

Vacuum radiation processes in strong electromagnetic fields

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Introduction

classical electrodynamics \iff Maxwell's equations
(linear)

$$\mathcal{L}_{\text{Max}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)$$

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QED vacuum \neq “empty space”



Vacuum fluctuations give rise to nonlinear phenomena

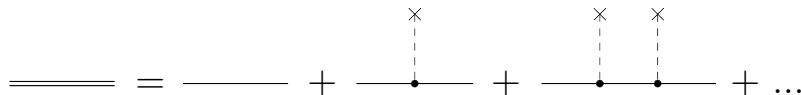
Introduction. Strong-field QED

QED Lagrangian:

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + eA^\mu\bar{\psi}\gamma_\mu\psi.$$

Replace $A^\mu \rightarrow A^\mu + \mathcal{A}^\mu$, where \mathcal{A}^μ is a classical external background

Furry picture:

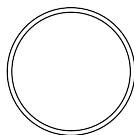


The diagram illustrates the Furry picture expansion. On the left, a double horizontal line represents the external background field. This is set equal to a sum of terms. The first term is a single horizontal line. The second term is a horizontal line with a single vertex (black dot) in the middle, from which a vertical dashed line extends upwards to a cross symbol. The third term is a horizontal line with two vertices (black dots) in the middle, each with a vertical dashed line extending upwards to a cross symbol. The series continues with an ellipsis.

$$== = \text{---} + \text{---} \cdot \text{---} \times + \text{---} \cdot \text{---} \cdot \text{---} \times \times + \dots$$

Introduction. Effective action

To zeroth order in α , all of the connected vacuum diagrams are given by



$$Z[\mathcal{A}^\mu] = \mathcal{N} \int \mathcal{D}\bar{\psi} \int \mathcal{D}\psi e^{iS} \equiv e^{iS_{\text{eff}}[\mathcal{A}^\mu]},$$

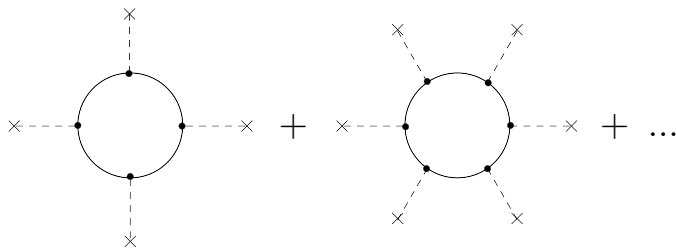
$$S_{\text{eff}}[\mathcal{A}^\mu] = S_{\text{Max}}[\mathcal{A}^\mu] + S^{(1)}[\mathcal{A}^\mu],$$

where $S^{(1)}[\mathcal{A}^\mu]$ is the one-loop effective action.

Constant field. Perturbative expansion

$$\mathcal{L}_{\text{eff}}(\mathcal{F}, \mathcal{G}) = -\mathcal{F} + \frac{8}{45} \frac{\alpha^2}{m^4} \mathcal{F}^2 + \frac{14}{45} \frac{\alpha^2}{m^4} \mathcal{G}^2 + \dots$$

Corrections to Maxwell's Lagrangian:



In external fields vacuum can be viewed as a **nonlinear** medium!

One-loop effective Lagrangian. Imaginary part

All-order result for \mathcal{L}_{eff} was obtained in

W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936)

For a purely electric field $\mathbf{E} = \text{const}$,

$$\text{Im } \mathcal{L}_{\text{eff}} = \frac{(eE)^2}{8\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-n\pi \frac{m^2}{eE}}.$$

Vacuum persistence amplitude = $\exp(iS_{\text{eff}})$.

Probability of vacuum decay:

$$P_{\text{decay}} = 1 - |e^{iS_{\text{eff}}}|^2 = 1 - e^{-2 \text{Im } S_{\text{eff}}}$$

J. Schwinger, Phys. Rev. **82**, 664 (1951)

$$P_{\text{decay}} \sim \exp\left(-\frac{\pi E_c}{E}\right)$$

Critical field strength:

$$E_c = \frac{m^2 c^3}{|e| \hbar} \approx 1.3 \times 10^{16} \text{ V/cm.}$$

Particle yield can be enhanced by introducing temporal oscillations or multiple laser pulses and taking into account the volume factor.

Nevertheless, it can hardly be non-negligible unless $E_0 \gtrsim 0.1 E_c$.

$E_0 = 0.1 E_c$ corresponds to $I_0 \sim 10^{27} \text{ W/cm}^2$.

Vacuum photon emission

Can one indirectly probe the Schwinger mechanism by measuring photons?

Photon number density:

$$n_{\mathbf{k},\lambda} = \langle f | c_{\mathbf{k},\lambda}^\dagger c_{\mathbf{k},\lambda} | f \rangle.$$

A. Di Piazza, K. Z. Hatsagortsyan, and C. H. Keitel, PRD **72**, 085005 (2005)

A. M. Fedotov and N. B. Narozhny, PLA **362**, 1 (2007)

F. Karbstein and R. Shaisultanov, PRD **91**, 113002 (2015)

A. Otto and B. Kämpfer, PRD **95**, 125007 (2017)

H. Gies, F. Karbstein, and C. Kohlfürst, PRD **97**, 036022 (2018)

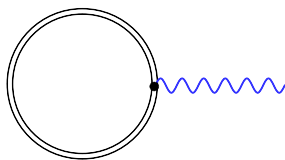
B. King, H. Hu, and B. Shen, PRA **98**, 023817 (2018)

I. A. Aleksandrov, G. Plunien, and V. M. Shabaev, PRD **100**, 116003 (2019)

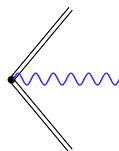
F. Karbstein, A. Blinne, H. Gies, and M. Zepf, PRL **123**, 091802 (2019)

Vacuum photon emission

Feynman diagrams within the Furry picture:



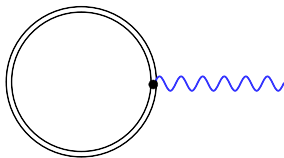
tadpole



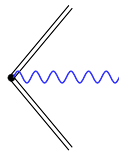
vertex

Vacuum photon emission

Feynman diagrams within the Furry picture:

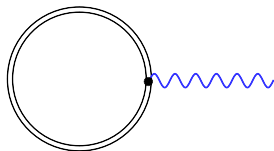


tadpole



vertex
our focus

Tadpole diagram



Gives rise to emission of photons similar to those constituting the external field or higher harmonics

A. Di Piazza, K. Z. Hatsagortsyan, and C. H. Keitel, PRD **72**, 085005 (2005)

A. M. Fedotov and N. B. Narozhny, PLA **362**, 1 (2007)

F. Karbstein and R. Shaisultanov, PRD **91**, 113002 (2015)

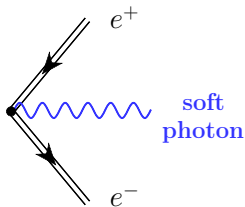
H. Gies, F. Karbstein, and C. Kohlfürst, PRD **97**, 036022 (2018)

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I. A. Aleksandrov, G. Plunien, and V. M. Shabaev, PRD **100**, 116003 (2019)

F. Karbstein, A. Blinne, H. Gies, and M. Zepf, PRL **123**, 091802 (2019)

Vertex diagram. Soft photons

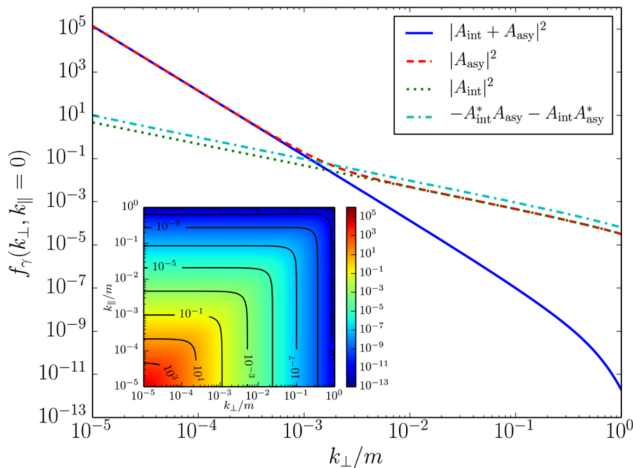


Predicts a huge amount of soft photons:

$$n_{\mathbf{k},\lambda} \sim \frac{1}{k_0^3}$$

A. Otto and B. Kämpfer, PRD **95**, 125007 (2017)

Vertex diagram. Soft photons



Sauter pulse $E(t) = E_0 \text{sech}^2(t/\tau)$ with $E_0 = 0.2E_c$ and $\tau = 2m^{-1}$

A. Otto and B. Kämpfer, PRD **95**, 125007 (2017)

Vertex diagram. Space-time-dependent fields

Vertex diagram was analyzed in the case of a standing wave background in [I. A. Aleksandrov, G. Plunien, and V. M. Shabaev, PRD **100**, 116003 (2019)].

- Qualitative behavior $1/k_0^3$ remains the same
- Soft photons are not emitted isotropically
- Anisotropy cannot be identified by means of local approximations, i.e., by averaging the results over the spatial coordinates

Photons in the initial state

What will change if the initial state contains (probe) photons?

$$|in\rangle = c_{\mathbf{q},\varkappa}^\dagger |0, in\rangle$$

Photons in the initial state

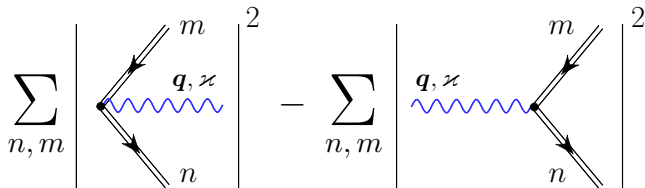
What will change if the initial state contains (probe) photons?

$$|\text{in}\rangle = c_{\mathbf{q},\varkappa}^\dagger |0, \text{in}\rangle$$

$n_{\mathbf{k},\lambda}$ contains now three contributions:

- 0th-order: one initial photon
- α^1 : vacuum terms (tadpole and vertex)
- α^1 : photon-induced contribution

Photon-induced contribution

$$\sum_{n,m} \left| \begin{array}{c} \text{diagram 1} \end{array} \right|^2 - \sum_{n,m} \left| \begin{array}{c} \text{diagram 2} \end{array} \right|^2$$


The first diagram shows an incoming photon (wavy line) interacting with a system (black dot) to produce two outgoing particles (double lines) labeled m and n . The second diagram shows an incoming photon (wavy line) interacting with a system (black dot) to produce two outgoing particles (double lines) labeled m and n . The diagrams are separated by a minus sign.

Photon lines correspond to the quantum numbers \mathbf{q} and \varkappa of the initial photon

\Rightarrow “stimulated emission” and “absorption”

vacuum contribution = “spontaneous emission”

Photon-induced contribution

$$\sum_{n,m} \left| \begin{array}{c} \text{diagram 1} \end{array} \right|^2 - \sum_{n,m} \left| \begin{array}{c} \text{diagram 2} \end{array} \right|^2$$

The first diagram shows an incoming photon (wavy blue line) with momentum q and polarization ε interacting with a fermion line (double black line) that splits into two outgoing fermions with momenta m and n . The second diagram is identical but with the photon momentum reversed to $-q$.

The second diagram is the first one with $k^\mu \rightarrow -k^\mu$

\Rightarrow even powers of $1/k_0$ change the sign:

$$(A/k_0^3 + B/k_0^2 + \dots) - (A/k_0^3 - B/k_0^2 + \dots) = 2B/k_0^2 + \dots$$

suppressed by k_0/m but still non-zero

Number of signal photons

- Introduce a smearing function for the initial photon state
- Integrate the density $n_{\mathbf{k},\lambda}$ over the small momentum volume V_q , where the initial photon was localized

Number of photons ($\mathbf{l} \equiv \mathbf{q}/q_0$):

$$N^{(1)} = 1 + \left(\frac{A_{\mathbf{l},\kappa}}{q_0^3} + \frac{B_{\mathbf{l},\kappa}}{q_0^2} + \dots \right) V V_q + \frac{2(2\pi)^3 B_{\mathbf{l},\kappa}}{q_0^2} + \dots$$

The **vacuum** term is proportional to V . The **photon-induced** term will be proportional to the initial number of photons N :

$$N^{(\text{ph})} = \frac{2(2\pi)^3 |B_{\mathbf{l},\kappa}|}{q_0^2} N.$$

One has to evaluate A and B .

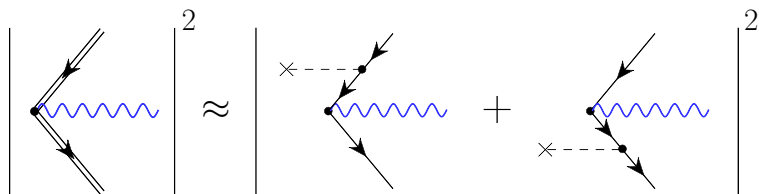
Numerical techniques

- Exact solutions of the Dirac equation in the case of the Sauter pulse \implies closed-form analytical expressions
- Furry-picture quantization + solving the Dirac equation in momentum space (exact numerical approach)
[\[I. A. A., G. Plunien, and V. M. Shabaev, PRD **100**, 116003 \(2019\)\]](#)
- Perturbation theory (PT)
- WKB approach for scalar QED

[I. Aleksandrov, A. Di Piazza, G. Plunien, V. Shabaev, PRD **105**, 116005 \(2022\)](#)

Perturbation theory

For sufficiently large field frequencies, one can use



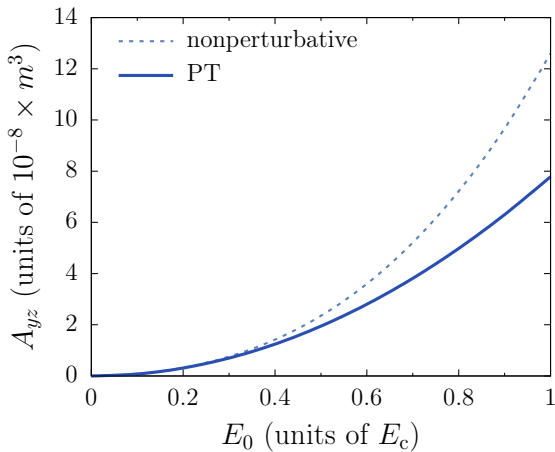
The diagram shows an equation between two squared amplitudes. On the left is a vertical line with a vertex from which two solid lines with arrows and a wavy blue line emerge. On the right is the sum of two diagrams, each preceded by a cross and a dashed line. The first diagram on the right has a vertex with one solid line and one wavy line, and a second vertex with two solid lines. The second diagram on the right has a vertex with one solid line and one wavy line, and a second vertex with one solid line and one wavy line. The entire right-hand side is enclosed in a square with a superscript 2.

$$\left| \text{Diagram 1} \right|^2 \approx \left| \text{Diagram 2} + \text{Diagram 3} \right|^2$$

We explicitly reveal the power-law behavior $A/k_0^3 + B/k_0^2 + \dots$ and extract A and B

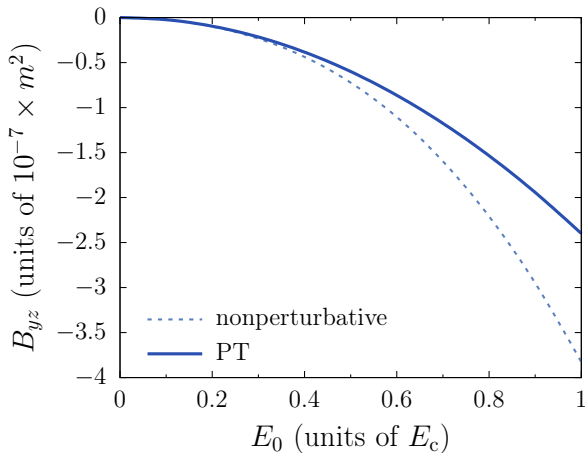
Perturbation theory. Sauter pulse $E(t) = E_0 \cosh^{-2} t/\tau$

Sauter pulse with $\tau = 1.0 m^{-1}$



Perturbation theory. Sauter pulse $E(t) = E_0 \cosh^{-2} t/\tau$

Sauter pulse with $\tau = 1.0 m^{-1}$



WKB analysis

In the case of the Sauter pulse (scalar* QED):

$$A_{\mathbf{n},\lambda}^{(\text{WKB})} = \frac{\alpha}{8\pi^5} \int d\mathbf{p} \frac{(\mathbf{p}, \mathbf{e}_\lambda)^2 p_0^2(\mathbf{P})}{[p_0^2(\mathbf{P}) - (\mathbf{P}, \mathbf{n})^2]^2} |\alpha_{\mathbf{p}}|^2,$$

$$B_{\mathbf{n},\lambda}^{(\text{WKB})} = \frac{\alpha}{8\pi^5} \int d\mathbf{p} \frac{(\mathbf{p}, \mathbf{e}_\lambda)^2 (\mathbf{P}, \mathbf{n})}{[p_0^2(\mathbf{P}) - (\mathbf{P}, \mathbf{n})^2]^2} |\alpha_{\mathbf{p}}|^2,$$

where $\mathbf{P} \equiv \mathbf{p} - e\mathbf{A}_0$, $p_0(\mathbf{p}) \equiv \sqrt{m^2 + \mathbf{p}^2}$, and $|\alpha_{\mathbf{p}}|^2$ is the number density of electrons/positrons produced.

* The results should be multiplied by 2 as the spin effects and statistics are not important here

[D. Sevostyanov, I. A. A., G. Plunien, V. Shabaev, PRD **104**, 076014 (2021)].

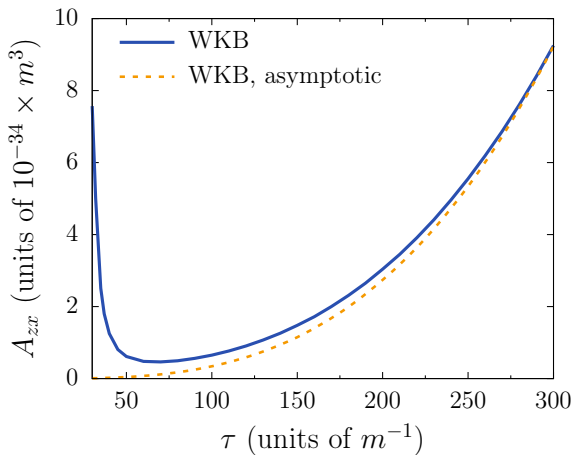
WKB analysis. Asymptotic behavior

For $m\tau \gg 1$ and $E_0 \ll E_c$:

$$A_{zx} \approx \frac{\alpha}{16\pi^6} m^3 (m\tau)^3 \left(\frac{E_0}{E_c} \right)^{11/2} e^{-\pi E_c/E_0},$$
$$B_{zx} \approx -\frac{\alpha}{16\pi^6} m^2 (m\tau)^2 \left(\frac{E_0}{E_c} \right)^{9/2} e^{-\pi E_c/E_0}.$$

WKB analysis

Sauter pulse with $E_0 = 0.05E_c$

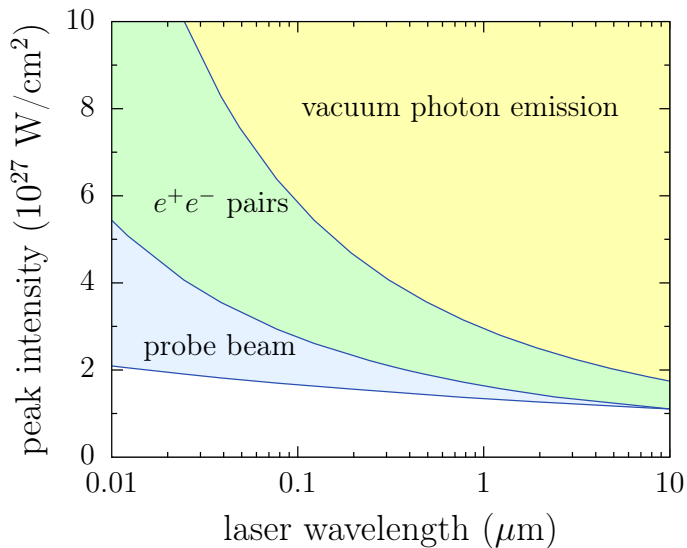


Comparison

Let us compare the threshold values of the laser intensity:

- Vacuum pair production: 10 pairs.
- Vacuum photon emission: 10 photons.
- Photon-induced contribution: relative change of the probe beam intensity on the level of 1%.

Comparison



Conclusions

- Strong external background can produce a huge amount of soft photons
- Instead of measuring the vacuum contribution, one can use an additional probe photon beam
- Measuring the intensity change of the probe photon beam, one can indirectly observe the Schwinger mechanism.

I. Aleksandrov, A. Di Piazza, G. Plunien, V. Shabaev, PRD **105**, 116005 (2022)

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Thank you for your attention!