Photon emission accompanying a vacuum instability under the action of a quasiconstant external electric field in graphene

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Dirac model for electronic excitations in graphene

Reduced QED$_{3,2}$:

At the charge neutrality point ($\Theta = \mu = 0$), low-energy electronic excitations in graphene behave as relativistic **Dirac massless fermions in** $2+1$ **dimensions** with the Fermi velocity $v_F \simeq \frac{c}{300}$ playing the role of the speed of light.

Electromagnetic field couples minimally to electrons.

In the reduced QED$_{3,2}$ - **two distinct velocities,** v_F **and** c.

There are actually $N_f = 4$ species of fermions in this model.

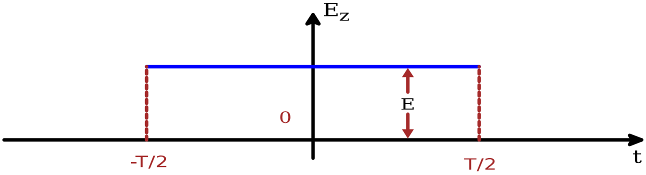
Two species of pseudo spin corresponding to excitations about the two distinct Dirac points in the Brillouin zone and a (real) spin degeneracy factor.

We consider an infinite flat graphene sample on which a uniform electric field is applied, directed along the plane XY of the sample. The applied field is the T-constant electric field.

T-constant electric field

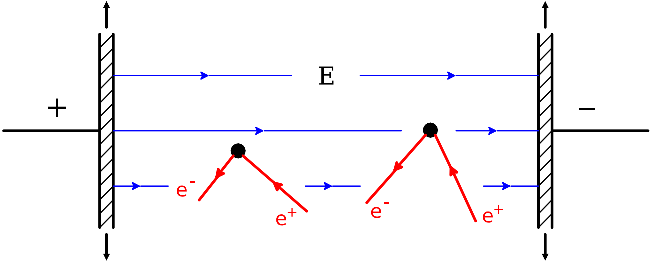
In QED, it was caculated the mean current, $\langle j^1(t)\rangle$, **and EMT of matter,** $\langle T_{\mu\nu}(t)\rangle$, **in one-loop approximation taking an exact account of the interaction with** T-const. **electric field** [Gavrilov, Gitman, PRD 78 (2008); PRL 101 (2008); Gavrilov, Gitman, Yokomizo, PRD 86 (2012)]

T-const. EF turns on to E at $-T/2 = t_{in}$ and turns off to 0 at $T/2 = t_{out}$.



$A_1(t) = -Et$, $t \in [t_{in}, t_{out}]$, being constant for $t \in (-\infty, t_{in})$ and $t \in (t_{out}, +\infty)$.

T-constant electric field in graphene.



In case $\Delta t_{st} < \Delta t < \Delta t_B$, ($\Delta t_{st}$ - non-linear regime, effects of particle-creation reach their asymptotic values; Δt_B - continuous Dirac model)

$$\Delta t_{st} = (e\,|E|\,v_F\,/\,\hbar)^{-1/2} \gg t_\gamma \simeq 0.24\text{fs}, \quad \Delta t_B = 2\pi\hbar(e\,|E|\,a)^{-1},$$
$$L_x \sim 1\mu\text{m}, \quad \Delta t \sim T_{bal} \sim 10^{-12}\text{s}, \quad V = EL_x > 7 \times 10^{-4}\,\text{V},$$

Δt_B is the Bloch time, $a \approx 0.142$ nm is the carbon-carbon distance.
[Gavrilov, Gitman, Yokomizo, PRD 86 (2012)].

Radiative processes

The electromagnetic field is not confined to the graphene surface XY, $z = 0$, but rather propagates in the ambient $3 + 1$ **dimensional space-time**, $e_z \perp XY$

I (classical field radiation):
Low-frequency, $\omega \lesssim T^{-1}$, crossed electric and magnetic fields are radiated in direction orthogonal to graphene plane by a mean current of pairs created from vacuum, $\langle j^1(t) \rangle_g$.
Theory: Maxwell equations.

II (photon emission/absorption):
High-frequency, $\omega \gg T^{-1}$, emission (absorption) of a photon due to a particle state transition. E. g., (1) emission by an electron in initial state, (2) emission with pair creation from vacuum.
Theory: Strong-field reduced $\mathrm{QED}_{3,2}$ in the presence of slowly varying external electric field.

Generalized Furry representation

The quantum Dirac field $\Psi(x)$ in the Heisenberg representation

$$\Psi(x) = \sum_n \left[a_n(in) \; {}_+\psi_n(x) + b_n^\dagger(in) \; {}_-\psi_n(x) \right]$$
$$= \sum_n \left[a_n(out) \; {}^+\psi_n(x) + b_n^\dagger(out)) \; {}^-\psi_n(x) \right].$$

In- and *out*-solutions with given quantum numbers n are related by a linear transformation of the form:

$${}^\zeta\psi_n(x) = g({}_+|^\zeta) \; {}_+\psi_n(x) + g({}_-|^\zeta) \; {}_-\psi_n(x) \; ,$$

where the g's are some complex coefficients. Then a linear canonical transformation (Bogolyubov transformation) between *in*- and *out*- operators is defined by these coefficients. E.g., **differential mean number N_n of particles created**:

$$N_n = \langle 0, in | a_n^\dagger(out) a_n(out) | 0, in \rangle = |g({}_-|^+)|^2$$

Generalized Furry representation

The vacuum mean values of the current density vector $\langle j^{\mu}(t)\rangle$ in the Heisenberg representation

$$\begin{aligned}\langle j^{\mu}(t)\rangle &= \langle 0, in|j^{\mu}|0, in\rangle\,,\\ \langle j^{\mu}(t)\rangle &= \delta_{1}^{\mu}\langle j^{1}(t)\rangle,\ \ \langle j^{1}(t)\rangle \sim n^{cr}\,(t-t_{in})\end{aligned}$$

in-vacuum and *out*-vacuum definitions:

$$\begin{aligned}a_n(in)|0, in\rangle = b_n(in)|0, in\rangle = 0,\ \forall n,\\ a_n(out)|0, out\rangle = b_n(out)|0, out\rangle = 0,\ \forall n.\end{aligned}$$

Here we stress the time-dependence of these mean values, which does exist due to the time-dependence of the external field. The renormalized value $\langle j^{\mu}(t)\rangle$ is **source in equations of motion for mean field**, respectively. In particular, **complete description of the backreaction is related to the calculation of this mean value for any** t.

One-loop mean current in graphene

Multiplied by a degeneracy factor of four, describe, respectively:

$$n_g^{cr} = r_g^{cr} T, \quad r_g^{cr} = \pi^{-2} \left(v_F \hbar^3 \right)^{-1/2} |eE|^{3/2};$$
$$\langle j^1(t) \rangle_g = \mathrm{sgn}(E) 2 e v_F r_g^{cr}$$

These results hold true for all t that satisfy **the stabilization condition** $T > \Delta t_{st}$.

$\langle j^1(t) \rangle_g \sim |E|^{3/2}$ **is a key formula in the study of the conductivity in the graphene at low carrier density beyond the linear response in dc.** It describes **the mean electric current of coherent carriers** produced by the applied electric field.

Dirac model in the Fock space (a reduced QED_3,2)

Nonrelativistic Schrödinger wavefunctions in the three-dimensional space:

$$\phi_\alpha(t,\mathbf{r},z) = \psi_\alpha(t,\mathbf{r})\varphi(z)\,\mathrm{e}^{ip_z z/\hbar}\,,$$

$\varphi(z)$ represents the width of the material and $\mathbf{r}=(x,y)$ is two-dimensional vector on the graphene plane, $z=0$. Each component $\psi_\alpha(t,\mathbf{r})$, $\alpha=1,2$ of the Dirac spinor is a wavefunction with support in a specific sublattice of the honeycomb lattice of graphene. The effective Dirac equation:

$$i\hbar\partial_t\psi(t,\mathbf{r}) = H^{\mathrm{ext}}\psi(t,\mathbf{r})\,,$$
$$H^{\mathrm{ext}} = v_F\gamma^0\left\{\gamma\left[\mathbf{p}+\frac{e}{c}\mathbf{A}^{\mathrm{ext}}(t,\mathbf{r})\right]+mv_F\right\}$$

The interaction with an external electric field is taken into account exactly; [Fradkin,Gitman,Shvartsman, QED with Unstable Vacuum (Springer 1991)].

Dirac model in the Fock space

In the usual dipole approximation, z-dependence of the QED Hamiltonian can be integrated out:

$$\widehat{\mathcal{H}}(t) = \widehat{\mathcal{H}}_{e,\mathbf{A}^{\text{ext}}} + \widehat{\mathcal{H}}_{e,\gamma} + \widehat{\mathcal{H}}_{\gamma} \,,$$

$$\widehat{\mathcal{H}}_{e,\mathbf{A}^{\text{ext}}} = \int \hat{\Psi}^{\dagger}(t,\mathbf{r})\, H^{\text{ext}} \hat{\Psi}(t,\mathbf{r})\, d\mathbf{r},$$

$$\widehat{\mathcal{H}}_{e,\gamma} = -\int \hat{\mathbf{j}}(t,\mathbf{r}) \hat{\mathbf{A}}(t,\mathbf{r}) d\mathbf{r} \,,$$

$$\hat{\mathbf{j}}(t,\mathbf{r}) = -\frac{ev_F}{2c}\left[\hat{\Psi}^{\dagger}(t,\mathbf{r}), \gamma^0 \boldsymbol{\gamma} \hat{\Psi}(t,\mathbf{r})\right]_{-} \,,$$

$\widehat{\mathcal{H}}_{e,\mathbf{A}^{\text{ext}}}$ is the Hamiltonian of the electrons and holes interacting with an electric field given by the time-dependent potential $\mathbf{A}^{\text{ext}}(t,\mathbf{r})$ and $\widehat{\mathcal{H}}_{e,\gamma}$ is the Hamiltonian of the electron-photon interaction.

Dirac model in the Fock space

Quantum electromagnetic field in the representation with the standard annihilation and creation operators of photons, $C_{\mathbf{k}\vartheta}$ and $C^{\dagger}_{\mathbf{k}\vartheta}$:

$$\hat{\mathbf{A}}(t,\mathbf{r}) = c\sum_{\mathbf{K},\vartheta}\sqrt{\frac{2\pi\hbar}{\varepsilon V\omega}}\boldsymbol{\epsilon}_{\mathbf{K}\vartheta}\left[C_{\mathbf{K}\vartheta}\, e^{i(\mathbf{kr}-\omega t)} + C^{\dagger}_{\mathbf{K}\vartheta}\, e^{-i(\mathbf{kr}-\omega t)}\right],$$

$\vartheta = 1, 2$ is a polarization index, the $\boldsymbol{\epsilon}_{\mathbf{K}\vartheta}$ are unit polarization vectors transversal to each other and to three-dimensional wave vector $\mathbf{K} = (\mathbf{k}, k_z)$. $\omega = cK$, $K = |\mathbf{K}|$, V is the volume of the box regularization, and ε is the relative permittivity ($\varepsilon = 1$ for graphene suspended in vacuum).

The initial and final states with definite numbers of charged particles and photons can be generally written in the following way:

$$\begin{aligned} |\mathrm{in} &> = C^{\dagger}\ldots b^{\dagger}(\mathrm{in})\ldots a^{\dagger}(\mathrm{in})\ldots|0,\mathrm{in}\rangle, \\ |\mathrm{out} &> = C^{\dagger}\ldots b^{\dagger}(\mathrm{out})\ldots a^{\dagger}(\mathrm{out})\ldots|0,\mathrm{out}\rangle. \end{aligned}$$

Dirac model in the Fock space

In the Furry representation: The exact probability amplitude for transition from an initial to a final state

$$W =< \mathrm{out}|\mathcal{S}|\mathrm{in} >$$

The generating functional of mean values:

$$< \mathcal{F}\left(t\right) >=< \mathrm{in}|\mathcal{S}^{-1}\mathcal{T}\mathcal{F}\left(t\right)\mathcal{S}|\mathrm{in} >$$

The scattering matrix

$$\mathcal{S} = \mathcal{T}\exp\left\{-\frac{i}{\hbar}\int_{-\infty}^{\infty}\mathcal{H}_{\mathrm{int}}dt\right\},\ \ \mathcal{H}_{\mathrm{int}} \approx -\int \mathbf{j}(t,\mathbf{r})\mathbf{A}(t,\mathbf{r})d\mathbf{r}\ ,$$

$$\mathbf{j}(t,\mathbf{r}) = -\frac{ev_F}{2c}\left[\Psi^{\dagger}\left(t,\mathbf{r}\right),\gamma^0\gamma\Psi\left(t,\mathbf{r}\right)\right]_{-}\ ,$$

Dirac model in the Fock space

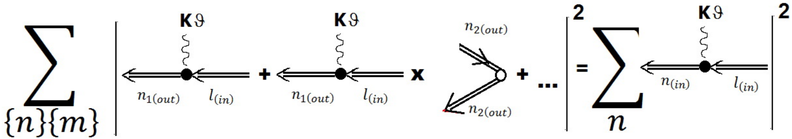
In the first-order approximation

$$\mathcal{S} \approx 1 + i\Upsilon^{(1)}, \ \Upsilon^{(1)} = \frac{1}{\hbar} \int \mathbf{j}(t,\mathbf{r}) \mathbf{A}(t,\mathbf{r}) d\mathbf{r} dt$$
$$\Longrightarrow < \mathrm{out}|\mathcal{S}|\mathrm{in} > \approx i < \mathrm{out}|\Upsilon^{(1)}|\mathrm{in} > .$$

Assuming that $\langle \mathrm{out}|$- and $|\mathrm{in}\rangle$- states are orthogonal for the matrix element of the photon emission.
In general, the emission of a single photon by an electron is accompanied by the creation of $M \geq 0$ electron-hole pairs from the vacuum by the quasiconstant electric field:

$$\mathcal{P}_M \left(\mathbf{K}, \vartheta | \overset{+}{l} \right) = \sum_{\{m\}\{n\}} \left[M! \, (M+1)! \right]^{-1} \left| \langle 0, \mathrm{out} | \, b_{n_M} \, (\mathrm{out}) \dots b_{n_1} \, (\mathrm{ou} \right.$$
$$\left. \times \, a_{m_{M+1}} \, (\mathrm{out}) \dots a_{m_1} \, (\mathrm{out}) \, C_{\mathbf{K}\vartheta} i \Upsilon^{(1)} a_l^\dagger (\mathrm{in}) |0, \mathrm{in} \rangle \right|^2 .$$

Total probability

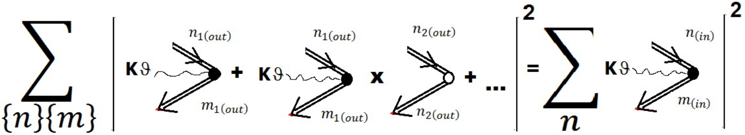


Optical theorem: photon emission by an electron

The total probability of the emission of the given photon

$$\mathcal{P}\left(\mathbf{K},\vartheta|\overset{+}{l}\right)=\sum_{M=0}^{\infty}\mathcal{P}_M\left(\mathbf{K},\vartheta|\overset{+}{l}\right).$$

Total probability



Optical theorem: pair creation with photon emission

$$\mathcal{P}\left(\mathbf{K},\vartheta\right) = \sum_{M=1}^{\infty}\mathcal{P}_{M}\left(\mathbf{K},\vartheta\right),$$

$$\mathcal{P}_{M}\left(\mathbf{K},\vartheta\right) = \sum_{\{m\}\{n\}}\left(M!\right)^{-2}\left|\left\langle 0,\mathrm{out}\right| b_{n_M}\left(\mathrm{out}\right)\ldots b_{n_1}\left(\mathrm{out}\right)\right.$$

$$\left.\times\, a_{m_M}\left(\mathrm{out}\right)\ldots a_{m_1}\left(\mathrm{out}\right) C_{\mathbf{K}\vartheta} i\Upsilon^{(1)}\left|0,\mathrm{in}\right\rangle\right|^{2}$$

Optical theorem

These probabilities can be represented as a trace of the operators $C_{\mathbf{K}\vartheta}\mathcal{S}\left|\mathrm{in}\right\rangle\left\langle\mathrm{in}\right|\mathcal{S}^{-1}C_{\mathbf{K}\vartheta}^{\dagger}$ with respect to the out-basis,

$$\mathcal{P}\left(\mathbf{K},\vartheta|\,\mathrm{in}\right)=\mathrm{tr}\left[C_{\mathbf{K}\vartheta}\mathcal{S}\left|\mathrm{in}\right\rangle\left\langle\mathrm{in}\right|\mathcal{S}^{-1}C_{\mathbf{K}\vartheta}^{\dagger}\right]\ ,$$

where $|\mathrm{in}>:\quad|0,\mathrm{in}\rangle$, $a_l^{\dagger}(\mathrm{in})|0,\mathrm{in}\rangle$, or $b_l^{\dagger}(\mathrm{in})|0,\mathrm{in}\rangle$.
The unitary transformation relates the in and out- Fock spaces, $|\mathrm{in}\rangle=V|\mathrm{out}\rangle$.
This trace can be written as a mean value of the photon number operator,

$$\mathcal{P}\left(\mathbf{K},\vartheta|\,\mathrm{in}\right)=\left\langle\mathrm{in}\right|\mathcal{S}^{-1}C_{\mathbf{k}\vartheta}^{\dagger}C_{\mathbf{k}\vartheta}\mathcal{S}\left|\mathrm{in}\right\rangle$$

In the normal form

$$\mathbf{j}(t,\mathbf{r})=:\mathbf{j}(t,\mathbf{r}):+\langle\mathbf{j}(t,\mathbf{r})\rangle_{\mathrm{in}}\ ,\quad\langle\mathbf{j}(t,\mathbf{r})\rangle_{\mathrm{in}}=\langle 0,\mathrm{in}\left|\mathbf{j}(t,\mathbf{r})\right|0,\mathrm{in}\rangle$$

Total probability

The probability of one photon emission which is accompanied by a pair production from the vacuum (the number of species $N_f = 4$):

$$\mathcal{P}_{N_f}(\mathbf{K},\vartheta) = N_f \mathcal{P}(\mathbf{K},\vartheta), \; \mathcal{P}(\mathbf{K},\vartheta) = \sum_l \mathcal{P}(l;\, \mathbf{K},\vartheta|\, 0),$$

$$\mathcal{P}(l;\, \mathbf{K},\vartheta|\, 0) = \sum_n \left| w_{\text{in}}^{(1)}\left(\overset{-}{n}\overset{+}{l};\, \mathbf{K},\vartheta|\, 0\right)\right|^2 ,$$

$$w_{\text{in}}^{(1)}\left(\overset{-}{n},\overset{+}{l};\, \mathbf{K},\vartheta|\, 0\right) = \frac{ic}{\hbar}\sqrt{\frac{2\pi\hbar}{\varepsilon V\omega}} \int \epsilon_{\mathbf{K}\vartheta}\mathbf{j}_{\text{in}}\left(\overset{-}{n},\overset{+}{l}\middle|\, 0\right) e^{i(\omega t - \mathbf{kr})} dt d\mathbf{r} ,$$

$$\mathbf{j}_{\text{in}}\left(\overset{-}{n},\overset{+}{l}\middle|\, 0\right) = \langle 0, \text{in}\,| b_n(\text{in}) a_l(\text{in}) : \mathbf{j}(t,\mathbf{r}) :|\, 0, \text{in}\rangle ,$$

$$\mathbf{j}_{\text{in}}\left(\overset{-}{n},\overset{+}{l}\middle|\, 0\right) = -\frac{ev_F}{c}\, {}_{+}\bar{\psi}_l(t,\mathbf{r})\, \gamma\, {}_{-}\psi_n(t,\mathbf{r})$$

Total probability

The probability of one photon emission from a single-electron (hole) state:

$$\mathcal{P}\left(\mathbf{K},\vartheta|\overset{\pm}{l}\right)=\sum_{n}\left|w_{\mathrm{in}}^{(1)}\left(\overset{\pm}{n};\mathbf{K},\vartheta|\overset{\pm}{l}\right)\right|^{2},$$

$$w_{\mathrm{in}}^{(1)}\left(\overset{\pm}{n};\mathbf{K},\vartheta|\overset{\pm}{l}\right)=\frac{ic}{\hbar}\sqrt{\frac{2\pi\hbar}{\varepsilon V\omega}}\int\boldsymbol{\epsilon}_{\mathbf{K}\vartheta}\mathbf{j}_{\mathrm{in}}\left(\overset{\pm}{n}\middle|\overset{\pm}{l}\right)e^{i(\omega t-\mathbf{kr})}dtd\mathbf{r},$$

$$\mathbf{j}_{\mathrm{in}}\left(\overset{+}{n}\middle|\overset{+}{l}\right)=\langle 0,\mathrm{in}|\,a_{n}\,(\mathrm{in}):\mathbf{j}(t,\mathbf{r}):a_{l}^{\dagger}(\mathrm{in})|0,\mathrm{in}\rangle,$$

$$\mathbf{j}_{\mathrm{in}}\left(\overset{-}{n}\middle|\overset{-}{l}\right)=\langle 0,\mathrm{in}|\,b_{n}\,(\mathrm{in}):\mathbf{j}(t,\mathbf{r}):b_{l}^{\dagger}(\mathrm{in})|0,\mathrm{in}\rangle,$$

$$\mathbf{j}_{\mathrm{in}}\left(\overset{\pm}{n}\middle|\overset{\pm}{l}\right)=\mp\frac{ev_{F}}{c}\,{}_{\pm}\bar{\psi}_{n}\left(t,\mathbf{r}\right)\gamma\,{}_{\pm}\psi_{l}\left(t,\mathbf{r}\right)$$

Total probability

Using the parametrization , $d\mathbf{K} = c^{-3}\omega^2 d\omega d\Omega$, the probabilties of the emission per unit frequency and solid angle $d\Omega$ are

$$\frac{d\mathcal{P}\left(\mathbf{p};\,\mathbf{K},\vartheta|\,0\right)}{d\omega d\Omega} = \frac{\alpha}{\varepsilon}\left(\frac{v_F}{c}\right)^2 \frac{\omega \Delta t_{st}^2}{(2\pi)^2} \left|M^0_{\mathbf{p}'\mathbf{p}}\right|^2 \Big|_{\mathbf{p}'=\mathbf{p}-\hbar\mathbf{k}},$$

$$\frac{d\mathcal{P}\left(\mathbf{K},\,\vartheta|\,\overset{\pm}{\mathbf{p}}\right)}{d\omega d\Omega} = \frac{\alpha}{\varepsilon}\left(\frac{v_F}{c}\right)^2 \frac{\omega \Delta t_{st}^2}{(2\pi)^2} \left|M^{\pm}_{\mathbf{p}'\mathbf{p}}\right|^2 \Big|_{\mathbf{p}'=\mathbf{p}-\hbar\mathbf{k}}$$

Total probability

$$M^{0}_{\mathbf{p}'\mathbf{p}} = -\exp\left(i\omega\frac{p_x+p'_x}{2eE}\right)\exp\left[-\frac{\pi\left(\lambda+\lambda'\right)}{8}\right]$$
$$\times\left\{i\chi^{0,1}_{\vartheta}\tilde{Y}_{00}+(2eE\hbar)^{-1}v_F(mv_F+i\zeta p'_y)(mv_F+i\zeta p_y)\chi^{1,0}_{\vartheta}\tilde{Y}_{11}\right.$$
$$\left.+\,e^{-i\pi/4}\sqrt{\frac{v_F}{2eE\hbar}}\left[-(mv_F+i\zeta p'_y)\chi^{1,1}_{\vartheta}\tilde{Y}_{10}+(mv_F+i\zeta p_y)\chi^{0,0}_{\vartheta}\tilde{Y}_{01}\right]\right\}$$

$$M^{+}_{\mathbf{p}'\mathbf{p}} = -\exp\left(i\omega\frac{p_x+p'_x}{2eE}\right)\exp\left[-\frac{\pi\left(\lambda+\lambda'\right)}{8}\right]$$
$$\times\left\{\chi^{0,0}_{\vartheta}Y_{00}+(2eE\hbar)^{-1}v_F(mv_F+i\zeta p'_y)(mv_F-i\zeta p_y)\chi^{1,1}_{\vartheta}Y_{11}\right.$$
$$\left.+\sqrt{\frac{v_F}{2eE\hbar}}e^{i\pi/4}\left[(mv_F+i\zeta p'_y)\chi^{1,0}_{\vartheta}Y_{10}-i(mv_F-i\zeta p_y)\chi^{0,1}_{\vartheta}Y_{01}\right]\right\}$$

Total probability

Fourier transformations of the product of two WPC functions:

$$Y_{j'j} = \int_{u(t_1)}^{u(t_2)} D_{-\nu'-j'}[-(1+i)u_-]D_{\nu-j}[-(1-i)u_+]e^{iu_0 u}du,$$

$$\tilde{Y}_{j'j} = \int_{u(t_1)}^{u(t_2)} D_{-\nu'-j'}[-(1+i)u_-]D_{-\nu-j}[-(1+i)u_+]e^{iu_0 u}du,$$

$$u_\pm = u \pm u_x/2,\ u_0 = \Delta t_{st}\omega,\ \nu = \frac{i\lambda}{2},\ \ \lambda = \frac{v_F p_y^2 + m^2 v_F^3}{eE\hbar}$$

$$u(t) = \sqrt{\frac{v_F}{eE\hbar}}\left[eEt - \frac{1}{2}(p_x + p'_x)\right],\quad u_x = \sqrt{\frac{v_F}{eE\hbar}}(p'_x - p_x),$$

$$\chi_\vartheta^{(1-s')/2,(1-s)/2} = U_{s'}^\dagger \gamma^0 \overrightarrow{\gamma} \cdot \vec{\epsilon}_{\mathbf{K}\vartheta} U_s,\ \nu' = \frac{i\lambda'}{2},\quad \lambda' = \lambda|_{p_y \to p'_y}\ .$$

U_s is the constant orthonormalized spinors, $\sigma_x U_s = sU_s$.

$$M^-_{\mathbf{p}'\mathbf{p}} = M^+_{\mathbf{p}\mathbf{p}'}\ .$$

Total probability

To evaluate the angular matrix element $\chi_{\vartheta}^{j',j}$, we define an orthonormal triple

$$\mathbf{K}/K = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)\ ,$$
$$\epsilon_{\mathbf{K}1} = \mathbf{e}_z \times \mathbf{K}/\left|\mathbf{e}_z \times \mathbf{K}\right|\ , \quad \epsilon_{\mathbf{K}2} = \mathbf{K}\times\epsilon_{\mathbf{K}1}/\left|\mathbf{K}\times\epsilon_{\mathbf{K}1}\right|$$

$$\begin{aligned}
\epsilon_{\mathbf{K}1} &= (-\sin\phi,\ \cos\phi,\ 0)\ ,\\
\epsilon_{\mathbf{K}2} &= (-\cos\theta\cos\phi,\ -\cos\theta\sin\phi,\ \sin\theta)
\end{aligned}$$

For $\mathbf{K}$ in the upper spatial region, $k_z \geq 0$:

$$\chi_1^{1,1} = -\chi_1^{0,0} = \sin\phi,\ \chi_1^{0,1} = -\chi_1^{1,0} = i\zeta\cos\phi\ ;$$
$$\chi_2^{1,1} = -\chi_2^{0,0} = \cos\theta\cos\phi,\ \chi_2^{0,1} = -\chi_2^{1,0} = -i\zeta\cos\theta\sin\phi$$

Total probability

These forms can be simplified using the hyperbolic coordinates ρ and φ,

$$\rho = \sqrt{u_0^2 - u_x^2}, \; \tanh\varphi = \frac{u_x}{u_0} \text{ if } u_0^2 - u_x^2 > 0$$

In any frequency range

$$\frac{|u_x|}{u_0} = \frac{|k_x|}{K}\frac{v_F}{c} \leq \frac{v_F}{c}$$

This feature of photon emission is due to the fact that the Fermi velocity v_F in graphene is much smaller than the speed of light c.

Total probability

Exact results:

$$
\begin{aligned}
Y_{j'j} &= \exp\left[\left(i\frac{\lambda'-\lambda}{2}+j'+j-1\right)\varphi\right]\mathcal{J}_{j',j}(\rho),\\
\mathcal{J}_{j',j}(\rho) &= (-1)^{j}\sqrt{\frac{2}{\pi}}\Gamma\left(\nu-j+1\right)e^{i\pi(\nu'+j'-1)/2}\sinh\frac{\pi\lambda}{2}I_{j',1-j}(\rho);\\
\tilde{Y}_{j'j} &= \exp\left[\left(i\frac{\lambda'-\lambda}{2}+j'-j\right)\varphi\right]\tilde{\mathcal{J}}_{j',j}(\rho),\\
\tilde{\mathcal{J}}_{j',j}(\rho) &= e^{i\pi(\nu+\nu'+j+j')/2}I_{j',j}(\rho);\\
I_{j',j}(\rho) &= \sqrt{\pi}\exp\left[\left(\ln\frac{\rho}{\sqrt{2}}-\frac{i\pi}{4}\right)(\nu-\nu'+j-j')+i\frac{\rho^2}{2}-\frac{i\pi}{4}\right]\\
&\quad\times\Psi\left(\nu+j,1+\nu-\nu'+j-j';-i\frac{\rho^2}{2}\right)
\end{aligned}
$$

Natural limits of parameters

A big characteristic time scale in the graphene physics,

$$\Delta t_{st} = \left(|eE|\, v_F \,/\, \hbar\right)^{-1/2} \gg t_\gamma \ .$$

and $t_\gamma = \hbar/\gamma \simeq 0.24\mathrm{fs}$ is the microscopic time scale.

$$\begin{aligned} E &= aE_0, \quad E_0 = 1\times 10^6\,\mathrm{V/m}, \quad 7\times 10^{-4} \ll a \ll 8, \\ V &= EL_x, \ 7\times 10^{-4}\,\mathrm{V} \ll V \ll 8\,\mathrm{V} \end{aligned}$$

Characteristic frequency

$$\omega_{sc} = \Delta t_{st}^{-1} \approx \sqrt{a}\times 0.39\times 10^{14}\mathrm{s}^{-1}$$

The natural range of the very low frequency of emission, $\omega \lesssim \omega^{\mathrm{IR}} = 2\pi T^{-1}$.
It is enough to restrict the applicability of the perturbation theory by the condition $\omega > \omega^{\mathrm{IR}}$,

$$u_0 = \frac{\omega}{\omega_{sc}} > u_0^{\mathrm{IR}}, \ u_0^{\mathrm{IR}} = \frac{\omega^{\mathrm{IR}}}{\omega_{sc}} = 2\pi\frac{\Delta t_{st}}{T}$$

The high frequency case

The wide high frequency range:

$$1 < u_0 = \frac{\omega}{\omega_{sc}} < u_0^{\max}, \quad u_0^{\max} \approx \frac{2T}{\Delta t_{st}}$$

At the saddle-point, $u = \rho/2$, $P_x(t) \approx P'_x(t)$:

$$v_F\left[\left|P_x(t)\right| + \left|P'_x(t)\right|\right] = \hbar\omega,$$
$$P_x(t) = p_x - eEt < 0, \; P'_x(t) = p'_x - eEt < 0$$

The saddle-point - the center of the formation interval $\Delta t \sim \Delta t_{st}$:

$$t_c = \frac{1}{2}\left(\Delta t_{st} u_0 + \frac{p_x + p'_x}{eE}\right)$$

Photon with such a frequency is emitted during the formation interval preceding the moment t_2 of switching off the electric field

$$\omega_2 \approx \frac{2\left|P_x(t_2)\right|}{eE\Delta t_{st}}\omega_{sc}, \; \omega_{\max} \approx \frac{2T}{\Delta t_{st}}\omega_{sc}$$

Natural limits of parameters

Choosing $T = T_{bal} \sim 10^{-12}\mathrm{s}$, we find

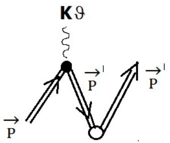
$$u_0^{\max} \approx 78\sqrt{a},\ \omega_{\max} \approx 3.0 \times 10^{15} a\,\mathrm{s}^{-1}\ .$$

Characteristic wavelength scale is:

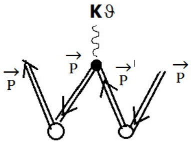
$$l_{sc} = \frac{2\pi c}{\omega_{sc}} \approx \frac{48}{\sqrt{a}} \times 10^{-6}\mathrm{m}$$

For example, in the case of the typical voltage $\sim 1\,\mathrm{V}$ $(a \sim 1)$ the corresponding high-frequency range is a mid-wavelength infrared. We stress that terahertz-field induced spontaneous optical emission in the range of 340–600 nm was observed from a monolayer graphene on a glass substrate [I.V. Oladyshkin and etal, Phys. Rev. B 96, 155401 (2017)]

Total probability



emission by an electron



pair creation with photon emission

Angular and polarization distribution

The leading contributions:

$$\left|M^{+}_{\mathbf{p}'\mathbf{p}}\right|^2\Big|_{\mathbf{p}'=\mathbf{p}-\hbar\mathbf{k}} \approx f\left(\lambda,\lambda'\right)\left|\chi^{0,1}_{\vartheta}\right|^2, \quad \left|M^{-}_{\mathbf{p}'\mathbf{p}}\right|^2 = \left|M^{+}_{\mathbf{p}\mathbf{p}'}\right|^2,$$

$$f\left(\lambda,\lambda'\right) = 2\pi \sinh\frac{\pi\lambda}{2}\exp\left[-\frac{\pi}{4}\left(5\lambda+7\lambda'\right)\right];$$

$$\left|M^{0}_{\mathbf{p}'\mathbf{p}}\right|^2\Big|_{\mathbf{p}'=-\mathbf{p}-\hbar\mathbf{k}} \approx \tilde{f}\left(\lambda,\lambda'\right)\left|\chi^{0,1}_{\vartheta}\right|^2, \quad \tilde{f}\left(\lambda,\lambda'\right) = e^{-\pi\left(\lambda+\lambda'\right)};$$

$$\left|\chi^{0,1}_{1}\right|^2 = \cos^2\phi, \quad \left|\chi^{0,1}_{2}\right|^2 = \cos^2\theta\sin^2\phi,$$

$$\cos^2\phi = \frac{k_x^2}{k_x^2+k_y^2}, \quad \cos^2\theta = \frac{k_z^2}{K^2},$$

$$\lambda = \left(v_F\Delta t_{st}\right)^2\left|p_y/\hbar\right|^2, \quad \lambda' = \left(v_F\Delta t_{st}\right)^2\left|p_y/\hbar - k_y\right|^2$$

Angular and polarization distribution

Unpolarized emission

$$\left|\chi_1^{0,1}\right|^2 + \left|\chi_2^{0,1}\right|^2 = 1 - \sin^2\phi\left(1 - \cos^2\theta\right) = 1 - \frac{k_y^2}{K^2}$$

Total probabilities at high frequencies

$$\frac{d\mathcal{P}\left(\mathbf{K}|\overset{\pm}{\mathbf{p}}\right)}{d\omega d\Omega} = \frac{\alpha}{\varepsilon}\left(\frac{v_F}{c}\right)^2 \frac{\omega\Delta t_{st}^2}{(2\pi)^2}\left|M^{\pm}_{\mathbf{p}'\mathbf{p}}\right|^2\Big|_{\mathbf{p}'=\mathbf{p}-\hbar\mathbf{k}},$$

$$\frac{d\mathcal{P}\left(\mathbf{K},\vartheta|\overset{\pm}{\mathrm{in}}\right)}{d\omega d\Omega} = \frac{S}{(2\pi\hbar)^2}\int \frac{d\mathcal{P}\left(\mathbf{K},\vartheta|\overset{\pm}{\mathbf{p}}\right)}{d\omega d\Omega} N_{\mathbf{p}}^{(\pm)}(\mathrm{in})\,dp_x dp_y;$$

$$\frac{d\mathcal{P}\left(\mathbf{p};\,\mathbf{K},\vartheta|\,0\right)}{d\omega d\Omega} \approx \frac{\alpha}{\varepsilon}\left(\frac{v_F}{c}\right)^2 \frac{\omega\Delta t_{st}^2}{(2\pi)^2}\left|M^{0}_{\mathbf{p}'\mathbf{p}}\right|^2\Big|_{\mathbf{p}'=\mathbf{p}-\hbar\mathbf{k}},$$

$$\frac{d\mathcal{P}\left(\mathbf{K},\vartheta\right)}{d\omega d\Omega} = \frac{N_f S}{(2\pi\hbar)^2}\int_D \frac{d\mathcal{P}\left(\mathbf{p};\,\mathbf{K},\vartheta|\,0\right)}{d\omega d\Omega}\,dp_x dp_y$$

Total probability at high frequencies

The total probability of the emission which accompanies the pair productionfrom the initial vacuum state per unit frequency and solid angle:

$$\frac{d\mathcal{P}\left(\mathbf{K},\vartheta\right)}{d\omega d\Omega} \approx \mathcal{R}\left(\mathbf{K},\vartheta\right) ST,\ \mathcal{R}\left(\mathbf{K},\vartheta\right) = d\left(\omega,\omega_y\right)\left|\chi_{\vartheta}^{0,1}\right|^2,$$

$$d\left(\omega,\omega_y\right) = \frac{\alpha N_f}{\varepsilon\sqrt{2}\left(2\pi\right)^2}\frac{\omega}{\omega_{sc} l_{sc}^2}\exp\left[-\frac{\pi}{2}\left(\frac{v_F\omega_y}{c\omega_{sc}}\right)^2\right],$$

where $l_{sc} = \frac{2\pi c}{\omega_{sc}}$ is the characteristic wavelength scale.
The maximum total emission probability which accompanies the pair production from the vacuum:

$$\mathcal{P}_{\max} \approx \frac{\alpha}{\varepsilon}\frac{2^{3/2} N_f S}{3\pi^2 l_{sc}^2}\left(\frac{T}{\Delta t_{st}}\right)^3.$$

The typical quantity is $Sl_{sc}^{-2} \sim a\left(48\right)^{-2}$. The restriction $\mathcal{P}_{\max} < 1$ may impose an essential limit on the applicability.

The main new results obtained

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- We analyze the applicability of the presented calculations to the graphene physics in laboratory conditions. In fact, we are talking about a possible observation of the Schwinger effect in these conditions.

The main new results obtained. Generalizations

- In a high frequency approximation the variation of the external electric field acting on the particle within the formation length can be neglected, which justify the applicability of the locally constant field approximation. Thus, the developed approach can be easily extended to study the emission in any slowly varying field configuration.

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- S.P. Gavrilov and D.M. Gitman, *Photon emission in the graphene under the action of a quasiconstant external electric field*, arXiv:2210.03223.

Thanks

The end