The process $e^+e^- \to \Lambda\bar{\Lambda}$ in the vicinity of charmonium $\psi(3770)$

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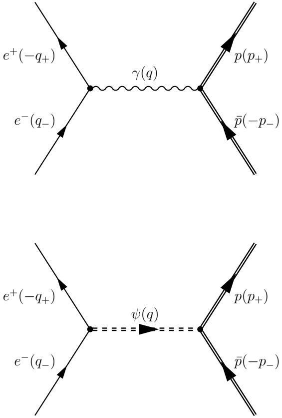
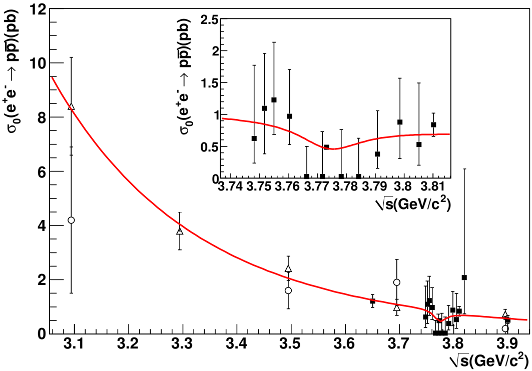
Charmonium $\psi(3770)$ production at BES

Large statistics of J/ψ, $\psi(2S)$ and $\psi(3770)$ samples have been obtained in recent years by BEPCII/BESIII facility [BESIII Collaboration, M. Ablikim *et al.*, Phys.Lett. **B710**, 594 (2012)]. It provides the possibility to study many decay channels of J/ψ, $\psi(2S)$ and $\psi(3770)$ resonances.

In a profound work BESIII has measured the phase angle ϕ between the continuum and the resonant amplitude [M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014)].



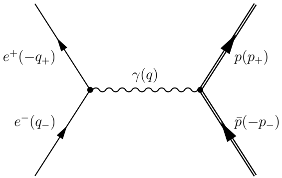


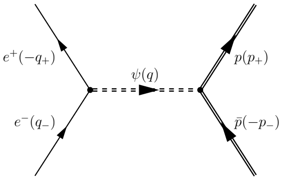
Charmonium $\psi(3770)$ production at BES

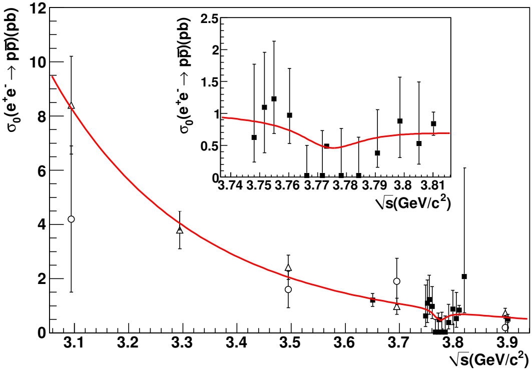
Fit parameterization was like

$$\sigma\left(s\right)=\left|A_{con}+A_{\psi}\,e^{i\phi}\right|^{2}=\left|\sqrt{\sigma_{con}\left(s\right)}+\sqrt{\sigma_{\psi}}\frac{M_{\psi}\Gamma_{\psi}}{s-M_{\psi}^{2}+iM_{\psi}\Gamma_{\psi}}e^{i\phi}\right|^{2}, \quad (1)$$

$$\sigma_{con}\left(s\right)=\frac{4\pi\alpha^{2}\beta}{3s}\left(1+\frac{2M_{p}^{2}}{s}\right)\left|G\left(s\right)\right|^{2}. \quad (2)$$







$$\sigma(s) = \left|A_{con} + A_\psi\, e^{i\phi}\right|^2 = \left|\sqrt{\sigma_{con}(s)} + \sqrt{\sigma_\psi}\,\frac{M_\psi \Gamma_\psi}{s - M_\psi^2 + iM_\psi\Gamma_\psi} e^{i\phi}\right|^2, \qquad (3)$$

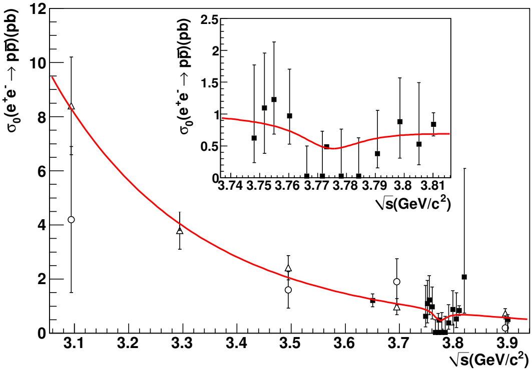
Fitting procedure found two possible solutions:

Solution	σ_ψ (pb)	ϕ ($^\circ$)
(1)	$0.059 \pm 0.032 \pm 0.012$	$255.8 \pm 37.9 \pm 4.8$
(2)	$2.57 \pm 0.12 \pm 0.12$	$266.9 \pm 6.1 \pm 0.9$

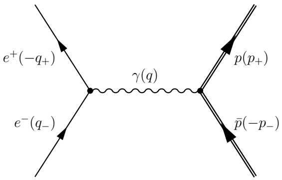
Effective proton form factor was parameterized as:

$$|G(s)| = \frac{C}{s^2 \ln^2(s/\Lambda^2)},$$

with $\Lambda = 0.3$ GeV is the QCD scale parameter and $C = (62.6 \pm 4.1)$ GeV^4.



Born approximation



We consider the mechanisms of creation of a $p\bar{p}$ pair in electron–positron collisions which proceeds through virtual photon intermediate state

$$\mathcal{M}_B = \frac{1}{s} J_\mu^{e\bar{e}\to\gamma}(q)\, J^\mu_{\gamma\to p\bar{p}}(q), \tag{4}$$

where lepton $J_\mu^{e\bar{e}\to\gamma}$ and proton $J^\mu_{\gamma\to p\bar{p}}$ currents have a form:

$$J_\mu^{e\bar{e}\to\gamma}(q) = -e\left[\bar{v}(q_+)\gamma_\mu u(q_-)\right], \tag{5}$$

$$J_\mu^{\gamma\to p\bar{p}}(q) = e\left[\bar{u}(p_+)\Gamma_\mu(q)v(p_-)\right], \tag{6}$$

where proton electromagnetic vertex $\Gamma_\mu(q)$ is parameterized in a standard way:

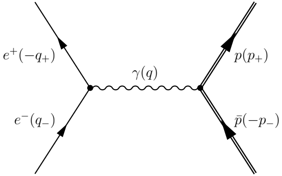
$$\Gamma_\mu(q) = F_1(q^2)\gamma_\mu - \frac{F_2(q^2)}{4M_p}\left(\gamma_\mu\hat{q} - \hat{q}\gamma_\mu\right), \tag{7}$$

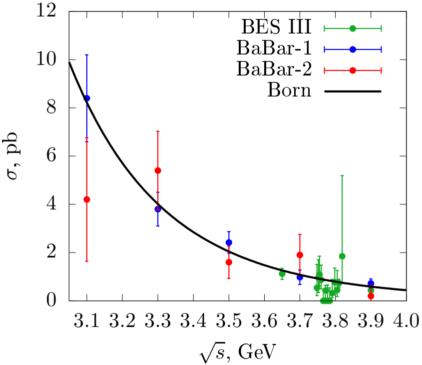
where form factors $F_{1,2}(q^2)$ are related to electric $G_E(q^2)$ and magnetic $G_M(q^2)$ form factors of the proton. We use effective form factor approach and assume that $|G_E| = |G_M|$ and thus:

$$F_1(s) = |G(s)| = \frac{C}{s^2\log^2\left(s/\Lambda^2\right)}, \qquad F_2(s) = 0, \tag{8}$$

where $\Lambda = 0.3$ GeV is the QCD scale parameter and $C = (62.6 \pm 4.1)$ GeV^4 was fitted in [M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014)].

Born approximation



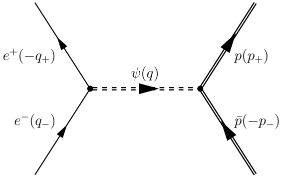


The corresponding contribution to the differential cross section then reads as

$$\frac{d\sigma}{d\cos\theta_p} = \frac{\pi\alpha^2\beta_p}{2s}\left(2 - \beta_p^2 \sin^2\theta_p\right)|F_1(s)|^2\,, \tag{9}$$

where $s = q^2 = (q_+ + q_-)^2 = 4E^2$ is the total invariant energy of the process, E is the electron beam energy in the center-of-mass reference frame, β_p is the proton velocity ($\beta_p^2 = 1 - M_p^2/E^2$). The total cross section then is equal to:

$$\sigma_B(s) = \frac{2\pi\alpha^2}{3s}\beta_p\left(3 - \beta_p^2\right)|F_1(s)|^2\,. \tag{10}$$



The second mechanism describes the conversion of electron–positron pair to $\psi(3770)$ with the subsequent conversion to the proton–antiproton pair. For this aim we put the whole matrix element as

$$\mathcal{M} = \mathcal{M}_B + \mathcal{M}_\psi, \tag{11}$$

where the contribution with $\psi(3770)$ intermediate state is

$$\mathcal{M}_\psi = \frac{g^{\mu\nu} - q^\mu q^\nu / M_\psi^2}{s - M_\psi^2 + iM_\psi \Gamma_\psi} J_\mu^{e\bar{e}\to\psi}(q) J_\nu^{\psi\to p\bar{p}}(q). \tag{12}$$

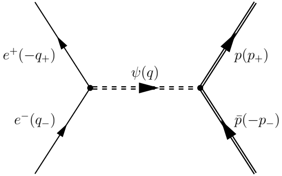
Here we assumed that vertex $\psi \to e^+e^-$ has the same structure as $\gamma \to e^+e^-$, i.e.:

$$J_\mu^{e\bar{e}\to\psi}(q) = g_e \left[\bar{v}(q_+)\gamma_\mu u(q_-)\right], \tag{13}$$

and the constant g_e is defined via $\psi \to e^+e^-$ decay

$$|g_e| = \sqrt{\frac{12\pi\Gamma_{\psi\to e^+e^-}}{M_\psi}} = 1.6 \cdot 10^{-3}. \tag{14}$$

We should also note that in paper [E.A. Kuraev, Yu.M. Bystritskiy and E. Tomasi-Gustafsson, Nucl. Phys. A **920**, 45 (2013).] it was shown that the vertexes of this type do not generate large imaginary part and thus we can neglect their contribution to the relative phase ϕ.



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where the contribution with $\psi(3770)$ intermediate state is

$$\mathcal{M}_\psi = \frac{g^{\mu\nu} - q^\mu q^\nu / M_\psi^2}{s - M_\psi^2 + iM_\psi \Gamma_\psi} J_\mu^{e\bar{e}\to\psi}(q) J_\nu^{\psi\to p\bar{p}}(q). \tag{16}$$

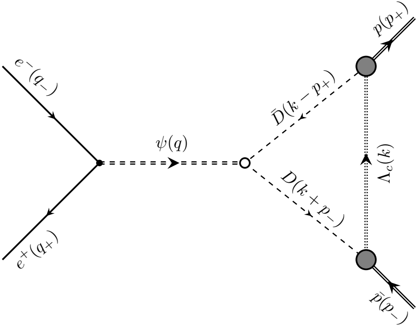
Thus we can present the interference contribution to the total cross section in the form:

$$\sigma_{int}(s) = \mathrm{Re}\left(\frac{S_i(s)}{s - M_\psi^2 + iM_\psi \Gamma_\psi}\right), \tag{17}$$

where $S_i(s)$ contains all the dynamics of the transformation of charmonium into proton-antiproton pair and has the following explicit form:

$$S_i(s) = \frac{e g_e \beta_p}{48\pi s} \int\limits_{-1}^{1} d\cos\theta_p \sum_{s'} \left(J^\alpha_{\gamma\to p\bar{p}}\right)^* J_\alpha^{\psi\to p\bar{p}}. \tag{18}$$

D-meson loop mechanism



The main contribution seems to be generated by D-meson loop mechanism:

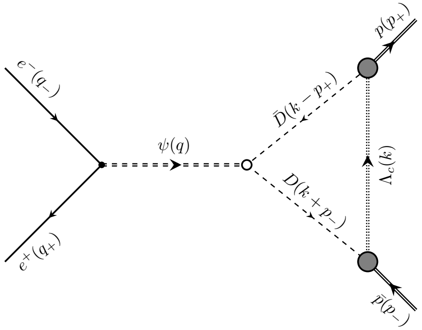
$$S_D(s) = \frac{\alpha\, g_e}{2^4\, 3\pi^2}\beta_p\, F_1(s)\, Z_D(s)\,,$$

$$Z_D(s) = \frac{1}{s}\int \frac{dk}{i\pi^2} G_{\psi D\bar{D}}(s,(k+p_-)^2,(k-p_+)^2)\times$$
$$\times\, \frac{SpD(s,k^2)}{\left(k^2-M_\Lambda^2\right)\left((k-p_+)^2-M_D^2\right)\left((k+p_-)^2-M_D^2\right)}\times$$
$$\times\, G_{\Lambda DP}\left(k^2,(k-p_+)^2\right) G_{\Lambda DP}\left(k^2,(k+p_-)^2\right),$$

and $SpD(s,k^2)$ is the trace of γ-matrices over the baryon line:

$$SpD(s,k^2) = \mathrm{Sp}\left[(\hat{p}_+ + M_p)\gamma_5(\hat{k}+M_\Lambda)\gamma_5(\hat{p}_- - M_p)(\hat{k}-M_p)\right]. \qquad (19)$$

D-meson loop mechanism

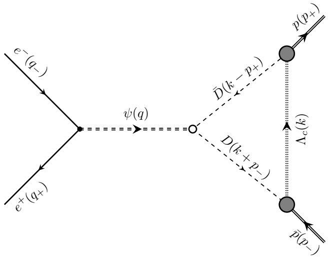


The calculation is done using dispersion relation with one subtraction and the imaginary part of Z_D is found by using Cutkosky rule:

$$\frac{1}{(k+p_-)^2 - M_D^2} \longrightarrow -2\pi i\ \delta\left((k+p_-)^2 - M_D^2\right),$$
$$\frac{1}{(k-p_+)^2 - M_D^2} \longrightarrow -2\pi i\ \delta\left((k-p_+)^2 - M_D^2\right),$$

which gives the following result for the $\mathrm{Im}\, Z_D\,(s)$ which is non-zero above the threshold ($s > 4M_D^2$):

$$\mathrm{Im}\, Z_D\,(s) = -\frac{2\pi}{s^{3/2}} G_{\psi D\bar{D}}(s, M_D^2, M_D^2) \times$$
$$\times \int\limits_{C_k^{(1)}}^{1} \frac{dC_k}{\sqrt{D_1}} \sum_{i=1,2} \frac{k_{(i)}^2}{k_{(i)}^2 + M_\Lambda^2}\ SpD(s, -k_{(i)}^2)\ G_{\Lambda DP}^2\left(-k_{(i)}^2, M_D^2\right).$$



The dynamical dependence of function $G_{\psi D\bar{D}}(s, M_D^2, M_D^2)$ we propose following to G.P. Lepage, S.J. Brodsky, Phys. Lett. B **87**, 359 (1979) in the form:

$$G_{\psi D\bar{D}}\left(s, M_D^2, M_D^2\right) = g_{\psi D\bar{D}} \frac{M_\psi^2}{s} \frac{\log\left(M_\psi^2/\Lambda_D^2\right)}{\log\left(s/\Lambda_D^2\right)},$$

where constant $g_{\psi D\bar{D}}$ is fixed by the $\psi \to D\bar{D}$ decay width:

$$g_{\psi D\bar{D}} \equiv G_{\psi D\bar{D}}(M_\psi^2, M_D^2, M_D^2) = 4\sqrt{\frac{3\pi\,\Gamma_{\psi\to D\bar{D}}}{M_\psi\,\beta_D^3}} = 18.4,$$

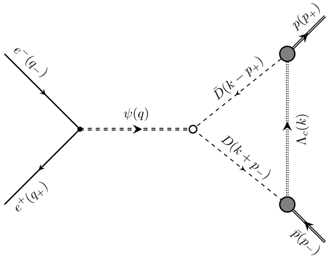
where $\beta_D = \sqrt{1 - 4M_D^2/M_\psi^2}$ is the D-meson velocity in this decay. For ΛDP-vertex with the off-mass-shell D-meson we follow the results of L. Reinders, H. Rubinstein, S. Yazaki, Phys. Rept. **127**, 1 (1985):

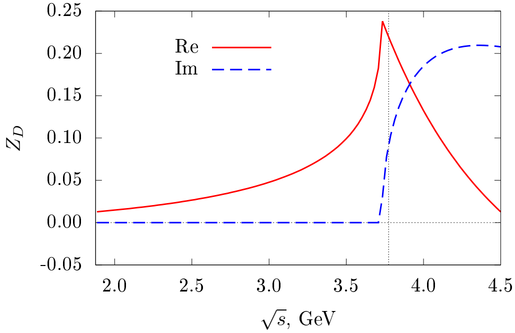
$$g_D(p^2) = \frac{2M_D^2 f_D}{m_u + m_c} \frac{g_{DN\Lambda}}{p^2 - M_D^2}, \qquad (20)$$

where $f_D \approx 180$ MeV. The constant $g_{DN\Lambda} \approx 6.74$ was estimated in F. Navarra, M. Nielsen, Phys. Lett. B **443**, 285 (1998) in scattering regime, i.e. for $p^2 < 0$. Thus in our calculation in we use the following expression:

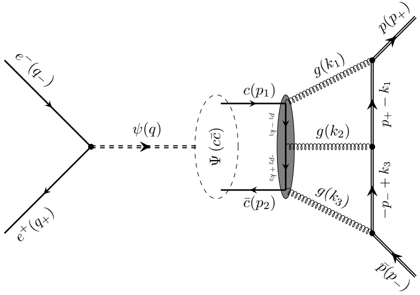
$$G_{\Lambda DP}(-k_{(i)}^2, M_D^2) = \frac{f_D\, g_{DN\Lambda}}{m_u + m_c}. \qquad (21)$$

D-meson loop mechanism





Three gluon mechanism



In paper A.I. Ahmadov, Yu.M. Bystritskiy, E.A. Kuraev and P. Wang, Nucl. Phys. B **888**, 271-283 (2014) it was shown that important possible contribution to relative phase comes from OZI-violated three gluon mechanism:

$$S_{3g}(s) = \frac{\alpha\,\alpha_s^3}{2^3\,3} g_e\, g_{col}\,\phi\,\beta_p\,F_1(s)\;G_\psi(s)\,Z_{3g}(s)\,,$$

where quantity ϕ is related with the charmonium wave function:

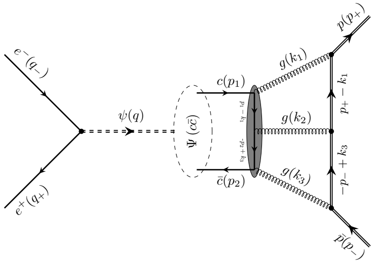
$$\phi = \frac{|\Psi(\mathbf{r}=\mathbf{0})|}{M_\psi^{3/2}} = \frac{\alpha_s^{3/2}}{3\sqrt{3\pi}},$$

and

$$Z_{3g}(s) = \frac{4}{\pi^5 s}\int \frac{dk_1}{k_1^2}\,\frac{dk_2}{k_2^2}\,\frac{dk_3}{k_3^2}\,\frac{Sp3g\;\delta(q-k_1-k_2-k_3)}{((p_+-k_1)^2-M_p^2)((p_--k_3)^2-M_p^2)}, \qquad (22)$$

where the quantity $Sp3g$ is the product of traces over proton and c-quark lines:

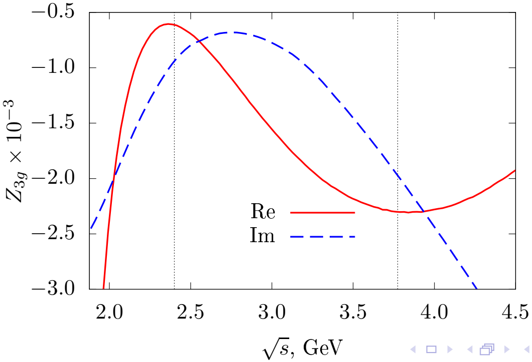
$$Sp3g = \mathrm{Sp}\left[\hat{Q}_{\alpha\beta\gamma}(\hat{p}_1+m_c)\gamma^\mu(\hat{p}_2-m_c)\right]\times$$
$$\times\,\mathrm{Sp}\left[(\hat{p}_++M_p)\gamma^\alpha(\hat{p}_+-\hat{k}_1+M_p)\gamma^\beta(-\hat{p}_-+\hat{k}_3+M_p)\gamma^\gamma(\hat{p}_--M_p)\gamma_\mu\right],$$



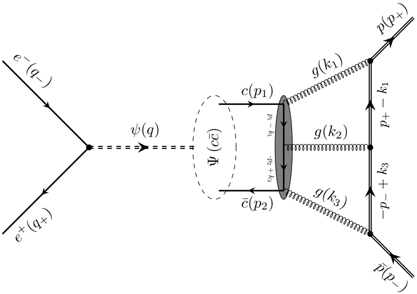
Again we use dispersion relation with one subtraction and the imaginary part of Z_{3g} is found by using Cutkosky rule:

$$\frac{1}{k_1^2 + i0}\,\frac{1}{k_2^2 + i0}\,\frac{1}{k_3^2 + i0} \longrightarrow$$
$$\longrightarrow (-2\pi i)^3\,\delta(k_1^2)\,\delta(k_2^2)\,\delta(k_3^2),$$

which gives the following result for the Im $Z_{3g}\,(s)$:



Three gluon mechanism

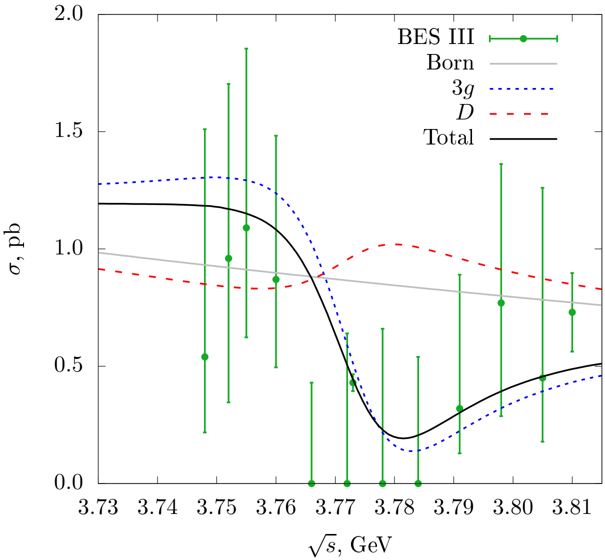


Let us remind that interference contains some number of form factors:

$$S_{3g}(s) = \frac{\alpha\,\alpha_s^3}{2^3\,3} g_e\, g_{col}\, \phi\, \beta_p\, F_1(s)\; G_\psi(s)\, Z_{3g}(s)\,,$$

where $F_1(s)$ is the electromagnetic form factor of the proton in time-like region while $G_\psi(s)$ is some similar form factor which describes the transition of three gluons into proton–antiproton final state:

$$\left|G_\psi(s)\right| = \frac{C_\psi}{s^2\log^2\left(s/\Lambda^2\right)}.$$



Fitting the total curve with BES data gives us:

$$\left|G_\psi(s)\right| = \frac{C_\psi}{s^2 \log^2\left(s/\Lambda^2\right)},$$

with

$$C_\psi = (45 \pm 9) \ \text{GeV}^4.$$

Compare to data of $e^+e^- \to p\bar{p}$ process

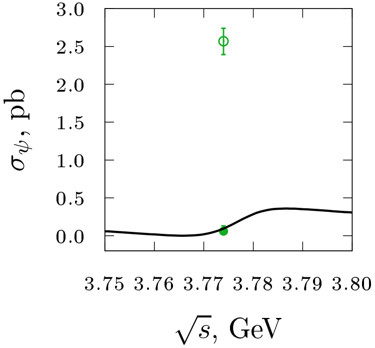
Now we recall the fittings of BES III collaboration from M. Ablikim *et al.* [BESIII Collaboration], Phys. Lett. B **735**, 101 (2014) where they used empirical formula:

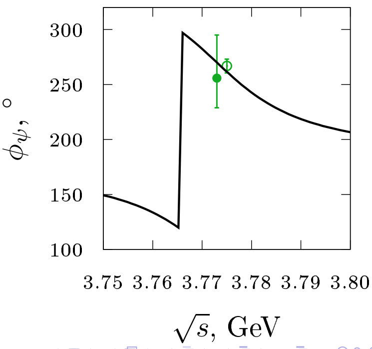
$$\sigma\left(s\right) = \left|A_{con} + A_\psi\, e^{i\phi}\right|^2 = \left|\sqrt{\sigma_{con}\left(s\right)} + \sqrt{\sigma_\psi}\frac{M_\psi \Gamma_\psi}{s - M_\psi^2 + i M_\psi \Gamma_\psi} e^{i\phi_\psi}\right|^2, \qquad (23)$$

and obtained two possible solutions:

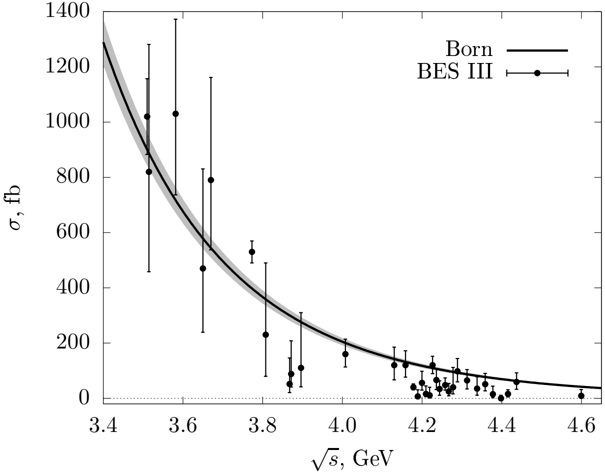
Solution	σ_ψ (pb)	ϕ_ψ ($^\circ$)
(1)	$0.059 \pm 0.032 \pm 0.012$	$255.8 \pm 37.9 \pm 4.8$
(2)	$2.57 \pm 0.12 \pm 0.12$	$266.9 \pm 6.1 \pm 0.9$

So one can see that our fit ($\sigma_\psi = 0.075$ pb, $\phi_\psi = 270^\circ$) prefers one of these solutions:





The process $e^+e^- \to \Lambda\bar{\Lambda}$ at BES III



The data for the cross section is taken from BESIII, M. Ablikim *et al.*, Phys. Rev. D **104**, L091104 (2021).

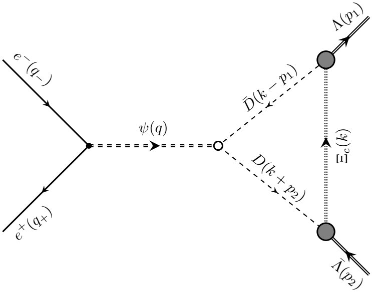
We fit this data with the standard Born cross section with the electromagnetic formfactor of Λ-baryon:

$$|G_\Lambda(s)| = \frac{C_\Lambda}{s^2 \log^2(s/\Lambda^2)},$$

and get

$$C_\Lambda = (43.1 \pm 1.4)\ \text{GeV}^4.$$

The process $e^+e^- \to \Lambda\bar{\Lambda}$: Two mechanisms

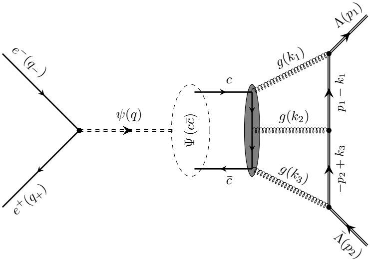


Here we use the same form factor:

$$G_{\Lambda D\Xi}(k^2, M_D^2) = \frac{f_D\, g_{\Lambda D\Xi}}{m_u + m_c}, \qquad k^2 < 0,$$

where

$$g_{\Lambda D\Xi} \approx g_{DN\Lambda} = 6.7 \pm 2.1.$$



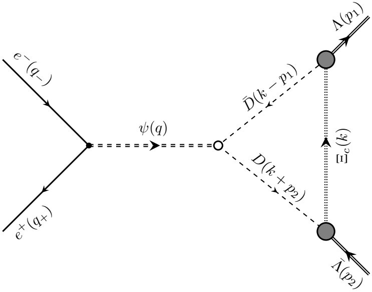
The factor of three gluon transition into $\Lambda\bar{\Lambda}$ has the form:

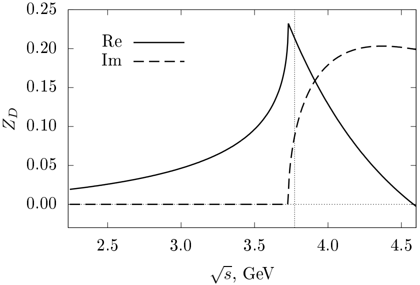
$$\left|G_\psi(s)\right| = \frac{C_\psi}{s^2 \log^2\left(s/\Lambda^2\right)},$$

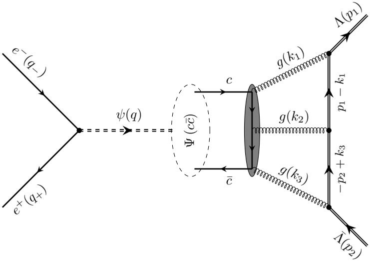
and the constant C_ψ here is the same as in the case of proton–antiproton production since gluons do not feel the flavour of the quarks in the final baryons:

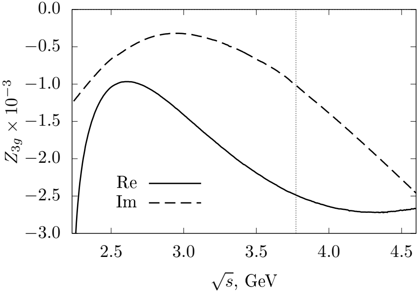
$$C_\psi = (45 \pm 9)\ \text{GeV}^4.$$

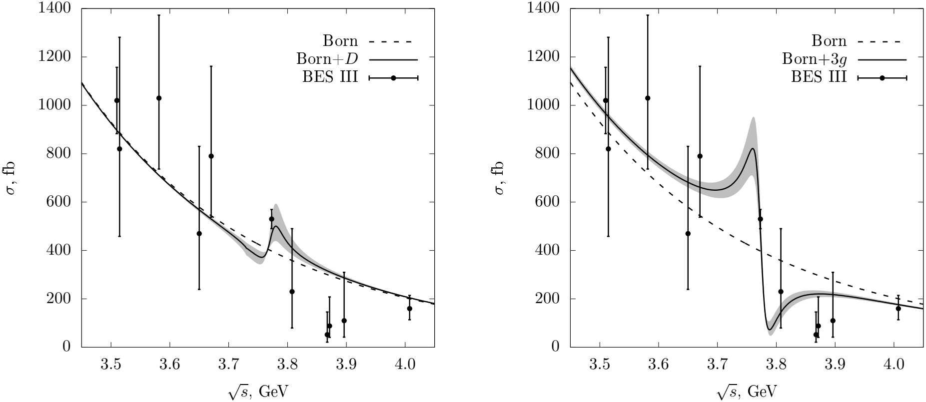
The process $e^+e^- \to \Lambda\bar{\Lambda}$: Two mechanisms





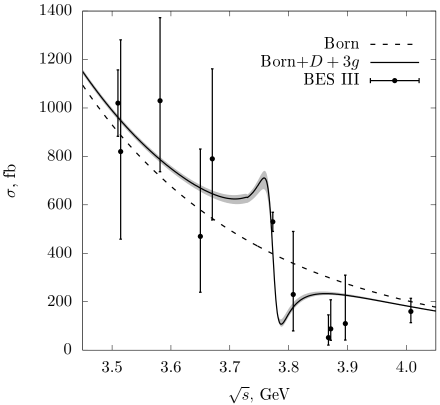


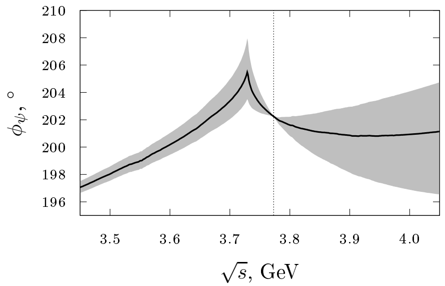




The data are from BESIII, M. Ablikim *et al.*, Phys. Rev. D **104**, L091104 (2021).

The process $e^+e^- \to \Lambda\bar{\Lambda}$: Comparison to data





The data are from BESIII, M. Ablikim *et al.*, Phys. Rev. D **104**, L091104 (2021).

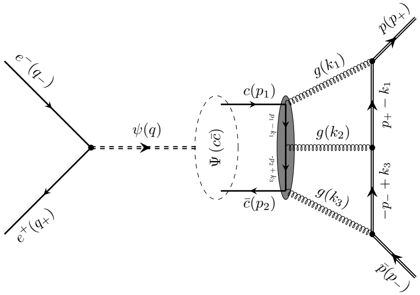
Conclusions

- Three gluon mechanism of charmonium $\psi(3770)$ transition into proton–antiproton pair gives dominant contribution and may explain cross section behavior in the vicinity of the resonance.
- For $\psi(3770)$ transition into $\Lambda\bar{\Lambda}$ pair both mechanisms contribute noticeably, but still three gluon one seems to be responsible for the "shoulders" around the charmonium peak.
- The large phase between Born and charmonium contributions is mostly generated by three gluon mechanism.

Publications

- Yu.M. Bystritskiy, Phys. Rev. D **103**, no.11, 116029 (2021).
- Yu.M. Bystritskiy and A.I. Ahmadov, Phys. Rev. D **105**, no.11, 116012 (2022).

Backup slide: The value of α_s



We note that our calculation of three gluon mechanism contains the QCD coupling constant α_s at charmonium scale (i.e. at $s \sim M_c^2$) in rather high degree:

$$S_{3g}\,(s) = \frac{\alpha\,\alpha_s^3}{2^3\,3} g_e\, g_{col}\, \phi\, \beta_p\, F_1\,(s)\; G_\psi(s)\, Z_{3g}\,(s)\,,$$

and thus it is very sensitive to its value.

We use the value $\alpha_s(M_c) = 0.28$ which is expected by the QCD evolution of α_s from the b-quark scale to the c-quark scale.
We should note that this value differs from the one for J/ψ charmonium for which one must use much smaller value of parameter $\alpha_s(M_c) = 0.19$ (see H. Chiang, J. Hufner, H. Pirner, Phys. Lett. B **324**, 482 (1994)).

Backup slide: The dispersion relation

The dispersion relations we use can be obtained from the standard dispersion relation written over the variable s with subtraction at the point $s = 0$:

$$\text{Re}\, Z_i(s) = \text{Re}\, Z_i(0) + \frac{s}{\pi}\mathcal{P}\int\limits_{s_{\text{min}}}^{\infty} \frac{ds_1}{s_1\,(s_1 - s)}\ \text{Im}\, Z_i(s_1), \tag{24}$$

where s_{min} is the minimal threshold at which Im $Z_i(s_1)$ becomes non-zero. The substraction constant Re $Z_i(0)$ vanishes (i.e. Re $Z_i(0) = 0$) since there are no open charm in the proton and thus in the Compton limit the vertex $\psi \to p\bar{p}$ must vanish.
Next we move to the β_p:

$$\beta_1 = \sqrt{1 - \frac{4M_p^2}{s_1}}, \quad \longrightarrow \quad ds_1 = \frac{8M_p^2\beta_1}{\left(1 - \beta_1^2\right)^2} d\beta_1,$$

and get the final form of dispersion relation:

$$\text{Re}\, Z_i(\beta) = \frac{1}{\pi}\left\{\text{Im}\, Z_i(\beta)\log\left|\frac{1-\beta^2}{\beta_{\text{min}}^2 - \beta^2}\right| + \int\limits_{\beta_{\text{min}}}^{1}\frac{2\beta_1 d\beta_1}{\beta_1^2 - \beta^2}\left[\text{Im}\, Z_i(\beta_1) - \text{Im}\, Z_i(\beta)\right]\right\}. \tag{25}$$

We note that for the D-meson loop contribution the imaginary part Im $Z_D(\beta)$ is non-zero above threshold ($s > 4M_D^2$), thus the lower limit of integration is $\beta_{\text{min}} = \sqrt{1 - M_p^2/M_D^2}$. For the three gluon contribution the threshold for the imaginary part Im $Z_{3g}(\beta)$ coincides with the threshold of the reaction, i.e. $s_{\text{min}} = 4M_p^2$ and thus lower limit of integration is $\beta_{\text{min}} = 0$.