

# BRST Symmetry Method for Cubic Interacting Vertices for Unconstrained Integer Higher Spin Fields

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based on I.L. Buchbinder, A.R, General Cubic Interacting Vertex for Massless Integer Higher Spin Fields, Physics Letters B 820 (2021) 136470, [arXiv:2105.12030],

A.R. Towards the structure of a cubic interaction vertex for massless integer higher spin fields [arXiv:2205.00488] PEPAN (2022) ,

I.L. Buchbinder, A.R, General Cubic Interacting Vertex for Massless and Massive Integer HS Fields, in progress

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## Abstract

We consider the massless & massive theory of higher spin (HS) fields as natural candidates for possible new particles for the matter ( $s = n + 1/2$ ), interactions ( $s = n$ ) HS fields closely related with (Super)SFT and should be revealed at High Energy after Universe birthday.

There are hundreds results on free dynamics for HS fields, and dozens on cubic and  $n -$ adic interactions for HS fields on Minkowski and AdS spaces To derive it many approaches exist to describe free and interacting dynamics dividing on **metric-like**, **frame-like** (Lorentz frame) formulations. and superfield approach

E. Wigner, M. Fierz, W. Pauli; V. Ginzburg; E. Fradkin; L. Singh, C. Hagen, C. Fronsdall, J. Fang, M. Vasiliev, R. Metsaev, Labastida, Yu. Zinoviev, J. Buchbinder, A. Pashnev, V. Krykhtin

One of the most powerful tools are light-cone and BRST (Becchi, Rouet, Stora, Tyutin 1974) method applied for inverse problem of finding Lagrangians from the non-Lagrangian equations elaborating as the constraints on HS field  $R^{1,d}$  ( $AdS_d$ ) with trivial  $H$  within generalized canonical formalism .

Talk devoted to construction a (off-shell) covariant general Lagrangian cubic vertices to the irreducible HS fields on the  $R^{1,d}$ . To this end, we develop a concept of deformation Noether's procedure of free gauge theory on a base of BRST approach with complete BRST operator (J. Buchbinder, A. Pashnev, M. Tsulaia, V. Krykhtin, A.R. 1998-2010).

# BRST approach with complete $Q$ to Lagrangians for HS fields on $R^{1,d-1}$

$$(m, s) \quad (\partial^\nu \partial_\nu + m^2, \partial^{\mu_1}, \eta^{\mu_1 \mu_2}) \phi_{\mu(s)} = (0, 0, 0) \quad \Longleftrightarrow \quad (1)$$

$$(l_0, l_1, l_{11}, g_0 - d/2) |\phi\rangle = (0, 0, 0, s) |\phi\rangle.$$

$diag \eta^{\mu\nu} = (+, -, \dots, -)$ , basic vector  $|\phi\rangle$  &  $l_0, l_1, l_{11}, g_0$  are determined:

$$|\phi\rangle = \sum_{s \geq 0} \frac{i^s}{s!} \phi^{\mu(s)} \prod_{i=1}^s a_{\mu_i}^+ |0\rangle, \quad [a_\nu, a_\mu^+] = -\eta_{\mu\nu}, \quad (2)$$

$$(l_0, l_1, l_{11}, g_0) = (\partial^\nu \partial_\nu, -ia^\nu \partial_\nu, \frac{1}{2} a^\mu a_\mu, -\frac{1}{2} \{a_\mu^+, a^\mu\}).$$

$$Q = \eta_0 l_0 + \eta_1^+ l_1 + l_1^+ \eta_1 + \eta_{11}^+ \widehat{L}_{11} + \widehat{L}_{11}^+ \eta_{11} + i\eta_1^+ \eta_1 \mathcal{P}_0, \quad Q^2|_\Sigma = 0$$

$$\mathcal{S}_{0|s}[\phi, \phi_1, \dots] = \mathcal{S}_{0|s}[|\chi\rangle_s] = \int d\eta_{0s} \langle \chi | KQ | \chi \rangle_s, \quad \delta(|\chi\rangle_s, |\Lambda\rangle_s) = Q(|\Lambda\rangle_s, |\Lambda^1\rangle_s)$$

with trace  $\widehat{L}_{11}$  (and its dual)  $\widehat{L}_{11}^+$  selecting  $ISO(1, d-1)$  irreps with integer helicities  $s$ . The operator of cubic vertex as 3-vector satisfies to the properties of BRST and spin closeness

$$\sum_i Q^{(i)} |V^{(3)}\rangle_{(s)_3}^{(m)_3} = 0, \quad \sigma^{(i)} |V^{(3)}\rangle_{(s)_3}^{(m)_3} = 0, \quad (3)$$

We covariantize the cubic vertices  $|V^{(3)}\rangle_{(s)_3}^{(m)_3}$  firstly found in light-cone [R. Metsaev, 2006] with preserving the irreducibility for the fields on the interacting level for each copy of interacting HS fields. (for  $i = 1, 2, 3$  enumerating the copy of fields, masses  $(m)_3 = (m_1, m_2, m_3)$  and spins  $(s)_3 = (s_1, s_2, s_3)$ )

. As compared to the covariant form of the vertices obtained with using BRST approach with incomplete BRST operator  $Q_c$  [R. Metsaev, PLB 720 (2013) 237], the interacting theory with  $Q$  leads to new contributions to the vertex that contain additional terms with fewer space-time derivatives of fields, as well as with multiple trace contributions.

## Contents

- Interaction vertices in the gauge theories
- Lagrangians for HS fields from SFT
- BRST-BFV approach for Lagrangians for HS fields
- BRST approach with complete BRST operator to Lagrangians for free massless HS fields on  $R^{1,d-1}$ ;
- Including interaction through systems of equations for cubic vertices;
- General solution for the cubic vertices for unconstrained of helicities  $s_1, s_2, s_3$  HS fields
  - ① BRST-closed linear on oscillators operators  $\mathcal{L}_{k_i}^{(i)}$ ;
  - ② BRST-closed cubic on oscillators operators  $\mathcal{Z}$ ;
  - ③ BRST-closed trace operators  $U_{j_i}^{(s_i)}$ ;
- General (covariant) and partial solutions for the cubic vertices;
- Correspondence with Metsaev's results on cubic vertices ;
- Conclusion & Outlook

## Known results on cubic vertices

- metric formalism F. Berends, J. Van Reisen, NPB164 (1980), Berends, G. Burgers, H Van Dam, Nucl. Phys. B271 (1986); A. K. H. Bengtsson, I. Bengtsson, L. Brink, NPB (1983), E.S. Fradkin, M.A. Vasiliev, NPB 291 (1987), R. Manvelyan, K. Mkrtchyan, W. Ruhl, PLB 696 (2011), [arXiv:1009.1054 [hep-th]], E. Joung, M. Taronna, NPB 861 (2012) 145, arXiv:1110.5918[hep-th], I. Buchbinder, V. Krykhtin, M. Tsulaia, D. Weissman, Cubic Vertices for  $\mathcal{N} = 1$ , NPB 967 (2021), arXiv:2103.08231;
- (half)integer spin for  $ISO(1, d - 1)$  in the light-cone R.R. Metsaev, NPB 759 (2006) hep-th/0512342, NPB 859 (2012) [arXiv:0712.3526[hep-th]]; 4d [arXiv:2206.13268[hep-th]] ;
- within constrained (with algebraic constraints, cov.) BRST approach for integer spins -R.R. Metsaev, PL B 720 (2013) arXiv:1205.3131 [hep-th];
- first, in BRST approach for reducible reps  $ISO(1, d - 1)$ ,  $SO(2, d - 1)$  I.L. Buchbinder, A. Fotopoulos, A. Petkou, M. Tsulaia, PRD 74 (2006) 105018, [arXiv:hep-th/0609082];
- in frame-like approach M. Vasiliev, Cubic Vertices for Symmetric higher spin Gauge Fields in (A)dS<sub>d</sub>, NPB 862 (2012) 341 , arXiv:1108.5921[hep-th] arXiv:2208.02004, M. Khabarov, Yu. Zinoviev. JHEP 02 (2021);
- Cov. cubic vertex in BRST approach for unconstrained HS fields (irreps) not found

# Interaction vertices in the gauge theories

## Gauge theory of 1 -stage reducibility

$S_0[A]$  – classical action of fields  $A^i, i = 1, \dots, n = (n_+, n_-)$ ,  $\varepsilon(A^i) = \varepsilon_i$ ,

$$\delta_0 S_0 = 0, \quad \delta_0 A^i = R_{0\alpha_0}^i \xi^{\alpha_0}, \quad \alpha_0 = 1, \dots, m_0 = (m_{0+}, m_{0-}), \implies$$

$$\overleftarrow{\partial}_i S_0 R_{0\alpha_0}^i = 0 \quad \text{rank} \left\| R_{0\alpha_0}^i \right\| |_{\partial_i S_0 = 0} = m < m_0, \Rightarrow \delta_0^{(0)} \xi^{\alpha_0} = Z_{0\alpha_1} \xi^{\alpha_1} :$$

$$\text{so that } R_{0\alpha_0}^i Z_{0\alpha_1} \xi^{\alpha_0} |_{\partial_i S_0 = 0} = 0 \text{ and } \alpha_1 = 1, \dots, m_1 = (m_{1+}, m_{1-}) = (m_0 - m) \\ \varepsilon(\xi^{\alpha_s}) = \varepsilon_{\alpha_s}, \quad s = 0, 1$$

Deformation of  $k$  copies of LF for fields  $A^{i(p)}$ ,  $p = 1, \dots, k$  with quadratic  $\sum_p S_0^{(p)}[A^{(p)}]$  of free fields  $A^{i(p)}$  with rank condition  $\boxed{N = k(n - m)}$

$$S_{int} = \sum_{p=1}^k S_0^{(p)}[A^{(p)}] + g^1 S_1 + g^2 S_2 + \dots + g^r S_r, \quad \boxed{\deg_A S_r = r + 2},$$

$$\delta_{[l]} A^{i(p)} = \delta_0 A^{i(p)} + g \delta_1 A^{i(p)} + \dots + g^l \delta_l A^{i(p)} = R_{[l]\alpha_0(t)}^{i(p)} \xi^{\alpha_0(t)}, \quad \boxed{\deg_A R_{l\alpha_0(t)}^{i(p)} = l}$$

$$\delta_{[l]}^{(0)} \xi^{\alpha_0(p)} = \left\{ \delta_0^{(0)} + g \delta_1^{(0)} + \dots + g^l \delta_l^{(0)} \right\} \xi^{\alpha_0(p)} = Z_{[l]\alpha_1(t)}^{\alpha_0(p)} \xi^{\alpha_1(t)}, \quad \boxed{\deg_A Z_{l\alpha_1(t)}^{\alpha_0(p)} = l},$$

$$R_{0\alpha_0(t)}^{i(p)} \equiv R_{0\alpha_0}^i \delta_t^p, \quad Z_{0\alpha_1(t)}^{\alpha_0(p)} \equiv Z_{0\alpha_1} \delta_t^p.$$

# Interaction vertices in the gauge theories: deformation procedure

Noether's identities as system in powers of  $g$  from  $\delta_{\Sigma} S_{int} = 0$ :  $\boxed{\delta_{\Sigma} \equiv \sum_{l=0}^{\infty} \delta_l}$

$$g^1 : \quad \delta_0 S_1 + \delta_1 \bar{S}_0 = 0, \quad (4)$$

$$g^2 : \quad \delta_0 S_2 + \delta_1 S_1 + \delta_2 \bar{S}_0 = 0, \quad \bar{S}_0 \equiv \sum_{p=1}^k S_0^{(p)}$$

for the gauge transforms of 0 level from  $\delta_{\Sigma}^{(0)} \delta_{\Sigma} A^{i(p)}|_{\partial S_{int}=0} = 0$ :

$$\begin{aligned} g^1 : \quad & \left( \delta_1^{(0)} \delta_0 A^{i(p)} + \delta_0^{(0)} \delta_1 A^{i(p)} \right) |_{\partial S_{[1]}=0} = 0, \\ g^2 : \quad & \left( \delta_2^{(0)} \delta_0 + \delta_1^{(0)} \delta_1 + \delta_0^{(0)} \delta_2 \right) A^{i(p)} |_{\partial S_{[2]}=0} = 0, \end{aligned} \quad (5)$$

for the cubic vertex for GTh of 1 reducibility level

$$S_{int} = \sum_{p=1}^3 S_0^{(p)} [A^{(p)}] + g^1 S_1, \quad \boxed{\deg_A S_1 = 3},$$

$$\delta_{[1]} A^{i(p)} = \delta_0 A^{i(p)} + g \delta_1 A^{i(p)} = R_{[1]}{}_{\alpha_0(t)}^{i(p)} \xi^{\alpha_0(t)}, \quad (6)$$

$$\delta_{[1]}^{(0)} \xi^{\alpha_0(p)} = \left\{ \delta_0^{(0)} + g \delta_1^{(0)} \right\} \xi^{\alpha_0(p)} = Z_{[1]}{}_{\alpha_1(t)}^{\alpha_0(p)} \xi^{\alpha_1(t)}. \quad (7)$$

The equations (4)–(5) pass to

$$g^1 : \quad \delta_0 S_1 + \delta_1 \sum_{p=1}^3 S_0^{(p)} = 0, \quad (8)$$

$$g^2 : \quad \left( \delta_1^{(0)} \delta_0 A^{i(p)} + \delta_0^{(0)} \delta_1 A^{i(p)} \right) |_{\partial S_{[1]}=0} = 0, \quad p = 1, 2, 3. \quad (9)$$

# Lagrangians for HS fields from SFT

A construction of **covariant cubic vertex** for HS fields of spins  $(0, s_1), (0, s_2), (0, s_3) \dots [(m_1, s_1), (m_2, s_2), (m_3, s_3) \dots]$  on  $\mathbb{R}^{1, d-1}$  within LF for totally symmetric tensor fields without holonomic constraintsit is suggested within BRST approach with complete BRST operator (following from SFT).

A connection of massless HS fields with SFT : (E. Witten (1986);

C. Thorn(1989))  $\Rightarrow$  follows from tensionless limit of BRST operator  $\mathcal{Q}$  for  $(\alpha' \rightarrow \infty)$ : (A. Bengtsson (1983) G.Bonelli (2003), A. Sagnotti, M. Tsulaia, NPB (2004)).

$$\boxed{\mathcal{Q} \xrightarrow{\alpha' \rightarrow \infty} \{\infty\} \text{ set of HS fields in string spectrum}}$$

for reducible  $ISO(1, d - 1)$  reps: (in representation on  $\mathcal{H}$ :  $(a_k^\mu, \mathcal{P}_0, \mathcal{P}_k, \eta_k)|0\rangle = 0$ )

$$\boxed{\lim_{\alpha' \rightarrow \infty} \mathcal{Q} = Q_c = \eta_0 l_0 + \sum_{k>0}^{\infty} [\eta_k^+ l_k + \eta_k l_k^+ + i\eta_k^+ \eta_k \mathcal{P}_0] = \eta_0 l_0 - i\mathcal{P}_0 M + \Delta Q_c,}$$

$$(l_k, l_k^+) = -i\partial_\mu (a_k^\mu, a_k^{+\mu}); [l_0, l_k^{(+)}] = 0, [l_k, l_k^+] = l_0 : [a_k^\mu, a_l^{+\nu}] = -\delta_{kl} \eta^{\mu\nu}$$

$$\boxed{(d = 26 : ) \mathcal{Q}|\chi\rangle = 0, \delta|\chi\rangle = \mathcal{Q}|\Lambda_0\rangle, \delta|\Lambda_0\rangle = \mathcal{Q}|\Lambda_1\rangle, \dots ; (\varepsilon, gh)|\Lambda_k\rangle = (k+1, -k-1)}$$

$$\boxed{(\forall d : ) Q_c|\chi\rangle = 0, \delta|\chi\rangle = Q_c|\Lambda_0\rangle, \delta|\Lambda_0\rangle = Q_c|\Lambda_1\rangle, \dots}$$

# Lagrangians for HS fields from SFT

## Consequences:

EoM - Lagrangian, determine free gauge theory of  $(k-1)$ -reducibility stage & follows

$$S[\Phi, \dots] = \int d\eta_0 \langle \chi | Q_c | \chi \rangle, \quad \delta |\chi\rangle = Q_c |\Lambda_0\rangle, \dots \delta |\Lambda_{k-1}\rangle = Q_c |\Lambda_k\rangle, \text{gh}(|\chi\rangle, |\Lambda_i\rangle) = (0, -i - 1) \quad (10)$$

For TS  $\Phi_{\mu(s)}$  due to  $gh$ -homogeneity

$$\begin{aligned} |\varphi_1\rangle_s &= \Phi_{\mu(s)} a^{\mu_1+} \dots a^{\mu_s+} |0\rangle + \eta_1^+ \mathcal{P}_1^+ D_{\mu(s-2)} a^{\mu_1+} \dots a^{\mu_{s-2}+} |0\rangle, \\ |\varphi_2\rangle_s &= \mathcal{P}_1^+ C_{\mu(s-1)} a^{\mu_1+} \dots a^{\mu_{s-1}+} |0\rangle, \quad |\Lambda_0\rangle_s = \mathcal{P}_1^+ \Lambda_{\mu(s-1)} a^{\mu_1+} \dots a^{\mu_{s-1}+} |0\rangle. \end{aligned}$$

1) Follows *triplet formulation* in terms  $\Phi_{\mu(s)}, C_{\mu(s-1)}, D_{\mu(s-2)}$  ( $m=0$ ) reducible  $ISO(1, d-1)$  representations (Francia, Sagnotti 2003) with HS fs  $(s, s-2, \dots, 1/0)$  in oscillator form,

$$\boxed{\begin{array}{l} l_0 |\Phi\rangle_s - l_1^+ |C\rangle_{s-1} = 0, \quad l_1 |\Phi\rangle_s - l_1^+ |D\rangle_{s-2} = |C\rangle_{s-1}, \\ l_0 |D\rangle_{s-2} - l_1 |C\rangle_{s-1} = 0 \\ \delta \left( |\Phi\rangle_s, |C\rangle_{s-1}, |D\rangle_{s-2} \right) = \left( l_1^+, l_0, l_1 \right) |\Lambda\rangle_{s-1} \end{array}} \quad (11)$$

sign "s" in  $|\Phi\rangle_s, |\chi\rangle_s$ :  $\sigma_c |\chi\rangle_s = (s + \frac{d-2}{2}) |\chi\rangle_s$  for the spin operator

$$\boxed{\sigma_c = (g_0 + \eta_1^+ \mathcal{P}_1 - \eta_1 \mathcal{P}_1^+)}, \quad g_0 = -\frac{1}{2} \{ a_\mu^+ \cdot a^\mu \}$$

Lagrangian property (11) follows from the action (given on Fock space  $\mathcal{H}$ ):

# Lagrangians for HS fields from SFT

$$\boxed{S[\Phi, C, D] = \int d\eta_0 s \langle \chi | Q_c | \chi \rangle_s, \delta | \chi \rangle_s = Q_c | \Lambda_0 \rangle_s, \text{gh}(|\chi\rangle, |\Lambda_0\rangle) = (0, -1)} \quad (12)$$

2) LF for HS field from **irreducible representation** ( $m = 0, s$ ) includes trace condition ( $l_{11}|\Phi\rangle_s = 0$ ) in the form of BRST-extended constraint  $\mathcal{L}_{11}$  ( $[Q_c, \mathcal{L}_{11}] = 0$ ,  $[Q_c, \sigma_c] = 0$ ) imposed on  $|\chi\rangle, |\Lambda_0\rangle$

$$\boxed{\begin{aligned} S_{0|s}[\Phi, C, D] &= \int d\eta_0 s \langle \chi | Q_c | \chi \rangle_s, \quad \delta | \chi \rangle_s = Q_c | \Lambda_0 \rangle_s, \\ \mathcal{L}_{11}(|\chi\rangle, |\Lambda_0\rangle) &= (l_{11} + \eta_1 P_1)(|\chi\rangle, |\Lambda_0\rangle) = (0, 0), \quad \text{c } l_{11} = 1/2a^\mu a_\mu \end{aligned}}. \quad (13)$$

(13)  $\Rightarrow$  **Fronsdal formulation** with  $\Lambda_{(\mu)s-1}$  ( $\Lambda^{\mu}{}_{\mu\dots} = 0$ ) &  $\Phi_{(\mu)s}$  ( $\Phi^{\mu\nu}{}_{\mu\nu\dots} = 0$ ).

(13) - *constrained BRST Lagrangian formulation* with external holonomic constraint (Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev Grigoriev, Tipunin, 2008).

$$S_{[1]}[\chi_c^{(1)}, \chi^{(2)}, \chi^{(3)}] = \sum_{i=1}^3 S_{0|s_i} + g \int \prod_{e=1}^3 d\eta_0^{(e)} \left( {}_{s_e} \langle \chi^{(e)} | V^{M(3)} \rangle_{(s)_3} + h.c. \right),$$

$$\begin{aligned} \delta_{[1]} |\chi_c^{(i)}\rangle_{s_i} &= Q_c^{(i)} |\Lambda_c^{(i)}\rangle_{s_i} - g \int \prod_{e=1}^2 d\eta_0^{(i+e)} \left( {}_{s_{i+1}} \langle \Lambda_c^{(i+1)} | {}_{s_{i+2}} \langle \chi_c^{(i+2)} | \right. \\ &\quad \left. + (i+1 \leftrightarrow i+2) \right) |V^{M(3)}\rangle_{(s)_3}, \quad \boxed{[(Q_c^{tot}, \mathcal{L}_{11}^{(i)}) | V^{M(3)} \rangle_{(s)_3} = 0]} \end{aligned}$$

Inclusion into the system a ( $l_{11}|\Phi\rangle=0$ ) equally with the rest differential constraints, in order to the all irrep conditions extracting the particle ( $m = 0, s$ );

# BRST-BFV approach for Lagrangians for HS fields

A. Pashnev, M. Tsulaia, MPLA (1998);

I. Buchbinder, A. R., NPB 2012, [arXiv:1110.5044[hep-th]]

$$\mathcal{S}_{0|s}[\Phi, \phi_1, \dots] = \mathcal{S}_{0|s}[|\chi\rangle_s] = \int d\eta_{0s} \langle \chi | KQ | \chi \rangle_s, \quad (15)$$

$$\delta |\chi\rangle_s = Q |\Lambda\rangle_s, \quad \delta |\Lambda\rangle_s = Q |\Lambda^1\rangle_s, \quad \delta |\Lambda^1\rangle_s = 0. \quad (16)$$

An equivalence of the Lagrangian formulations with incomplete & complete BRST operators for any irrep with discrete spin on  $\mathbb{R}^{1,d-1}$  is established in A. R, JHEP (2018) arXiv:1803.04678[hep-th], but for interacting theory of the same HS field this question has not yet been solved .

Basic result for the cubic vertex for TS HS fields on  $\mathbb{R}^{1,d-1}$  was obtained for 5d, 6d in light-cone by R. Metsaev (hep-th/0512342, arxiv:0712.3526), for covariant form in constrained BRST approach in -R. Metsaev, PLB 2013.

## BRST approach for Lagrangians for HS fields

In BRST approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, I. Buchbinder, V. Krykhtin, A.R.) It is developed an algorithm instead of **direct problem** of generalized canonical quantization for dynam. system subject to constraints **inverse problem** of constructing GI LF for HS fields with  $(m, s)$

$$\boxed{\begin{array}{l} \text{irreps conditions} \\ \text{ISO}(1,d-1), \text{SO}(2,d-1) \end{array}} \xrightarrow{\text{SFT}} \boxed{\begin{array}{l} (\text{super})\text{algebra}\{o_I(x)\} : \mathcal{H} \\ [o_I, o_J] = f^K_{IJ}(o)o_k + \Delta_{ab}(g_0) \end{array}}$$

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$$\xrightarrow{\text{conversion}} \boxed{\begin{array}{c} O_I = o_I + o'_I : \mathcal{H} \otimes \mathcal{H}' \\ [O_I, O_J] = F_{IJ}^K(o', O)O_K \end{array}}$$

A.Pashnev, J.Buchbinder, A.R.

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A.Pashnev, J.Buchbinder, A.R.

$$\xrightarrow{\text{BFV}} \boxed{\begin{array}{c} \text{BRST operator for } \{O_I\}: Q'(x) \\ Q' = C^I O_I + \frac{1}{2} C^I C^J F_{IJ}^K \mathcal{P}_K + \text{more} \end{array}}$$

M.Henneaux

# BRST approach for Lagrangians for HS fields

In BRST approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, I. Buchbinder, V. Krykhtin, A.R.) It is developed an algorithm instead of **direct problem** of generalized canonical quantization for dynam. system subject to constraints **inverse problem** of constructing GI LF for HS fields with  $(m, s)$

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$$\xrightarrow{\text{BFV}} \boxed{\begin{array}{c} \text{M. Henneaux} \\ \text{BRST operator for } \{O_I\}: Q'(x) \\ Q' = C^I O_I + \frac{1}{2} C^I C^J F_{IJ}^K \mathcal{P}_K + \text{more} \end{array}}$$

$$\xrightarrow{\text{JF}} \boxed{\begin{array}{c} Q' = Q + (g_0 + h + \text{more})C_g + \dots : Q^2|_{\sigma|\chi\rangle=0} = 0 \\ \text{mass-shell: } Q|\chi\rangle = 0, gh(|\chi\rangle) = 0 \Rightarrow \text{action: } S = \int d\eta_0 \langle \chi | K Q | \chi \rangle \\ \text{spin: } (g_0 + \text{more})_{|\chi\rangle, |\Lambda\rangle, \dots} = -h(|\chi\rangle, |\Lambda\rangle, \dots) \\ \text{gauge symmetries: } \delta|\chi\rangle = Q|\Lambda\rangle, \delta|\Lambda\rangle = Q|\Lambda^1\rangle, \dots \end{array}}$$

$Q$  - BRST operator for 1-st class constraints  $\{O_\alpha\} \subset \{O_I\}$  without invertible  $g_0$ . **on 2-3 stages gauge and auxiliary fields** introduced automatically when getting LF for the basic field

# BRST approach with complete BRST operator for free massless (massive) HS fields on $R^{1,d-1}$

In BRST approach a dynamic of free field of helicity (spin)  $s$  is determined in the extended configuration space with GI action by  $\phi_{\mu(s)}$  and auxiliary fields  $\phi_{1\mu(s-1)}, \dots$ . All of them are included in  $|\chi\rangle_s$  described

$$\mathcal{S}_{0|s}[\phi, \phi_1, \dots] = \mathcal{S}_{0|s}[|\chi\rangle_s] = \int d\eta_{0s} \langle \chi | KQ | \chi \rangle_s, \quad (17)$$

$\mathcal{S}_{0|s}[|\chi\rangle_s]$  invariant w.r.t. reducible gauge transforms

$$\delta|\chi\rangle_s = Q|\Lambda\rangle_s, \quad \delta|\Lambda\rangle_s = Q|\Lambda^1\rangle_s, \quad \delta|\Lambda^1\rangle_s = 0. \quad (18)$$

with  $|\Lambda\rangle_s, |\Lambda^1\rangle_s$  gauge parameter vectors of 0- & 1-levels in Abelian gauge transforms (18).

# BRST approach with complete BRST operator to Lagrangians for free massless HS fields on $R^{1,d-1}$

$Q$  - BRST operator constructed w.r.t. HS symmetry algebra

$\{l_0, l_1, l_1^+, l_{11}, l_{11}^+ = \frac{1}{2}a^{+\nu}a_\nu^+\}$  ith Grassmann-odd ghost operators  
 $\eta_0, \eta_1^+, \eta_1, \eta_{11}^+, \eta_{11}, \mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_1^+, \mathcal{P}_{11}, \mathcal{P}_{11}^+$ ,

$$\begin{aligned} Q &= \eta_0 l_0 + \eta_1^+ l_1 + l_1^+ \eta_1 + \eta_{11}^+ \hat{L}_{11} + \hat{L}_{11}^+ \eta_{11} + \eta_1^+ \eta_1 \mathcal{P}_0 \\ &= \eta_0 l_0 + \Delta Q + \eta_1^+ \eta_1 \mathcal{P}_0, \end{aligned} \quad (19)$$

где

$$(\hat{L}_{11}, \hat{L}_{11}^+) = (L_{11} + \eta_1 \mathcal{P}_1, L_{11}^+ + \mathcal{P}_1^+ \eta_1^+). \quad (20)$$

$$L_{11} = l_{11} + (b^+ b + h) b - 1/2d^2, \quad L_{11}^+ = l_{11}^+ + b^+ - 1/2d^{+2}$$

и  $(\epsilon, gh)Q = (1, 1)$ . Algebra  $l_0, l_1 + md, l_1^+ + md^+, L_{11}, L_{11}^+, G_0$  is determined

$$\begin{aligned} [l_0, l_1^{(+)}] &= 0, \quad [l_1, l_1^+] = l_0 \quad [L_{11}, L_{11}^+] = G_0, \quad [G_0, L_{11}^+] = 2L_{11}^+, \\ [l_1, L_{11}^+] &= -l_1^+, \quad [l_1, G_0] = l_1 \end{aligned}$$

with extended

$$G_0 = g_0 + 2b^+ b + d^+ d + h, \quad b, b^+, d, d^+ \quad [b, b^+] = [d, d^+] = 1$$

- auxiliary conversion oscillators generating  $\mathcal{H}'$ .

# BRST approach with complete BRST operator to Lagrangians for free massless HS fields on $R^{1,d-1}$

conversion parameter when constructing Verma module for  $so(1, 2)$

$$h = h(s) = -s - \frac{d-6}{2}.$$

$$\{\eta_0, \mathcal{P}_0\} = i, \quad \{\eta_1, \mathcal{P}_1^+\} = \{\eta_1^+, \mathcal{P}_1\} = \{\eta_{11}, \mathcal{P}_{11}^+\} = \{\eta_{11}^+, \mathcal{P}_{11}\} = 1.$$

theory is characterized by spin operator  $\sigma$ :

$$\boxed{\sigma = G_0 + \eta_1^+ \mathcal{P}_1 - \eta_1 \mathcal{P}_1^+ + 2(\eta_{11}^+ \mathcal{P}_{11} - \eta_{11} \mathcal{P}_{11}^+).} \quad (21)$$

$\sigma$  selects vectors with definite spin  $s$

$$\sigma(|\chi\rangle_s, |\Lambda\rangle_s, |\Lambda^1\rangle_s) = (0, 0, 0), \quad (22)$$

with periodic  $\varepsilon$  and decreasing  $gh$   $(0, 0)$ ,  $(1, -1)$ ,  $(0, -2)$  respectively.

All operators act in total Hilbert space  $\mathcal{H}_{tot} = \mathcal{H} \otimes \mathcal{H}_{gh} \otimes \mathcal{H}'$  c

$$\langle \chi | \psi \rangle = \int d^d x \langle 0 | \chi^*(a, b; \eta_1, \mathcal{P}_1, \eta_{11}, \mathcal{P}_{11}) \psi(a^+, b^+; \eta_1^+, \mathcal{P}_1^+, \eta_{11}^+, \mathcal{P}_{11}^+) | 0 \rangle.$$

Operators  $Q, \sigma$  supercommute and Hermitian (see, I.L. Buchbinder, A. Pashnev, M. Tsulaia, PLB (2001), I. Buchbinder, A. R., NPB (2012) arXiv:1110.5044)

$$Q^2 = \eta_{11}^+ \eta_{11} \sigma, \quad [Q, \sigma] = 0; \quad (23)$$

$$(Q^+, \sigma^+) K = K(Q, \sigma), \quad K = \sum_{n=0}^{\infty} \frac{1}{n!} (b^+)^n |0\rangle \langle 0 | b^n \prod_{i=0}^{n-1} (i + h(s)),$$

# BRST approach with complete BRST operator to Lagrangians for free massless HS fields on $R^{1,d-1}$

$Q^2|_{\sigma \tilde{\mathcal{H}}_{tot}=0} = 0$  is nilpotent on subspace with zero values for  $\sigma$  (22).

Field  $|\chi\rangle_s$ ,  $|\Lambda\rangle_s$ ,  $|\Lambda^1\rangle_s$  (as the result of spin condition) are defined in the form

$$\begin{aligned} |\chi\rangle_s &= |\Phi\rangle_s + \eta_1^+ \left( \mathcal{P}_1^+ |\phi_2\rangle_{s-2} + \mathcal{P}_{11}^+ |\phi_{21}\rangle_{s-3} + \eta_{11}^+ \mathcal{P}_1^+ \mathcal{P}_{11}^+ |\phi_{22}\rangle_{s-6} \right) \\ &\quad + \eta_{11}^+ \left( \mathcal{P}_1^+ |\phi_{31}\rangle_{s-3} + \mathcal{P}_{11}^+ |\phi_{32}\rangle_{s-4} \right) + \eta_0 \left( \mathcal{P}_1^+ |\phi_1\rangle_{s-1} + \mathcal{P}_{11}^+ |\phi_{11}\rangle_{s-2} \right. \\ &\quad \left. + \mathcal{P}_1^+ \mathcal{P}_{11}^+ \left[ \eta_1^+ |\phi_{12}\rangle_{s-4} + \eta_{11}^+ |\phi_{13}\rangle_{s-5} \right] \right), \end{aligned} \quad (24)$$

$$\begin{aligned} |\Lambda\rangle_s &= \mathcal{P}_1^+ |\xi\rangle_{s-1} + \mathcal{P}_{11}^+ |\xi_1\rangle_{s-2} + \mathcal{P}_1^+ \mathcal{P}_{11}^+ \left( \eta_1^+ |\xi_{11}\rangle_{s-4} \right. \\ &\quad \left. + \eta_{11}^+ |\xi_{12}\rangle_{s-5} \right) + \eta_0 \mathcal{P}_1^+ \mathcal{P}_{11}^+ |\xi_{01}\rangle_{s-3}, \end{aligned} \quad (25)$$

$$|\Lambda^1\rangle_s = \mathcal{P}_1^+ \mathcal{P}_{11}^+ |\xi^1\rangle_{s-3}. \quad (26)$$

with  $|\phi...\rangle_{...} \equiv |\phi(a^+, b^+, d^+)...\rangle_{...}$ : .  $|\Phi\rangle_s|_{(b^+=d^+=0)} = |\phi\rangle_s$

# Including interaction through systems of equations for cubic vertices

Cubic vertex for HS fields  $(s_1, s_2, s_3)$  within BRST approach includes 3 copies of vectors  $|\chi^{(i)}\rangle_{s_i}$ ,  $|\Lambda^{(i)}\rangle_{s_i}$ ,  $|\Lambda^{(i)1}\rangle_{s_i}$  with  $|0\rangle^i$  and oscillators  $a^{(i)\mu+} \dots$ ,  $i = 1, 2, 3$ . Deformed action and gauge transformations

$$S_{[1](s)_3}[\chi^{(1)}, \chi^{(2)}, \chi^{(3)}] = \sum_{i=1}^3 \mathcal{S}_{0|s_i} + g \int \prod_{e=1}^3 d\eta_0^{(e)} \left( {}_{s_e} \langle \chi^{(e)} K^{(e)} | V^{(3)} \rangle_{(s)_3} + h.c. \right),$$

$$\delta_{[1]} |\chi^{(i)}\rangle_{s_i} = Q^{(i)} |\Lambda^{(i)}\rangle_{s_i} - g \int \prod_{e=1}^2 d\eta_0^{(i+e)} \left( {}_{s_{i+1}} \langle \Lambda^{(i+1)} K^{(i+1)} | {}_{s_{i+2}} \langle \chi^{(i+2)} K^{(i+1)} | \right. \\ \left. + (i+1 \leftrightarrow i+2) \right) |\tilde{V}^{(3)}\rangle_{(s)_3},$$

$$\delta_{[1]} |\Lambda^{(i)}\rangle_{s_i} = Q^{(i)} |\Lambda^{(i)1}\rangle_{s_i} - g \int \prod_{e=1}^2 d\eta_0^{(i+e)} \left( {}_{s_{i+1}} \langle \Lambda^{(i+1)1} K^{(i+1)} | {}_{s_{i+2}} \langle \chi^{(i+2)} K^{(i+1)} | \right. \\ \left. + (i+1 \leftrightarrow i+2) \right) |\hat{V}^{(3)}\rangle_{(s)_3}$$

with unknown  $|V^{(3)}\rangle_{(s)_3}$ ,  $|\tilde{V}^{(3)}\rangle_{(s)_3}$ ,  $|\hat{V}^{(3)}\rangle_{(s)_3}$ .

# Including interaction through systems of equations for cubic vertices

$x$ -locality  $|V^{(3)}\rangle, |\tilde{V}^{(3)}\rangle, |\hat{V}^{(3)}\rangle$  and momenta conversation law:

$$|V^{(3)}\rangle_{(s)3} = \prod_{i=2}^3 \delta^{(d)}(x_1 - x_i) V^{(3)} \prod_{j=1}^3 \eta_0^{(j)} |0\rangle, \quad |0\rangle \equiv \otimes_{e=1}^3 |0\rangle^e$$

$$p_\mu^{(1)} + p_\mu^{(2)} + p_\mu^{(3)} = 0. \quad (s)_3 \equiv (s_1, s_2, s_3) \quad [i+3 \simeq i]$$

invariance  $S_{[1]}$  w.r.t  $\delta_{[1]}|\chi^{(i)}\rangle_{s_i}, i = 1, 2, 3, \rightarrow$  system of equations:

$$g^0 : Q^{(i)} Q^{(i)} |\Lambda^{(i)}\rangle_{s_i} = \eta_{11}^{(i)+} \eta_{11}^{(i)} \sigma^{(i)} |\Lambda^{(i)}\rangle_{s_i} \equiv 0, \quad i = 1, 2, 3,$$

$$g^1 : \int \prod_{e=1}^3 d\eta_0^{(e)}|_{s_j} \langle \Lambda^{(j)} K^{(j)}|_{s_{j+1}} \langle \chi^{(j+1)} K^{(j+1)}|_{s_{j+2}} \langle \chi^{(j+2)} K^{(j+2)}| \mathcal{Q}(V^3, \tilde{V}^3) = 0,$$

$$\mathcal{Q}(V^3, \tilde{V}^3) = \sum_{k=1}^3 Q^{(k)} |\tilde{V}^{(3)}\rangle_{(s)3} + Q^{(j)} \left( |V^{(3)}\rangle_{(s)3} - |\tilde{V}^{(3)}\rangle_{(s)3} \right), \quad j = 1, 2, 3.$$

preservation the form of gauge transformations  $|\chi^{(i)}\rangle_{s_i}$  w.r.t  $\delta_{[1]}|\Lambda^{(i)}\rangle_{s_i} \Leftrightarrow |\Lambda^{(i)1}\rangle_{s_i} \rightarrow$

$$g^0 : Q^{(i)} Q^{(i)} |\Lambda^{(i)1}\rangle_{s_i} = \eta_{11}^{(i)+} \eta_{11}^{(i)} \sigma^{(i)} |\Lambda^{(i)1}\rangle_{s_i} \equiv 0, \quad i = 1, 2, 3,$$

$$g^1 : \int \prod_{e=1}^2 d\eta_0^{(e)}|_{s_{j+1}} \langle \Lambda^{(j+1)1} K^{(j+1)}|_{s_{j+2}} \langle \chi^{(j+2)} K^{(j+2)}| \left( \mathcal{Q}(\tilde{V}^3, \hat{V}^3) - Q^{(j+2)} |\hat{V}^{(3)}\rangle \right) = 0,$$

$$c \quad \mathcal{Q}(\tilde{V}^3, \hat{V}^3) = \sum_{k=1}^3 Q^{(k)} |\hat{V}^{(3)}\rangle_{(s)3} + Q^{(j)} \left( |\tilde{V}^{(3)}\rangle_{(s)3} - |\hat{V}^{(3)}\rangle_{(s)3} \right), \quad j = 1, 2, 3.$$

# Including interaction through systems of equations for cubic vertices

should fulfill **spin condition** as consequence of spin equations on each vectors (22)  
 $|\chi^{(i)}\rangle_{s_i}$ ,  $|\Lambda^{(i)}\rangle_{s_i}$ ,  $|\Lambda^{(i)1}\rangle_{s_i}$ :

$$\sigma^{(i)} \left( |V^{(3)}\rangle_{(s)3}, |\tilde{V}^{(3)}\rangle_{(s)3}, |\hat{V}^{(3)}\rangle_{(s)3} \right) = 0, \quad i = 1, 2, 3 \quad (27)$$

deformed gauge transformations should form closed algebra

$$[\delta_{[1]}^{\Lambda_1}, \delta_{[1]}^{\Lambda_2}] |\chi^{(i)}\rangle = -g \delta_{[1]}^{\Lambda_3} |\chi^{(i)}\rangle, \quad (28)$$

For simplicity we suppose  $|\tilde{V}^{(3)}\rangle_{(s)3} = |V^{(3)}\rangle_{(s)3} = |\hat{V}^{(3)}\rangle_{(s)3}$ . then the total set of equations

$$\boxed{(Q^{tot}, \sigma^{(i)}) |V^{(3)}\rangle_{(s)3} = \vec{0}}, \quad Q^{tot} = \sum_{k=1} Q^{(k)}$$

# General solution for the cubic vertices for unconstrained of helicities $s_1, s_2, s_3$ HS fields

we seek ( $Q^{tot}$ -BRST and  $\sigma^{(i)}$ - closed) solution in the form of products of specific homogeneous in oscillators operators

, (1),  $Q^{tot}$ -BRST- closed forms  $\mathcal{L}_{k_i}^{(i)}$ ,  $i = 1, 2, 3$ ,  $k_i = 1, \dots, s_i$  &  $\mathcal{Z}$  constructed from  $L^{(i)}$ , Z R.R. Metsaev, (2013) arXiv:1205.3131 [hep-th],

where  $\deg_{(a^+, \eta^+)} L^{(i)} = 1$ ,  $\deg_{(a^+, \eta^+)} Z = 3$ ,

$$\mathcal{L}_{k_i}^{(i)} = (L^{(i)})^{k_i-2} \left( (L^{(i)})^2 - \frac{ik_i!}{2(k_i-2)!} \eta_{11}^{(i)+} [2\mathcal{P}_0^{(i+1)} + 2\mathcal{P}_0^{(i+2)} - \mathcal{P}_0^{(i)}] \right), \quad (29)$$

$$L^{(i)} = (p_\mu^{(i+1)} - p_\mu^{(i+2)}) a^{(i)\mu+} - i(\mathcal{P}_0^{(i+1)} - \mathcal{P}_0^{(i+2)}) \eta_1^{(i)+}, \quad p_\mu^{(i)} = -i\partial_\mu^{(i)} \quad (30)$$

$$Z = L_{11}^{(12)+} L^{(3)} + L_{11}^{(23)+} L^{(1)} + L_{11}^{(31)+} L^{(2)}. \quad (31)$$

we have used momenta conservation -law and non-invariance  $L^{(i)}$  when acting by trace operator:  $(\widehat{\mathcal{L}}_{11}^{(i)} (L^{(i)})^2 |0\rangle \neq 0)$  and

$$L_{11}^{(ii+1)+} = a^{(i)\mu+} a_\mu^{(i+1)+} - \frac{1}{2} \mathcal{P}_1^{(i)+} \eta_1^{(i+1)+} - \frac{1}{2} \mathcal{P}_1^{(i+1)+} \eta_1^{(i)+}. \quad (32)$$

# BRST-closed cubic on oscillators operators $\mathcal{Z}$

(2), one has new 2-, 4- , ...,  $[s_i/2]$  forms in powers in  $\rightarrow$

$$U_{j_i}^{(s_i)}(\eta_{11}^{(i)+}, \mathcal{P}_{11}^{(i)+}) := (\widehat{L}_{11}^{(i)+})^{(j_i-2)} \{ (\widehat{L}_{11}^{(i)+})^2 - j_i(j_i-1)\eta_{11}^{(i)+}\mathcal{P}_{11}^{(i)+} \}, \quad i = 1, 2, 3.$$

First, we should check (30) and (45) -  $Q^{tot}$ -BRST-closed as extensions of BRST operators  $Q_c^{tot}$  in Alkalaev Grigoriev, Tipunin, 2008, 2011, R.R. Metsaev, Phys. Lett. B 720 (2013) in view of trace operators.

$\mathcal{Z}_j$  is determined from the condition of BRST closeness, e.g. for  $j = 1$

$$\begin{aligned} \mathcal{Z} \prod_{j=1}^3 \mathcal{L}_{k_j}^{(j)} &= Z \prod_{j=1}^3 \mathcal{L}_{k_j}^{(j)} - \sum_{i=1}^3 k_i \frac{b^{(i)+}}{h^{(i)}} [[L_{11}^{(i)}, Z}], L^{(i)} \} \prod_{j=1}^3 \mathcal{L}_{k_j - \delta_{ij}}^{(j)} \\ &+ \sum_{i \neq e}^3 k_i k_e \frac{b^{(i)+} b^{(e)+}}{h^{(i)} h^{(e)}} [L_{11}^{(e)}, [[L_{11}^{(i)}, Z}], L^{(i)} \} L^{(e)} \} \prod_{j=1}^3 \mathcal{L}_{k_j - \delta_{ij} - \delta_{ej}}^{(j)} \\ &- \sum_{i \neq e \neq o}^3 k_i k_e k_o \frac{b^{(i)+} b^{(e)+} b^{(o)+}}{h^{(i)} h^{(e)} h^{(o)}} [L_{11}^{(o)}, [L_{11}^{(e)}, [[L_{11}^{(i)}, Z}], L^{(i)} \} L^{(e)} \} L^{(0)} \} \prod_{j=1}^3 \mathcal{L}_{k_j - 1}^{(j)}. \end{aligned} \quad (33)$$

For  $j > 1$  expression for  $\mathcal{Z}_j$  is deduced analogously.

# BRST-closed cubic on oscillators operators $\mathcal{Z}$

For  $j = 2$

$$\begin{aligned}
 \mathcal{Z}_2 \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} &= Z\mathcal{Z} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} - \sum_{i_1=1}^3 \frac{b^{(i_1)+}}{h^{(i_1)}} \left[ \left[ \widehat{L}_{11}^{(i_1)}, Z \right], \mathcal{Z} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} \right] \\
 &+ \sum_{i_1 \neq e_1}^3 \frac{b^{(i_1)+} b^{(e_1)+}}{h^{(i_1)} h^{(e_1)}} \left[ \widehat{L}_{11}^{(e_1)}, \left[ \left[ \widehat{L}_{11}^{(i_1)}, Z \right], \mathcal{Z} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} \right] \right] \\
 &- \sum_{i_1 \neq e_1 \neq o_1}^3 \frac{b^{(i_1)+} b^{(e_1)+} b^{(o_1)+}}{h^{(i_1)} h^{(e_1)} h^{(o_1)}} \left[ \widehat{L}_{11}^{(o_1)}, \left[ \widehat{L}_{11}^{(e_1)}, \left[ \left[ \widehat{L}_{11}^{(i_1)}, Z \right], \mathcal{Z} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} \right] \right] \right]. \tag{34}
 \end{aligned}$$

for  $j \geq 1$  we have by induction

$$\begin{aligned}
 \mathcal{Z}_j \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} &= Z\mathcal{Z}_{j-1} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} - \sum_{i_{j-1}=1}^3 \frac{b^{(i_{j-1})+}}{h^{(i_{j-1})}} \left[ \left[ \widehat{L}_{11}^{(i_{j-1})}, Z \right], \mathcal{Z}_{j-1} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} \right] \\
 &+ \sum_{i_{j-1} \neq e_{j-1}}^3 \frac{b^{(i_{j-1})+} b^{(e_{j-1})+}}{h^{(i_{j-1})} h^{(e_{j-1})}} \left[ \widehat{L}_{11}^{(e_{j-1})}, \left[ \left[ \widehat{L}_{11}^{(i_{j-1})}, Z \right], \mathcal{Z}_{j-1} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} \right] \right] - \\
 &\quad \sum_{i_{j-1} \neq e_{j-1} \neq o_{j-1}}^3 \frac{b^{(i_{j-1})+} b^{(e_{j-1})+} b^{(o_{j-1})+}}{h^{(i_{j-1})} h^{(e_{j-1})} h^{(o_{j-1})}} \left[ \widehat{L}_{11}^{(o_{j-1})}, \left[ \widehat{L}_{11}^{(e_{j-1})}, \left[ \left[ \widehat{L}_{11}^{(i_{j-1})}, Z \right], \mathcal{Z}_{j-1} \prod_{p=1}^3 \mathcal{L}_{k_p}^{(p)} \right] \right] \right].
 \end{aligned}$$

# General (covariant) and partial solutions for the cubic vertices

general solution for covariant cubic vertex preserving irreps of  $ISO(1, d - 1)$  for HS fields  $(s_1, s_2, s_3)$  (thus correct degrees of freedom) when passing to interacting theory

$$|V^{(3)}\rangle_{(s)_3} = |V^{M(3)}\rangle_{(s)_3} + \sum_{(j_1, j_2, j_3) > 0}^{([s_1/2], [s_2/2], [s_3/2])} U_{j_1}^{(s_1)} U_{j_2}^{(s_2)} U_{j_3}^{(s_3)} |V^{M(3)}\rangle_{(s)_3 - 2(j)_3}, \quad (36)$$

$|V^{M(3)}\rangle_{(s)_3 - 2(j)_3}$  determined in R.R. Metsaev, PLB 720 (2013) with modified forms respecting trace  $\mathcal{L}_{k_i}^{(i)}$ , (29) и  $\mathcal{Z}_j$

$$|V^{M(3)}\rangle_{(s)_3 - 2(j)_3} = \sum_k \mathcal{Z}_{1/2\{(s-2J)-k\}} \prod_{i=1}^3 \mathcal{L}_{s_i - 2j_i - 1/2(s-2J-k)}^{(i)} |0\rangle, \quad (37)$$

$$(s, J) = (\sum_i s_i, \sum_i j_i). \quad (38)$$

and enumerated by naturals  $(k, j_1, j_2, j_3)$  satisfying to the equations

$$\boxed{s - 2J - 2s_{\min} \leq k \leq s - 2J, \quad k = s - 2J - 2p, \quad p \in \mathbb{N}_0, \quad 0 \leq j_i \leq [s_i/2].} \quad (39)$$

General vertex (45) besides modified terms (45) contains new ones. These are linear in trace  $U_{1_i}^{(s_i)} = \widehat{L}_{11}^{(i)+}$  for each field copy

$$\sum_{(j_1, j_2, j_3) > 0}^{(1, 1, 1)} (\widehat{L}_{11}^{(1)+})^{j_1} (\widehat{L}_{11}^{(2)+})^{j_2} (\widehat{L}_{11}^{(3)+})^{j_3} |V^{M(3)}\rangle_{(s)_3 - 2(j)_3}. \quad (40)$$

as differed from  $|V^{M(3)}\rangle_{(s)_3 - 2(j)_3}$  in vertex:  $b^{(i)+}, \eta_{11}^{(i)+} \mathcal{P}_{11}^{(i)+}$ ,  $i = 1, 2, 3$ .

## Correspondence with Metsaev's results on cubic vertices

Thus, for even  $s_i$ ,  $i = 1, 2, 3$  vertex for  $j_i = [s_i/2]$  be with  $s_i/2$  traces for initial fields  $|\phi\rangle_{s_i}$ ,  $\prod_i Tr^{s_i/2} \phi^{(i)}$ , without derivatives (!)

$$|\bar{V}^{(3)}\rangle_{(s)_3} = \prod_{i=1}^3 U_{[s_i/2]}^{(s_i)} |0\rangle = \prod_{i=1}^3 (\hat{L}_{11}^{(i)+})^{(j_i-2)} \{(\hat{L}_{11}^{+(i)})^2 - j_i(j_i-1)\eta_{11}^{(i)+} \mathcal{P}_{11}^{(i)+}\} |0\rangle. \quad (41)$$

For 1, 2 or for all odd  $s_1, s_2, s_3$ , the vertices with minimal number of derivatives will contain 1, 2 or 3 derivatives:

$$|V_1^{(3)}\rangle_{(s)_3} = U_{[s_1-1/2]}^{(s_1)} U_{[s_2/2]}^{(s_2)} U_{[s_3/2]}^{(s_3)} L^{(1)} |0\rangle, \quad (42)$$

$$|V_2^{(3)}\rangle_{(s)_3} = U_{[s_1-1/2]}^{(s_1)} U_{[s_2-1/2]}^{(s_2)} U_{[s_3/2]}^{(s_3)} L^{(1)} L^{(2)} |0\rangle, \quad (43)$$

$$|V_3^{(3)}\rangle_{(s)_3} = \prod_{i=1}^3 U_{[s_i-1/2]}^{(s_i)} \{ \prod_{i=1}^3 L^{(i)} + Z \} |0\rangle. \quad (44)$$

for  $d > 4$  the number of independent (even parity invariant) vertices in  $\forall |V^{M(3)}\rangle_{(s)_3-2(j)_3}$  enumerated by  $k$  & equal  $(s_{\min} - 2j_{\min} + 1)$ , but for  $d = 4$  it reduced to 2, i.e.  $k = s - 2J, s - 2J - 2s_{\min}$  due to proportionality for  $k$ :  $s - 2J - 2s_{\min} < k < s - 2J$ , to terms with  $\partial^2$  (**R. Metsaev, PLB 720 (2013)**). For  $s_1 = s_2 = s_3$  we have selfinteracting vertex (45)  $\phi_{\mu(s)}$ .

$|V^{(3)}\rangle \neq |V^{M(3)}\rangle$  not identical:  $L_{11}^{(i)}|V^{M(3)}\rangle \neq 0$ .

## Correspondence

1)  $|V_{irrep}^{M(3)}\rangle = |V^{M(3)}\rangle / L_{11}^{(i)}|V^{M(3)}\rangle$ ,

2) reducing  $\mathcal{M}_{un} \rightarrow \mathcal{M}_c$ :  $|V^{(3)}\rangle_{\mathcal{M}_{un}} \rightarrow \mathcal{M}_c = |\check{V}^{(3)}\rangle$

Then  $|V_{irrep}^{M(3)}\rangle = |\check{V}^{(3)}\rangle!$

# General solution for the vertex for 1 massive $(0, s_1), (0, s_2), (m, s_3)$

the general solution for the parity invariant vertex has the form

$$|V_m^{(3)}\rangle_{(s)_3} = |V_m^{M(3)}\rangle_{(s)_3} + \sum_{(j_1, j_2, j_3) > 0}^{([s_1/2], [s_2/2], [s_3/2])} U_{j_3}^{(s_1)} U_{j_3}^{(s_2)} U_{j_3}^{(s_3)} |V_m^{M(3)}\rangle_{(s)_3 - 2(j)_3},$$

where the vertex  $|V_m^{M(3)}\rangle_{(s)_3 - 2(j)_3}$  was defined constrained formulation but with modified forms  $\mathcal{L}_k^{(3)}$ , and  $\mathcal{L}_{11|\sigma^{(i+2)}}^{(ii+1)+}$  instead of

$$L_{11}^{(12)+} = a^{(1)\mu+} a_\mu^{(2)+} + \frac{1}{2m_3^2} L^{(1)} L^{(2)} - \frac{1}{2} \mathcal{P}_1^{(1)+} \eta_1^{(2)+} - \frac{1}{2} \mathcal{P}_1^{(2)+} \eta_1^{(1)+}$$

$$L_{11}^{(23)+} = a^{(2)\mu+} a_\mu^{(3)+} - \frac{1}{2m_3^2} L^{(2)} L^{(3)} + \frac{1}{2m_3} d^{(3)+} L^{(2)} - \frac{1}{2} \mathcal{P}_1^{(2)+} \eta_1^{(3)+} - \frac{1}{2} \mathcal{P}_1^{(3)+} \eta_1^{(2)+}$$

$$L_{11}^{(13)+} = a^{(1)\mu+} a_\mu^{(3)+} - \frac{1}{2m_3^2} L^{(1)} L^{(3)} - \frac{1}{2m_3} d^{(3)+} L^{(1)} - \frac{1}{2} \mathcal{P}_1^{(1)+} \eta_1^{(3)+} - \frac{1}{2} \mathcal{P}_1^{(3)+} \eta_1^{(1)+}$$

$$\mathcal{L}_k^{(3)} = \sum_{j=0}^{[k/2]} (-1)^j (L^{(3)})^{k-2j} (\hat{p}^{(3)})^{2j} \frac{k!}{j! 2^j (k-2j)!} \frac{(b^{(3)+})^j}{\prod_{p=0}^{j-1} (h^{(3)} + p)}.$$

$$V_{m|(s)_3 - 2(j)_3}^{M(3)} = \sum_k \mathcal{L}_k^{(3)} \prod_{i=1}^3 \mathcal{L}_{11|\sigma_{i+2}}^{(ii+1)+}, (s, J) = \left( \sum_i s_i, \sum_i j_i \right).$$

and is  $(3+1)$ -parameter family to be enumerated by the natural parameters  $(i)_{3,0}$  and  $k$ .

$$\begin{aligned} \sigma_i &= \frac{1}{2}(s - 2J - k) - s_i, \quad i = 1, 2; \quad \sigma_3 = \frac{1}{2}(s + k) - s_3, \\ \max\left(0, (s_3 - 2j_3) - \sum_{i=1}^2 (s_i - 2j_i)\right) &\leq k \leq s_3 - 2j_3 - |s_1 - 2j_1 - (s_2 - 2j_2)|, \\ 0 \leq 2j_i &\leq s_i, \quad s - 2J - k = 2p, \quad p \in \mathbb{N}_0. \end{aligned}$$

# Conclusion

- It is found general cubic interacting vertex (off-shell) for irreducible interacting HS fields with integer helicities  $s_1, s_2, s_3$  on Minkowski  $\mathbb{R}^{1,d-1}$  space;
- Construction (off-shell) covariant cubic interaction vertex massless and massive irreducible HS fields with  $(0, s_1)$ ,  $(0, s_2)$ ,  $(m, s_3)$  with some lower spin component examples within BRST approach (in progress);
- BRST approach is developed for constructing cubic vertices by deformation of 3 copies of free 1-st stage reducible gauge theories without any constraints for massless TS HS fields up to interacting 1-st stage reducible gauge theory with accuracy up to the 1-st order in  $g$ . Entangled system of equations are derived and solved on cubic vertex from deformed free actions and reducible gauge transformations with preservation of number of physical degrees of freedom;
- BRST-closed cubic vertex generalizes CV from [arXiv:1205.3131 [hep-th]] for massless HS fields and appears by covariant analog of even-parity vertex suggested in the light-cone formalism [hep-th/0512342] and reproduces new inputs into the vertex with traces and less numbers of space-time derivatives, including the terms without derivatives.

# Outlook

- Construction (off-shell) covariant cubic vertex for irreducible HS fields with integer and half-integer spins  $s_1, n_2 + \frac{1}{2}, n_3 + \frac{1}{2}$  on a base of BRST approach;
- Construction (off-shell) covariant cubic vertex for irreducible mixed-symmetric HS fields subject to  $Ys_1^1, \dots, s_1^k, Ys_2^1, \dots, s_2^l, Ys_3^1, \dots, s_3^m$  integer spins  $\bar{s}_1^k, \bar{s}_2^l, \bar{s}_3^m$ ;
- Construction (off-shell) covariant cubic supersymmetric vertex within BRST approach for irreducible irreps with (half)integer superspins;
- Derivation system of equations for quartic vertex and its solution for irreducible  $ISO(1, d - 1)$  massless representations with integer helicities  $s_1, s_2, s_3, s_4$  and study its (non)local realization .

- Construction (off-shell) covariant cubic vertex for irreducible HS fields with integer and half-integer spins  $s_1, n_2 + \frac{1}{2}, n_3 + \frac{1}{2}$  on a base of BRST approach;
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- Construction (off-shell) covariant cubic supersymmetric vertex within BRST approach for irreducible irreps with (half)integer superspins;
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**Thank you very much**