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The graph diameter of a distributed system with a given dominant set

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Диаметр графа распределенной системы с заданным доминирующим множеством

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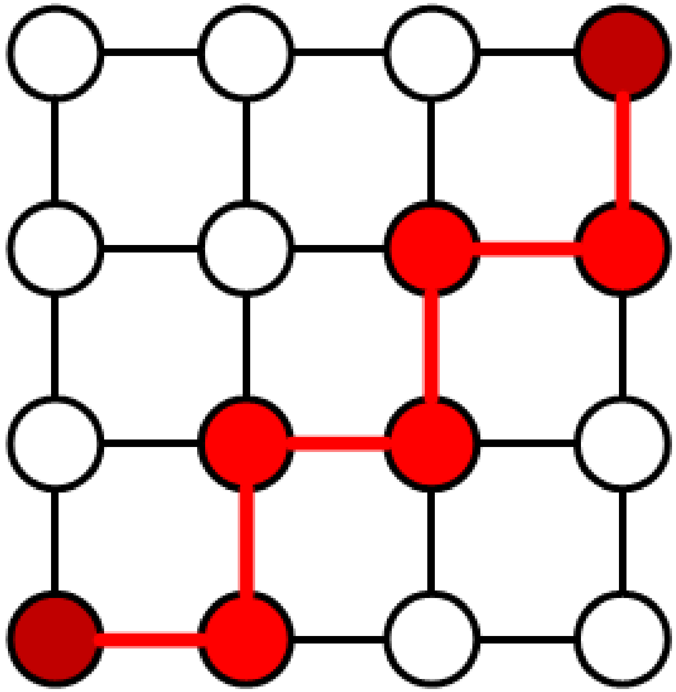
Structured distributed systems can have many control or coordinating nodes to ensure fault tolerance. This is achieved by using certain network topologies and allocating several types of nodes that perform various functions: calculation, storage, coordination and management.

The role of control or coordination nodes can be performed by elements that "cover" all other nodes, that is ,the "dominant set" of vertices of the graph.

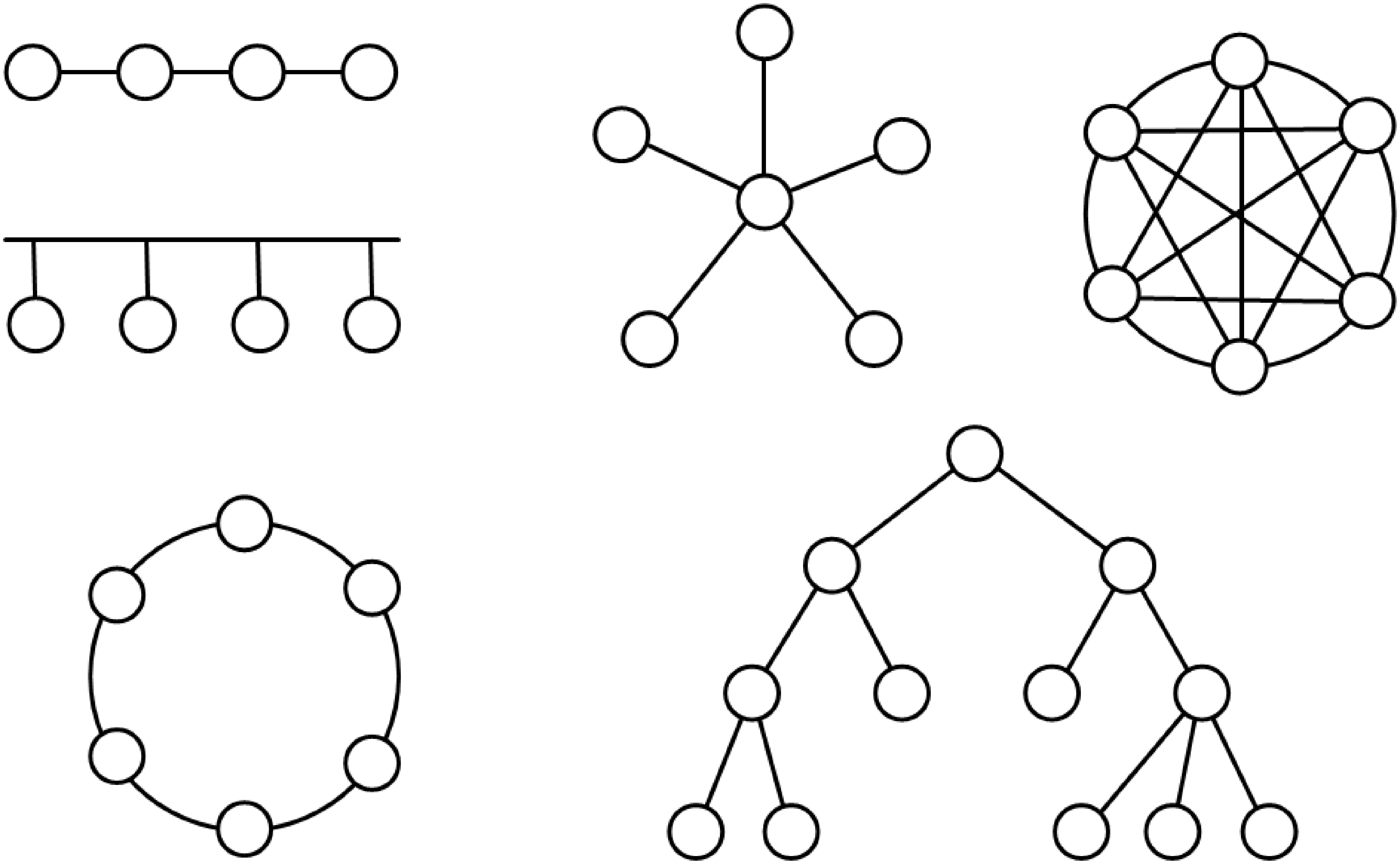
In addition, an important characteristic of a telecommunications network graph is the length of the maximum shortest chain over all pairs of vertices or the diameter. The diameter of the graph allows you to estimate the longest message transmission time in the network.

Distance between nodes



Max distance(diameter) = less than 4-5 edges in route

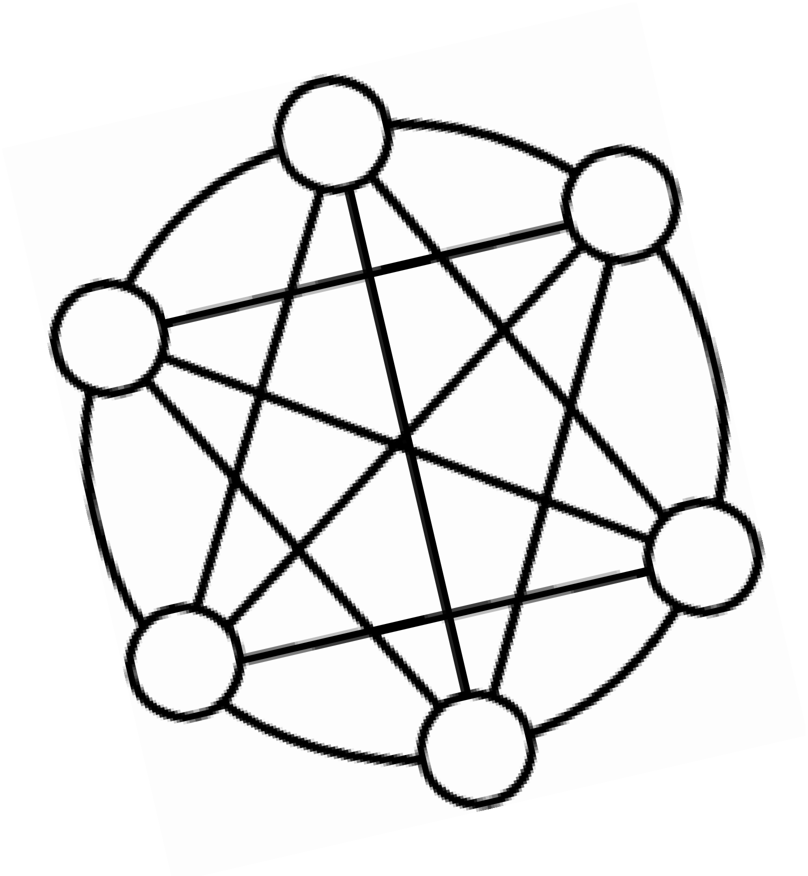
Base networks topologies



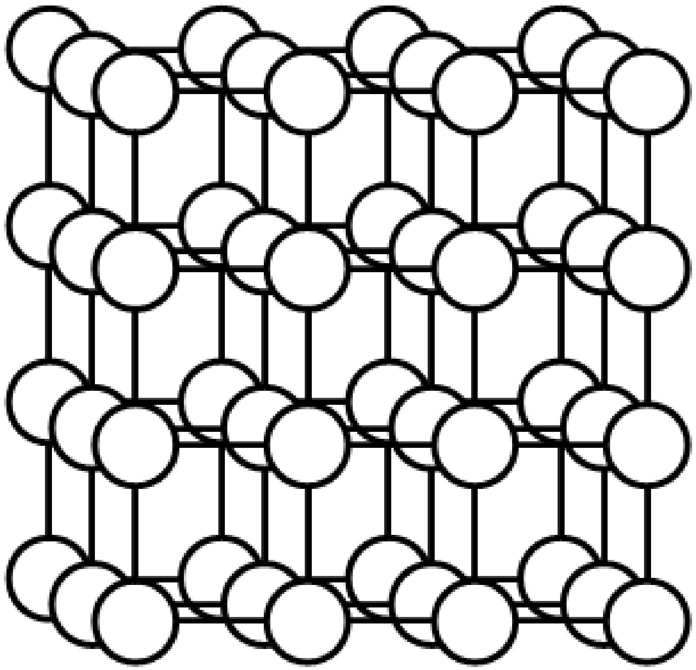
Network topologies for big corporative networks (datacenter, supercomputer et al.)

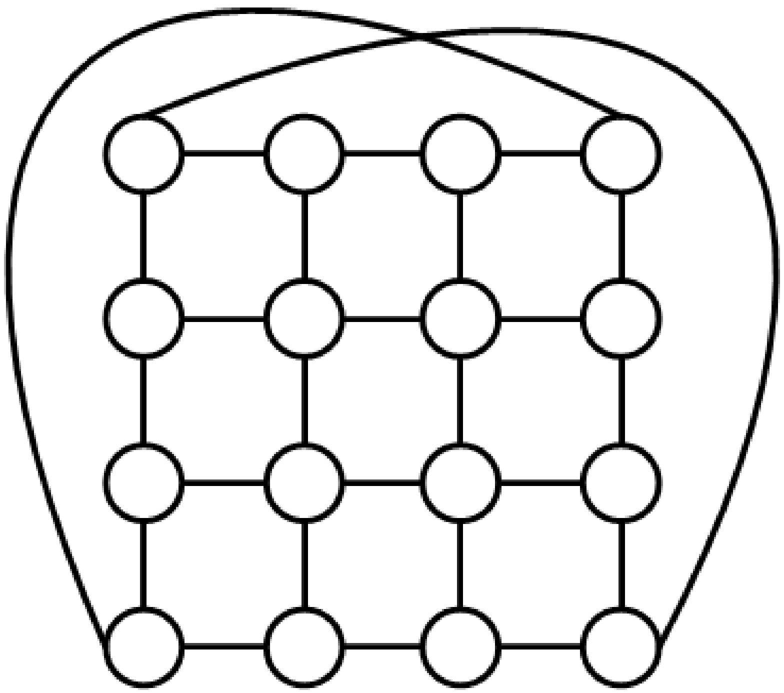
- **Fat tree**
- N-dimensional torus
- N-dimensional grid
- Hypercube
- **Dragonfly**
- **Jellyfish**
- All-connected topology

All-connected network topology

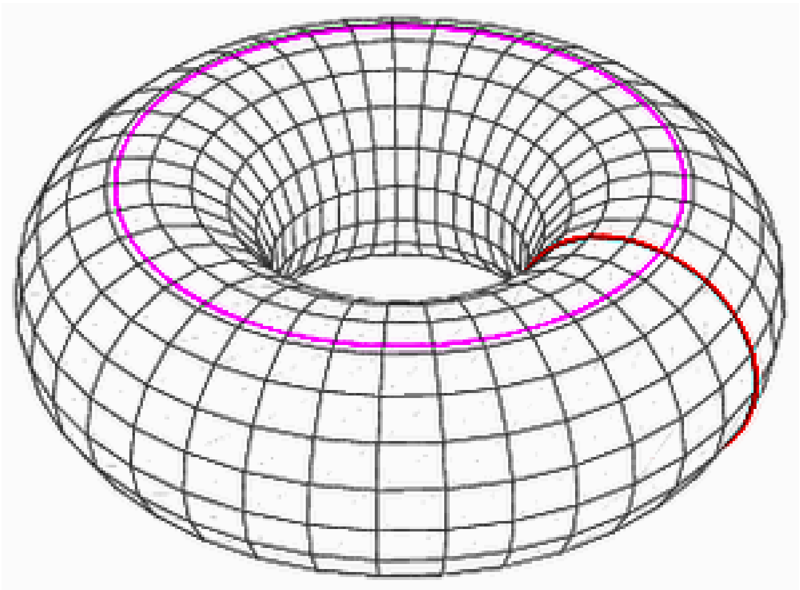


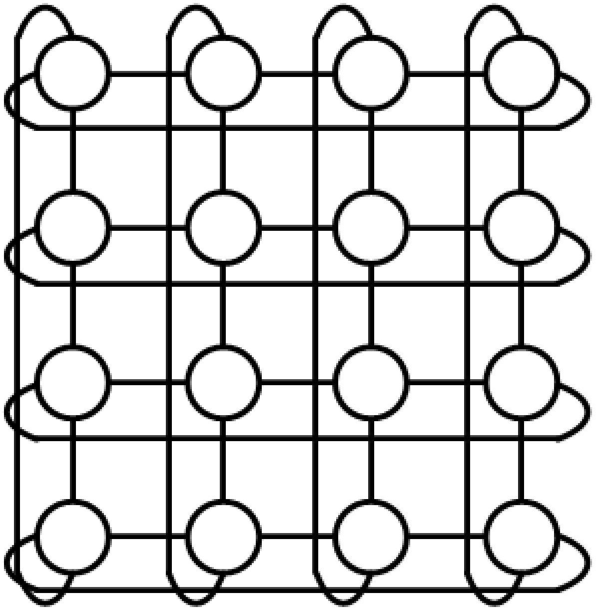
Grid





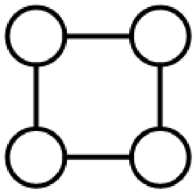
Torus



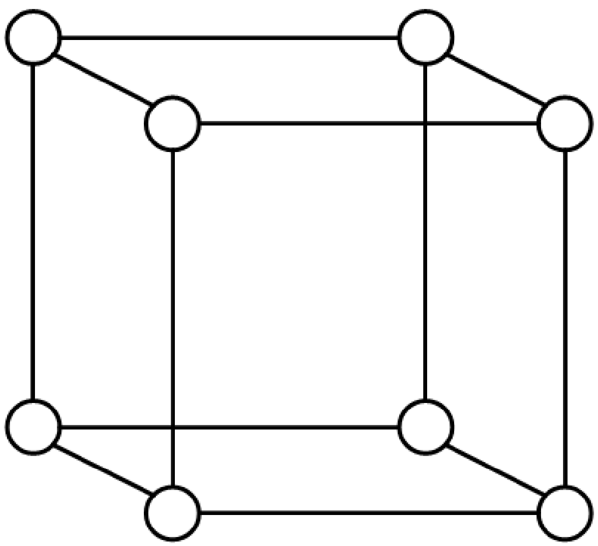


Hypercube

2



3



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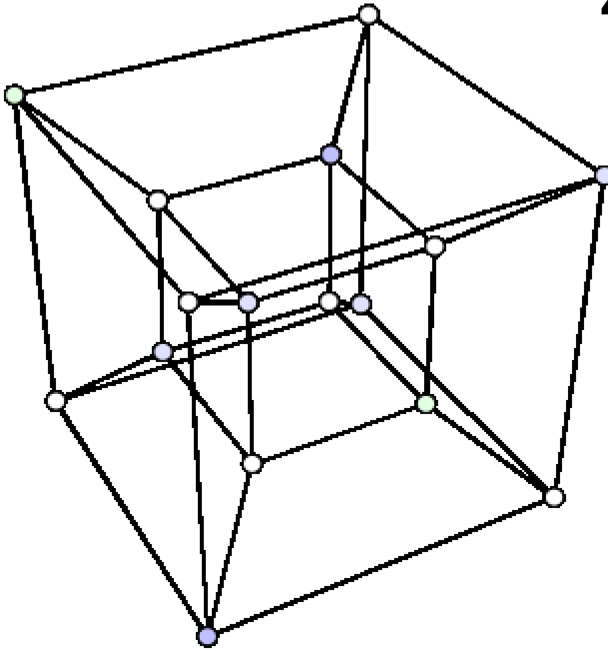

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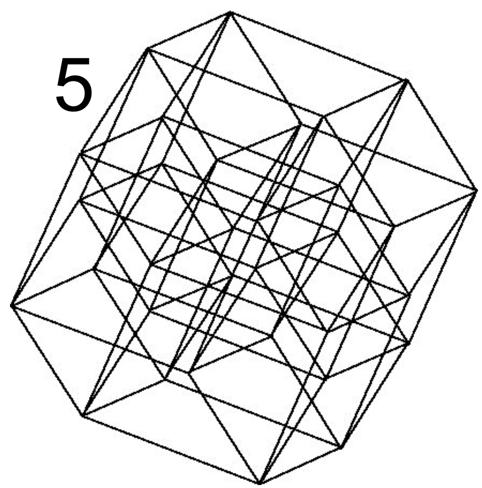

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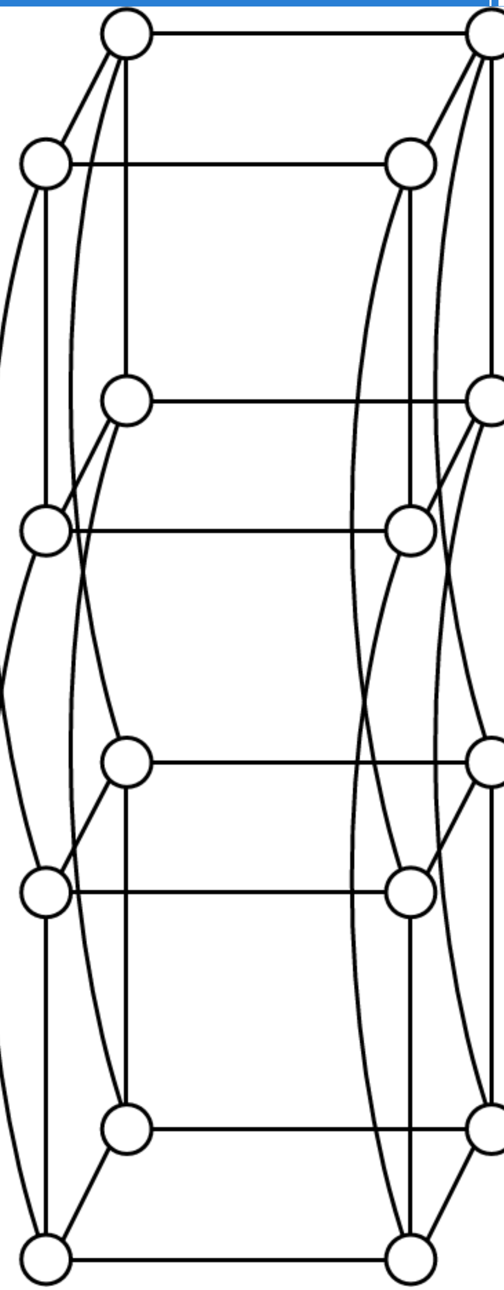
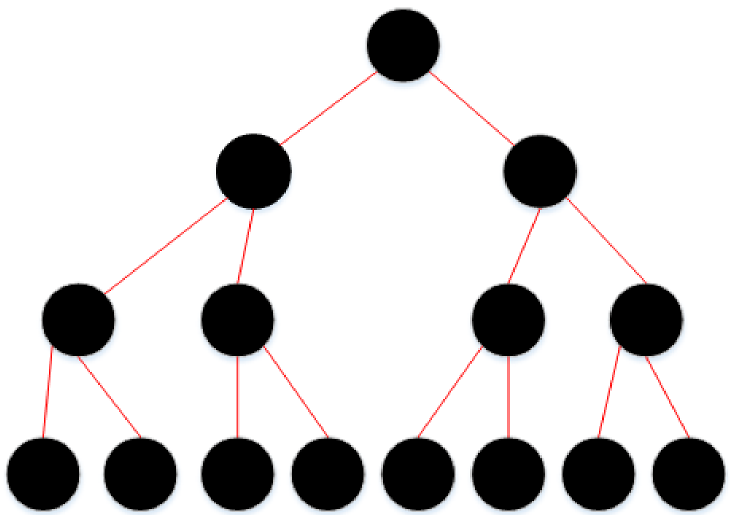
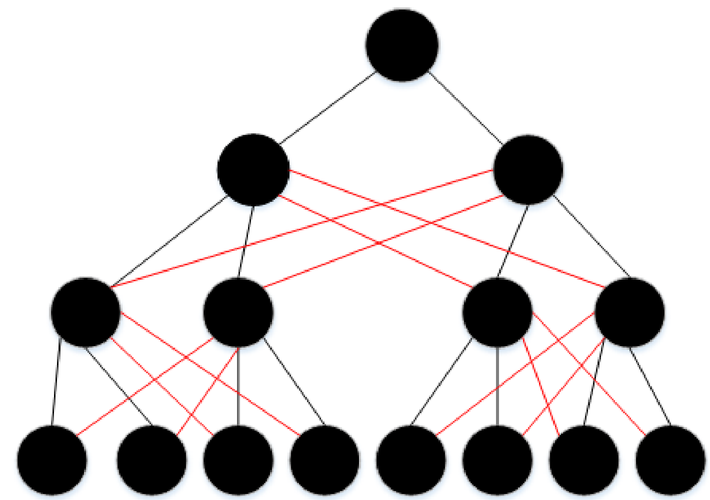


FatTree



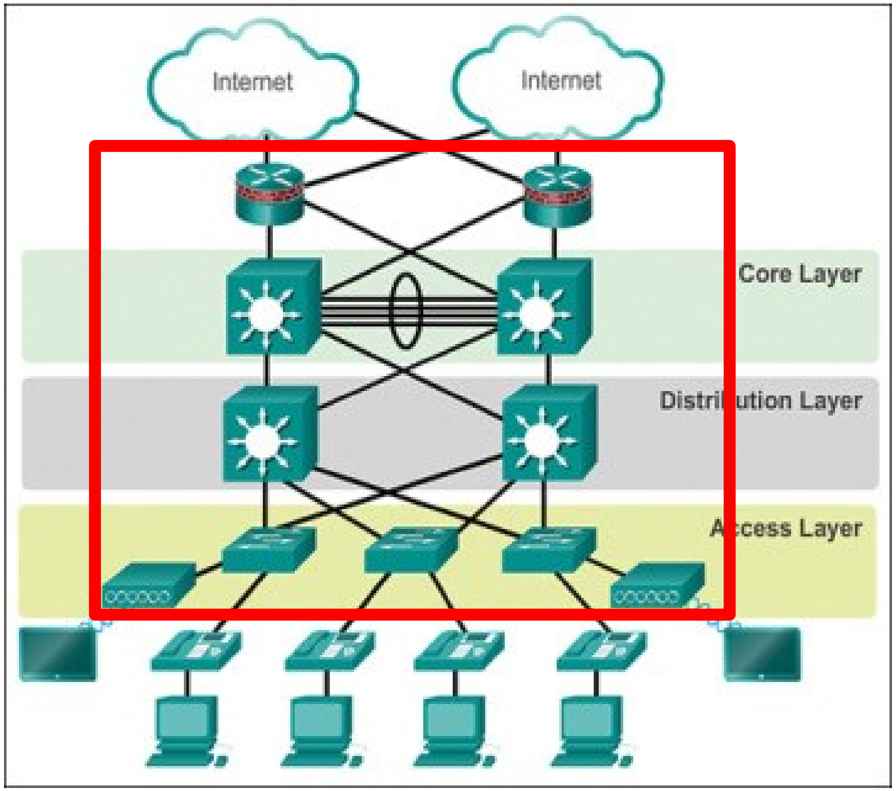
1

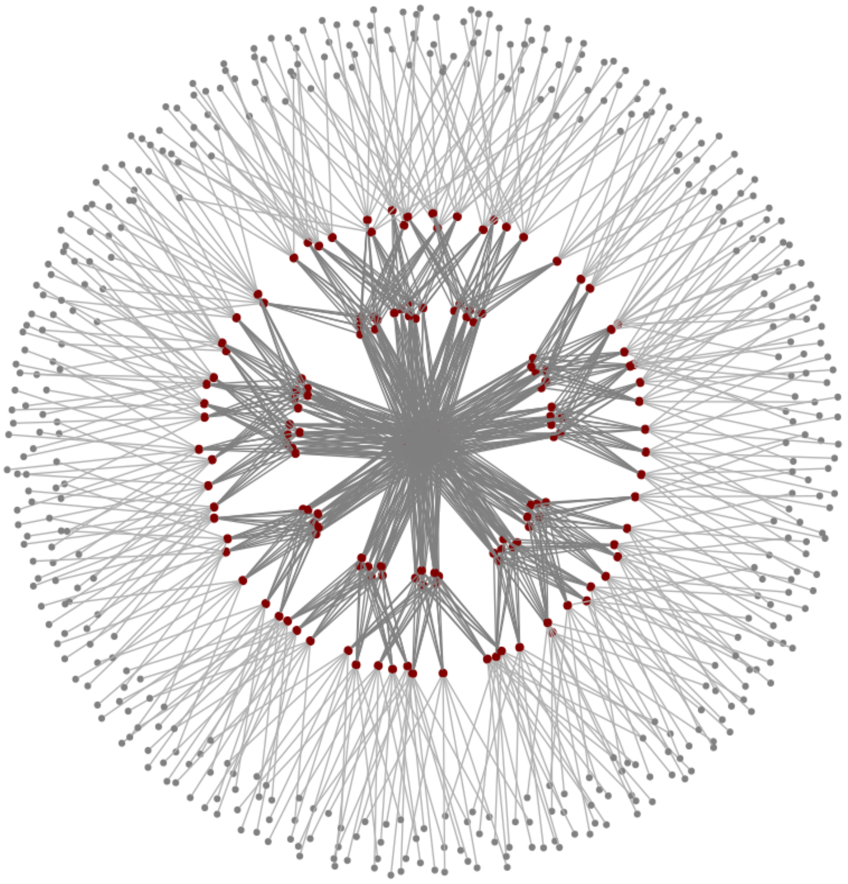


2

- All-known and useful network topology
- Better for big networks than classical tree
- Restriction of vertex order is possible
- The fault tolerance network topology but is sensitive to multiple failures

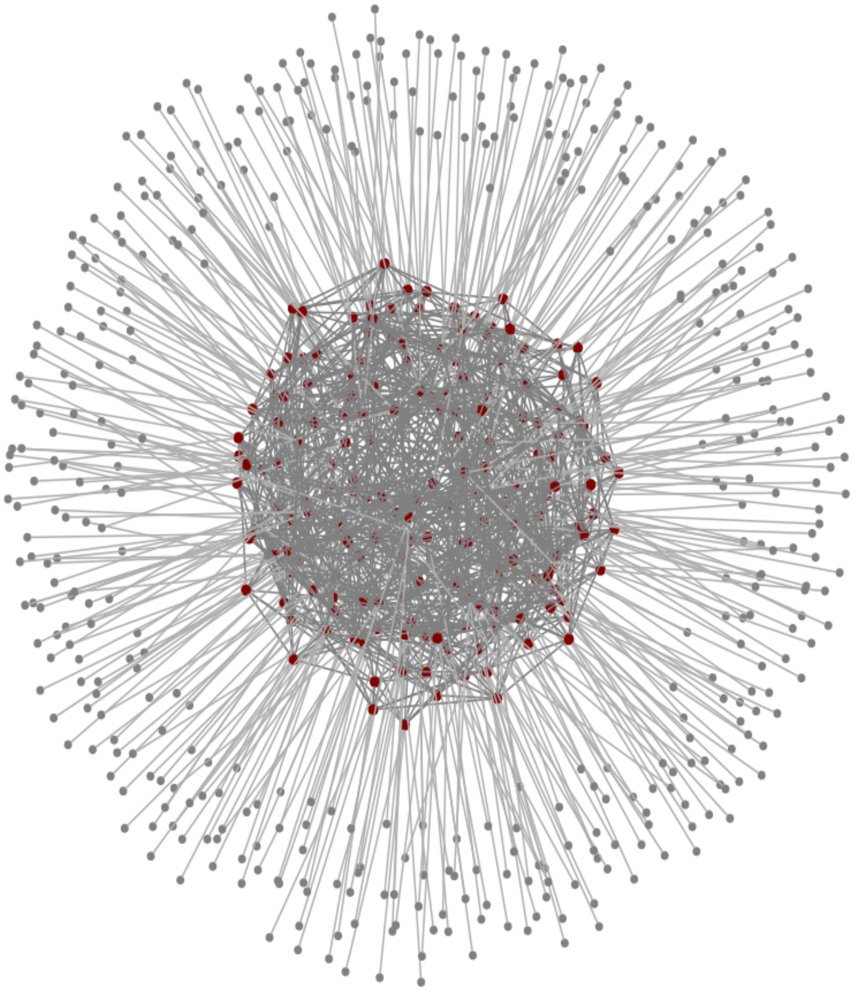
FatTree



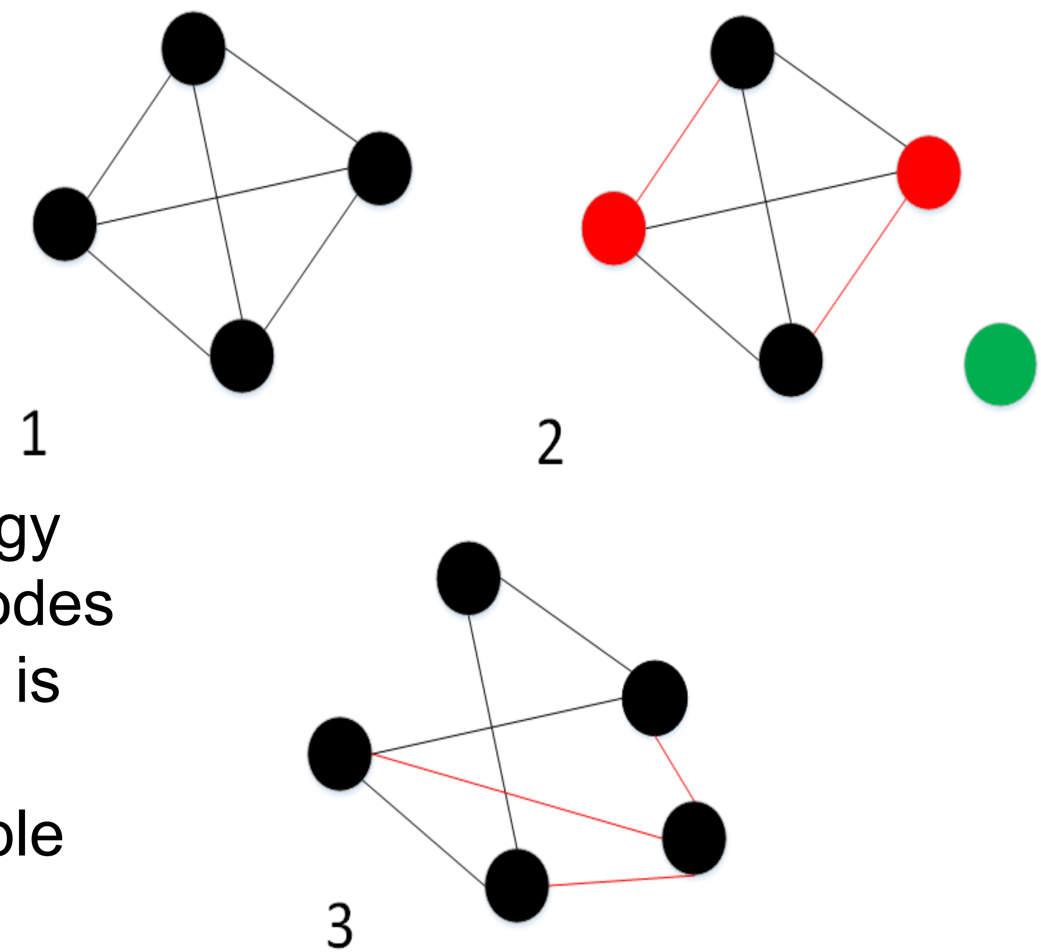


* from ciscopress.com

JellyFish

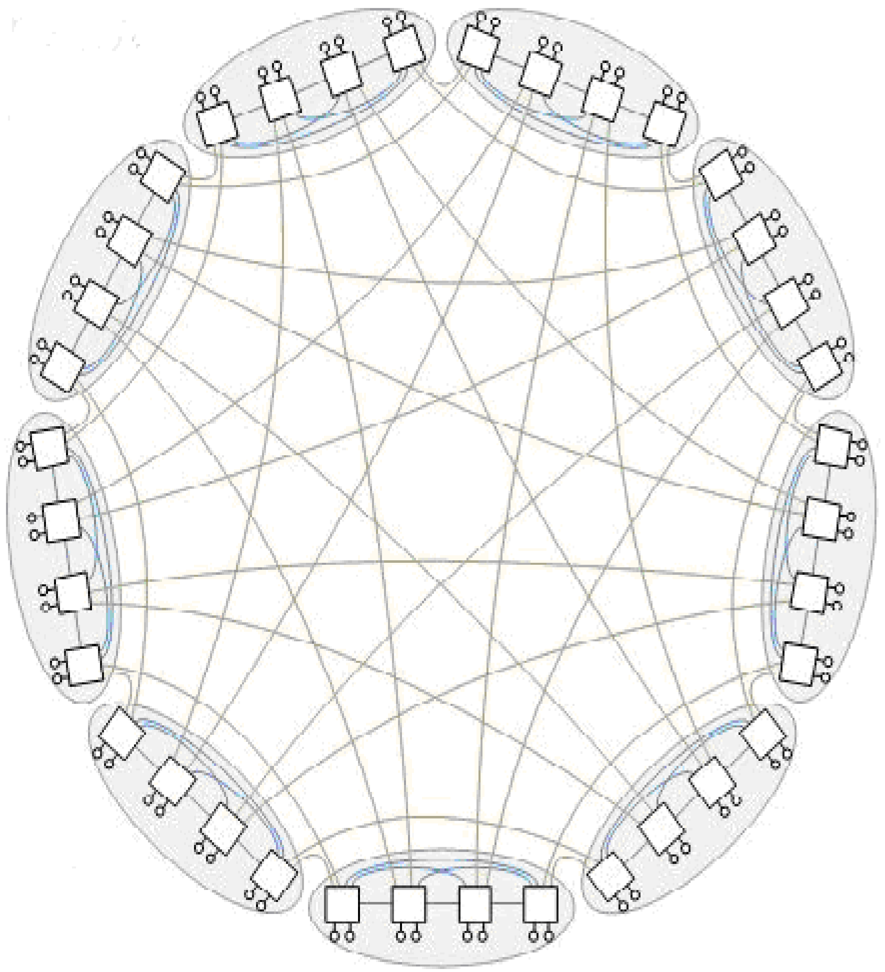


JellyFish



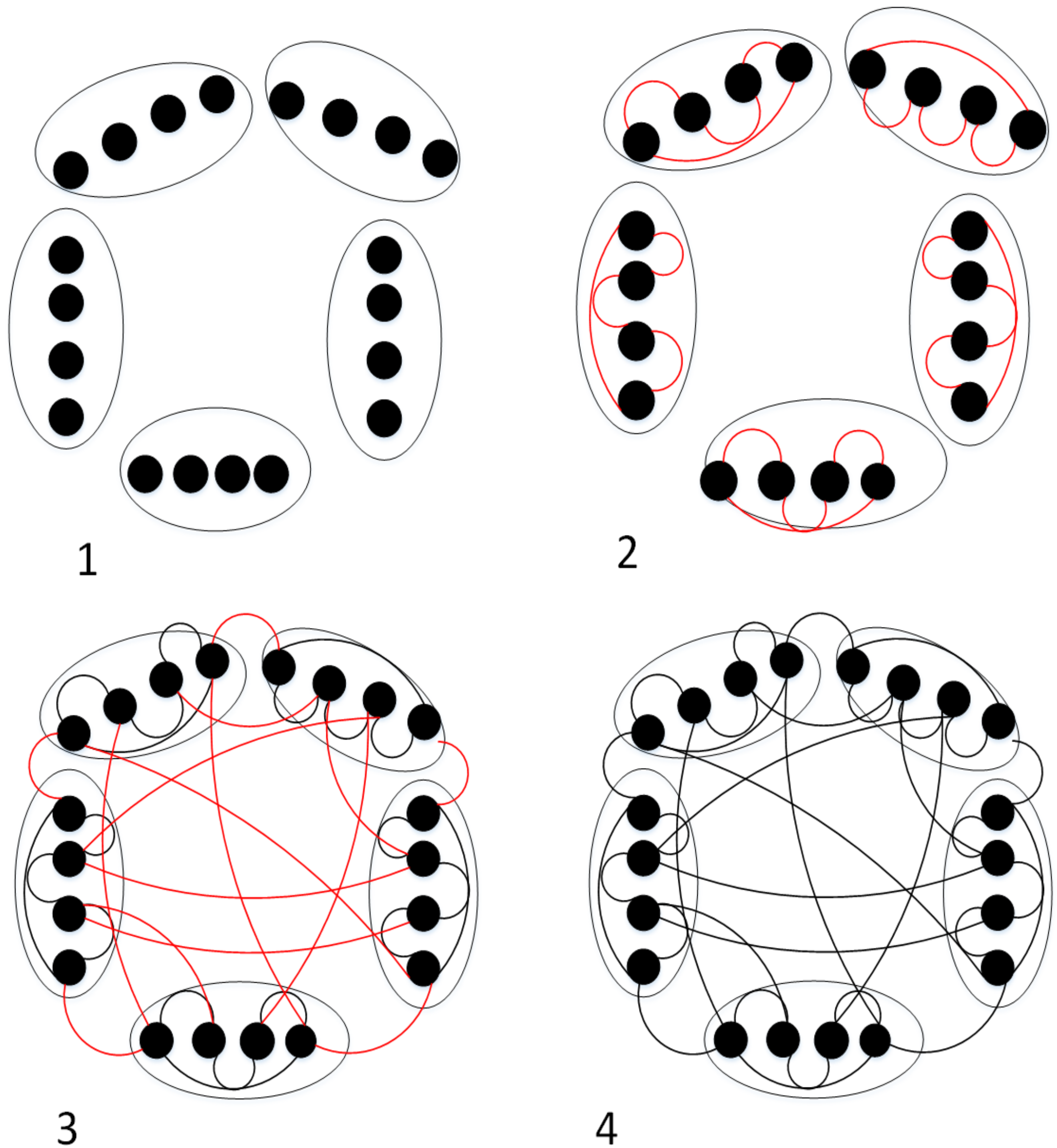
- Stochastic network topology
- Simple to add or delete nodes
- Restriction of vertex order is possible
- Fault tolerance and scalable network topology

DragonFly

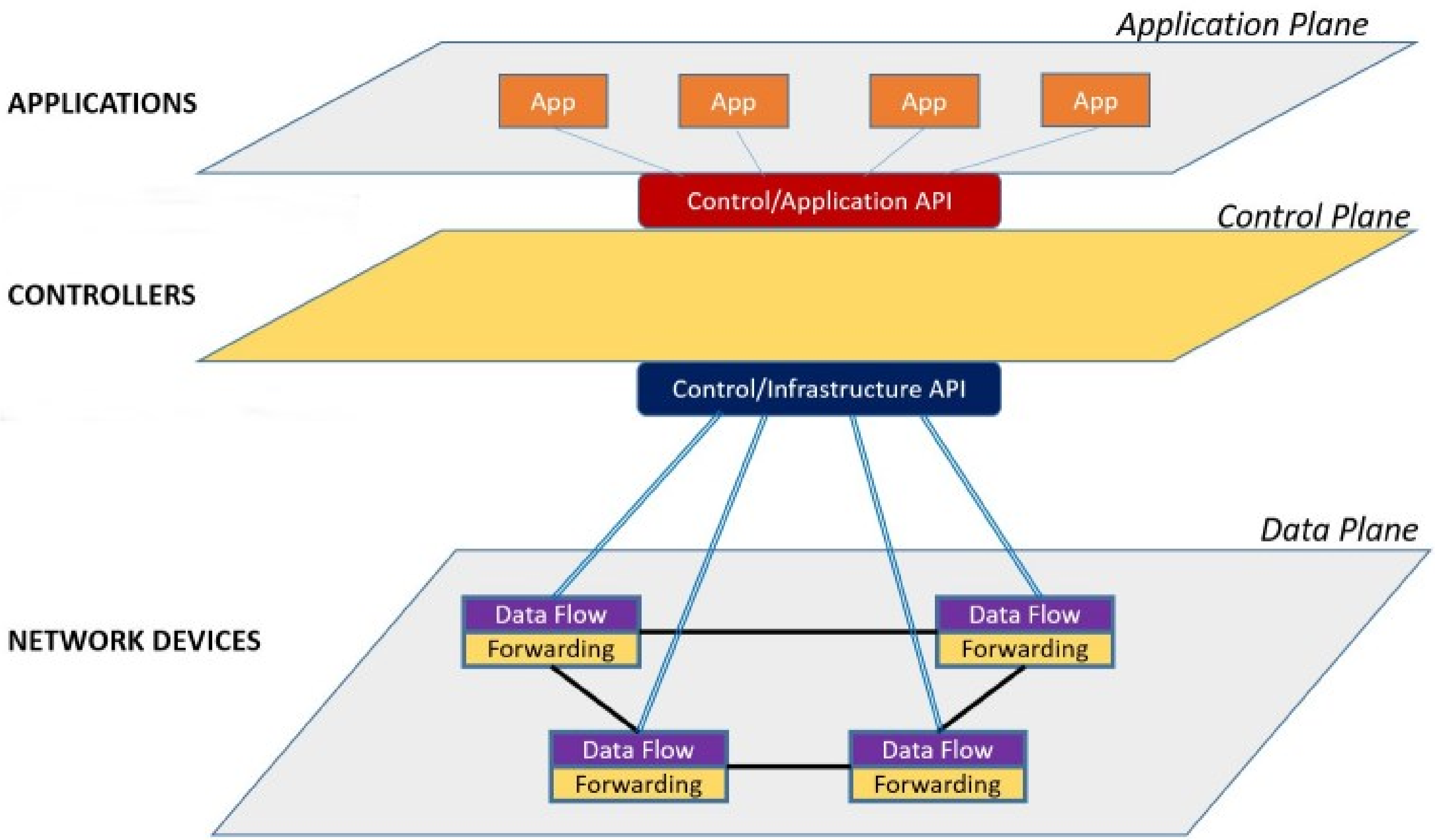


DragonFly

- Hierarchical network topology
- Restriction of vertex order is possible
- Fault tolerance and scalable network topology

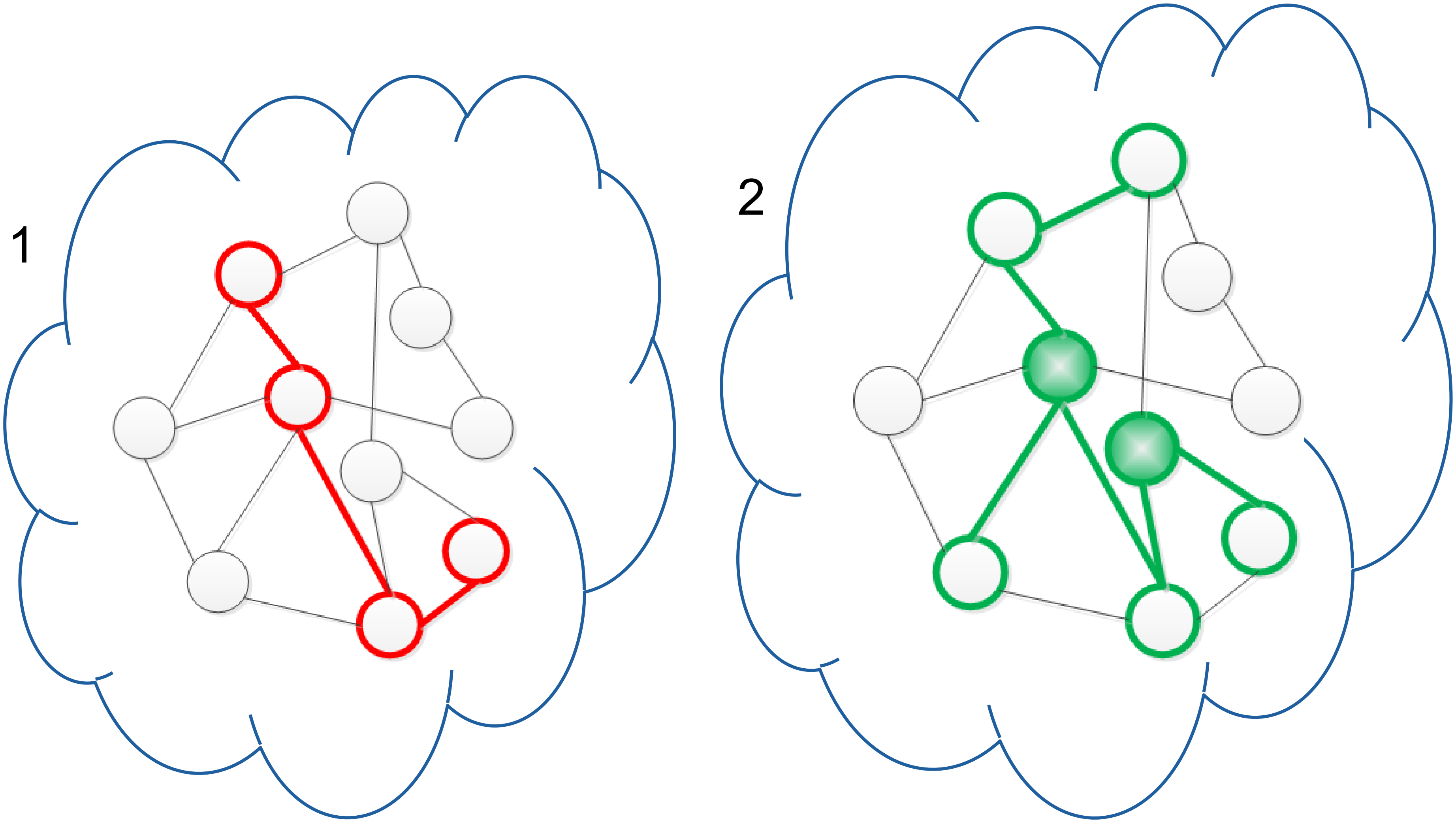


Software Defined Network (SDN)



A software-defined network is a data transmission network in which the network management(control) level is separated from data transmission devices and implemented programmatically.

Organization of a virtual network infrastructure



Network design

Research of the metric characteristics of the network graph under restrictions on its structure

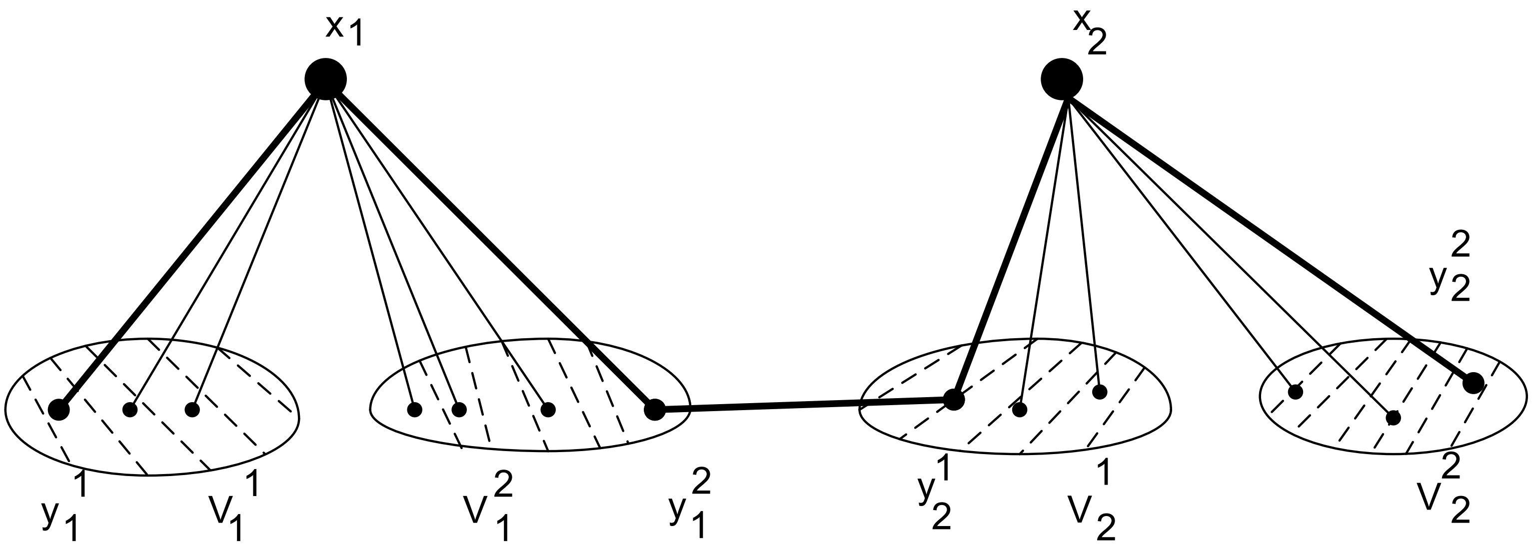
A method for constructing a network with a maximum diameter and estimating the number of elements in it at a fixed size of the dominant set is proposed. To do this, we construct a graph of a special form in which the "dominated" subsets of vertices satisfy certain structural conditions. Based on this construction, the problem of maximizing the length of the shortest chain (finding the diametric chain) is solved.

Let $D = \{x_i, i = \overline{1,s}\}$ be some dominant set of the desired graph *G*. In it, the vertices $D \subset V$ of are not adjacent $(x_i., x_j) \notin E, i\ , j = \overline{1,s}$. We define $V_i = N(x_i)$ in disjoint "dominated" subsets of vertices $V \setminus D$ - the neighborhood of the vertex x_i and the corresponding subgraphs $G_i = G_i(V_i, E_i)$, by the construction $V_i \cap V_j = \varnothing$ $E_i \cap E_j = \varnothing$ $i, j = \overline{1,s}$

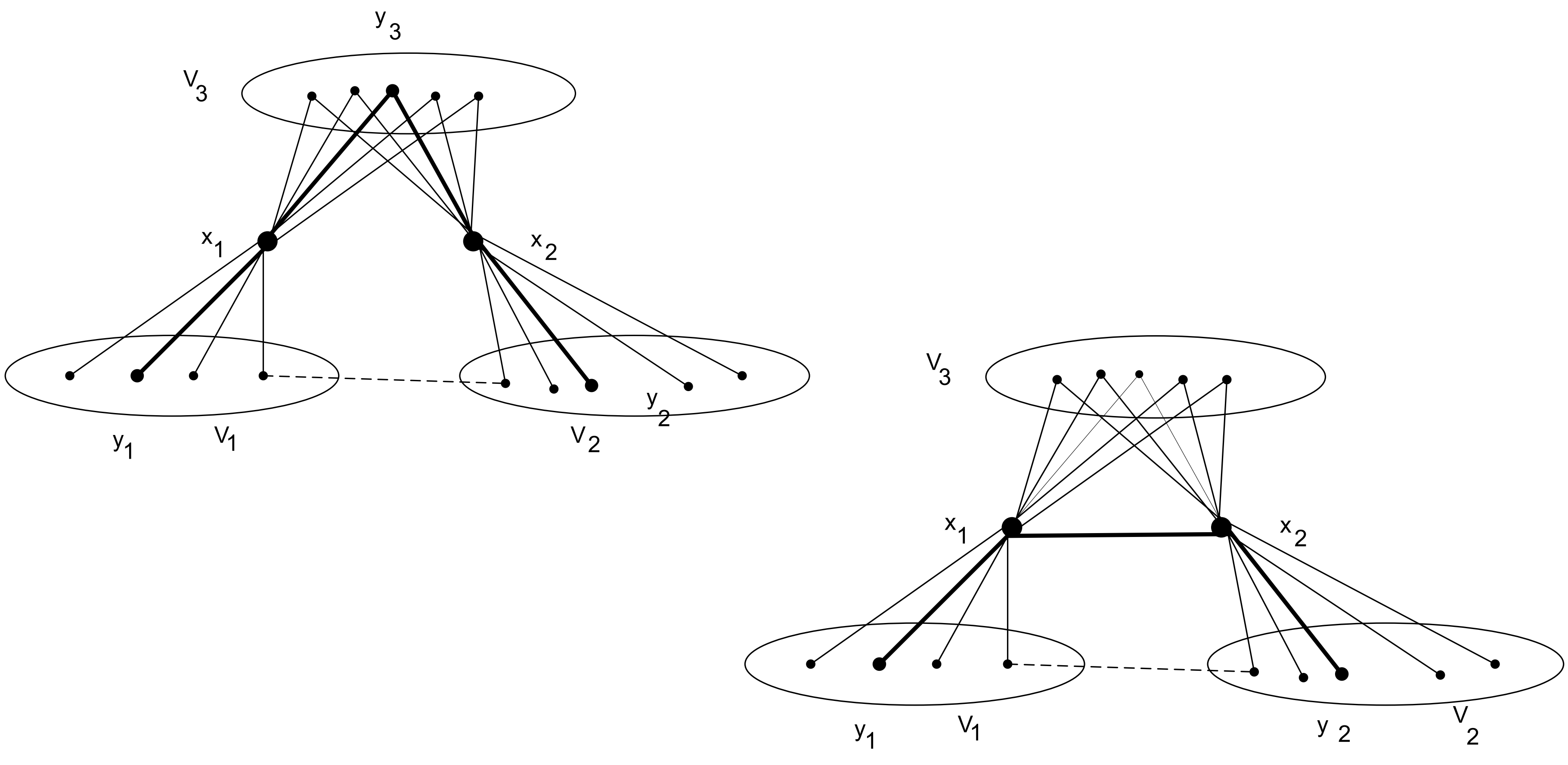
Theorem: If a graph *G=G(V,E)* with a dominant set $D=\{x_i,\ i=1..s\}$ satisfies the conditions, then the length of the maximum shortest chain (diametric) in it is equal to *3s-1*. It has the form: $\{y_1^1, x_1, y_1^2, y_2^1, x_2, y_2^2, \ldots, y_{s-1}^1, x_{s-1}, y_s^2, x_s, y_s^1\}$, where $y_i^1 \in V_i^1$ $y_i^2 \in V_i^2$

On the set of all graphs with such a dominant set *D*, the diameter is $\delta \leq 3s - 1$

A graph with a dominant set of 2 vertices



a diametrical chain $\{ y_1^1 ,\ x_1 ,\ y_1^2 ,\ y_2^1 ,\ x_2 ,\ y_2^2 \}$ of length 5 is marked


y3
V3
x1
x2
y1
V1
V2
y2
V3
x1
x2
y1
V1
y 2
V2

Conclusions

- The method for constructing a graph with a maximum diameter on a set of graphs with a given size of the dominant set is proposed;
- Using this method of constructing a telecommunications network graph allows you to avoid excessive centralization;
- It becomes possible to estimate the time of data transmission in the network;
- The obtained results are concretized for graphs with two dominant vertices;
- The proposed approach allows for further development for constructing graphs of telecommunications networks with a known dominant set and a certain diameter value.

Thank you for your attention

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