Lepton Flavor Universality Violation (LFUV) in B-decays

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Introduction

The Standard Model (SM) has been tested and confirmed by many experiments.

Nowadays, the focus has shifted beyond the SM by seeking new particles and new interactions.

So far no new particles were observed directly at the Large Hadron Collider (LHC) at CERN.

Fortunately, there are indirect hints for new physics (NP)

Introduction

Semileptonic B-meson decays via charged current $(b \to c\ell\nu_\ell)$

Rare B-meson decays via flavor changing neutral current $(b \to s\ell^+\ell^-)$

The difference of the forward-backward asymmetry in the decays $B \to D^*\mu\nu$ vs $B \to D^*e\nu$ $(\Delta A_{\rm FB})$

All these observables admit an interpretation in terms of Lepton Flavor Universality Violation (LFUV).

Anomalies and LFUV

Three generations of leptons in the Standard Model (SM):

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}.$$

The SM assumes that the interactions of leptons are universal, i.e. the same for the three generations, and differ only because of their different masses.

Therefore, if we refer to LFUV we mean interactions with different couplings to electrons, muons and tau leptons that directly distinguish among the leptons at the Lagrangian level.

Anomalies and LFUV

$b \to c\tau\nu$

This charged current transition is already mediated at tree-level in the SM and the corresponding decays have significant branching ratios ($\mathcal{O}(10^{-3})$).

- The differential decay rate, $\mathrm{d}\Gamma$, for semileptonic decays involving $D^{(*)}$ mesons depends on both m_ℓ^2 and q^2, the invariant mass squared of the lepton pair

$$\frac{\mathrm{d}\Gamma^{\mathrm{SM}}(\bar{B} \to D^{(*)}\ell^- \bar{\nu}_\ell)}{\mathrm{d}q^2} = \underbrace{\frac{G_F^2\,|V_{cb}|^2\,|p^*_{D^{(*)}}|\,q^2}{96\pi^3 m_B^2}\left(1 - \frac{m_\ell^2}{q^2}\right)^2}_{\text{universal and phase space factors}}$$

$$\times \underbrace{\left[(|H_+|^2 + |H_-|^2 + |H_0|^2)\left(1 + \frac{m_\ell^2}{2q^2}\right) + \frac{3m_\ell^2}{2q^2}|H_s|^2\right]}_{\text{hadronic effects}} .$$

- The four helicity amplitudes $H_\pm, H_0, H_s$ capture the impact of hadronic effects. They depend on the spin of the charm meson and on q^2: $m_\ell^2 \leq q^2 \leq (m_B - m_{D^*})^2$.

Anomalies and LFUV

$b \to c\tau\nu$

Measurements of the ratios of semileptonic branching fractions remove the dependence on $|V_{cb}|$, lead to a partial cancellation of theoretical uncertainties related to hadronic effects, and reduce of the impact of experimental uncertainties.

Here the ratios

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}, \quad D^{(*)} = D \text{ or } D^*, \quad \ell = e \text{ or } \mu$$

Current SM predictions

$$R^{\mathrm{SM}}(D) = 0.299 \pm 0.003, \qquad R^{\mathrm{SM}}(D^*) = 0.258 \pm 0.005$$

to be compared with the averages of the experimantal measurements

$$R(D) = 0.340 \pm 0.030, \qquad R(D^*) = 0.295 \pm 0.014.$$

There is deviation around 3σ.

Anomalies and LFUV

$b \to c\tau\nu$

The LHCb collaboration reported about measurement of the ratio of semileptonic branching fractions $R(J/\psi)$:

$$R(J/\psi) = \frac{\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)} = 0.71 \pm 0.25.$$

R. Aaij et al. [LHCb], Phys. Rev. Lett. 120, no.12, 121801 (2018) [arXiv:1711.05623 [hep-ex]]

This result lies within 2σ deviations of the range of central values predicted from the Standard Model, 0.24 to 0.29. Thereby, the above results are supported by this measurement.

Anomalies and LFUV

$b \to s\ell^+\ell^-$

The flavor changing neutral current processes $b \to s\ell^+\ell^-$ transitions are loop and CKM suppressed in the SM, resulting in branching ratios which are of 10^{-6}

The ratios

$$R(K^{(*)}) = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)}$$

are particularly prominent. They are measured by LHCb and Belle and their theory predictions are very clean (within the SM) since the dependence on the form factors drops out to an excellent approximation. The experimental results obtained for several bins are compatible with the SM expectations at the level of $2.1 - 2.5\ \sigma$.

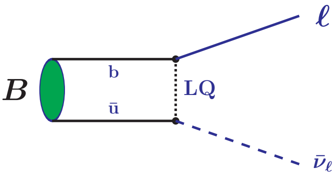
There are some $b \to s\ell^+\ell^-$ transitions that deviate from the SM predictions:

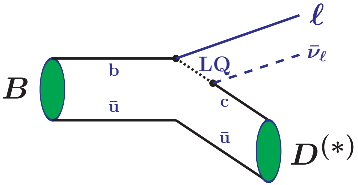
- the optimized angular decay observable $P_5^{\prime\mu}$
- the total branching ratios $\mathcal{B}(B \to K^*\mu^+\mu^-)$, $\mathcal{B}(B \to K\mu^+\mu^-)$ and $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$
- the purely leptonic decay $B_s \to \mu^+\mu^-$ displays a tension.

Theoretical attempts to explain

- **A charged Higgs boson H^- in two-Higgs doublet models. The H^- would mediate weak decays, similar to the W^-, but couple differently to leptons of different mass.**

- **LeptoQuarks = LQ, hypothetical particles with both electric and color charges that allow transitions from quarks to leptons and vice versa.**





Effective theories

Weak decays of hadrons are mediated by weak interactions of their quark constituents.

The goal is to derive an effective theory of quarks at energy scale of the order 1 GeV.

The theoretical framework is provided by the operator product expansion (OPE).

It allows to separate short- and long- distance dynamics.

Effective theories

The short-distance effects can be treated perturbatively in terms of four-fermion operators.

The long-distance effects are encoded in hadronic matrix elements of these operators. Their calculation requires information about the structure of hadrons and therefore cannot be done in perturbation theory.

A variety of theoretical approaches have been applied to this problem.

Example: $b \to c\bar{u}s$-quark transitions

The W-exchange tree-level amplitude:

$$-\frac{g_2^2}{8} V_{us}^\dagger V_{cb}\ (\bar{s} O^\mu u) \left[\frac{-g_{\mu\nu}}{M_W^2 - k^2} \right] (\bar{c} O^\nu b)$$

The momentum transfer $|k| << M_W$:

$$\frac{-g_{\mu\nu}}{M_W^2 - k^2} \longrightarrow \frac{-g_{\mu\nu}}{M_W^2} \equiv -\left(\frac{8}{g_2^2}\right)\left(\frac{G_F}{\sqrt{2}}\right) g_{\mu\nu}$$

Thus we arrive at effective Hamiltonian

$$\mathcal{H}_{\rm eff}^{\rm tree} = \frac{G_F}{\sqrt{2}} V_{us}^\dagger V_{cb}\, Q_2, \qquad Q_2 \equiv (\bar{s}_\alpha O^\mu u_\alpha)(\bar{c}_\beta O_\mu b_\beta)$$

By taking into account QCD corrections:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{us}^\dagger V_{cb} \left[C_1(\mu) Q_1 + C_2(\mu) Q_2 \right],$$

$$Q_1 \equiv (\bar{s}_\alpha O^\mu u_\beta)(\bar{c}_\beta O_\mu b_\alpha)$$

Effective Hamiltonian

Using the operator product expansion (OPE) formalism and renormalization group techniques, the effective Hamiltonian of the weak decays is derived.

$$A(f \to i) = \langle f | \mathcal{H}_{\text{eff}} | i \rangle = \frac{G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_k \underbrace{C_k(\mu)}_{\text{SD}} \underbrace{\langle f | Q_k(\mu) | i \rangle}_{\text{LD}}$$

- SD = Short-Distance contributions
- LD = Long-Distance contributions
- The Wilson coefficients $C_i(\mu)$ are calculated by using "matching" the full and effective theories, and the renormalization group.
- $Q_k(\mu)$ are the local operators generated by electroweak interactions and QCD
- The problem is to evaluate the matrix elements $\langle f | Q_k(\mu) | i \rangle$

Covariant confined quark model

Quark currents

$$J_M(x) = \int dx_1 \int dx_2\, F_M(x;x_1,x_2) \cdot \bar{q}^a_{f_1}(x_1)\, \Gamma_M\, q^a_{f_2}(x_2) \qquad \text{Meson}$$

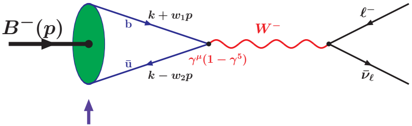
$$\begin{aligned} J_B(x) &= \int dx_1 \int dx_2 \int dx_3\, F_B(x;x_1,x_2,x_3) \qquad \text{Baryon} \\ &\times \Gamma_1\, q^{a_1}_{f_1}(x_1) \left[\varepsilon^{a_1a_2a_3} q^{T\,a_2}_{f_2}(x_2)\, C\, \Gamma_2\, q^{a_3}_{f_3}(x_3)\right] \end{aligned}$$

$$\begin{aligned} J_T(x) &= \int dx_1 \ldots \int dx_4\, F_T(x;x_1,\ldots,x_4) \qquad \text{Tetraquark} \\ &\times \left[\varepsilon^{a_1a_2c} q^{T\,a_1}_{f_1}(x_1)\, C\Gamma_1\, q^{a_2}_{f_2}(x_2)\right] \cdot \left[\varepsilon^{a_3a_4c} \bar{q}^{T\,a_3}_{f_3}(x_3)\, \Gamma_2 C\, \bar{q}^{a_4}_{f_4}(x_4)\right] \end{aligned}$$

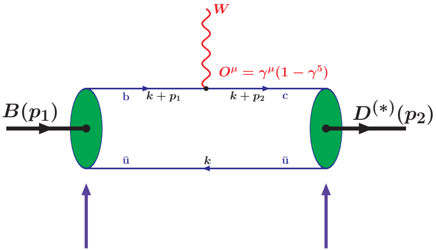
Vertex functions

$$F_H(x;x_1,\ldots,x_n) = \delta\left(x - \sum_{i=1}^{n} w_i x_i\right) \Phi_H\left(\sum_{i<j}(x_i - x_j)^2\right), \qquad w_i = \frac{m_i}{\sum\limits_{j=1}^{n} m_j}$$

Matrix elements



$\phi_B\left(-k^2\right)$ $\quad w_1 = \frac{m_b}{m_b+m_u}, \quad w_2 = 1 - w_1$



$\phi_B\left(-\left(k + w_{bu}\, p_1\right)^2\right)$ $\quad \phi_{D^{(*)}}\left(-\left(k + w_{cu}\, p_2\right)^2\right)$

$w_{bu} = \frac{m_u}{m_b+m_u}$ $\quad w_{cu} = \frac{m_u}{m_c+m_u}$

Leptonic decay constants

Pseudoscalar mesons

$$N_c\, g_P \int \frac{d^4k}{(2\pi)^4 i}\, \widetilde{\Phi}_P(-k^2)\, \mathrm{tr}\Big[O^{\mu} S_1(k + w_1 p)\gamma^5 S_2(k - w_2 p)\Big] = f_P p^{\mu}$$

Vector mesons

$$N_c\, g_V \int \frac{d^4k}{(2\pi)^4 i}\, \widetilde{\Phi}_V(-k^2)\, \mathrm{tr}\Big[O^{\mu} S_1(k + w_1 p)\; \not\epsilon_V S_2(k - w_2 p)\Big] = m_V f_V \epsilon_V^{\mu}$$

Analyzing New Physics in the decays $B \to D^{(*)}\tau\nu_\tau$

M.A. Ivanov, J.G. Körner and C.T. Tran, Phys. Rev. D 94, no.9, 094028 (2016)

Effective Hamiltonian for the quark-level transition $b \to c\tau^-\bar{\nu}_\tau$:

$$\boxed{\mathcal{H}_{eff} \propto G_F V_{cb}\left[(1+V_L)\mathcal{O}_{V_L} + V_R\mathcal{O}_{V_R} + S_L\mathcal{O}_{S_L} + S_R\mathcal{O}_{S_R} + T_L\mathcal{O}_{T_L}\right]}$$

where the four-fermion operators are written as

$$\mathcal{O}_{V_L} = (\bar{c}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu P_L\nu_\tau) \qquad \mathcal{O}_{V_R} = (\bar{c}\gamma^\mu P_R b)(\bar{\tau}\gamma_\mu P_L\nu_\tau)$$

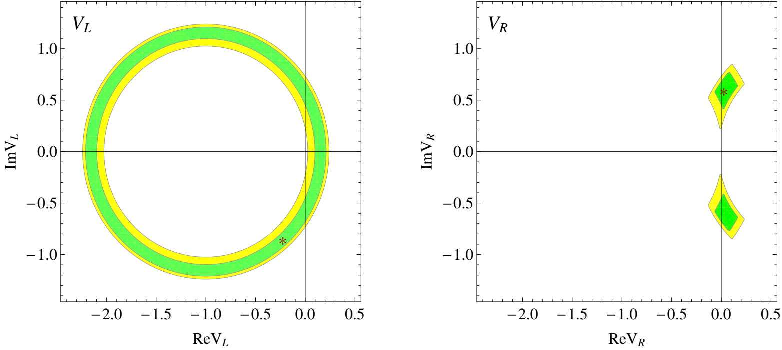
$$\mathcal{O}_{S_L} = (\bar{c}P_L b)(\bar{\tau}P_L\nu_\tau) \qquad \mathcal{O}_{S_R} = (\bar{c}P_R b)(\bar{\tau}P_L\nu_\tau)$$

$$\mathcal{O}_{T_L} = (\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\tau}\sigma_{\mu\nu}P_L\nu_\tau)$$

- Here, $\sigma_{\mu\nu} = i\,[\gamma_\mu,\gamma_\nu]\,/2$, $\quad P_{L,R} = (1\mp\gamma_5)/2$.
- $V_{L,R}$, $S_{L,R}$, and T_L - complex Wilson coefficients governing NP.
- In the SM: $V_{L,R} = S_{L,R} = T_L = 0$.
- Neutrino is always left handed.
- NP only affects leptons of the third generation.

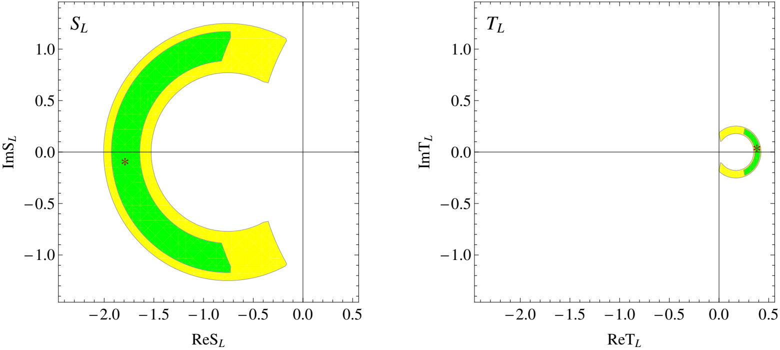
Allowed regions for NP couplings

Assuming that besides the SM contribution, only one of the NP operators is switched on at a time, and NP only affects the tau modes.



The allowed regions of the Wilson coefficients $V_{L,R}$ within 2σ (green, dark) and 3σ (yellow, light).
The best fit value in each case is denoted with the symbol $*$.

Allowed regions for NP couplings



The allowed regions of the Wilson coefficients S_L, and T_L within 2σ (green, dark) and 3σ (yellow, light).

Allowed regions for NP couplings

- It is important to note that while determining these regions, we also take into account a theoretical error of 10% for the ratios $R(D^{(*)})$.
- The operator $\mathcal{O}_{S_R}$ is excluded at 2σ and is not presented here.
- In each allowed region at 2σ we find the best-fit value for each NP coupling.

$$\begin{aligned} V_L &= -1.33 + i\,1.11, & V_R = 0.03 - i\,0.60, \\ S_L &= -1.79 - i\,0.22, & T_L = 0.38 - i\,0.06. \end{aligned}$$

Form-factor-independent test of lepton universality

S. Groote, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli and C. T. Tran,

Phys. Rev. D 103, no.9, 093001 (2021)

Generic differential $(q^2, \cos\theta)$ distribution for the semileptonic decays

$$\bar{B}^0 \to D^{(*)+}\ell^-\bar{\nu}_\ell, \qquad B_c^- \to J/\psi(\eta_c)\ell^-\bar{\nu}_\ell, \qquad \Lambda_b \to \Lambda_c\ell^-\bar{\nu}_\ell$$

is written as

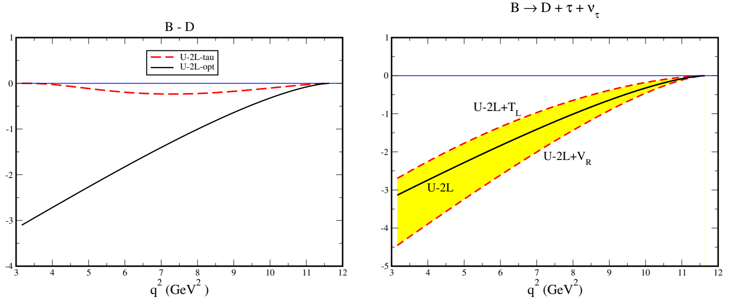
$$\frac{d^2\Gamma(q^2,\ell)}{dq^2\, d\cos\theta} \propto \, v^2\Big(A_0(q^2,\ell) + A_1(q^2,\ell)\cos\theta + v\, A_2(q^2)\cos^2\theta\Big)$$

The velocity type factor $v = 1 - m_\ell^2/q^2$ factors out in the quadratic $\cos^2\theta$ coefficient. Therefore one can define an optimized partial rates: $d\Gamma^{\rm opt}(q^2)/dq^2 \propto A_2(q^2)$ which are the same in the SM for all three (e,μ,τ) modes in the common phase space $m_\tau^2 < q^2 \le (m_1 - m_2)^2$:

$$d\Gamma^{\rm opt}(q^2)|_e = d\Gamma^{\rm opt}(q^2)|_\mu = d\Gamma^{\rm opt}(q^2)|_\tau$$

This equality is form-factor independent. In this way one can test μ/e, τ/μ and τ/e lepton universality regardless of form-factor effects. New Physics (NP) contributions designed to strengthen the τ rate will clearly lead to a violation of these equalities.

Form-factor-independent test of lepton universality



Left panel: q^2 dependence of the optimized partial rate $d\Gamma^{\rm optd}_{U-2L}/dq^2$ (solid curve) and $d\Gamma_{U-2L}/dq^2 = v^3\, d\Gamma^{\rm optd}_{U-2L}/dq^2$ (τ-mode, dashed curve) in units of 10^{-15} GeV^{-1}

Right panel: $P \to P'$ (V) semileptonic transitions taking into account NP effects for the τ mode.

The optimized partial rate $\Gamma^{\rm optd}_{U-2L}$ in units of 10^{-14} GeV

$q^2_{\rm min}$	$B \to D$	$B_c \to \eta_c$	$B \to D^*$	$B_c \to J/\psi$	$\Lambda_b - \Lambda_c$
m_τ^2	-1.14	-1.21	-0.73	-0.49	-0.90
4 GeV^2	-0.89	-0.93	-0.54	-0.36	-0.71

The decays $B_c \to J/\psi + \bar{\ell}\nu_\ell$ and $B_c \to J/\psi + \pi(K)$

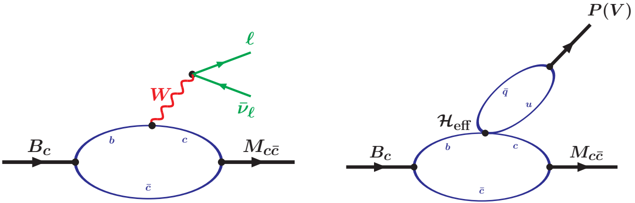
A. Issadykov and M.A. Ivanov, Phys. Lett. B 783, 178-182 (2018)

The LHCb Collaboration reported on the measurements of the ratios of the branching fractions:

$$\mathcal{R}_{\pi^+/\mu^+\nu} = \frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)} = 0.0469 \pm 0.0028(\mathrm{stat}) \pm 0.0046(\mathrm{syst})$$

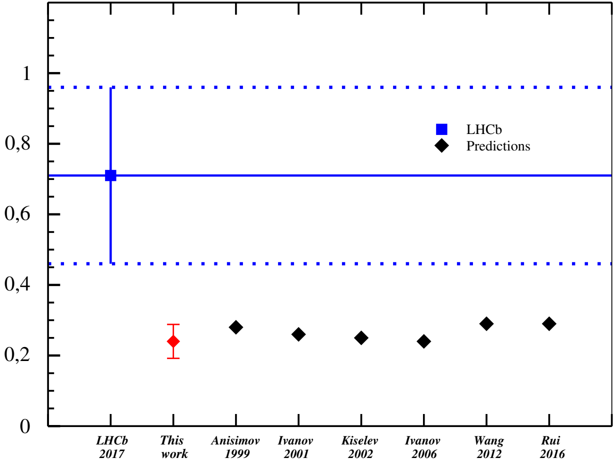
$$\mathcal{R}_{K^+/\pi^+} = \frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = \begin{cases} 0.069 \pm 0.019(\mathrm{stat}) \pm 0.005(\mathrm{syst}) \\ 0.079 \pm 0.007(\mathrm{stat}) \pm 0.003(\mathrm{syst}) \end{cases}$$

$$\mathcal{R}_{J/\psi} = \frac{\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)} = 0.71 \pm 0.17(\mathrm{stat}) \pm 0.18(\mathrm{syst})$$



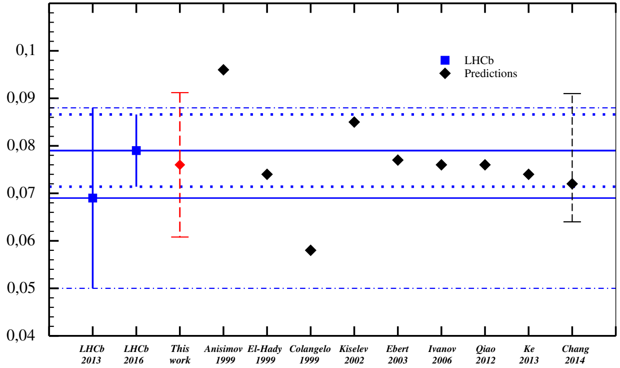
Pictorial representation of the semileptonic and nonleptonic B_c decays in the CCQM.

Theoretical predictions vs. LHCb data for the ratio $\mathcal{R}_{\mathcal{J}/\psi}$



Solid line-central experimental value, dotted lines–experimental error bar.

Theoretical predictions vs. LHCb data for the ratio $\mathcal{R}_{K^+/\pi^+}$



Two solid lines- central experimental values, dash-dotted and dotted lines –experimental error bar from two LHCb-experiments.

Some observations

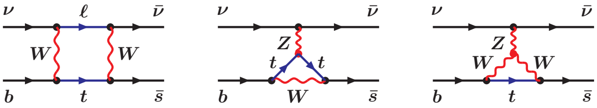
The theoretical predictions for the ratio $\mathcal{R}_{J/\psi}$ are more than $2\ \sigma$ less than the experimental data.

At the same time the predictions for the ratio $\mathcal{R}_{K/\pi}$ are well consistent with the experimental data.

This might be very important since it may imply that the new physics has strong couplings to the leptons but not hadrons.

The rare decay $B \to K(K^*) + \nu\bar{\nu}$

One loop diagrams giving the leading contributions to the transition $b \to s + \nu\bar{\nu}$



Effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\frac{\alpha_{em}}{2\pi} \frac{X_t}{\sin^2\theta_W} \right] (\bar{s} O_\mu b)(\bar{\nu} O^\mu \nu) + \text{h.c.}$$

The function X_t is calculated in the leading order (LO) plus subleading contributions. The results of such precision calculations are given by

$$X_t = 1.469 \pm 0.017\,.$$

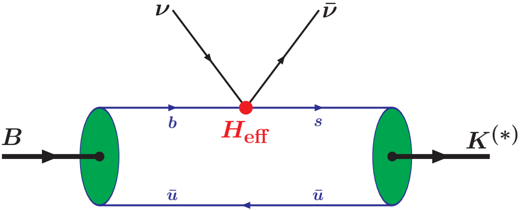
Matrix elements and branching fractions

Matrix elements:

$$M(B \to K^{(*)}\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{2\pi}\frac{X_t}{\sin^2\Theta_W} V_{tb}\, V_{ts}^{*} \langle K^{(*)} \,|\, \bar{s}\, O^{\mu}\, b\, | B\rangle \langle \bar{\nu}^{s_1}\, O_{\mu}\, \nu^{s_2}\rangle .$$

where $K^{(*)} = K$ or K^{*}.

The quark diagram describing the decay $B \to K^{(*)}\nu\bar{\nu}$



Branching fractions

The differential branching fraction is written as

$$\frac{d\mathcal{B}(B^+ \to K^{(*)+} + \nu\bar{\nu})}{dq^2} = 3\tau_{B^+} \frac{(G_F \lambda_t \alpha_{em})^2}{3(2\pi)^5} \left(\frac{X_t}{\sin^2\theta_W} \right)^2 \times \frac{|\mathbf{p}_2|}{4m_1^2} \left(\tilde{H}_+^2 + \tilde{H}_-^2 + \tilde{H}_0^2 \right).$$

A factor of 3 at the beginning of the formula results from the summation by neutrino flavors: ν_e, ν_μ, ν_τ.
The scaled helicity amplitudes $\tilde{H}$ are written down

$B \to K$ transition:

$$\tilde{H}_\pm = 0, \qquad \tilde{H}_0 = 2\, m_1\, |\mathbf{p}_2|\, F_+.$$

$B \to K^*$ transition:

$$\tilde{H}_\pm = \frac{\sqrt{q^2}}{m_1 + m_2} \Big(-Pq\, A_0 \pm 2\, m_1\, |\mathbf{p}_2|\, V \Big),$$

$$\tilde{H}_0 = \frac{1}{m_1 + m_2} \frac{1}{2\, m_2} \Big(-Pq\, (Pq - q^2)\, A_0 + 4\, m_1^2\, |\mathbf{p}_2|^2\, A_+ \Big).$$

Numerical results

The branching fractions of the decays $B \to K^{(*)}\nu\bar{\nu}$

	CCQM	Buras [1]	BaBar [2]	Belle [3]
$10^6\,\mathcal{B}(K^+)$	4.96 ± 0.74	3.98 ± 0.47	< 17 (90% CL)	–
$10^6\,\mathcal{B}(K^{*\,+})$	9.57 ± 1.43	9.19 ± 0.99	–	< 40 (90% CL)

1. A. J. Buras, J. Girrbach-Noe, C. Niehoff and D. M. Straub, JHEP 02, 184 (2015)
2. J. P. Lees et al. [BaBar], Phys. Rev. D 87, no.11, 112005 (2013)
3. O. Lutz et al. [Belle], Phys. Rev. D 87, no.11, 111103 (2013)

Summary

The observed enhancements of the tauonic mode in the (semi)leptonic $B(B_c)$-meson decay may indicate a violation of lepton universality.

We have provided an analysis of possible NP in the decays $\bar{B}^0 \to D^{(*)}\tau^-\bar{\nu}_\tau$ using the form factors obtained from our covariant quark model. Starting with a general effective Hamiltonian including NP operators, we have derived the full angular distribution and defined a large set of physical observables which helps discriminate between NP scenarios.

In addition, searches for lepton universality violation in semileptonic decays of Λ_b-baryons are being planned.

We have proposed a form-factor-independent test of lepton universality for semileptonic B meson, B_c meson, and Λ_b baryon decays by analyzing the two-fold $(q^2, \cos\theta)$ decay distribution.

Summary

The calculated branching fractions $\mathcal{R}_{\pi^+/\mu^+\nu}$ and $\mathcal{R}_{K^+/\pi^+}$ are in good agreement with the LHCb data and other theoretical approaches. At the same time the theoretical predictions for the ratio $\mathcal{R}_{J/\psi}$ are more than $2\ \sigma$ less than the experimental data. This may indicate on the possibility of New physics effects in this decay.

At present, the measurements are limited by the available experimental uncertainties

The future data will show whether the obtained results are an indication of beyond-the-SM physics or the result of larger-than-expected statistical or systematic deviations.

A confirmation of new physics contributions in these decays would change our understanding of matter and trigger an intense program of experimental and theoretical research.