Odderon and three-pomeron one-dimensional reggeon models

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Outline

- Motivation
- Pomeron-odderon model
- Three-pomeron model
- Results

Motivation

Effective theory of reggeon interaction, or Gribov model, was introduced as a set of diagram rules [1]. Then, in [2] it was reimagined as Feynman rules for euclidean field theory with the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}\left(\Phi^{+}\partial_{y}\Phi - \Phi\partial_{y}\Phi^{+}\right) - \mu\Phi^{+}\Phi - \alpha'\vec{\nabla}\Phi^{+}\cdot\vec{\nabla}\Phi + i\lambda\Phi^{+}\left(\Phi + \Phi^{+}\right)\Phi.$$

Here $\Phi(y, \vec{b})$ – reggeon complex field, $\Phi^{+}(y, \vec{b})$ – conjugated field, y – rapidity, $\vec{b}$ – impact parameter vector. Theory parameters $\mu = \alpha(0) - 1$ and $\alpha' = \alpha'(0)$ are determined by pomeron Regge trajectory $j = \alpha(t) + 1$.

The model can be simplified by omitting transverse dimensions and thus making the model 1-dimensional. Fields then become $\Phi(y)$, $\Phi^+(y)$. Canonical quantization in imaginary time gives $[\Phi, \Phi^+] = 1$, allowing to interpret Φ^+ as a creation operator, and Φ as an annihilation operator. Eucledian Hamiltonian is then [3]

$$H = -\mu\Phi^+\Phi + i\lambda\left(\Phi^+\Phi^+\Phi + \Phi^+\Phi\Phi\right).$$

It is non-Hermitian, but its spectrum is real positive[3, 4, 5, 6, 7] and can be obtained numerically [8, 9]. There also exists exactly solvable Schwimmer model [10] (only fan diagrams).

Pomeron-odderon model

As an analogy to Gribov model

$$H = -\left(\Phi^+\partial_y\Phi + \Psi^+\partial_y\Psi - L\right) = -\mu_P\Phi^+\Phi - \mu_O\Psi^+\Psi + \\ i\lambda\left(\Phi^+\Phi^+\Phi + \Phi^+\Phi\Phi + 2\Psi^+\Psi\Phi + 2\Phi^+\Psi^+\Psi - \Phi^+\Psi\Psi + \Psi^+\Psi^+\Phi\right)$$

with the following commutation relations:

$$[\Phi, \Phi^+] = 1, \quad [\Psi, \Psi^+] = 1, \quad [\Phi, \Psi^+] = 0, \quad [\Psi, \Phi^+] = 0.$$

Here Φ is C-even pomeron field with the positive signature, Ψ is C-odd odderon field with the negative signature, both of which depend only on y.

The equations for the evolution of pomeron and odderon with rapidity was derived in QCD [11]. For the local pomeron $N(\boldsymbol{k}, y)$ and odderon $O(\boldsymbol{k}, y)$, which do not depend on $\boldsymbol{k}$ and depend only on rapidity y, the equations simplify to

$$\begin{aligned}\frac{\partial N(y)}{\partial y} =& \mu_P N(y) - \bar{\alpha}_s N^2(y) + \bar{\alpha}_s O^2(y),\\ \frac{\partial O(y)}{\partial y} =& \mu_O O(y) - 2\bar{\alpha}_s N(y) O(y),\end{aligned}$$

where the mass parameters μ_P and μ_O have a sense of the pomeron and odderon intercepts minus 1, respectively. If one chooses $\lambda = N_c \alpha_s / \pi$ then this equations completely coincide with the ones obtained from the Hamiltonian in the fan approximation, if $\Phi = -iN(y), \Psi = -iO(y)$. Also, as in [12], $\alpha_O(0) = 1$ and $\mu_O = 0$, and signs correspond to those in [13]

Passage to real Hamiltonian

Fock-Bargmann representation:

$$u \equiv i\Phi^+, \quad v = \frac{\partial}{\partial u} \equiv -i\Phi, \qquad w \equiv i\Psi^+ = -\Psi^+, \quad z = \frac{\partial}{\partial w} \equiv -i\Psi.$$

In terms of these operators

$$H = -\mu_P u \frac{\partial}{\partial u} - \mu_O w \frac{\partial}{\partial w} + \lambda u^2 \frac{\partial}{\partial u} - \lambda u \frac{\partial^2}{\partial u^2} \\ - 2\lambda w \frac{\partial^2}{\partial u \partial w} + 2\lambda u w \frac{\partial}{\partial w} + \lambda u \frac{\partial^2}{\partial w^2} + \lambda w^2 \frac{\partial}{\partial u}.$$

Evolution equation is

$$\frac{\partial F(y; u, w)}{\partial y} = -HF(y; u, w).$$

Interaction with a complex object is described by Glauber states:

$$F^{(in)} = 1 - e^{-ig_P^{(i)}\Phi^+ + g_O^{(i)}\Psi^+}, \quad F^{(fin)} = 1 - e^{-ig_P^{(f)}\Phi - g_O^{(f)}\Psi}$$

Amplitude is then

$$i\mathcal{A}(y) = \left\langle 0 | F^{(fin)}(\Phi, \Psi) e^{-Hy} F_{y=0}(\Phi^+, \Psi^+) | 0 \right\rangle .$$

$$i\mathcal{A}(y) = -F_y(u, w)\Big|_{u=g_P^{(f)}, w=-ig_O^{(f)}},$$

Propagators

$$P(y) = \frac{\partial F(u, w)}{\partial u}\Big|_{u=0, w=0}, \quad O(y) = \frac{\partial F(u, w)}{\partial w}\Big|_{u=0, w=0}$$

Calculation approaches

- Power expansion approach: $F_y(u,w) = \sum g_{nm}(y) u^n w^m$.
- Point-evolution approach

1st substitution

$$\phi = \frac{\Phi + i\Psi}{\sqrt{2}}, \quad \phi^+ = \frac{\Phi^+ - i\Psi^+}{\sqrt{2}}, \quad \tilde{\phi} = \frac{\Phi - i\Psi}{\sqrt{2}}, \quad \tilde{\phi}^+ = \frac{\Phi^+ + i\Psi^+}{\sqrt{2}}.$$

2nd substitution

$$\phi^+ = -iu, \quad \phi = i\frac{\partial}{\partial u} = iv, \quad \tilde{\phi}^+ = -iw, \quad \tilde{\phi} = \frac{\partial}{\partial w} = iz.$$

$$\begin{aligned} H = &-\frac{1}{2}\mu_P(u+w)\Big(\frac{\partial}{\partial u} + \frac{\partial}{\partial w}\Big) \\ &-\lambda\sqrt{2}\Big(u\frac{\partial^2}{\partial u^2} - u^2\frac{\partial}{\partial u} + w\frac{\partial^2}{\partial w^2} - w^2\frac{\partial}{\partial w}\Big) \\ &-\frac{\lambda}{\sqrt{2}}(u-w)^2\Big(\frac{\partial}{\partial u} + \frac{\partial}{\partial w}\Big). \end{aligned}$$

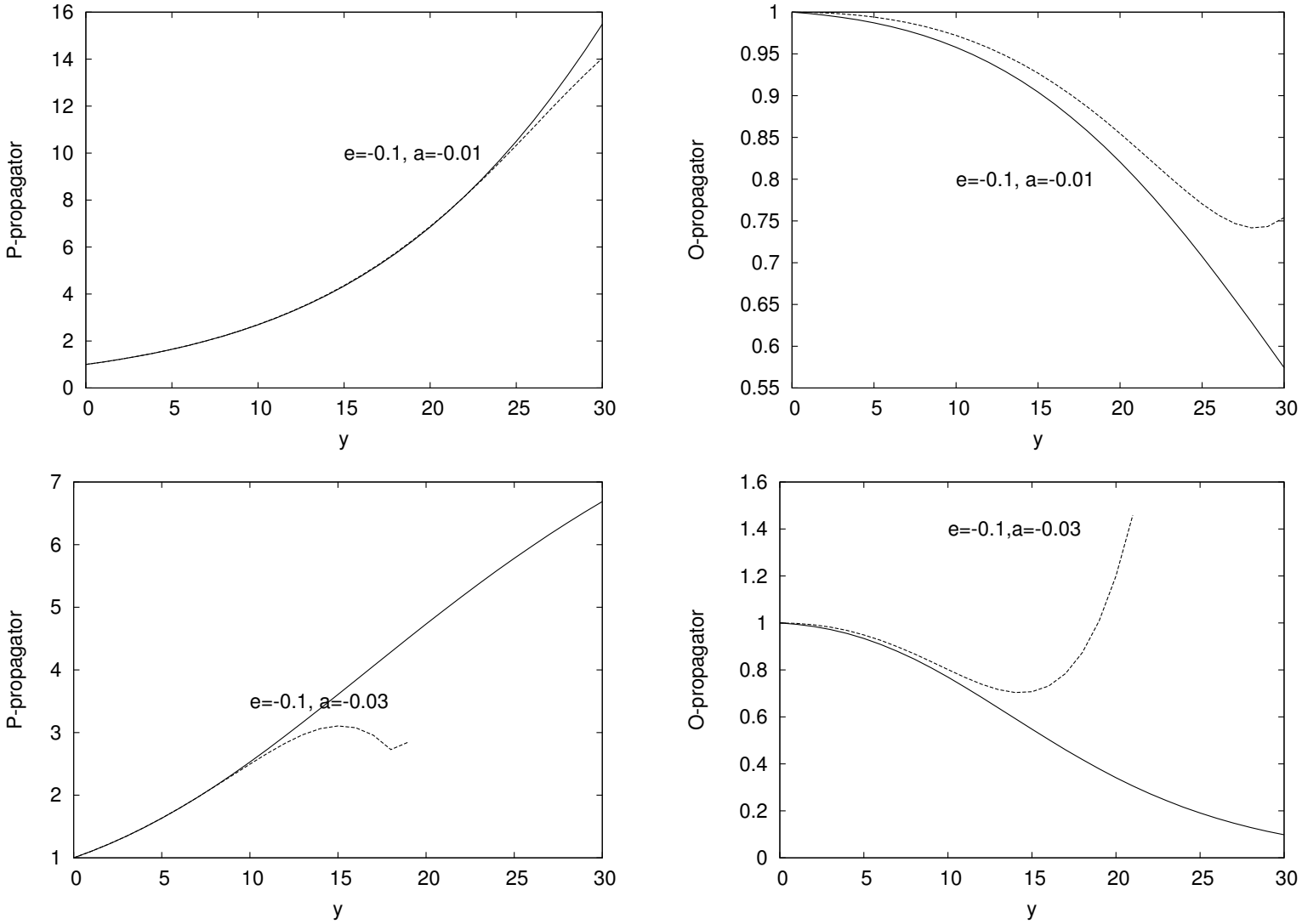

Figure: Pomeron (left) and odderon (right) propagators, calculated by power expansion (dashed curves) and point evolution (solid curves). In all panels $\mu = 0.1$. In upper panels $\lambda = 0.01$, in the lower $\lambda = 0.03$.

Results — power expansion

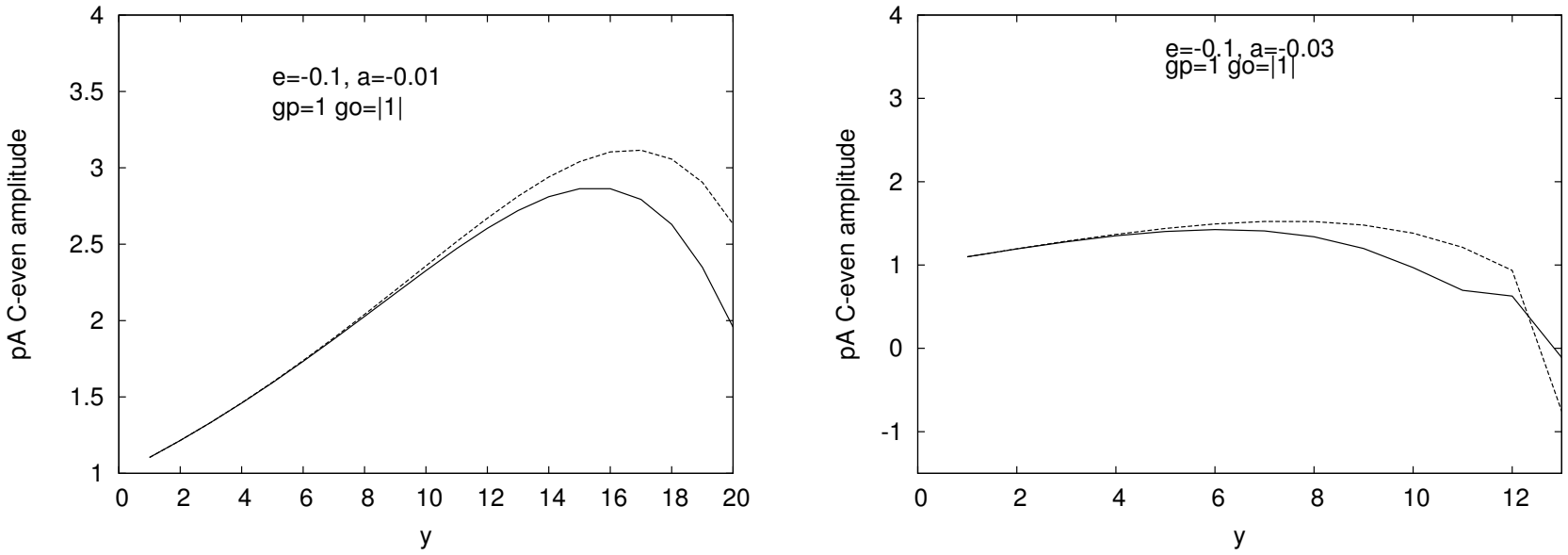

Figure: pA C-even amplitudes at different rapidities for $\mu_p = 0.1$, $\lambda = 0.01, 0.03$ and $g_P = |g_O| = 1$. The upper curves correspond to summation of the fan diagrams.

Results — power expansion

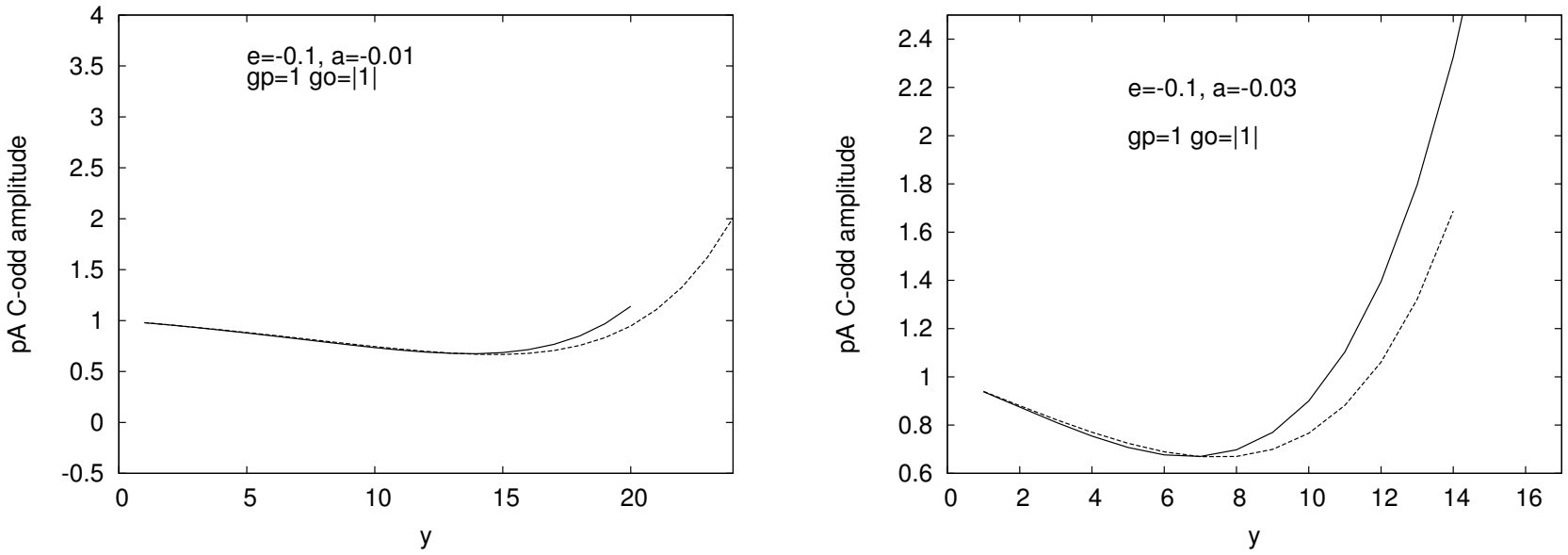

Figure: pA C-odd amplitudes at different rapidities for $\mu_p = 0.1$, $\lambda = 0.01, 0.03$ and $g_P = |g_O| = 1$. The lower curves correspond to summation of the fan diagrams.

Results — point evolution

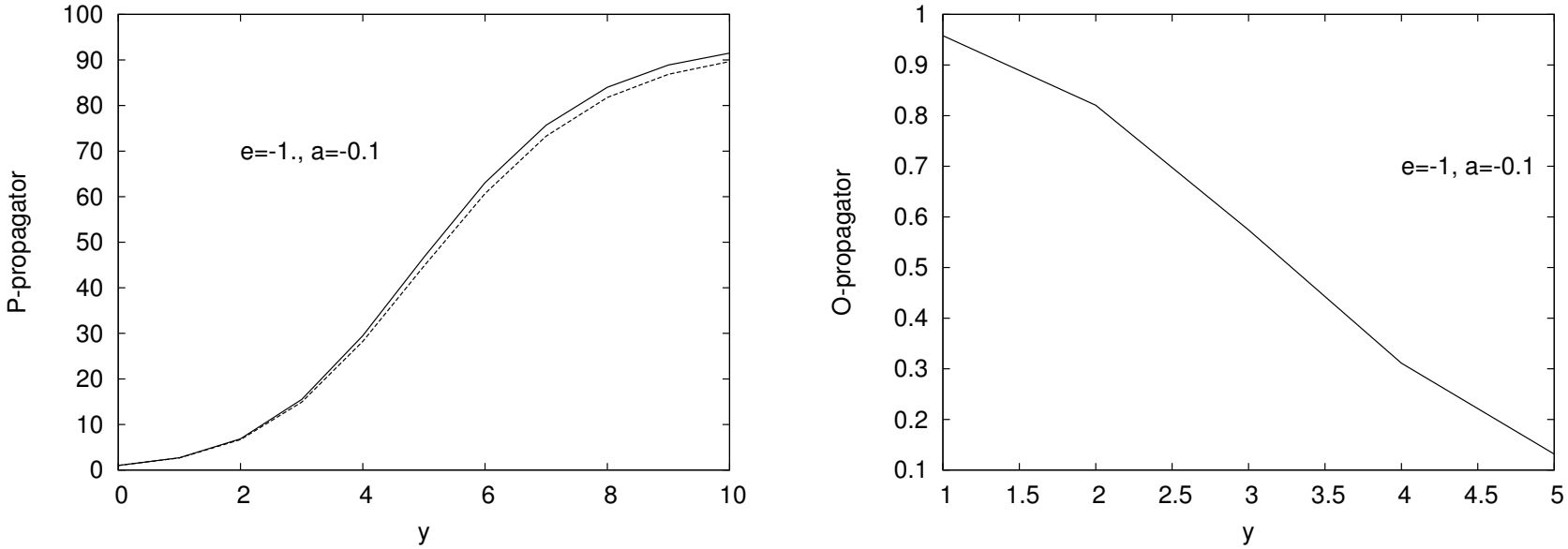

Figure: The solid curves show pomeron (left panel) and odderon (right panel) propagators as functions of rapidity for $\mu = 1$, $\lambda = 0.1$. The dashed curve in the left panel shows the pomeron propagator in absence of the odderon (only pomeron loops).

Results — point evolution

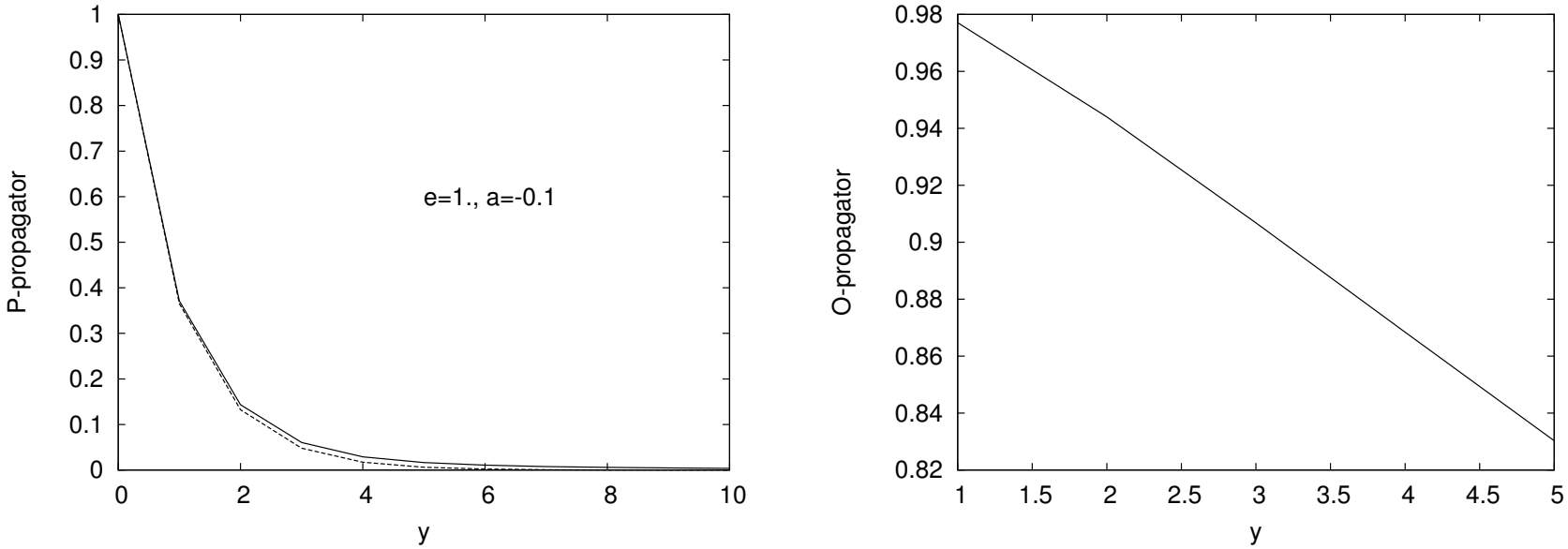

Figure: The solid curves show pomeron (left panel) and odderon (right panel) propagators as functions of rapidity for $\mu = -1$, $\lambda = 0.1$. The dashed curve in the left panel shows the pomeron propagator in absence of the odderon (only pomeron loops).

Results — point evolution

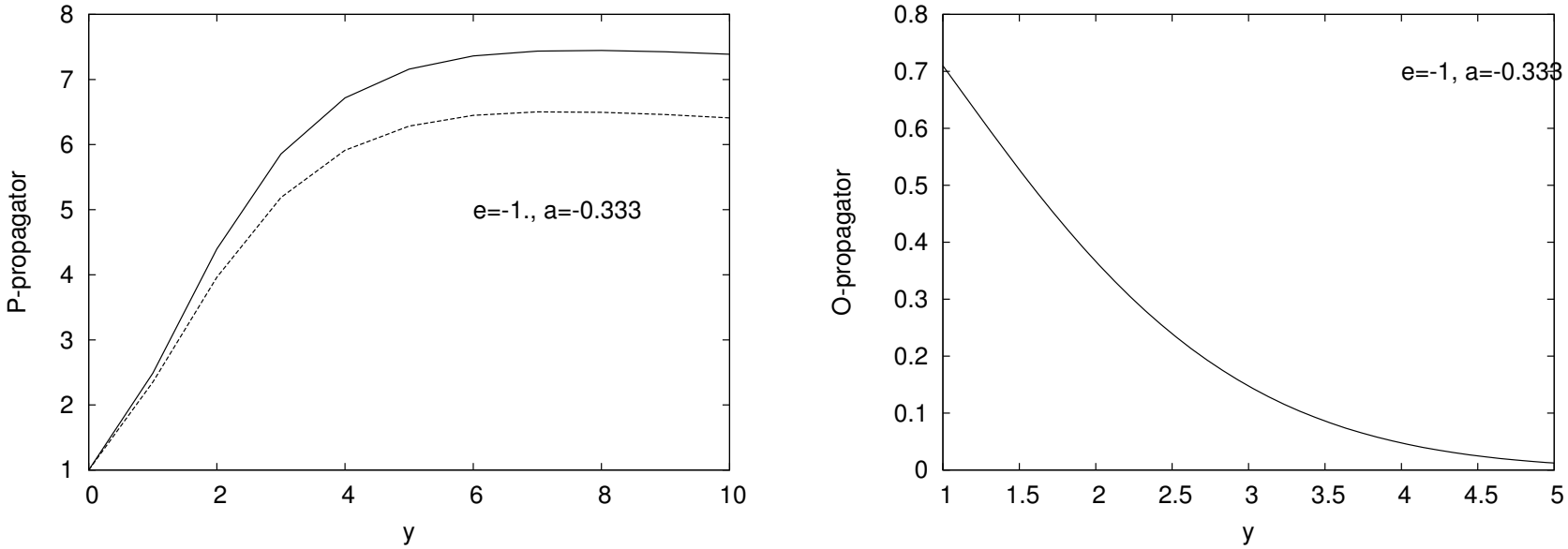

Figure: The solid curves show pomeron (left panel) and odderon (right panel) propagators as functions of rapidity for $\mu = 1$, $\lambda = 1/3$. The dashed curve in the left panel shows the pomeron propagator in absence of the odderon (only pomeron loops).

Results — point evolution

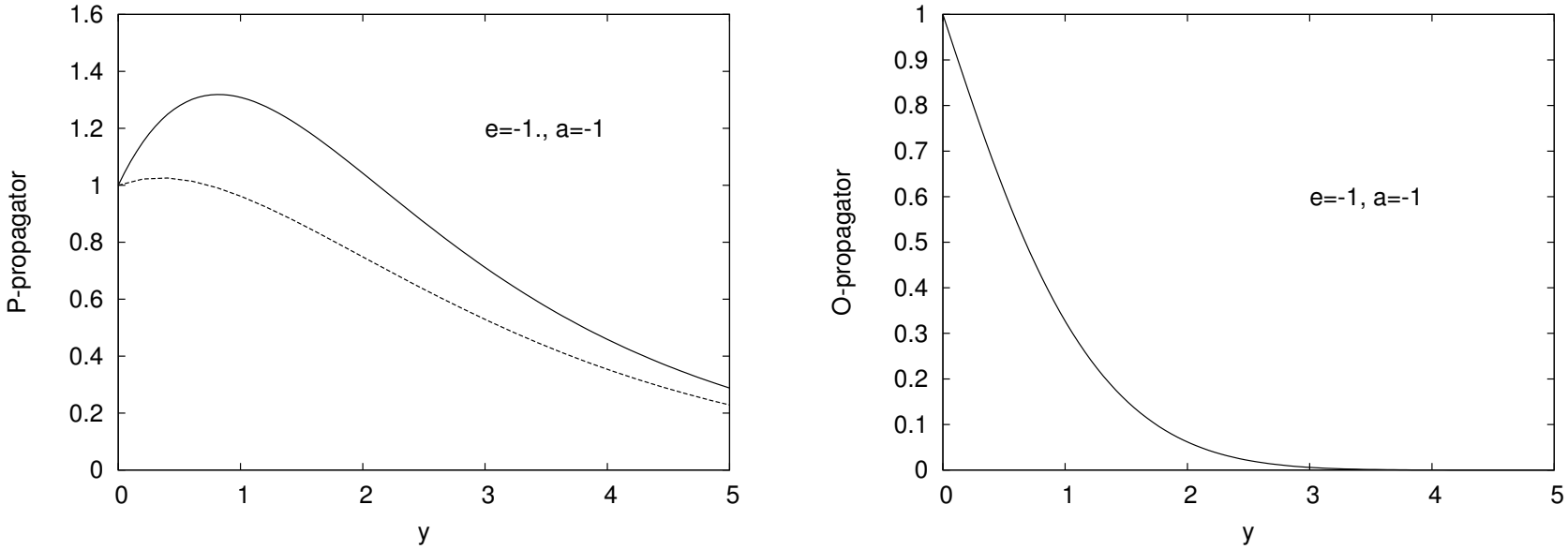

Figure: The solid curves show pomeron (left panel) and odderon (right panel) propagators as functions of rapidity for $\mu = 1$, $\lambda = 1$. The dashed curve in the left panel shows the pomeron propagator in absence of the odderon (only pomeron loops).

Results — point evolution

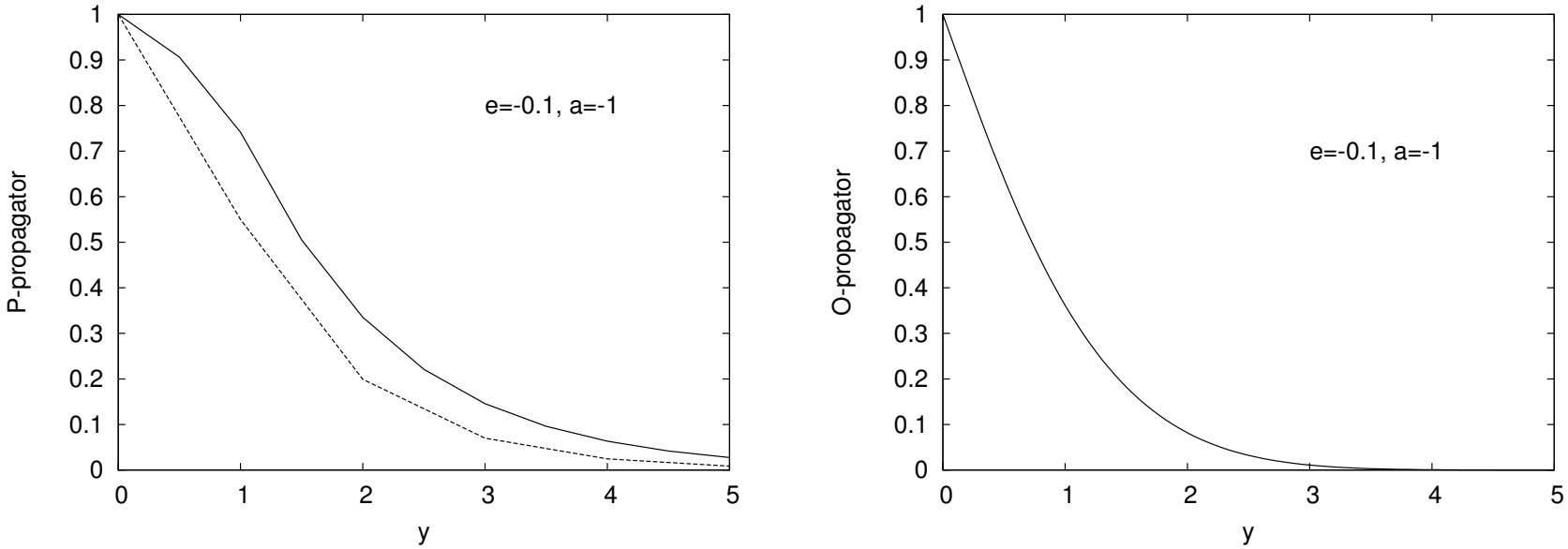

Figure: The solid curves show pomeron (left panel) and odderon (right panel) propagators as functions of rapidity $\mu = 0.1$, $\lambda = 1$. The dashed curve in the left panel shows the pomeron propagator in absence of the odderon (only pomeron loops).

Two-pomeron model

If we consider the following Hamiltonian

$$H = -\mu\Phi^+\Phi - \mu_1\Psi^+\Psi + \\ i\lambda\left(\Phi^+\Phi^+\Phi + \Phi^+\Phi\Phi + 2\Psi^+\Psi\Phi + 2\Phi^+\Psi^+\Psi - \Phi^+\Psi\Psi - \Psi^+\Psi^+\Phi\right)$$

where Φ is C-even pomeron and Ψ is C-odd pomeron, both with positive signatures, and perform the substitution

$$\varphi = \frac{\Phi + i\Psi}{\sqrt{2}}, \quad \varphi^+ = \frac{\Phi^+ - i\Psi^+}{\sqrt{2}}$$

$$\tilde{\varphi} = \frac{\Phi - i\Psi}{\sqrt{2}}, \quad \tilde{\varphi}^+ = \frac{\Phi^+ + i\Psi^+}{\sqrt{2}}$$

Then, with $\mu_1 = 0$,

$$H = H_0 + V = -\frac{\mu}{2}\left(\varphi^+\varphi + \tilde{\varphi}^+\tilde{\varphi}\right) - \frac{\mu}{2}\left(\varphi^+\tilde{\varphi} + \tilde{\varphi}^+\varphi\right) + \\ + i\sqrt{2}\,\lambda\left(\varphi^+\varphi\varphi + \varphi^+\varphi^+\varphi\right) + i\sqrt{2}\,\lambda\left(\tilde{\varphi}^+\tilde{\varphi}\tilde{\varphi} + \tilde{\varphi}^+\tilde{\varphi}^+\tilde{\varphi}\right)$$

where

$$H_0 = h(\phi, \phi^+) + h(\tilde{\phi}, \tilde{\phi}^+),$$
$$V = -\frac{\mu}{2}\left(\phi^+\tilde{\phi} + \tilde{\phi}^+\phi\right)$$

and h being Gribov model Hamiltonian.

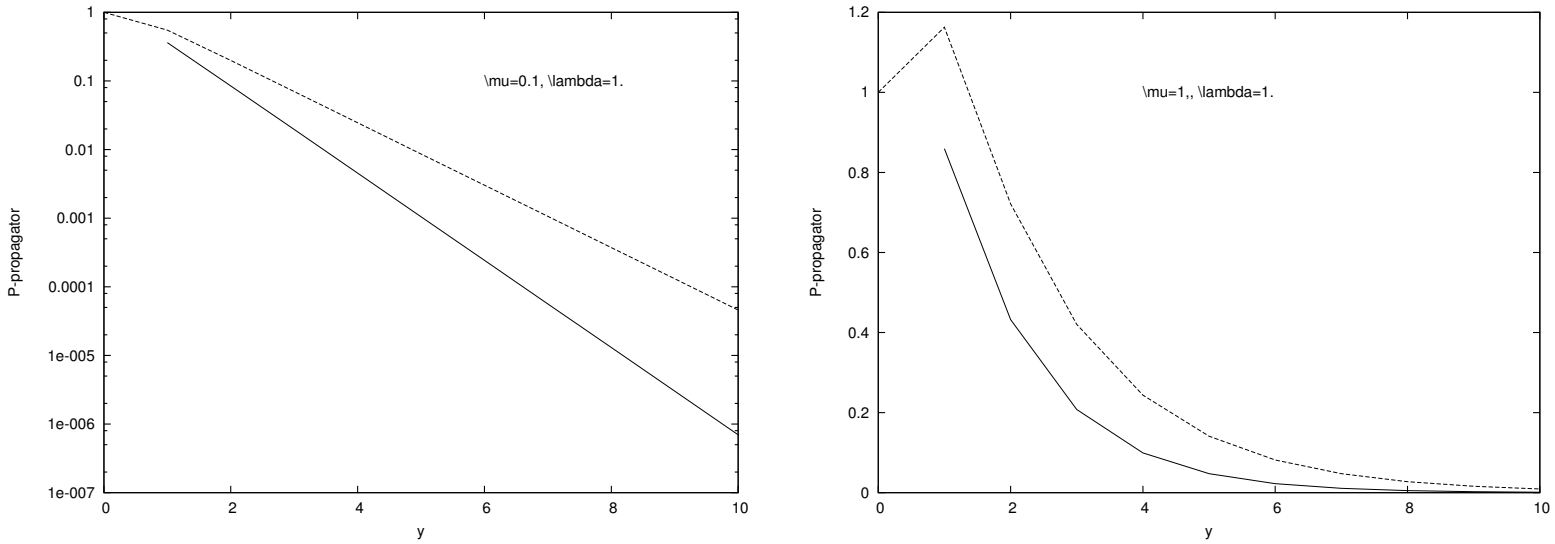

Figure: The solid curve shows the pomeron propagator as a function of rapidity for $\mu = 0.1$ (left) and $\mu = 1$ (right), $\lambda = 1$. The dashed curve shows the pomeron propagator in absence of the C-odd pomeron.

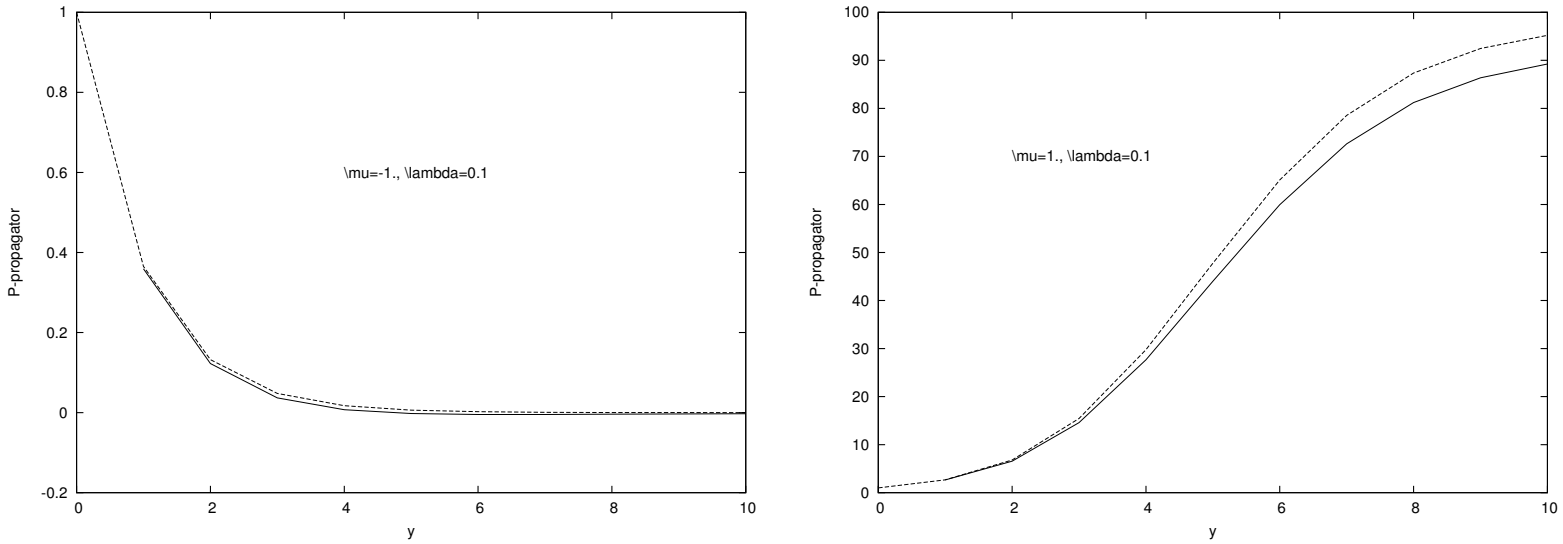

Figure: The solid curve shows the pomeron propagator as a function of rapidity for $\mu = -1$ (left) and $\mu = 1$ (right), $\lambda = 0.1$. The dashed curve shows the pomeron propagator in absence of the C-odd pomeron.

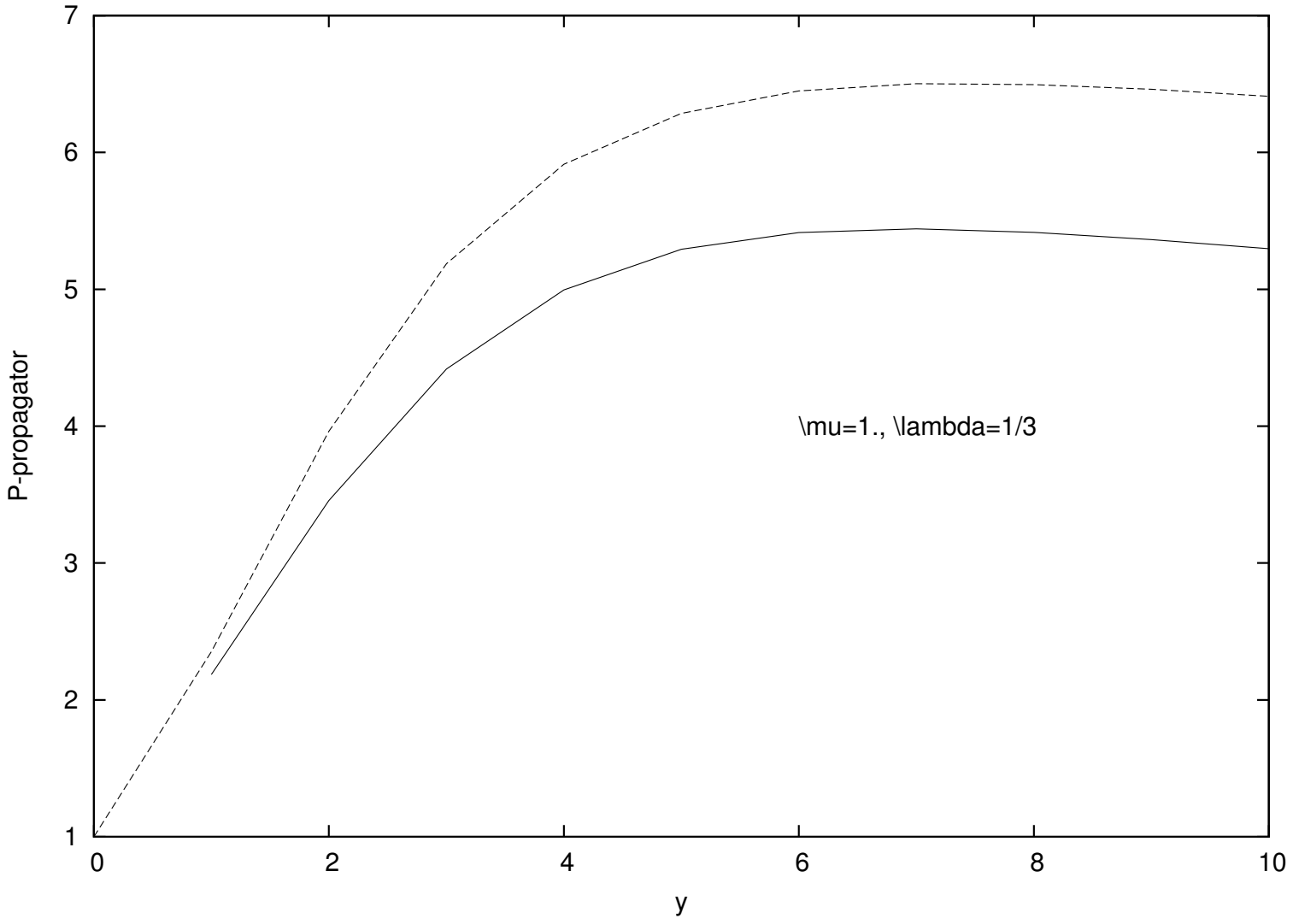

Figure: The solid curve shows the pomeron propagator as a function of rapidity for $\mu = 1$, $\lambda = 0.333$. The dashed curve shows the pomeron propagator in absence of the C-odd reggeon.

Three-pomeron model

Balitsky-Kovchegov equation [14, 15]

$$\frac{\partial \Phi(\boldsymbol{k}, y)}{\partial y} = \bar{\alpha}_s H^{BFKL} \Phi(\boldsymbol{k}, y) - \bar{\alpha}_s \Phi^2(\boldsymbol{k}, y).$$

H^{BFKL} eigenvalues

$$\mu(l, \nu) = \bar{\alpha}_s \Big[2\psi(1) - \psi\Big(\frac{1+|l|}{2} + i\nu\Big) - \psi\Big(\frac{1+|l|}{2} - i\nu\Big)\Big], \quad \bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}.$$

Conformal spin l eigenstates expansion of Φ

$$\Phi(\mathbf{k}, y) = \sum_{l, even} \Phi_l(k, y) e^{il\varphi}$$

$$\mu(0,0) = 4\bar{\alpha}_s \ln 2, \quad \mu(2,0) = 4\bar{\alpha}_s (\ln 2 - 1), \quad \mu(4,0) = 4\bar{\alpha}_s (\ln 2 - 4/3).$$

In units $4\bar{\alpha}_s$ the values are respectively $+0.693$, -0.307 and -0.640, so with increase in l contributions of Φ_l decrease. Thus, if we consider the first pomerons in the series, we obtain

$$H = -\mu_0 \Phi_0^\dagger \Phi_0 - \mu_2 \left(\Phi_{-2}^\dagger \Phi_2 + \Phi_2^\dagger \Phi_{-2} \right) + i\lambda \left(\Phi_0^\dagger \left(\Phi_0^2 + 2\Phi_2 \Phi_{-2} \right) + 2\Phi_2^\dagger \Phi_{-2} \Phi_0 + 2\Phi_{-2}^\dagger \Phi_2 \Phi_0 + \left(\Phi_0^{\dagger 2} + 2\Phi_2^\dagger \Phi_{-2}^\dagger \right) \Phi_0 + 2\Phi_2^\dagger \Phi_0^\dagger \Phi_{-2} + 2\Phi_{-2}^\dagger \Phi_0^\dagger \Phi_2 \right)$$

Here $(\Phi_2)^\dagger = \Phi_{-2}^\dagger$, $(\Phi_{-2})^\dagger = \Phi_2^\dagger$.

Passing to real fields

By redefining

$$\Phi_0^\dagger = -iu, \quad \Phi_2^\dagger = -iw, \quad \Phi^\dagger_{-2} = -iq$$
$$\Phi_0 = i\frac{\partial}{\partial u}, \quad \Phi_2 = i\frac{\partial}{\partial w}, \quad \Phi_{-2} = i\frac{\partial}{\partial q}$$

we obtain

$$H = -\mu_0 u\frac{\partial}{\partial u} - \mu_2(w\frac{\partial}{\partial w} + q\frac{\partial}{\partial q}) - \\ - \lambda\left(u\frac{\partial^2}{\partial u^2} - u^2\frac{\partial}{\partial u} + 2u\frac{\partial^2}{\partial w\partial q} - 2wq\frac{\partial}{\partial u} + \right. \\ \left. 2w\frac{\partial^2}{\partial w\partial u} - 2uw\frac{\partial}{\partial w} + 2q\frac{\partial^2}{\partial q\partial u} - 2uq\frac{\partial}{\partial q}\right)$$

Transition to two fields

The Hamiltonian commutes with the operator of spin

$$I = 2\left(\Phi_2^+\Phi_{-2} - \Phi_{-2}^+\Phi_2\right).$$

Fock space basis element is $u^k w^m q^p$ with $m - p = \frac{I}{2}$. If we denote $z = wq$, then basis element becomes $u^k w^{\frac{I}{2}} z^p$. Then

$$H = -\mu_0 u\frac{\partial}{\partial u} - 2\mu_2 z\frac{\partial}{\partial z} - \frac{I\mu_2}{2} - \lambda\left(u\frac{\partial^2}{\partial u^2} - u^2\frac{\partial}{\partial u} + I\left(\frac{\partial}{\partial u} - u\right) + (I+2)u\frac{\partial}{\partial z} - 2z\frac{\partial}{\partial u} + 2uz\frac{\partial^2}{\partial z^2} + 4z\frac{\partial^2}{\partial z\partial u} - 4uz\frac{\partial}{\partial z}\right).$$

Substituting $z = \frac{1}{2}\xi^2$ we obtain

$$H = -\mu_0 u \frac{\partial}{\partial u} - \mu_2 \xi \frac{\partial}{\partial \xi} - \frac{l\mu_2}{2} - \lambda \left(u \frac{\partial^2}{\partial u^2} - u^2 \frac{\partial}{\partial u} + l \left(\frac{\partial}{\partial u} - u \right) + \frac{(l+1)u}{2\xi} \frac{\partial}{\partial \xi} + \frac{1}{2} u \frac{\partial^2}{\partial \xi^2} - 2\xi^2 \frac{\partial}{\partial u} + 2\xi \frac{\partial^2}{\partial u \partial \xi} - 2u\xi \frac{\partial}{\partial \xi} \right)$$

Composite fields

Substituting

$$u = \frac{u+\xi}{\sqrt{2}}, \quad w = \frac{u-\xi}{\sqrt{2}}$$

Hamiltonian then becomes

$$H = -\frac{1}{2}\mu_0(u+w)\left(\frac{\partial}{\partial u}+\frac{\partial}{\partial w}\right) - \frac{1}{2}\mu_2(u-w)\left(\frac{\partial}{\partial u}-\frac{\partial}{\partial w}\right) - \frac{l\mu_2}{2} -$$
$$\sqrt{2}\lambda\left(u\frac{\partial^2}{\partial u^2} - u^2\frac{\partial}{\partial u} + w\frac{\partial^2}{\partial w^2} - w^2\frac{\partial}{\partial w}\right) -$$
$$\frac{l+1}{\sqrt{2}}\lambda\frac{u+w}{u-w}\left(\frac{\partial}{\partial u}-\frac{\partial}{\partial w}\right) +$$
$$\frac{l}{\sqrt{2}}\lambda\left((u+w) - \left(\frac{\partial}{\partial u}+\frac{\partial}{\partial w}\right)\right)$$

Propagators

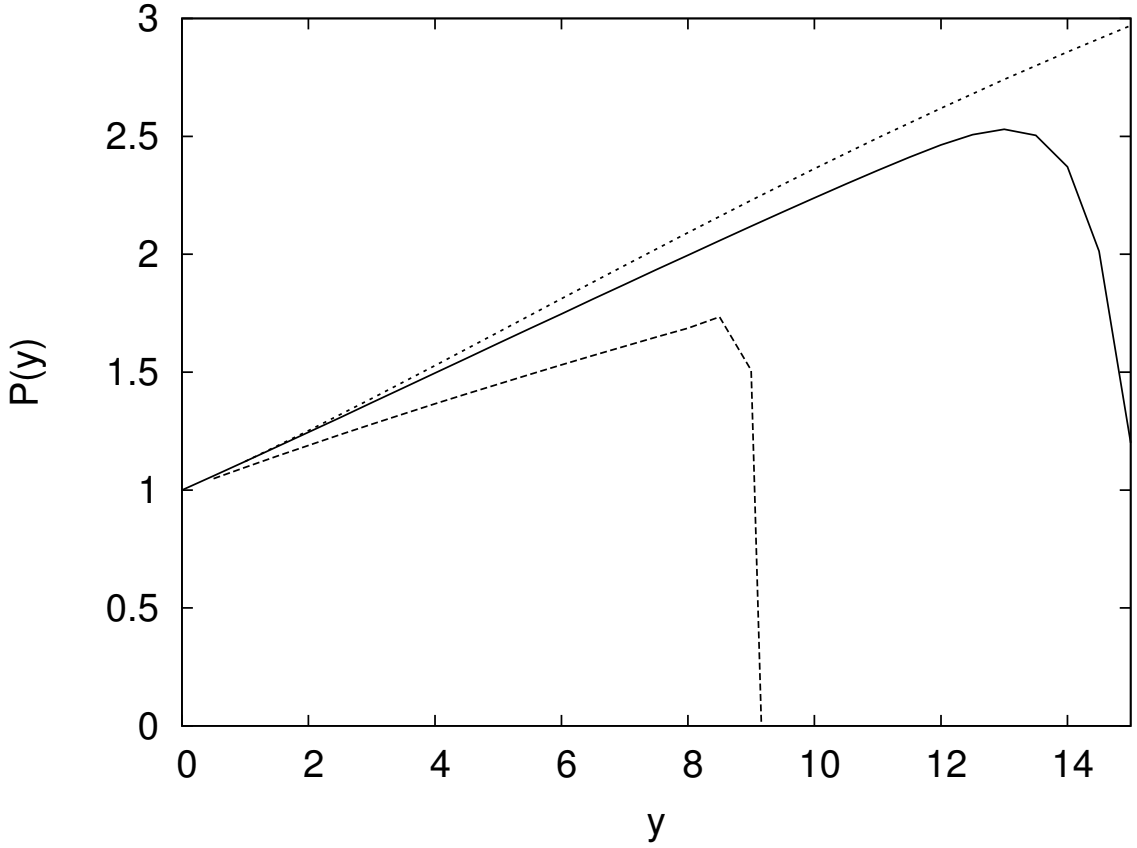

Figure: The propagator of the pomeron $l = 0$ at different rapidities with three pomerons $l = 0, \pm 2$ obtained by power expansion (middle curve),direct point-like evolution (lower curve) and two-field point-like evolution with composite fields (upper curve). $\mu_0 = 0.12$, $\mu_2 = -0.0531$, $\bar{\alpha}_s = 0.0433$.

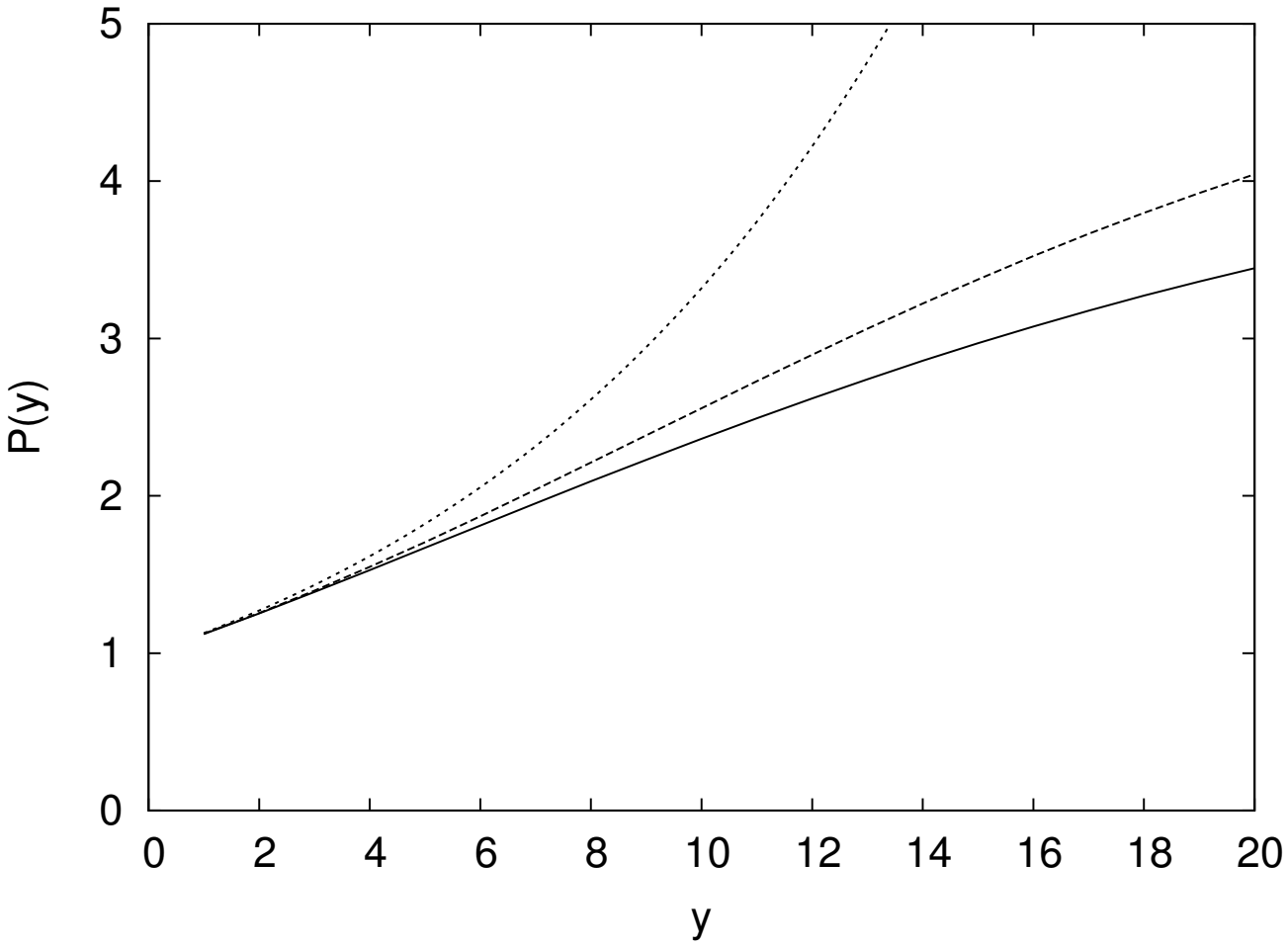

Figure: The propagator of the pomeron $l = 0$ at different rapidities with three pomerons $l = 0, \pm 2$ obtained by two-field point-like evolution with composite fields (lower curve) and without the subdominant pomerons (middle curve). $\mu_0 = 0.12$, $\mu_2 = -0.0531$, $\bar{\alpha}_s = 0.0433$. The upper curve corresponds to the free propagator $\exp(0.12y)$.

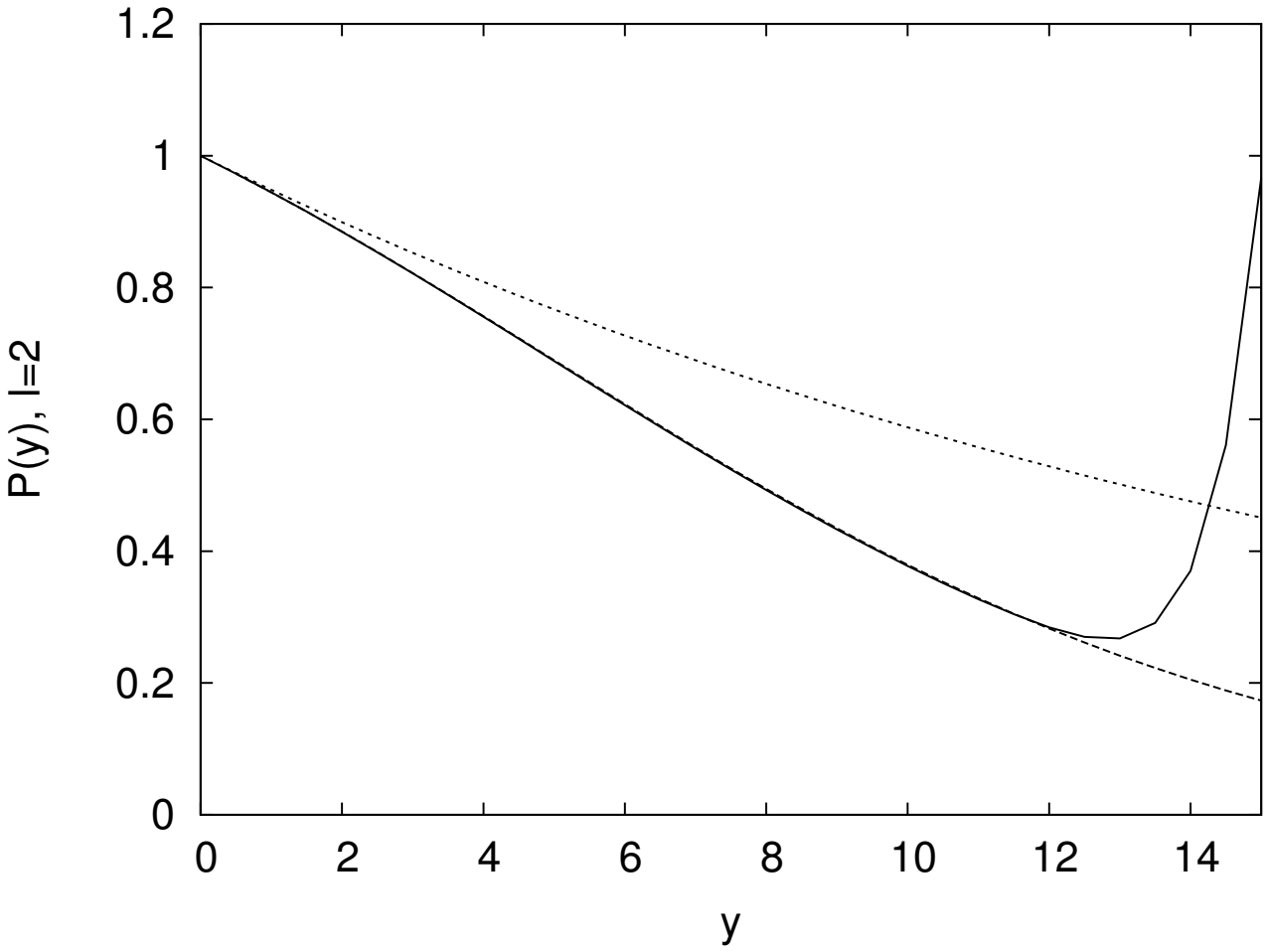

Figure: The propagator of the subdominant pomeron $l = 2$ at different rapidities with three pomerons $l = 0, \pm 2$ obtained by power evolution (middle curve) and two-field point-like evolution with composite fields (lower curve). $\mu_0 = 0.12$, $\mu_2 = -0.0531$, $\bar{\alpha}_s = 0.0433$. The upper curve corresponds to the free propagator $\exp(-0.0531y)$.

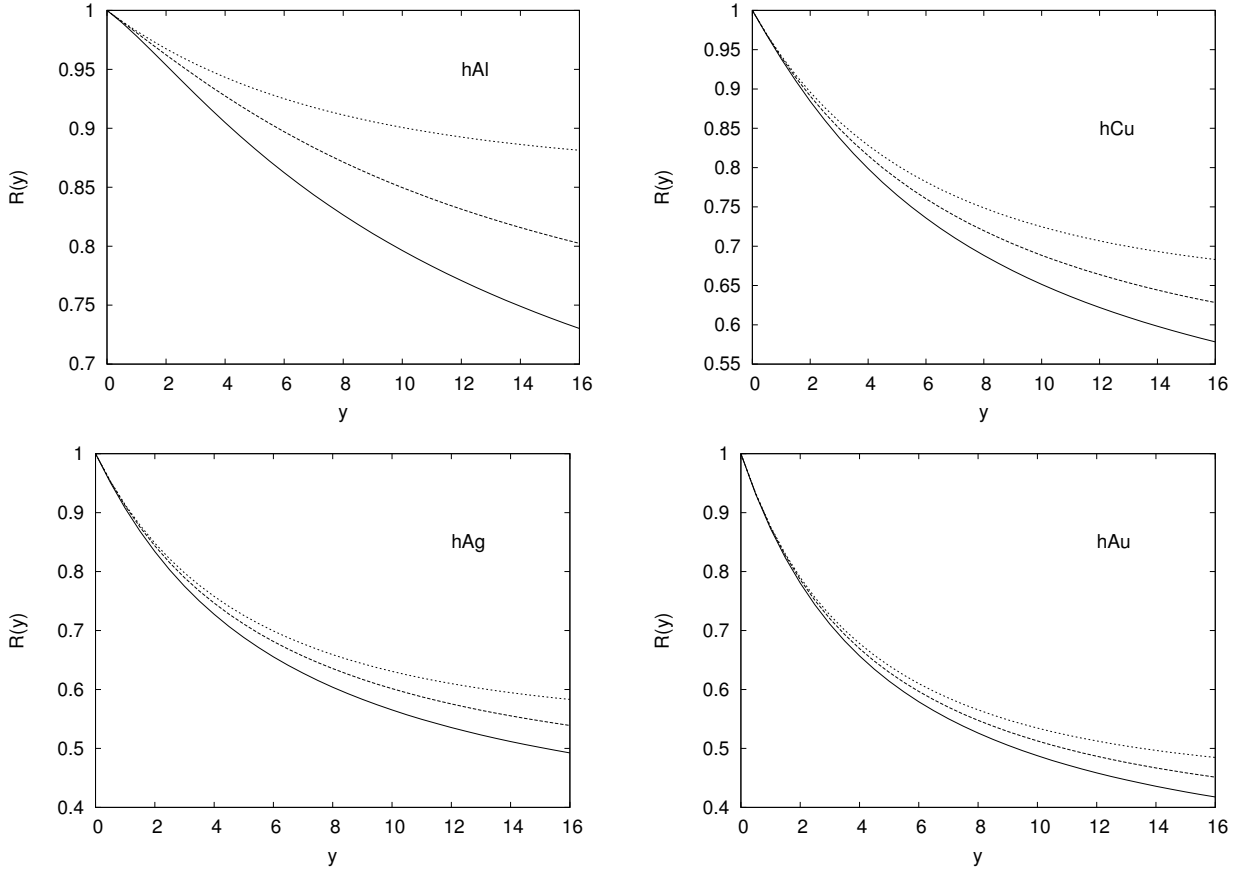

Figure: The y-dependence of the hA cross-sections calculated by point-like evolution with composite operators (the lower curves), without the subdominant pomerons $l = \pm 2$ (the middle curves) and without loops (fan diagrams, the upper curves). Panels correspond to the scattering on Aluminium (upper left), Copper (upper right), Silver (bottom left) and Gold (bottom right). In all cases $\mu_0 = 0.12$, $\mu_2 = -0.0531$, $\bar{\alpha}_s = 0.0433$, $R(y) = \frac{\sigma_A(y)}{\sigma_A(y=0)}$

Conclusion

- Several one-dimensional reggeon models are suggested, that allow numerical analysis by solving the evolution equation
- Inclusion of odderon increases propagators and amplitudes
- Inclusion of the C-odd subdominant pomeron decreases propagators
- Inclusion of subdominant pomerons with conformal spins $l = \pm 2$ decreases propagators and amplitudes

This talk is built upon the following works:

M.A.Braun, E.M.Kuzminskii, M.I.Vyazovsky, Eur.Phys.Jour. **C 81** (2021) :676

M.A.Braun, E.M.Kuzminskii, M.I.Vyazovsky, Eur.Phys.Jour. **C 82** (2022) :595

Bibliography

1. V.N. Gribov, Sov. Phys. JETP **26** (1968) 414.
2. A.A. Migdal, A.M. Polyakov, K.A. Ter-Martirosyan, Phys. Lett. **48 B** (1974) 239.
3. M. Ciafaloni, M. Le Bellac, G.C. Rossi, Nucl. Phys. **B 130** (1977) 388.
4. D. Amati, L. Caneschi, R. Jengo, Nucl. Phys. **B 101** (1975) 397.
5. V. Alessandrini, D. Amati, R. Jengo, Nucl. Phys. **B 108** (1976) 425.
6. R. Jengo, Nucl. Phys. **B 108** (1976) 447.
7. D. Amati, M. Le Bellac, G. Marchesini, M. Ciafaloni, Nucl. Phys. **B 112** (1976) 107.
8. S. Bondarenko, Eur. Phys. Jour. **C 71** (2011) 1587.
9. M.A. Braun, E.M. Kuzminskii, A.V. Kozhedub, A.M. Puchkov and M.I. Vyazovsky, Eur. Phys. Jour. **C 79** (2019) :664.
10. A.Schwimmer, Nucl. Phys. **B 94** (1975) 445.
11. Y. Hatta, E. Iancu, K. Itakura and L. McLerran, Nucl.Phys. **A 760** (2005) 172.
12. J. Bartels, L.N. Lipatov and G.P. Vacca, Phys. Lett. B **477** (2000) 178.
13. J.Bartels, C.Contreras, G.P.Vacca, Phys.Rev. **D 95** (2017) 014013.
14. I. Balitsky, Nucl.Phys. **B 463** (1996) 99.
15. Yu.V. Kovchegov, Phys.Rev. **D 60** (1999) 034008.

Thanks for your attention!