Models in Quantum Field Theory'22

Four-point amplitudes in fishnet theories and dual conformal integrals

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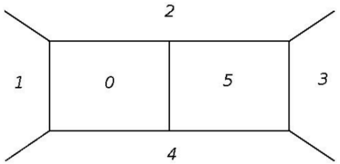
Setup

1611.02179

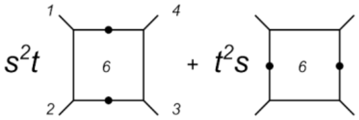
$$\left.\frac{\mathcal{M}_{3d}^{(2)}}{\mathcal{M}_{3d}^{tree}}\right|_{f_{CS}} = \left.\frac{\mathcal{M}_{4d}^{(1)}}{\mathcal{M}_{4d}^{tree}}\right|_{f_{SYM}} + \text{const} + O(\epsilon)$$



Set of indices for double-box integrals

No.	ν_1	ν_2	ν_3	ν_4	ν_5	ν_6	ν_7	α_1	α_2
(1)	1	2	1	1	2	1	2	2	2
(2)	2	1	2	2	1	2	1	1	4
(3)	1	1	1	1	1	1	3	1	2
(4)	2	1	1	2	1	1	2	1	3
(5)	1	1	2	1	1	2	2	1	3
(6)	3	1	1	3	1	1	1	1	4
(7)	1	1	3	1	1	3	1	1	4
(8)	1	2	2	1	2	2	1	2	3
(9)	2	2	1	2	2	1	1	2	3
(10)	1	3	1	1	3	1	1	3	2

Single loop box-type amplitude:



$$e^{\epsilon(\gamma_E-2)}\mathcal{M}_4^{(1)} = -\frac{1}{2\epsilon^2}\left(\left(\frac{\mu^2}{s}\right)^{\epsilon} + \left(\frac{\mu^2}{t}\right)^{\epsilon}\right) + \frac{1}{4}\ln^2\frac{s}{t} + \frac{\pi^2}{3} + \frac{1}{8}$$

One-loop regularized action for minimal M2-brane

$$iS_{reg} = -\frac{N}{4\pi\epsilon}\left(2S_{div,s} + 2S_{div,t} + \frac{1}{4}\ln^2\left(\frac{s}{t}\right) + 2\zeta(2) - 8\ln^2 2 - \frac{3}{2}\right)$$

Dual conformal symmetry

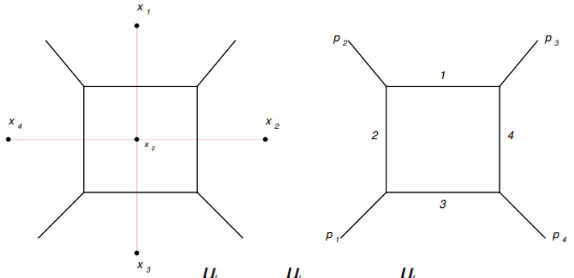
Tree level

$$I[x_i^\mu] = \frac{x_i^\mu}{x_i^2}, \quad I[x_{ij}^2] = \frac{x_{ij}^2}{x_i^2 x_j^2}.$$

One-loop box-type integrals

$$Box_4^{d=4}(s,t) = \int d^4x_0 \frac{x_{12}^2 x_{23}^2}{x_{10}^2 x_{20}^2 x_{30}^2 x_{40}^2}$$

$$Box_4^{d=6}(s,t) = \int d^6x_0 \frac{x_{13}^4 x_{24}^2}{(x_{10}^2)^2 x_{20}^2 (x_{30}^2)^2 x_{40}^2}.$$



$$x_i^\mu - x_{i+1}^\mu = p_i^\mu$$

Conformal boost

$$K^\nu = \sum_{i=1}^{4} \left(2x_i^\nu (x_i \partial_i) - x_i^2 \partial_i^\nu\right)$$

Conformal symmetry is preserved

$$K^\nu \; Box_4^{d=4}(s,t) = 0$$

Conformal symmetry is broken

$$K^\nu \; F_4 = \gamma_{cusp}(g) \sum_{i=1}^{4} x_i^\nu L\left(\frac{x_{i,i+2}^2}{x_{i-1,i+1}^2}\right).$$

Higher dimensional amplitudes

Single box integral in alpha-representation:

$$\mathcal{I}(s,t,\nu_1,\nu_2,\nu_3,\nu_4,d) = (-1)^{\nu}\frac{\pi^{d/2}e^{i\frac{\pi}{2}(\nu+h(1-d/2))}}{\prod_{i=1}^{4}\Gamma(\nu_i)}\int_0^{\infty}\prod_{i=1}^{4}d\alpha_i\;\prod_{i=1}^{4}\alpha_i^{\nu_i-1}\mathcal{U}^{-d/2}e^{i\mathcal{V}/\mathcal{U}}$$

$$\begin{aligned}\mathcal{V} &= t\alpha_1\alpha_3 + s\alpha_2\alpha_4\\ \mathcal{U} &= \alpha_1+\alpha_2+\alpha_3+\alpha_4\end{aligned}$$

$$-\pi\frac{\partial}{\partial t}\mathcal{I}(s,t,1,1,1,1,d) = \mathcal{I}(s,t,2,1,2,1,d+2)$$

$$-\pi\frac{\partial}{\partial s}\mathcal{I}(t,s,1,1,1,1,d) = \mathcal{I}(t,s,1,2,1,2,d+2)$$

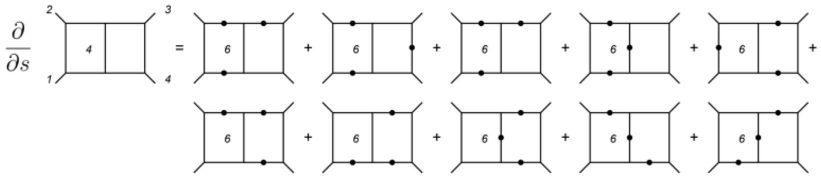
Simple relation between single box integrals in the same channel

$$Box_4^{d=6}(s,t) = -\pi\left(t\frac{\partial}{\partial t}-1\right)Box_4^{d=4}(s,t)$$

The same is for double boxes

$$DBox_4^{d=6,1}(s,t) = -\pi\left(t\frac{\partial}{\partial t}-1\right)DBox_4^{d=4}(s,t)$$

Set of double box integrals, some of individual integrals are non-conformal:



Iterative structure?

Iterative BDS-exponent in N=4 SYM 4D

$$M_4^{d=4} = \sum_{l=0}^{\infty} g^l M_4^{d=4,(l)} = \exp\left[\sum_l g^l \left(f^{(l)}(\epsilon) M_4^{d=4,(1)}(l\epsilon) + C^{(l)} + E_4^{(l)}(\epsilon) \right)\right]$$

Iterative BDS-exponent in ?-SYM 6D-theory

$$\begin{aligned} M_4^{d=6} &= \sum_{l=0}^{\infty} {g_6}^l \tilde{M}_4^{d=6,(l)} = \sum_{l=0}^{\infty} {g_6}^l \left(\hat{\mathcal{D}}_{st} M_4^{d=4,(l)} - (l+1) M_4^{d=4,(l)} \right) \\ &= \left(-g\frac{\partial}{\partial g} - 1 + \omega(u) + \omega(v) \right) M_4^{d=4} \Big|_{g \mapsto g_6} + O(m^2). \end{aligned}$$

Amplitude computed in minimal volume calculations must contain non-trivial prefactor

$$\sum_{l=0}^{\infty} M_4^{d=6,(l)} \sim exp\left(\mathcal{S}_s + \mathcal{S}_t + \frac{\gamma(g_4)}{8} L^2\left(\frac{s}{t}\right) \right)\Big|_{g \mapsto g_6}$$

Regge trajectory:

$$s\frac{\partial}{\partial s} log \left[\sum_{l=0}^{\infty} g^l M_4^{d=4,(l)} \right] = \omega(v),$$

$$\omega(v) = 1 + \frac{\gamma_{cusp}(g)}{4} L(v) - \tilde{G}(g).$$

Conformal derivative operator

$$\hat{\mathcal{D}}_{st} = s\frac{\partial}{\partial s} + t\frac{\partial}{\partial t}.$$

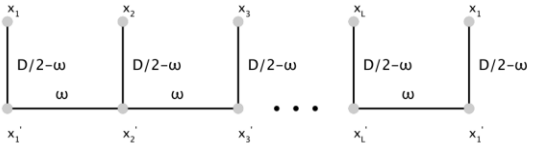
The theory is too complicated

Fishnets in 6D

Fishnet Lagrangian for arbitrary dimensions with
additional isotropic parameter

$$\mathcal{L}_{main} = N_c \ \mathrm{tr} \left(\ \phi_1^*(\partial^2)^{\omega}\phi_1 + \phi_2^*(\partial^2)^{D/2-\omega}\phi_2 + (4\pi)^{D/2} g^2 \ \phi_1^*\phi_2^*\phi_1\phi_2 \ \right)$$

$$\mathcal{L}_{main} = N_c \ \mathrm{tr} \left(\ \phi^*(\partial^2)\phi + \chi^*(\partial^4)\chi + (4\pi)^3 g^2 \ \phi^*\chi^*\phi\chi \ \right)$$



Double-trace counter-term for finiteness

$$\mathcal{L}_{c.t.}^{(D,\omega)} = \sum_i \alpha_i(g) \mathrm{tr} \left(\mathcal{O}_2^{(i)} \right) \mathrm{tr} \left(\tilde{\mathcal{O}}_2^{(i)} \right)$$

$$\mathcal{L}_{c.t.}^{(6,1)}/(4\pi)^3 = g \left(\mathrm{tr}\left(\phi\chi\right) \mathrm{tr}\left(\phi^*\chi^*\right) + \mathrm{tr}\left(\phi^*\chi\right) \mathrm{tr}\left(\phi\chi^*\right) \right) + \text{possible } \chi \text{ interactions}$$

$$\mu(\nu, J) \ = \ 16\pi \frac{\nu^2(4\nu^2 + (J+1)^2)(J+1)}{2^J},$$

Eigenvalue of graph-building operator

After LSZ-reduction of correlation function we can get a
colour-ordered amplitude:

$$A_4^{D=6}(z,g) = \frac{\partial}{\partial z} A_4^{D=4}(z,g) = \int_C \frac{dJ}{2i\sin(\pi J)} \int_{-\infty}^{+\infty} d\nu \frac{\mu(\nu, J)}{h(\nu, J) - g^4} \frac{\partial \Omega_{\nu,J}(z)}{\partial z}$$

$$h(\mu, J) \ = \ \left(\nu^2 + \frac{J^2}{4}\right)\left(\nu^2 + \frac{(J+2)^2}{4}\right),$$

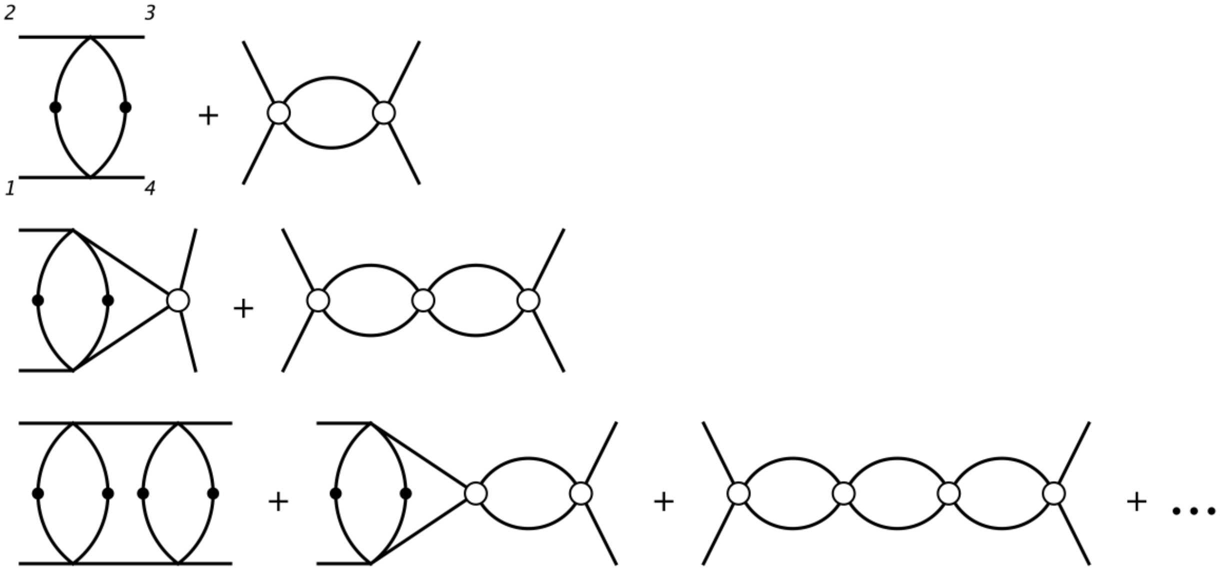
Kinematical part (reduced conformal
block)

$$\Omega_{\nu,J}(z) = \frac{2^J}{\pi^2} \sinh^2(\pi\nu + i\pi J/2) \sum_{k=0}^{J} \frac{P_k(z) P_{J-k}(z)}{(J/2-k)^2 + \nu^2}$$

Fishnet diagrams

Single trace contribution

Double trace contribution

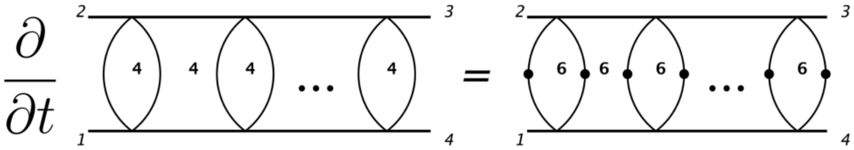


Weak coupling

Bootstrap to find individual integrals in any order by g

$$\mathcal{B}^{(l)}(-1+y) = \frac{\sum_{\vec{a}} C^{(1)}_{\vec{a}} H_{a_1,\ldots a_{w_2 l}}(-1+y)}{y} + \frac{\sum_{\vec{a}} C^{(2)}_{\vec{a}} H_{a_1,\ldots a_{w_2 l}}(-1+y)}{-1+y}$$

4D-6D correspondence



$$\mathcal{B}^{(0)}(x) = \frac{1}{x},\ \mathcal{B}^{(1)}(x) = \frac{\log(x)^2+\pi^2}{2(1+x)}.$$

Leading logs in high energy limit

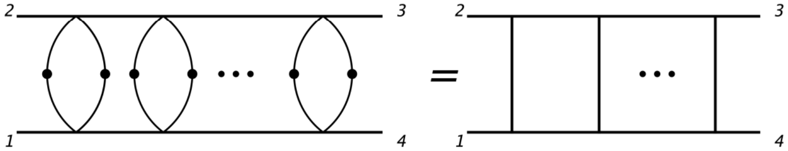
$$a^{LLA}_{(l)} = \frac{1}{l!(l+1)!},$$

$$a^{NLA}_{(l)} = \frac{2l(l-1)}{l!(l+1)!},$$

$$a^{NNLA}_{(l)} = \frac{2l(l-1)(l+2)+\pi^2(l+1)}{l!(l+2)!},$$

$$a^{NNNLA}_{(l)} = \frac{2l\left(l(l+1)\left(2l^2+2l+3\pi^2-13\right)+6(l-4)\zeta_3+18\right)}{3l!(l+2)!}.$$

6D-6D correspondence: bubbles and 6d SYM-boxes



Strong coupling limit

Strong g contribution comes from large spins:

$$A^{D=4,u}(z,g) = \frac{1}{2i}\int_C \frac{d(gJ)}{\sin(\pi gJ)} \int_{-\infty}^{+\infty} d\nu \frac{\mu(\nu, gJ)}{h(\nu, gJ) - g^4}\Omega_{\nu,gJ}(z)$$

Amplitude in high energy (Regge) limit:

$$A_4^{D=4,u}(z,1/g) = g^{-1/2}\frac{4\pi\,\pi^{1/2}\,\mathbf{L}\exp\left(\frac{2}{g}\sqrt{\pi^2+\mathbf{L}^2}\right)}{i\sqrt{z^2-1}\,(\pi^2+\mathbf{L}^2)^{7/4}\sin\left(\frac{2\pi\mathbf{L}}{g\sqrt{\pi^2+\mathbf{L}^2}}\right)} + \ldots$$

Korchemsky result:

$$A_4^{D=4,u}(z,1/g) \sim \frac{z^{2/g-1}}{\log^{3/2}(z)} + \ldots$$

Amplitude in z=1 limit:

$$A_4^{D=4,u}(z=\pm 1, 1/g) = \sqrt{g}\frac{8\pi}{2\pi^3 i}\exp(2\pi/g)\,.$$

Minimal volume calculations is needed!

Open questions and related theories

- [x] Beta-deformed fishnet theory (biscalar+Yukawa coupling, easy to generalize on 6D, computable)
- [ ] Triscalar fishnet theory in 6d
- [ ] Bifermion fishnet theory
- [ ] Multipoint (5pt,6pt...) amplitudes/correlators behavior (Basso-Dixon-Derkachov conjecture)

Conclusions:

- We propose possible iterative structure in 6d ?-theory based on dual conformal symmetry
- We studied more simple 6d fishnet theory and calculated amplitudes in different kinematical limits and coupling regimes
- Relations between 4d fishnet/6d fishnet theories and 6d fishnet/6d SYM theories have been found

Thank you for attention!