

# Quantum-quasiclassical analysis of CM nonseparability in atom due to relativistic effects stimulated by laser field

Vladimir S. Melezhik

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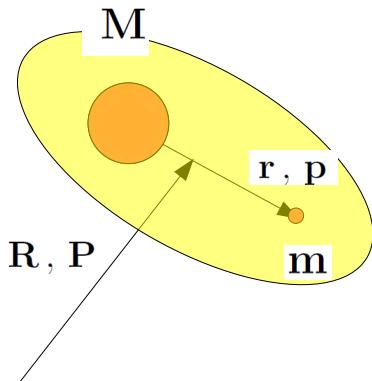


supported by Grant of Russian Science Foundation No. 20-11-20257

MQFTP-22, St-Petersburg, 10-14 October 2022



# quantum-quasiclassical approach - idea



$$M \gg m$$

$$P = MV \gg p = mv$$

$$i\hbar \frac{\partial}{\partial t} |\psi(\mathbf{r}, t)\rangle = [H_0(\mathbf{r}) + V(\mathbf{r}, \mathbf{R}(t))]|\psi(\mathbf{r}, t)\rangle$$

$$H_{cl}(\mathbf{P}, \mathbf{R}, t) = \frac{\mathbf{P}^2}{2M} + \langle \psi(\mathbf{r}, t) | V(\mathbf{r}, \mathbf{R}(t)) | \psi(\mathbf{r}, t) \rangle$$

$$\frac{d}{dt} \mathbf{P} = -\frac{\partial}{\partial \mathbf{R}} H_{cl}(\mathbf{P}, \mathbf{R}, t)$$

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PHYSICAL REVIEW LETTERS

28 FEBRUARY 2000

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## Quantum Energy Flow in Atomic Ions Moving in Magnetic Fields

V. S. Melezhik<sup>1,\*</sup> and P. Schmelcher<sup>2</sup>

K. J. McCann and M. R. Flannery, Chem. Phys. Lett. **35**, 124 (1975); J. Chem. Phys. **63**, 4695 (1975).  
G. D. Billing, Chem. Phys. **9**, 359 (1975).

# quantum-quasiclassical approach - results

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Recent Progress in Treatment of Sticking and Stripping with Time-Dependent Approach

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PHYSICAL REVIEW A **103**, 053109 (2021)

Improving efficiency of sympathetic cooling in atom-ion and atom-atom confined collisions

Vladimir S. Melezhik<sup>1</sup> \*

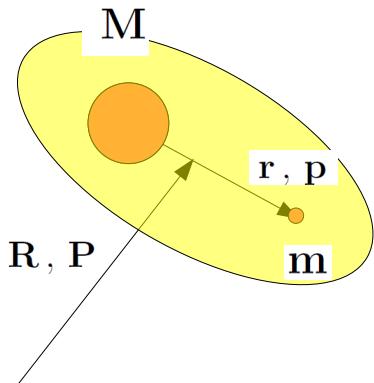
Eur. Phys. J. A (2022) 58:34  
<https://doi.org/10.1140/epja/s10050-022-00684-z>

THE EUROPEAN  
PHYSICAL JOURNAL A

Investigation of low-lying resonances in breakup of halo nuclei  
within the time-dependent approach

Dinara Valiolda<sup>1,2,3</sup>, Daniyar Janseitov<sup>1,2,3,a</sup> , Vladimir Melezhik<sup>3,4,b</sup>

# hydrogen atom in strong laser field

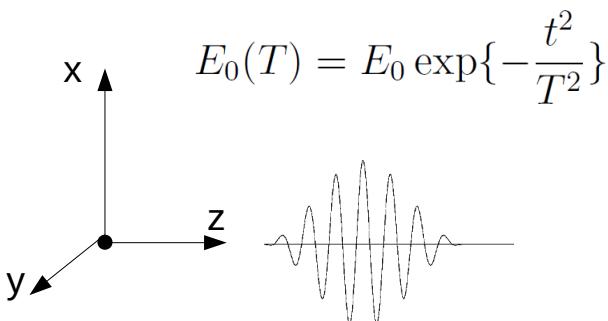


$$i\hbar \frac{\partial}{\partial t} |\psi(\mathbf{r}, \mathbf{R}, t)\rangle = [H_0(\mathbf{r}) + V(\mathbf{r}, \mathbf{R}(t))]|\psi(\mathbf{r}, \mathbf{R}, t)\rangle$$

$$H_0(\mathbf{r}) = \frac{p^2}{2\mu} - \frac{1}{r} - E_0(T) \cos(\omega t)x + \alpha[\dots]$$

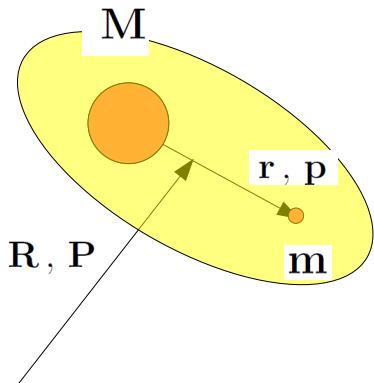
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$$P = MV \gg p = mv$$



$$\alpha = \frac{1}{c} = \frac{1}{137}$$

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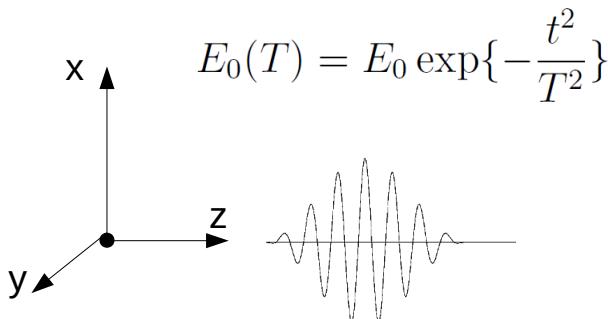


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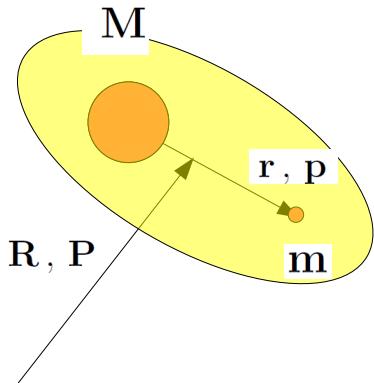
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$$V(\mathbf{r}, \mathbf{R}) = -\alpha\omega E_0(T) \sin(\omega t)[xZ(t) + zX(t)] - \alpha E_0(T) \cos(\omega t)X(t)p_z$$

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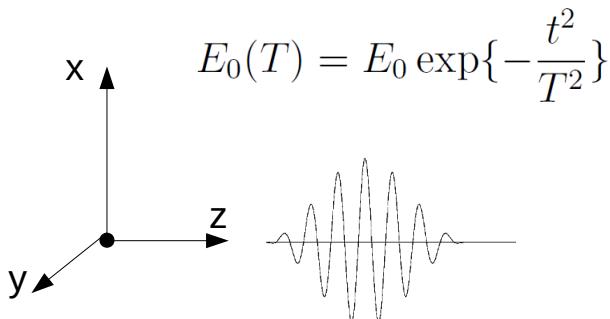


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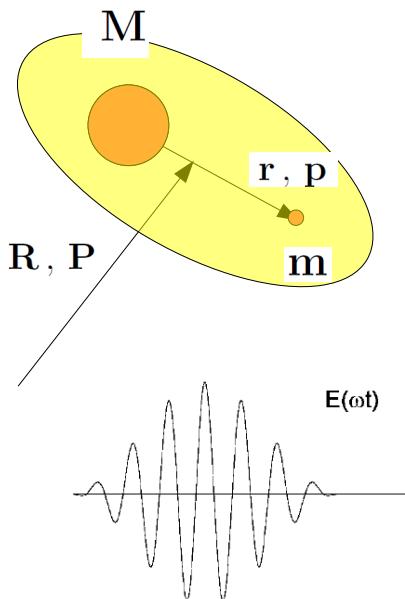


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how integrate 6D TDSE ? !!!

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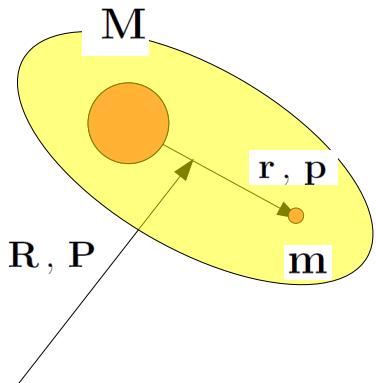
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A. Bray, U. Eichmann, S. Patchkovskii, PRL 124 (2020)

With additional artificial trapping potential the problem was reduced to effective 3D

They proposed to use CM-velocity spectroscopy as a «build-in» classical monitoring devise for observing internal quantum dynamics in strong external laser fields

# hydrogen atom in strong laser field



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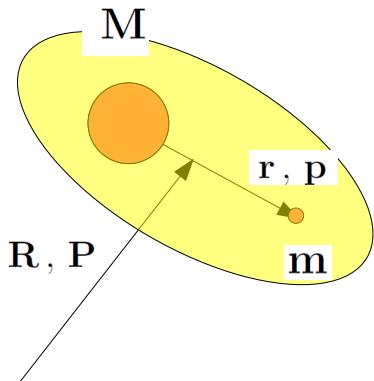
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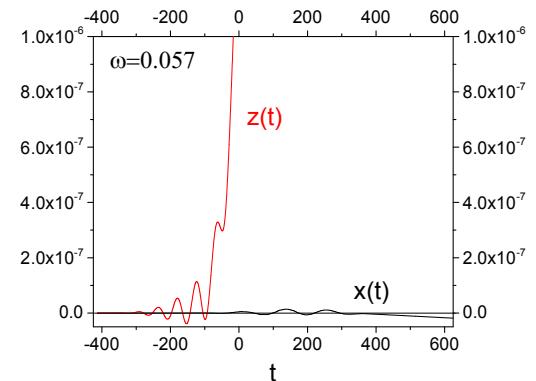
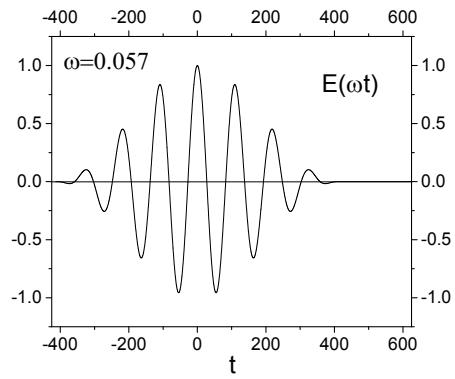
$$\frac{d}{dt} \mathbf{R} = \frac{\partial}{\partial \mathbf{P}} H_{cl}(\mathbf{P}, \mathbf{R}, t)$$

$$\langle \psi(\mathbf{r}, t) | \mathbf{r}(t) | \psi(\mathbf{r}, t) \rangle \quad \langle \psi(\mathbf{r}, t) | \mathbf{p}(t) | \psi(\mathbf{r}, t) \rangle \quad \mathbf{R}(t) \quad \mathbf{P}(t)$$

$$\langle |E_{kin}(t)| \rangle = \frac{1}{2T} \int_{-T}^T \frac{P^2(t)}{2M} dt \sim \int |P(\omega)|^2 dt \sim \int \{|P_x(\omega)|^2 + |P_y(\omega)|^2 + |P_z(\omega)|^2\} dt$$

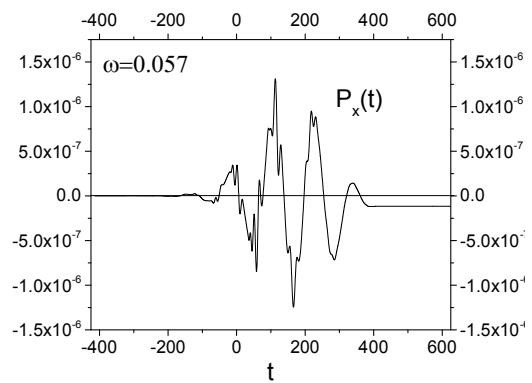
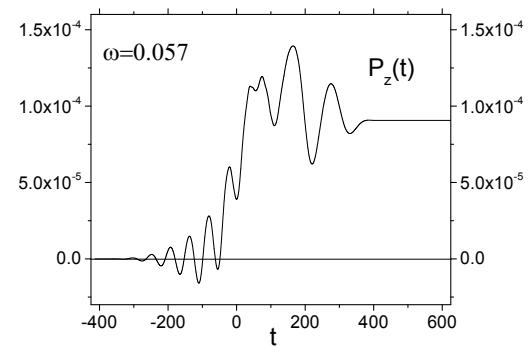
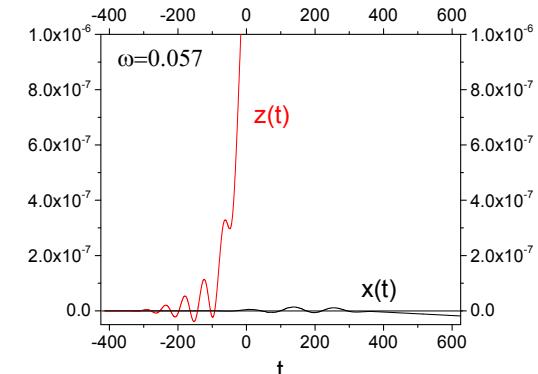
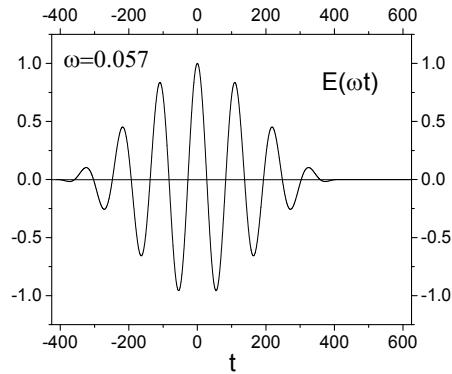
$$I = 10^{14} \frac{W}{cm^2} \quad \lambda = 800nm \quad T = 5.3fs$$

$$\mathbf{R}(-T) = \mathbf{P}(-T) = 0$$



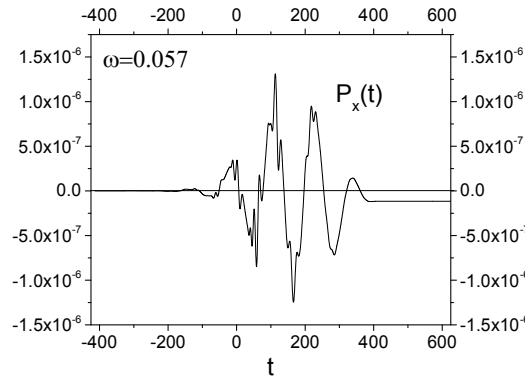
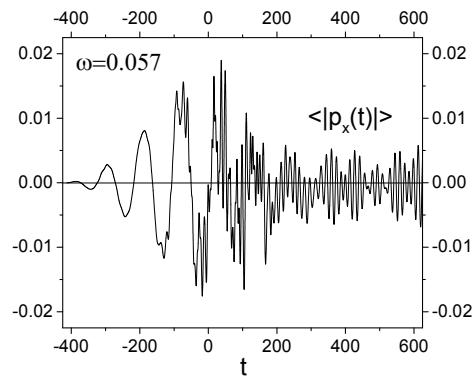
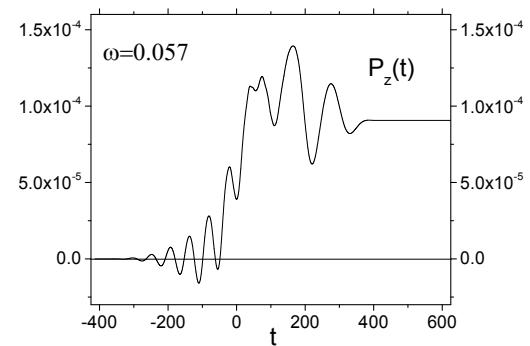
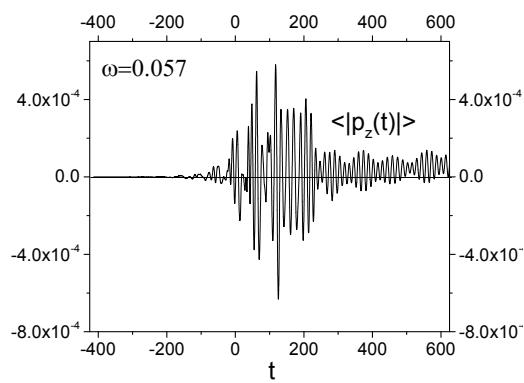
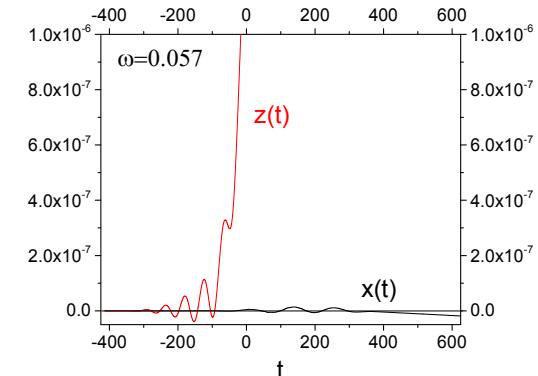
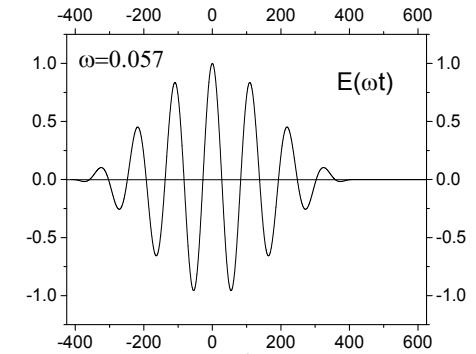
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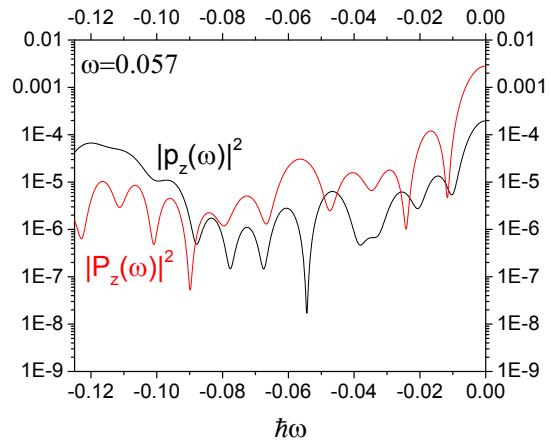
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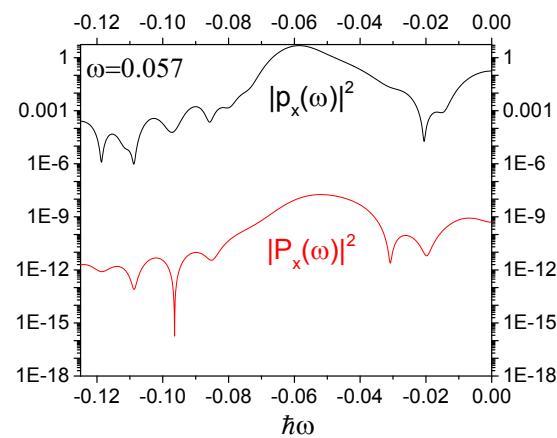
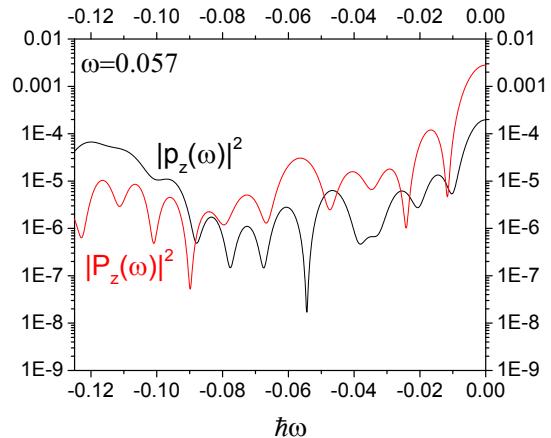
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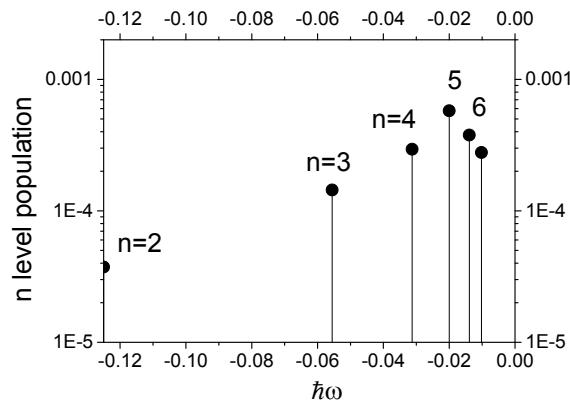
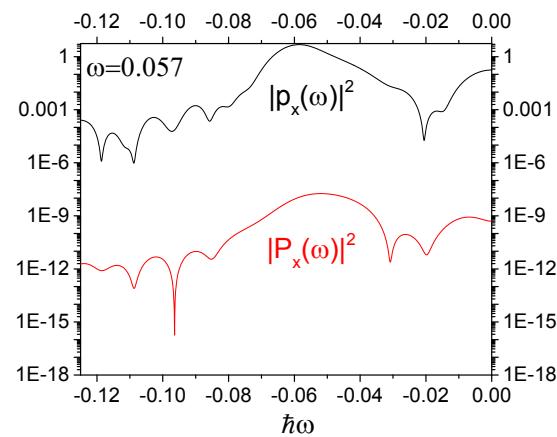
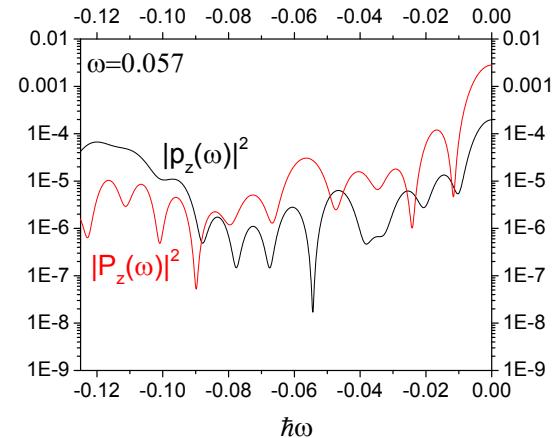
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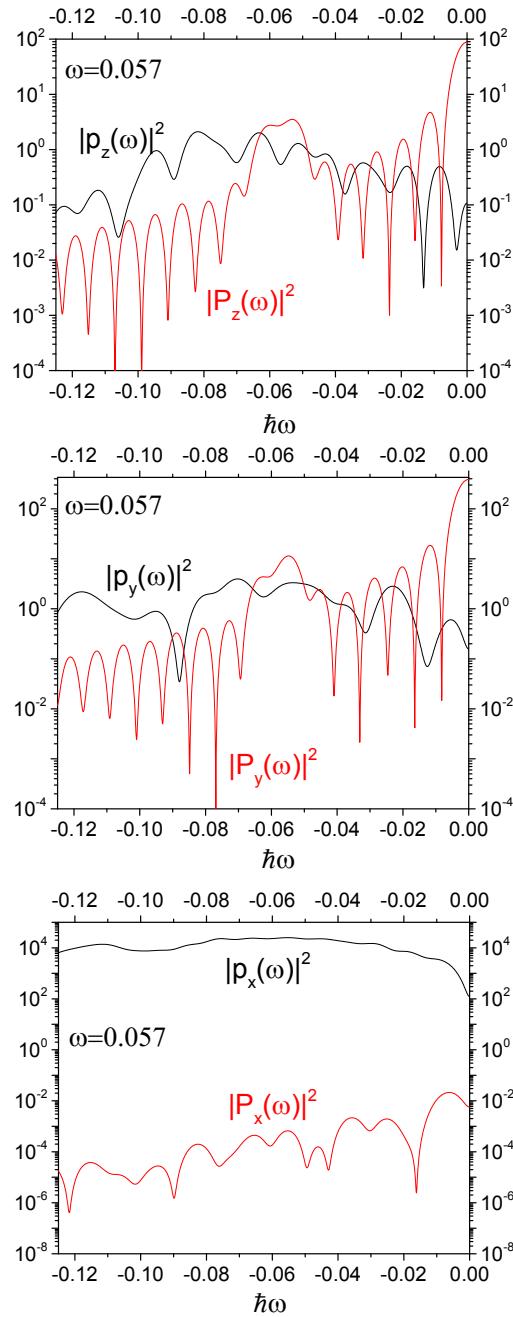
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$$X(-T) = Z(-T) = 0 , \quad Y(-T) = \frac{w_0}{2}$$

$$\mathbf{P}(-T) = 0$$

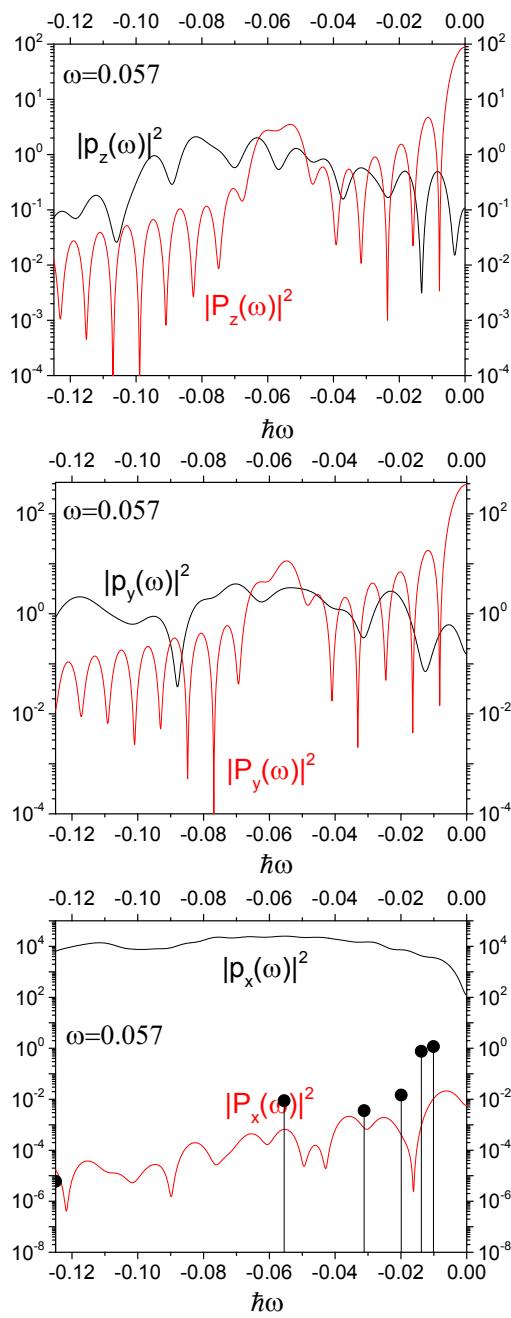


$$E_0(T) = E_0 \exp\left\{-\frac{t^2}{T^2}\right\} \exp\left\{-\frac{\rho^2}{w_0^2}\right\}$$

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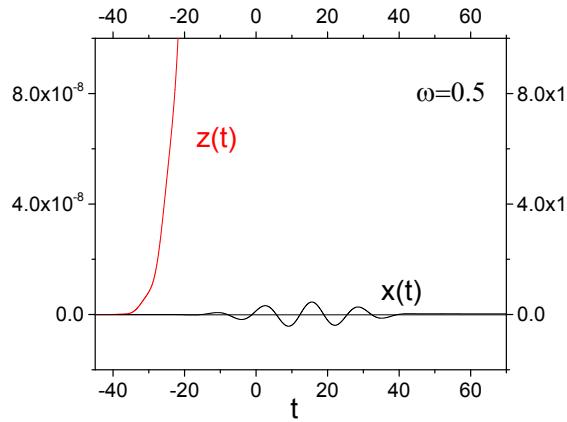
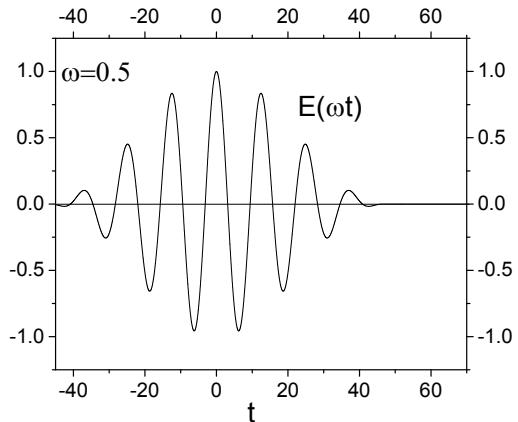
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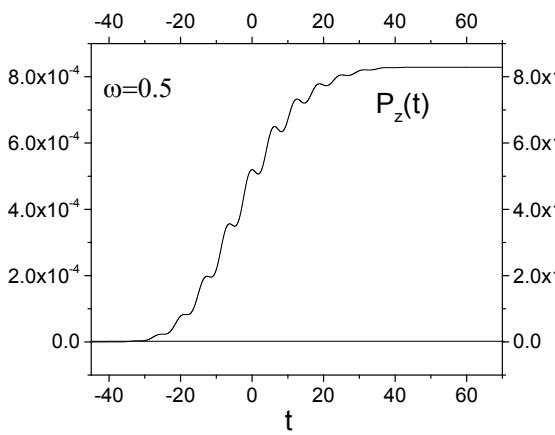
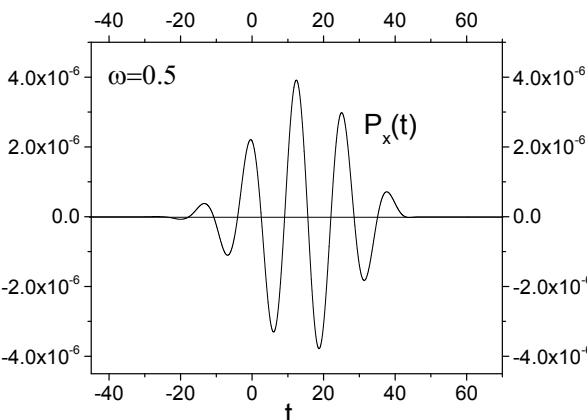
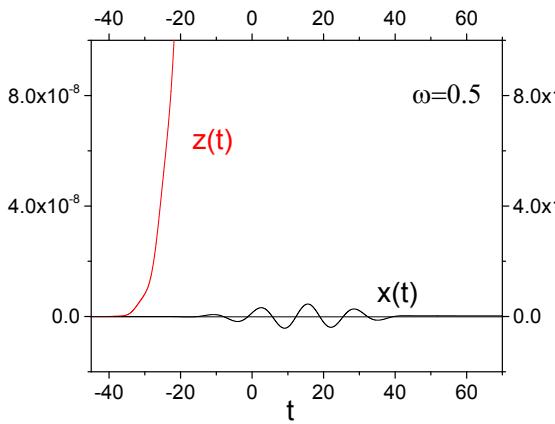
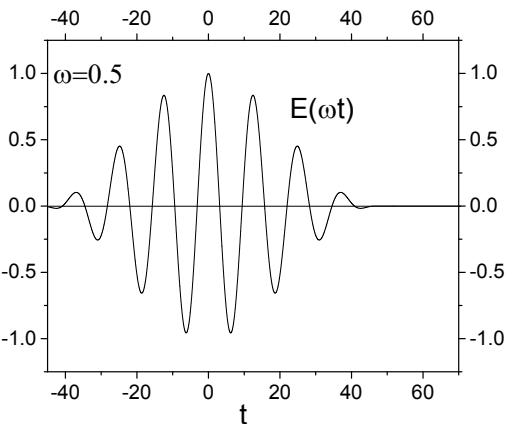
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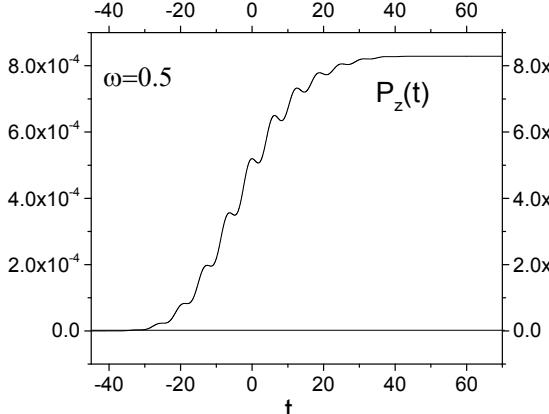
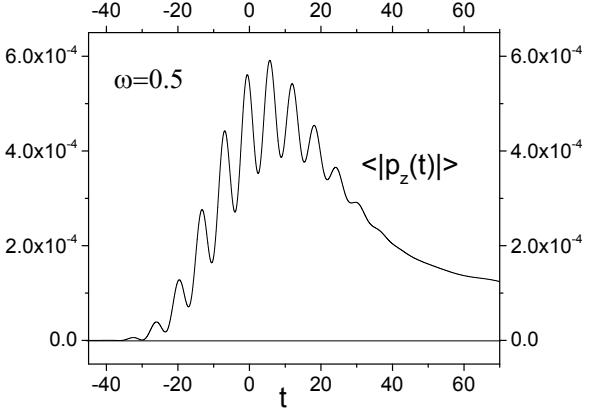
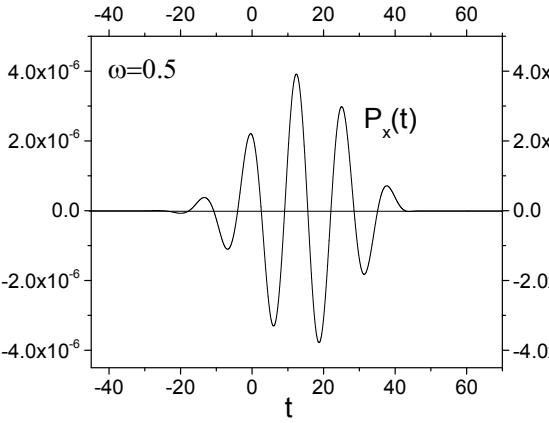
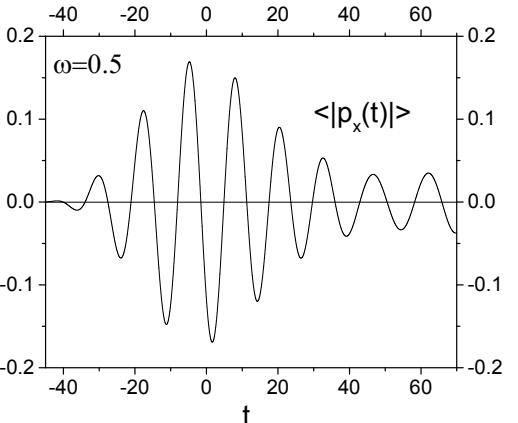
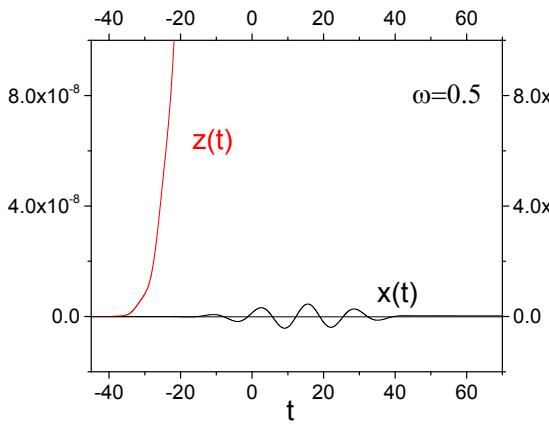
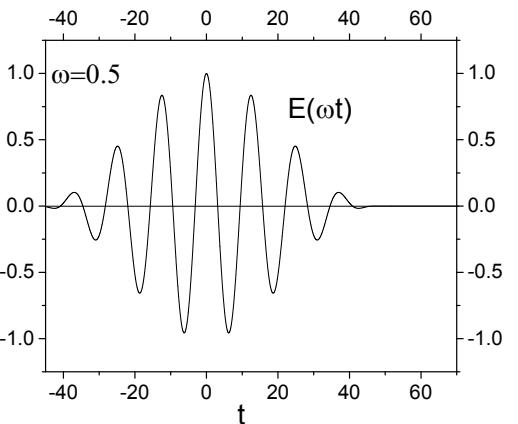
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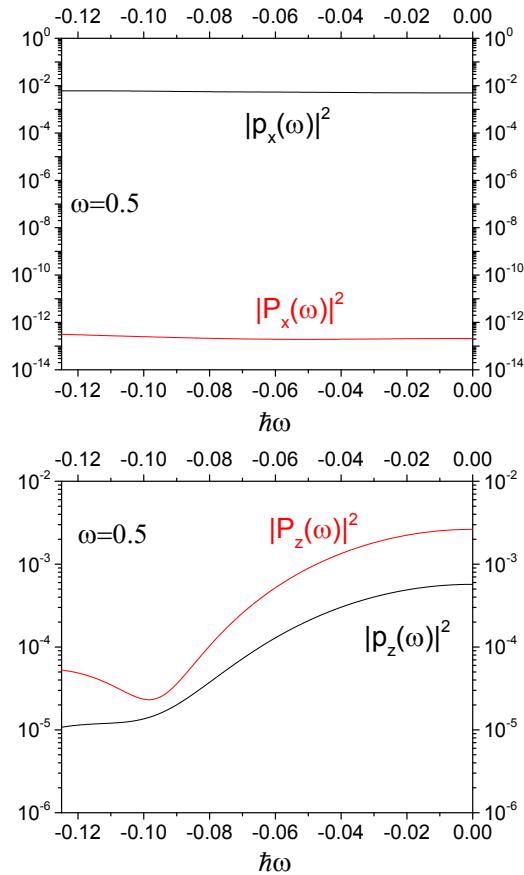
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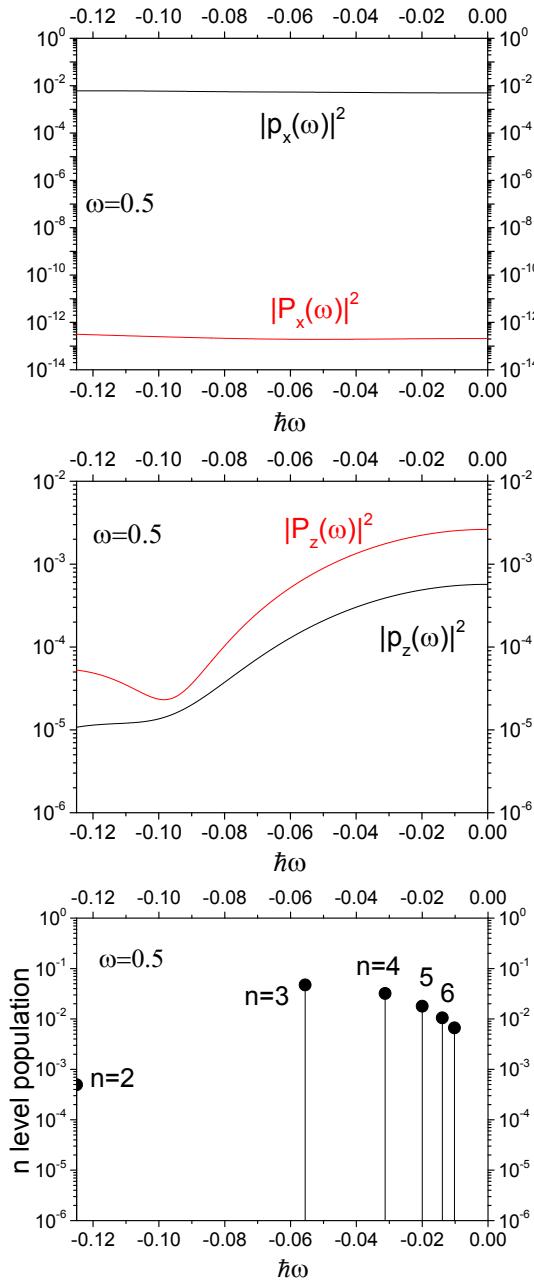
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# Conclusion & Outlook

- *it was confirmed with quantum-quasicalassical approach  
the correlation between internal and CM dynamics  
in hydrogen atom in strong laser fields*
- *by using CM-velocity spectroscopy as the «build-up» classical set up  
we obtain information about internal quantum dynamics of atoms  
in intense laser fields*