Probing holographic model of a rotating $\mathcal{N} = 4$ SYM quark-gluon plasma

based on work with

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also on JHEP 04 (2021) 169 with I.Ya. Aref'eva and E.Gourgoulhon;

Nucl.Phys.B 979 (2022) 115786 with E.Gourgoulhon and M. Usova

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The AdS/CFT conjecture

The strongest version of the conjecture

$4d$ $\mathcal{N}=4$ SYM with $SU(N)$ is dynamically equivalent to type IIB superstring theory(contains strings and D-branes) on $AdS_5 \times S^5$ with a string length $\ell_s = \sqrt{\alpha'}$ and coupling constant g_s with the radius L and N units of $F_{(5)}$ flux on S^5.

$$g_{YM}^2 = 2\pi g_s, \quad 2g_{YM}^2 N = \frac{L^4}{\alpha'^2}, \quad \lambda = g_{YM}^2 N.$$

Forms of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence

	$\mathcal{N}=4$ SYM	IIB theory on $AdS_5 \times S^5$
Strongest form	any N and λ	Quantum string theory, $g_s \neq 0$, $\alpha'/L^2 \neq 0$
Strong form	$N \to \infty$,λ fixed but arbitrary	Classical string theory, $g_s \to 0$,$\alpha'/L^2 \neq 0$
Weak form	$N \to \infty$,λ large	Classical supergravity, $g_s \to 0$,$\alpha'/L^2 \to 0$

Classical (super)strings in asymptotically AdS_5 can predict results for strongly coupled 4d $\mathcal{N}=4$ SYM with $SU(N)$, $N \to \infty$.

Comments on asymptotically AdS backgrounds

- "the starting object" (Maldacena'97): 10d D_3 brane with $N = \frac{1}{2}$ SUSY (g_{ij} and F_5) reduces to $AdS_5 \times S^5$
- the isometry group $SO(2,4)$ of AdS_5 is a symmetry group of the dual CFT
- field theory "lives" on the boundary of the gravity background
- flat boundary $\Leftrightarrow$ CFT on R^4; spherical boundary $\Leftrightarrow$ CFT on cylinder $R \times \mathbb{S}^3$

  Example: Poincare patch of AdS_5

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}, \quad z \to 0 : ds^2_{bnd} = -dt^2 + d\vec{x}^2$$

  Example: global AdS_5

$$ds^2 = -(1 + y^2\ell^2)dT^2 + y^2(d\Theta^2 + \sin^2\Theta d\Phi^2 + \cos^2\Theta d\Psi^2) + \frac{dy^2}{1 + y^2\ell^2}.$$

  Boundary: $y \to \infty$, $R \times \mathbb{S}^3$: $ds^2 = -\ell^2 dT^2 + d\Theta^2 + \sin^2\Theta d\Phi^2 + \cos^2\Theta d\Psi^2$.

Holography at finite temperature

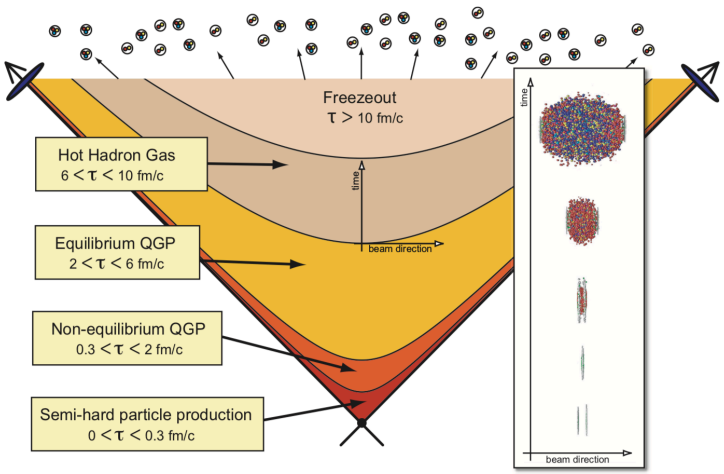
- Pure $AdS_5 \Leftrightarrow T = 0$ 4d $\mathcal{N} = 4$ SYM at strong coupling with $SU(N)$ (Maldacena'97)
- AdS_5 BH $\Leftrightarrow$ thermal ensemble of $\mathcal{N} = 4$ SYM $SU(N)$ at strong coupling (Witten'98)
  - *Note: for SUSY black holes $T_H = 0$, so ordinary AdS BH are used*
  - T of the thermal ensemble of CFT is identified with the Hawking temperature T_H of black hole
  - The Hawking-Page phase transition in the BH = The first order phase transition in the dual theory

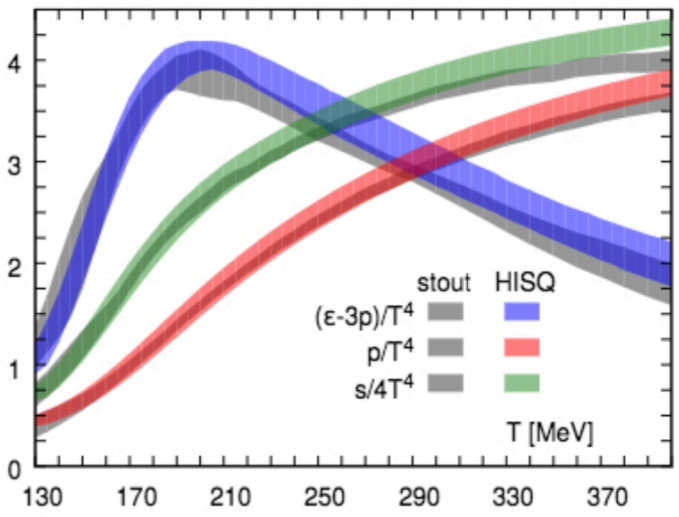
Sundborg'00: free $\mathcal{N} = 4$ SYM on $R \times \mathbb{S}^3$ at $T \neq 0$ has a phase transition at the Hagedorn temperature
Harmark et al.'18'20 the Hagedorn temperature at any value of the 't Hooft coupling

Outline

Quark Gluon Plasma

- QGP is a deconfined phase of QCD at high T (at strong coupling)
- The critical temperature T in the non-rotating case ($\mu = 0$) is $T_c \approx 170$ MeV.
- QCD at high T has a quasi-conformal behaviour ($T^\mu_\mu = 0$) (lattice)





QGP and holography

- The viscosity-to-entropy ratio for QGP from holography $\frac{\eta}{s} = \frac{1}{4\pi}$
  Policastro, Son, Starinets, Phys.Rev.Lett,2001
  checked on RHIC I. Arsene et al. [BRAHMS Collaboration],Nucl. Phys. A 757,1, 2005.
- Light "baryon" diffusion $D = 1/2\pi T$ hep-th/0205052
- 2'nd order transport coefficients arXiv:0712.2451, arXiv:0712.2456
- Aref'va, Phys. Usp. 57, 527 (2014).

Rotating quark-gluon plasma

- It is produced in non-central heavy-ion collisions.
- There is a nonzero total angular momentum related to colliding nuclei.
- The angular momentum remains in the QGP (conserved in time).

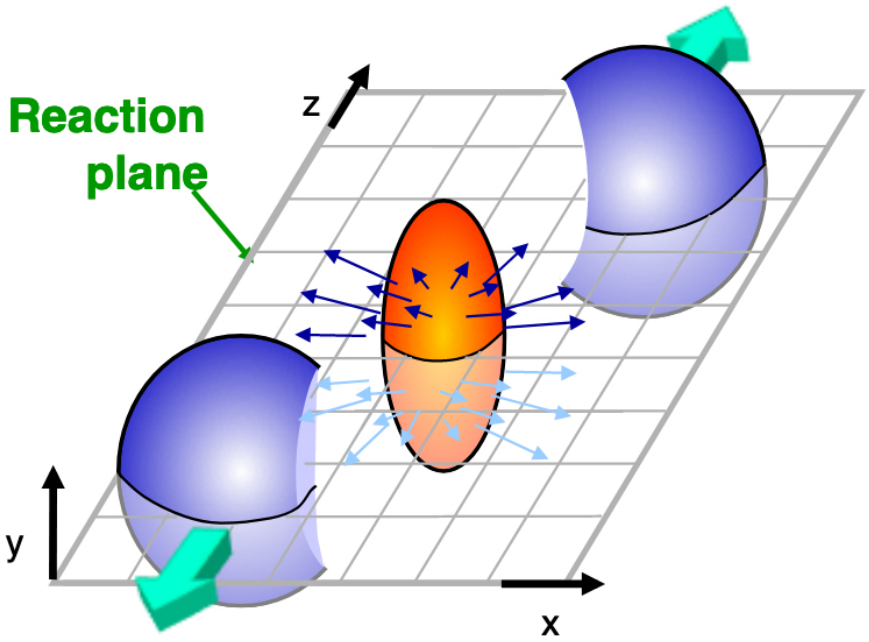


Figure: Geometry of non-central heavy ion collision (Pic. from B.Muller `arXiv:1309.7616`)

Rotating quark-gluon plasma

- The measurements of the Λ, $\bar{\Lambda}$ hyperon polarization by STAR predict the angular velocity $\Omega \sim 6 \pm 1$ MeV.
  L. Adamczyk et al. (STAR Collaboration),*Nature* 548, 62 (2017).
- The value of Ω obtained in the hydrodynamic simulations is $\Omega \sim 20 - 40$ MeV.
  Y. Jiang, Z.-W. Lin, J. Liao, *Phys. Rev.***C 94**, 044910 (2016).
  M. Baznat, K. Gudima, A. Sorin and O.Teryaev, Phys. Rev. C 93, 031902(2016).
- Lattice calculations " The critical temperature of the confinement-deconfinement transition in gluodynamics increases with increasing angular velocity"
  V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev and A. A. Roenko, *Phys. Rev.* D **103** (2021) no.9, 094515; `arXiv:2102.05084 [hep-lat]`.
- Effective models of rotating QGP: Ebihara et.al., Phys. Lett. B **764** (2017), 94-99, Chernodub and Gongyo, JHEP **01** (2017), 136, Chernodub, Phys. Rev. D **103** (2021) no.5, 054027, Fujimoto et.al, 2101.09173, it was found out the rotation decreases the deconfinement temperature.

Rotating QGP and holography

- Bhattacharyya et.al.'08: Holographic fluid/gravity correspondence for rotating black holes.
- Nata Atmaja and Schalm'10: Rotating QGP $\cong$ Kerr-AdS black hole ($d = 4$)
- Romaschke et.al.'19:Heavy ion collisions and Kerr-AdS black holes
- X. Chen et.al. "Gluodynamics and deconfinement phase transition under rotation from holography", *JHEP* 2021, 132 (2021);
- Garbiso and Kaminski, "Hydrodynamics of simply spinning black holes & hydrodynamics for spinning quantum fluids," *JHEP* **12** (2020), 112.
- V. Cardoso, et.al., Holographic thermalization, quasinormal modes and superradiance in Kerr-AdS, JHEP 04(2014)183.
- J.B. Amado et.al., *On the Kerr-AdS/CFT correspondence*, JHEP 08(2017)094.
- I.Y. Aref'eva, A.A. Golubtsova, E. Gourgoulhon, Holographic drag force in 5d Kerr-AdS black hole, *JHEP* 4 (169), 2021.
- A.A. Golubtsova, E. Gourgoulhon, M.K. Usova, Heavy quarks in rotating plasma via holography, *Nucl. Phys.* B 979, 115786, 2022.

Outline

Holographic backgrounds

Solutions to Einstein equations with S^3-symmetry:

- Anti-de Sitter-Schwarzschild black hole (M)

$$ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\Omega_3^2, \quad f = \ell^2 + \frac{1}{r^2} - \frac{2M}{r^4}$$

- Kerr-Anti-de Sitter black hole (M, J)

$$\begin{aligned} ds^2 \quad &\simeq \quad -(1+y^2)dT^2 + \frac{dy^2}{1+y^2-\frac{2M}{\Delta^2 y^2}} + y^2(d\Theta^2 + \sin^2\Theta d\Phi^2 + \cos^2\Theta d\Psi^2) \\ &+ \quad \frac{2M}{\Delta^3 y^2} dT^2 + \frac{2Ma^2 \sin^4\Theta}{\Delta^3 y^2} d\Phi^2 + \frac{2Mb^2\cos^4\Theta}{\Delta^3 y^2} d\Psi^2 - \\ &- \quad \frac{4Ma\sin^2\Theta}{\Delta^3 y^2} dT d\Phi - \frac{4Mb\cos^2\Theta}{\Delta^3 y^2} dT d\Psi + \frac{4Mab\sin^2\Theta\cos^2\Theta}{\Delta^3 y^2} d\Phi d\Psi, \end{aligned}$$

where $\Delta = 1 - a^2\ell^{-2}\sin^2\Theta - b^2\ell^{-2}\cos^2\Theta$, $J \to 0$, –AdS-Schwarzschild

The conformal boundary of 5d AdS BH is 4d $R \times \mathbb{S}^3$ at $r \to \infty$ ($y \to \infty$):

$$ds^2 = -dT^2 + d\Theta^2 + \sin^2\Theta d\Phi^2 + \cos^2\Theta d\Psi^2.$$

The dependence of T on r_h

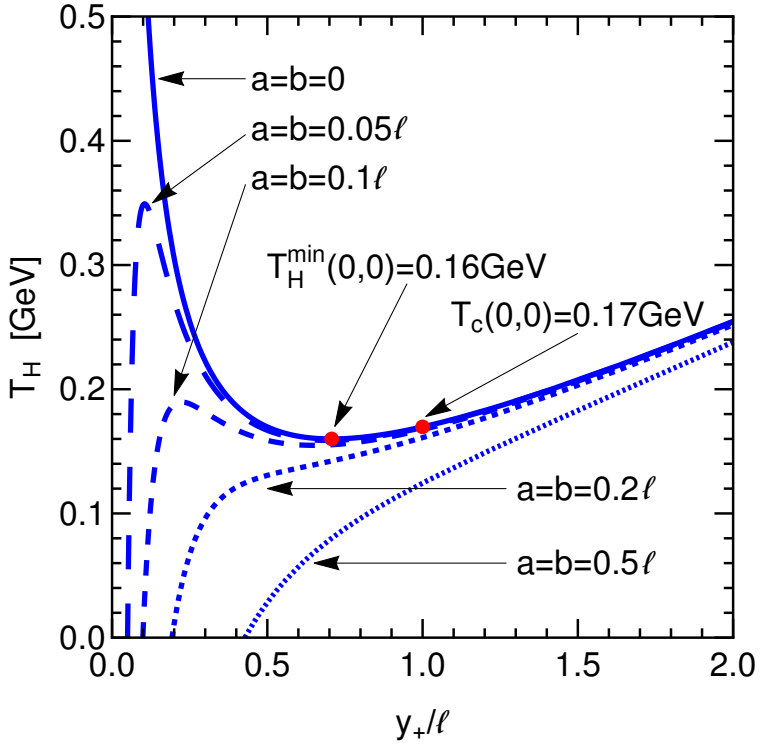


Figure: The Hawking temperature dependence on the horizon y_+

The Schwarzschild-AdS_5 and Kerr-AdS_5 BHs have minima of $T_H^{\min}$. Aref'eva,AG, Gourgoulhon'20

Holographic Wilson loops

- $d = 4$ $\mathcal{N} = 4$ SYM with $SU(N)$

$$W(\mathcal{C}) = \frac{1}{N}\mathrm{Tr}\mathcal{P}\exp\left(\oint ds A_\mu \dot{x}^\mu + |\dot{x}^i|\Phi_i\theta^i\right)$$

- The AdS/CFT duality (Maldacena'98): NG action of an open string in AdS_5

$$\langle W(\mathcal{C})\rangle = e^{-S_{NG,\min}-S_0}$$

The Nambu-Goto action

$$S_{NG} = \frac{1}{2\pi\alpha'}\int \mathrm{d}\sigma\mathrm{d}\tau\sqrt{-\det(g_{\alpha\beta})},$$

where $(g_{\alpha\beta})$ is the induced metric on the string worldsheet

$$g_{\alpha\beta} = G_{MN}\partial_\alpha X^M \partial_\beta X^N,$$

G_{MN}– spacetime metric, X^M – embedding coordinates, α, β – WS indices.

Holographic Wilson loops

- The interquark potential is related to the expectation value of the static temporal Wilson loop
$$\langle W(\mathcal{C})\rangle \sim e^{-\mathcal{T}V(L)},$$
the distance between quarks L and the temporal extent of the Wilson loop $\mathcal{T} \to \infty$.
- The quark-antiquark potential can be found in the following way
$$V_{q\bar{q}} = \frac{S_{NG}}{\mathcal{T}}|_{\mathcal{T}\to\infty}.$$
- The Cornell potential
$$V_{q\bar{q}} = \sigma L - \frac{\kappa}{L},$$
κ and σ are the Coulomb strength and string tension parameters
- in the confined phase the expectation value of the Wilson loop reproduces an area law
$$\langle W(\mathcal{C})\rangle \sim e^{-\sigma LT} = e^{-\sigma \mathrm{Area}(\mathcal{C})}.$$

Wilson loop in Kerr-AdS_5 black hole

$$
\begin{aligned}
ds^2 \quad &\simeq \quad -(1+y^2)dT^2 + \frac{dy^2}{1+y^2-\frac{2M}{\Delta^2 y^2}} + y^2(d\Theta^2 + \sin^2\Theta d\Phi^2 + \cos^2\Theta d\Psi^2) \\
&+ \quad \frac{2M}{\Delta^3 y^2}dT^2 + \frac{2Ma^2\sin^4\Theta}{\Delta^3 y^2}d\Phi^2 + \frac{2Mb^2\cos^4\Theta}{\Delta^3 y^2}d\Psi^2 - \\
&- \quad \frac{4Ma\sin^2\Theta}{\Delta^3 y^2}dTd\Phi - \frac{4Mb\cos^2\Theta}{\Delta^3 y^2}dTd\Psi + \frac{4Mab\sin^2\Theta\cos^2\Theta}{\Delta^3 y^2}d\Phi d\Psi,
\end{aligned}
$$

where $\Delta = 1 - a^2\ell^{-2}\sin^2\Theta - b^2\ell^{-2}\cos^2\Theta$.

The horizon is defined by $1 + y^2\ell^{-2} - \dfrac{2M}{\Delta^2 y^2} = 0$. The Hawking temperature reads

$$
T_H = \frac{1}{2\pi}\left(y_+(1+y_+^2\ell^{-2})\left(\frac{1}{y_+^2+a^2} + \frac{1}{y_+^2+b^2}\right) - \frac{1}{y_+}\right).
$$

The worldsheet parametrization

$$
\boxed{\tau = T, \qquad \sigma = \Phi, \qquad y = y(\Phi), \qquad \Phi \in [0, 2\pi L_\Phi].}
$$

The boundary conditions $y\left(-\dfrac{L_\Phi}{2}\right) = y\left(\dfrac{L_\Phi}{2}\right) = 0$.

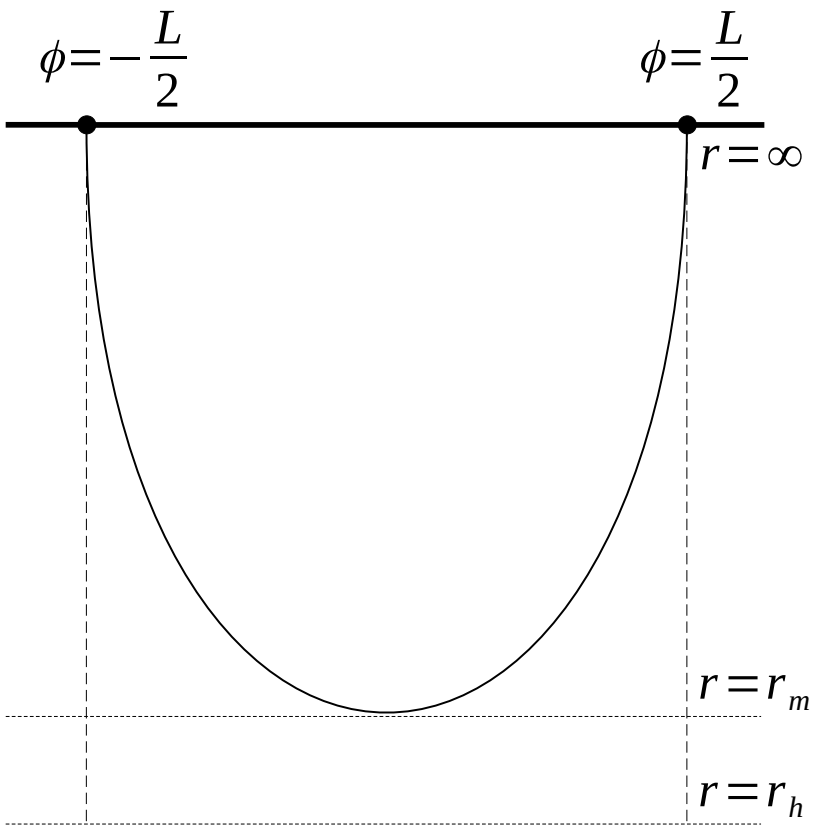


FIGURE: The string endpoints at $\Phi = -\frac{L}{2}$ and $\Phi = \frac{L}{2}$ and static straight strings(dashed lines)

Wilson loop in Kerr-AdS_5 black hole

The Nambu-Goto action is

$$S_{NG} = \frac{T}{2\pi\alpha'} \int_{-\frac{L_\Phi}{2}}^{\frac{L_\Phi}{2}} d\Phi \sqrt{y'^2 \frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)} + y^2 F_{\Delta^3}(y) \sin^2\Theta},$$

where we redefine

$$f_{\Delta^2}(y) \equiv 1 + y^2\ell^{-2} - \frac{2M}{\Delta^2 y^2}, \qquad f_{\Delta^3}(y) \equiv 1 + y^2\ell^{-2} - \frac{2M}{\Delta^3 y^2}$$

$$F_{\Delta^3}(y) = f_{\Delta^3}(y) + \frac{2Ma^2\sin^2\Theta}{y^4\Delta^3}\left(1 + y^2\ell^{-2}\right).$$

The integral of motion

$$\mathcal{H} = -\frac{y^2 F_{\Delta^3}(y)\sin^2\Theta}{\sqrt{y'^2 \frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)} + y^2 F_{\Delta^3}(y)\sin^2\Theta}}.$$

The turning point is defined by $y' = 0$, so we have

$$-y \sin\Theta \sqrt{F_{\Delta^3}(y)}\Big|_{y=y_m} = -\frac{\ell}{C}, \quad \text{with} \quad y_m = y(\Phi_m).$$

The equation of motion is

$$y'^2 = y^2 F_{\Delta^3}(y) \frac{f_{\Delta^2}(y)}{f_{\Delta^3}(y)} \sin^2\Theta \left[\frac{C^2}{\ell^2} \sin^2\Theta y^2 F_{\Delta^3}(y) - 1 \right].$$

The distance between quarks L_Φ:

$$\frac{L_\Phi}{2} = \int_{y_m}^{\infty} dy \, \frac{\ell}{\sin\Theta y \sqrt{F_{\Delta^3}(y)} \sqrt{C^2 \sin^2\Theta y^2 F_{\Delta^3}(y) - \ell^2}} \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}.$$

The holographic renormalization

Coming to the integration in terms of y we obtain

$$S_{NG} = \frac{T}{\pi\alpha'} \int_{y_m}^{\infty} dy \, \frac{C \sin\Theta y \sqrt{F_{\Delta^3}(y)}}{\sqrt{C^2 \sin^2\Theta y^2 F_{\Delta^3}(y) - \ell^2}} \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}.$$

The holographic renormalization is a subtraction of the self-energy of two free quarks which corresponds to the action of two straight strings from $y = \infty$ up to the horizon y_+

$$S_0 = \frac{T}{\pi\alpha'} \int_{y_+}^{\infty} dy \, \sqrt{-G_{TT} G_{yy}} = \frac{T}{\pi\alpha'} \left(\int_{y_m}^{\infty} + \int_{y_+}^{y_m} \right) \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}} dy.$$

The renormalized NG action and the quark-antiquark potential

$$\boxed{V_{q\bar{q}} = \frac{S_{NG}^{ren}}{T} = \frac{\sqrt{\lambda}}{\pi\ell^2} \left[\int_{y_m}^{\infty} dy \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}} \left(\frac{C \sin\Theta y \sqrt{F_{\Delta^3}(y)}}{\sqrt{C^2 \sin^2\Theta y^2 F_{\Delta^3}(y) - \ell^2}} - 1 \right) - \int_{y_+}^{y_m} dy \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}} \right].}$$

$J = 0 \ (a = b = 0)$

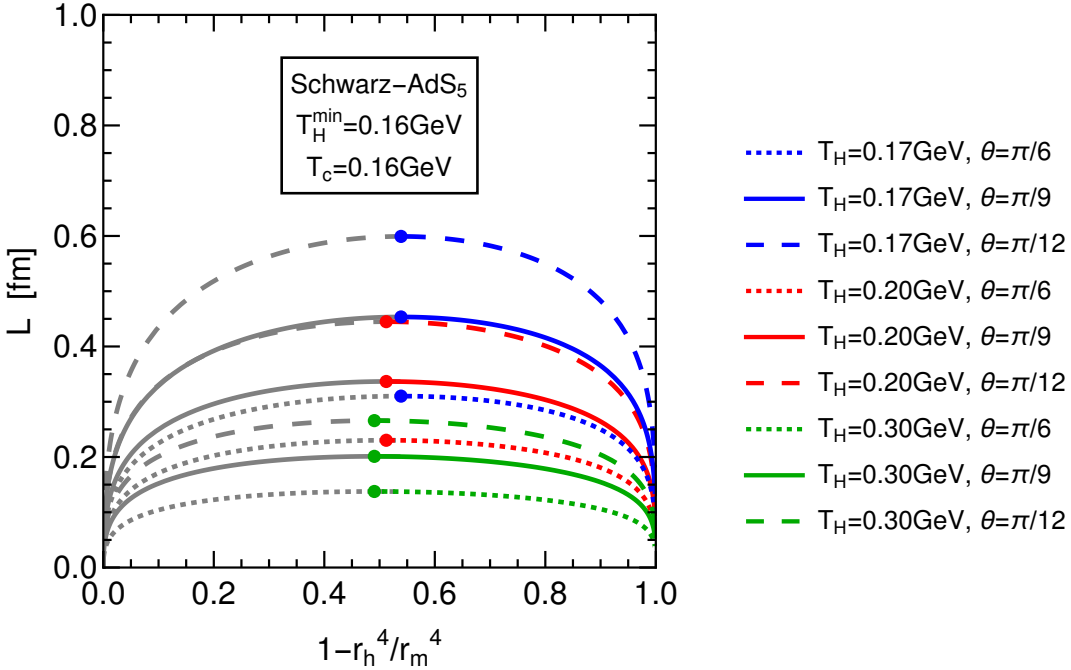


FIGURE: The distance L between quark and antiquark, depending on the string turning point r_m

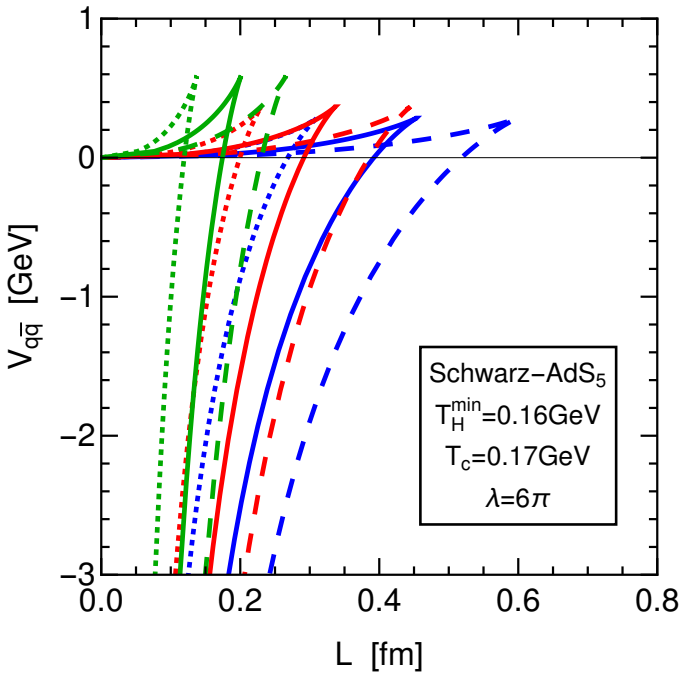


FIGURE: Numerical results for the dependence of V_{qq} on the distance between them L

$J \neq 0$

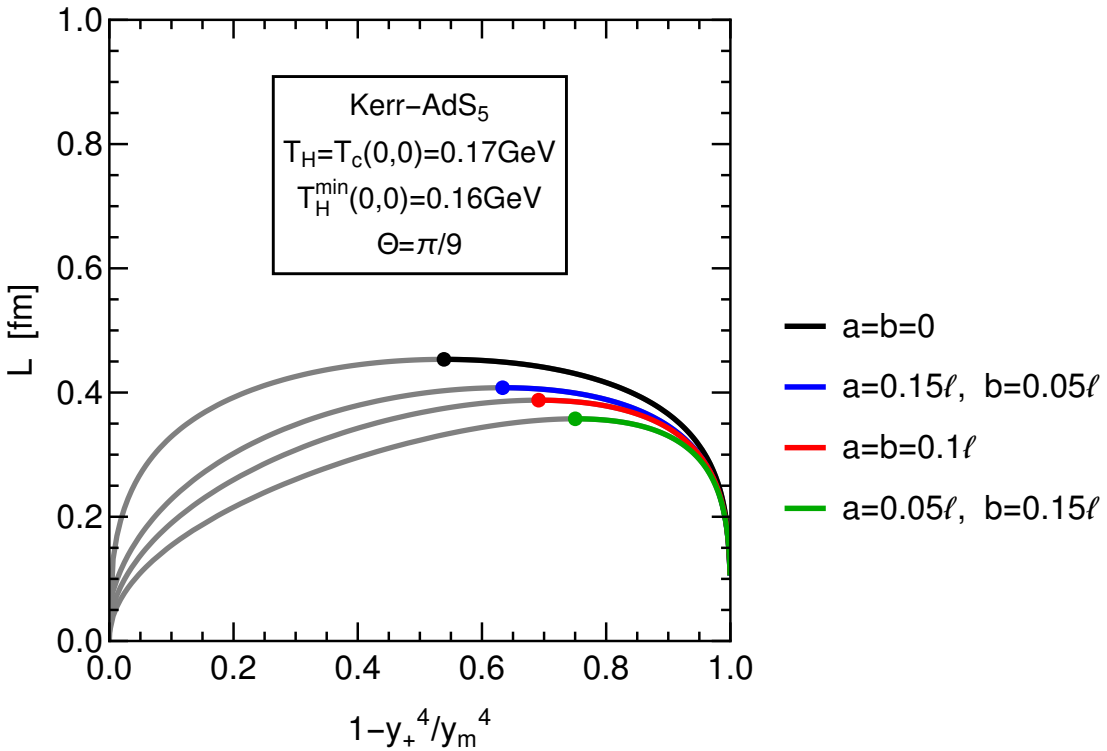


FIGURE: The distance between quark-antiquark L, depending on the string turning point y_m. The maximum distances – *the screening lengths* – are depicted by dots

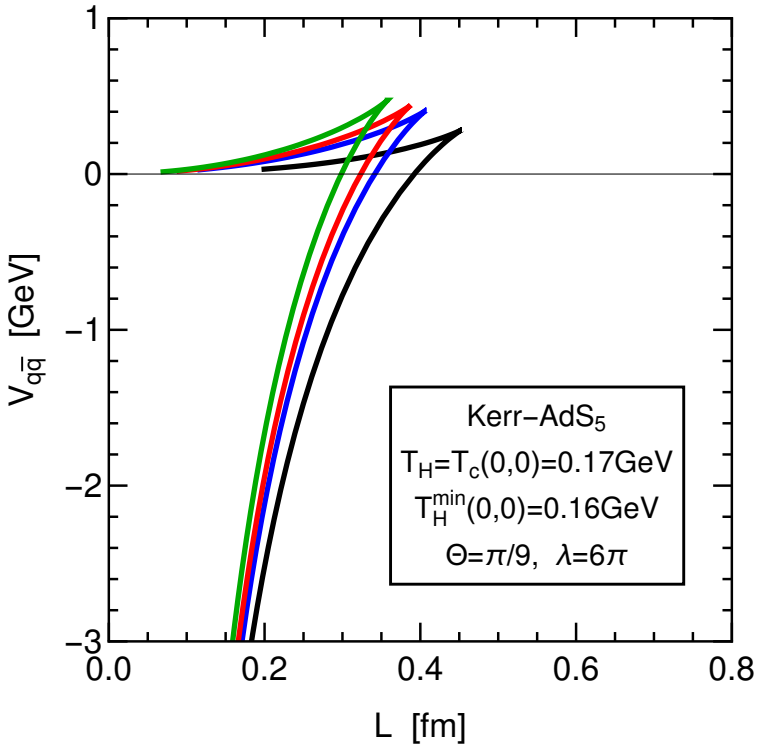


FIGURE: Numerical results of the heavy quark-antiquark potential V_{qq} dependence on the distance L

The relation between S_{NG}^{ren} and L_ϕ

The relation between the string action and the quark-antiquark distance

$$S_{NG} = \frac{T}{\pi\alpha'} I_1(y_m, C), \quad \frac{L_\Phi}{2} = I_2(y_m, C).$$

We have the following relation :

$$\frac{\partial I_2(y_m, C)}{\partial C} = \frac{C}{\ell} \frac{\partial I_1(y_m, C)}{\partial C}.$$

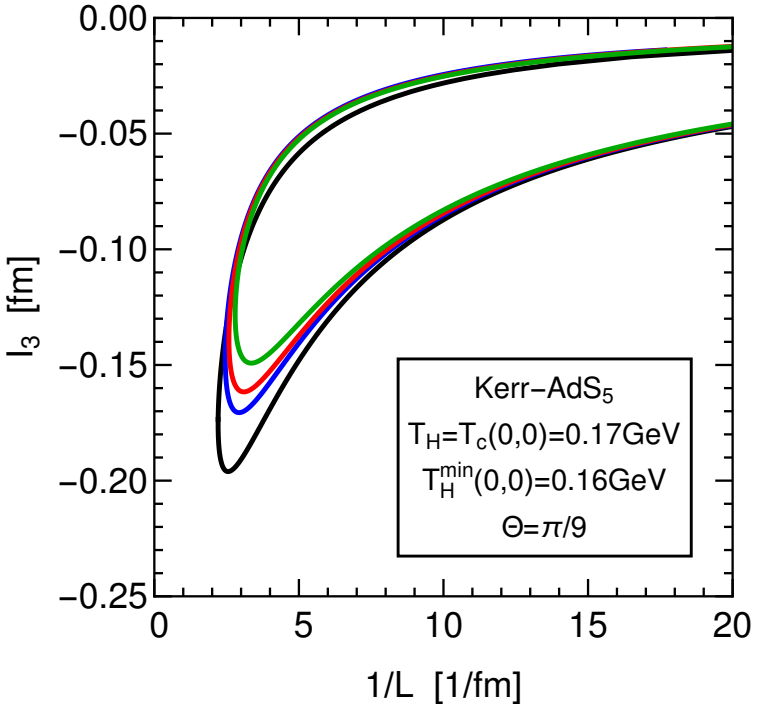
$$S_{NG} = \frac{\ell}{C} \frac{T}{\pi\alpha'} \left(\frac{L_\Phi}{2} + I_3 \right),$$

where

$$I_3 = \int_{y_m}^{\infty} dy \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}} \left(\frac{\sqrt{C^2 \sin^2\Theta y^2 F_{\Delta^3}(y) - \ell^2}}{y \sin\Theta \sqrt{F_{\Delta^3}(y)}} - C \right) - \frac{C}{\ell} \int_{y_+}^{y_m} dy \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}$$

We find for the quark-antiquark potential

$$V_{q\bar{q}} = \frac{\sqrt{\lambda}}{\pi\ell^2} y_m \sin\Theta \sqrt{F_{\Delta^3}(y_m)} \left(\frac{L_\Phi}{2} + I_3 \right).$$



$$V_{q\bar{q}}(L) = \sigma L - \frac{\kappa}{L} + \frac{\chi}{\sqrt{L}}.$$

T, GeV	θ	a/ℓ	b/ℓ	σ, GeV/fm	κ, GeV·fm	χ, GeV·fm$^{1/2}$
0.17	$\pi/9$	0	0	2.21704	1.28893	1.55392
		0.15	0.05	2.76058	1.30234	1.71247
		0.1	0.1	2.91683	1.31574	1.80647
		0.05	0.15	3.13669	1.33658	1.95819

TABLE: Fitting coefficients at temperatures $T = 0.17$ GeV, angle $\theta = \pi/9$

Light-like WL and jet-quenching parameter

The JQ parameter $\hat{q}$ gives the squared average transverse momentum exchange between the medium and highly energetic parton per unit path length.
The expectation value of the the light-like WL on the contour $\mathcal{C}$ in the adjoint representation and the jet-quenching parameter $\hat{q}$ for a fast parton Rajagopal et. al.'06

$$\langle W^A(\mathcal{C})\rangle \approx \exp\Big[-\frac{1}{4\sqrt{2}}\hat{q}L^- L^2\Big],$$

where L^- is a large side of the rectangular contour $\mathcal{C}$ and L is a short side. At the same time it is known that the Wilson loop operator in the adjoint representation is related to the the Wilson loop operator in the fundamental representation as follows

$$\langle W^A(\mathcal{C})\rangle \approx \langle W^F(\mathcal{C})\rangle^2 .$$

Following the holographic dictionary, we have

$$\langle W^F(\mathcal{C})\rangle = e^{-S_{NG}}.$$

planar AdS $$\hat{q} = \frac{\pi^2\sqrt{\lambda}T^3}{\beta} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3.$$

Light-like Wilson loop in Schwarzschild-AdS_5

"Light-cone" coordinates

$$dx^+ = \ell^2(dt - \ell d\phi), \quad dx^- = \ell^2(dt + \ell d\phi).$$

The string parametrization

$$\tau = x^-, \quad \sigma = \psi, \quad x^\mu = x^\mu(\sigma), \quad \theta(\sigma) = const, \quad x^+(\sigma) = const.$$

The Nambu-Goto action is

$$S = \frac{L^-}{2\pi\alpha'} \int_{-L/2}^{L/2} d\psi \, \frac{r}{2\ell^2} \sqrt{\left(\frac{f(r)}{r^2} - \ell^{-2} r^2 \sin^2\theta\right)\left(\cos^2\theta + \frac{r'^2}{f(r)}\right)}, \quad r' \equiv \partial r/\partial\psi$$

The first integral is given by

$$\mathcal{H} = -\frac{\cos^2\theta \sqrt{f(r) - r^4 \ell^{-2} \sin^2\theta}}{2\ell^2 \sqrt{\cos^2\theta + \frac{r'^2}{f(r)}}}.$$

The equation for $r(\sigma)$

$$r'^2 = \frac{f(r)\cos^2\theta}{4C^2\ell^6}[\cos^2\theta(f(r)\ell^2 - r^4\sin^2\theta) - 4C^2\ell^6], \quad C = const$$

The holographic regulazation

$$S^{reg} = \frac{L^-}{\pi\alpha'} \int_{r_H+\epsilon}^{\infty} dr \, \frac{\sqrt{f(r)\ell^2 - r^4 \sin^2\theta}}{2\ell^3\sqrt{f(r)}} \left(\frac{\cos\theta\sqrt{f(r)\ell^2 - r^4\sin^2\theta}}{\sqrt{\cos^2\theta(f(r)\ell^2 - r^4\sin^2\theta) - 4C^2\ell^6}} - 1 \right).$$

Expanding for small C (in the low energy limit)

$$S^{reg} = \frac{L^-}{\pi\alpha'} \frac{\ell^2 C^2}{\cos^2\theta} \mathcal{I}, \quad \mathcal{I} = \int_{r_m}^{\infty} \frac{dr}{\sqrt{f(r)}\sqrt{f(r) - r^4\ell^{-2}\sin^2\theta}}$$

and r_m is defined as a positive real solution to the equation

$$r^2 + r^4\ell^{-2}\cos^2\theta - 2M = 0.$$

To find the relation between L and C we remember that $r(\pm L/2) = \infty$

$$\frac{L}{2} = \int_{r_H}^{\infty} \frac{dr}{r'} = \frac{2C\ell^3}{\cos\theta} \int_{r_H}^{\infty} \frac{dr}{\sqrt{f(r)}\sqrt{\cos^2\theta(f(r)\ell^2 - r^4\sin^2\theta) - 4C^2\ell^6}}.$$

For small C we have

$$\frac{L}{2} = \frac{2\ell^2 C}{\cos^2\theta}\mathcal{I}.$$

Then we come to

$$S^{reg} = \frac{L^-}{\pi\alpha'}\frac{L^2\cos^2\theta}{16\ell^2\int_{r_m}^{\infty}\frac{dr}{\sqrt{f(r)}\sqrt{f(r)-r^4\ell^{-2}\sin^2\theta}}}.$$

Wilson loop and jet-quenching parameter (Rajagopal'06)

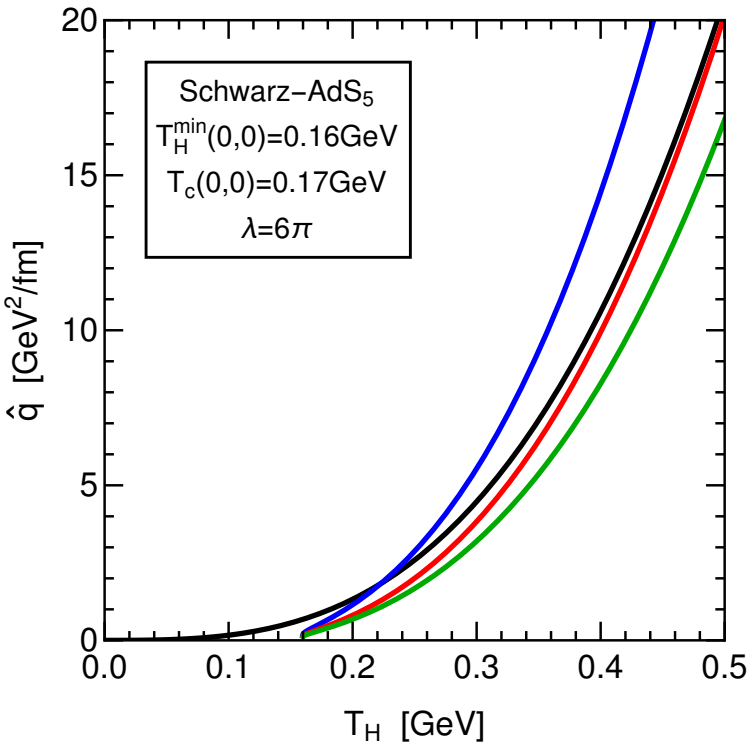
$$\langle W^A(\mathcal{C})\rangle \approx \exp\Big[-\frac{1}{4\sqrt{2}}\hat{q}L^- L^2\Big].$$

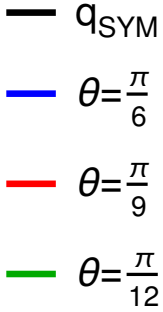
The jet-quenching parameter is

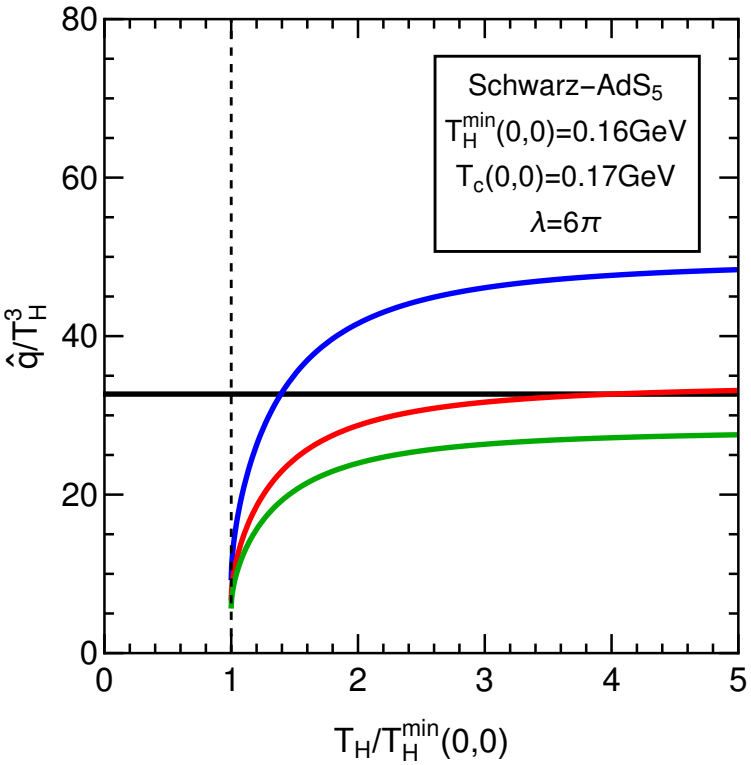
$$\hat{q} = \frac{\sqrt{\lambda}}{\sqrt{2}\pi}\frac{\cos^2\theta}{\ell^4\int_{r_m}^{\infty}\frac{dr}{\sqrt{f(r)}\sqrt{f(r)-r^4\ell^{-2}\sin^2\theta}}},$$

$$\hat{q} = \frac{\pi^2\sqrt{\lambda}T^3}{\beta} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3.$$

The jet-quenching parameter $J = 0 (a = b = 0)$







The jet-quenching parameter $J \neq 0 (a \neq b \neq 0)$

The "light-cone" coordinates

$$\mathrm{d}x^+ = dT - a\mathrm{d}\Phi, \qquad \mathrm{d}x^- = \mathrm{d}T + a\mathrm{d}\Phi.$$

$$S^{reg} = \frac{L^-}{\pi\alpha'} C^2 \mathcal{I} + \mathcal{O}(C^4), \qquad \frac{L}{2} = 2C\mathcal{I} + \mathcal{O}(C^3),$$

where for convenience we introduce

$$\mathcal{I} = \int_{y_+}^{\infty} dy \, \frac{\sqrt{\tilde{\eta}(y) + \frac{2M}{\Delta^3 y^2}\cos^4\Theta}}{\cos^2\Theta\left(\tilde{\eta}(y)\frac{2M}{\Delta^3 y^2} b^2 \cos^2\Theta + \tilde{\eta}(y) y^2 + \frac{2M}{\Delta^3}\cos^4\Theta\right)\sqrt{f_{\Delta^2}(y)}},$$

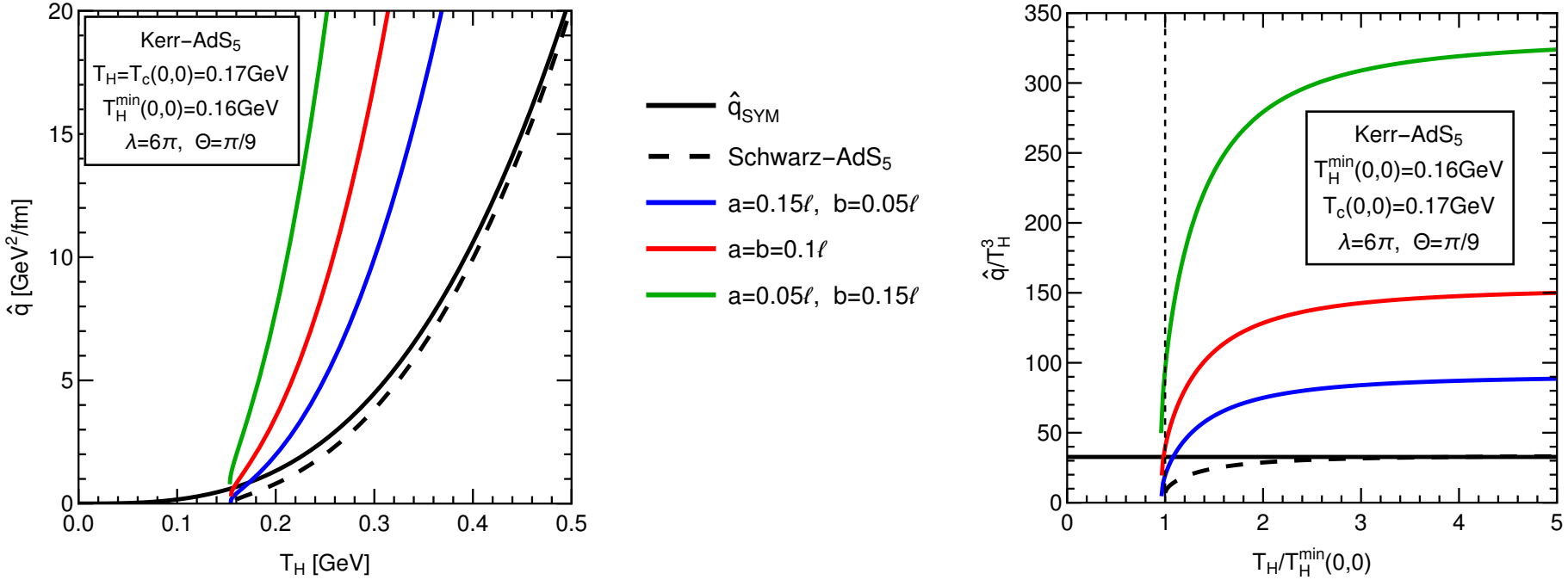
$$\beta(y) = \cos^2\Theta(\tilde{\eta}(y)\frac{2M}{\Delta^3 y^2} b^2 \cos^2\Theta + \tilde{\zeta}(y) y^2), \tilde{\eta}(y) = 1 + y^2 - \frac{y^2}{a^2}\sin^2\Theta, \tilde{\zeta}(y) = \tilde{\eta} - \frac{2M}{\Delta^3 y^2}\cos^4\Theta.$$

$$S^{reg} = \frac{L^-}{\pi\alpha'} \frac{L^2}{16\mathcal{I}}.$$

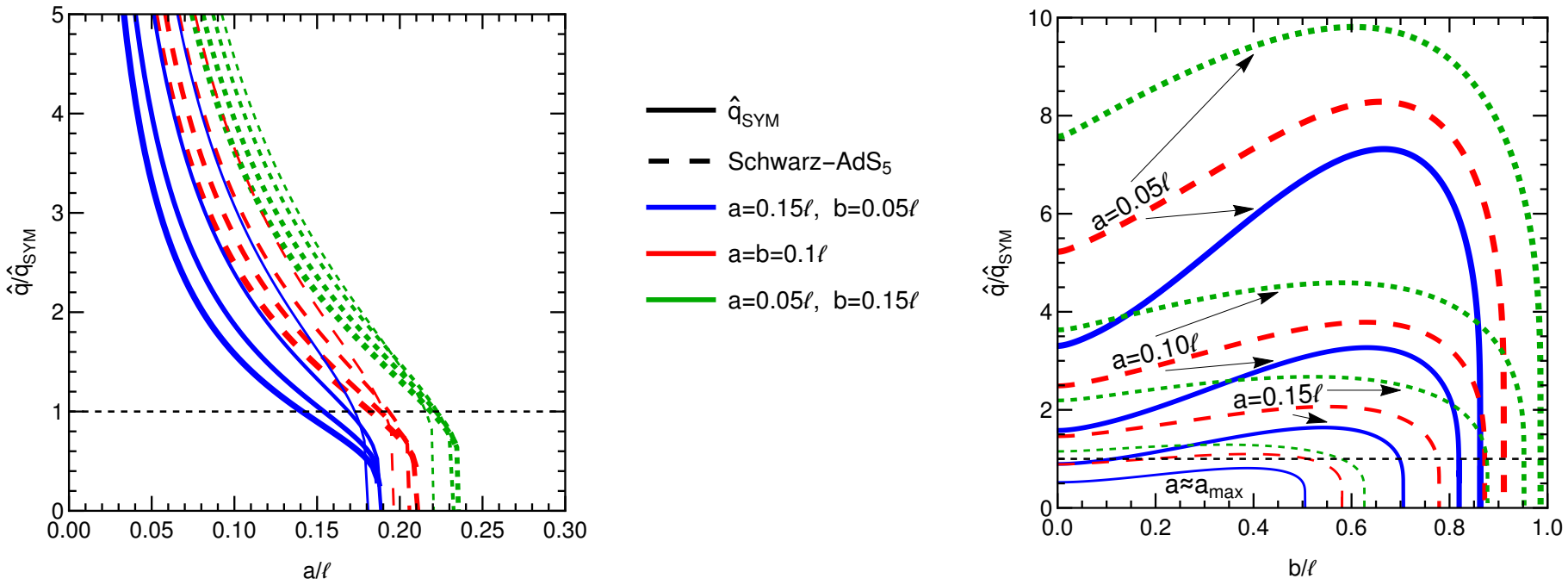
The jet-quenching parameter can be read off as follows

$$\hat{q} = \frac{\sqrt{\lambda}}{\sqrt{2}\pi\mathcal{I}}.$$

The jet-quenching parameter $J \neq 0 (a \neq b \neq 0)$



The jet-quenching parameter $J \neq 0 (a \neq b \neq 0)$



Thank you for attention!