Massless 6D infinite spin fields

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based on

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Motivation

Relativistic symmetry associates elementary particles in D-dimensional space-time with unitary irreps of the Poincaré group $ISO(1, D-1)$.

$$[P_a, P_b] = 0\,, \qquad [P_a, M_{bc}] = i\,(\eta_{ca}P_b - \eta_{ba}P_c)\,,$$
$$[M_{ab}, M_{cd}] = i\,(\eta_{ac}M_{bd} - \eta_{bc}M_{ad} + \eta_{bd}M_{ac} - \eta_{ad}M_{bc})\,.$$

Irreps are defined as eigenvectors of Casimir operators.

Second order Casimir operators of $iso(1, D-1)$ is

$$C_2 := P^a P_a\,.$$

In the case of physically interesting unitary irreducible representations, this Casimir operator takes the values

$$P^a P_a \geq m^2\,,$$

where m is mass of corresponding states.

D-dimensional Poincare irreps are induced from irreps of stability subgroup.

	$P^2 = m^2$	Stability subgroup
Massive irreps.	$m^2 \neq 0$	compact group $SO(D-1)$
Massless irreps.	$m^2 = 0$	non-compact group $ISO(D-2)$

Since stability subgroup is compact in massive case, irreducible representations (corresponding fields) are finite-component in this case.

In the massless case, stability subgroup is non-compact.

- Massless finite spin (helicity) representations.
  In this case the Euclidean $(D-2)$-translations are realized trivially.
  Irreps are described by usual finite-component fields.
- Massless infinite (continuous) spin representations.
  In this case the Euclidean $(D-2)$-translations are realized non-trivially.
  Such irreps describe a tower of infinite number of massless states.

In this talk we will consider massless infinite (continuous) spin representations.

First more detail studies of infinite spin representations were carried out only in the 1970s in [J.Yngvason, 1970; G.Iverson, G.Mack, 1971].

Next stage of research on these representations began in fact in the 2000s and was initiated by [L.Brink, at.al., 2002; J.Mund, at.al., 2004] and [X.Bekaert, J.Mourad, 2006; X.Bekaert, N.Boulanger, 2006].

Although physical status of unitary infinite (continuous) spin representations of the Poincaré group is still not very clear, interest in them is caused

- by an identical spectrum of states of infinite spin theory and higher-spin theory [M.Vasiliev, 1989,... and other], which led to the formation of a certain research branch mainly in the context of the theory of higher spin fields;
- by its potential relation to string theory (with infinite number of physical states) as candidates for Quantum Gravity Theory.

These properties of continuous spin particles are very attractive.
Lately a lot of research has been carried out on continuous spin particles [.....].

In this talk, I present our study of irreducible massless representations of the $6D$ Poincaré group focusing on the infinite spin representations.

In particular, in this talk there will be presented twistorial formulation of $D = 6$ continuous spin representations. As follows from basic hypotheses of twistor approach, this formulation is an alternative (or additional) to space-time formulation, which is used in most researches. The obtained twistor formulation of $6D$ infinite spin representations is a certain generalization of the twistor formulation of such representations in $4D$ case, which we found earlier [I.Buchbinder, SF, A.Isaev, A.Rusnak, 2018-2019].

Besides, in this talk there will be presented light-front description of $6D$ infinite spin fields. A pleasant bonus in constructing such a formulation was the appearance of harmonics in the theory obtained.

We begin with definition of the infinite (continuous) spin representations in $6D$ Minkowski space.

Casimir operators and $6D$ irreducible massless representations

$$P_a\,, \qquad W_{abc}=\varepsilon_{abcdeg}P^dM^{eg}\,, \qquad \Upsilon_a=\varepsilon_{abcdeg}P^bM^{cd}M^{eg}\,.$$

commute with P_a. Then, the operators

$$C_2=P^aP_a\,, \qquad C_4=\frac{1}{24}\,W^{abc}W_{abc}\,, \qquad C_6=\frac{1}{64}\,\Upsilon^a\Upsilon_a$$

are the second, fourth and sixth order Casimir operators of $iso(1,5)$.

$$\begin{aligned}
C_2 &= P^aP_a\,,\\
C_4 &= \Pi^a\Pi_a - \frac{1}{2}M^{ab}M_{ab}\,C_2\,,\\
C_6 &= -\Pi^bM_{ba}\,\Pi_cM^{ca} + \frac{1}{2}\Big(M^{ab}M_{ab}-8\Big)\,C_4\\
&\quad + \frac{1}{8}\Big[M^{cd}M_{cd}\Big(M^{ab}M_{ab}-8\Big)+2M^{ab}M_{bc}M^{cd}M_{da}\Big]\,C_2\,,
\end{aligned}$$

where

$$\Pi_a := P^b\,M_{ba}\,, \qquad [\Pi_a,\ \Pi_b] = -i\,M_{ab}\,C_2\,.$$

Eigenvalues of the Casimir operators in massless case:

- Finite spin (helicity) representations.

$$C_2 = C_4 = C_6 = 0\,.$$

- Infinite (continuous) spin representations.

$$C_4 \;=\; -\mu^2\,, \qquad \mu \neq 0\,.$$

$$C_6 \;=\; -\mu^2\, s(s+1)\,,$$

where s is fixed (half-)integer number

The field description of $D = 6$ infinite spin representations with additional vector variables was considered in [X.Bekaert, J.Mourad, 2006; X.Bekaert, N.Boulanger, 2006].

Here we propose different field descriptions of such representations.

Twistorial formulation of infinite spin particle

i) Twistor consists of two commuting Lorentz spinors.

ii) Twistors are the phase space coordinates.

iii) In twistors, momentum operators are expressed as a bilinear combinations of twistors.

iv) In twistor space, conformal symmetry is realized by linear transformations.

$6D$ twistors

The $D=(1+5)$ twistor $\quad (\mathcal{A}=1,\ldots,8,\ I=1,2,\ \alpha=1,2,3,4)$

$$Z^I_{\mathcal{A}} = \begin{pmatrix} \pi^I_\alpha \\ \omega^{\beta I} \end{pmatrix}$$

$$(\pi^I_\alpha)^\dagger = \epsilon_{IJ} B_{\dot\alpha}{}^\beta \pi^J_\beta \,, \qquad (\omega^{\alpha I})^\dagger = \epsilon_{IJ}\omega^{\beta J}(B^{-1})_\beta{}^{\dot\alpha}$$

$$\left[\pi^I_\alpha\,,\omega^\beta_J\right] = i\delta^\beta_\alpha \delta^I_J\,.$$

The operators ω^α_I can be realized by differential operators

$$\omega^\alpha_I = -i\,\frac{\partial}{\partial \pi^I_\alpha}\,.$$

The quantities $X_{[\mathcal{A}\mathcal{B}]} := Z^I_{\mathcal{A}} Z^J_{\mathcal{B}} \epsilon_{IJ}$ form the $so(2,6) \simeq so^*(8)$ algebra.

Basic properties of the twistor formulation:

$$P_a = \pi^I_\alpha (\tilde{\sigma}_a)^{\alpha\beta} \pi_{\beta I}$$

is light-like $P_a P^a = 0$ automatically due to $(\tilde{\sigma}^a)^{\alpha\beta} (\tilde{\sigma}_a)^{\gamma\delta} = 2\varepsilon^{\alpha\beta\gamma\delta}$.

The generators M_{ab} of the Lorentz algebra $so(1,5)$

$$M_{ab} = -i\pi^I_\alpha (\tilde{\sigma}_{ab})^\alpha{}_\beta \frac{\partial}{\partial \pi^I_\beta} \,.$$

But, in this one-twistor case $C_4 = C_6 = 0$.

Thus, the one-twistor model describes only massless finite spin representations.

For description of massless infinite spin irreps it is necessary to use two or more twistors.

We introduce the second twistor

$$Y^A_{\mathcal{A}} = \begin{pmatrix} \lambda^A_\alpha \\ \eta^{\beta A} \end{pmatrix} .$$

$$\lambda^A_\alpha\,, \qquad (\lambda^A_\alpha)^* = \epsilon_{AB} B_{\dot\alpha}{}^{\beta} \lambda^B_\beta\,; \qquad \eta^{\alpha A}\,, \qquad (\eta^{\alpha A})^* = \epsilon_{AB}\eta^{\beta B}(B^{-1})_\beta{}^{\dot\alpha}\,.$$

In these expressions, $A = 1, 2$ – SU(2) index.

$$\left[\lambda^A_\alpha\,, \eta^\beta_B\right] = i\delta^\beta_\alpha \delta^A_B\,.$$

$$\eta^\alpha_A = -i\,\frac{\partial}{\partial\lambda^A_\alpha}\,.$$

Physical states are described by the $6D$ two-twistor field

$$\Psi = \Psi(\pi^I_\alpha, \lambda^A_\alpha)\,,$$

which is a function of upper halves of both twistors.

Twistorial infinite spin fields

Bitwistor representation of $6D$ Poincare algebra

$$P_a = \pi^I_\alpha (\tilde\sigma_a)^{\alpha\beta} \pi_{\beta I}\,,$$

$$M_{ab} \;=\; -i\pi^I_\alpha (\tilde\sigma_{ab})^\alpha{}_\beta \frac{\partial}{\partial \pi^I_\beta} - i\lambda^A_\alpha (\tilde\sigma_{ab})^\alpha{}_\beta \frac{\partial}{\partial \lambda^A_\beta}\,.$$

Fourth-order Casimir operator

$$C_4 = \Pi^a \Pi_a = 2\left(\epsilon^{\alpha\beta\gamma\delta}\pi^K_\alpha \pi_{\beta K} \lambda^C_\gamma \lambda_{\delta C}\right)\epsilon^{IJ}\epsilon^{AB}\left(\pi_I \frac{\partial}{\partial\lambda^A}\right)\left(\pi_J \frac{\partial}{\partial\lambda^B}\right)\,.$$

Twistor equations

a) $\left(\epsilon^{\alpha\beta\gamma\delta}\pi^K_\alpha \pi_{\beta K} \lambda^C_\gamma \lambda_{\delta C} \;-\; \mu^2\right)\Psi = 0\,,$ b) $\left(\pi_{\alpha I}\frac{\partial}{\partial \lambda^A_\alpha} \;-\; \frac{i}{2}\,\epsilon_{IA}\right)\Psi = 0\,.$

Six-order Casimir operator

$$C_6 \Psi = -\mu^2 \, J_{\rm i} J_{\rm i} \, \Psi \,,$$

where

$$J_{\rm i} \;:=\; \frac{1}{2}\,\pi^I_\alpha (\sigma_{\rm i})_I{}^J \frac{\partial}{\partial \pi^J_\alpha} \;+\; \frac{1}{2}\,\lambda^A_\alpha (\sigma_{\rm i})_A{}^B \frac{\partial}{\partial \lambda^B_\alpha}$$

The twistor field $\Psi(\pi, \lambda)$ must obey the following condition:

$$J_{\rm i} J_{\rm i} \, \Psi \;=\; s(s+1)\, \Psi \,.$$

So the quantum number s of infinite spin irreps coincides with the spin of the diagonal $\mathsf{SU}(2)$ automorphism subgroup.

Twistor field with quantum number s can be described by means of the completely symmetric $2s$ rank spin-tensor field (I_i are $\mathsf{SU}(2)$-indices)

$$\Psi_{I_1 \ldots I_{2s}}(\pi, \lambda) = \Psi_{(I_1 \ldots I_{2s})}(\pi, \lambda)$$

and is represented by an infinite series in the spinors π^I_α and λ^A_α.

Field twistor transform and space-time infinite spin fields

Field twistor transform links twistor field formulation with space-time one.

For it, we construct the fields

$$\pi_{\alpha_1}^{I_1} \ldots \pi_{\alpha_{2s}}^{I_{2s}} \, \Psi_{I_1 \ldots I_{2s}}(\pi, \lambda)\,,$$

which are SU(2) scalars.

Then, performing the following integral transform

$$\Phi_{\alpha_1 \ldots \alpha_{2s}}(x, \lambda) = \int \mu(\pi)\, e^{ix^a p_a}\, \pi_{\alpha_1}^{I_1} \ldots \pi_{\alpha_{2s}}^{I_{2s}} \, \Psi_{I_1 \ldots I_{2s}}(\pi, \lambda)\,,$$

where $\mu(\pi)$ is the integration measure in the "π-space" and $p_a = \pi_\alpha^I (\tilde{\sigma}_a)^{\alpha\beta} \pi_{\beta I}$, we obtain a completely symmetric space-time field $\Phi_{\alpha_1 \ldots \alpha_{2s}}(x, \lambda)$, which depends on the space-time coordinates x^a and additional spinor variables λ_α^I.

The field $\Phi_{\alpha_1\ldots\alpha_{2s}}(x,\lambda)$ automatically satisfies

$$i\frac{\partial}{\partial x^a}(\tilde{\sigma}^a)^{\beta\alpha_1}\Phi_{\alpha_1\ldots\alpha_{2s}}=0\,,\qquad \frac{\partial}{\partial x^a}\frac{\partial}{\partial x_a}\Phi_{\alpha_1\ldots\alpha_{2s}}=0\,.$$

and

$$\left(i\frac{\partial}{\partial x^a}\;\lambda^K_\beta(\tilde{\sigma}^a)^{\beta\gamma}\lambda_{\gamma K}\;+\;2\mu^2\right)\Phi_{\alpha_1\ldots\alpha_{2s}}=0\,,$$

$$\left(i\frac{\partial}{\partial x^a}\;\frac{\partial}{\partial\lambda^K_\beta}(\sigma^a)_{\beta\gamma}\frac{\partial}{\partial\lambda_{\gamma K}}\;-\;2\right)\Phi_{\alpha_1\ldots\alpha_{2s}}=0\,.$$

In addition, the space-time field $\Phi_{\alpha_1\ldots\alpha_{2s}}(x,\lambda)$ obeys

$$\lambda^I_\beta\;(\sigma_{\rm i})_I{}^K\;\frac{\partial}{\partial\lambda^K_\beta}\;\Phi_{\alpha_1\ldots\alpha_{2s}}=0\,.$$

Thus, we derive space-time formulation (with additional spinor variables λ^I_α) of the $6D$ infinite spin fields which is a generalization of our formulation of the $4D$ infinite spin fields.

Light-front description of $6D$ infinite spin fields

Another space-time formulation of infinite spin representations, more appropriate for light-front field description.

Operator conditions in space-time formulation

We consider the representations in the space of states $|\Psi\rangle$. The basic operators

$$x^a\,, \quad p_a\,; \qquad \xi^I_\alpha\,, \quad \rho^{\alpha I}\,.$$

$$(x^a)^\dagger = x^a\,, \qquad (p_a)^\dagger = p_a\,, \qquad (\xi^I_\alpha)^\dagger = \epsilon_{IJ} B_{\dot\alpha}{}^\beta \xi^J_\beta\,, \qquad (\rho^{\alpha I})^\dagger = \epsilon_{IJ}\rho^{\beta J}(B^{-1})_\beta{}^{\dot\alpha}\,,$$

$$[x^a, p_b] = i\delta^a_b\,, \qquad \left[\xi^I_\alpha\,, \rho^\beta_J\right] = i\delta^\beta_\alpha \delta^I{}_J\,.$$

In this representation

$$C_4 = -\tilde{\ell}\,\ell\,,$$

$$\ell := \frac{1}{2}\,\rho_I^{\alpha}(p_a\sigma^a)_{\alpha\beta}\rho^{\beta I}\,, \qquad \tilde{\ell} := \frac{1}{2}\,\xi_{\alpha}^{I}(p_a\tilde{\sigma}^a)^{\alpha\beta}\xi_{\beta I}\,.$$

$$C_6 \ = \ -\mu^2\,J_{\mathrm{i}}J_{\mathrm{i}}\,,$$

$$J_{\mathrm{i}} \ := \ \frac{i}{2}\,\xi_{\alpha}^{I}(\sigma_{\mathrm{i}})_I{}^J\rho_J^{\alpha}\,.$$

Infinite spin states are defined by the constraints

$$\ell\,|\Psi\rangle \ = \ \mu\,|\Psi\rangle\,, \qquad \tilde{\ell}\,|\Psi\rangle \ = \ \mu\,|\Psi\rangle\,,$$

$$J_{\mathrm{i}}J_{\mathrm{i}}\,|\Psi\rangle \ = \ s(s+1)\,|\Psi\rangle\,.$$

We describe the infinite spin vectors $|\Psi\rangle$ in terms of appropriate fields.

Infinite spin fields in the light-cone frame

Light-cone frame: $p^0 = p^5 = k\,, \quad p^{\hat a} = 0\,,\ \hat a = 1,2,3,4\,.$

Represent $\mathrm{SU}^*(4)$ spinors ξ^I_α, $\rho^{\alpha I}$ as:

$$\xi^I_\alpha = (\xi^I_i, \xi^I_{\underline{i}})\,, \qquad \rho^\alpha_I = (\rho^i_I, \rho^{\underline{i}}_I)\,,$$

where $i = 1,2$ and $\underline{i} = 1,2$.

In the light-cone frame

$$\tilde\ell \;=\; k\,\epsilon^{ij}\epsilon_{IJ}\,\xi^I_i\xi^J_j\,, \qquad \ell \;=\; k\,\epsilon_{\underline{ij}}\epsilon^{IJ}\,\rho^{\underline{i}}_I\rho^{\underline{j}}_J$$

are written in the form

$$\epsilon_{IJ}u^I_i u^J_j = \epsilon_{ij}\,, \qquad \epsilon_{IJ}v^I_{\underline{i}}v^J_{\underline{j}} = \epsilon_{\underline{ij}}\,,$$

$$u^I_i := \sqrt{2k/\mu}\,\xi^I_i\,; \qquad v^I_{\underline{i}} := \sqrt{2k/\mu}\,\rho^I_{\underline{i}}\,, \qquad \rho^I_{\underline{i}} = \epsilon_{\underline{ij}}\epsilon^{IJ}\rho^{\underline{j}}_J\,.$$

$$(u^I_i)^* = -\epsilon_{IJ}\epsilon^{ij}u^J_j\,, \qquad (v^I_{\underline{i}})^* = -\epsilon_{IJ}\epsilon^{\underline{ij}}v^J_{\underline{j}}\,.$$

As a result, in light-cone frame u_i^I and $v_{\underline{i}}^I$ are elements of the $\mathrm{SU}(2)$ groups and parameterize compact space. Analogously to [GIKOS, 1984], we will use the notation:

$$u_i^1 = u_i^+ \,, \quad u_i^2 = u_i^- \,, \qquad v_{\underline{i}}^1 = v_{\underline{i}}^+ \,, \quad v_{\underline{i}}^2 = v_{\underline{i}}^- \,.$$

We consider differential realization for last spinor operators:

$$\rho_I^i = -i\,\frac{\partial}{\partial \xi_i^I} = -i\sqrt{2k/\mu}\,\frac{\partial}{\partial u_i^I} \,, \qquad \xi_{\underline{i}}^I = i\,\frac{\partial}{\partial \rho_I^{\underline{i}}} = i\sqrt{2k/\mu}\,\epsilon_{ij}\epsilon^{IJ}\,\frac{\partial}{\partial v_j^J} \,.$$

In such a representation

$$J_\pm = D_u^{\pm\pm} + D_v^{\pm\pm} \,, \qquad J_3 = \frac{1}{2}\left(D_u^0 + D_v^0\right) \,, \quad \text{where}$$

$$D_u^{\pm\pm} := u_i^\pm \frac{\partial}{\partial u_i^\mp} \,, \quad D_u^0 := u_i^+ \frac{\partial}{\partial u_i^+} - u_i^- \frac{\partial}{\partial u_i^-}$$

are harmonic derivatives [GIKOS, 1984].

As a solution to $J_i J_i \Psi = s(s+1)\Psi$ we take the highest weight vector $\Psi^{(2s)}$:

$$J_+ \Psi^{(2s)} = 0\,, \qquad (J_3 - s)\Psi^{(2s)} = 0\,.$$

This field also obeys the conditions

$$\ell\,\Psi^{(2s)} = \mu\,\Psi^{(2s)}\,, \qquad \tilde{\ell}\,\Psi^{(2s)} = \mu\,\Psi^{(2s)}\,.$$

It is natural to present the solution by using δ-functions:

$$\Psi^{(2s)}(u^\pm, v^\pm) = \delta(\ell - \mu)\delta(\tilde{\ell} - \mu)\,\Phi^{(2s)}(u^\pm, v^\pm)\,,$$

where the field $\Phi^{(2s)}(u^\pm, v^\pm)$ satisfies the conditions

a) $\left(D_u^{++} + D_v^{++}\right)\Phi^{(2s)}(u^\pm, v^\pm) = 0\,,$ b) $\left(D_u^0 + D_v^0 - 2s\right)\Phi^{(2s)}(u^\pm, v^\pm) = 0\,.$

Equation b) means the $\mathrm{U}(1)$ covariance of the field $\Phi^{(2s)}(u^\pm, v^\pm)$:

$$\Phi^{(2s)}(e^{\pm i\varphi}u^\pm, e^{\pm i\alpha}v^\pm) = e^{2si\alpha}\Phi^{(2s)}(u^\pm, v^\pm)\,.$$

The field $\Phi^{(2s)}(u^\pm, v^\pm)$ is in a one-to-one correspondence with the function on the coset space $[\mathrm{SU}(2)\otimes\mathrm{SU}(2)]/\mathrm{U}(1)$.

Note that different type $(\mathrm{SU}_L(2)/\mathrm{U}_L(1)\otimes\mathrm{SU}(2)_R/\mathrm{U}_R(1))$ of the bi-harmonic space was used in [Ivanov, Sutulin, 1994].

General solution

$$\Phi^{(2s)}(u^{\pm}, v^{\pm}) = \sum_{r=0}^{\infty} \Phi^{(2s)}_{k(r)\,\underline{l}(r)}(u^{+}, v^{+})\, y^{k(r)\,\underline{l}(r)}\,, \qquad \text{where}$$

$$\Phi^{(2s)}_{k(r)\,\underline{l}(r)}(u^{+}, v^{+}) = \sum_{\substack{p,q=0,\\ p+q=2s}}^{2s} \phi^{i(p)\,\underline{j}(q)}_{k(r)\,\underline{l}(r)}\, u^{+}_{i(p)} v^{+}_{\underline{j}(q)}\,.$$

These expressions use the following concise notation for the monomials:

$$u^{+}_{i(r)} := u^{+}_{i_1} \ldots u^{+}_{i_r}\,, \qquad v^{+}_{\underline{i}(r)} := v^{+}_{\underline{i}_1} \ldots v^{+}_{\underline{i}_r}\,, \qquad y^{i(r)\,\underline{j}(r)} := y^{i_1\underline{j}_1} \ldots y^{i_r\underline{j}_r}\,.$$

$$y^{i\underline{j}} := u^{i\,+} v^{\underline{j}\,-} - u^{i\,-} v^{\underline{j}\,+}$$

Thus the field $\Phi^{(2s)}(u^{\pm}, v^{\pm})$ is a linear combination with the constant coefficients $\phi^{i(p)\,\underline{j}(q)}_{k(r)\,\underline{l}(r)}$ of an infinite number of basis states $u^{+}_{i(p)} v^{+}_{\underline{j}(q)} y_{k(r)\,\underline{l}(r)}$ and defines the irreducible infinite spin $iso(1,5)$ representation in the light-cone frame.

Light-front field theory

The light-front [Dirac, 1949] is defined as surface $x^+ = (x^0 + x^5)/\sqrt{2}$ in $6D$ Minkowski space $\mathbb{R}^{1,5}$. Coordinate x^+ is interpreted as a "time" evolution parameter.

The role of the Hamiltonian is played by

$$H = P^- \,.$$

Infinite spin fields in the light-front coordinates have the light-cone frame form, where the coefficients $\phi^{i(p)\underline{i}(q)}_{k(r)\,\underline{l}(r)}$ are functions of $x^\pm = (x^0 \pm x^5)/\sqrt{2}$ and $x^{\hat{a}}$:

$$\Phi^{(2s)}(x^\pm, x^{\hat{a}}, u^\pm, v^\pm) \;=\; \sum_{\substack{p,q=0,\\ p+q=2s}}^{2s} \sum_{r=0}^{\infty} \phi^{i(p)\underline{i}(q)}_{k(r)\,\underline{l}(r)}(x^\pm, x^{\hat{a}})\, u^+_{i(p)} v^+_{\underline{j}(q)}\, y^{k(r)\,\underline{l}(r)} \,.$$

Equation of motion is the Schrödinger-type equation

$$\left(-i\frac{\partial}{\partial x^+} - H\right)\Phi^{(2s)}(x^\pm, x^{\hat{a}}, u^\pm, v^\pm) = 0\,, \qquad\qquad H = \frac{p_{\hat{a}}p_{\hat{a}}}{2p^+}\,.$$

Equation of motion has equivalent form

$$\Box\,\Phi^{(2s)}(x^{\pm},x^{\hat a},u^{\pm},v^{\pm}) = 0\,, \quad \text{where} \quad \Box \;:=\; 2\frac{\partial}{\partial x^{+}}\frac{\partial}{\partial x^{-}} - \frac{\partial}{\partial x^{\hat a}}\frac{\partial}{\partial x^{\hat a}}\,.$$

This equation is the equation of motion corresponding to the action

$$S \;=\; \int d^{\,6}x\,du\,dv\;\bar\Phi^{(-2s)}\Box\,\Phi^{(2s)}\,,$$

$d^{\,6}x = dx^{+}dx^{-}d^{\,4}x$ and $dudv$ is bi-harmonic space measure [GIKOS, 1984].
$\bar\Phi^{(-2s)}$ – complex conjugation of $\Phi^{(2s)}$: $\bar\Phi^{(-2s)} \;=\; (\Phi^{(2s)})^{*}\,.$

Derived harmonic light-front approach opens a possibility to construct an interacting theory for $6D$ infinite spin fields. Hamiltonian should go to

$$H \quad\to\quad H + H_{\text{int}}\,.$$

To preserve zero harmonic charge of the action, H_{int} should have zero harmonic charge as well. For example, self-interaction of charged fields $\Phi^{(2s)}$, $s \neq 0$ can only be of an even order, such as $\sim \bar\Phi^{(-2s)}\bar\Phi^{(-2s)}\Phi^{(2s)}\Phi^{(2s)}$.
For fields with different charges there is an additional choices similarly as
$\sim \bar\Phi_1^{(-2s)}\left(\Phi_2^{(0)} + \bar\Phi_2^{(0)}\right)\Phi_1^{(2s)}$ or $\sim\left(\Phi_1^{(q_1)}\Phi_2^{(q_2)}\Phi_3^{(q_3)} + c.c.\right)$ at
$q_1 + q_2 + q_3 = 0$.

Conclusion

In this talk, new results in the description of $6D$ massless infinite spin representations were presented.

- Explicit expressions are found for the Casimir operators of the algebra $iso(1,5)$. It is proved that the infinite spin representation is described by one real parameter μ and one integer or half-integer number s.
- It is shown that the massless infinite spin representation is realized on the two-twistor fields. We present a full set of equations of motion for two-twistor fields.
- A field twistor transform is constructed and infinite spin fields are found in the space-time formulation with an additional spinor coordinate.
- We present a new $6D$ infinite spin field theory in the light-front formulation. For it, we obtain infinite-spin fields in the light-cone frame which depend on two sets of the $\mathrm{SU}(2)$-harmonic variables.

Thank you very much for your attention !