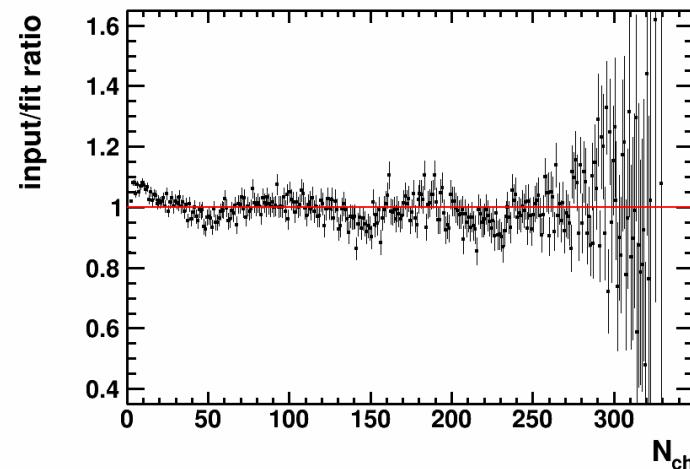
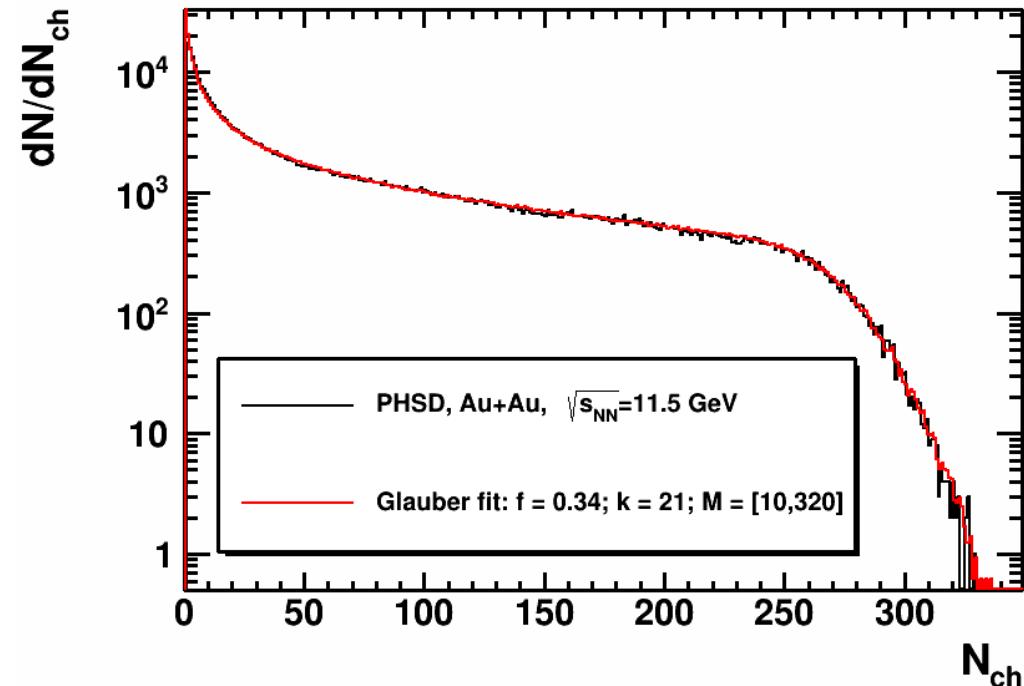
$f=0$, $k=24$, $\mu=0.27$, $\chi^2=1.75\pm 0.09$, $M=[10,265]$



MC Glauber fit is in the good agreement with simulated input for the large multiplicity region

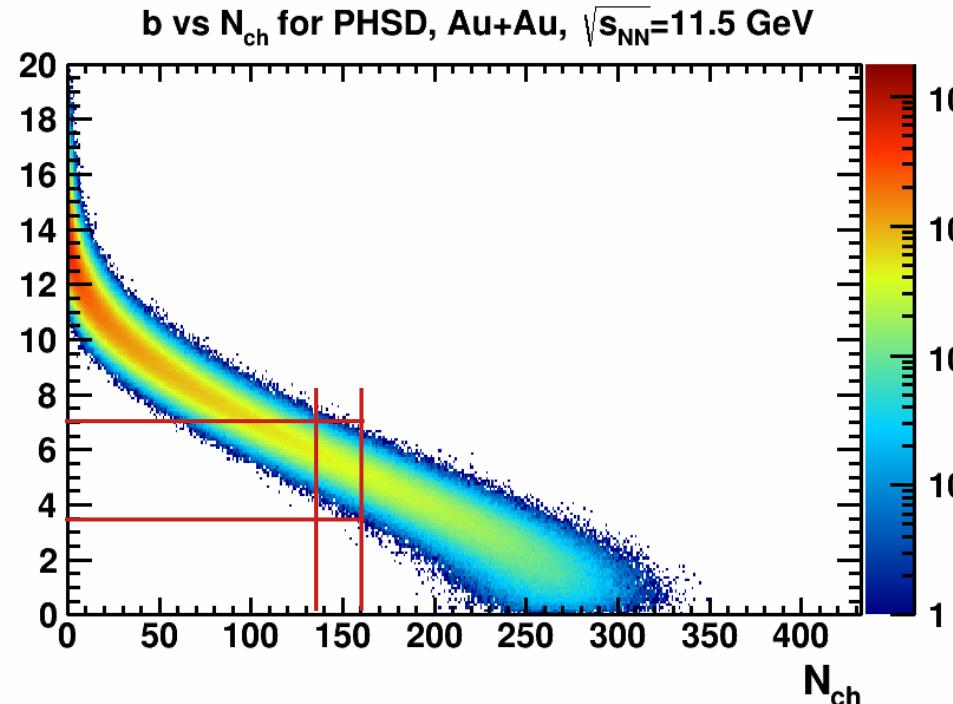
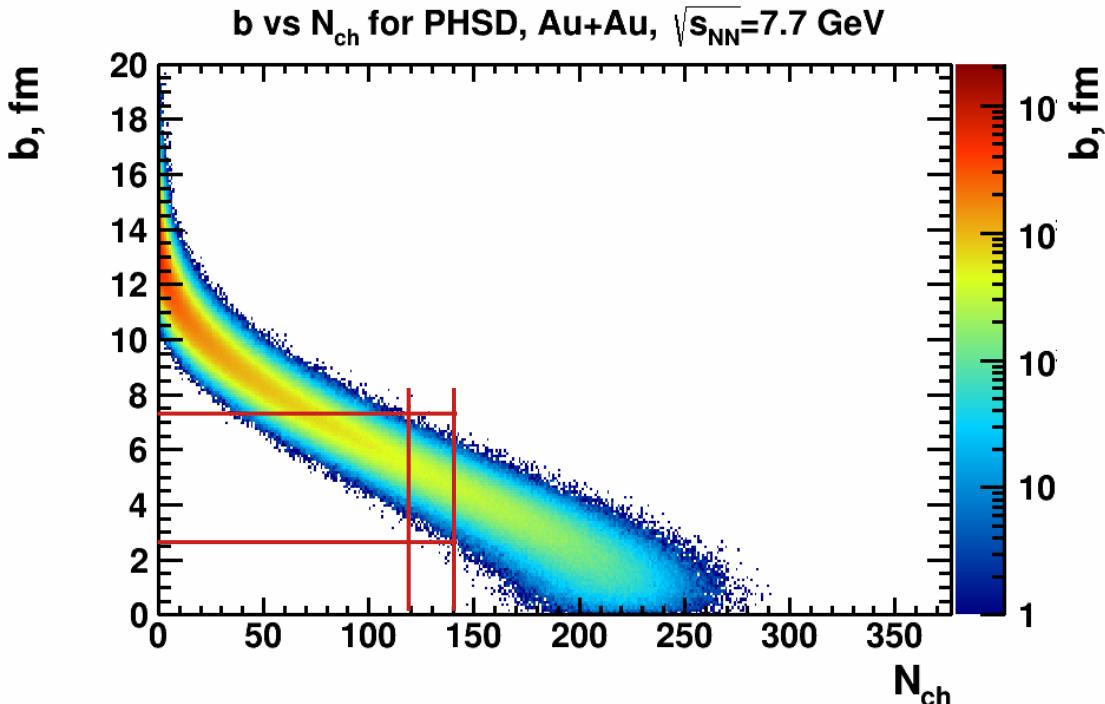
MC Glauber fit: h^\pm multiplicity

$f=0.34, k=21, \mu=0.39, \chi^2=1.47\pm0.08, M=(10,320)$



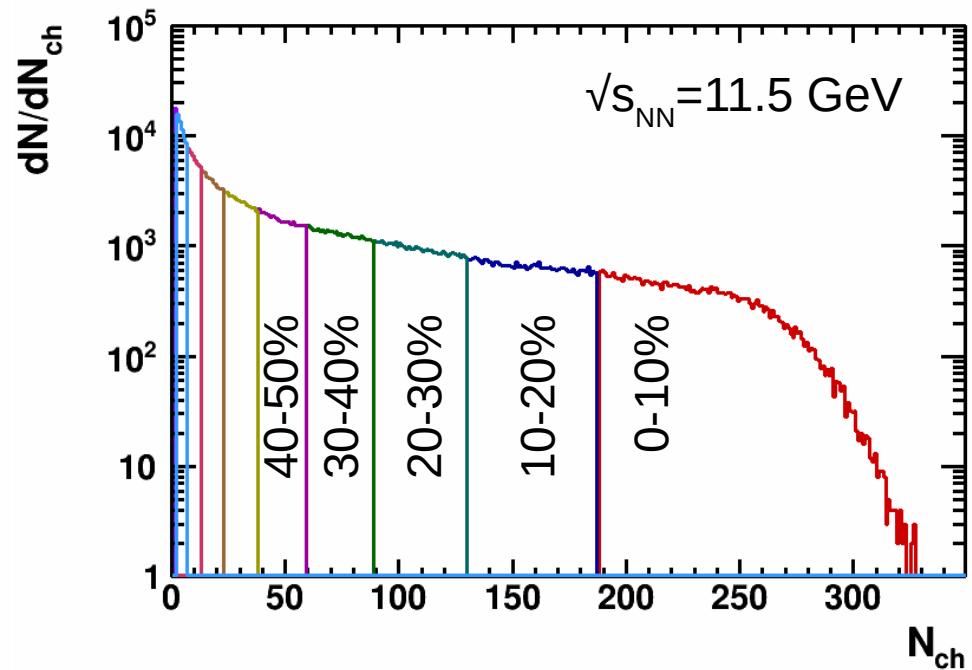
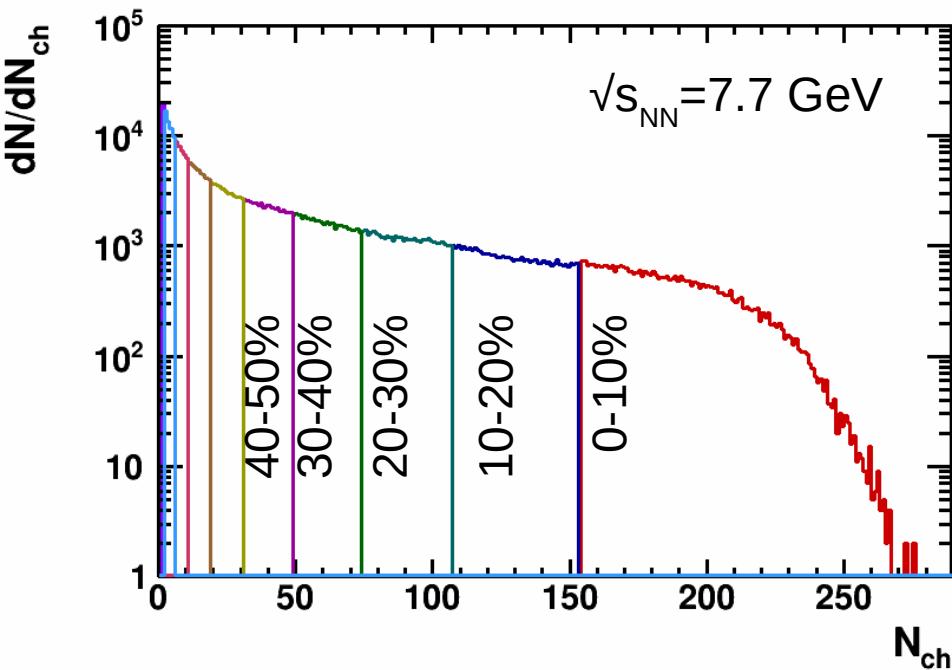
MC Glauber fit is in the good agreement with simulated input for the large multiplicity region

b vs. multiplicity correlation

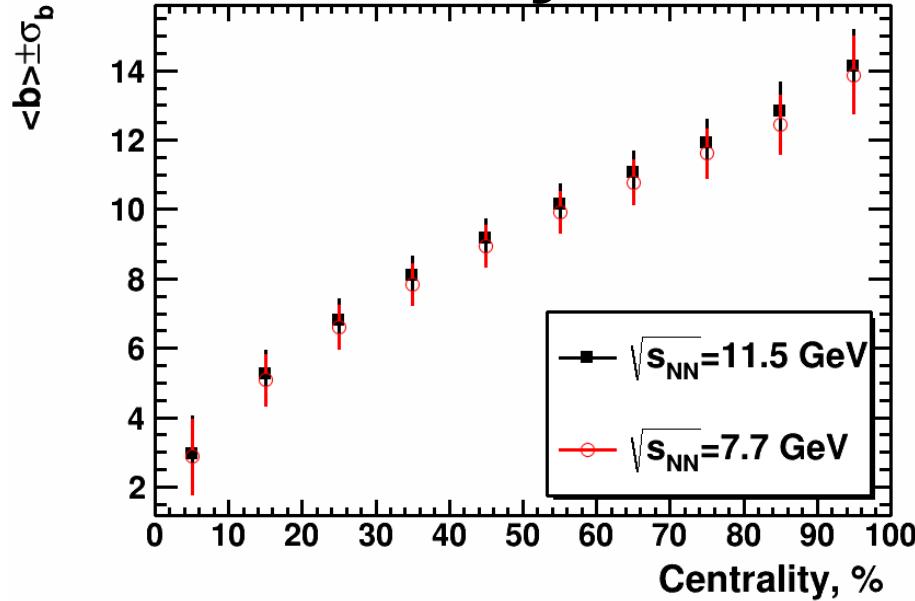
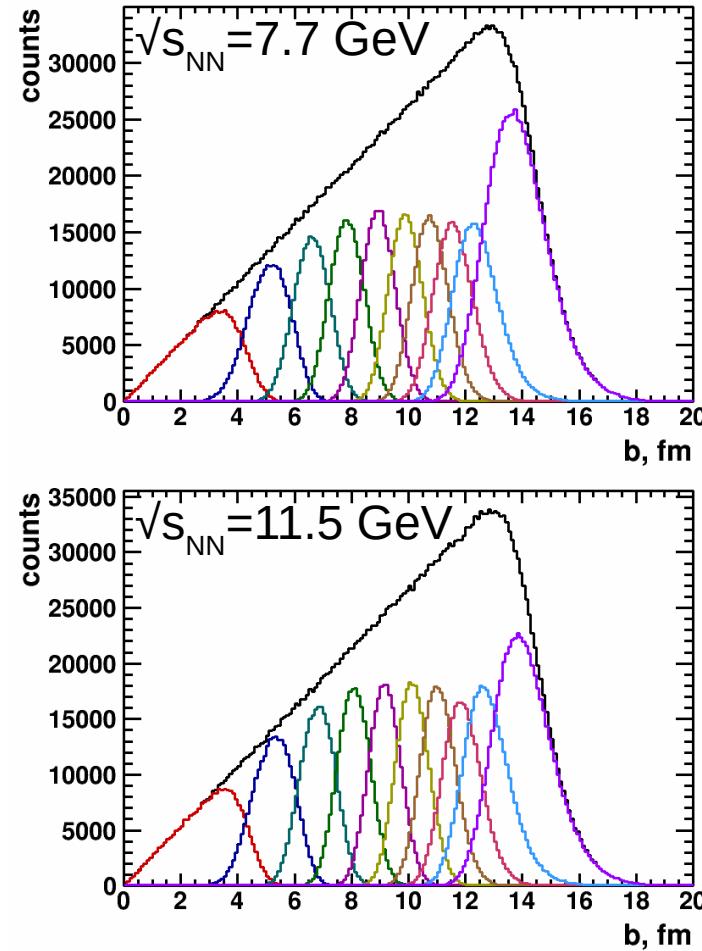


Events in multiplicities $M \pm \Delta M$ have impact parameter in range $b \pm \sigma_b$

N_{ch} distribution in centrality classes

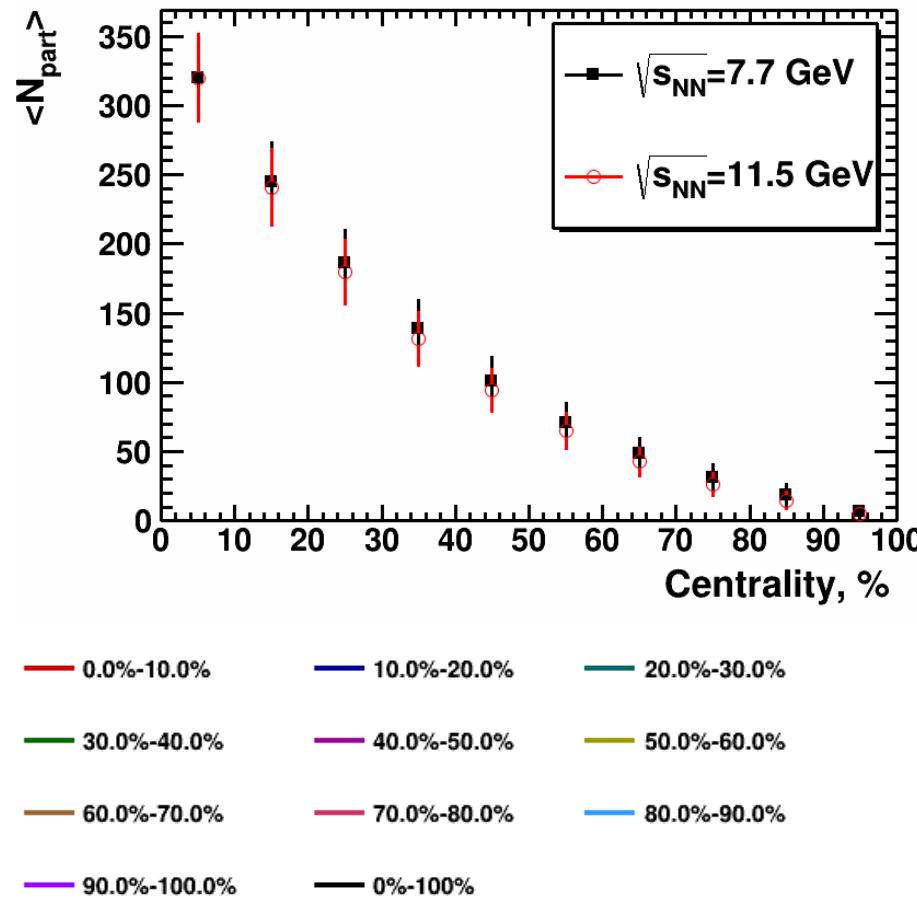
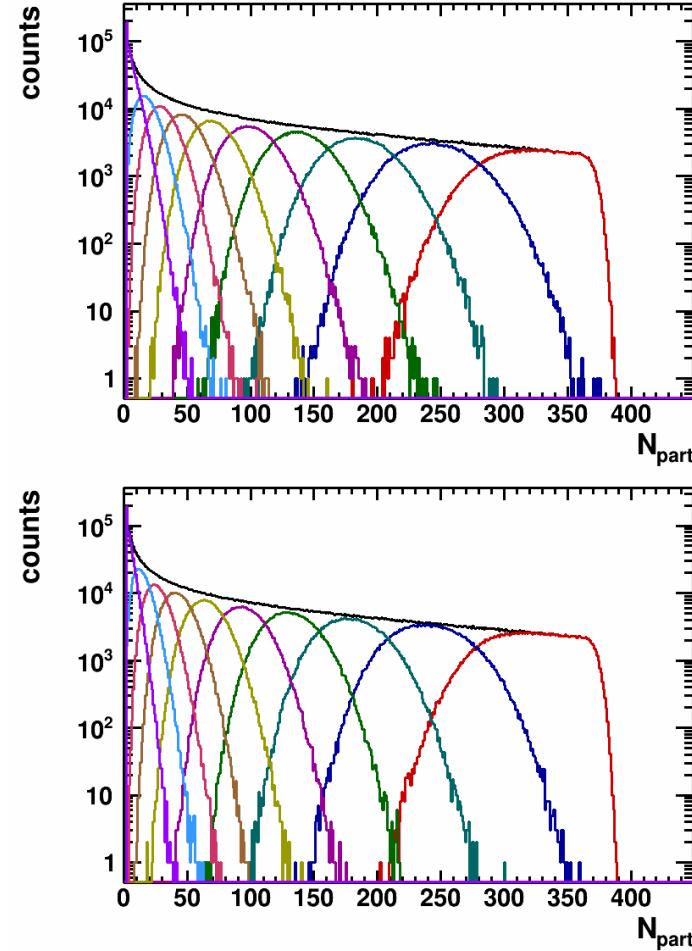


b distribution in centrality classes

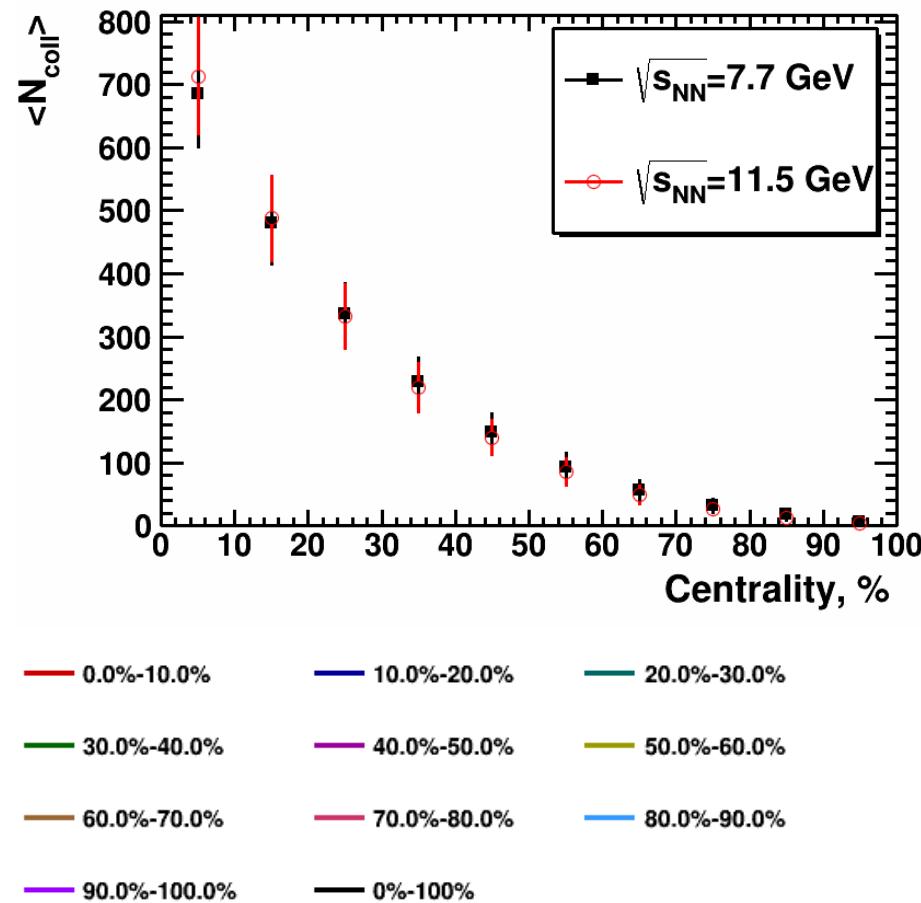
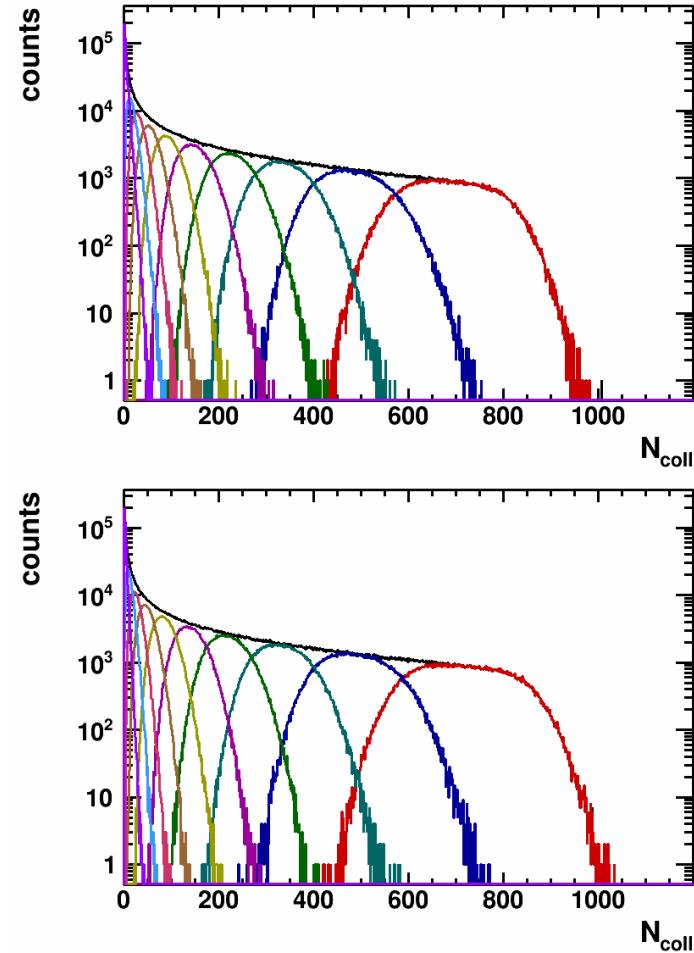


- 0.0%-10.0% — 10.0%-20.0% — 20.0%-30.0%
- 30.0%-40.0% — 40.0%-50.0% — 50.0%-60.0%
- 60.0%-70.0% — 70.0%-80.0% — 80.0%-90.0%
- 90.0%-100.0% — 0%-100%

N_{part} distribution in centrality classes

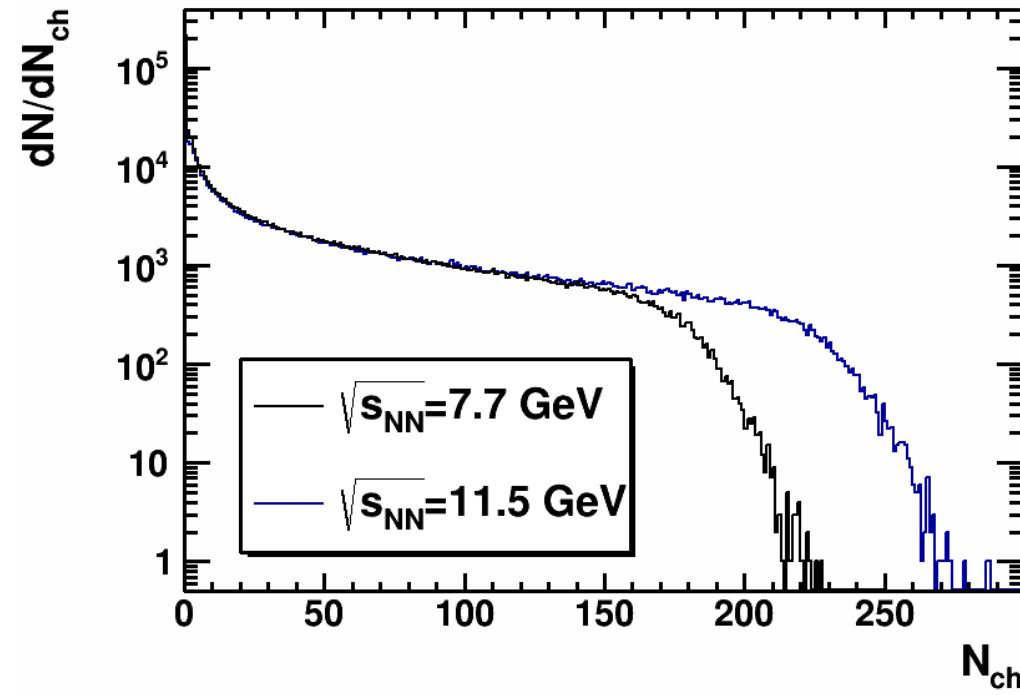


N_{coll} distribution in centrality classes



Centrality framework results for SMASH

Charged particle multiplicity in SMASH



Generated data:

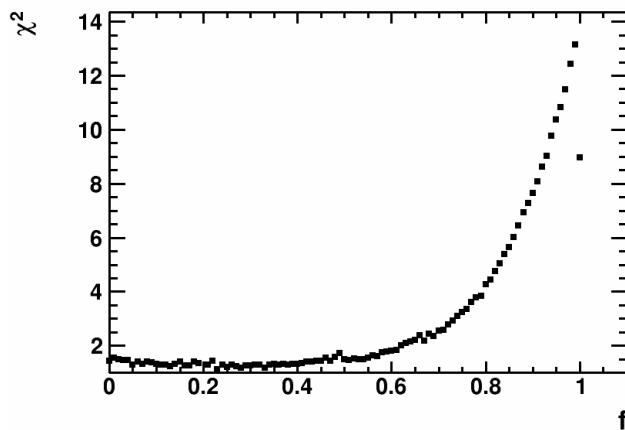
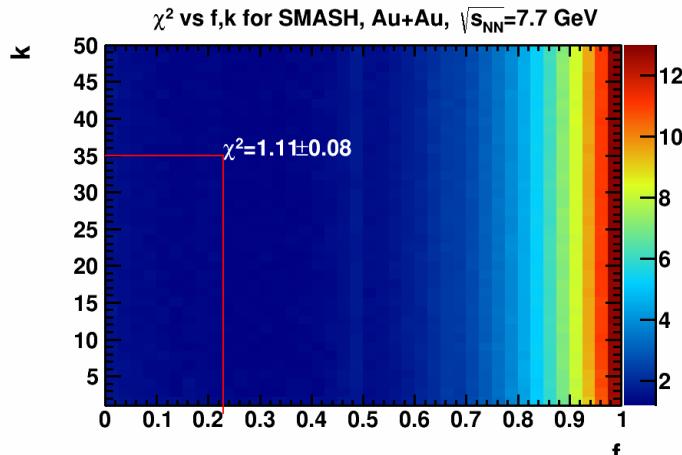
- SMASH v1.6 simulation
 - Au+Au, $N_{ev}=500k$,
 $\sqrt{s_{NN}}=7.7, 11.5 \text{ GeV}$

Used particle selection:

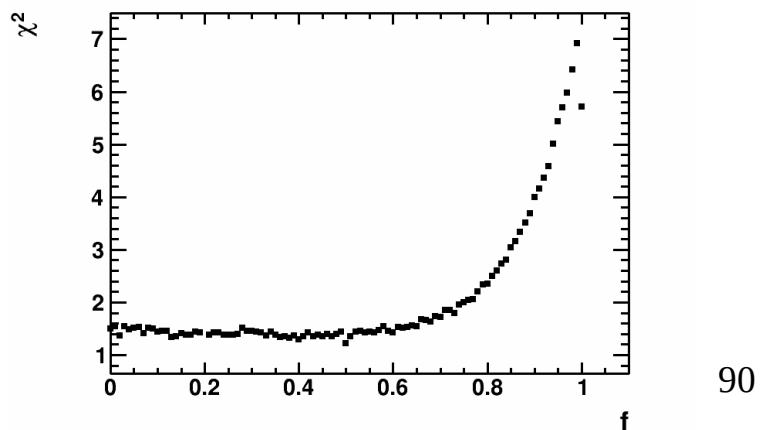
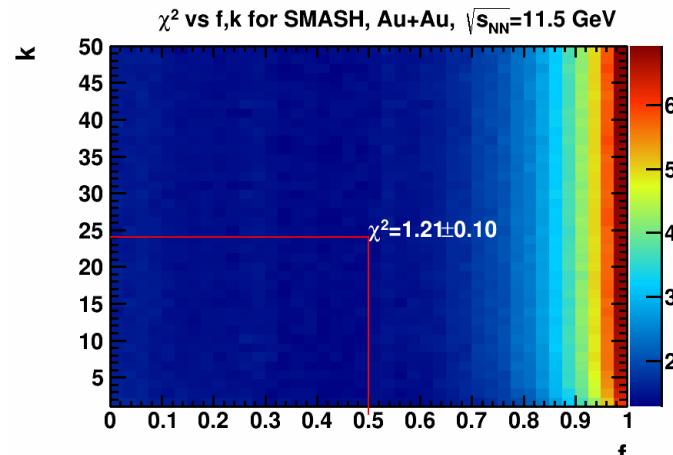
- $|\eta| < 0.5$
- $p_T > 0.15 \text{ GeV}/c$

Fit parameters f,k vs χ^2

$F=0.23, k=35, \mu=0.24, \chi^2=1.11\pm 0.08, M=(5,225)$

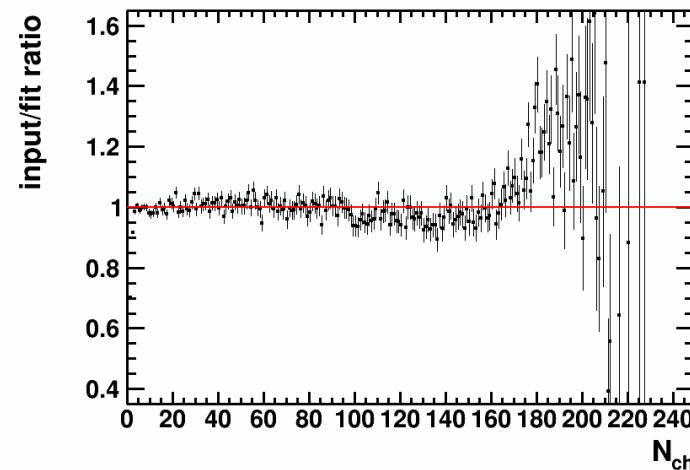
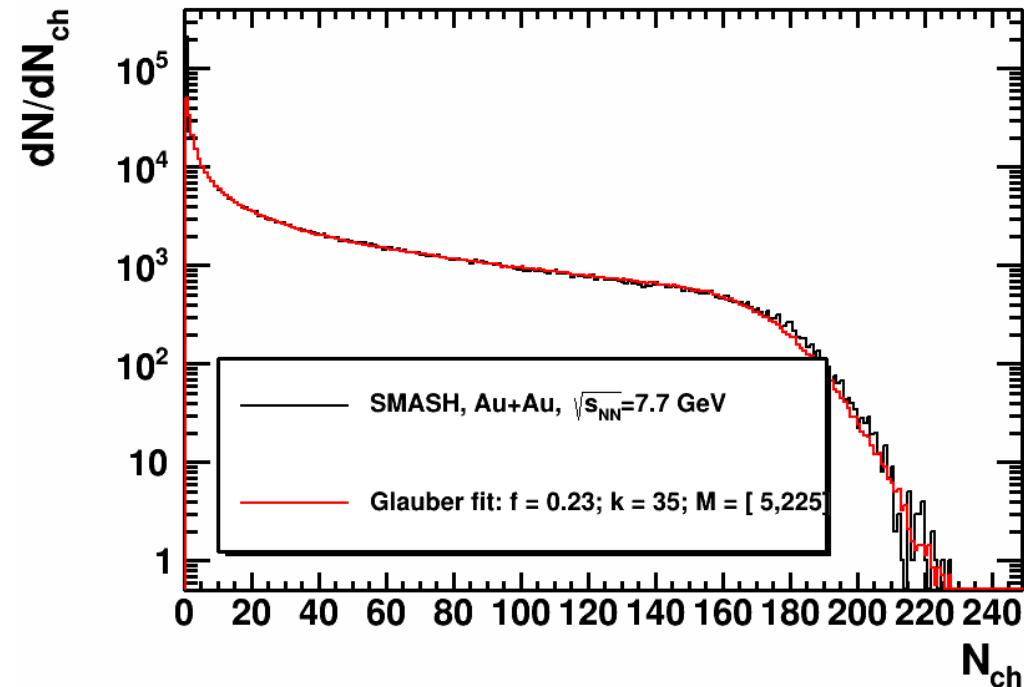


$f=0.5, k=24, \mu=0.36, \chi^2=1.21\pm 0.1, M=(10,265)$



MC Glauber fit: h^\pm multiplicity

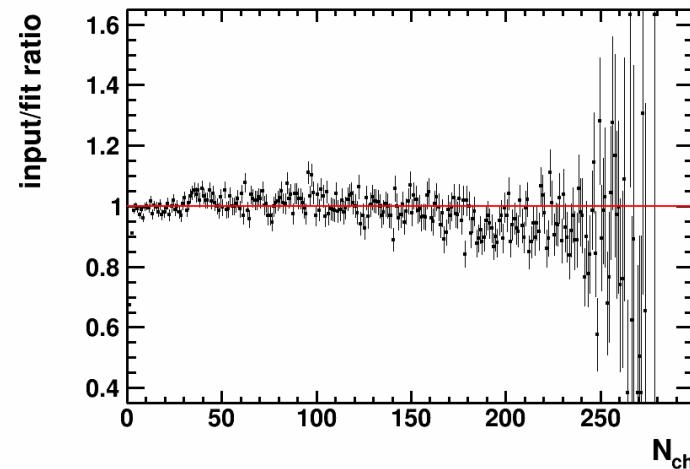
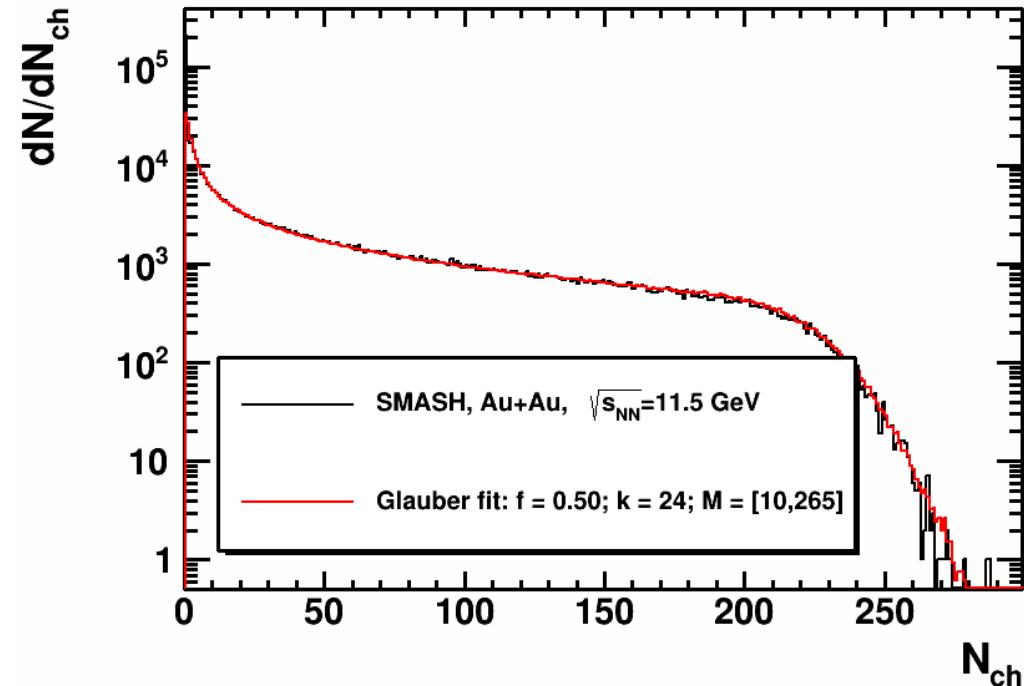
$F=0.23, k=35, \mu=0.24, \chi^2=1.11\pm0.08, M=(5,225)$



MC Glauber fit is deviate from SMASH data for large multiplicity region

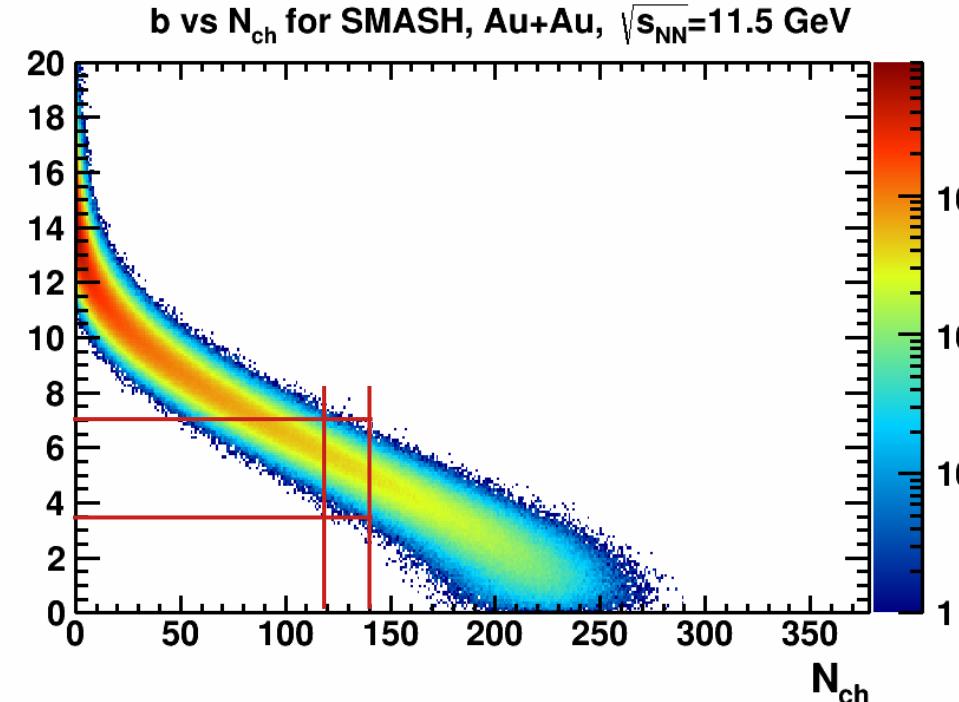
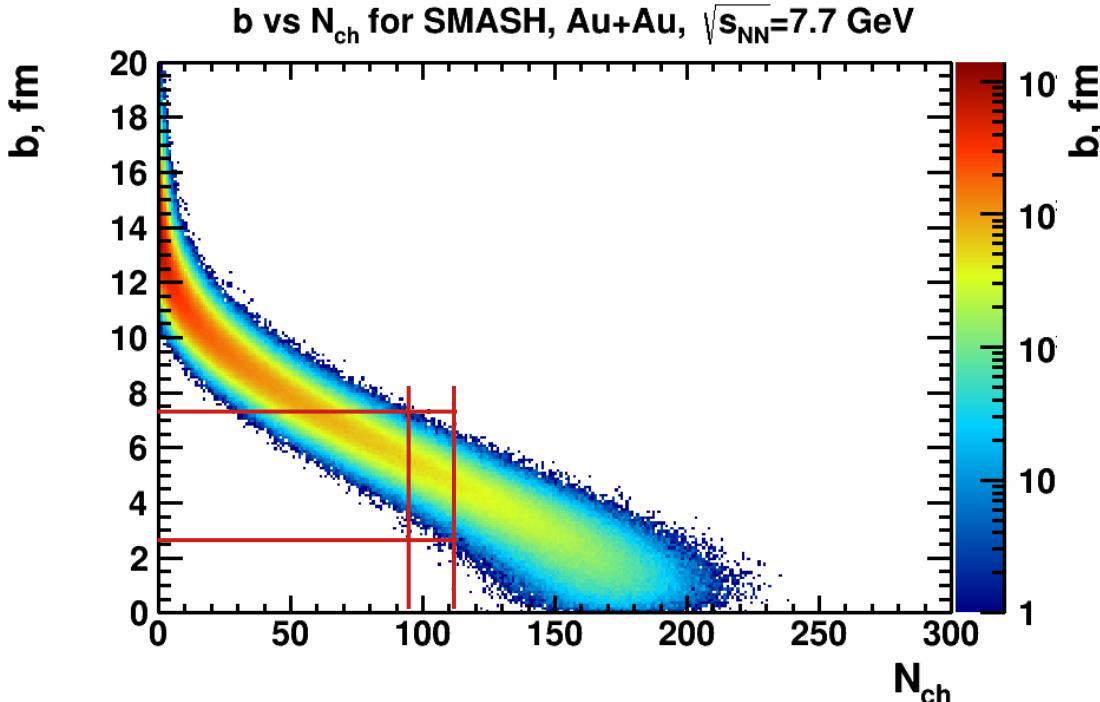
MC Glauber fit: h^\pm multiplicity

$f=0.5, k=24, \mu=0.36, \chi^2=1.21\pm0.1, M=(10,265)$



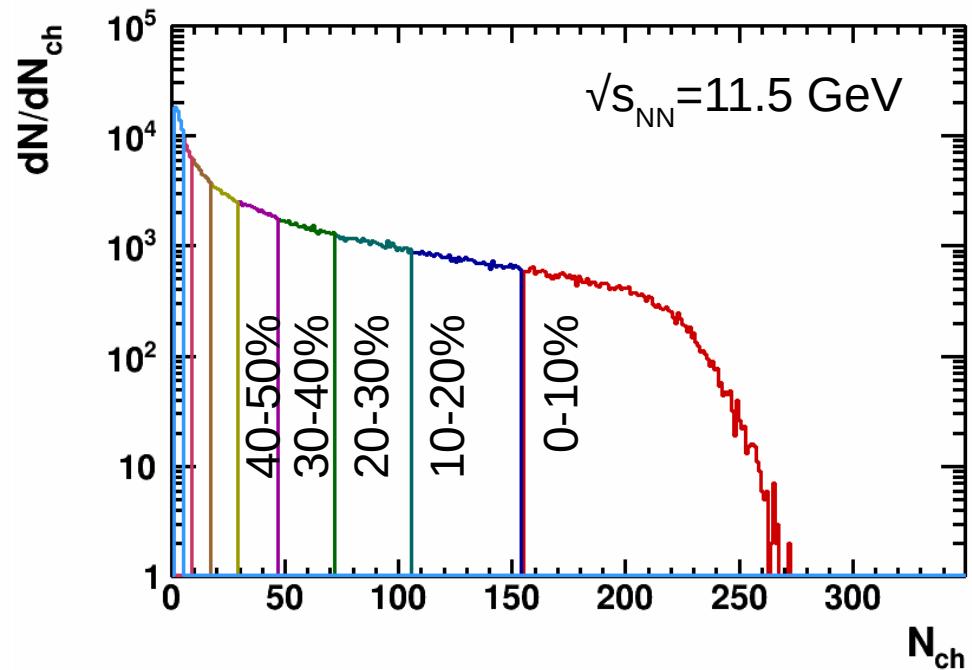
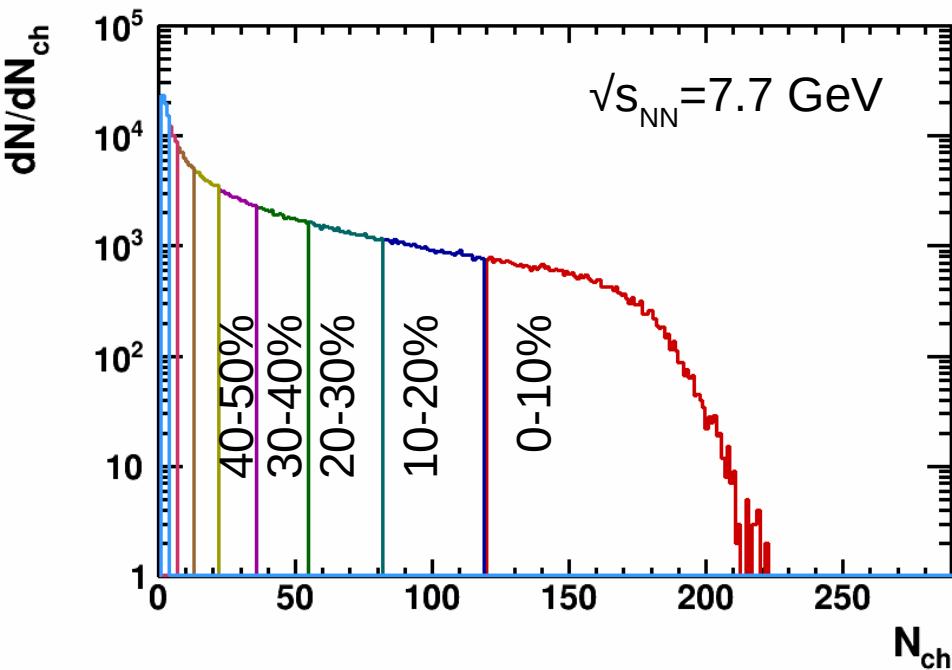
MC Glauber fit is in the good agreement with simulated input for the large multiplicity region

b vs. multiplicity correlation

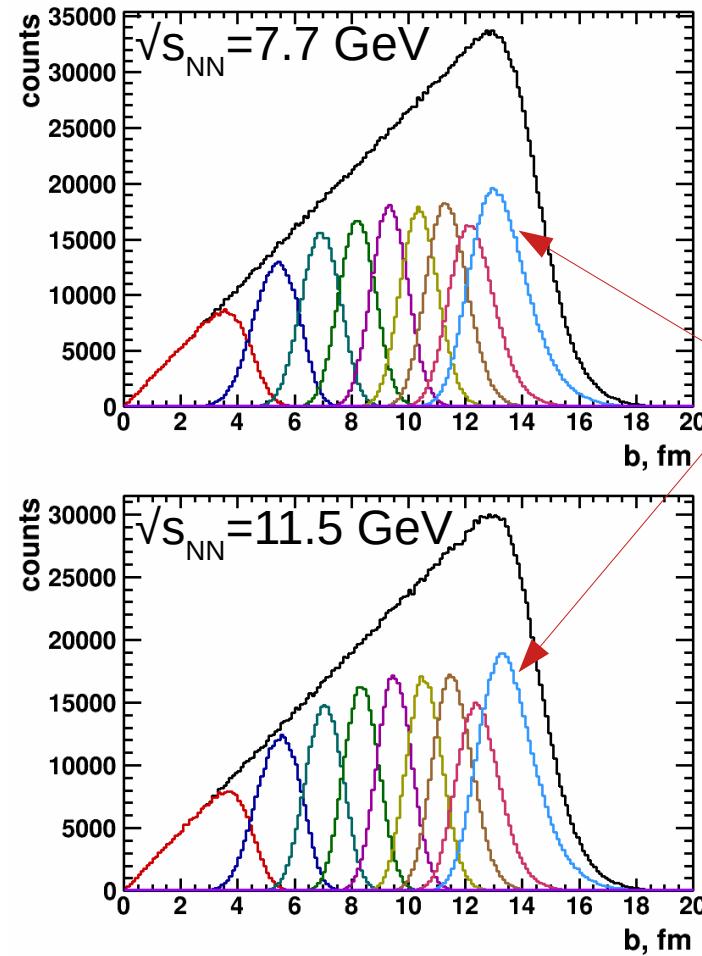


Events in multiplicities $M \pm \Delta M$ have impact parameter in range $b \pm \sigma_b$

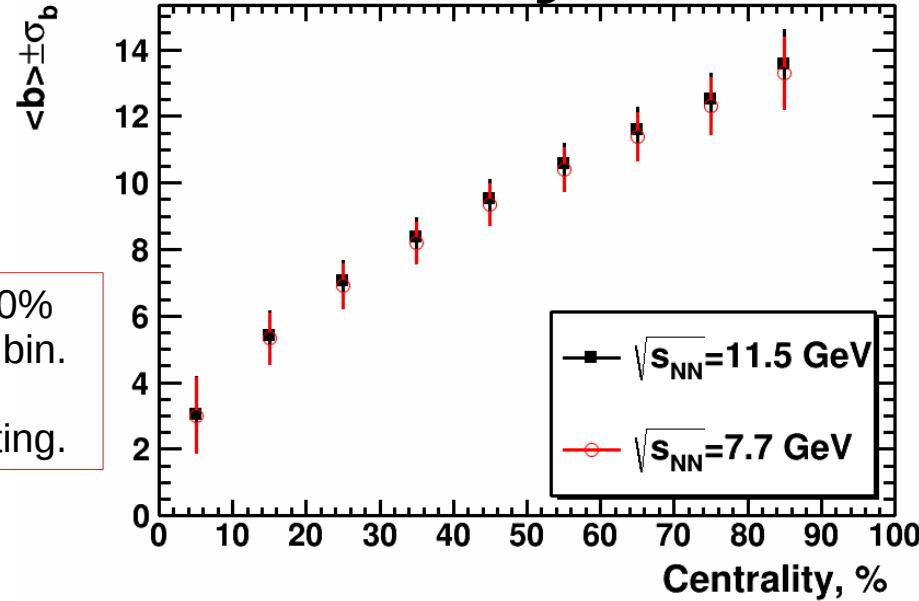
N_{ch} distribution in centrality classes



b distribution in centrality classes

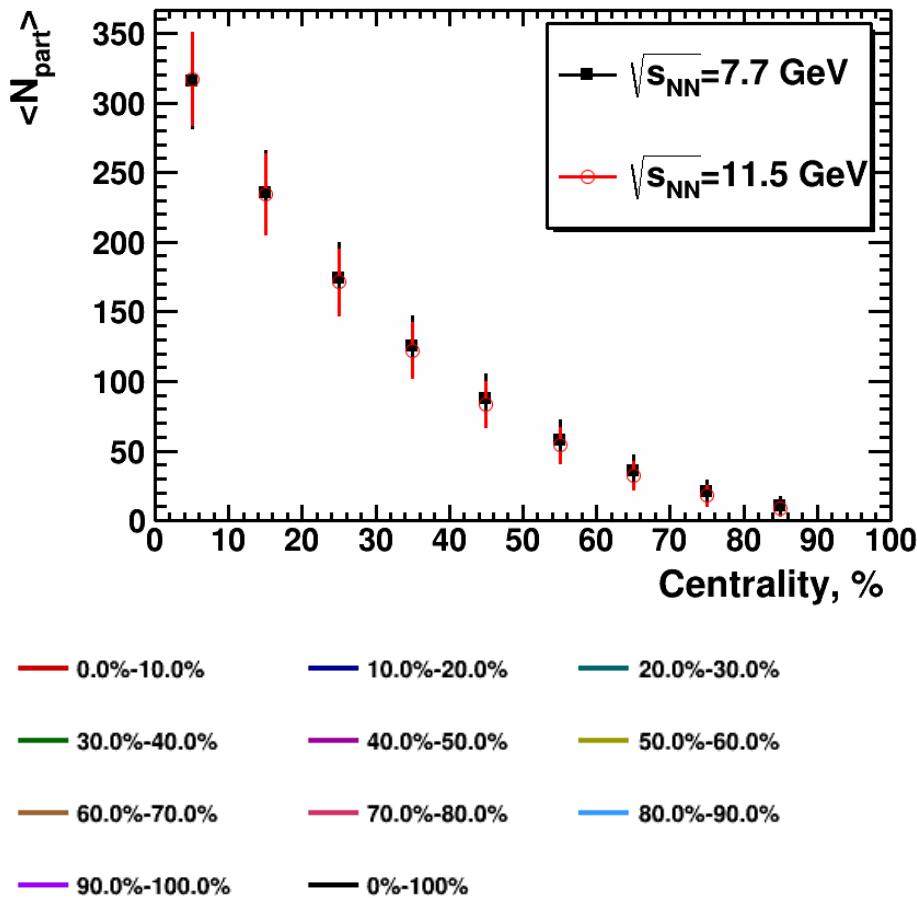
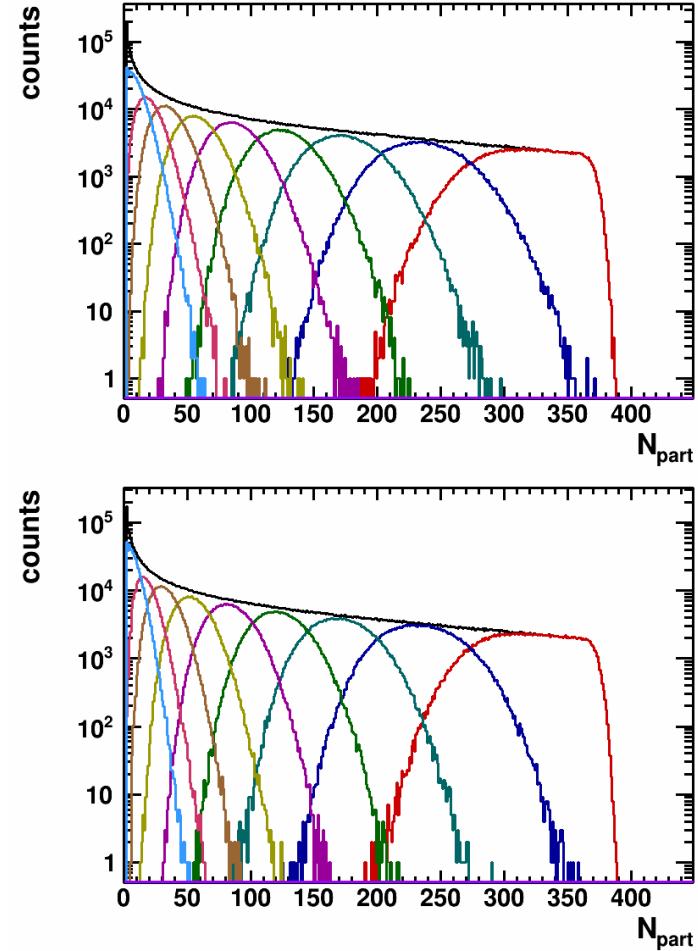


No 90-100% centrality bin.
Investigating.



- | | | |
|----------------|---------------|---------------|
| — 0.0%-10.0% | — 10.0%-20.0% | — 20.0%-30.0% |
| — 30.0%-40.0% | — 40.0%-50.0% | — 50.0%-60.0% |
| — 60.0%-70.0% | — 70.0%-80.0% | — 80.0%-90.0% |
| — 90.0%-100.0% | — 0%-100% | |

N_{part} distribution in centrality classes



N_{coll} distribution in centrality classes

