Green’s function technique for precise calculation of false vacuum decay rate.

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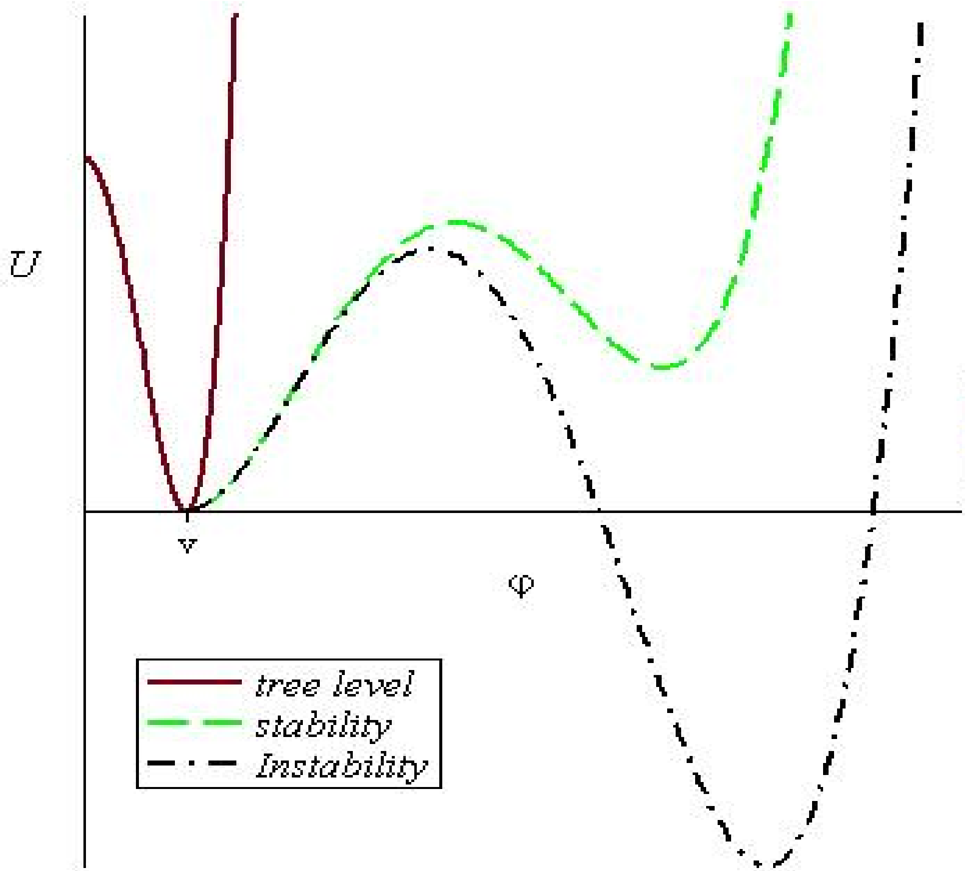
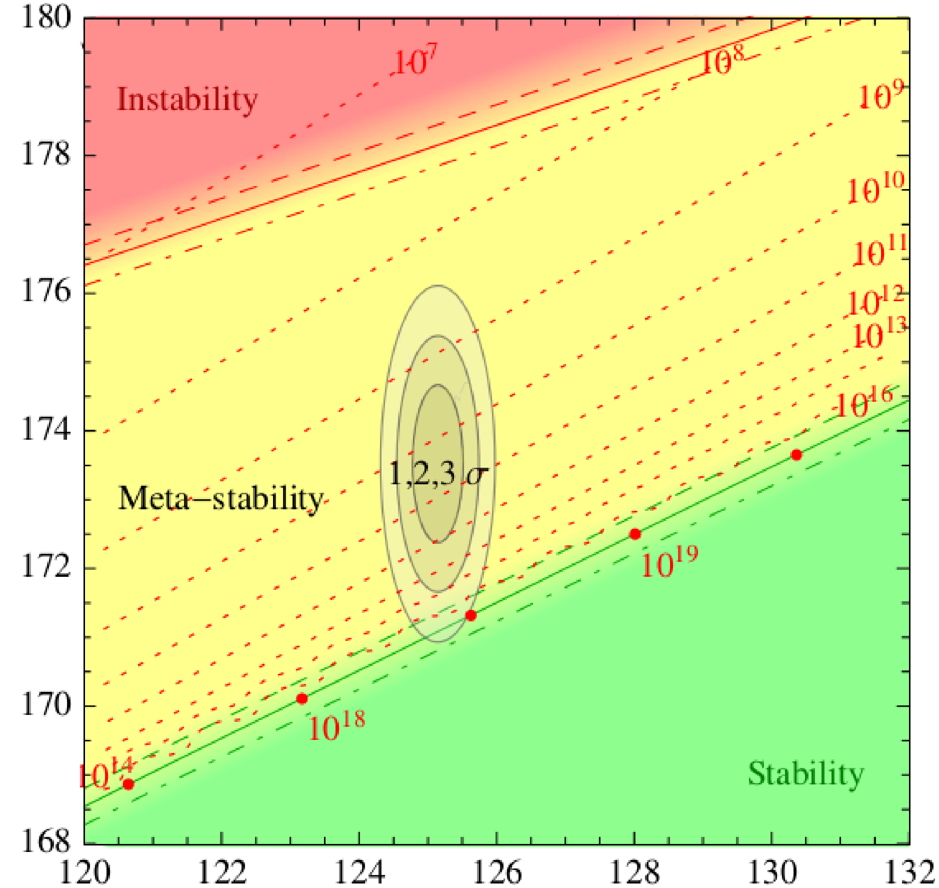
Vacuum metastability of the Standard Model

$$U(\varphi) = \lambda\left(\varphi^2 - v^2\right)^2 + U_{corr}$$



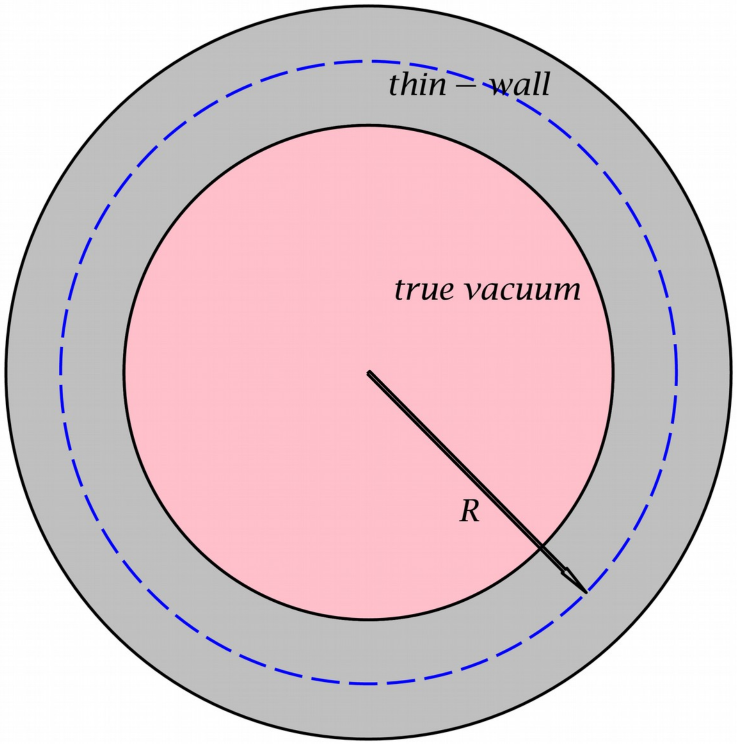


Mass of the t quark (GeV)

Mass of the Higgs boson (GeV)

Buttazzo D., Degrassi G., Giardino P.P.,Giudice G.F., Sala F., Salvio A. and Strumia A., JHEP **1312** (2013) 089

False vacuum decay in 4 dimensional space



$$t \to -i\tau$$ Wick rotation

$$\frac{\Gamma}{V} = \underbrace{\left(\frac{S_c}{2\pi}\right)^2 \left[\frac{\det'\left(-\frac{d^2}{d\tau^2} - \Delta + U''(\varphi_c)\right)}{\det\left(-\frac{d^2}{d\tau^2} - \Delta + U''(v)\right)}\right]^{-\frac{1}{2}}}_{\text{One loop}} \exp\left(\frac{-\overbrace{S_b}^{\text{Action}}}{\hbar}\right) \underbrace{\left\{1 + \hbar I_2 + \mathcal{O}(\hbar^2)\right\}}_{\text{NNLO and + corrections}}$$

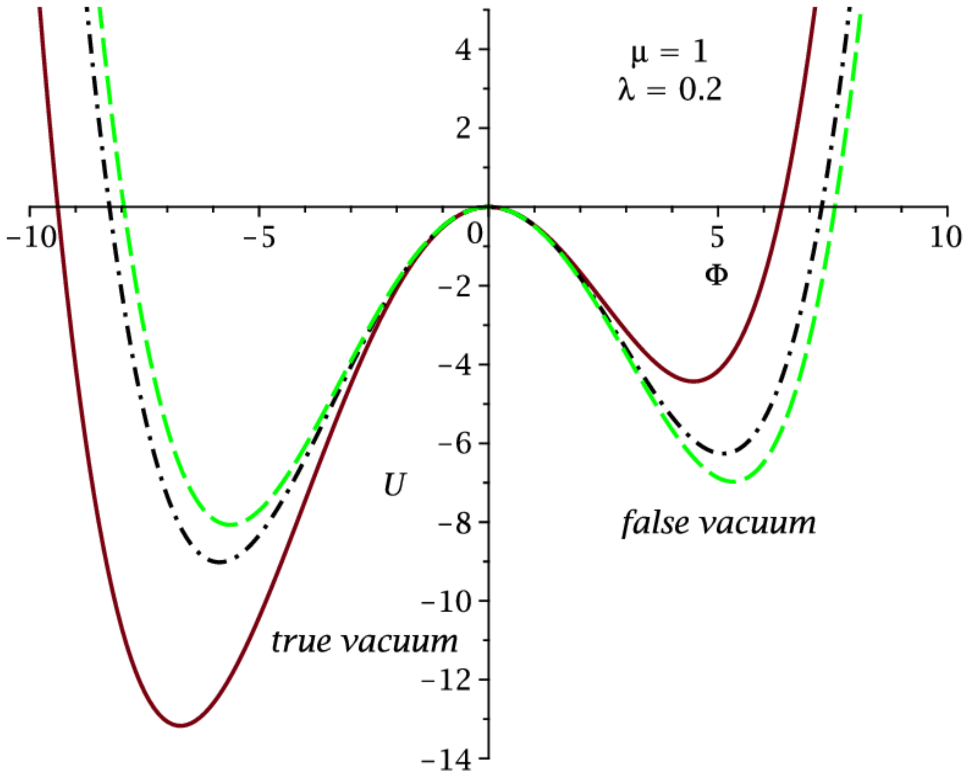
False vacuum decay in 4 dimensional space

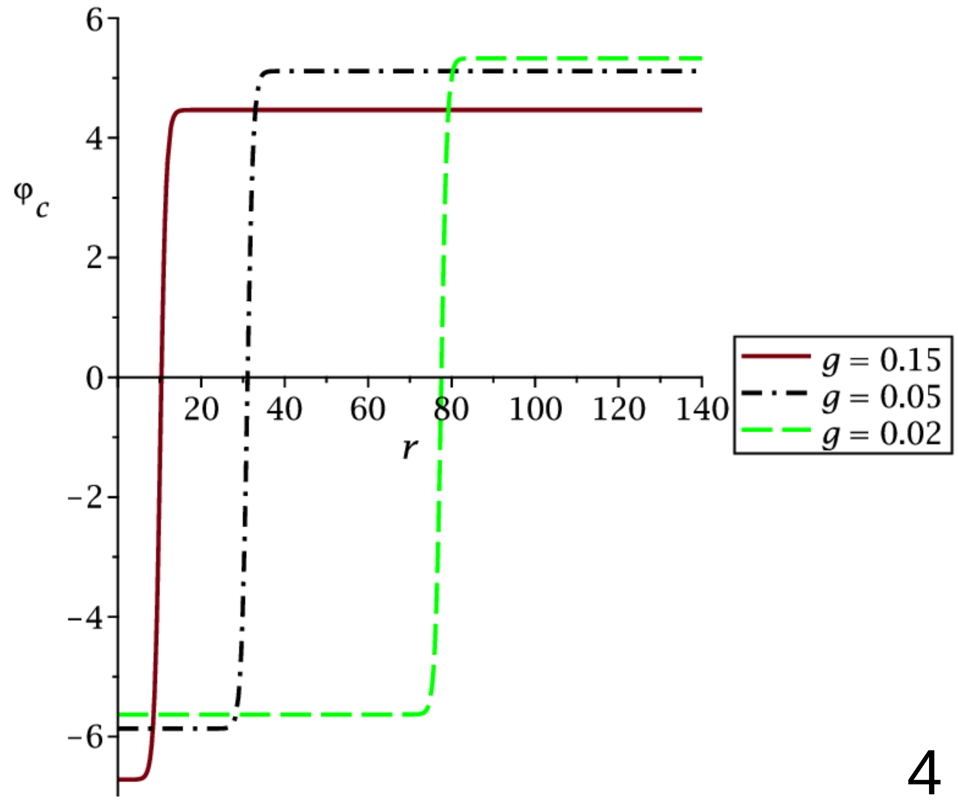
$$r = \sqrt{(\tau - \tau_0)^2 + (\vec{r} - \vec{r}_0)^2},$$ τ_0 , $\vec{r}_0$ -collective coordinates

$$\frac{d^2\varphi_c}{dr^2} + \frac{3}{r}\frac{d\varphi_c}{dr} - U'(\varphi_c) = 0$$

$$U = -\frac{m^2}{2!}\varphi^2 + \frac{g}{3!}\varphi^3 + \frac{\lambda}{4!}\varphi^4$$

Bounce solution:



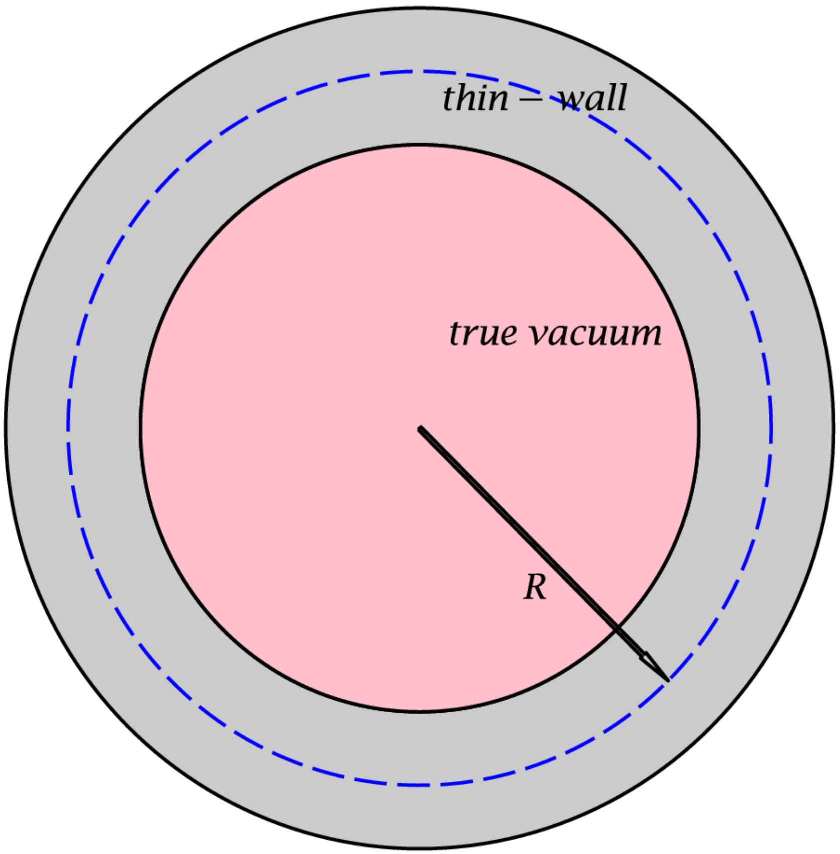


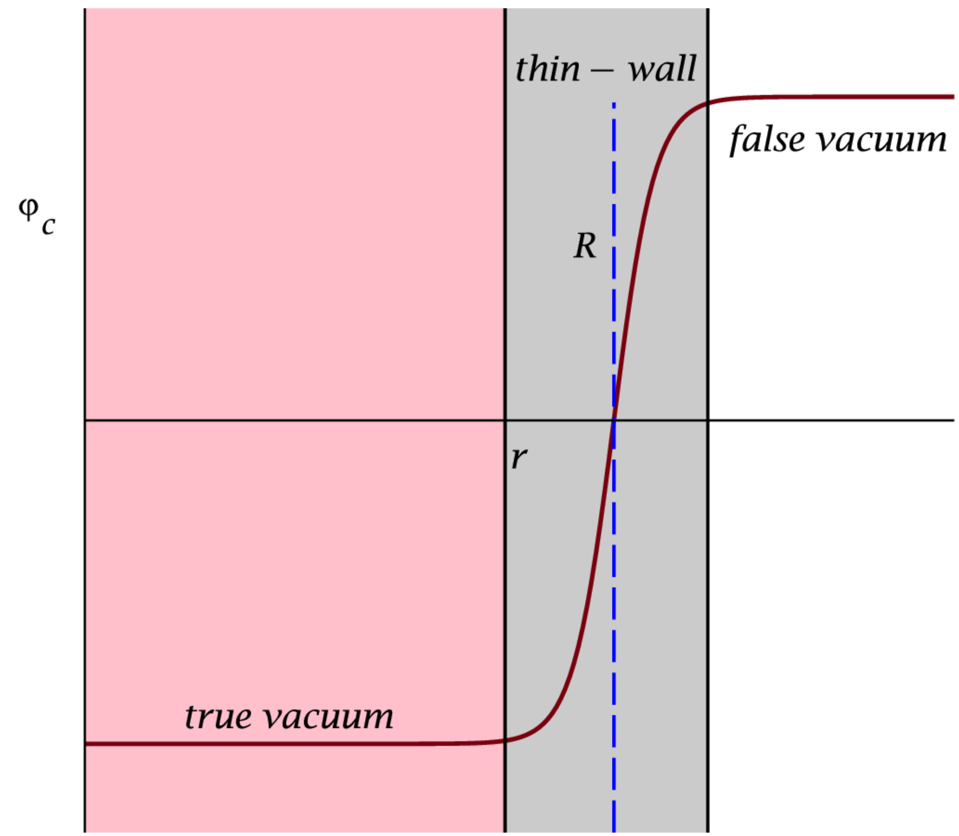
Thin-wall approximation

$$U = -\frac{m^2}{2!}\varphi^2 + \frac{g}{3!}\varphi^3 + \frac{\lambda}{4!}\varphi^4 \quad \Longrightarrow \quad U = -\frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

$$\frac{d^2\varphi_c}{dr^2} + \frac{3}{r}\frac{d\varphi_c}{dr} - U'(\varphi_c) = 0 \quad \Longrightarrow \quad \frac{d^2\varphi_c}{dr^2} - U'(\varphi_c) = 0$$

$$\varphi_c(r) = -\sqrt{\frac{6m^2}{\lambda}}\tanh\left(\frac{m}{\sqrt{2}}(r-R)\right), \qquad R = \sqrt{\frac{6\lambda}{g^2}}$$

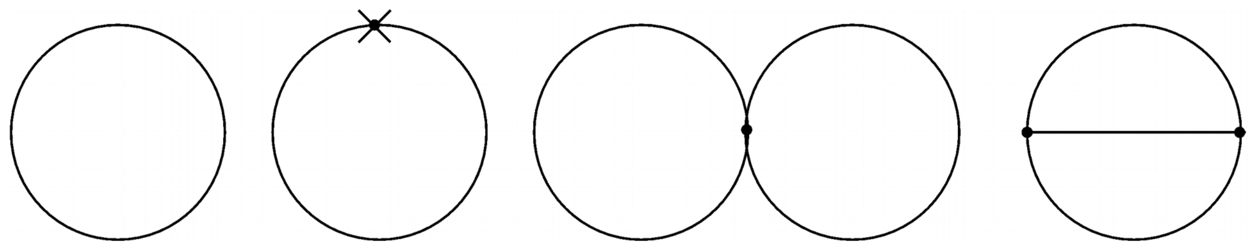




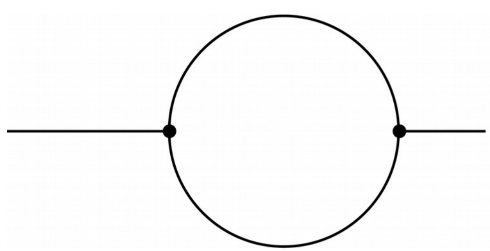
Renormalization

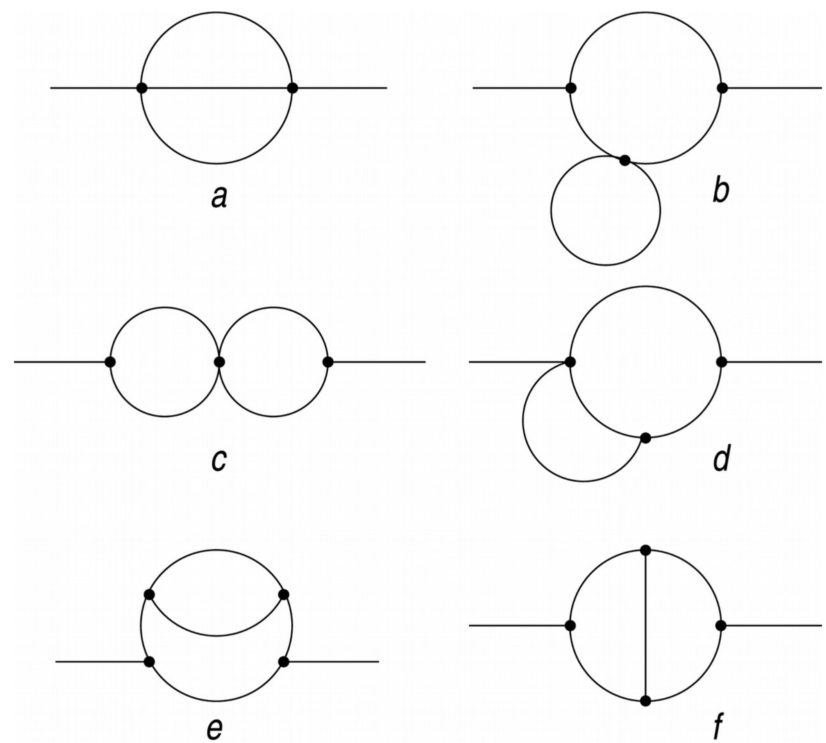
$$\mathcal{L}_{\text{counterterms}} = \frac{1}{2}\delta\mu^2\Phi^2 + \frac{\delta\lambda}{4!}\Phi^4 + \frac{\delta Z}{2}(\partial_\mu\Phi)^2$$

Diagrams contributing to effective potential:



Diagrams contributing to wave function renormalization:

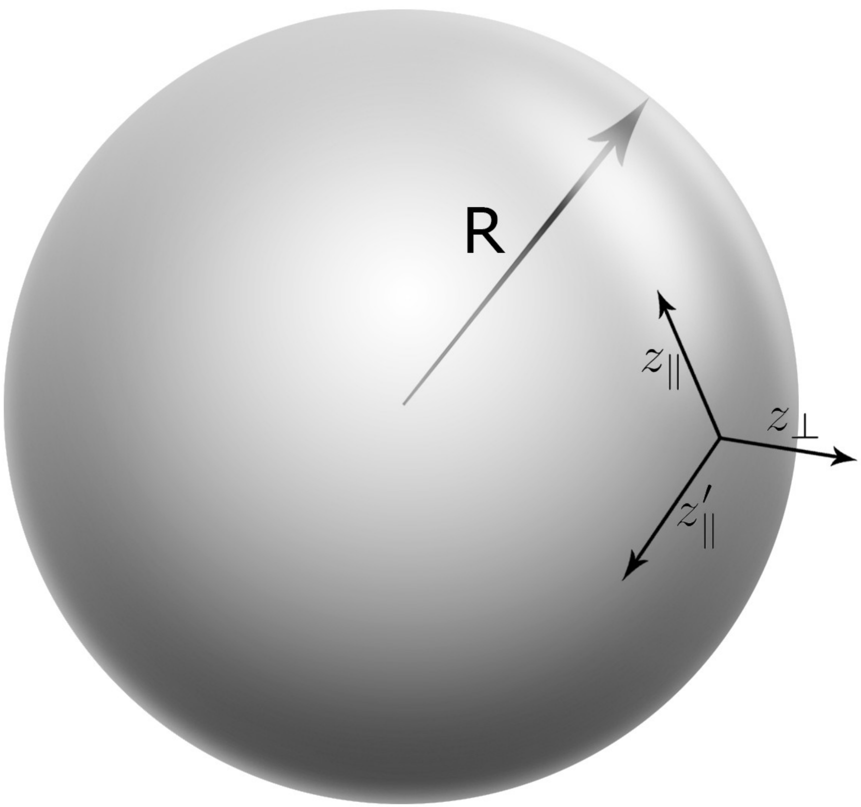




4-dimensional Green function

$$(-\Delta^{(4)} + U''(\varphi_c(\vec{x})))G(\vec{x},\vec{x}') = \delta(\vec{x}-\vec{x}') - \sum_{i=1}^{4} \varphi_0^i(\vec{x})\varphi_0^i(\vec{x}')$$

$$G(\vec{x},\vec{x}') = \int \frac{d^{(3)}\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{z}_\parallel - \mathbf{z}'_\parallel)} G(z_\perp, z'_\perp, m(\mathbf{k}))$$



$$\varphi_c(\vec{x}) = \varphi_c(z_\perp)$$

One loop solution

$$\frac{\Gamma}{V} = \left(\frac{S_b}{2\pi}\right)^2 \frac{(2\gamma)^5 R}{\sqrt{3}} \exp(-S_b + \hbar I^{(1)})$$

Coleman-Weinberg renormalization scheme:

$$I^{(1)}_{CW} = S_b \left(\frac{3\lambda}{16\pi^2}\right) \left(\frac{\pi}{3\sqrt{3}} + 21 + \boxed{\frac{1}{6}}\right) = 0.413604\lambda S_c$$

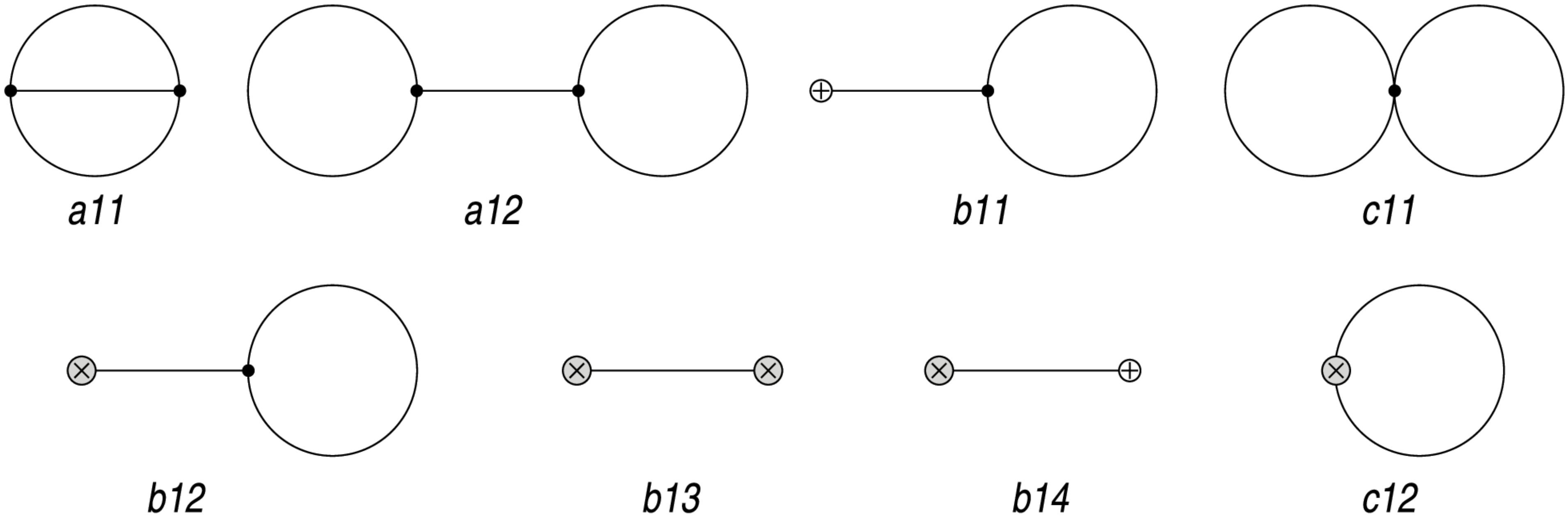
$\overline{MS}$ renormalization scheme:

$$I^{(1)}_{\overline{MS}} = \frac{3S_b\lambda}{(4\pi)^2}\left[\frac{\pi}{3\sqrt{3}} - 2 + \log\left(\frac{4\gamma^2}{\mu^2_{\overline{MS}}}\right)\right]$$

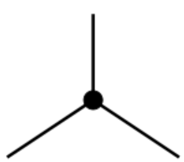
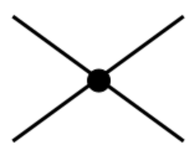
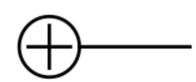
RG equation:

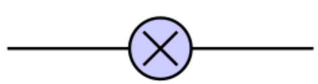
$$\frac{d}{d\log\mu_{\overline{MS}}}\left(-\frac{1}{\hbar}S_b + I^{(1)} + \mathcal{O}(\hbar)\right) = 0$$

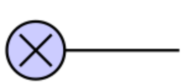
Feynman rules and diagrams



 $= \int \frac{d^{(d-1)}\mathbf{k}}{(2\pi)^{d-1}} e^{i\mathbf{k}(\mathbf{z}_\parallel - \mathbf{z}'_\parallel)} G(z_\perp, z'_\perp, \mathbf{k})$

 $= 2\gamma\sqrt{3\lambda}x$,  $= -\lambda,$  $= \frac{2x(1-x^2)}{S_b^2}$

 $= -\delta\mu^2 - \frac{\delta\lambda}{2} v^2 x^2 + 2\gamma^2(3x^2 - 1)\delta Z$

 $= vx\left(\delta\mu^2 + \frac{\delta\lambda}{3!} v^2 x^2 - 2\gamma^2 \delta Z(2x^2 - 1)\right),$

Non Gaussian effects

$$\frac{\Gamma}{V} = \left(\frac{S_b}{2\pi}\right)^2 \frac{(2\gamma)^5 R}{\sqrt{3}} \exp(-S_b + \hbar I^{(1)} + \hbar^2 I^{(2)})$$

Coleman-Weinberg renormalization scheme:

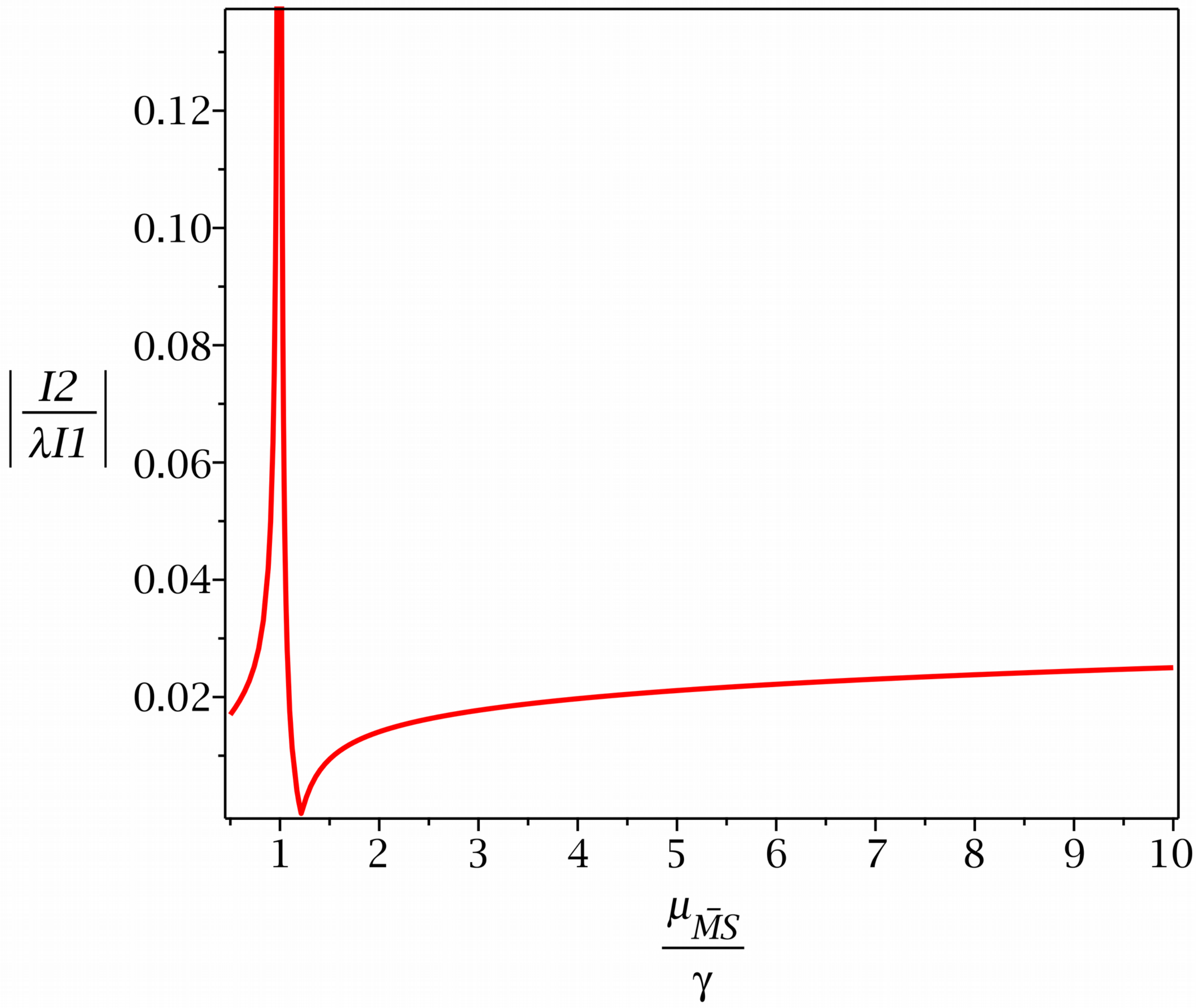
$$I^{(2)}_{CW} = \frac{S_b \lambda^2}{\pi^4}\left[-\frac{89\ 779}{49\ 152} - \frac{143\pi}{2048\sqrt{3}} + \frac{5003\pi^2}{1\ 146\ 880} + \frac{365}{36\ 864\sqrt{3}}\Im\left[2(e^{\frac{i\pi}{3}})\right] - \right.$$
$$\left. - \frac{3\sqrt{3}}{2560}\left\{\pi\log 3 - 6i\mathrm{Li}_2\left(\frac{3-i\sqrt{3}}{6}\right) + 6i\mathrm{Li}_2\left(\frac{3+i\sqrt{3}}{6}\right)\right\} + \frac{s_0}{8}\right]$$

$\overline{MS}$ renormalization scheme:

$$I^{(2)}_{\overline{MS}} = \frac{S_b \lambda^2}{8\pi^4}\left[-\frac{3}{4} + \frac{7\sqrt{3}\pi}{160} - \frac{197\pi^2}{8960} - \frac{142 - 3\sqrt{3}\pi}{384}\log\left(\frac{4\gamma^2}{\mu_{\overline{MS}}{}^2}\right) + \right.$$
$$\left. + \frac{9}{256}\log^2\left(\frac{4\gamma^2}{\mu_{\overline{MS}}{}^2}\right) - \frac{3\sqrt{3}}{320}\left\{\pi\log 3 - 6i\mathrm{Li}_2\left(\frac{3-i\sqrt{3}}{6}\right) + 6i\mathrm{Li}_2\left(\frac{3+i\sqrt{3}}{6}\right)\right\} + s_0\right]$$

From sunset: $s_0 \approx 0.71$

Significance of two loop correction



Two-loop contribution is of order 2 − 4% of one-loop result in both schemes and as such could be safely neglected if we are not interested in this type of accuracy.

Thank you for your attention!