

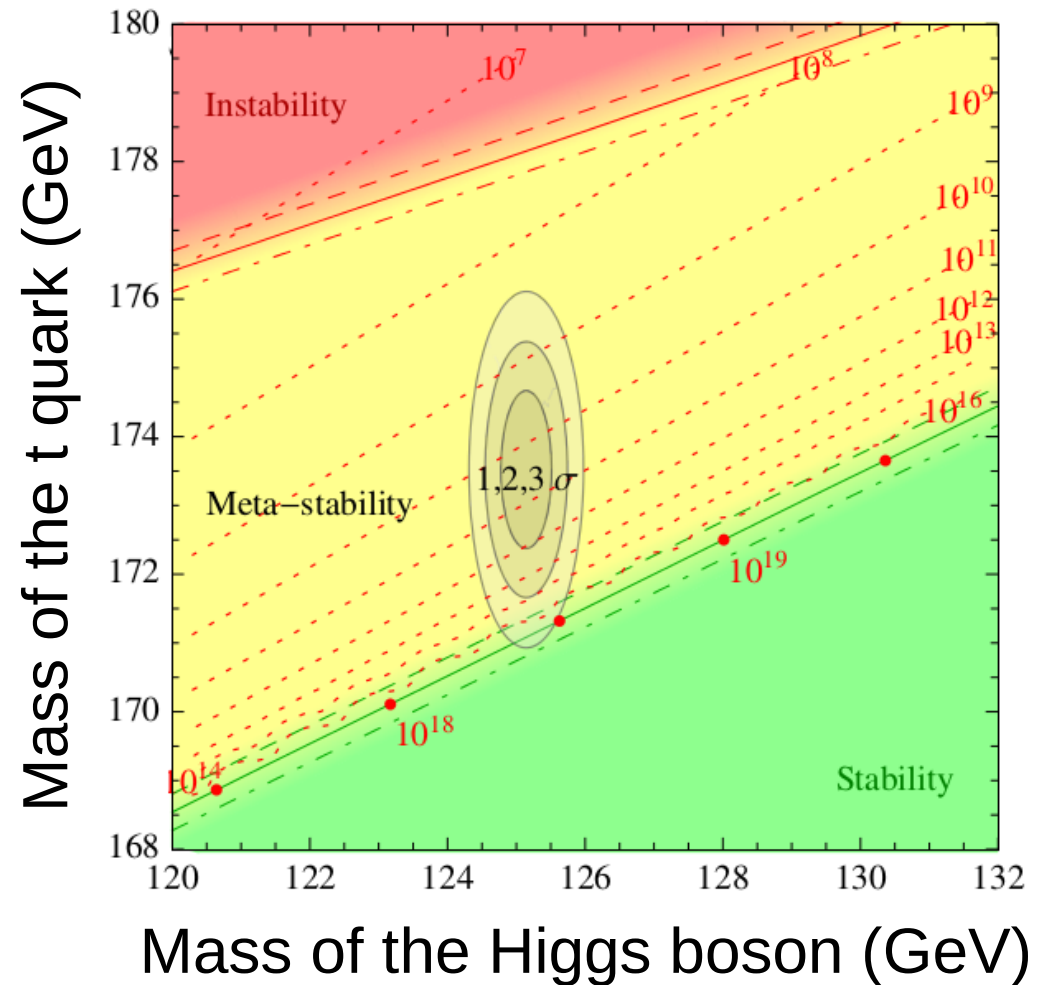
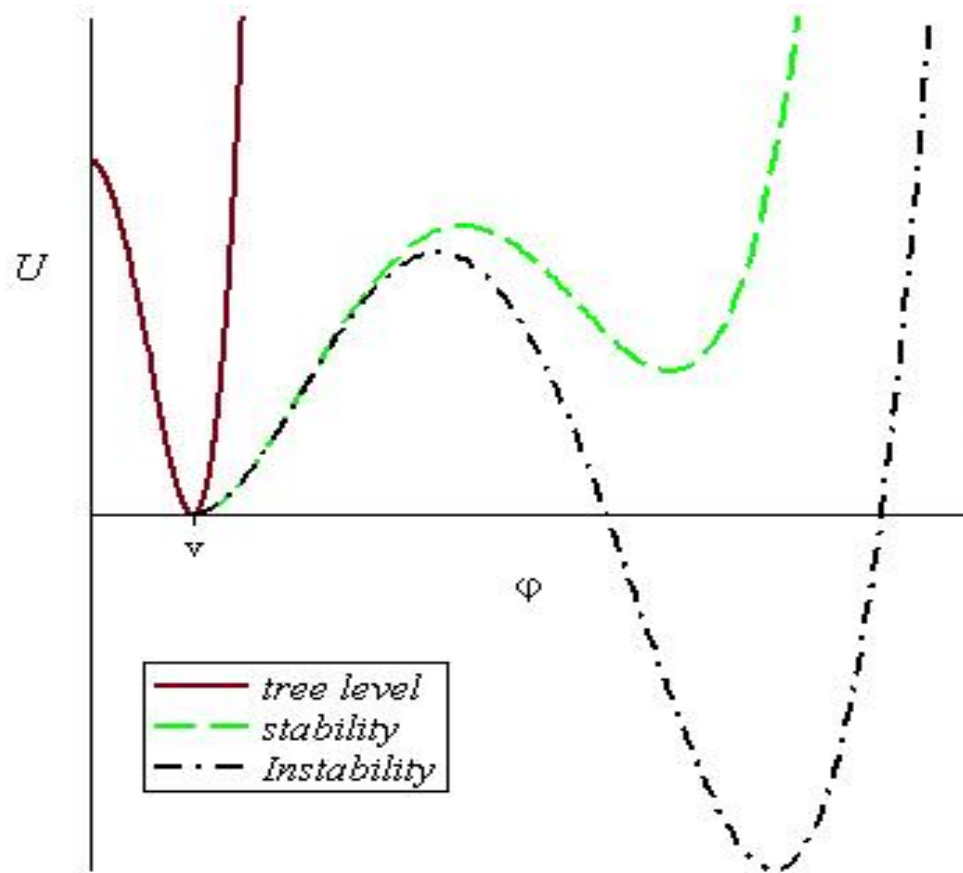
Green's function technique for precise calculation of false vacuum decay rate.

Maxim Bezuglov (JINR)

Andrei Onishchenko (JINR)

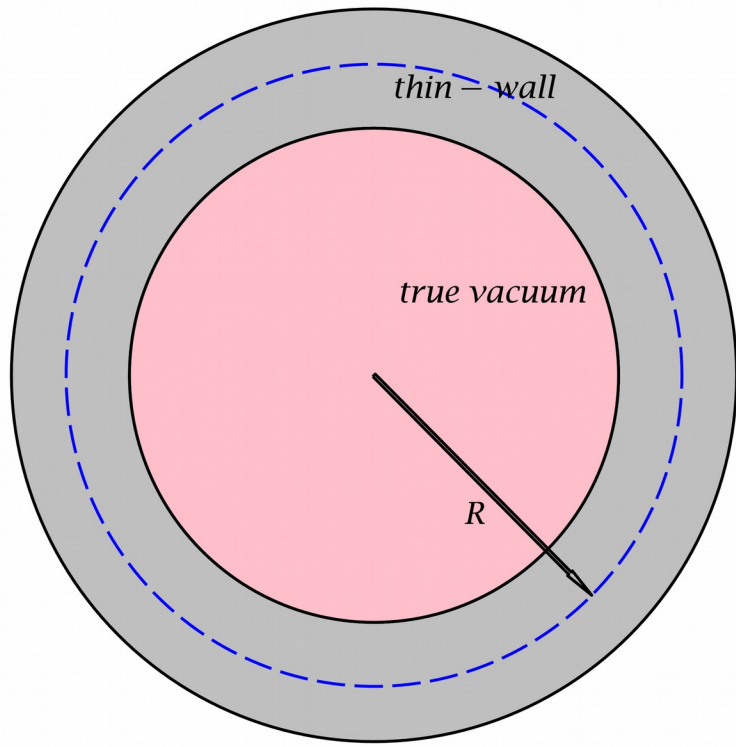
# Vacuum metastability of the Standard Model

$$U(\varphi) = \lambda (\varphi^2 - v^2)^2 + U_{\text{corr}}$$



Buttazzo D., Degrandi G., Giardino P.P., Giudice G.F., Sala F., Salvio A. and Strumia A., JHEP **1312** (2013) 089

# False vacuum decay in 4 dimensional space



$t \rightarrow -i\tau$  Wick rotation

$$\frac{\Gamma}{V} = \underbrace{\left(\frac{S_c}{2\pi}\right)^2 \left[ \frac{\det' \left( -\frac{d^2}{d\tau^2} - \Delta + U''(\varphi_c) \right)}{\det \left( -\frac{d^2}{d\tau^2} - \Delta + U''(v) \right)} \right]^{-\frac{1}{2}}}_{\text{One loop}} \exp \left( \overbrace{\frac{-S_b}{\hbar}}^{\text{Action}} \right) \underbrace{\left\{ 1 + \hbar I_2 + \mathcal{O}(\hbar^2) \right\}}_{\text{NNLO and + corrections}}$$

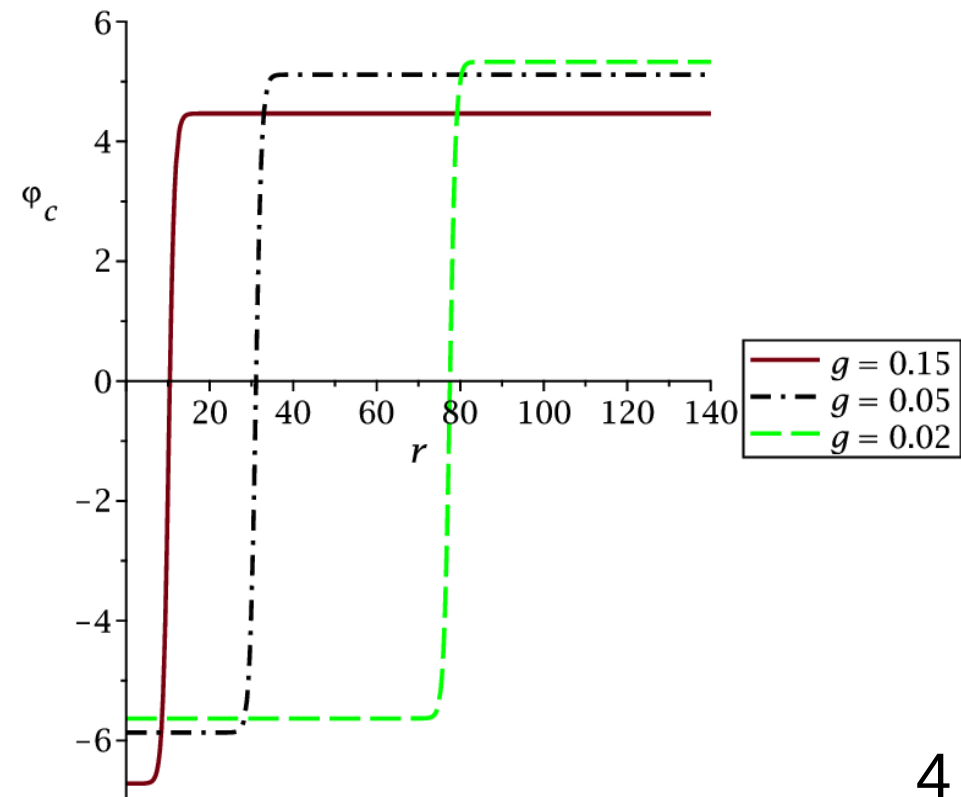
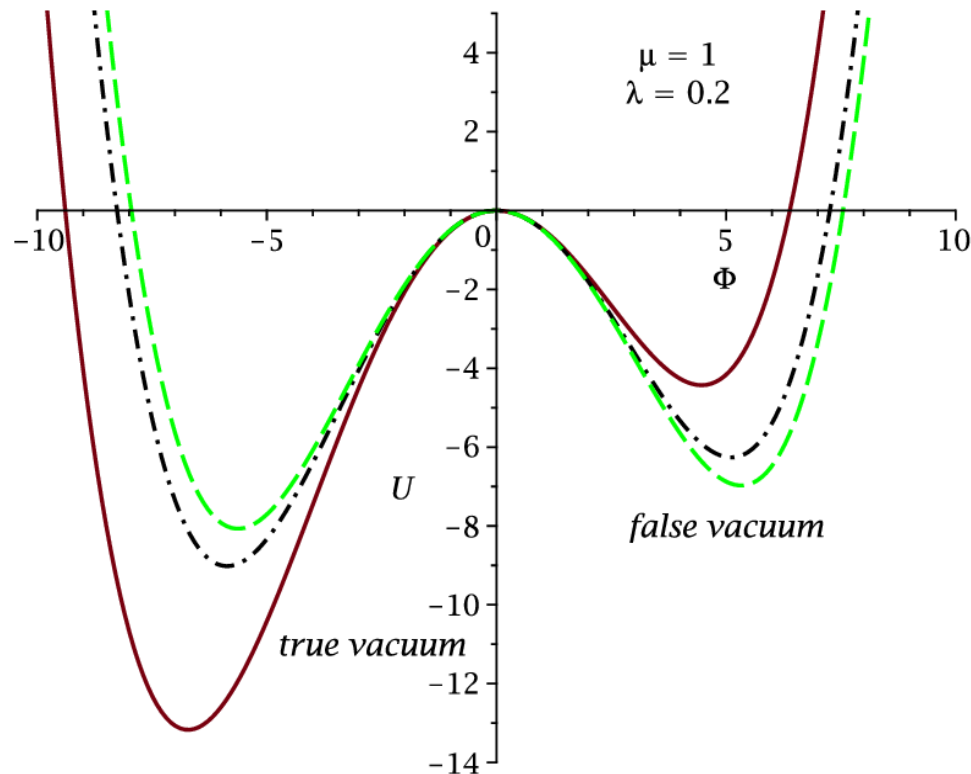
# False vacuum decay in 4 dimensional space

$$r = \sqrt{(\tau - \tau_0)^2 + (\vec{r} - \vec{r}_0)^2}, \quad \tau_0, \vec{r}_0 \text{ -collective coordinates}$$

$$\frac{d^2 \varphi_c}{dr^2} + \frac{3}{r} \frac{d\varphi_c}{dr} - U'(\varphi_c) = 0$$

$$U = -\frac{m^2}{2!} \varphi^2 + \frac{g}{3!} \varphi^3 + \frac{\lambda}{4!} \varphi^4$$

Bounce solution:

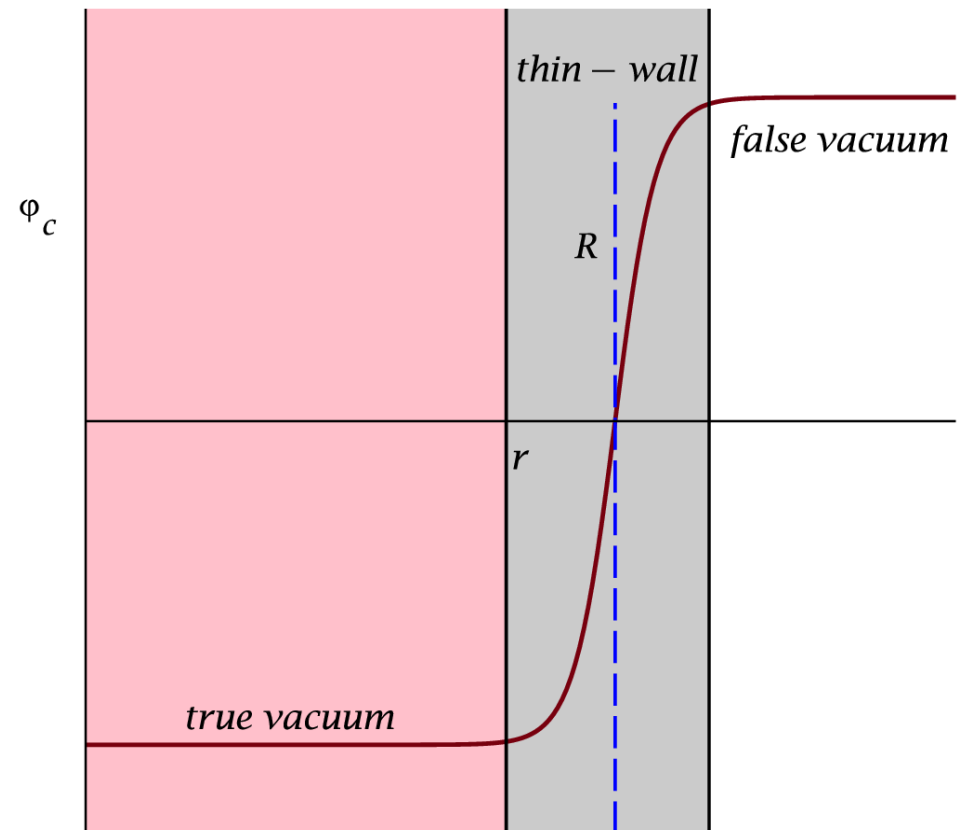
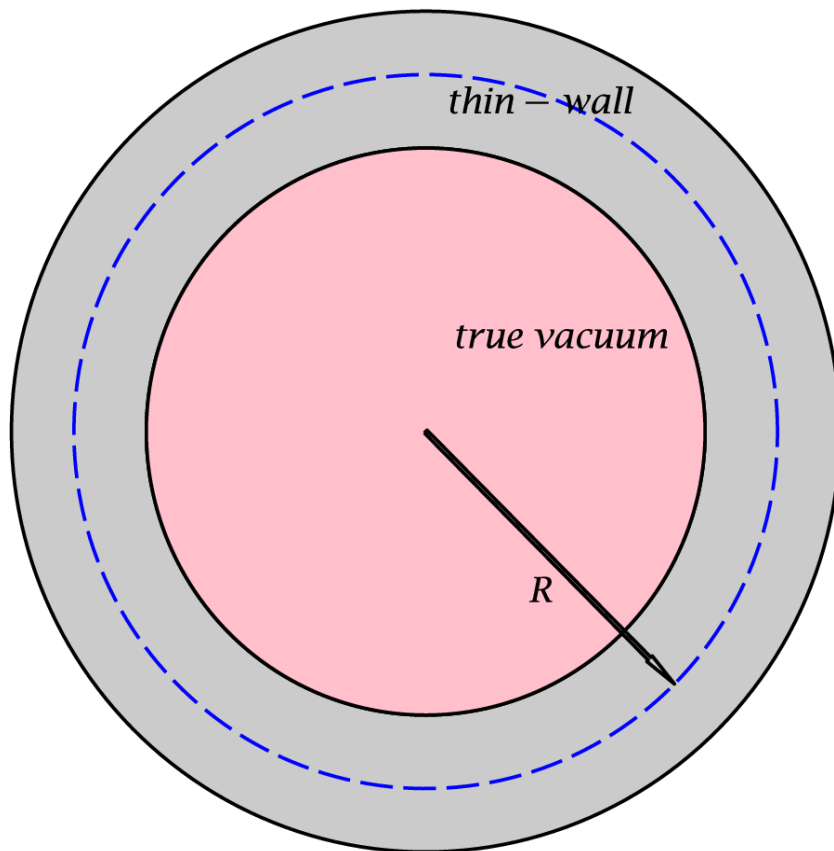


# Thin-wall approximation

$$U = -\frac{m^2}{2!}\varphi^2 + \frac{g}{3!}\varphi^3 + \frac{\lambda}{4!}\varphi^4 \quad \Rightarrow \quad U = -\frac{m^2}{2!}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

$$\frac{d^2\varphi_c}{dr^2} + \frac{3}{r} \frac{d\varphi_c}{dr} - U'(\varphi_c) = 0 \quad \Rightarrow \quad \frac{d^2\varphi_c}{dr^2} - U'(\varphi_c) = 0$$

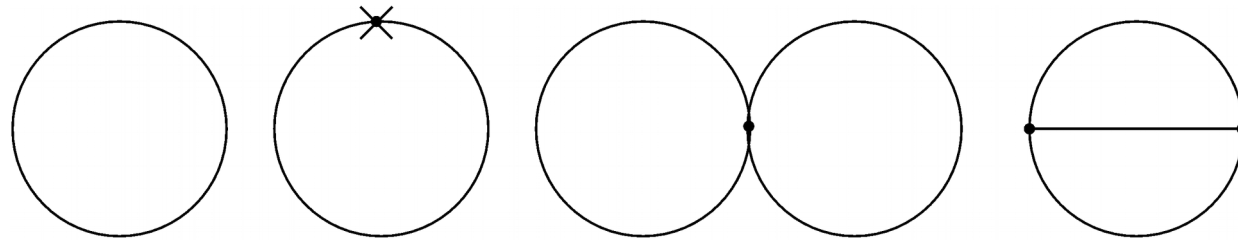
$$\varphi_c(r) = -\sqrt{\frac{6m^2}{\lambda}} \tanh\left(\frac{m}{\sqrt{2}}(r - R)\right), \quad R = \sqrt{\frac{6\lambda}{g^2}}$$



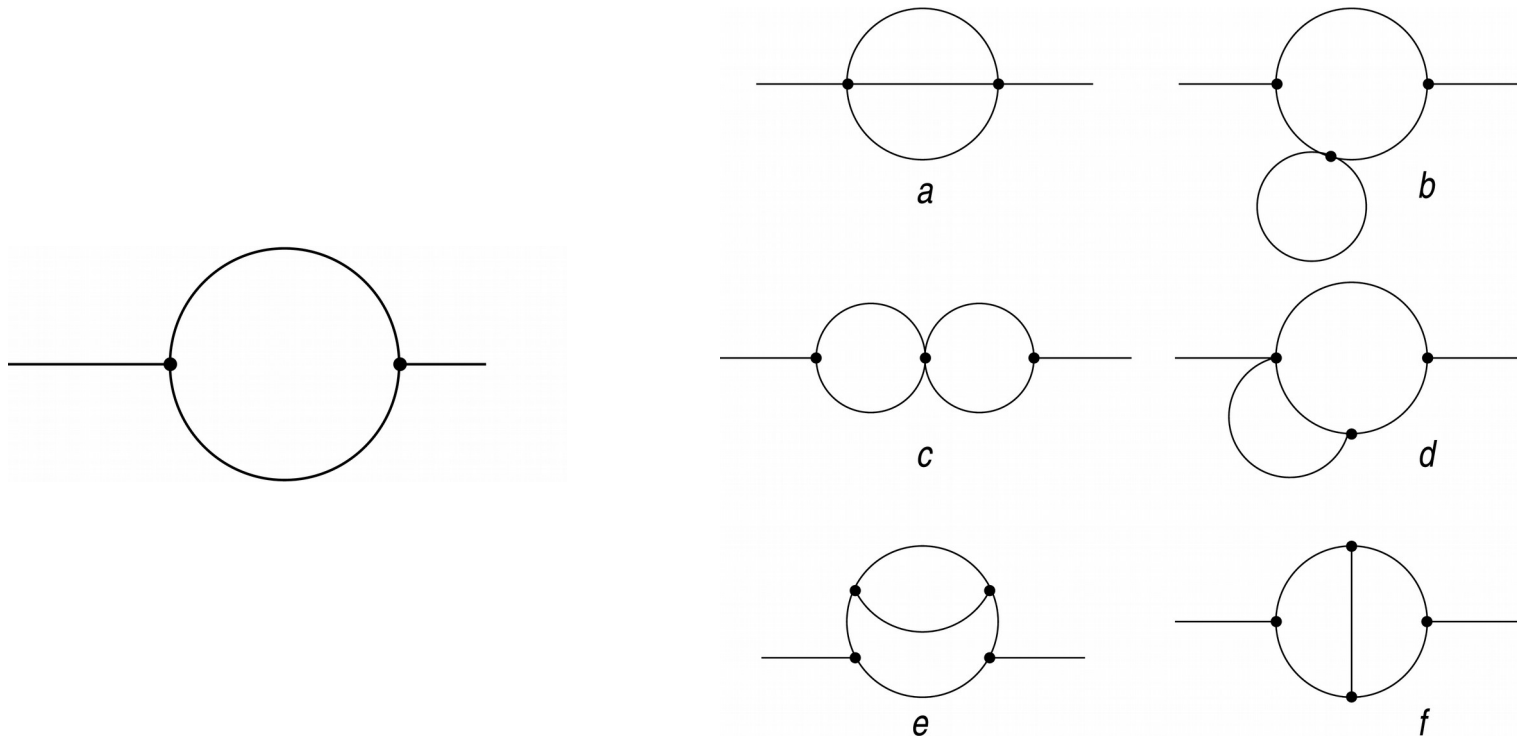
# Renormalization

$$\mathcal{L}_{\text{counterterms}} = \frac{1}{2}\delta\mu^2\Phi^2 + \frac{\delta\lambda}{4!}\Phi^4 + \frac{\delta Z}{2}(\partial_\mu\Phi)^2$$

Diagrams contributing to effective potential:



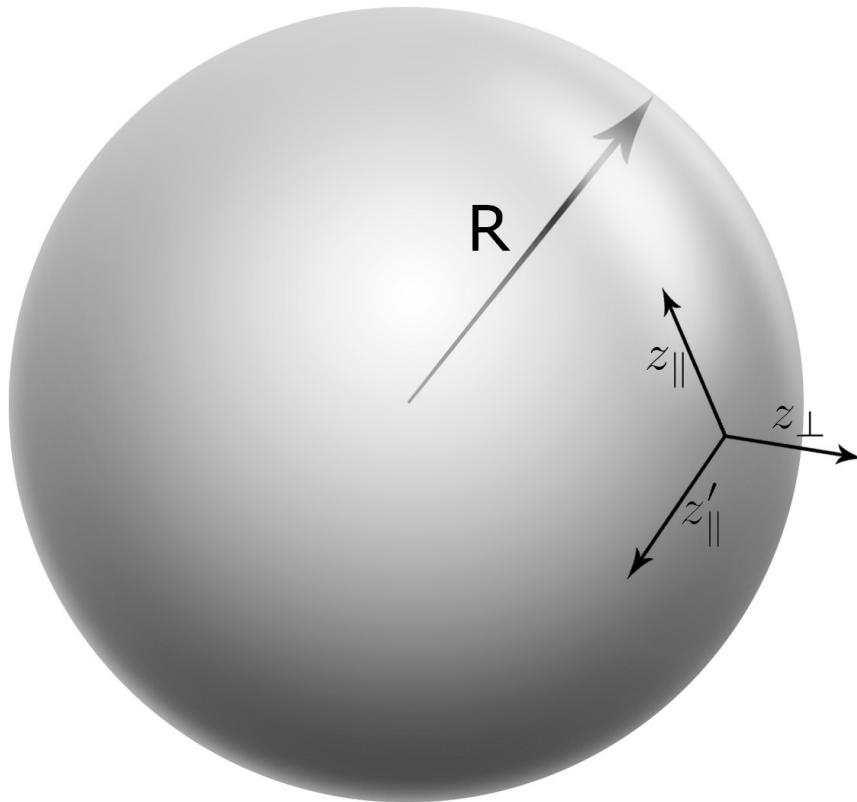
Diagrams contributing to wave function renormalization:



# 4-dimensional Green function

$$(-\Delta^{(4)} + U''(\varphi_c(\vec{x})))G(\vec{x}, \vec{x}') = \delta(\vec{x} - \vec{x}') - \sum_{i=1}^4 \varphi_0^i(\vec{x})\varphi_0^i(\vec{x}')$$

$$G(\vec{x}, \vec{x}') = \int \frac{d^{(3)}\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{z}_{\parallel} - \mathbf{z}'_{\parallel})} G(z_{\perp}, z'_{\perp}, m(\mathbf{k}))$$



$$\varphi_c(\vec{x}) = \varphi_c(z_{\perp})$$

# One loop solution

$$\frac{\Gamma}{V} = \left( \frac{S_b}{2\pi} \right)^2 \frac{(2\gamma)^5 R}{\sqrt{3}} \exp(-S_b + \hbar I^{(1)})$$

Coleman-Weinberg renormalization scheme:

$$I_{CW}^{(1)} = S_b \left( \frac{3\lambda}{16\pi^2} \right) \left( \frac{\pi}{3\sqrt{3}} + 21 + \boxed{\frac{1}{6}} \right) = 0.413604\lambda S_c$$

$\overline{MS}$  renormalization scheme:

$$I_{\overline{MS}}^{(1)} = \frac{3S_b\lambda}{(4\pi)^2} \left[ \frac{\pi}{3\sqrt{3}} - 2 + \log \left( \frac{4\gamma^2}{\mu_{\overline{MS}}^2} \right) \right]$$

RG equation:

$$\frac{d}{d \log \mu_{\overline{MS}}} \left( -\frac{1}{\hbar} S_b + I^{(1)} + \mathcal{O}(\hbar) \right) = 0$$



A diagram consisting of two identical circles positioned side-by-side. A horizontal line segment connects the rightmost point of the left circle to the leftmost point of the right circle. The line segment is tangent to both circles at their respective points of contact.

 $\oplus$ 

A Venn diagram consisting of two overlapping circles. The intersection of the two circles is shaded in gray. The circles are drawn with thin black outlines.

A diagram of a simple graph consisting of two vertices, each represented by a circle with an 'X' inside, connected by a single horizontal edge.

$$\bullet \text{---} \bullet = \int \frac{d^{(d-1)}\mathbf{k}}{(2\pi)^{d-1}} e^{i\mathbf{k}(\mathbf{z}_{\parallel} - \mathbf{z}'_{\parallel})} G(z_{\perp}, z'_{\perp}, \mathbf{k})$$

$$= 2\gamma\sqrt{3\lambda}x\,,$$

$$= -\lambda,$$

$$\bigoplus \text{---} = \frac{2x(1-x^2)}{S_b^2}$$

$$\text{---} \bigcirc \times \text{---} = -\delta\mu^2 - \frac{\delta\lambda}{2} v^2 x^2 + 2\gamma^2 (3x^2 - 1) \delta Z$$

$$\textcircled{\times} \text{---} = vx \left( \delta\mu^2 + \frac{\delta\lambda}{3!} v^2 x^2 - 2\gamma^2 \delta Z (2x^2 - 1) \right),$$

# Non Gaussian effects

$$\frac{\Gamma}{V} = \left( \frac{S_b}{2\pi} \right)^2 \frac{(2\gamma)^5 R}{\sqrt{3}} \exp(-S_b + \hbar I^{(1)} + \hbar^2 I^{(2)})$$

Coleman-Weinberg renormalization scheme:

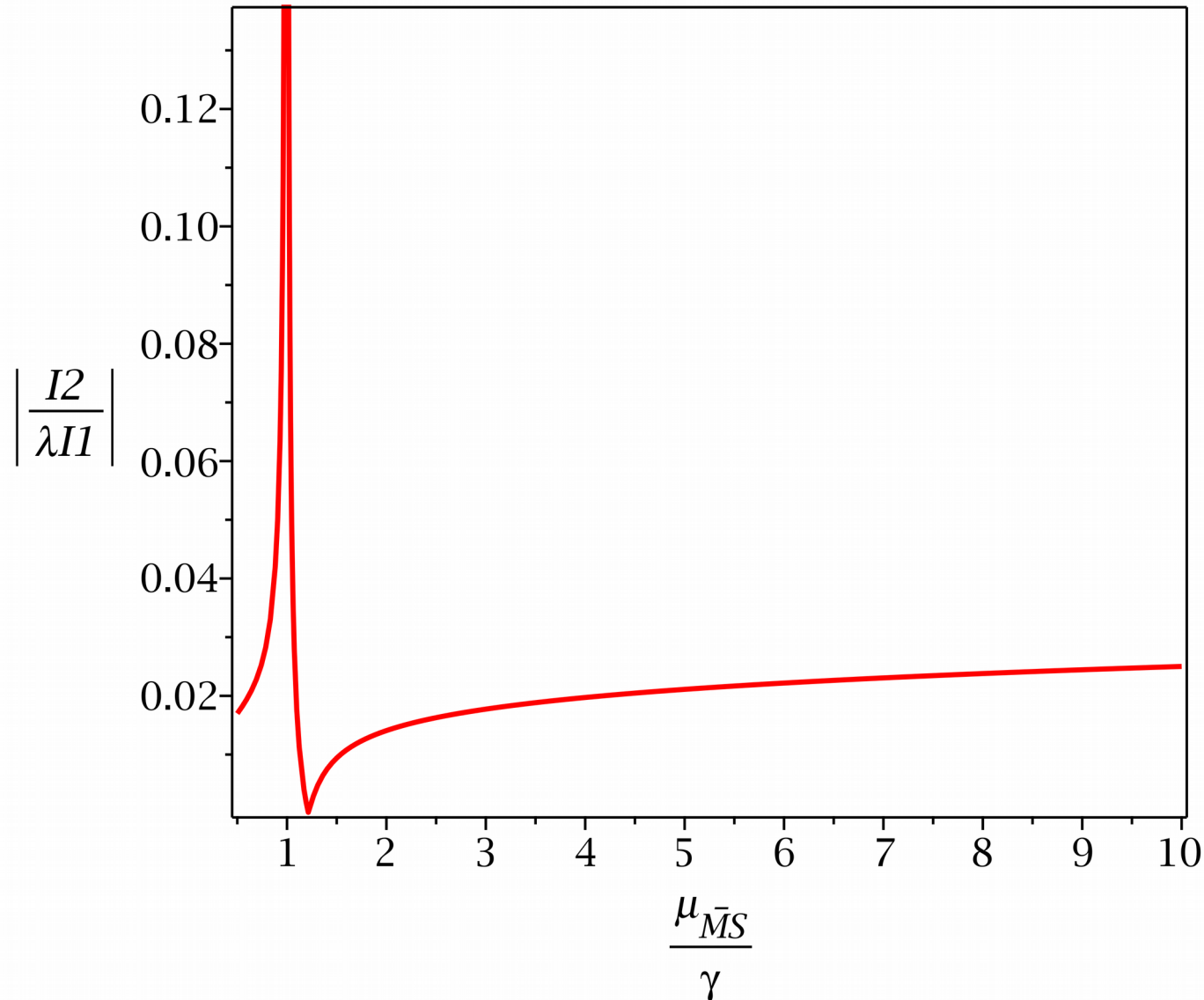
$$I_{CW}^{(2)} = \frac{S_b \lambda^2}{\pi^4} \left[ -\frac{89\,779}{49\,152} - \frac{143\pi}{2048\sqrt{3}} + \frac{5003\pi^2}{1\,146\,880} + \frac{365}{36\,864\sqrt{3}} \Im \left[ 2(e^{\frac{i\pi}{3}}) \right] - \right. \\ \left. - \frac{3\sqrt{3}}{2560} \left\{ \pi \log 3 - 6i \text{Li}_2 \left( \frac{3 - i\sqrt{3}}{6} \right) + 6i \text{Li}_2 \left( \frac{3 + i\sqrt{3}}{6} \right) \right\} + \frac{s_0}{8} \right]$$

$\overline{MS}$  renormalization scheme:

$$I_{\overline{MS}}^{(2)} = \frac{S_b \lambda^2}{8\pi^4} \left[ -\frac{3}{4} + \frac{7\sqrt{3}\pi}{160} - \frac{197\pi^2}{8960} - \frac{142 - 3\sqrt{3}\pi}{384} \log \left( \frac{4\gamma^2}{\mu_{\overline{MS}}^2} \right) + \right. \\ \left. + \frac{9}{256} \log^2 \left( \frac{4\gamma^2}{\mu_{\overline{MS}}^2} \right) - \frac{3\sqrt{3}}{320} \left\{ \pi \log 3 - 6i \text{Li}_2 \left( \frac{3 - i\sqrt{3}}{6} \right) + 6i \text{Li}_2 \left( \frac{3 + i\sqrt{3}}{6} \right) \right\} + s_0 \right]$$

From sunset:  $s_0 \approx 0.71$

# Significance of two loop correction



Two-loop contribution is of order 2 – 4% of one-loop result in both schemes and as such could be safely neglected if we are not interested in this type of accuracy.

Thank you for your attention!