# Thermalization with non–zero anomalous quantum averages

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## I - Plan

Plan of presentation :

1) Introduction

2) Method of equation constructing

3) First order analysis

4) Kinetic equations and solution

# Introduction

Study of quantum field theory dynamics over strong field backgrounds demands consideration of correlation functions over quantum states with non–zero anomalous quantum expectation values. By this we mean such value :

$$\mathrm{Tr}[\hat{\rho} a_{\vec{q}} a_{\vec{q}'}] \equiv \langle a_q a_{q'} \rangle \tag{1}$$

In fact, even for zero initial values anomalous averages are generated dynamically in loop corrections over strong background fields. This happens e.g. in expanding universe, during collapse process, in the presence of strong electric fields. Furthermore, anomalous averages play the key role in BCS theory.

# Formulation of the problem

We consider four–dimensional real massive selfinteracting scalar field theory in flat space–time:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mathrm{m}^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4. \tag{2}$$

To have an analytic headway we consider spatially homogeneous states :

$$\left\langle \mathrm{a}_{\mathrm{q}}^{+} \mathrm{a}_{\mathrm{q}'} \right\rangle \sim \mathrm{n}_{\mathrm{q}}^{0}\, \delta(\vec{\mathrm{q}} - \vec{\mathrm{q}}') \tag{3}$$

$$\left\langle \mathrm{a}_{\mathrm{q}} \mathrm{a}_{\mathrm{q}'} \right\rangle \sim \chi_{\mathrm{q}}^{0}\, \delta(\vec{\mathrm{q}} + \vec{\mathrm{q}}') \tag{4}$$

# Mode decomposition

We work with the standard mode decomposition:

$$\varphi(\mathrm{x},\mathrm{t}) = \int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3}\left[\mathrm{a_p}\frac{\mathrm{e}^{\mathrm{i}(\vec{\mathrm{p}}\vec{\mathrm{x}}-\epsilon_\mathrm{p}\mathrm{t})}}{\sqrt{2\epsilon_\mathrm{p}}} + \mathrm{a}_\mathrm{p}^{+}\frac{\mathrm{e}^{-\mathrm{i}(\vec{\mathrm{p}}\vec{\mathrm{x}}-\epsilon_\mathrm{p}\mathrm{t})}}{\sqrt{2\epsilon_\mathrm{p}}}\right], \quad \epsilon_\mathrm{p} = \sqrt{\vec{\mathrm{p}}^{\,2}+\mathrm{m}^2}, \tag{5}$$

and consider dynamics of such initial state:

$$\left\langle \mathrm{a}_\mathrm{q}^{+}\mathrm{a}_{\mathrm{q}'}\right\rangle = \mathrm{n}_\mathrm{q}^0\,\delta\left(\vec{\mathrm{q}}-\vec{\mathrm{q}}'\right) \quad \text{and} \quad \left\langle \mathrm{a}_\mathrm{q}\mathrm{a}_{\mathrm{q}'}\right\rangle = \chi_\mathrm{q}^0\,\delta\left(\vec{\mathrm{q}}+\vec{\mathrm{q}}'\right). \tag{6}$$

# In - state

Example of such initial state, which satisfies Wick's theorem, is thermal state for " wrong" mode decomposition :

$$\varphi(\mathrm{x},\mathrm{t}) = \int \frac{d^3p}{(2\pi)^3}\left[\mathrm{b_p}\,\frac{\mathrm{f_p}(\mathrm{t},\mathrm{x})}{\sqrt{2\epsilon_\mathrm{p}}} + \mathrm{b_p^+}\,\frac{\mathrm{f_p^*}(\mathrm{t},\mathrm{x})}{\sqrt{2\epsilon_\mathrm{p}}}\right], \tag{7}$$

where:

$$\mathrm{f_p}(\mathrm{t},\mathrm{x}) = \mathrm{u_p}e^{\mathrm{i}(\vec{\mathrm{p}}\vec{\mathrm{x}}-\epsilon_\mathrm{p}\mathrm{t})} + \mathrm{v_p}e^{-\mathrm{i}(\vec{\mathrm{p}}\vec{\mathrm{x}}-\epsilon_\mathrm{p}\mathrm{t})}. \tag{8}$$

# Evolution

We describe evolution with the Heisenberg's equations:

$$\frac{\mathrm{d}}{\mathrm{dt}}\left\langle \mathrm{a}_{\mathrm{q}}^{+}\mathrm{a}_{\mathrm{q}'}\right\rangle = \mathrm{i}\left\langle \left[\mathrm{H}_{\mathrm{int}}(\mathrm{t}),\, \mathrm{a}_{\mathrm{q}}^{+}\mathrm{a}_{\mathrm{q}'}\right] \right\rangle, \quad \frac{\mathrm{d}}{\mathrm{dt}}\langle \mathrm{a}_{\mathrm{q}}\mathrm{a}_{\mathrm{q}'}\rangle = \mathrm{i}\left\langle \left[\mathrm{H}_{\mathrm{int}}(\mathrm{t}),\, \mathrm{a}_{\mathrm{q}}\mathrm{a}_{\mathrm{q}'}\right] \right\rangle. \quad (9)$$

# First order

Equations in the first order of coupling constant :

$$\frac{dn_q}{dt} = \frac{i\lambda}{2} \int \frac{d^3p}{4(2\pi)^3\epsilon_q\epsilon_p} \Bigg[ \Big( \chi_q\chi_p e^{-2i(\epsilon_q+\epsilon_p)t} - \chi_q^*\chi_p^* e^{2i(\epsilon_q+\epsilon_p)t} \Big) +$$

$$(1+2n_p)\Big( \chi_q e^{-2i\epsilon_q t} - \chi_q^* e^{2i\epsilon_q t} \Big) + \Big( \chi_p^*\chi_q e^{2i(\epsilon_q-\epsilon_q)t} - \chi_p\chi_q^* e^{2i(\epsilon_q-\epsilon_p)t} \Big) \Bigg], \tag{10}$$

$$\frac{d\chi_q}{dt} = \frac{-i\lambda}{2} \int \frac{d^3p}{4(2\pi)^3\epsilon_q\epsilon_p} \Bigg[ 2\chi_q(1+2n_p) + (1+2n_q)\chi_p^* e^{2i(\epsilon_q+\epsilon_p)t} + (1+2n_q)\chi_p e^{2i(\epsilon_q-\epsilon_p)t}$$

$$+(1+2n_q)(1+2n_p)e^{2i\epsilon_q t} + 2\chi_q\chi_p^* e^{2i\epsilon_p t} + 2\chi_q\chi_p e^{-2i\epsilon_p t} \Bigg]. \tag{11}$$

# Mass renormalization

Considering n and $\chi$ as slow functions, we neglect oscillating terms and obtain:

$$\epsilon_{\mathrm{q}} \frac{\mathrm{d}\chi_{\mathrm{q}}}{\mathrm{dt}} = \chi_{\mathrm{q}} \frac{-\mathrm{i}\lambda}{2} \int \frac{\mathrm{d}^3\mathrm{p}}{4(2\pi)^3\epsilon_{\mathrm{p}}} \left[2\left(1 + 2\,\mathrm{n}_{\mathrm{p}}\right)\right]. \tag{12}$$

Which is nothing but mass renormalization.
Therefore there is not qualitative dynamics.

# Constructing of kinetic equations

The next step is to obtain kinetic equations. Now we write Heisenberg's equations twice, once again for the RHS of (9). One of the terms for example:

$$\frac{\mathrm{d}}{\mathrm{d}t'}\left\langle a^{+}_{q} a^{+}_{p_2} a^{+}_{p_3} a_{p_4} \right\rangle = i \left\langle \left[ H_{int}(t'),\, a^{+}_{q} a^{+}_{p_2} a^{+}_{p_3} a_{p_4} \right] \right\rangle, \tag{13}$$

# Kinetic equations

Finally we obtain:

$$\epsilon_{\mathrm{q}} \frac{\mathrm{d}}{\mathrm{dt}} \mathrm{n_q} = \frac{\lambda^2}{16} \int \frac{\mathrm{d^3 p_1 d^3 p_2 d^3 p_3}}{(2\pi)^9 \, \epsilon_1 \epsilon_2 \epsilon_3} \, \delta^4(\underline{\mathrm{q}} + \underline{\mathrm{p}}_1 - \underline{\mathrm{p}}_2 - \underline{\mathrm{p}}_3) \times$$
$$\times \Big[ (1+\mathrm{n_q})(1+\mathrm{n_1})\mathrm{n_2 n_3} - \mathrm{n_q n_1}(1+\mathrm{n_2})(1+\mathrm{n_3}) \Big], \tag{14}$$

$$\epsilon_{\mathrm{q}} \frac{\mathrm{d}}{\mathrm{dt}} \chi_{\mathrm{q}} = \frac{\lambda^2}{16} \int \frac{\mathrm{d^3 p_1 d^3 p_2 d^3 p_3}}{(2\pi)^9 \, \epsilon_1 \epsilon_2 \epsilon_3} \, \delta^4(\underline{\mathrm{q}} + \underline{\mathrm{p}}_1 - \underline{\mathrm{p}}_2 - \underline{\mathrm{p}}_3) \times$$
$$\times \Big\{ \chi_{\mathrm{q}} \Big[ (1+\mathrm{n_1})\mathrm{n_2 n_3} - \mathrm{n_1}(1+\mathrm{n_2})(1+\mathrm{n_3}) \Big] + 2\,\chi_1^* \chi_2 \chi_3 \Big[ \mathrm{n_q} - (1+\mathrm{n_q}) \Big] \Big\}. \tag{15}$$

# Near the equilibrium

Near the equilibrium and for the small $\chi$ , linearize the last equation:

$$\epsilon_{\mathrm{q}} \frac{\mathrm{d}}{\mathrm{dt}} \chi_{\mathrm{q}} \approx \chi_{\mathrm{q}} \frac{\lambda^2}{16} \int \frac{\mathrm{d}^3\mathrm{p}_1 \mathrm{d}^3\mathrm{p}_2 \mathrm{d}^3\mathrm{p}_3}{(2\pi)^9\, \epsilon_1 \epsilon_2 \epsilon_3}\, \delta^4(\underline{\mathrm{q}} + \underline{\mathrm{p}}_1 - \underline{\mathrm{p}}_2 - \underline{\mathrm{p}}_3) \Big[(1+\mathrm{n}_1)\mathrm{n}_2\mathrm{n}_3 - \mathrm{n}_1(1+\mathrm{n}_2)(1+\mathrm{n}_3)\Big]. \qquad (16)$$

# Solution and Conclusions

Finally we obtain :

$$\chi_{\mathrm{q}}(\mathrm{t}) = \mathrm{C}_{\mathrm{q}} \mathrm{e}^{-\Gamma_{\mathrm{q}} \mathrm{t}}, \tag{17}$$

where $\Gamma_{\mathrm{q}} > 0$:

$$\epsilon_{\mathrm{q}} \Gamma_{\mathrm{q}} = \frac{\lambda^2}{16} \int \frac{\mathrm{d}^3 \mathrm{p}_1 \mathrm{d}^3 \mathrm{p}_2 \mathrm{d}^3 \mathrm{p}_3}{(2\pi)^9 \, \epsilon_1 \epsilon_2 \epsilon_3} \, \delta^4(\underline{\mathrm{q}} + \underline{\mathrm{p}}_1 - \underline{\mathrm{p}}_2 - \underline{\mathrm{p}}_3) \Big[ \mathrm{n}_1 (1 + \mathrm{n}_2)(1 + \mathrm{n}_3) - (1 + \mathrm{n}_1) \mathrm{n}_2 \mathrm{n}_3 \Big]. \tag{18}$$

As we can see $\chi$ relaxes down to zero, so thermalization has a place to be even for states with non–zero (but small) anomalous averages.

Thank you for your attention!