Strongly intensive variable in the model of high-energy pp-interactions with the string clusters formation

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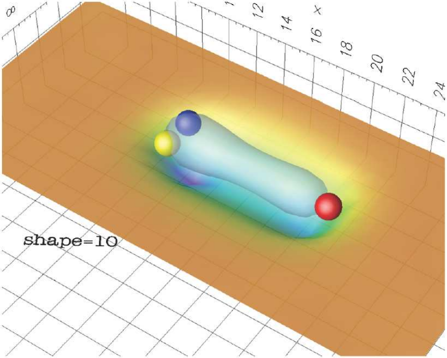
Strings as color flux tubes

Color flux-tubes (gluon, chromo-electric flux-tubes):
A.B. Kaidalov (QGSM), Phys. Lett. B **116**, 459 (1982)
A.B. Kaidalov, K.A. Ter-Martirosyan, Phys. Lett. B **117**,247 (1982)

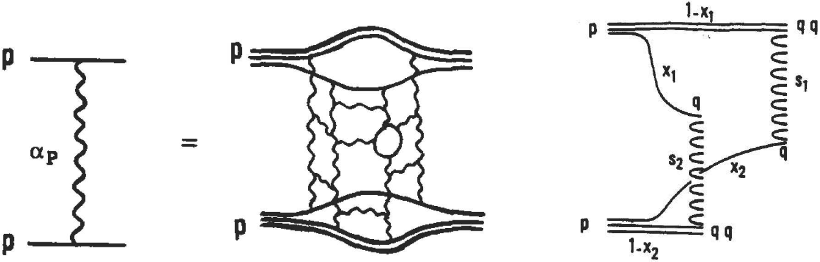
Confirmed by lattice QCD simulations:
F. Bissey, A. I. Signal, D. B. Leinwebe, Phys. Rev. D **80**, 114506 (2009)



Strings as a cut pomeron

Pomeron as a cylindrical structure (in the large color number limit):
G. 't Hooft, Nucl. Phys. B **72** (1974) 461 - 't Hooft's 1/Nc expansion
G. Veneziano, Nucl. Phys. B **117** (1976) 519 - Veneziano's topol.expan.

Cut pomeron as two strings (color reconnection of dipoles):
A. Capella, U.P. Sukhatme, C.–I. Tan, J. Tran Thanh Van (DPM)
Phys. Lett. B **81**, 68 (1979); Phys. Rep. **236**, 225 (1994)
K. Werner (VENUS,EPOS), Phys. Rep. **232**, 87 (1993)



Fragmentation of strings

Schwinger mechanizm in QED:

J. Schwinger, Phys. Rev. 82, 664 (1951)

A.I. Nikshov, Nucl. Phys. B21, 346 (1970)

T.D. Cohen and D.A. McGady, Phys.Rev.D 78, 036008 (2008)

Schwinger based picture in QCD:

E.G. Gurvich, Phys.Lett. 87B (1979) 386

A. Casher, H. Neunberg and S. Nussinov, Phys. Rev. D20 (1979) 179

M. Gyulassy and A. Iwazaki, Phys. Lett. B165 (1985) 157

A. Bialas, Phys. Lett. B 466 (1999) 301

Geometrical approach to sting fragmentation:

X. Artru, *Phys. Rep.* **97** (1983) 147

K. Werner (VENUS,EPOS), *Phys. Rept.* **232** (1993) 87

V.V.V., *Proceedings of the Baldin ISHEPP XIX vol.1*, JINR, Dubna (2008) 276-281; arXiv:0812.0604.

Various versions of the string model are used in such Monte Carlo (MC) event generators as PYTHIA, VENUS, HIJNG, AMPT, EPOS, DIPSY etc., to describe soft processes in strong interactions when perturbation QCD does not work.

Locality of soft part of strong interaction in rapidity

At large energy all these approaches lead to a homogeneous distribution in rapidity of particles produced from a string fragmentation and to a dependence of two-particle correlation functions only on differences of arguments.

Recall the Gribov-Regge approach $\Rightarrow$ the locality of interaction in rapidity (pseudorapidity)

$$y = \frac{1}{2}\ln\frac{k_+}{k_-} = \frac{1}{2}\ln\frac{k_0 + k_z}{k_0 - k_z} \qquad \eta = \frac{1}{2}\ln\frac{|\mathbf{k}| + k_z}{|\mathbf{k}| - k_z} = -\ln \mathrm{tg}\frac{\theta^*}{2} \tag{1}$$

$\Rightarrow$ Translation invariance of the particle spectra in rapidity at central domain:

$$\rho(\eta) \equiv \frac{dN_{ch}}{d\eta} = \rho_0 \ , \qquad \rho_2(\eta_1,\eta_2) \equiv \frac{d^2 N_{ch}}{d\eta_1\, d\eta_2} = \rho_2(\Delta\eta) \ . \tag{2}$$

$$\Delta\eta = \eta_1 - \eta_2$$

String fusion effects.

pp $\rightarrow$ pA $\rightarrow$ AA - the increase of the string density in transverse plain leads to the string fusion (color ropes or string cluster formation)

T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. B **245**, 449 (1984)
A. Bialas, W. Czyz, Nucl. Phys. B **267**, 242 (1986)
M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993)

Collective effects are observed also in high-multiplicity pp events at LHC

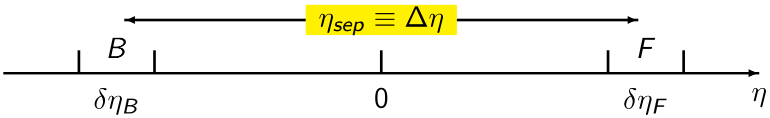
String fusion in pp collisions with increasing energy and centrality

$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.
$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).

N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, Phys.Rev.Lett. **73**, 2813 (1994).

The same ideas in DIPSY:
C. Bierlich, G. Gustafson, L. Lonnblad, A. Tarasov JHEP **03** (2015) 148
Effects of Overlapping Strings in pp Collisions

Forward-Backward (FB) Rapidity Correlations



Forward-Backward (FB) Rapidity Correlations: $\quad (k_z, \mathbf{k}_\perp) \Rightarrow (y, \mathbf{k}_\perp)$

$$y \equiv \frac{1}{2} \ln \frac{k_0 + k_z}{k_0 - k_z} \,, \qquad \eta \equiv \frac{1}{2} \ln \frac{|\mathbf{k}| + k_z}{|\mathbf{k}| - k_z} = -\ln \mathrm{tg} \left(\frac{\theta^*}{2} \right)$$

The correlation coefficient:

$$b_{BF} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{cov(F, B)}{D_F} \tag{3}$$

Short- and long-range rapidity correlations

Traditional Observables

Traditional FB correlation:
B, F $\Rightarrow$ n_B, n_F - the extensive variables $\Rightarrow$ b_{nn}
A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)
The locality of strong interaction in rapidity $\Rightarrow$
Short-Range FB Correlations (SRC),
between particles from a same source (string).
$z - \eta$ correspondence, *X.Artru,Phys.Rept.**97**(1983)147,*
V.V.V.,arXiv:0812.0604
Event-by-event variance in the number of cut pomerons (strings) $\Rightarrow$
Long-Range FB Correlations (LRC) at large η_{sep}
(the trivial "volume" fluctuations).

We'll look for observables, which is not sensitive to the fluctuation in the number of sources (strings), but is sensitive to the fluctuation in the quality of sources (e.g. to the formation of string clusters by string fusion).

Two-particle correlation function C_2

$$C_2(\eta_1, \eta_2) \equiv \frac{\rho_2(\eta_1, \eta_2)}{\rho(\eta_1)\rho(\eta_2)} - 1 \ ,$$

where

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} \ , \qquad \rho_2(\eta_1, \eta_2) = \frac{d^2 N_{ch}}{d\eta_1 \, d\eta_2}$$

This enables to extract from experimental data the absolute value of the $C_2(\eta_1, \eta_2)$ without any commonly used event mixing procedure, including the cases when a translation invariance in rapidity is absent.

If the translation invariance in rapidity is valid
(symmetric reaction, mid-rapidities, LHC energies) then:

$$\rho(\eta) = \rho_0 = const \ , \qquad \rho_2(\eta_1, \eta_2) = \rho_2(\eta_1 - \eta_2) = \rho_2(\Delta\eta)$$

$$C_2(\eta_1, \eta_2) = C_2(\Delta\eta) = \rho_2(\Delta\eta)/\rho_0^2 - 1 \ .$$

Relation of b_{nn} to the two-particle correlation function C_2

For the traditional n-n FB correlation coefficient b_{nn} we have for small observation windows (see e.g.V.V.,Nucl.Phys.A939(2015)21):

$$b_{nn} = \frac{\langle n_F\rangle\langle n_B\rangle}{D_{n_F}} \frac{\mathrm{cov}(n_F, n_B)}{\langle n_F\rangle\langle n_B\rangle} \to \frac{\langle n_F\rangle\langle n_B\rangle}{D_{n_F}} C_2(\eta_F, \eta_B)$$

For arbitrary symmetrical observation windows with translation invariance in rapidity: $\langle n_F\rangle = \langle n_B\rangle = \langle n\rangle$, $D_{n_F} = D_{n_B} = D_n$, $\omega_n \equiv D_n/\langle n\rangle$, and

$$b_{nn} = \frac{\langle n\rangle \mathrm{cov}(n_F, n_B)}{\omega_n} = \frac{\langle n\rangle\, I_{FB}}{1 + \langle n\rangle\, I_{FF}} \quad \to \frac{\langle n\rangle\, C_2(\eta_{sep})}{1 + \langle n\rangle\, C_2(0)}$$

where

$$I_{FF} = \frac{1}{\delta\eta_F^2} \int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2\;\, C_2(\eta_1 - \eta_2) \qquad \to C_2(0)$$

$$I_{FB} = \frac{1}{\delta\eta_F \delta\eta_B} \int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_B} d\eta_2\;\, C_2(\eta_1 - \eta_2) \qquad \to C_2(\eta_{sep})$$

The last limit is valid for the small windows: $\delta\eta_F = \delta\eta_B = \delta\eta \ll \eta_{corr}$.

Advanced Observables

The string fusion processes under consideration affects both LRC and SRC.
The LRC is sensitive to fluctuations in both quantity and type of sources.
The SRC is sensitive to the properties of a single source (string) and its modification in a process of string fusion into string clusters.

Unfortunately for traditional observables
the n_F-n_B correlation is strongly influenced by the "volume" fluctuations.

We can suppress the influence of these trivial "volume" fluctuations compared to the contribution of string fusion processes:

1) for LRC going from traditional extensive variables n_F and n_B to new intensive ones, e.g. event-mean transverse momenta p_F and p_B of all particles (n_F and n_B) in the intervals $\delta\eta_F$ and $\delta\eta_B$ (see e.g. [V.V., EPJ Web of Conf. 125, 04022 (2016)]).

2) for SRC going from b_{nn} to more sophisticated correlation observables, e.g. to the strongly intensive observable $\Sigma(n_F, n_B)$ (see e.g. [E. Andronov, V.V., Eur.Phys.J.A 55(2019)14, V.V., EPJ Web Conf. 191(2018)04011]).

The strongly intensive observable $\Sigma(n_F, n_B)$

The strongly intensive quantities
[*M.I.Gorenstein,M.Gazdzicki,Phys.Rev.C84(2011)014904*].
We define the strongly intensive observable $\Sigma(n_F, n_B)$ between multiplicities in forward (n_F) and backward (n_B) windows
[*E.V.Andronov,Theor.Math.Phys.185(2015)1383*] as

$$\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F \rangle + \langle n_B \rangle}[\langle n_F \rangle\, \omega_{n_B} + \langle n_B \rangle\, \omega_{n_F} - 2\,\mathrm{cov}(n_F\, n_B)] \; , \qquad (4)$$

where

$$\mathrm{cov}(n_F, n_B) \equiv \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle \; , \qquad (5)$$

and ω_{n_F} and ω_{n_B} are the corresponding scaled variances of the multiplicities:

$$\omega_n \equiv \frac{D_n}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} \; . \qquad (6)$$

Σ in the model with independent identical strings

The fundamental characteristics of a string:
one- and two-particle rapidity distributions from a single string decay:

$$\lambda(\eta) = \mu_0, \qquad \lambda_2(\eta_1, \eta_2) = \lambda_2(\eta_1 - \eta_2) = \lambda_2(\Delta\eta)$$

$\Lambda(\Delta\eta)$ - two-particle correlation function of a string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\Delta\eta)}{\mu_0^2} - 1 = \Lambda(\Delta\eta) \ .$$

$\delta\eta$ - the width of the observation windows ($\delta\eta_F = \delta\eta_B = \delta\eta$) (in simple case: $\delta\eta \ll \eta_{corr}$), $\Delta\eta = \eta_{sep}$ - distance between the observation windows
.

$$\boxed{\Sigma(n_F, n_B) = 1 + \mu_0\, \delta\eta\, [J_{FF} - J_{FB}] \to 1 + \mu_0 \delta\eta [\Lambda(0) - \Lambda(\Delta\eta)]}$$

$$J_{FF} = \frac{1}{\delta\eta_F^2} \int_{\delta\eta_F} dy_1 \int_{\delta\eta_F} dy_2\ \Lambda(y_1 - y_2) \to \Lambda(0)$$

$$J_{FB} = \frac{1}{\delta\eta_F \delta\eta_B} \int_{\delta\eta_F} dy_1 \int_{\delta\eta_B} dy_2\ \Lambda(y_1 - y_2) \to \Lambda(\Delta\eta)$$

Vechernin V 2018 Eur.Phys.J.: Web of Conf. 191 04011
Andronov E, Vechernin V 2019 Eur.Phys.J. A 55 14
Vechernin V, Andronov E 2019 Universe 5 15

Properties of Σ in model with independent identical strings

- We see that in the model with identical strings the $\Sigma(\Delta\eta)$ is a really strongly intensive quantity. It does not depend nor on the mean number of strings $\langle N\rangle$, nor on their event-by-event fluctuations $\omega_N \equiv D_N/\langle N\rangle$.
It depends ONLY on string parameters: μ_0 and $\Lambda(\Delta\eta)$.

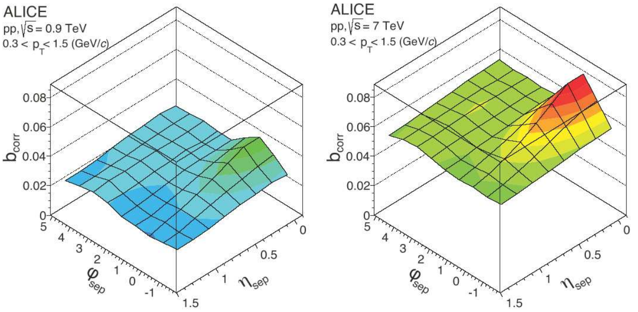
$\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B) = \Sigma(\Delta\eta)$ vs e.g. $C_2(\Delta\eta, \Delta\phi) = \frac{\omega_N + \Lambda(\Delta\eta,\Delta\phi)}{\langle N\rangle}$

V.V.,Nucl.Phys.A939(2015)21

- The $\Sigma(0) = 1$ and increases with the gap between windows, $\Delta\eta$, as the $\Lambda(\Delta\eta)$ decrease to 0 with $\Delta\eta$, since the correlations in a string go off with increase of $\Delta\eta$.
- The rate of the $\Sigma(\Delta\eta)$ growth with $\Delta\eta$ is proportional to the width of the observation window $\delta\eta$ and μ_0 - the multiplicity produced from one string.
- The model predicts saturation of the $\Sigma(\Delta\eta)$ on the level
$\Sigma(\Delta\eta) = 1 + \mu_0\delta\eta\,\Lambda(0) = \omega_\mu = D_\mu/\langle\mu\rangle$ at large $\Delta\eta$,
since $\Lambda(\Delta\eta) \to 0$ at the $\Delta\eta \gg \eta_{corr}$,
where the η_{corr} is a string correlation length.

The ALICE data on b_{nn} in pp

ALICE collab., JHEP 05(2015)097



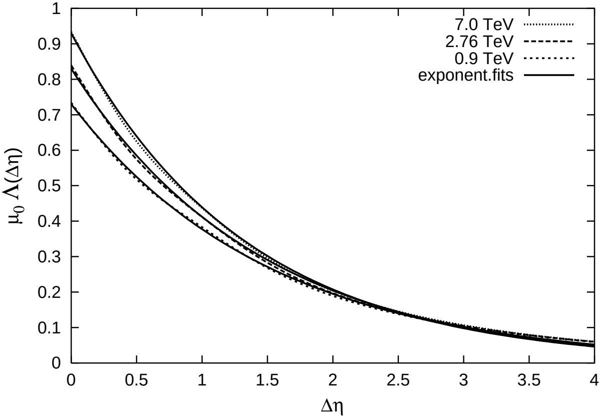
$$b_{nn} = \frac{\mu_0 \delta\eta\,[\omega_N + \Lambda(\Delta\eta,\Delta\phi)]}{1+\mu_0\delta\eta\,[\omega_N+\Lambda(0,0)]} \;\Rightarrow\; \omega_N,\, \Lambda(\Delta\eta,\Delta\phi) \;\Rightarrow\; \Lambda(\Delta\eta)$$

V.V.,Nucl.Phys.A939(2015)21; V.V., EPJ Web Conf. 191(2018)04011.
E. Andronov, V.V., Eur.Phys.J.A 55(2019)14,

The string correlation function $\Lambda(\Delta\eta)$

Then we find $\Lambda(\Delta\eta)$ integrating over azimuth:

$$\Lambda(\eta_{sep}) = \frac{1}{\pi}\int_0^{\pi} \Lambda(\eta_{sep}, \phi_{sep})\, d\phi_{sep} \ .$$



The string correlation function $\Lambda(\Delta\eta)$

The obtained dependencies in this fugure for three initial energies are well approximated by the exponent:

$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}} , \tag{7}$$

with the parameters presented in the table:

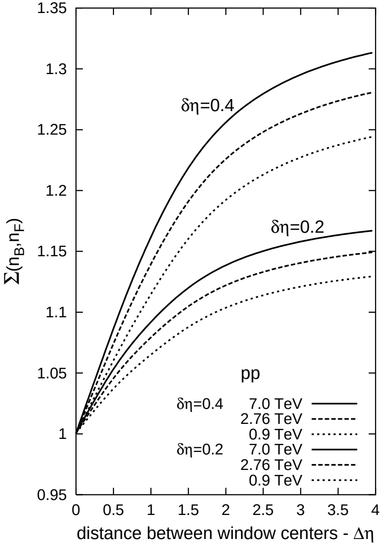
$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

[V.V., EPJ Web Conf. 191(2018)04011]

We see that the correlation length, η_{corr}, decreases with the increase of collision energy.

This can be interpreted as a signal of an increase with energy of the admixture of strings of a new type - the fused strings in pp collisions.

The predictions for the $\Sigma(n_F, n_B)$ in the model with independent identical strings



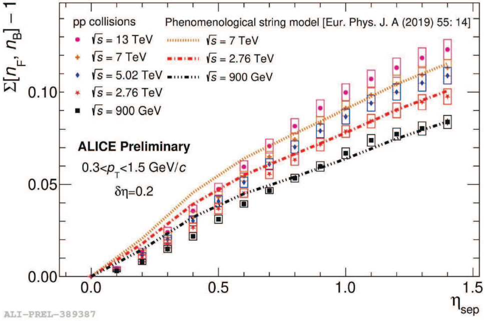
V.Vechernin, EPJ WoC 191 (2018) 04011, E.Andronov, V.Vechernin, Eur.Phys.J. A55 (2019) 14

Using the $\Lambda(\Delta\eta, \Delta\phi)$, extracted in V.Vechernin,Nucl.Phys.A939(2015)21 from the ALICE pp data on FB correlations in small acceptance windows, separated in azimuth and rapidity [ALICE collab., JHEP05(2015)097]

The string parameters occur dependent on initial energy (!?)
The hint on the increase of the string cluster contribution to $\Sigma(n_F, n_B)$ with collision energy in pp collisions at LHC energies.

Comparing the $\Sigma(n_F, n_B)$ with preliminary ALICE data

The comparison of the string model predictions with preliminary ALICE data for the $\Sigma(n_F, n_B)$ in pp collisions at energies 0.9 - 7 TeV [Andrey Erokhin (for the ALICE Collaboration) "Forward-backward multiplicity correlations with strongly intensive observables in pp collisions", The VI-th International Conference on the Initial Stages of High-Energy Nuclear Collisions (IS2021), 10-15 January 2021]:



"Phenomenological string model from [Eur.Phys.J.A55.1(2019),p.14] reproduces the quantitative behavior better than PYTHIA"

$\Sigma(n_F, n_B)$ in the model with string fusion

In the model with string fusion on transverse grid we find [*S.N. Belokurova, V.V.V., Theor.Math.Phys. 200(2019)1094*]:

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B) \ , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} \ , \tag{8}$$

where k is a degree of string overlapping and $\langle n^{(k)} \rangle$ is a mean number of particles produced from areas with such overlapping. $\sum \alpha_k = 1$.
Here $\Sigma_k(\mu_F, \mu_B)$ is the variable Σ for the cluster formed by k strings:

$$\Sigma_k(\mu_F, \mu_B) = \Sigma_k(\Delta\eta) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\Delta\eta)] \ ,$$

where $\mu_0^{(k)}$ and $\Lambda_k(\Delta\eta)$ are the corresponding parameters of the string cluster.

$$\Lambda_k(\Delta\eta) = \Lambda_0^{(k)} exp[-|\Delta\eta|/\eta_{corr}^{(k)}]$$

$\Sigma(n_F, n_B)$ in the model with string fusion

For such string cluster, formed by k fused strings, we expect, basing on the string decay picture
[*V.V.*, Baldin ISHEPP XIX v.1(2008)276; arXiv:0812.0604]:

1) larger multiplicity from one string, $\mu_0^{(k)} > \mu_0$,
2) smaller correlation length, $\eta_{corr}^{(k)} < \eta_{corr}$.

This corresponds to the analysis of the net-charge fluctuations in the framework of the string model for pp and AA collisions *[A.Titov, V.V., PoS(Baldin ISHEPP XXI)047(2012)]*.

Both factors lead to the steeper increase of $\Sigma_k(\Delta\eta)$ with $\Delta\eta$ and its saturation at a higher level

That is in accordance with the energy dependence obtained above for $\Sigma(n_F, n_B)$ from the ALICE pp data.

$\Sigma(n_F, n_B)$ in the model with string fusion

[*M.A.Braun,C.Pajares* Nucl.Phys.B 390 (1993) 542]

$$\mu_0^{(k)} = \mu_0^{(1)}\sqrt{k}\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)} = const\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)}/\sqrt{k}\ ,$$

which is instructive to compare with

$$\mu_0^{(k)} = \mu_0^{(1)}\, k\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)}/k\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)} = const\ .$$

for the case without string fusion in a given transverse cell.

(In last case $\Sigma(n_F, n_B) = \Sigma_1(\mu_F, \mu_B)$ and does not depends on α_k.)

The values of the parameters $\Lambda_0^{(1)} = 0.8$ and $\eta_{corr}^{(1)} = 2.7$ were chosen so that to obtain a correspondence with the values of the $\Sigma(n_F,\ n_B)$ obtained in [Vechernin V 2018 Eur.Phys.J.:Web of Conf. 191 04011].

Note that in that paper the $\Sigma(n_F,\ n_B)$ was calculated on the base of the string pair correlation function, $\Lambda(\Delta\eta)$, extracted in [V.Vechernin, Nucl.Phys.A939(2015)21] from the ALICE data on the FB correlations [ALICE collab.,JHEP05(2015)097] in the approx. of IDENTICAL strings.

MC calculations of $\Sigma(n_F, n_B)$ in the model with string clusters formation

[V.V. Vechernin, S.N. Belokurova, J.Phys.:Conf.Ser. 1690(2020)012088, arXiv:2012.07682, S. Belokurova, Phys.Part.Nucl.53(2022)154, arXiv:2011.10434]

- Modelling the initial string distribution in the impact parameter plane of pp collisions for different initial energies to take into account string fusion processes. Like in [*V. Vechernin, I. Lakomov. Proceedings of Science (Baldin ISHEPP XXI) (2013) 072.*].
- Monte Carlo simulations of string configurations and calculation of weighting factors α_k as a function of centrality and initial energy of pp collision.

$$\alpha_k = \frac{\langle n^{(k)}\rangle}{\sum_{k=1}^{\infty}\langle n^{(k)}\rangle} = \frac{\langle m^{(k)}\rangle \mu_0^{(k)}\delta\eta}{\sum_{k=1}^{\infty}\langle m^{(k)}\rangle \mu_0^{(k)}\delta\eta} = \frac{\langle m^{(k)}\rangle \sqrt{k}}{\sum_{k=1}^{\infty}\langle m^{(k)}\rangle \sqrt{k}} ,$$

where the $\langle m^{(k)}\rangle$ is the mean number of clusters with k fused strings, which we take from our MC simulations of the string configurations.

- Calculation the $\Sigma(n_F, n_B)$ for different centralities of pp collision at few LHC energies using the relation (8).

Various versions of string fusion

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$\langle n\rangle_k = \mu_0\sqrt{k}\, S_k/\sigma_0$, $\langle p_t^2\rangle_k = p_0^2\sqrt{k}$, $k = 1, 2, 3, \dots$

global fusion (clusters)

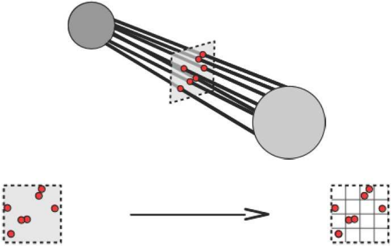
M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

$\langle p_t^2\rangle_{cl} = p_0^2\sqrt{k_{cl}}$, $\langle n\rangle_{cl} = \mu_0\sqrt{k_{cl}}S_{cl}/\sigma_0$, $k_{cl} = k\sigma_0/S_{cl}$

the version of SFM with the finite lattice (grid) in transverse plane

Vechernin V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136

Braun M.A., Kolevatov R.S., Pajares C., Vechernin V.V., Eur.Phys.J. C32 (2004) 535



Domains in transverse area

The approach with string fusion on a transverse lattice (grid) was exploited later for a description of various phenomena (correlations, anisotropic azimuthal flows, the ridge) in high energy hadronic collisions in
ALICE collaboration et al., J. Phys. G **32** 1295 (2006), [Sect. 6.5.15]
V.V.V., Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858
M.A. Braun, C. Pajares, Eur. Phys. J. C **71**, 1558 (2011)
M.A. Braun, C. Pajares, V.V.V., Nucl. Phys. A **906**, 14 (2013)
V.N. Kovalenko, Phys. Atom. Nucl. **76**, 1189 (2013)
M.A. Braun, C. Pajares, V.V.V., Eur. Phys. J. A **51**, 44 (2015)
V.V.V., Theor. Math. Phys. 184 (2015) 1271
V.V.V., Theor. Math. Phys. 190 (2017) 251
It leads to the splitting of the transverse area into domains with different, fluctuating values of color field within them.
What was also considered in the CGC approach
A.Kovner., M. Lublinsky, Phys.Rev. D **83**, 034017 (2011)

Distribution of strings in the transverse plane

pp interactions

$$w_{str}(\vec{s},\vec{b}) \sim T(\vec{s}-\vec{b}/2)\,T(\vec{s}+\vec{b}/2)/\sigma_{pp}(b) \qquad (9)$$

$\sigma_{pp} = \int \sigma_{pp}(b)\ d^2\vec{b}$ - non-diffractive pp cross section

$T(\vec{s}) = \int\limits_{-\infty}^{+\infty} \rho(\vec{s},z)\ dz$ - parton profile function of nucleon

$$\rho(r) = \frac{1}{\pi^{3/2}\alpha^3} e^{-r^2/\alpha^2}\ , \qquad T(s) = \frac{e^{-s^2/\alpha^2}}{\pi\alpha^2}\ , \qquad (10)$$

$$w_{str}(\vec{s},\vec{b}) \sim e^{-(\vec{s}+\vec{b}/2)^2/\alpha^2} e^{-(\vec{s}-\vec{b}/2)^2/\alpha^2}/\sigma_{pp}(b) = e^{-2s^2/\alpha^2} e^{-b^2/2\alpha^2}/\sigma_{pp}(b)$$

b–s factorization $\Rightarrow$

$$\langle N_{str}(b)\rangle \sim e^{-b^2/2\alpha^2}/\sigma_{pp}(b) \qquad (11)$$

Event-by-event fluctuations of the number of cut pomerons

$\boxed{P(N,b) = e^{-\overline{N}(b)}\overline{N}(b)^N/N!}$ -Poisson, $\quad P(0,b) = e^{-\overline{N}(b)}$

$\boxed{\widetilde{P}(N,b) = P(N,b)/[1-P(0,b)]}$ -modified Poisson, $\sum_{N=1}\widetilde{P}(N,b) = 1$

$$\langle N(b)\rangle = \sum_{N=1} N\,\widetilde{P}(N,b) = \overline{N}(b)/[1-P(0,b)] \tag{12}$$

$$\sigma_{pp}^{ND}(b) = 1 - P(0,b) = 1 - e^{-\overline{N}(b)} \tag{13}$$

$$\langle N(b)\rangle = \overline{N}(b)/\sigma_{pp}^{ND}(b)$$

$$\boxed{\overline{N}(b) = N_0 e^{-b^2/2\alpha^2}} \qquad \boxed{\langle N(b)\rangle = \overline{N}(b)/[1-\exp(-\overline{N}(b))]}$$

$N_{str} = 2N$, $\quad N$ - the number of cut pomerons in a given event

$$\boxed{\langle N(b)\rangle = N_0 e^{-b^2/2\alpha^2}/\sigma_{pp}(b)}$$

Probability to have N cut pomerons in a non-diffractive pp collision

Integration over the impact parameter b leads to

$$\boxed{w_N = \frac{2\pi\alpha^2}{\sigma_{pp} N}\left[1 - e^{-N_0}\sum_{k=0}^{N-1} N_0^k/k!\right] = \frac{\sigma_N}{\sigma_{pp}^{ND}}}$$

where we have introduced the σ_N by

$$\boxed{\sigma_N \equiv \frac{2\pi\alpha^2}{N}\left[1 - e^{-N_0}\sum_{k=0}^{N-1} N_0^k/k!\right]}$$

$$\sum_{N=1}^{\infty}\sigma_N = 2\pi\alpha^2[E_1(N_0) + \gamma + \ln N_0] = \sigma_{pp}^{ND}$$

where σ_{pp}^{ND} is the non-diffractive pp cross section.

$$E_1(z) = \int_1^{\infty} e^{-zt}\,\frac{dt}{t}\,, \qquad \gamma = 0.577...$$

Comparison with quasi-eikonal and Regge approaches

Now we see that our formula for the σ_N coincides with the well known result for the cross-section σ_N of N cut-pomeron exchange, obtained in the quasi-eikonal and Regge approaches:

$$\sigma_N = \frac{4\pi\lambda}{C\,N}\left[1 - e^{-z}\sum_{k=0}^{N-1} z^k/k!\right]$$

where

$$z = \frac{2\gamma C}{\lambda}\exp(\Delta\xi)\ , \qquad \lambda = R^2 + \alpha'\xi\ , \qquad \xi = \ln(s/1GeV^2)\ .$$

Here Δ and α' are the residue and the slope of the pomeron trajectory. The parameters γ and R characterize the coupling of the pomeron trajectory with initial hadrons. The quasi-eikonal parameter C is related to the small-mass diffraction dissociation of incoming hadrons.

K.A. Ter-Martirosyan Phys. Lett. B **44**, 377 (1973).
A.B. Kaidalov, K.A. Ter-Martirosyan Yad. Fiz. **39**, 1545 (1984); **40**, 211 (1984).
V.A. Abramovsky, V.N. Gribov, O.V. Kancheli Yad. Fiz. **18**, 595 (1973).

Comparison with the Regge approach

This enables to connect the parameters N_0 and α of our model with the parameters of the pomeron trajectory and its couplings to hadrons. Comparing we have

$$N_0 = z = \frac{2\gamma C}{\lambda} \exp(\Delta\xi) \ , \quad \alpha = \sqrt{\frac{2\lambda}{C}} \ , \quad \lambda = R^2 + \alpha' \xi \tag{14}$$

The numerical values of the parameters in the paper:

G.H.Arakelyan,A.Capella,A.B.Kaidalov,Yu.M.Shabelski Eur.Phys.J.C**26**,81(2002)

$$\Delta = 0.139, \quad \alpha' = 0.21 \ GeV^{-2},$$
$$\gamma_{pp} = 1.77 \ GeV^{-2}, \quad R^2 = 3.18 \ GeV^{-2}, \quad C = 1.5 \ ,$$

Our values of the parameters:

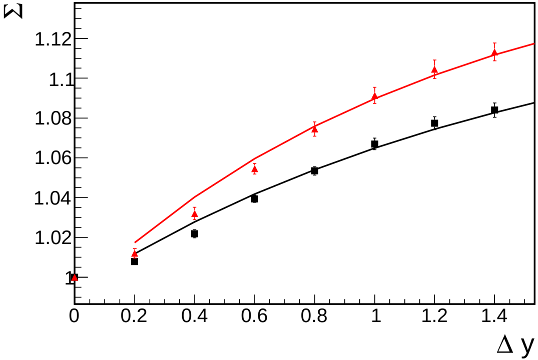
$$\Delta = 0.2, \quad \alpha' = 0.05 \ GeV^{-2},$$
$$\gamma_{pp} = 1.035 \ GeV^{-2}, \quad R^2 = 3.3 \ GeV^{-2}, \quad C = 1.5 \ .$$

Soft and Hard Pomeron:

J. Bleibel, L.V. Bravina, E.E. Zabrodin. Phys. Rev. D 93, 114012 (2016)

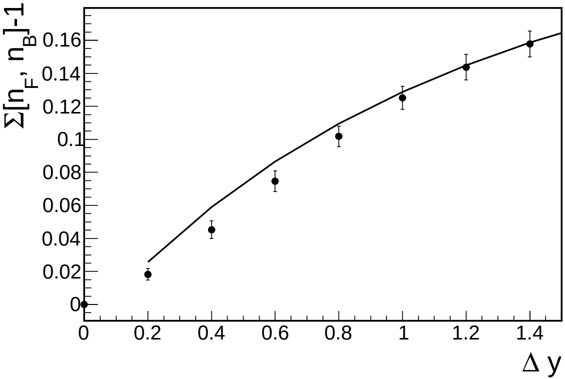
Comparison with the ALICE experimental data

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B)$$



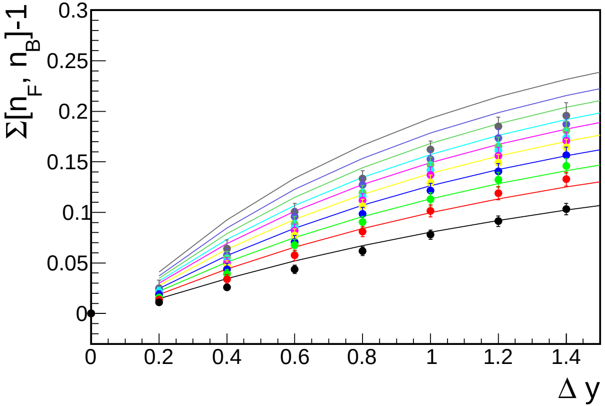
The $\Sigma(n_F, n_B)$ for windows of width δy =0.2 for two initial energies 0.9 TeV (lower curve) and 7 TeV (top curve). Points - experimental values obtained in [A.Erokhin (for the ALICE Collaboration) reported at IS2021] for particles with transverse momenta 0.3-1.5 GeV/c at the energies 0.9 TeV (■) and 7 TeV(▲).

Comparison with the ALICE experimental data



The $\Sigma(n_F, n_B)$ for windows of width δy =0.2 for initial energy 13 TeV (the curve). Points - experimental values obtained in [A.Erokhin (for the ALICE Collaboration) reported at IS2021] for particles with transverse momenta 0.2-2.0 GeV/c at the energy 13 TeV (•).

Comparison with the ALICE experimental data



The $\Sigma(n_F, n_B)$ for different pp-collision centrality classes at initial energy 13 TeV, Experimental points from [A.Erokhin (for the ALICE Collaboration) reported at IS2021]. Curves - our results in the model with the formation of string clusters. The centrality classes defined as follows (top down): 0–1%, 1–5%, 5-10%, 10-15%, 15-20%, 20-30%, 30-40%, 40-50%, 50-70%, 70-100%.

The model with independent identical strings

- In this version of the model the variable $\Sigma(n_F, n_B)$ depends only on the individual characteristics of a string and is independent of both the mean number of strings and its fluctuation, which reflects its strongly intensive character.
- So the studies of this observable enable to extract from the experimental data these fundamental characteristics of an individual string - a mean number of particles per unit of rapidity, μ_0, and the pair correlation function, $\Lambda(\Delta\eta, \Delta\phi)$, for particles produced from a fragmentation of a single string.
- However in this version of the model the string parameters occur dependent on collision energy. This fact can be considered as a signal that with increasing of the initial energy of a pp collision due to the string fusion the formation of the sources with new properties] - the string clusters - takes place

The model with string fusion and string clusters formation

- In this case the observable $\Sigma(n_F, n_B)$ is equal to a weighted average of its values for different string clusters, $\Sigma_k(\mu_F, \mu_B)$, with weight factors, α_k, which are proportional to the mean number of the particles, produced from all clusters formed by the k fused strings.
- The $\Sigma(n_F, n_B)$, through these weight factors, α_k becomes dependent on collision conditions - its energy and centrality.
- Analyzing these dependencies of the $\Sigma(n_F, n_B)$ we can extract from the experimental data the information on the individual characteristics of the string clusters - the multiplicity density, $\mu_0^{(k)}$, and the pair correlation function, $\Lambda_k(\Delta\eta)$, for particles, produced from a decay of a given cluster.
- In the framework of this approach it was shown that the overall increase of the $\Sigma(n_F, n_B)$ in pp collisions with collision energy and centrality can be explained by the formation of string clusters with new properties.

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Backup

Backup slides

C_2 through multiplicities in two small windows

For two small windows $\delta\eta_1$ and $\delta\eta_2$ around η_1 and η_2 we have

$$\rho(\eta) = \frac{\langle n \rangle}{\delta\eta} \ , \qquad \rho_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\delta\eta_1 \, \delta\eta_2} \ , \tag{15}$$

$$C_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} - 1 \ , \tag{16}$$

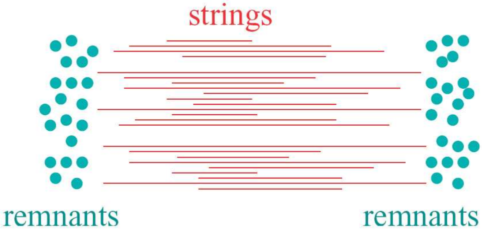
where n_1 and n_2 are the event multiplisities in these windows $\delta\eta_1$ and $\delta\eta_2$. Note that when $\eta_1 = \eta_2 = \eta$, $\eta_{sep} = 0$, we have to use

$$\rho_2(\eta, \eta) = \frac{\langle n(n-1) \rangle}{\delta\eta^2} \ , \qquad C_2(0) = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1 = \frac{\omega_n - 1}{\langle n \rangle} \ , \tag{17}$$

where n is the number of particles in small window $\delta\eta$ around the point η. (see e.g. [C.Pruneau,S.Gavin,S.Voloshin,Phys.Rev.C66(2002)044904] or [V.V.,Nucl.Phys.A939(2015)21]).

Initial state in the EPOS event generator

K. Werner, Collective phenomena in AuAu@RHIC and pp@LHC, ALICE Club, 21 Nov 2008, CERN, 2008.



One flux tube is the result of merging many individual strings

Epos: initial energy density obtained from strings, not partons

Fitting the parameters of the initial string distribution in the impact parameter plane of pp collisions

Table: The non-diffractive cross section, the multiplicity density at mid-rapidity and the mean number of initial strings in *pp* collisions at different initial energies.

$\sqrt{s}(GeV)$	$\sigma_{th}^{ND}(mb)$	$\sigma_{MC}^{ND}(mb)$	dN^{ND}/dy	$\langle N_{str} \rangle$
60	24.9	24.9	2.44	4.2
900	39.9	39.9	3.76	7.8
7000	52.5	52.4	5.44	13.4
13000	56.5	56.6	6.03	16.0

$$\sigma_{MC\ \ simulations}^{ND} = \frac{n_{sim}(N{=}0)}{n_{sim}(N{\geq}0)} S_b$$

$$\mu_0^{(k)} = \mu_0^{(1)} \sqrt{k} \text{ with } \mu_0^1 = 0.7$$

The parametrization of the single correlation function

The parametrization for the pair correlation function $\Lambda(\eta,\phi)$ of a single string (reflecting the Schwinger mechanism of a string decay, was suggested in [V.V.,Nucl.Phys.A939(2015)21]:

$$\Lambda(\eta,\phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\varphi^2}{\varphi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}} \ . \tag{18}$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and φ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and φ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (18)

$$|\varphi| \le \pi \ . \tag{19}$$

If $|\varphi| > \pi$, then we use the replacement $\varphi \to \varphi + 2\pi k$, so that (19) was fulfilled. With such completions the $\Lambda(\eta,\phi)$ meets the following properties

$$\Lambda(-\eta,\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta;-\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta,\phi+2\pi k) = \Lambda(\eta,\phi) \tag{20}$$

Fitting the model parameters by FBC in small windows

$\Lambda(\eta_{sep}, \phi_{sep})$ was fitted by the ALICE b_{nn} pp data with FB windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$, separated in azimuth and rapidity *[ALICE collab.,JHEP 05(2015)097]*. It gives for the parameters:

$\sqrt{s}$, TeV		0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i=1$ corresponds to the nearside and $i=2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.

[V.V.,Nucl.Phys.A939(2015)21]

$\Sigma(n_F, n_B)$ in windows separated in azimuth and rapidity

For small observation windows:

$$\boxed{\Sigma(\Delta\eta,\Delta\phi) = 1 + \frac{\delta\eta\,\delta\phi}{2\pi}\mu_0\,[\Lambda(0,0) - \Lambda(\Delta\eta,\Delta\phi)]}$$

$$\Delta\eta \equiv \eta_{sep}, \qquad \Delta\phi \equiv \phi_{sep}$$

For observation windows of an arbitrary width $\delta\eta_F\delta\phi_F$ and $\delta\eta_B\delta\phi_B$:

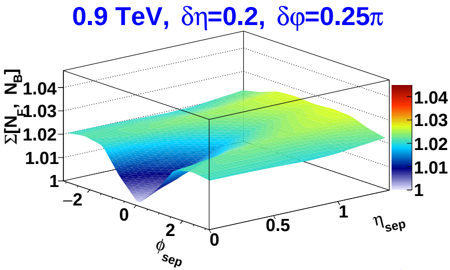
$$\Lambda(\Delta\eta,\Delta\phi) \to J_{FB}(\Delta\eta,\Delta\phi) = \frac{1}{\delta\eta_F\delta\phi_F\delta\eta_B\delta\phi_B}\times$$

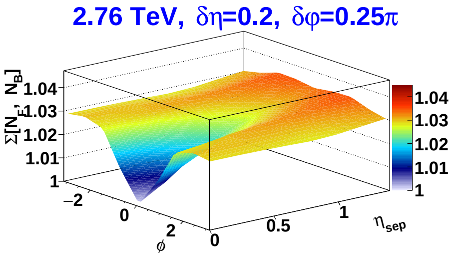
$$\times\int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_B\delta\phi_B} d\eta_2 d\phi_2\,\Lambda(\eta_1-\eta_2;\phi_1-\phi_2)\ ,$$

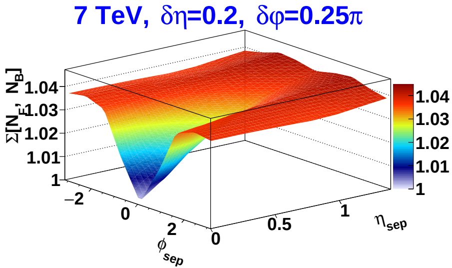
$$\Lambda(0,0) \to J_{FF} = \frac{1}{(\delta\eta_F\delta\phi_F)^2}\int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_F\delta\phi_F} d\eta_2 d\phi_2\,\Lambda(\eta_1-\eta_2;\phi_1-\phi_2)\ .$$

V.Vechernin, Nucl.Phys.A 939 (2015) 21

Σ for $\delta\eta$ $\delta\phi$ windows separated in azimuth and rapidity







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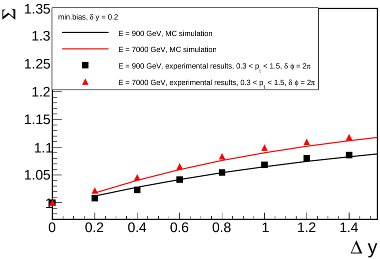
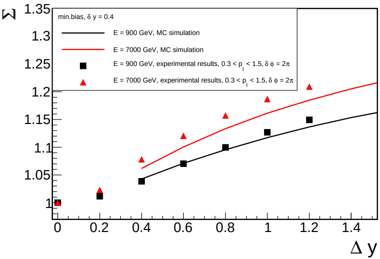
$\Sigma(n_F, n_B)$ for two widths of windows at 0.9 and 7 TeV





The strongly intensive observable $\Sigma(n_F, n_B)$ for pp collisions as a function of the rapidity distance $\Delta\eta = \Delta y$ between the centers of the FB observation windows, for two widths of windows: $\delta\eta$=0.2 (left panel) and $\delta\eta$=0.4 (right panel), and for two initial energies: 0.9 TeV (dashed lines) and 7 TeV (solid lines), calculated for particles with transverse momenta in the interval 0.3-1.5 GeV/c. Experimental points from [A.Erokhin (for the ALICE Collaboration) reported at IS2021].

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with energy is caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.

Centrality (multiplicity) dependence

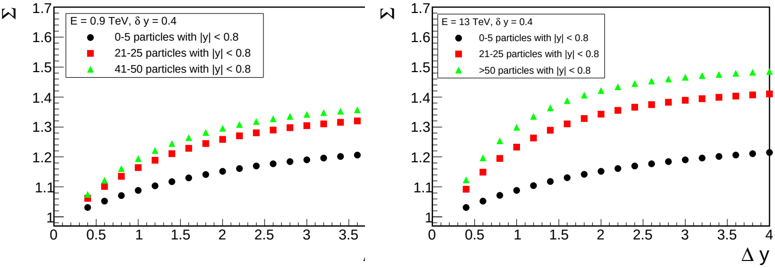


Figure: The strongly intensive variable $\Sigma(n_F,\ n_B)$ at different centralities as a function of the rapidity distance between the observation windows $\Delta\eta = \Delta y$ for *pp* collisions at energies 900 and 13000 GeV for the width of the observation windows $\delta\eta = 0.4$.

The increase of the $\Sigma(n_F, n_B)$ in pp collisions with the collision centrality is also caused by the increasing contribution of string fusion processes and the formation of string clusters with new properties.

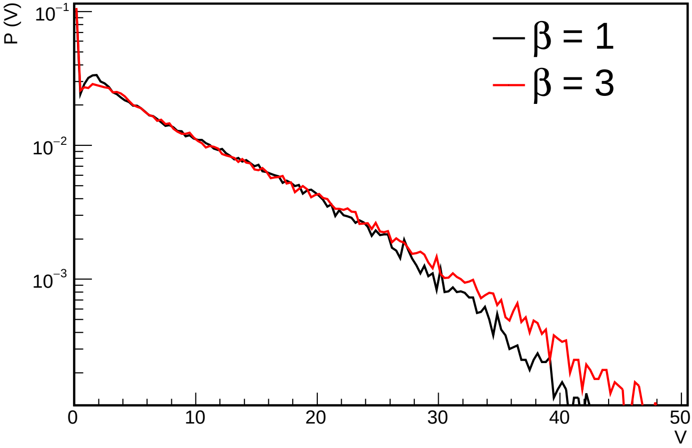
Modeling the centrality determinaion by V0 ALICE detector

To imitate this signal in our MC model calculations we carried out a preliminary MC simulation of 1000 000 min-bias pp events, determining the multiplicity N in a fixed rapidity interval, corresponding to the total acceptance of V0A and V0C detectors.
Then to generate the continuous signal, V0M, corresponding to this multiplicity N we use a detector response function:

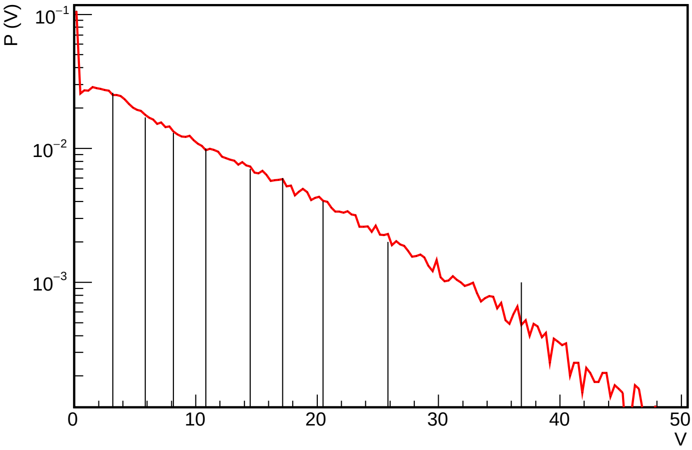
$$P_N(V) = C\,\theta(V)\exp\left[-\frac{(V-\gamma N)^2}{2\beta\,\gamma N}\right]\ ,$$

which is typical for detectors of this kind (see e.g. [Kurepin A.B.; Litvinenko A.G.; Litvinenko E.I. Phys. Atom. Nucl. 83 (2020) 1359].
The average value of the signal, $\langle V\rangle$, is proportional to the number of particles N that hit the detector, and the parameter β characterizes the magnitude of the signal smear around this average value. Note also that the relative error, δ_V, decreases with N as $1/\sqrt{N}$.

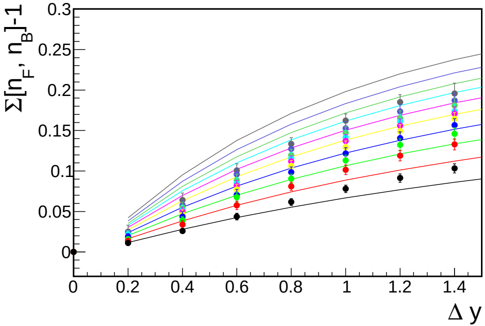
Modeling the centrality determinaion by V0 ALICE detector



Modeling the centrality determinaion by V0 ALICE detector

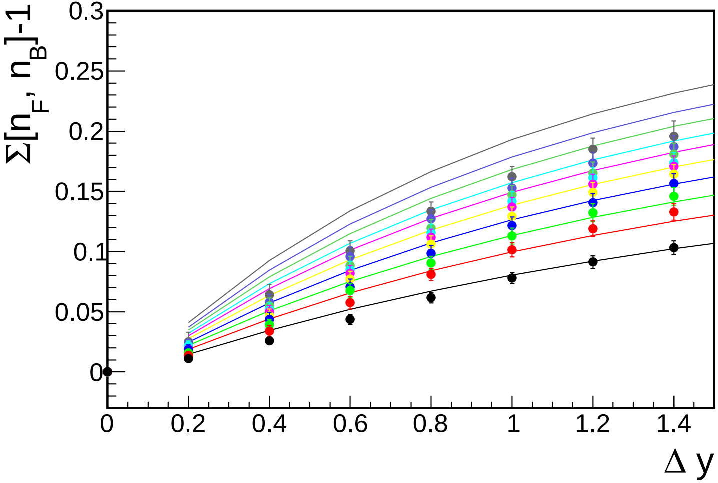


Comparison with the ALICE experimental data



The $\Sigma(n_F, n_B)$ for different pp-collision centrality classes at initial energy 13 TeV, Points - experimental values obtained in [A.Erokhin (for the ALICE Collaboration) reported at IS2021] Curves - our results in the model with the formation of string clusters. The centrality classes defined as follows (top down): 0–1%, 1–5%, 5-10%, 10-15%, 15-20%, 20-30%, 30-40%, 40-50%, 50-70%, 70-100%. $\beta = 1$

Comparison with the ALICE experimental data



The same but for $\beta = 3$.