The International Conference “Distributed Computing and Grid-technologies in Science and Education” (GRID 2021)

MIPT

The grid-characteristic method for applied dynamic problems

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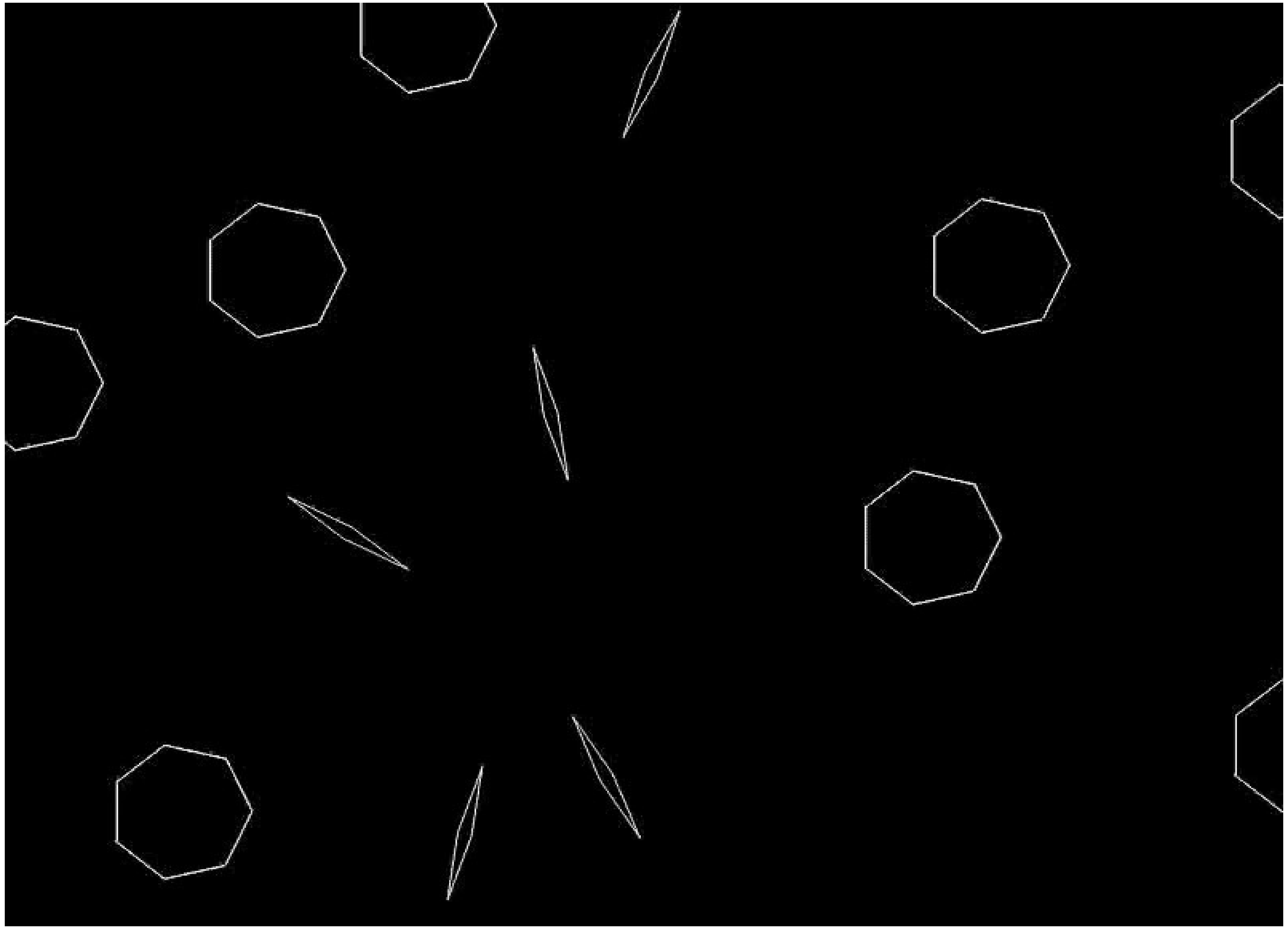
Hyperbolic Problems = Waves

- Seismic Survey Process (Direct and Inverse Problems)
- Seismic Activity Simulation (Earthquakes, Tsunami)
- Non-Destructing Control (Novel Composite Materials)
- Ultrasound Diagnostic of Diseases

Hyperbolic System. Isotropic Case

$$
\begin{cases}
\rho \dfrac{\partial V_x}{\partial t} = \dfrac{\partial T_{xx}}{\partial x} + \dfrac{\partial T_{xy}}{\partial y} + \dfrac{\partial T_{xz}}{\partial z}, \\
\rho \dfrac{\partial V_y}{\partial t} = \dfrac{\partial T_{yx}}{\partial x} + \dfrac{\partial T_{yy}}{\partial y} + \dfrac{\partial T_{yz}}{\partial z}, \\
\rho \dfrac{\partial V_z}{\partial t} = \dfrac{\partial T_{zx}}{\partial x} + \dfrac{\partial T_{zy}}{\partial y} + \dfrac{\partial T_{zz}}{\partial z}, \\
\dfrac{\partial T_{xx}}{\partial t} = (\lambda + 2\mu)\dfrac{\partial V_x}{\partial x} + \lambda(\dfrac{\partial V_y}{\partial y} + \dfrac{\partial V_z}{\partial z}), \\
\dfrac{\partial T_{yy}}{\partial t} = (\lambda + 2\mu)\dfrac{\partial V_y}{\partial y} + \lambda(\dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_z}{\partial z}), \\
\dfrac{\partial T_{zz}}{\partial t} = (\lambda + 2\mu)\dfrac{\partial V_z}{\partial z} + \lambda(\dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y}), \\
\dfrac{\partial T_{xy}}{\partial t} = \mu(\dfrac{\partial V_x}{\partial y} + \dfrac{\partial V_y}{\partial x}), \\
\dfrac{\partial T_{xz}}{\partial t} = \mu(\dfrac{\partial V_x}{\partial z} + \dfrac{\partial V_z}{\partial x}), \\
\dfrac{\partial T_{yz}}{\partial t} = \mu(\dfrac{\partial V_y}{\partial z} + \dfrac{\partial V_z}{\partial y}).
\end{cases}
$$



Elastic Parameters:

ρ – density, λ, μ – Lame parameters;

Unknowns: V – velocity; T – stress tensor

Numerical Grid-Characteristic Method

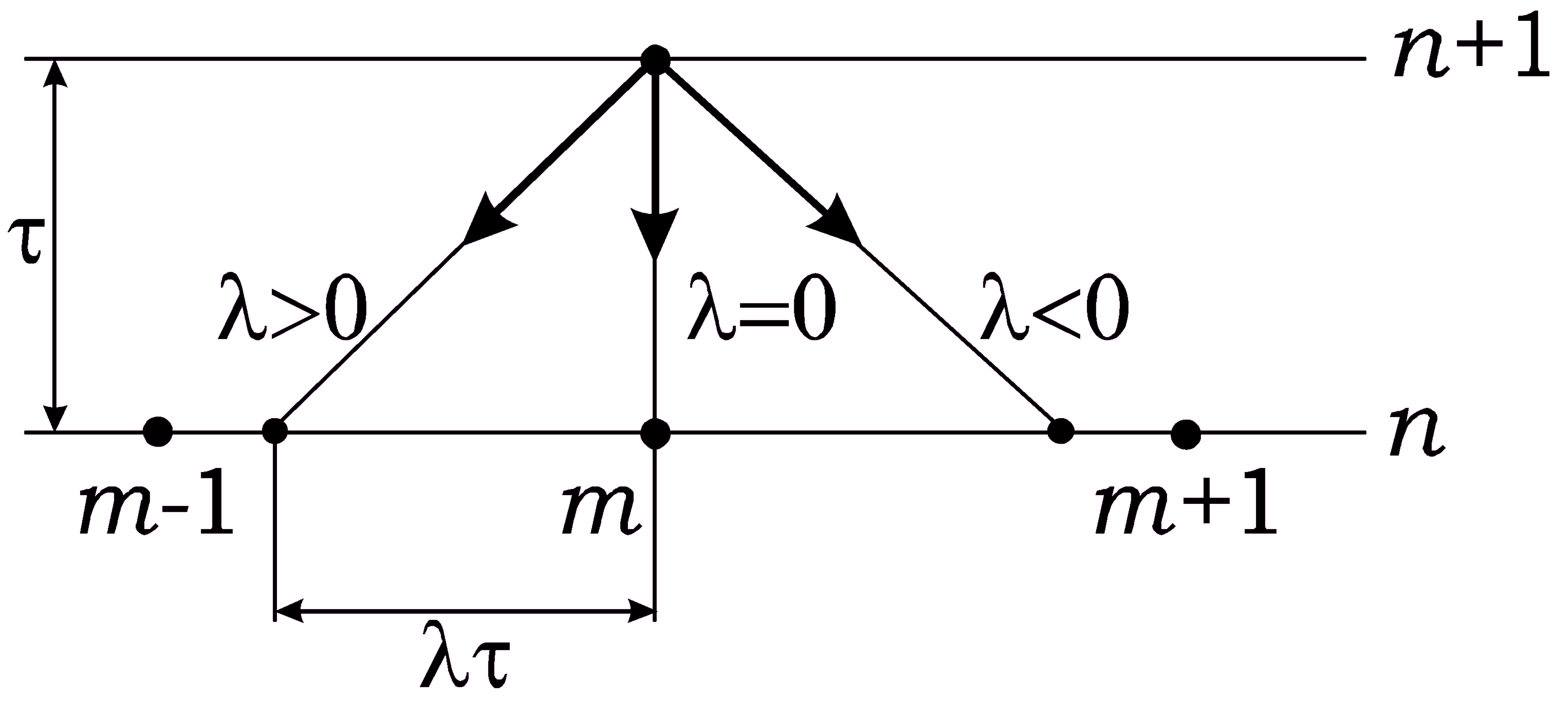
Hyperbolic System

$$\mathbf{A} = \mathbf{\Omega}^{-1}\mathbf{\Lambda}\mathbf{\Omega}$$

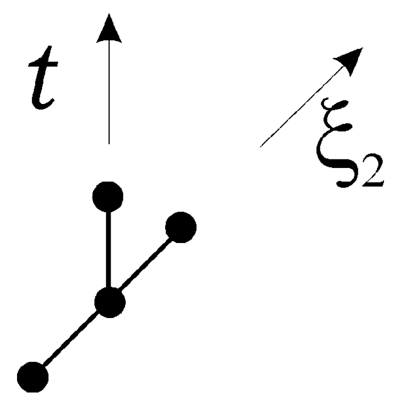
$$\frac{\partial \vec{u}}{\partial t} + \mathbf{\Omega}^{-1}\mathbf{\Lambda}\mathbf{\Omega}\frac{\partial \vec{u}}{\partial \xi} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \mathbf{\Lambda}\frac{\partial \vec{v}}{\partial \xi} = 0 \quad (\vec{v} \equiv \mathbf{\Omega}\vec{u})$$

$$v^{n+1}(\xi) = v^n(\xi - \lambda\tau)$$



Split Directions



$$\frac{\partial \vec{u}}{\partial t} + A_1\frac{\partial \vec{u}}{\partial \xi_1} = 0 \qquad \frac{\partial \vec{u}}{\partial t} + A_2\frac{\partial \vec{u}}{\partial \xi_1} = 0 \qquad \frac{\partial \vec{u}}{\partial t} + A_3\frac{\partial \vec{u}}{\partial \xi_3} = 0$$

$$\vec{u}' = \vec{u}^n - \tau A_1 \Delta_1 \vec{u}^n \rightarrow \vec{u}'' = \vec{u}' - \tau A_2 \Delta_2 \vec{u}' \rightarrow \vec{u}^{n+1} = \vec{u}'' - \tau A_3 \Delta_3 \vec{u}''$$

$$\boxed{\vec{u}^{n+1} = \vec{u}^n - \tau(A_1\Delta_1 + A_2\Delta_2 + A_3\Delta_3)\vec{u}^n + O(\tau^2)}$$

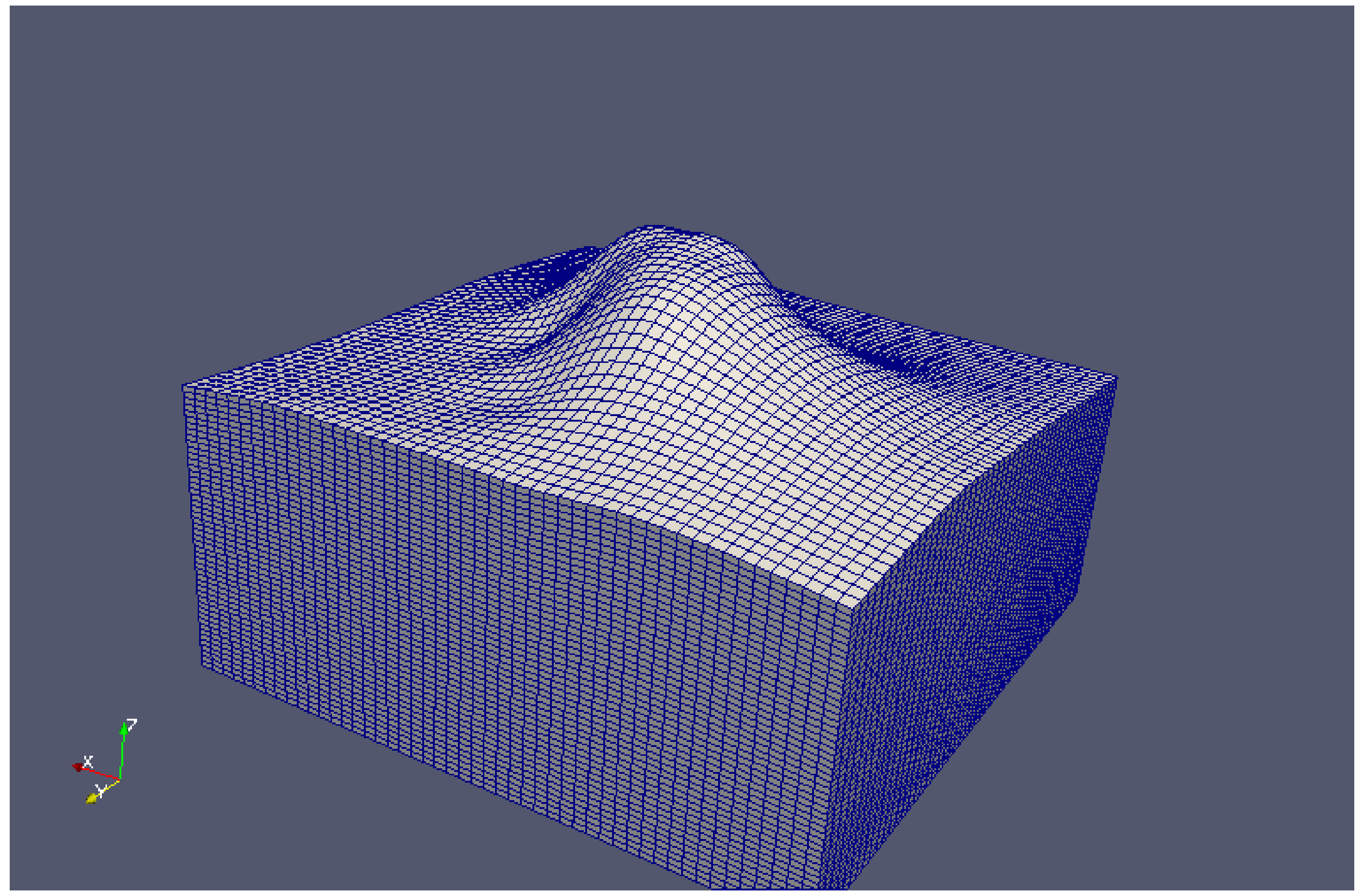
Research Software Developed

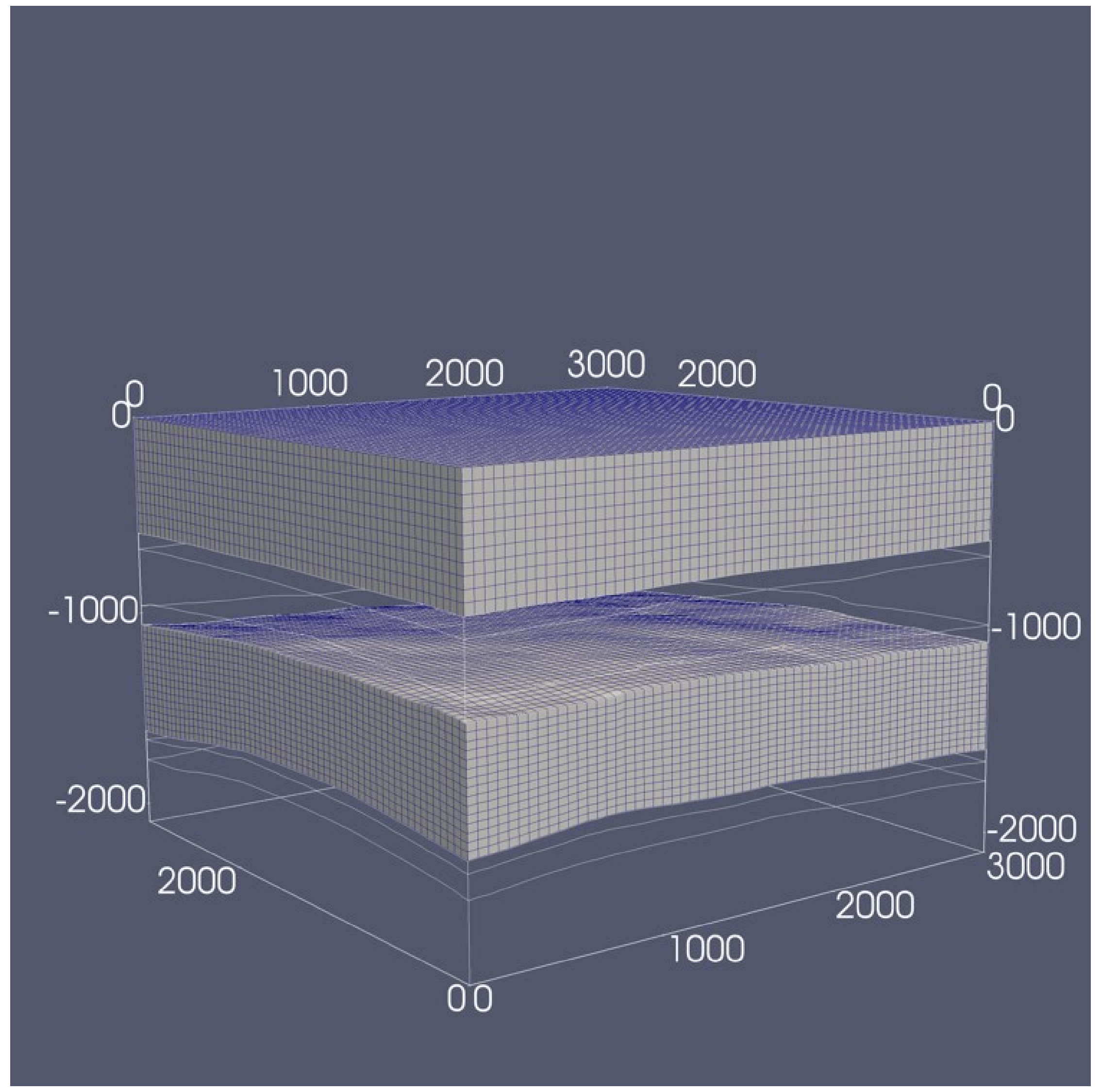
- Seismic waves simulation in elastic media
- Taking into account heterogeneities (cavities, layers, fractures)
- Discreet model of destruction (correction of stress tensor)
- C++, micro-optimisations (SIMD, SSE, AVX)
- Parallelization with OpenMP and MPI (~ 80 % up to 16 000 cores)

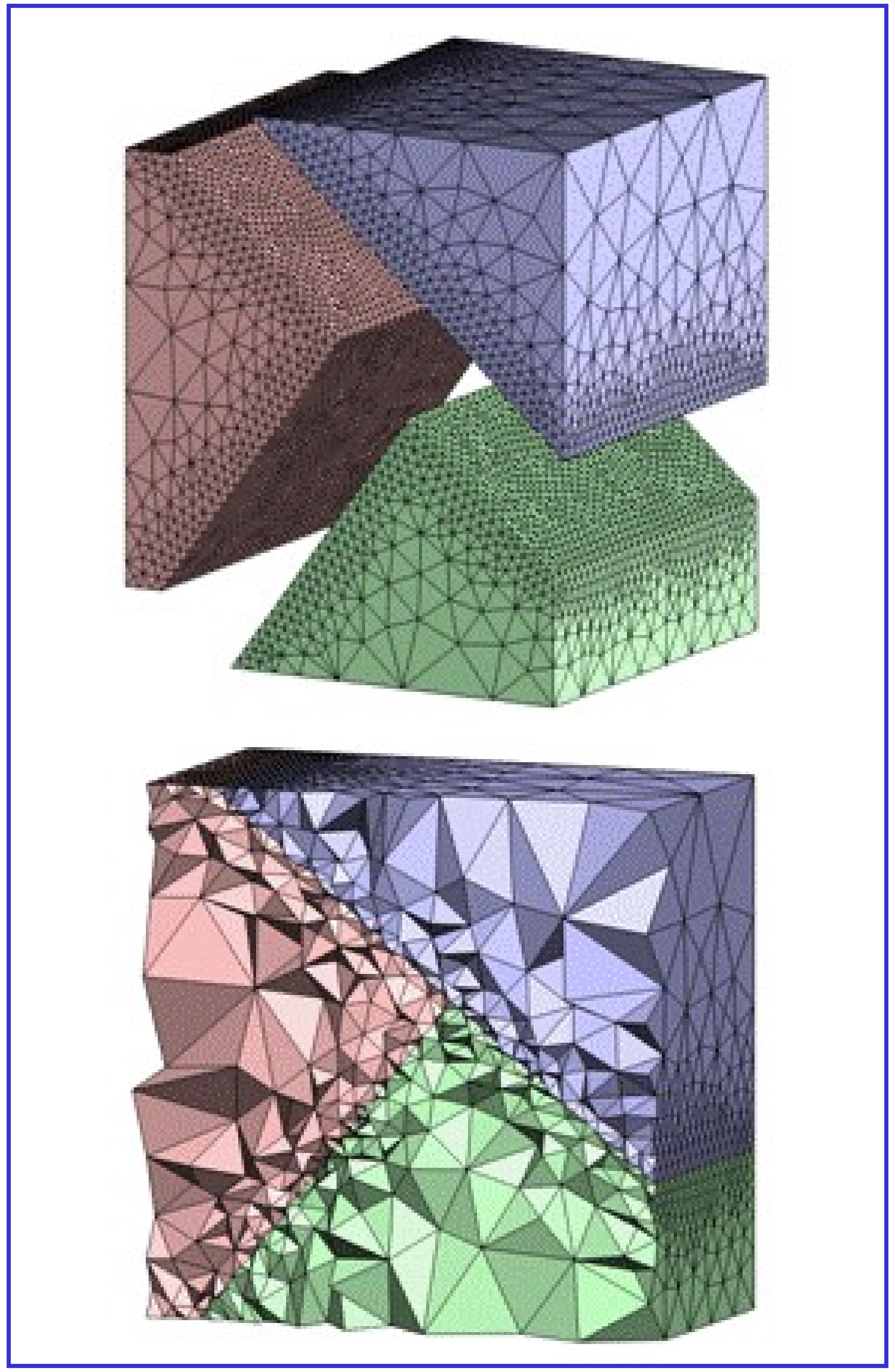
Khokhlov N.I., MIPT

Wide Range of Supported Meshes

- 2D
  - Rectangle
  - Curvilinear
  - Triangle
- 3D
  - Tetrahedral
  - Hexahedral
  - Hierarchical

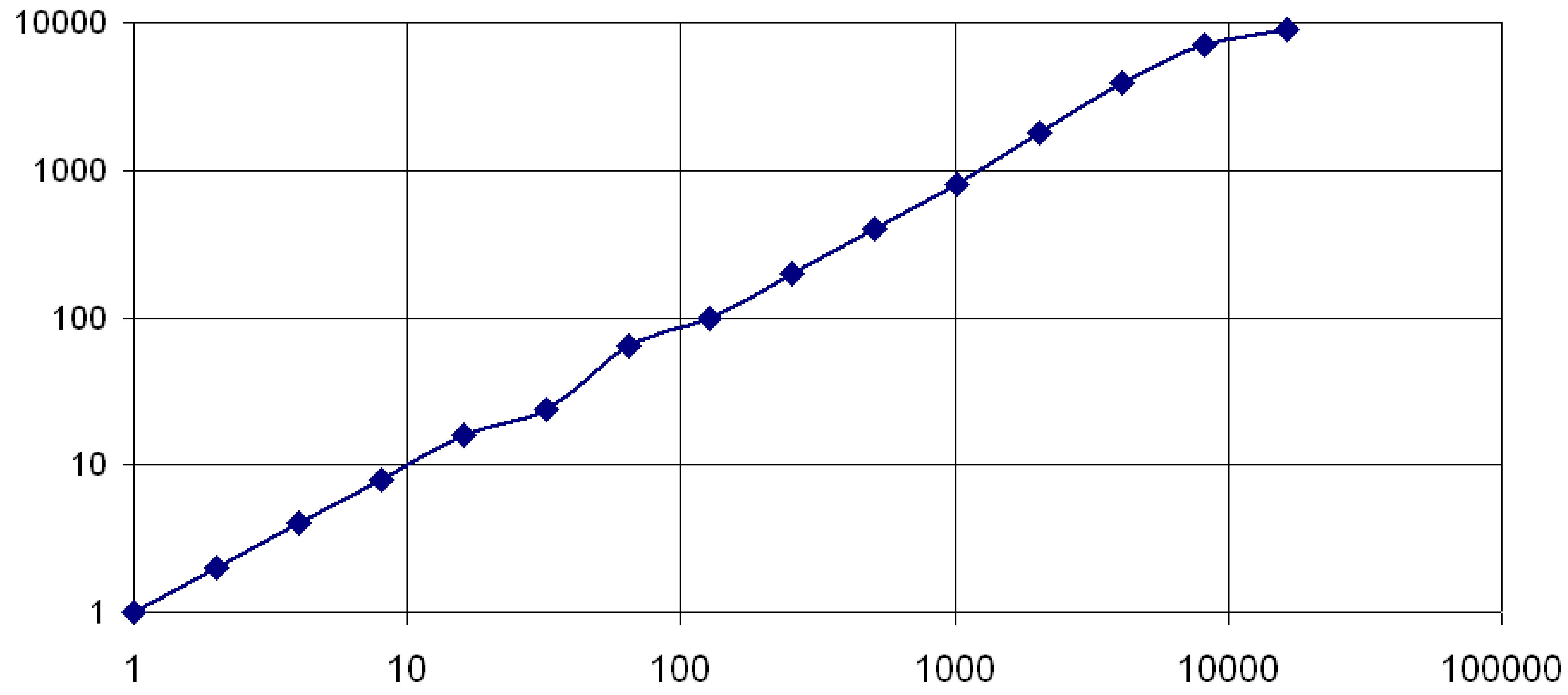






HPC Systems and Scalability

- HPC system: HECToR (Edinburgh)
- Up to 16 000 cores
- Efficiency ~ 80 %

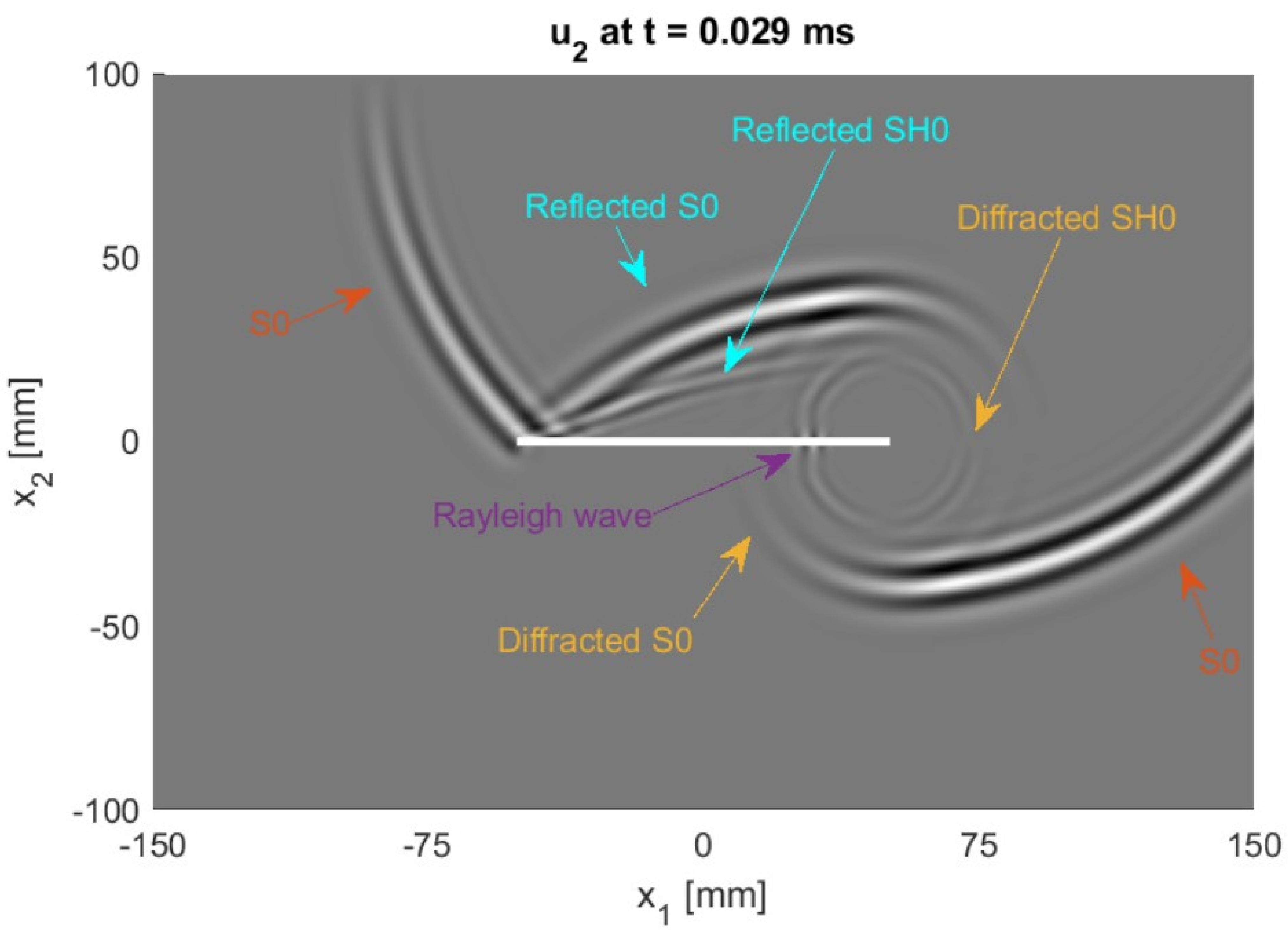


Parallelization Speedup

- MPI
  - HECToR – 80 % (up to 16k cores)
- CUDA
  - Tesla C2050 – 120x faster
  - Tesla K20c – 240x faster
- OpenMP:
  - AMD Opteron 6272 - 37x faster (64 cores)
  - AMD Opteron 8431 - 25x faster (48 cores)
  - Intel Xeon E5-2697 - 17x faster (24 cores)

Examples of Simulation

Waves in Thin Plates*



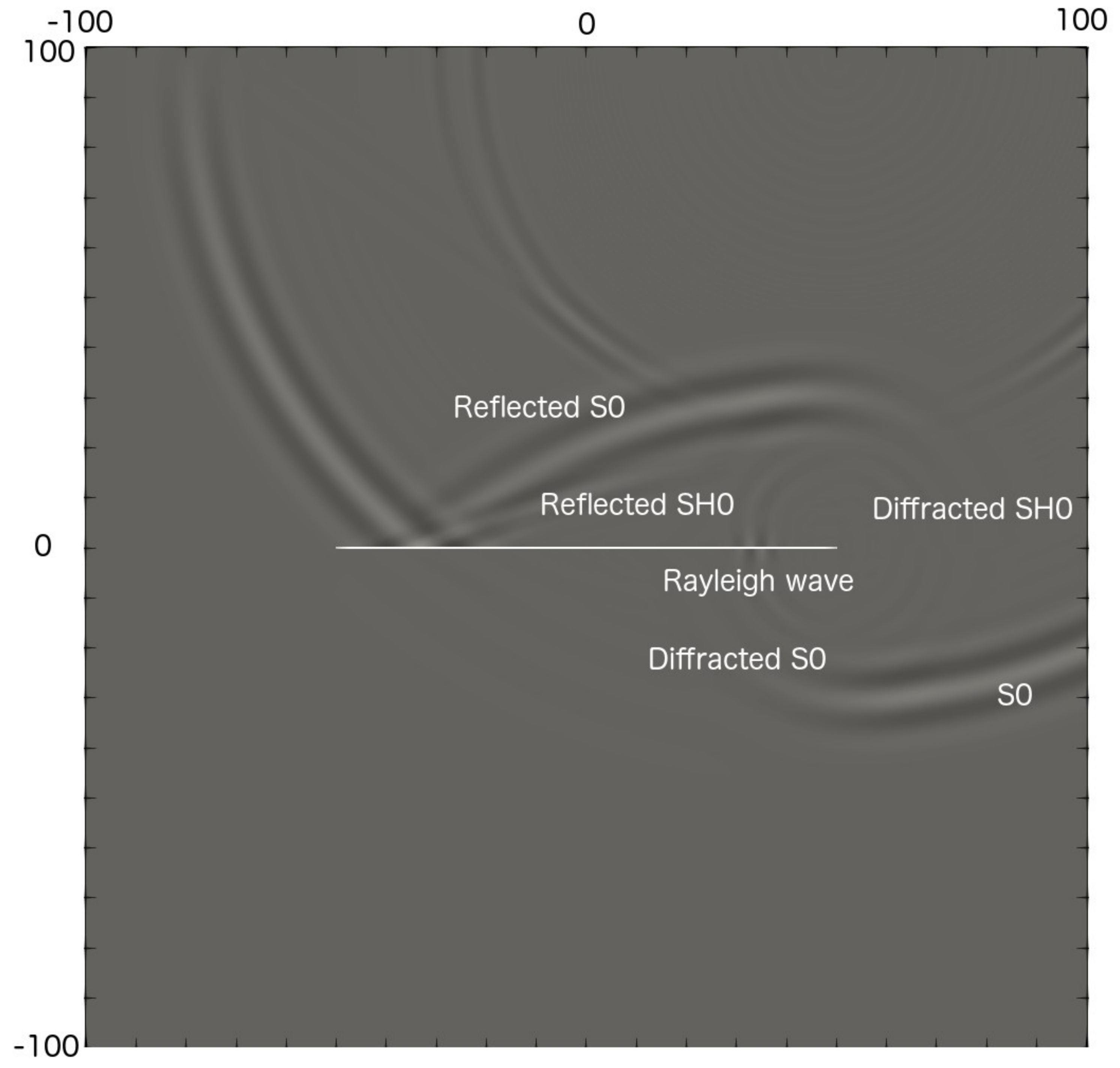
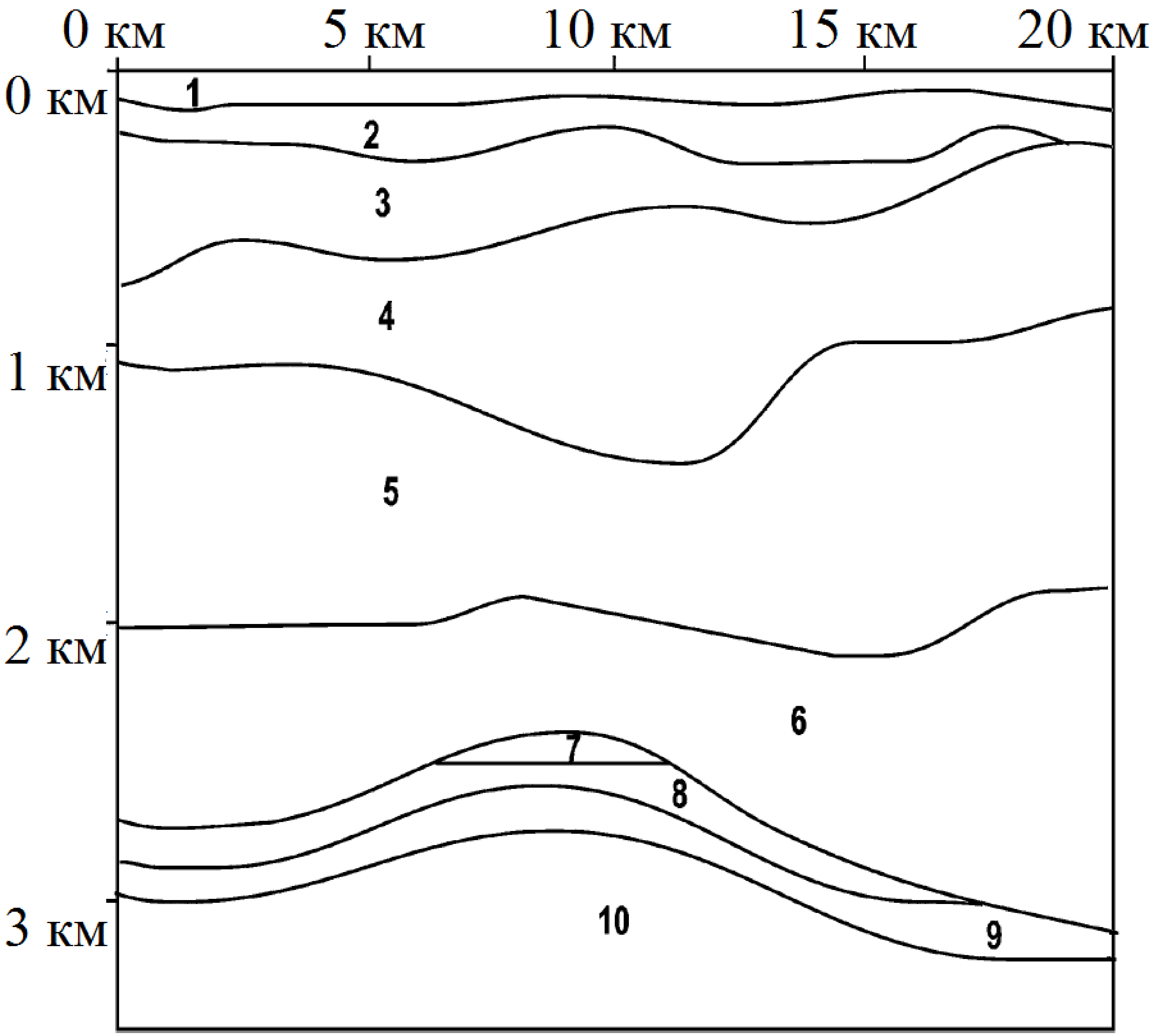
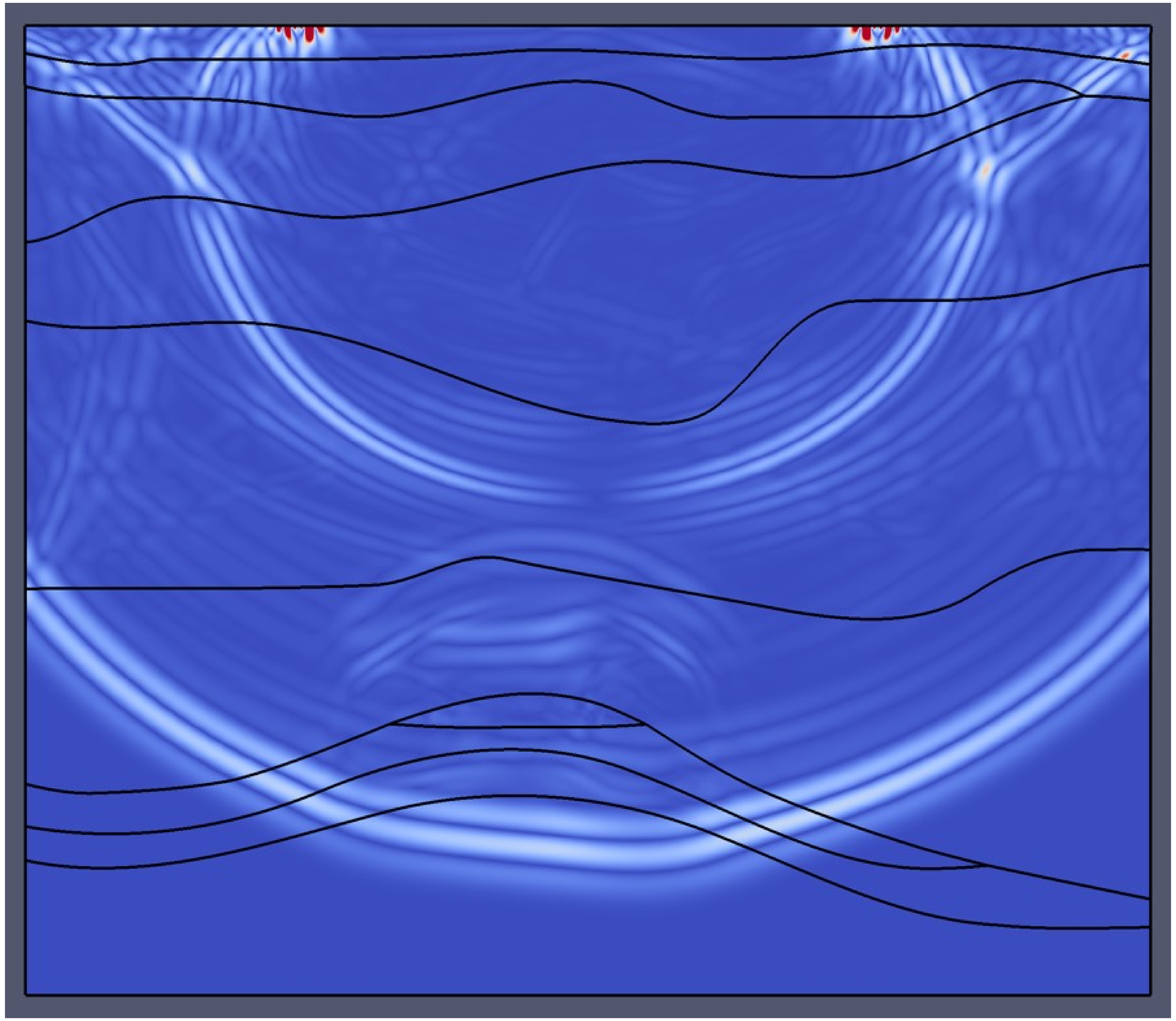


Fig. 2 presents the snapshots of displacement response of the cracked infinite plate after the S0 mode is excited by the applied tractions. At t = 0.015 ms, the initial wave packet of the S0 mode is propagating towards the crack. When it reaches the right crack tip, diffracted S0 mode and SH0 mode are generated, as well as diffracted Rayleigh-type waves propagating along crack surfaces at a speed slightly lower than that of the SH0 mode. When the initial S0 mode encounters the upper crack surface, reflected S0 and SH0 modes are generated. These diffracted and reflected waves can be seen clearly at t = 0.029 ms. SH0 mode waves are in-plane guided waves, so they do not have the out-of-plane displacement component and cannot be observed from the u3-displacement snapshot.

*Jun Li, Zahra Sharif Khodaei, and M.H. Aliabadi "Boundary Element Analysis of Lamb Wave Scattering by a Through-thickness Crack in a Plate" // International Conference on Fracture and Damage Mechanics, Mallorca, Spain, 14-17th September 2020

2D Geological Model





Transversely Isotropic Geological Media

1. We performed a numerical simulation of the propagation of seismic waves in the elastic Marmousi II model, into which the VTI anisotropy was added using the following relations

$$\varepsilon = 0.25\rho - 0.3,\ \delta = 0.125\rho - 0.1,$$

where density is in g/cm^3, ε and δ – Thomsen parameters. Rheological equations are

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{44} \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{pmatrix}$$

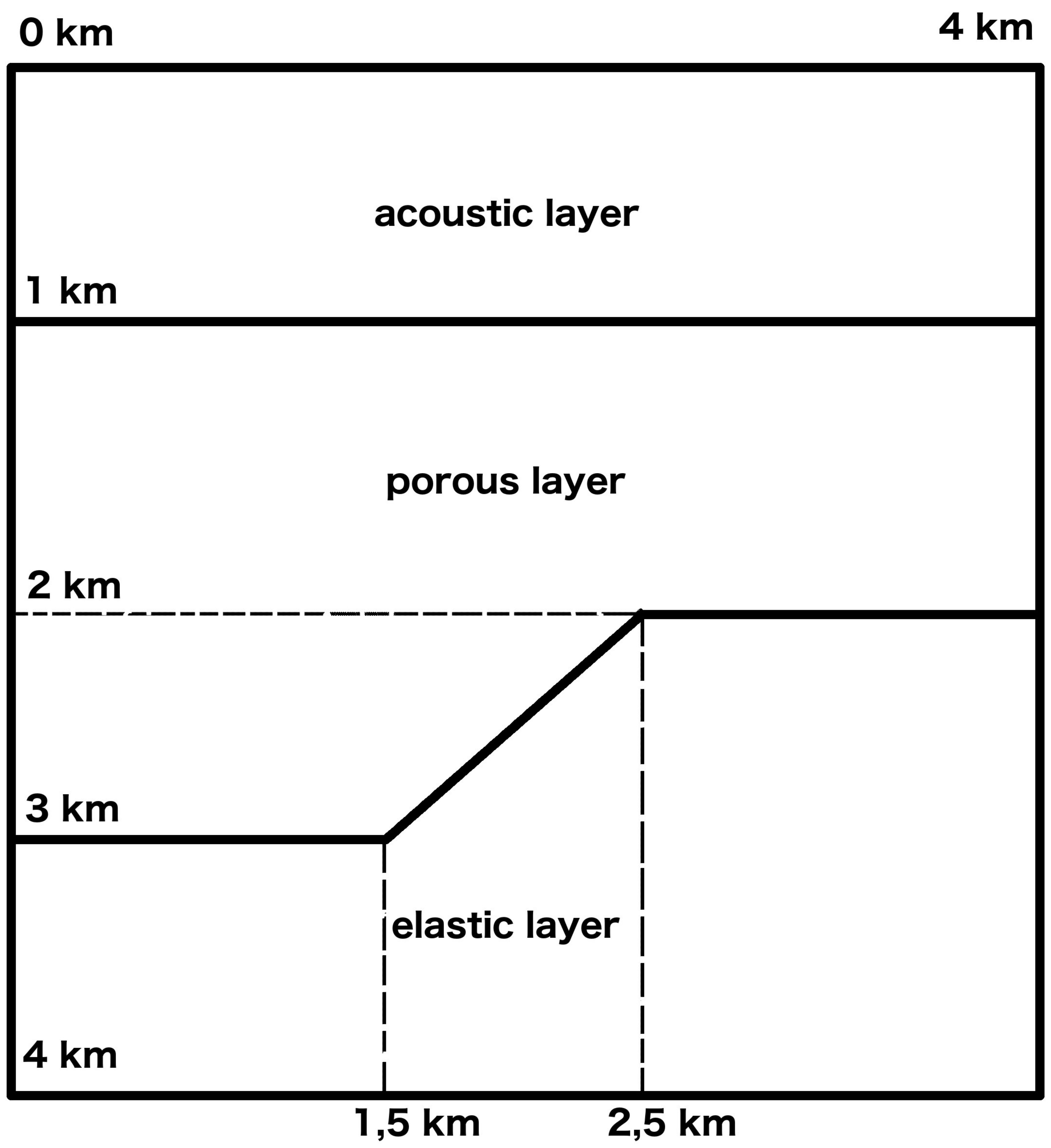
2. To cover the model with a computational mesh, a spatial step of 5 m and 3401 x 701 nodes were chosen. The time step was determined by the Courant condition and was 0.8 ms. All parameters of the anisotropic material were stored in each cell of the computational grid. The center of pressure located near the day surface was used as a disturbance.

Transversely Isotropic Geological Media

The layering of the geological model, the presence of sharp boundaries and anisotropy determine the complex wave pattern.

Seismic Waves in Fluid-Filled Porous Media

We considered the propagation of seismic waves in a model containing three different layers. The top one was the water column and was described by the acoustic approximation. The second one was the fluid-saturated medium. It can be bottom sediments or producing formations. The Dorovsky model was used for precise simulation of its dynamic behavior. The underlying rock massive was simulated in the frame of the elastic approximation. Dorovsky system is



$$\vec{u}_t + \frac{1}{\rho_s}(\nabla \cdot \mathbf{h})^T + \frac{1}{\rho_0}\nabla p = \vec{F},$$

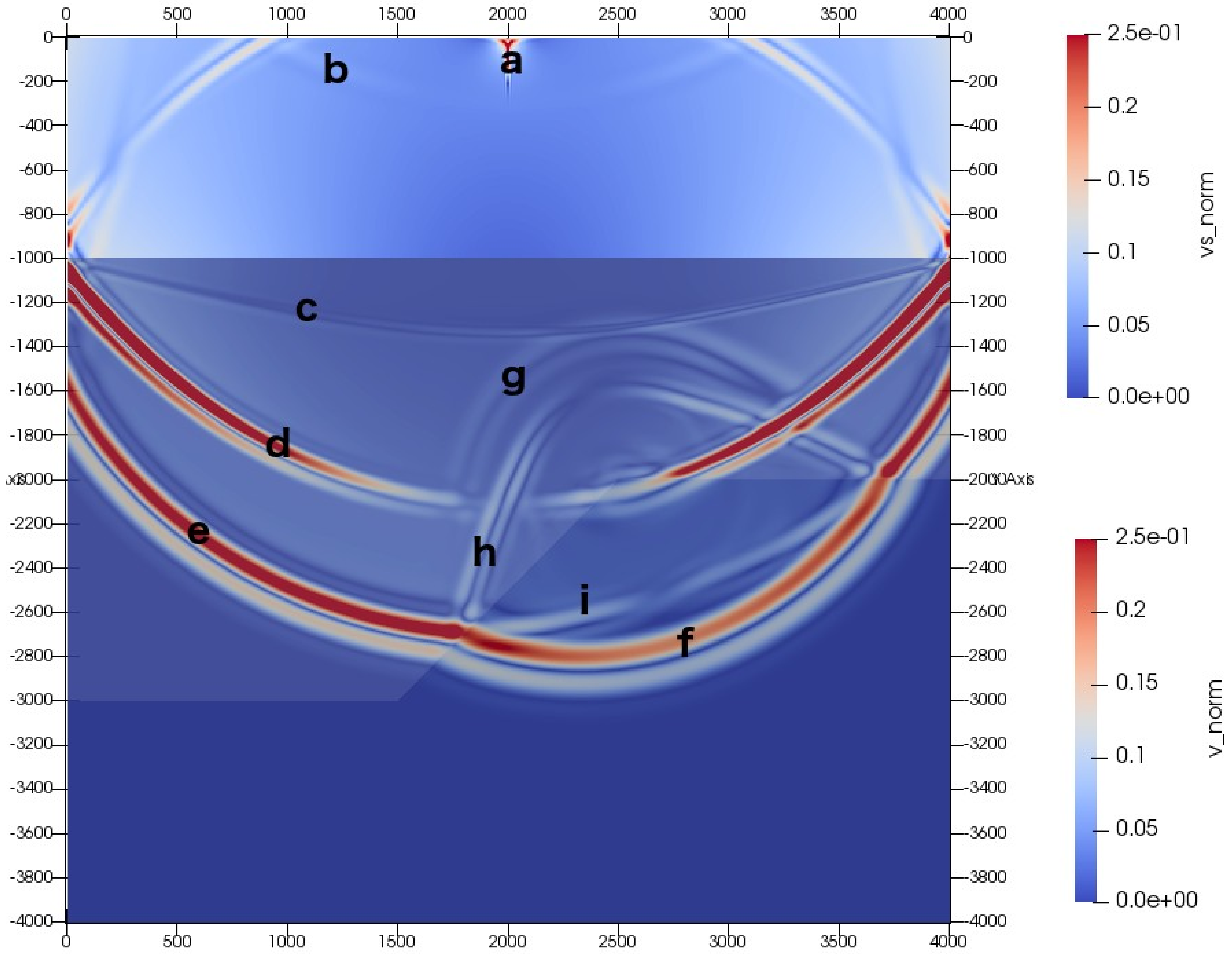
$$\vec{v}_t + \frac{1}{\rho_0}\nabla p = \vec{F},$$

$$p_t - (K - \alpha\rho_0\rho_s)(\nabla \cdot \vec{u}) + \alpha\rho_0\rho_f(\nabla \cdot \vec{v}) = 0.$$

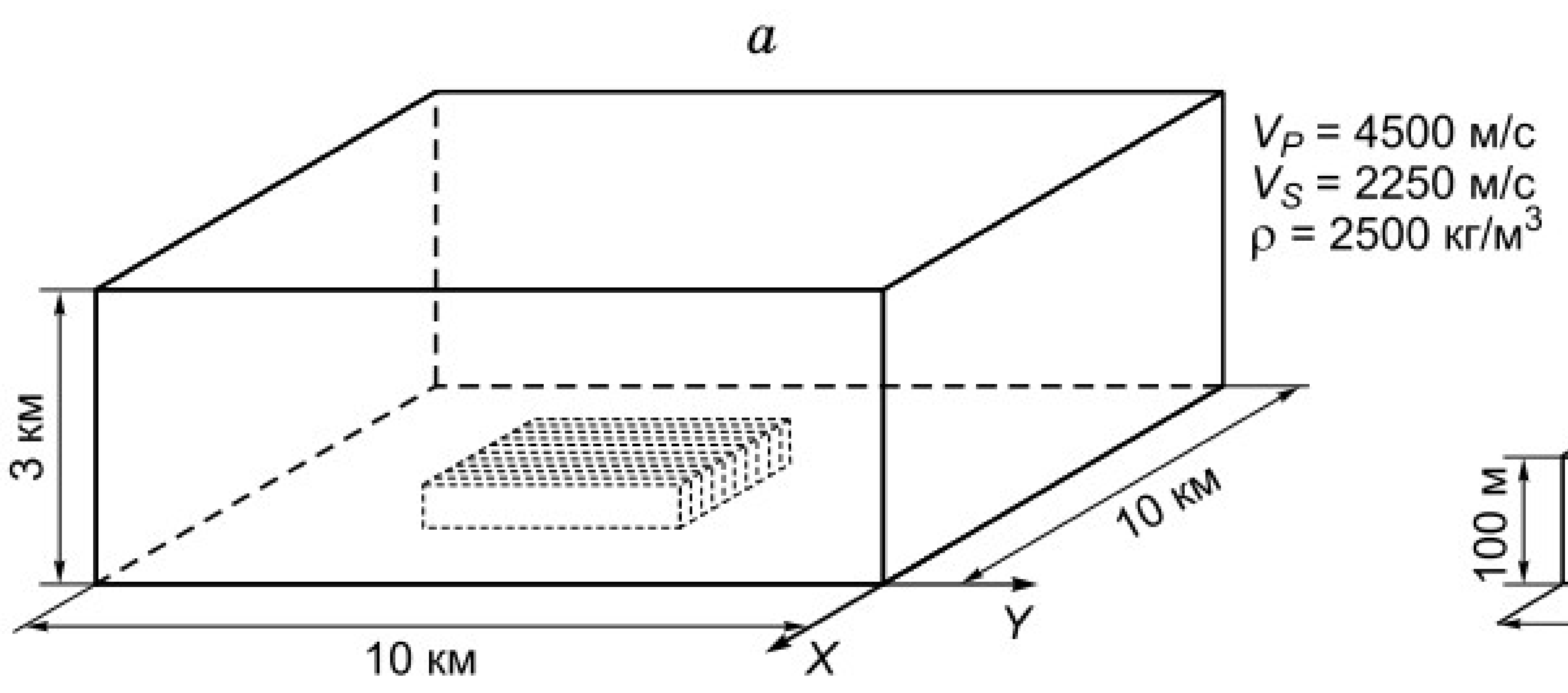
$$\mathbf{h}_t + \mu\left(\nabla \otimes \vec{u} + (\nabla \otimes \vec{u})^T\right) + \left[\left(\lambda - \frac{\rho_s}{\rho_0}K\right)(\nabla \cdot \vec{u}) - \frac{\rho_f}{\rho_0}K(\nabla \cdot \vec{v})\right]\mathbf{I} = 0,$$

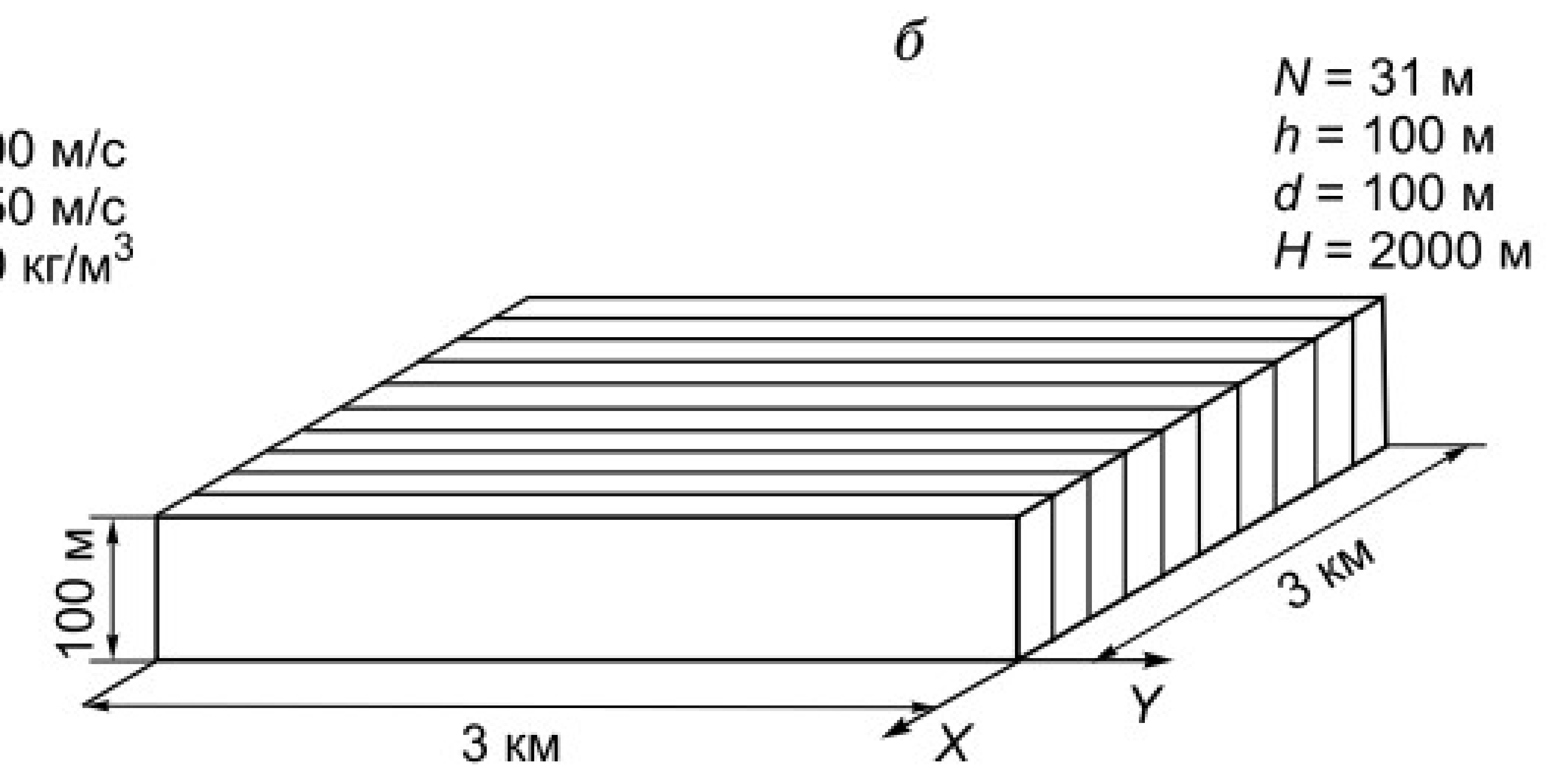
Seismic Waves in Fluid-Filled Porous Media

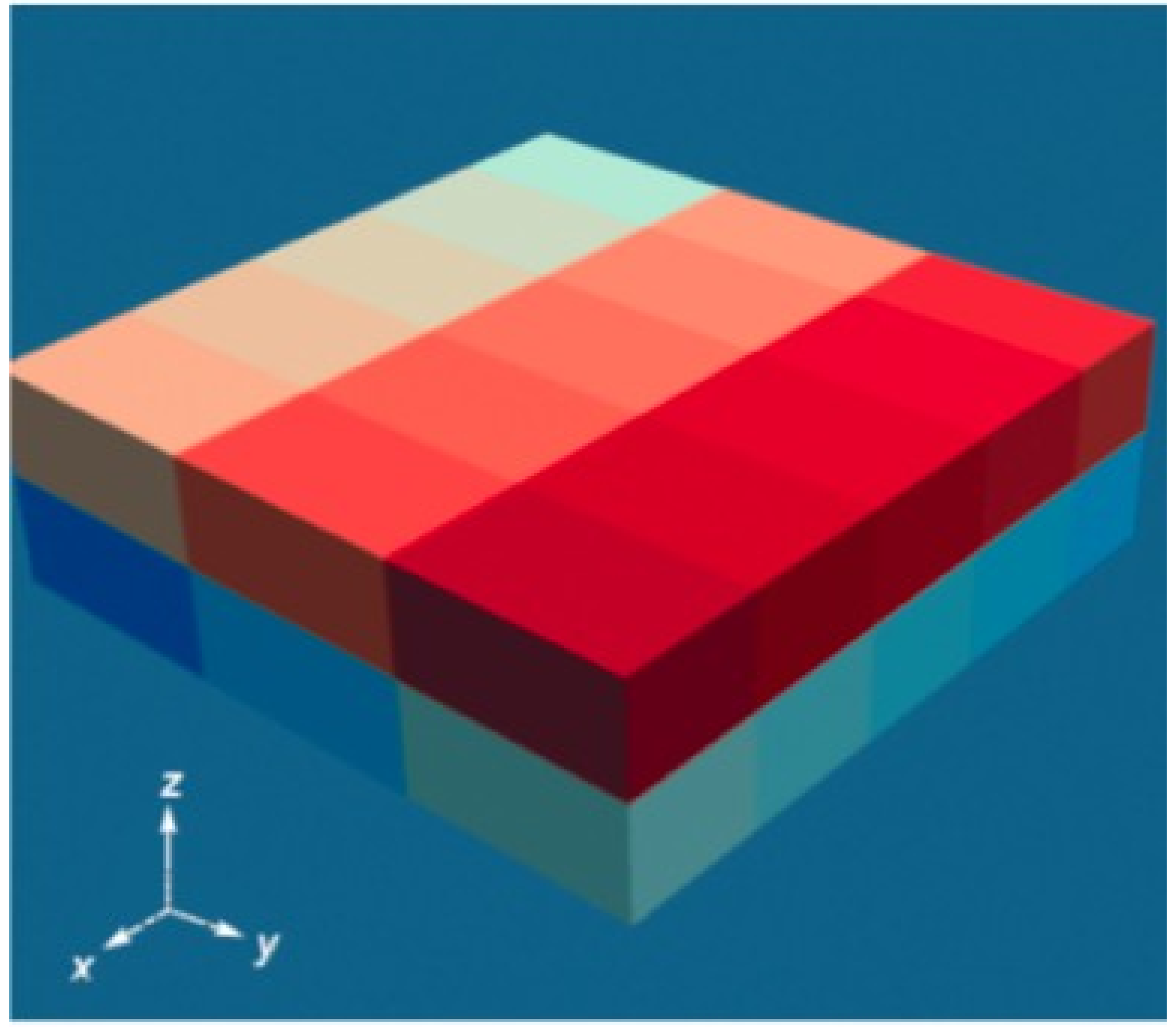
The simulated wavefield in the computational domain for the fixed time step is presented. Modulus of fluid and skeleton velocities are depicted. Different components of the seismic signal are denoted with letters from **a** to **g**. The source point (**a**) produces a stationary noise and can be easily filtered out. The wave **b** was initiated after the interaction with the day surface. Inside the porous layer, two P-waves (**e** and **c**) and one S-wave d are propagated. The last one demonstrates an obvious zero amplitude value right below the source point. The interaction of the P-wave (**e**) with the elastic layer leads to the occurrence of P (**f**) and S (**i**) transmitted waves and two reflected waves (**h** and **g**).

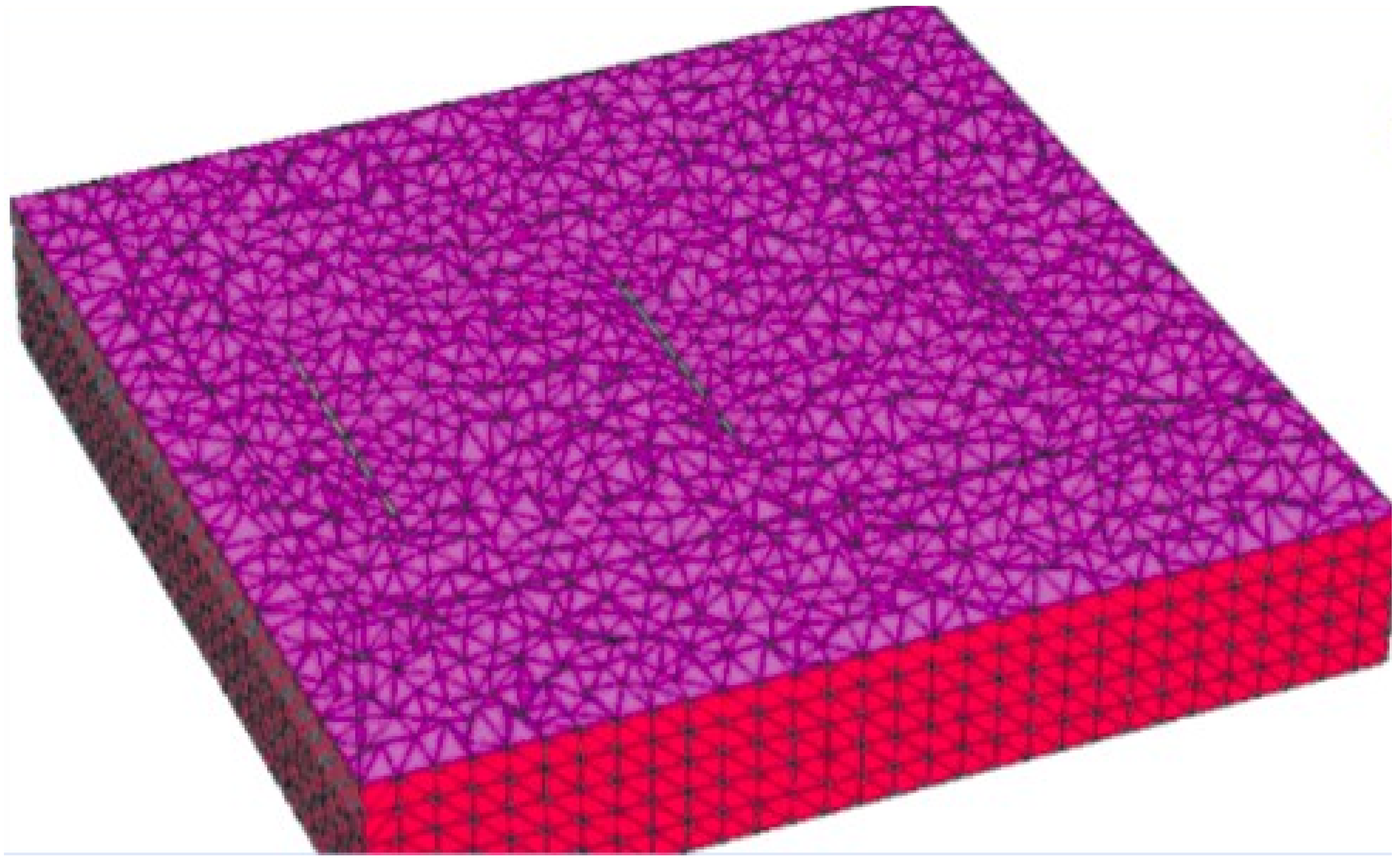


3D Full-Scaled Fractured Medium

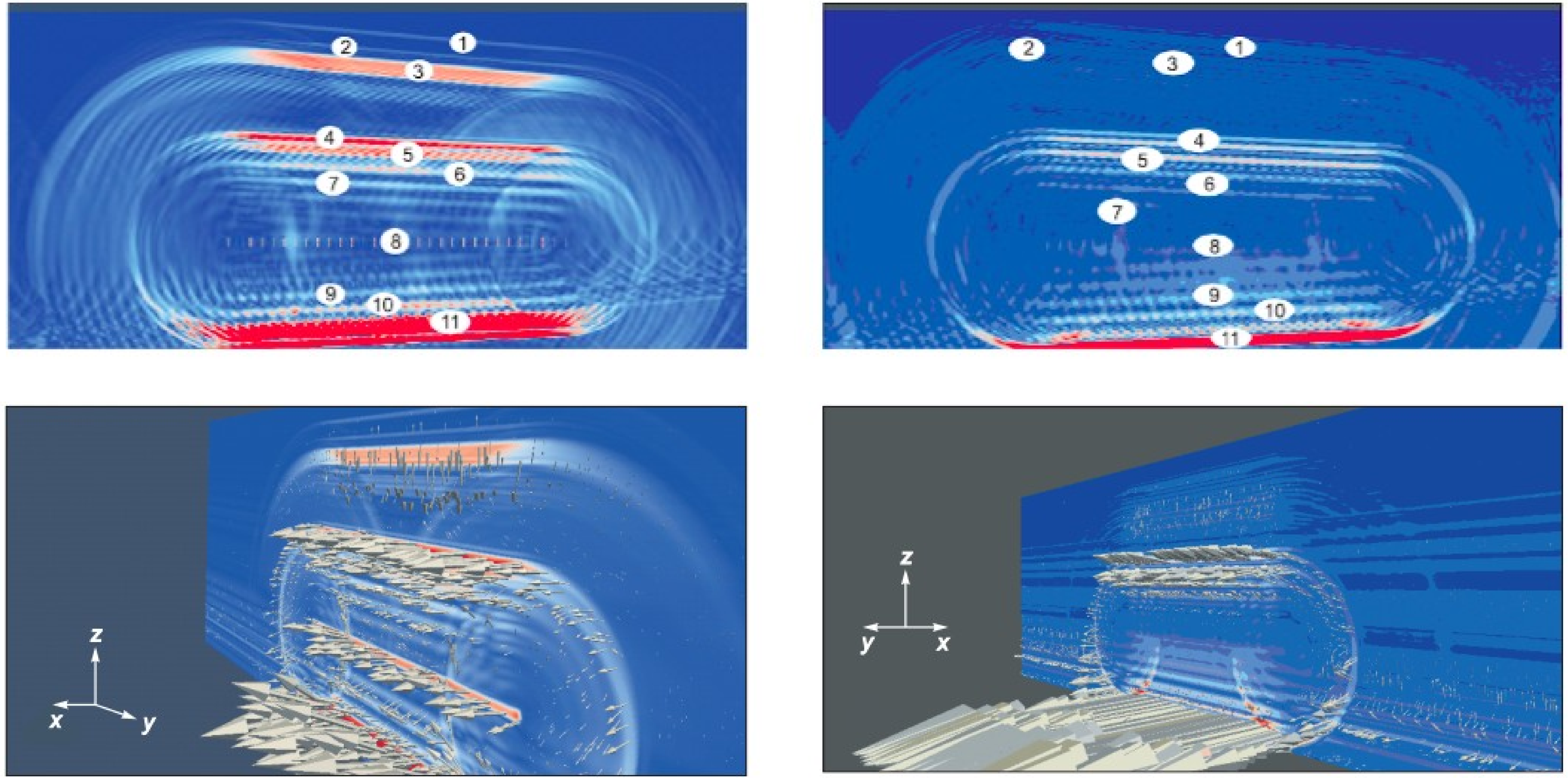








Slices of 3D Seismic Response



Modulus of velocity for empty cracks (left) and fluid-filled cracks (right)

Recent Scopus Publications

1. **BEKLEMYSHEVA, K., GOLUBEV, V., PETROV, I., VASYUKOV, A. Determining effects of impact loading on residual strength of fiber-metal laminates with grid-characteristic numerical method (2021) Chinese Journal of Aeronautics, 34 (7), pp. 1-12. - Q1**
2. Golubev, V.I., Shevchenko, A.V., Petrov, I.B. Application of the Dorovsky model for taking into account the fluid saturation of geological media (2021) Journal of Physics: Conference Series, 1715 (1), paper № 012056, .
3. Sharma, A., Golubev, V.I., Khare, R.K. Seismic Evaluation of Two-Storied Unreinforced Masonry Building with Rigid Diaphragm Using Nonlinear Static Analysis (2021) Smart Innovation, Systems and Technologies, 214, pp. 175-187.
4. Golubev, V.I., Vasyukov, A.V., Churyakov, M. Modeling Wave Responses from Thawed Permafrost Zones (2021) Smart Innovation, Systems and Technologies, 214, pp. 137-148.
5. Nikitin, I.S., Golubev, V.I., Golubeva, Y.A., Miryakha, V.A. Numerical Comparison of Different Approaches for the Fractured Medium Simulation (2021) Smart Innovation, Systems and Technologies, 214, pp. 87-99.
6. Golubev, V.I., Ekimenko, A.V., Nikitin, I.S., Golubeva, Y.A. Continuum model of layered medium for reservoir of Bazhenov formation (2021) Springer Geology, pp. 235-245.
7. Golubev, V., Nikitin, I., Golubeva, Y., Petrov, I. Numerical simulation of the dynamic loading process of initially damaged media (2020) AIP Conference Proceedings, 2309, paper № 0033949, .
8. Golubev, V., Nikitin, I., Ekimenko, A. Simulation of seismic responses from fractured MARMOUSI2 model (2020) AIP Conference Proceedings, 2312, paper № 050006, .
9. Nikitin, I.S., Golubev, V.I., Ekimenko, A.V., Anosova, M.B. Simulation of seismic responses from the 3D non-linear model of the Bazhenov formation (2020) IOP Conference Series: Materials Science and Engineering, 927 (1), paper № 012020, .
10. Shevchenko, A.V., Golubev, V.I., Ekimenko, A.V., Petrov, I.B. Simulation of seismic responses from oil saturated formations (2020) Geomodel 2020.
11. **Golubev, V., Shevchenko, A., Petrov, I. Simulation of Seismic Wave Propagation in a Multicomponent Oil Deposit Model (2020) International Journal of Applied Mechanics, 12 (8), статья № 2050084. – Q1**
12. Golubev, V.I., Khokhlov, N.I., Nikitin, I.S., Churyakov, M.A. Application of compact grid-characteristic schemes for acoustic problems (2020) Journal of Physics: Conference Series, 1479 (1), статья № 012058, .
13. Petrov, I.B., Golubev, V.I., Shevchenko, A.V. Problem of Acoustic Diagnostics of a Damaged Zone (2020) Doklady Mathematics, 101 (3), pp. 250-253.
14. Favorskaya, A.V., Golubev, V.I. Elastic and acoustic approximations for solving direct problems of human head ultrasonic study (2020) Procedia Computer Science, 176, pp. 2566-2575.
15. Favorskaya, A., Golubev, V. Study the elastic waves propagation in multistory buildings, taking into account dynamic destruction (2020) Smart Innovation, Systems and Technologies, 193, pp. 189-199.
16. Golubev, V.I., Muratov, M.V., Petrov, I.B. Different Approaches for Solving Inverse Seismic Problems in Fractured Media (2020) Smart Innovation, Systems and Technologies, 173, pp. 199-212.
17. Nikitin, I.S., Burago, N.G., Golubev, V.I., Nikitin, A.D. Methods for Calculating the Dynamics of Layered and Block Media with Nonlinear Contact Conditions (2020) Smart Innovation, Systems and Technologies, 173, pp. 171-183.

Thank you for your attention!

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