Baryon Asymmetry and Holographic Composed Higgs

Early Universe First Order Phase Transition due to the Composite Higgs Boson Dynamics in the Soft-wall Holographic Model

Oleg Novikov, Andrey Shavrin

St Petersburg University

Baryon Asymmetry

Sakharov conditions
(necessary conditions to produce the antisymmetry of matter and antimatter):

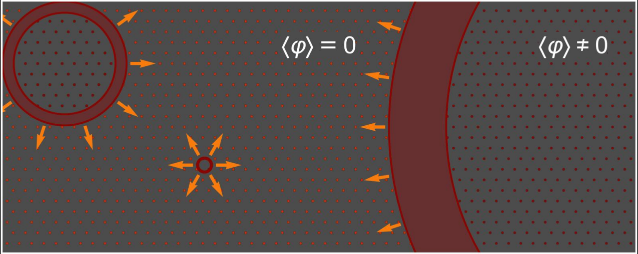
1. baryon number violation,
2. CP violation (particle - antiparticle),
3. CPT violation (violation of the thermodynamic equilibrium).

- Only the second is satisfied in perturbative Standard Model (CKM matrix; effect is too small).
- Counting non-perturbative effects (sphalerons) allows fulfilling the first two conditions.
- CTP theorem leads to "washing" of the asymmetry. It must be violated, for instance, with 1st order phase transition. In Standard Model ...

Electroweak Baryogenesis: Concept of "bubble" nucleation in SM

In SM: Phase Transition $\Rightarrow$ Electroweak Symmetry Breaking
(in the efficient potential of the electroweak model)

$$\mathsf{SU}(2)_\mathsf{L} \otimes \mathsf{U}(1)_\mathsf{Y} \to \mathsf{U}(1)_\mathsf{em}$$



$$m_{\text{Higgs B.}} < 80 \text{ GeV} \quad \Rightarrow \quad \text{1st order (wanted)}$$
$$m_{\text{Higgs B.}} = 125 \text{ GeV} \quad \Rightarrow \quad \text{crossover (in SM)}$$

Baryon asymmetry cannot be explained in the framework of Standard Model.

Composite Higgs model (CH)

Problem: EW symmetry breaking doesn't disturb the T.d. equilibrium.
Possible solution: to consider *inner* symmetry braking, e.g.: CH.

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{CH}} + \underbrace{W_\mu^\alpha J_L^{\alpha\,\mu} + B_\mu J_Y^\mu + \sum_r \bar{\psi}_r \mathcal{O}_r + \mathrm{h.c.}}_{\text{SM -- CH interaction}}$$

$\mathcal{L}_{\mathrm{CH}}$ is a new strongly coupled sector
$J_L^{\alpha\mu}$ and J_Y^μ are conserved currents of the new sector
$\mathcal{O}_r$ are composite operators of the new sector fields

Composite Higgs model (CH)

$$\begin{pmatrix}\mathcal{G} & \text{invariant} \\ & \text{vacuum}\end{pmatrix} \xrightarrow[\text{spontaneous}]{\text{breaking}} \begin{pmatrix}\mathcal{H} & \text{invarian} \\ & \text{vacuum}\end{pmatrix} \Rightarrow \text{Goldstone bosons} \ni \text{Higgs boson}$$

$$\begin{matrix}\text{physical model} \\ \text{must include EW}\end{matrix} : \mathcal{G} \supset \mathcal{H} \supseteq \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y} \Rightarrow \begin{matrix}\text{minimal} \\ \text{model}\end{matrix} : \mathcal{G} = \mathrm{SO}(5),\ \mathcal{H} = \mathrm{SO}(4)$$

Ψ_I are fundamental fields with $\mathrm{SO}(5)$ inner symmetry

$$\Sigma_{IJ} = \langle \bar{\Psi}_I \Psi_J \rangle = \xi^\top \left[\begin{pmatrix} 0_{4\times4} & 0 \\ 0 & X \end{pmatrix} + \eta_i \tilde{T}_i \right] \xi \xrightarrow[\text{low energy}]{\mathrm{SO}(5)\to\mathrm{SO}(4)} \begin{pmatrix} 0_{4\times4} & 0 \\ 0 & X \end{pmatrix} \Rightarrow \begin{matrix}\text{symmetry} \\ \text{breaking}\end{matrix}$$

$\xi = \exp(-i\pi_a T_a)$ are Goldstone bosons of the coset $T_a \in \mathrm{SO}(5)/\mathrm{SO}(4)$.
η_i are "radial" fluctuations along "unbroken" generators $\tilde{T}_i \in \mathrm{SO}(4)$
X is the background field. It gives main contribution in the effective potential.

Composite Higgs in AdS/CFT correspondence

$\mathcal{L}_{\rm CM}$ at $N \gg 1 \quad \Leftrightarrow \quad$ AdS/CFT correspondence

The condensate of $\mathcal{L}_{\rm CM}$: $\Sigma \sim \chi$ is dual to some filed in AdS

$$S_\chi = \int d^5x \sqrt{|g|}\, e^{\phi} \left(\mathrm{tr}\, g^{\mu\nu} (\partial_\mu \chi)^\top (\partial_\nu \chi) - 2 V_\chi(\Sigma) \right)$$

Extra motivation to use CHM: AdS/QCD
"AdS/CHM" has the same tunes.

CHM $\sim$ QCD $\Rightarrow$ we can use the results of AdS/QCD.
(i.e. there is reason to suppose self-consistent; but we should not be lazy and check it anyway)

Effective Potential and Holography

$$\mathcal{Z}[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] - \int d^4x \, \varphi(x)\, J(x)\right) \overset{\text{def}}{=} e^{-W[J]}$$

$$\langle\varphi\rangle = \left.\frac{\delta W[J]}{\delta J}\right|_{J=0}, \quad \Gamma[\langle\varphi\rangle] = W[J] - \int d^4x \, \frac{\delta W[J]}{\delta J(x)} J(x)$$ – Effective Action

$$\boxed{\text{EoM: } \frac{\delta\Gamma}{\delta\langle\varphi\rangle} = J}$$

extrema condition

Homogeneus Solution $\Rightarrow \langle\varphi\rangle = \text{const} \Rightarrow \Gamma = -\text{Vol}_4 \, V_{\text{eff}}$ — Effective Potential

---

$$\mathcal{Z}[J] \xlongequal[\text{correspondence}]{\text{AdS/CFT}} \mathcal{Z}_{\text{AdS}} \overset{\text{quasiclassical}}{\underset{\text{approximation}}{\approx}} \left. e^{-S_{\text{AdS}}} \right|_{\partial\text{AdS}}$$

$$\boxed{V_{\text{eff}} = -\frac{1}{\text{Vol}_4} \left. S_{\text{AdS}} \right|_{\partial\text{AdS}}}$$ — boundary term of the bulk theory defines *quantum* effective potential

Holographic Model and Solutions

$$S_\chi = \int d^5x\,\sqrt{|g|}\,e^{\phi}\,(\mathrm{tr}\,g^{\mu\nu}(\partial_\mu\chi)(\partial_\nu\chi) - 2V_\chi(\chi))$$ – dual to CH theory in the bulk without fluctuations and gauge fields

Potential parametrization : $V_\chi(\chi) = \frac{m^2}{2}\chi^2 - \frac{D}{4L^2}\lambda\chi^4 + \frac{\lambda^2\gamma}{6L^2}\chi^6 + o(\chi^8)$ is the expansion of a more general theory

---

Fixed geometry: AdS with black hole horizon at $z = z_{\mathrm{H}}$

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)\,dt^2 + \frac{dz^2}{f(z)} + d\vec{x}^2\right), \quad f(z) = 1 - \left(\frac{z}{z_{\mathrm{H}}}\right)^4; \quad \phi(z) = \phi_2 z^2.$$

$$\boxed{z_{\mathrm{H}} = 1, \quad T \sim \frac{1}{\sqrt{\phi_2}}}$$

All values are represented "in terms of *horizon units*"
Dilaton parameter plays role of the temperature.

"Extreme" Curve

$$\frac{\delta S_\chi}{\delta \chi} = 0 \;\Rightarrow\; \chi \xrightarrow{z\to 0} J\,z + \left(\sigma - \left(\frac{3}{2}J^3 + \phi_2 J\right)\log z\right)z^3 + o(z^5)$$ — give the sourses for CFT operators

Knowing the *extrema* of the effective potential and its *values* at these points, we can judge abut the phase transition

$$V_{\text{eff}} = -\frac{1}{\mathrm{Vol}_4} S_\chi\Big|_{\partial\mathrm{AdS}} \;\Rightarrow$$ from EoM for effective action : $\mathrm{Vol}_4 \dfrac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle} = J \;\Rightarrow$ extrema condition is absence of sources $\Rightarrow\; J = 0$

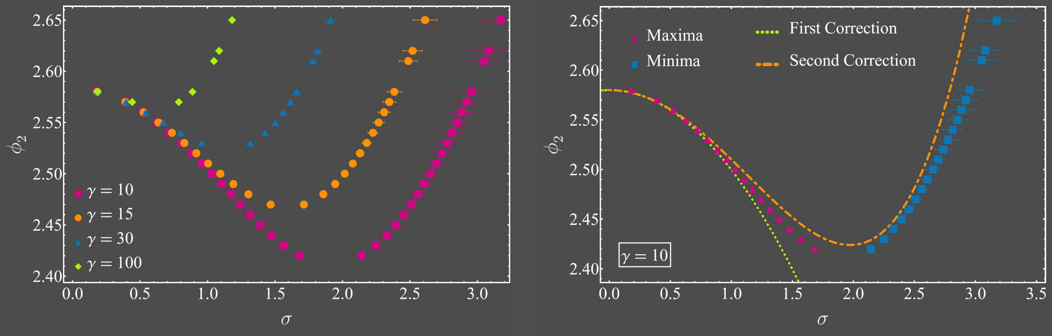
$$\overbrace{\chi \xrightarrow{z\to 0} \sigma z^3 + o(z^5)}^{\text{“extreme” solutions}} \text{ must give } \overbrace{\frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle}}^{\text{extrema}} = 0 \;\Rightarrow$$ a new condition for ϕ_2 and $\langle\varphi\rangle$

$$T \sim \frac{1}{\sqrt{\phi_2}}, \quad \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle} = 0 = \frac{\delta}{\delta\sigma} S_\chi\left[\chi_{\text{Sol.}}(z; J, \sigma)\right]\Big|_{J=0} \quad\Rightarrow\quad \{\sigma_1, \ldots, \sigma_n\} \text{ – extrema}$$

σ is (source) dual to $\langle\varphi\rangle$, vacuum average of the effective theory

Extrema of the Effective Potential ("Extreme" Solutions)

"Extreme" curves defines the positions σ of the effective potential extrema as functions of the parameter γ and temperature ϕ_2 ($T \sim \frac{1}{\sqrt{\phi_2}}$)



σ "is" average of the Higgs background $\sigma \sim \langle\varphi\rangle$ on the border.

Absolute value in physical units require the interaction with Standard Model.

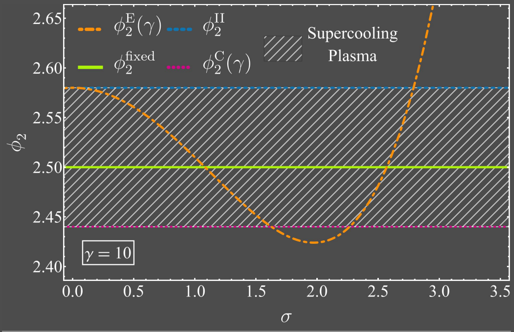
Currently, all values are *in terms of the horizon coordinate* $z_{\mathrm{H}} = 1$.

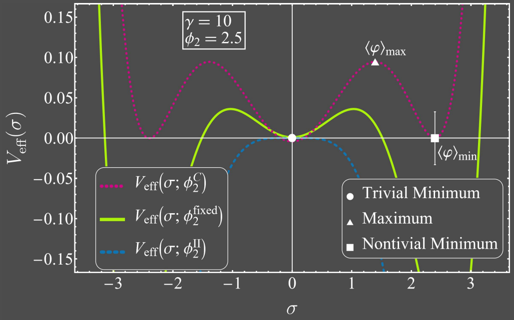
First Order Phase Transition and Effective Potential

values of V_{eff} in the extrema $\Rightarrow T = T_{\text{C}}$ $(\phi_2 = \phi_2^{\text{C}}) \Rightarrow$ nontrivial true vacuum appears.

At $T = T_{\text{II}}$ potential barrier vanishes.

First order PT is able in the range $T_{\text{II}} < T < T_{\text{C}}$ $(\phi_2^{\text{II}} > \phi_2 > \phi_2^{\text{C}}(\gamma))$





There is approximation $V_{\text{eff}} = a_0 + a_2\sigma^2 + a_4\sigma^4 + a_6\sigma^6$

with the points $(\sigma_{\max}, V_{\max}(\sigma_{\max}))$ and $(\sigma_{\min}, V_{\min}(\sigma_{\min}))$

Nucleation Ratio

Cosmological condition: Baryogenesis goes enough efficient
if there is one bubble per Hubble volume (i.e. gives observed asymmetry)

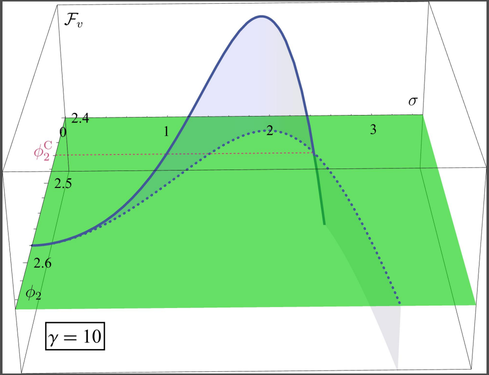
$$\underbrace{\text{Nucleation Ratio}: \quad A T^4 \exp\left(-\frac{F_C}{T}\right)}_{\substack{\text{Bubbles produced} \\ \text{per time} \times \text{space volume}}} \sim \underbrace{H^4(T) = \left(\frac{T^2}{M_{\mathrm{Pl}}}\right)^4 \quad \text{— Expansion of the Univerce}}_{1/(\text{Hubble time} \times \text{volume})}$$

$F = F[V_{\mathrm{eff}}(\langle\varphi\rangle), R]$ is free energy of the bubble;
R is the radius of the bubble, it appears with the sized defined by microscopic physic;
Critical value $\left.\frac{\partial F}{\partial R}\right|_{R_C \overset{\mathrm{def}}{=} R}$, $F_C \overset{\mathrm{def}}{=} F(R_C)$: if $R > R_C$, bubble grow. Otherwise, it bursts.

$$F[V_{\mathrm{eff}}] \xlongequal[\text{approximation}]{\text{thin walls}} 4\pi R^2 \mu - \frac{3\pi}{4} R^3 \left(\mathcal{F}_{\mathrm{out}} - \mathcal{F}_{\mathrm{in}}\right)$$

— Subtraction is extremely sensitive to accuracy!
Numerical appr. isn't efficient!

Effective Potential



Free energy density for non-trivial solution (line). Free energy density for trivial solution is zero (green field).

Free energy density with perturbation theory:

$$\mathcal{F} = \mathcal{F}_{(0)} + \lambda \mathcal{F}_{(1)} + \lambda^2 \mathcal{F}_{(2)} + \ldots$$

Free energy of bubble $F[V_{\text{eff}}]$ in processing ...

Next Step

$$A T^4 \exp\left(-\frac{F_C}{T}\right) \sim H^4(T) = \left(\frac{T^2}{M_{Pl}}\right)^4$$

- ~~Analytical Free Energy (Pertubation Theory)~~
- Fix the Parameters (Interaction with Standard Model – bulk gauge fields)
- Physical Units (Infrared Regularization and finite temperature – “radial” heavy fluctuations)

$$W^{\alpha}_{\mu} J^{\alpha\,\mu}_{L} + B_{\mu} J^{\mu}_{Y} \quad \Leftrightarrow \quad J^{\mu} \sim A^{M} \text{ – bulk } \mathcal{G} \text{ gauge field}$$

$$\Sigma_{IJ} = \langle \bar{\Psi}_I \Psi_J \rangle = \xi^{\top} \left[\begin{pmatrix} 0_{4\times4} & 0 \\ 0 & X \end{pmatrix} + \eta_i \tilde{T}_i \right] \xi \quad \Leftrightarrow \quad \frac{1}{T} \propto \sqrt{\phi_2} \sim \mu_{\mathrm{IR}} \sim m_{\eta} \gtrsim 10 \text{ TeV}$$

Conclusion

What we have done?

- We suggested an interacting Composed Higgs Model in soft-wall holography.
- First-order phase transition that could lead to the desired breaking thermodynamic equilibrium was found as a quantum effect through holography.
- Extrema solutions give finite results. That allows us to approximate the effective potential by its extrema.

What should we do now?

- count interaction of CHM with SM to find critical temperature T_C "in eV";
- find non-extrema values of the effective potential (here some extra UV renormalization is needed);
- count fluctuation and gauge interaction (adjoint representation of $\mathrm{SO}(5)$; it will give chemical potential).

What may we do with the model in the future? — search for PT in gravitational part "near-AdS"; — count gravity interaction (that breaks scale "invariance").

Thank you for your attention!

List of the Backup Slides:

- Nucleation Ratio
- First Order Phase Transition Criteria
- Units ("$z_\mathrm{H} = 1$" and physical "in eV")
- "Symmetries" of the CHM Potential
- CHM potential isn't "Tuned"

Nucleation Ratio

The next step is to consider
Baryogenesis generates enough asymmetry (enough efficient) if
there is one bubble per Hubble volume

$$\underbrace{\text{Nucleation Ratio}:\quad AT^4 e^{-\frac{F_C}{T}}}_{\substack{\text{Bubbles produced}\\ \text{per time}\times\text{space volume}}} \sim \underbrace{H^4(T) = \left(\frac{T^2}{M_{\mathrm{Pl}}}\right)^4 \quad \text{– Expansion of the Univerce}}_{1/\left(\text{Hubble time}\times\text{volume}\right)}$$

$F = F[\langle\varphi\rangle, R]$ – Free energy of the bubble; R is the radius of the bubble

Hubble horizon (time, volume, radius) — speed of receding object behind it is greater than the speed of light (Don't confuse with cosmological horizon)

Bubble appears with a certain size. It defines with "micro-physics".
If its radius is grater, then critical one $\left.\frac{\partial F}{\partial R}\right|_{R_{\mathrm{C}} \overset{\mathrm{def}}{=} R}$, the bubble grow. Otherwise, it bursts.

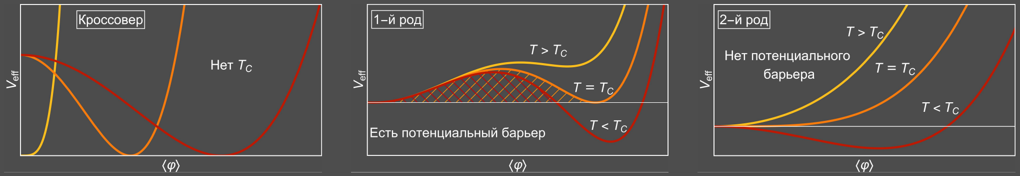
It gives $F_{\mathrm{C}} \overset{\mathrm{def}}{=} F(R_{\mathrm{C}})$ and defines nucleation ratio and "viability of the model".

First Order Phase Transition Criteria

$$\frac{\delta\Gamma}{\delta\langle\varphi\rangle} = \mathrm{Vol}_d \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle} = J, \quad J = 0 \quad \Leftrightarrow \quad \text{condition for all extrema}$$

"Physical" potential V_{eff} should have the trivial minimum (we can take $V_{\text{eff}}\big|_{\langle\varphi\rangle=0} = 0$)

and be even function $V_{\text{eff}}[\langle\varphi\rangle] = V_{\text{eff}}[-\langle\varphi\rangle]$ and give finite motion $V_{\text{eff}} \xrightarrow{\langle\varphi\rangle\to\pm\infty} \infty$



$$\frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle}\bigg|_{\langle\varphi\rangle=0} = 0, \quad \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle}\bigg|_{\langle\varphi\rangle=\langle\varphi\rangle_{\text{max}}>0} = 0, \quad \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle}\bigg|_{\langle\varphi\rangle=\langle\varphi\rangle_{\text{min}}>\langle\varphi\rangle_{\text{max}}} = 0,$$

$$T_C : \quad V_{\text{eff}}\big|^{T=T_C}_{\langle\varphi\rangle=\langle\varphi\rangle_{\text{min}}} = V_{\text{eff}}\big|^{\forall T}_{\langle\varphi\rangle=0} \overset{\text{fix}}{=} 0, \quad V_{\text{eff}}\big|^{T<T_C}_{\langle\varphi\rangle=\langle\varphi\rangle_{\text{max}}} > V_{\text{eff}}\big|^{T<T_C}_{\langle\varphi\rangle=\langle\varphi\rangle_{\text{min}}}$$

What Units are Used?

The scale has been fixed with the horizon coordinate in AdS $z_{\rm H} = 1$.
This fixed the energy/scale units of the all values. $z_{\rm H}$ recovering is needed to define "physical" units (eV, K, etc.)

"physical" dilaton parameter $\tilde{\phi}_2 = \frac{\phi_2}{z_{\rm H}^2}$; "physical" temperature $T = \frac{|f'(z_{\rm H})|}{4\pi} = \frac{1}{\pi z_{\rm H}}$

$\tilde{\phi}_2 = \text{const} \Rightarrow z_{\rm H} = \frac{\phi_2}{\tilde{\phi}_2} \Rightarrow$ "Temperatute" in "horizon" units $T = \frac{1}{\pi}\sqrt{\frac{\tilde{\phi}_2}{\phi_2}} \sim \frac{1}{\sqrt{\phi_2}}$

To recover $\tilde{\phi}_2$, we have to include gauge interaction of $\mathcal{G}$

$$\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma + [A_\mu, \Sigma]$$

and include terms of the interaction CHM with SM.

“Symmetries” of the CHM Potential

$V_\chi(\chi) = \frac{m^2}{2}\chi^2 - \frac{D}{4L^2}\lambda\chi^4 + \frac{\lambda^2\gamma}{6L^2}\chi^6$ is the expantion of a more general theory

Suggestions:

- The potential V_χ always has true vacuum with $E_{\min}$ ($V_\chi \xrightarrow{\chi\to\pm\infty} \infty$). So we may use any even power χ^n instead of the last term χ^6.
- The expansion of V_χ has certain sign of the second term $\lambda > 0$ (the first one m^2 chosen for the theory to be conformal in AdS).
- Higher orders of the expansion don't give new minima at the considered temperatures.

The certain parametrization has been chosen with respect to the “symmetries”

“Scale invariace”, defining the coefficents : $\begin{matrix} L \to L' \\ \chi \to \sqrt{\lambda}\chi \end{matrix}$; Conformality near the AdS border (“correct” conformal weights) : $\begin{matrix} \Delta_- = 1 \\ \Delta_+ = 3 \end{matrix} \Rightarrow m^2 = -\frac{D}{3L^2}$

D is for the Large D limit. But its usage doesn't give any results.
(to keep interaction constants finite at $D \to \infty$)

CHM potential isn't "Tuned"

$$V_\chi = a_2\chi^2 + a_4\chi^4 + a_6\chi^6, \quad a_2 < 0,\ a_4 < 0,\ a_6 > 0$$
$$V_{\text{eff}} = b_2\langle\varphi\rangle^2 + b_4\langle\varphi\rangle^4 + b_6\langle\varphi\rangle^6, \quad b_2 > 0,\ b_4 < 0,\ b_6 > 0$$

in details:

- $V_{\text{eff}} = V_{\text{eff}}[\langle\varphi\rangle]$ describes a quantum objects at the border. V_χ is a dual classical potential in the bulk.
- $V_{\text{eff}} = -\frac{1}{\text{Vol}_4} S_{\text{AdS}}\Big|_{\partial\text{AdS}}$ includes the solutions of the EoM $\frac{\delta S_\chi}{\delta\chi=0}$ in bulk. In other words, V_{eff} includes physics of AdS