Vacuum radiation processes in strong electromagnetic fields

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Introduction

classical electrodynamics $\Longleftrightarrow$ Maxwell's equations
(linear)

$$\mathcal{L}_{\mathsf{Max}} = -\frac{1}{4}\,F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}(\boldsymbol{E}^2 - \boldsymbol{B}^2)$$

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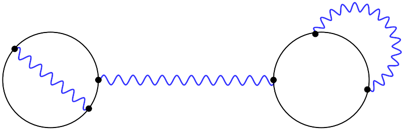
QED vacuum $\neq$ "empty space"



Vacuum fluctuations give rise to nonlinear phenomena

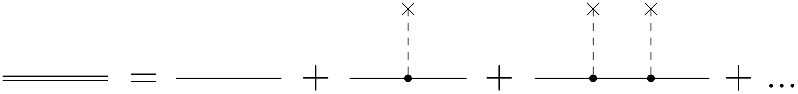
Introduction. Strong-field QED

QED Lagrangian:

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi + eA^{\mu}\bar{\psi}\gamma_{\mu}\psi.$$

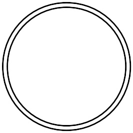
Replace $A^{\mu} \to A^{\mu} + \mathcal{A}^{\mu}$, where $\mathcal{A}^{\mu}$ is a classical external background

Furry picture:



Introduction. Effective action

To zeroth order in α, all of the connected vacuum diagrams are given by



$$Z\left[\mathcal{A}^{\mu}\right] = \mathcal{N}\int\mathcal{D}\bar{\psi}\int\mathcal{D}\psi\,\mathrm{e}^{iS} \equiv \mathrm{e}^{iS_{\mathsf{eff}}[\mathcal{A}^{\mu}]},$$

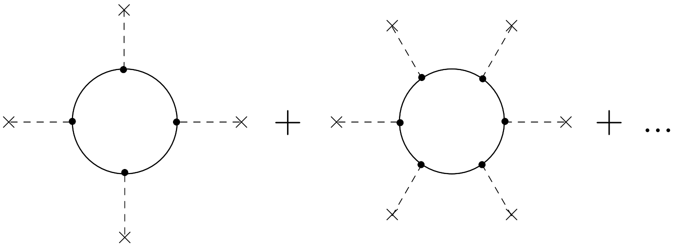
$$S_{\mathsf{eff}}[\mathcal{A}^{\mu}] = S_{\mathsf{Max}}[\mathcal{A}^{\mu}] + S^{(1)}[\mathcal{A}^{\mu}],$$

where $S^{(1)}[\mathcal{A}^{\mu}]$ is the one-loop effective action.

Constant field. Perturbative expansion

$$\mathcal{L}_{\text{eff}}(\mathcal{F},\mathcal{G}) = -\mathcal{F} + \frac{8}{45}\frac{\alpha^2}{m^4}\mathcal{F}^2 + \frac{14}{45}\frac{\alpha^2}{m^4}\mathcal{G}^2 + ...$$

Corrections to Maxwell's Lagrangian:



In external fields vacuum can be viewed as a nonlinear medium!

One-loop effective Lagrangian. Imaginary part

All-order result for $\mathcal{L}_{\text{eff}}$ was obtained in
W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936)

For a purely electric field $\boldsymbol{E} = \text{const}$,

$$\operatorname{Im}\,\mathcal{L}_{\text{eff}} = \frac{(eE)^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\,\mathrm{e}^{-n\pi\frac{m^2}{eE}}.$$

Vacuum persistence amplitude $= \exp(iS_{\text{eff}})$.

Probability of vacuum decay:

$$P_{\text{decay}} = 1 - |\mathrm{e}^{iS_{\text{eff}}}|^2 = 1 - \mathrm{e}^{-2\,\operatorname{Im}S_{\text{eff}}}$$

J. Schwinger, Phys. Rev. **82**, 664 (1951)

$$P_{\mathsf{decay}} \sim \exp\left(-\frac{\pi E_{\mathsf{c}}}{E}\right)$$

Critical field strength:

$$E_{\mathsf{c}} = \frac{m^2 c^3}{|e|\hbar} \approx 1.3 \times 10^{16}\ \mathsf{V/cm}.$$

Particle yield can be enhanced by introducing temporal oscillations or multiple laser pulses and taking into account the volume factor.

Nevertheless, it can hardly be non-negligible unless $E_0 \gtrsim 0.1 E_{\mathsf{c}}$.

$E_0 = 0.1 E_{\mathsf{c}}$ corresponds to $I_0 \sim 10^{27}$ W/cm^2.

Vacuum photon emission

Can one indirectly probe the Schwinger mechanism by measuring photons?

Photon number density:

$$n_{\boldsymbol{k},\lambda} = \langle f | c^{\dagger}_{\boldsymbol{k},\lambda} c_{\boldsymbol{k},\lambda} | f \rangle.$$

A. Di Piazza, K. Z. Hatsagortsyan, and C. H. Keitel, PRD **72**, 085005 (2005)

A. M. Fedotov and N. B. Narozhny, PLA **362**, 1 (2007)

F. Karbstein and R. Shaisultanov, PRD **91**, 113002 (2015)

A. Otto and B. Kämpfer, PRD **95**, 125007 (2017)

H. Gies, F. Karbstein, and C. Kohlfürst, PRD **97**, 036022 (2018)

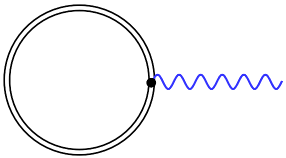
B. King, H. Hu, and B. Shen, PRA **98**, 023817 (2018)

I. A. Aleksandrov, G. Plunien, and V. M. Shabaev, PRD **100**, 116003 (2019)

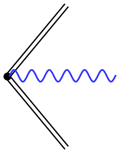
F. Karbstein, A. Blinne, H. Gies, and M. Zepf, PRL **123**, 091802 (2019)

Vacuum photon emission

Feynman diagrams within the Furry picture:



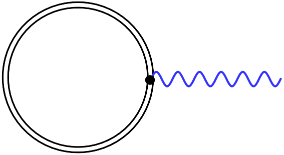
tadpole



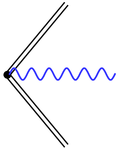
vertex

Vacuum photon emission

Feynman diagrams within the Furry picture:

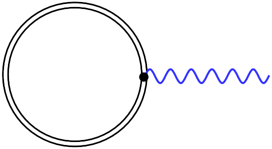


tadpole



vertex
our focus

Tadpole diagram



Gives rise to emission of photons similar to those constituting the external field or higher harmonics

A. Di Piazza, K. Z. Hatsagortsyan, and C. H. Keitel, PRD **72**, 085005 (2005)

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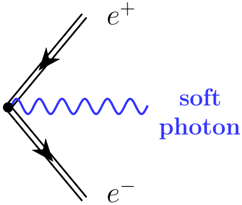
Vertex diagram. Soft photons

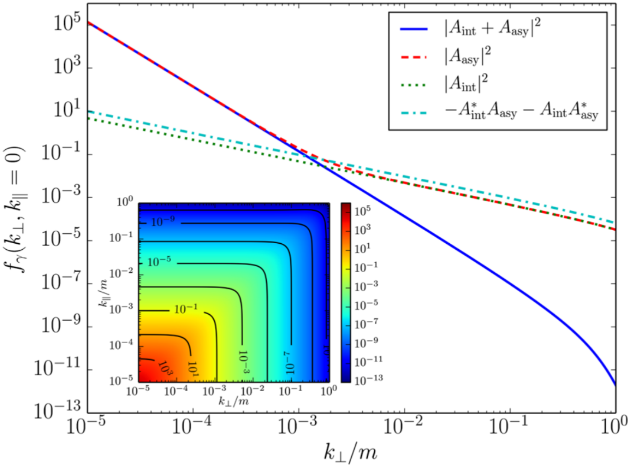


Predicts a huge amount of soft photons:

$$n_{\boldsymbol{k},\lambda} \sim \frac{1}{k_0^3}$$

A. Otto and B. Kämpfer, PRD **95**, 125007 (2017)

Vertex diagram. Soft photons



Sauter pulse $E(t) = E_0 \,\mathrm{sech}^2(t/\tau)$ with $E_0 = 0.2E_{\mathsf{c}}$ and $\tau = 2m^{-1}$

A. Otto and B. Kämpfer, PRD **95**, 125007 (2017)

Vertex diagram. Space-time-dependent fields

Vertex diagram was analyzed in the case of a standing wave background in [I. A. Aleksandrov, G. Plunien, and V. M. Shabaev, PRD **100**, 116003 (2019)].

- Qualitative behavior $1/k_0^3$ remains the same
- Soft photons are not emitted isotropically
- Anisotropy cannot be identified by means of local approximations, i.e., by averaging the results over the spatial coordinates

Photons in the initial state

What will change if the initial state contains (probe) photons?

$|\mathrm{in}\rangle = c^{\dagger}_{\boldsymbol{q},\varkappa}|0,\, \mathrm{in}\rangle$

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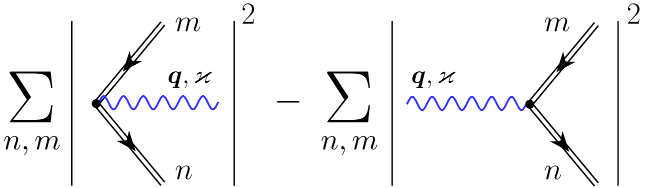
$|\text{in}\rangle = c^\dagger_{\boldsymbol{q},\varkappa}|0,\,\text{in}\rangle$

$n_{\boldsymbol{k},\lambda}$ contains now three contributions:

- 0th-order: one initial photon
- α^1: vacuum terms (tadpole and vertex)
- α^1: photon-induced contribution

Photon-induced contribution

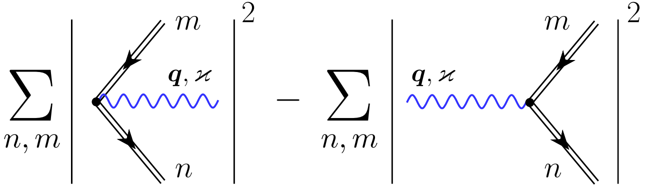


Photon lines correspond to the quantum numbers $\boldsymbol{q}$ and $\varkappa$ of the initial photon

$\Longrightarrow$ “stimulated emission” and “absorption”

vacuum contribution = “spontaneous emission”

Photon-induced contribution



The second diagram is the first one with $k^\mu \to -k^\mu$

$\Longrightarrow$ even powers of $1/k_0$ change the sign:

$$(A/k_0^3 + B/k_0^2 + ...) - (A/k_0^3 - B/k_0^2 + ...) = 2B/k_0^2 + ...$$

suppressed by k_0/m but still non-zero

Number of signal photons

- Introduce a smearing function for the initial photon state
- Integrate the density $n_{\boldsymbol{k},\lambda}$ over the small momentum volume V_q, where the initial photon was localized

Number of photons ($\boldsymbol{l} \equiv \boldsymbol{q}/q_0$):

$$N^{(1)} = 1 + \left(\frac{A_{\boldsymbol{l},\varkappa}}{q_0^3} + \frac{B_{\boldsymbol{l},\varkappa}}{q_0^2} + ... \right) V V_q + \frac{2(2\pi)^3 B_{\boldsymbol{l},\varkappa}}{q_0^2} + ...$$

The vacuum term is proportional to V. The photon-induced term will be proportional to the initial number of photons N:

$$N^{(\mathsf{ph})} = \frac{2(2\pi)^3 |B_{\boldsymbol{l},\varkappa}|}{q_0^2} N.$$

One has to evaluate A and B.

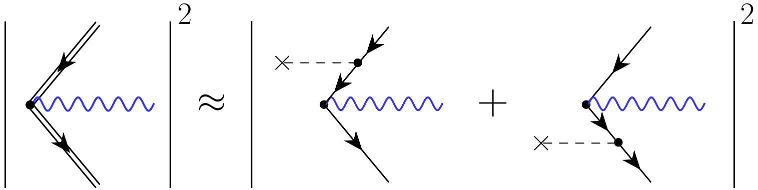
Numerical techniques

- Exact solutions of the Dirac equation in the case of the Sauter pulse $\Longrightarrow$ closed-form analytical expressions
- Furry-picture quantization + solving the Dirac equation in momentum space (exact numerical approach)
  [I. A. A., G. Plunien, and V. M. Shabaev, PRD **100**, 116003 (2019)]
- Perturbation theory (PT)
- WKB approach for scalar QED

I. Aleksandrov, A. Di Piazza, G. Plunien, V. Shabaev, PRD **105**, 116005 (2022)

Perturbation theory

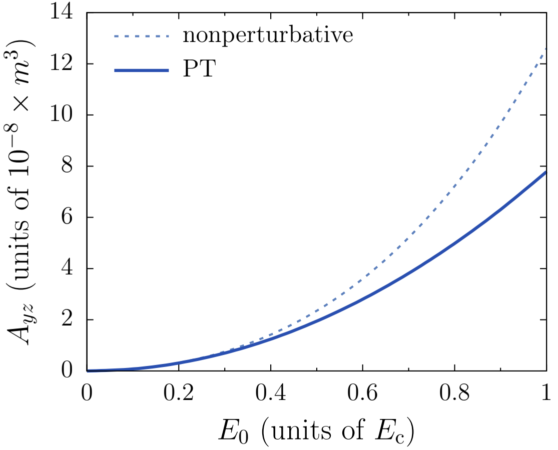
For sufficiently large field frequencies, one can use



We explicitly reveal the power-law behavior $A/k_0^3 + B/k_0^2 + \ldots$ and extract A and B

Perturbation theory. Sauter pulse $E(t) = E_0 \cosh^{-2} t/\tau$

Sauter pulse with $\tau = 1.0 m^{-1}$



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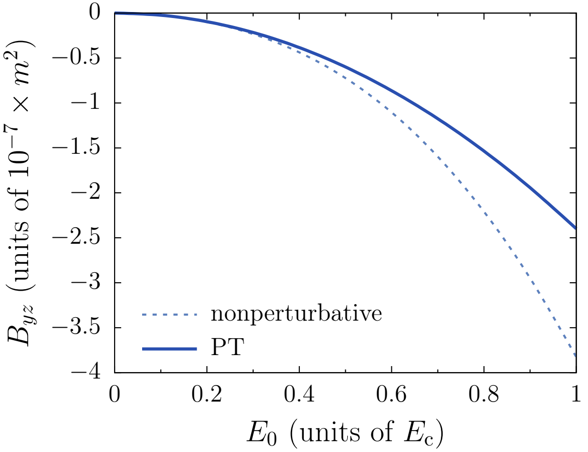


WKB analysis

In the case of the Sauter pulse (scalar* QED):

$$A_{\boldsymbol{n},\lambda}^{(\mathrm{WKB})} = \frac{\alpha}{8\pi^5} \int d\boldsymbol{p} \, \frac{(\boldsymbol{p}, \boldsymbol{e}_\lambda)^2 p_0^2(\boldsymbol{P})}{\left[p_0^2(\boldsymbol{P}) - (\boldsymbol{P}, \boldsymbol{n})^2\right]^2} \, |\alpha_{\boldsymbol{p}}|^2,$$

$$B_{\boldsymbol{n},\lambda}^{(\mathrm{WKB})} = \frac{\alpha}{8\pi^5} \int d\boldsymbol{p} \, \frac{(\boldsymbol{p}, \boldsymbol{e}_\lambda)^2 (\boldsymbol{P}, \boldsymbol{n})}{\left[p_0^2(\boldsymbol{P}) - (\boldsymbol{P}, \boldsymbol{n})^2\right]^2} \, |\alpha_{\boldsymbol{p}}|^2,$$

where $\boldsymbol{P} \equiv \boldsymbol{p} - e\boldsymbol{\mathcal{A}}_0$, $p_0(\boldsymbol{p}) \equiv \sqrt{m^2 + \boldsymbol{p}^2}$, and $|\alpha_{\boldsymbol{p}}|^2$ is the number density of electrons/positrons produced.

* The results should by multiplied by 2 as the spin effects and statistics are not important here

[D. Sevostyanov, I. A. A., G. Plunien, V. Shabaev, PRD **104**, 076014 (2021)].

WKB analysis. Asymptotic behavior

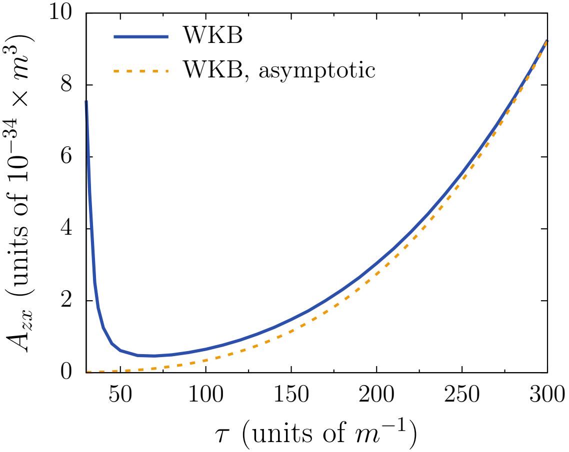
For $m\tau \gg 1$ and $E_0 \ll E_{\mathsf{c}}$:

$$A_{zx} \approx \frac{\alpha}{16\pi^6}\, m^3 (m\tau)^3 \left(\frac{E_0}{E_{\mathsf{c}}}\right)^{11/2} \mathrm{e}^{-\pi E_{\mathsf{c}}/E_0},$$

$$B_{zx} \approx -\frac{\alpha}{16\pi^6}\, m^2 (m\tau)^2 \left(\frac{E_0}{E_{\mathsf{c}}}\right)^{9/2} \mathrm{e}^{-\pi E_{\mathsf{c}}/E_0}.$$

WKB analysis

Sauter pulse with $E_0 = 0.05E_\mathsf{c}$

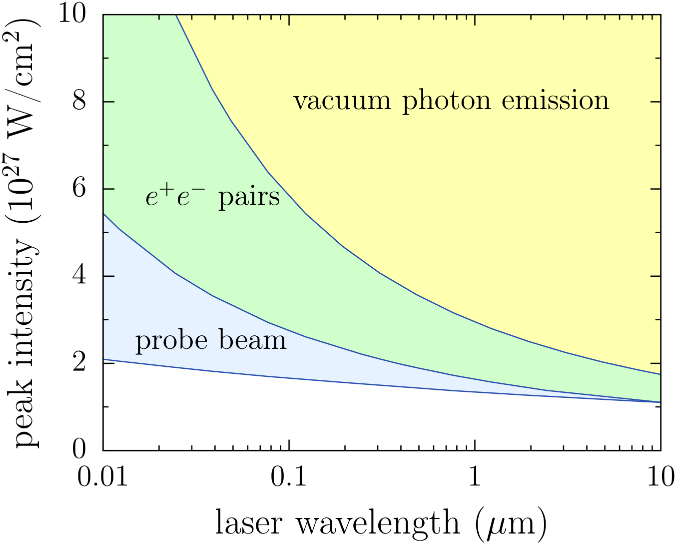


Comparison

Let us compare the threshold values of the laser intensity:

- Vacuum pair production: 10 pairs.
- Vacuum photon emission: 10 photons.
- Photon-induced contribution: relative change of the probe beam intensity on the level of 1%.

Comparison


vacuum photon emission
e^+e^- pairs
probe beam
peak intensity (10^{27} W/cm^2)
laser wavelength (μm)
0
2
4
6
8
10
0.01
0.1
1
10

Conclusions

- Strong external background can produce a huge amount of soft photons
- Instead of measuring the vacuum contribution, one can use an additional probe photon beam
- Measuring the intensity change of the probe photon beam, one can indirectly observe the Schwinger mechanism.

I. Aleksandrov, A. Di Piazza, G. Plunien, V. Shabaev, PRD **105**, 116005 (2022)

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Thank you for your attention!