VII International Conference "Models in Quantum Field Theory"

MQFT-2022

October 10–14, 2022, Saint Petersburg, Russia

O.V.Haneychuk, V.Yu.Shirokova, K.V.Stepanyantz

Moscow State University, Physical Faculty,
Department of Theoretical Physics

Three loop β-functions and the NSVZ relations for MSSM in the case of using the higher covariant derivative regularization

based on the paper O.H.,V.Shirokova, K.Stepanyantz

JHEP **09** (2022) 189

Since supersymmetry based theories are currently one of the candidates for describing physics "beyond the Standard Model", it is important to investigate supersymmetric models at the quantum level. In particular, the evolution of running gauge coupling constants in $\mathcal{N} = 1$ supersymmetric theories and in theories with softly broken supersymmetry is of considerable interest.

β-functions, which encode this behavior, in such theories can be related to the anomalous dimensions of the chiral matter superfields with the help of the exact Novikov, Shifman, Vainshtein, and Zakharov (NSVZ) β-function.

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. **B 277** (1986) 456; D.R.T.Jones, Phys.Lett. **B 123** (1983) 45.

$$\beta(\alpha,\lambda) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha,\lambda)/r\Big)}{2\pi(1 - C_2\alpha/2\pi)}.$$

Here α and λ are the gauge and Yukawa coupling constants, respectively, and we use the notation

$$\mathrm{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$
$$f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \qquad r \equiv \delta_{AA} = \dim G.$$

Introduction

The use of the NSVZ equations greatly simplifies calculations in higher orders, since NSVZ relations relate β-functions to anomalous dimensions in all previous orders. For example, knowing the two-loop anomalous dimensions, the NSVZ equations make it possible to obtain three-loop β-functions.

However, not every renormalization prescription belongs to the so-called NSVZ schemes, that is, schemes in which the NSVZ relation is satisfied. In particular, the most popular $\overline{\text{DR}}$-scheme, which implies that a theory is regularized by dimensional reduction and divergences are removed by modified minimal subtraction, is not NSVZ.

I. Jack, D. R. T. Jones and C. G. North, Phys. Lett. B **386** (1996) 138; I. Jack, D. R. T. Jones and C. G. North, Nucl. Phys. B **486** (1997) 479; I. Jack, D. R. T. Jones and A. Pickering, Phys. Lett. B **435** (1998) 61;

It was shown, that, starting from the three-loop approximation, the NSVZ relation is no longer satisfied for this renormalization prescription and its restoration requires a special finite renormalization in each order.

L.V.Avdeev, O.V.Tarasov, Phys.Lett. **112 B** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett **B386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

Moreover, dimensional reduction is not mathematically consistent,

W. Siegel, Phys. Lett. B **94** (1980), 37.

and can break supersymmetry in higher loops.

L. V. Avdeev, G. A. Chochia and A. A. Vladimirov, Phys. Lett. B **105** (1981), 272; L. V. Avdeev, Phys. Lett. B **117** (1982), 317; L. V. Avdeev and A. A. Vladimirov, Nucl. Phys. B **219** (1983), 262.

Here we will obtain the three-loop β-functions for the Minimal Supersymmetric Standard Model (MSSM) using the higher covariant derivative regularization. In the $\overline{\text{DR}}$-scheme the corresponding result has already been obtained earlier.

I. Jack, D. R. T. Jones and A. F. Kord, Annals Phys. **316** (2005), 213.

However, the use of regularization by higher covariant derivatives

A. A. Slavnov, Nucl. Phys. B **31** (1971) 301; A. A. Slavnov, Theor.Math.Phys. **13** (1972) 1064 [Teor. Mat. Fiz. 13 (1972) 174].

in supersymmetric theories offers certain advantages. It is is self-consistent and can be formulated in a manifestly supersymmetric way in terms of $\mathcal{N} = 1$ superfields.

V.K.Krivoshchekov, Theor.Math.Phys. **36** (1978) 745; P.West, Nucl.Phys. B **268**, (1986), 113.

Introduction

Residual one-loop divergences are removed due to the insertion of the Pauli–Villars determinants into the generating functional.

A.A.Slavnov, Theor.Math.Phys. **33**, (1977), 977.

Renormalization group functions (RGFs) defined in terms of the bare couplings satisfy the NSVZ relations in all orders under this regularization.

K. V. Stepanyantz, Nucl. Phys. B **909** (2016) 316; K. V. Stepanyantz, JHEP **1910** (2019) 011; K. Stepanyantz, Eur. Phys. J. C **80** (2020) no.10, 911.

RGFs defined in terms of the renormalized couplings can also obey the NSVZ equations when the HD+MSL renormalization prescription is used, which implies that theory is regularized by Higher Derivatives and divergences are removed by Minimal Subtractions of Logarithms.

A. L. Kataev and K. V. Stepanyantz, Nucl. Phys. B **875** (2013) 459.

For $N = 1$ supersymmetric theories with multiple gauge couplings regularized by higher covariant derivatives, the NSVZ equations are also satisfied for an arbitrary renormalization prescription for RGFs defined in terms of the bare couplings and for RGFs defined in terms of the renormalized couplings in the HD+MSL scheme.

D. Korneev, D. Plotnikov, K. Stepanyantz and N. Tereshina, JHEP 10 (2021), 046.

N=1supersymmetric theories with multiple gauge couplings

The classical action of a renormalizable N = 1 supersymmetric gauge theory in the massless limit can be written in the manifestly supersymmetric form

$$S = \sum_{K=1}^{n} Re\frac{1}{4}\int d^4x d^2\theta (W^a)^{A_K}(W_a)^{A_K} + \frac{1}{4}\int d^4x d^4\theta \phi^{*i}(e^{2V})_i^j \phi_j$$
$$+ (\frac{1}{6}{\lambda_0}^{ijk}\int d^4x d^2\theta \phi_i\phi_j\phi_k + c.c.),$$

where the subscript K numerates the factors G_K of the gauge group G, ${\lambda_0}^{ijk}$ are Yukawa couplings and W_a is the strength of the gauge superfield

$$V_j^i = \sum_K e_{0K} V^{A_K}(T^{A_K})_i^j.$$

The generators of the gauge group satisfy the commutation relations

$$[T^{A_K}, T^{B_K}] = if^{A_K B_K C_K} T^{C_K}.$$

MSSM is a softly broken N = 1 supersymmetric theory with the gauge group

$$G = SU(3) \times SU(2) \times U(1)_Y.$$

As a result, the theory contains three gauge coupling constants

$$\alpha_3 = \frac{{e_3}^2}{4\pi}; \quad \alpha_2 = \frac{{e_2}^2}{4\pi}; \quad \alpha_1 = \frac{5}{3} \cdot \frac{{e_1}^2}{4\pi},$$

where the factor $\frac{5}{3}$ is included into the definition of α_1 in order to have the gauge coupling unification condition in the form $\alpha_1 = \alpha_2 = \alpha_3$.

The chiral matter superfields include three generations of quarks and leptons, two Higgs fields H_u and H_d,and their superpartners as components. Their quantum numbers are representations for $SU(3)$ and $SU(2)$ groups, and the hypercharge Y for $U(1)$ group, which are presented in the table (a model without right neutrinos is under consideration). Subscripts 1, 2, 3 number generations.

superfield	Q_1, Q_2, Q_3	U_1, U_2, U_3	D_1, D_2, D_3	L_1, L_2, L_3	E_1, E_2, E_3	H_u	H_d
$SU(3)$	$\bar{3}$	3	3	1	1	1	1
$SU(2)$	2	1	1	2	1	2	2
$U(1)_Y$	$-1/6$	$2/3$	$-1/3$	$1/2$	-1	$-1/2$	$1/2$

The superpotential W is contained in the part of the MSSM action

$$\Delta S = \frac{1}{2}\int d^4x d^2\theta W + c.c.$$

and has the form

$$W = (Y_{0U})_{IJ}(\tilde{U} \quad \tilde{D})^a_I \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{u1} \\ H_{u2} \end{pmatrix} U_{aJ}$$
$$+ (Y_{0D})_{IJ}(\tilde{U} \quad \tilde{D})^a_I \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} D_{aJ}$$
$$+ (Y_{0E})_{IJ}(\tilde{N} \quad \tilde{E})_I \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} E_J + \mu_0 \begin{pmatrix} H_{u1} & H_{u2} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix}.$$

Here $\begin{pmatrix} \tilde{U} \\ \tilde{D} \end{pmatrix}$ and $\begin{pmatrix} \tilde{N} \\ \tilde{E} \end{pmatrix}$ describe left quarks and leptons, respectively, Y_{0U}, Y_{0D}, Y_{0E} are the dimensionless Yukawa matrices and the parameter μ_0 has the dimension of mass. Indices I and J numerate generations and take values from 1 to 3.

Higher derivative regularization for MSSM

The matter superfields ϕ_a are renormalized as

$$\phi_a = (\sqrt{Z})_a{}^b \phi_{b,R},$$

where $\phi_{b,R}$ are the renormalized superfields. Subscript a here numerates sets of chiral matter superfilds ϕ_a either belonging to a certain irreducible representation of the simple subgroup G_K or having certain charge q_{aK} with respect to $G_K = U(1)$. The renormalization of each coupling constant α_K and of the superfields ϕ_a is encoded in the corresponding β-function and anomalous dimension, respectively. For RGFs defined in terms of the bare couplings α_0 and λ_0 the derivatives are taken with respect to the dimensionful regularization parameter Λ at fixed values of the renormalized couplings.

$$\beta_K(\alpha_0, \lambda_0) \equiv \frac{d\alpha_{0K}}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}}; \qquad \gamma_a{}^b(\alpha_0, \lambda_0) \equiv -\frac{d\ln Z_a{}^b}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}},$$

For RGFs standardly defined in terms of the renormalized couplings α and λ the differentiations are made with respect to the renormalization point μ at fixed values of bare couplings.

$$\widetilde{\beta}_K(\alpha, \lambda) \equiv \frac{d\alpha_K}{d\ln\mu}\Big|_{\alpha_0,\lambda_0=\text{const}}; \qquad \widetilde{\gamma_a}^b(\alpha, \lambda) \equiv \frac{d\ln Z_a{}^b}{d\ln\mu}\Big|_{\alpha_0,\lambda_0=\text{const}}.$$

It should be noted that for $a \neq b$ $\gamma_a{}^b(\alpha_0, \lambda_0) = 0$ and $\gamma_a{}^b(\alpha, \lambda) = 0$, except when a and b correspond to different generations of the same superfield.

Higher derivative regularization for MSSM

We will use the background field method to quantize the theory and apply the higher derivatives regularization. To do this, we first need to replace $e^{2V} \to e^{2\mathcal{F}(V)} e^{2\boldsymbol{V}}$, where $\boldsymbol{V}$ and V are the background and quantum gauge superfields, respectively, and the function $\mathcal{F}(V)$ must contain nonlinear terms (cubic in the lowest approximation) with respect to the gauge superfield.

O. Piguet and K. Sibold, Nucl.Phys. B **197** (1982) 257; 272;
I.V.Tyutin, Yad.Fiz. **37** (1983) 761.

Next, the action should be modified by adding some terms with higher covariant derivatives

$$S \to S_{\text{reg}} = \sum_{K=1}^{n} \text{Re}\frac{1}{4}\int d^4x\, d^2\theta\, (W^a)^{A_K} \Big[\Big(e^{-2\boldsymbol{V}} e^{-2\mathcal{F}(V)} R\Big(-\frac{\bar{\nabla}^2\nabla^2}{16\Lambda^2}\Big)$$

$$\times e^{2\mathcal{F}(V)} e^{2\boldsymbol{V}}\Big)_{Adj} W_a\Big]^{A_K} + \frac{1}{4}\int d^4x\, d^4\theta\, \phi^{*i}\Big[F\Big(-\frac{\bar{\nabla}^2\nabla^2}{16\Lambda^2}\Big) e^{2\mathcal{F}(V)} e^{2\boldsymbol{V}}\Big]_i{}^j \phi_j$$

$$+\Big(\frac{1}{6}\int d^4x\, d^2\theta\, \lambda_0^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\Big),$$

where $\nabla_a = D_a$; $\bar{\nabla}_{\dot{a}} = e^{2\mathcal{F}(V)} e^{2\boldsymbol{V}} \bar{D}_{\dot{a}} e^{-2\boldsymbol{V}} e^{-2\mathcal{F}(V)}$ and the regulator functions $R(x)$ and $F(x)$ (the same for all subgroups G_K) rapidly grow at infinity and are equal to 1 at $x = 0$.

Higher derivative regularization for MSSM

The generating functional

$$Z[sources] = \int D\mu \prod_K \mathrm{Det}^{c_K}(PV, M_K)$$
$$\times \exp\left\{ i\Big(S_{\mathrm{reg}} + S_{\mathrm{gf}} + S_{\mathrm{FP}} + S_{\mathrm{NK}} + S_{\varphi} + S_{\mathrm{sources}} \Big) \right\}$$

includes the gauge fixing term S_{gf}, the action for the Faddeev-Popov ghosts S_{FP}, the action for the Nielsen-Kallosh ghosts S_{NK} and the Pauli-Villars determinants for removing residual one-loop divergences.

D. Korneev, D. Plotnikov, K. Stepanyantz and N. Tereshina, The NSVZ relations for N = 1 supersymmetric theories with multiple gauge couplings, JHEP **10** (2021) 046.

Here the action S_φ contains the Pauli-Villars superfields $\varphi_{1,K}$, $\varphi_{2,K}$ and $\varphi_{3,K}$ with the masses

$$M_{\varphi,K} = a_{\varphi,K}\Lambda.$$

These superfields lie in the adjoint representation of the subgroup G_K and in the trivial representations of the other subgroups and are introduced for canceling the one-loop divergences produced by gauge and ghost superfields.
The determinants $Det(PV,M_K)$ contain the Pauli-Villars superfields with the masses

$$M_K = a_K\Lambda$$

which remove the one-loop divergences produced by matter superfields.

The NSVZ equations for MSSM

RGFs defined in terms of the bare couplings depend on a regularization, but do not depend on a renormalization prescription.

A. L. Kataev and K. V. Stepanyantz, Nucl. Phys. B **875** (2013) 459.

In the case of using the higher covariant derivative regularization they satisfy the NSVZ equations which are generalized for the theory with multiple gauge couplings as follows:

$$\frac{\beta_K(\alpha_0,\lambda_0)}{\alpha_{0K}{}^2} = -\frac{1}{2\pi(1-C_2(G_K)\alpha_{0K}/2\pi)}\Big[3C_2(G_K) - \sum_a T_{aK}(1-\gamma_a{}^a(\alpha_0,\lambda_0))\Big],$$

where

$$C_2(G_K)\delta^{A_K B_K} = f^{A_K C_K D_K} f^{B_K C_K D_K}; \quad T_K(R_{aK})\delta^{A_K B_K} = (T_a^{A_K} T_a^{B_K})_{i_K}{}^{i_K};$$

$$T_{aK} = \begin{cases} \delta_{i_1}{}^{i_1} \cdot \ldots \cdot \delta_{i_{K-1}}{}^{i_{K-1}} T_K(R_{ak}) \delta_{i_{K+1}}{}^{i_{K+1}} \cdot \ldots \cdot \delta_{i_n}{}^{i_n}, & G_K \text{ is simple;} \\ \delta_{i_1}{}^{i_1} \cdot \ldots \cdot \delta_{i_{K-1}}{}^{i_{K-1}} q_{aK}^2 \delta_{i_{K+1}}{}^{i_{K+1}} \cdot \ldots \cdot \delta_{i_n}{}^{i_n}, & G_K = U(1). \end{cases}$$

Values of T_{aK} for all MSSM superfields are given in the table.

superfield	Q_1, Q_2, Q_3	U_1, U_2, U_3	D_1, D_2, D_3	L_1, L_2, L_3	E_1, E_2, E_3	H_u	H_d
$SU(3)$	1	1/2	1/2	0	0	0	0
$SU(2)$	3/2	0	0	1/2	0	1/2	1/2
$U(1)_Y$	1/6	4/3	1/3	1/2	1	1/2	1/2

The NSVZ equations for MSSM

The exact NSVZ expressions for the MSSM β-functions can be written in the form

$$\frac{\beta_3(\alpha_0,\lambda_0)}{{\alpha_{03}}^2} = -\frac{1}{2\pi(1-3\alpha_{03}/2\pi)}\Big[3+\mathrm{tr}\Big(\gamma_Q(\alpha_0,Y_0)+\frac{1}{2}\gamma_U(\alpha_0,Y_0)$$
$$+\frac{1}{2}\gamma_D(\alpha_0,Y_0)\Big)\Big];$$
$$\frac{\beta_2(\alpha_0,\lambda_0)}{{\alpha_{02}}^2} = -\frac{1}{2\pi(1-\alpha_{02}/\pi)}\Big[-1+\mathrm{tr}\Big(\frac{3}{2}\gamma_Q(\alpha_0,Y_0)+\frac{1}{2}\gamma_L(\alpha_0,Y_0)\Big)$$
$$+\frac{1}{2}\gamma_{H_u}(\alpha_0,Y_0)+\frac{1}{2}\gamma_{H_d}(\alpha_0,Y_0)\Big];$$
$$\frac{\beta_1(\alpha_0,\lambda_0)}{{\alpha_{01}}^2} = -\frac{3}{5}\frac{1}{2\pi}\Big[-11+\mathrm{tr}\Big(\frac{1}{6}\gamma_Q(\alpha_0,Y_0)+\frac{4}{3}\gamma_U(\alpha_0,Y_0)+\frac{1}{3}\gamma_D(\alpha_0,Y_0)$$
$$+\frac{1}{2}\gamma_L(\alpha_0,Y_0)+\gamma_E(\alpha_0,Y_0)\Big)+\frac{1}{2}\gamma_{H_u}(\alpha_0,Y_0)+\frac{1}{2}\gamma_{H_d}(\alpha_0,Y_0)\Big],$$

where traces imply summation over generation indices.

M. A. Shifman, Int. J. Mod. Phys. A **11** (1996), 5761; D. Korneev, D. Plotnikov, K. Stepanyantz and N. Tereshina, The NSVZ relations for $\mathcal{N}=1$ supersymmetric theories with multiple gauge couplings, JHEP **10** (2021) 046.

The two-loop anomalous dimension for MSSM

The expression for the two-loop anomalous dimension of the matter superfields defined in terms of the bare couplings for a theory with a single gauge coupling constant regularized by higher covariant derivatives can be written in the form

$$\gamma_i{}^j(\alpha_0,\lambda_0) = -\frac{d\ln Z_i{}^j}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = -\frac{\alpha_0}{\pi}C(R)_i{}^j + \frac{1}{4\pi^2}\lambda^*_{0imn}\lambda_0^{jmn}$$
$$+\frac{\alpha_0^2}{2\pi^2}\Big[[C(R)^2]_i{}^j - 3C_2C(R)_i{}^j(\ln a_\varphi + 1 + \frac{A}{2}) + T(R)C(R)_i{}^j(\ln a + 1 + \frac{A}{2})\Big]$$
$$-\frac{\alpha_0}{8\pi^3}\lambda^*_{0lmn}\lambda_0^{jmn}C(R)_i{}^l(1-B+A) + \frac{\alpha_0}{4\pi^3}\lambda^*_{0imn}\lambda_0^{jml}C(R)_l{}^n(1+B-A)$$
$$-\frac{1}{16\pi^4}\lambda^*_{0iac}\lambda_0^{jab}\lambda^*_{0bde}\lambda_0^{cde} + O(\alpha_0^3,\alpha_0^2\lambda_0^2,\alpha_0\lambda_0^4,\lambda_0^6),$$

where

$$C(R)_i{}^j \equiv (T^AT^A)_i{}^j; \quad tr(T^AT^B) = \delta^{AB}T(R); \quad T(Adj) \equiv C_2,$$
$$A \equiv \int_0^\infty dx\ln x\frac{d}{dx}\Big(\frac{1}{R(x)}\Big); \quad B \equiv \int_0^\infty dx\ln x\frac{d}{dx}\Big(\frac{1}{F^2(x)}\Big).$$

A. Kazantsev and K. Stepanyantz, Two-loop renormalization of the matter superfields and finiteness of $\mathcal{N}=1$ supersymmetric gauge theories regularized by higher derivatives, JHEP **06** (2020) 108.

The two-loop anomalous dimension for MSSM

The generalization of this expression for the case of a theory with several coupling constants can be presented in the form

$$\gamma_a{}^b(\alpha_0,\lambda_0) = -\frac{d\ln Z_a{}^b}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = -\sum_K \frac{\alpha_{0K}}{\pi} C(R_{aK})\delta_a{}^b + \frac{1}{4\pi^2}(\lambda_0^*\lambda_0)_a{}^b$$

$$+\sum_{KL}\frac{\alpha_{0K}\alpha_{0L}}{2\pi^2}C(R_{aK})C(R_{aL})\delta_a{}^b - \sum_K \frac{3\alpha_{0K}^2}{2\pi^2}C_2(G_K)C(R_{aK})(\ln a_{\varphi,K}+1+\frac{A}{2})\delta_a{}^b$$

$$+\sum_K \frac{\alpha_{0K}^2}{2\pi^2}C(R_{aK})\sum_c \mathbf{T}_{cK}(\ln a_K+1+\frac{A}{2})\delta_a{}^b - \sum_K \frac{\alpha_{0K}}{8\pi^3}(\lambda_0^*\lambda_0)_a{}^b C(R_{aK})$$

$$\times(1-B+A)+\sum_K\frac{\alpha_{0K}}{4\pi^3}(\lambda_0^* C_K\lambda_0)_a{}^b(1+B-A) - \frac{1}{16\pi^4}(\lambda_0^*[\lambda_0^*\lambda_0]\lambda_0)_a{}^b$$

$$+O(\alpha_0^3,\alpha_0^2\lambda_0^2,\alpha_0\lambda_0^4,\lambda_0^6),$$

where we use the notations

$$(T_a^{A_K}T_a^{A_K})_{i_K}{}^{j_K} = C(R_{aK})\delta_{i_K}{}^{j_K}; \quad (\lambda_0^*\lambda_0)_a{}^b\delta_{i_a}{}^{j_b} = \sum_{cd}\lambda^*_{0i_a m_c n_d}\lambda_0^{j_b m_c n_d};$$

$$(\lambda_0^* C_K\lambda_0)_a{}^b\delta_{i_a}{}^{j_b} = \sum_{cd}\lambda^*_{0i_a m_c n_d}C(R_{dK})\lambda_0^{j_b m_c n_d};$$

$$(\lambda_0^*[\lambda_0^*\lambda_0]\lambda_0)_a{}^b\delta_{i_a}{}^{j_b} = \sum_{cdefg}\lambda^*_{0i_a k_e l_f}\lambda_0^{j_b k_e p_g}\lambda^*_{0p_g m_c n_d}\lambda_0^{l_f m_c n_d}.$$

The two-loop anomalous dimension for MSSM

The two-loop anomalous dimensions in the case of using the higher covariant derivative regularization are too large. As an example we present here the one for the left quarks. In terms of the bare couplings (without the one-loop contribution) it is given by the expression

$$
\begin{aligned}
\gamma_Q(\alpha_0, Y_0)^T =& \frac{1}{2\pi^2}\left[\frac{1}{3600}\alpha_{01}^2 + \frac{9}{16}\alpha_{02}^2 + \frac{16}{9}\alpha_{03}^2 + \frac{1}{40}\alpha_{01}\alpha_{02} + \frac{2}{45}\alpha_{01}\alpha_{03}\right.\\
&+ 2\alpha_{02}\alpha_{03} - \frac{9}{2}\alpha_{02}^2(\ln a_{\varphi,SU(2)} + 1 + \frac{A}{2}) - 12\alpha_{03}^2(\ln a_{\varphi,SU(3)} + 1 + \frac{A}{2})\\
&+ \frac{11}{100}\alpha_{01}^2(\ln a_{U(1)} + 1 + \frac{A}{2}) + \frac{21}{4}\alpha_{02}^2(\ln a_{SU(2)} + 1 + \frac{A}{2})\\
&\left.+ 8\alpha_{03}^2(\ln a_{SU(3)} + 1 + \frac{A}{2})\right] + \frac{1}{8\pi^2}Y_{0U}Y_{0U}^\dagger\left[\frac{\alpha_{01}}{\pi}\left(\frac{1}{5} + \frac{13}{60}(B-A)\right)\right.\\
&\left.+ \frac{3}{4}\frac{\alpha_{02}}{\pi}(B-A) + \frac{4}{3}\frac{\alpha_{03}}{\pi}(B-A)\right]\\
&+\frac{1}{8\pi^2}Y_{0D}Y_{0D}^\dagger\left[\frac{\alpha_{01}}{\pi}\left(\frac{1}{10} + \frac{7}{60}(B-A)\right) + \frac{3}{4}\frac{\alpha_{02}}{\pi}(B-A) + \frac{4}{3}\frac{\alpha_{03}}{\pi}(B-A)\right]\\
&-\frac{1}{(8\pi^2)^2}\left[(Y_{0U}Y_{0U}^\dagger)^2 + (Y_{0D}Y_{0D}^\dagger)^2 + \frac{3}{2}tr(Y_{0U}Y_{0U}^\dagger)Y_{0U}Y_{0U}^\dagger\right.\\
&\left.+ \frac{3}{2}tr(Y_{0D}Y_{0D}^\dagger)Y_{0D}Y_{0D}^\dagger + \frac{1}{2}tr(Y_{0E}Y_{0E}^\dagger)Y_{0D}Y_{0D}^\dagger\right] + O(\alpha_0^3, \alpha_0^2 Y_0^2, \alpha_0 Y_0^4, Y_0^6).
\end{aligned}
$$

Other two-loop anomalous dimensions (defined both in terms of the bare and renormalized couplings) can be found in

JHEP **09** (2022) 189.

The two-loop anomalous dimension for MSSM

For the anomalous dimension standardly defined in terms of the renormalized couplings, the corresponding result can be written as follows:

$$
\begin{aligned}
\tilde{\gamma}_Q(\alpha,Y)^T = &-\frac{\alpha_1}{60\pi}-\frac{3\alpha_2}{4\pi}-\frac{4\alpha_3}{3\pi}+\frac{1}{8\pi^2}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right)+\frac{1}{2\pi^2}\left[\frac{1}{3600}\alpha_1^2+\frac{9}{16}\alpha_2^2\right.\\
&+\frac{16}{9}\alpha_3^2+\frac{1}{40}\alpha_1\alpha_2+\frac{2}{45}\alpha_1\alpha_3+2\alpha_2\alpha_3+\frac{11}{100}\alpha_1^2\left(\ln a_{U(1)}+1+\frac{A}{2}+g_{Q1}-b_{1,1}\right)\\
&+\frac{3}{4}\alpha_2^2\left(-6\ln a_{\varphi,SU(2)}+7\ln a_{SU(2)}+1+\frac{A}{2}+g_{Q2}-b_{1,2}\right)\\
&\left.-4\alpha_3^2\left(3\ln a_{\varphi,SU(3)}-2\ln a_{SU(3)}+1+\frac{A}{2}+g_{Q3}-b_{1,3}\right)\right]\\
&+\frac{1}{8\pi^2}Y_U Y_U^\dagger\left[\frac{\alpha_1}{\pi}\left(\frac{1}{5}+\frac{13}{60}(B-A+2g_{QU}-2j_{U1})\right)+\right.\\
&\left.+\frac{3}{4}\frac{\alpha_2}{\pi}(B-A+2g_{QU}-2j_{U2})+\frac{4}{3}\frac{\alpha_{03}}{\pi}(B-A+2g_{QU}-2j_{U3})\right]\\
&+\frac{1}{8\pi^2}Y_D Y_D^\dagger\left[\frac{\alpha_1}{\pi}\left(\frac{1}{10}+\frac{7}{60}(B-A+2g_{QD}-2j_{D1})\right)+\frac{3}{4}\frac{\alpha_2}{\pi}(B-A+2g_{QD}-2j_{D2})\right.\\
&\left.+\frac{4}{3}\frac{\alpha_3}{\pi}(B-A+2g_{QD}-2j_{D3})\right]-\frac{1}{(8\pi^2)^2}\left[(Y_U Y_U^\dagger)^2(1+3g_{QU}-3j_{UU})+(Y_D Y_D^\dagger)^2\right.\\
&\times(1+3g_{QD}-3j_{DD})+\frac{3}{2}tr(Y_U Y_U^\dagger)Y_U Y_U^\dagger(1+2g_{QU}-2j_{UtU})+\frac{3}{2}tr(Y_D Y_D^\dagger)Y_D Y_D^\dagger\\
&\times(1+2g_{QD}-2j_{DtD})+\frac{1}{2}tr(Y_E Y_E^\dagger)Y_D Y_D^\dagger(1+2g_{QD}-2j_{DtE})\\
&\left.+\frac{1}{2}\left(Y_U Y_U^\dagger Y_D Y_D^\dagger+Y_D Y_D^\dagger Y_U Y_U^\dagger\right)(g_{QU}+g_{QD}-j_{UD}-j_{DU})\right]+O(\alpha^3,\alpha^2Y^2,\alpha Y^4,Y^6).
\end{aligned}
$$

The two-loop anomalous dimension for MSSM

A renormalization prescription in the lowest approximations is specified by some finite constants entering into the relations between the bare and renormalized gauge couplings, into the expressions for the renormalization constants and into the renormalization of the Yukawa couplings as shown in these examples:

$$\frac{1}{\alpha_{03}} = \frac{1}{\alpha_3} + \frac{1}{2\pi}\left[3\left(\ln\frac{\Lambda}{\mu} + b_{1,3}\right) - \frac{11\alpha_1}{20\pi}\left(\ln\frac{\Lambda}{\mu} + b_{2,31}\right) - \frac{9\alpha_2}{4\pi}\left(\ln\frac{\Lambda}{\mu} + b_{2,32}\right)\right.$$
$$\left. - \frac{7\alpha_3}{2\pi}\left(\ln\frac{\Lambda}{\mu} + b_{2,33}\right) + \frac{1}{4\pi^2}tr(Y_U Y_U^\dagger)\left(\ln\frac{\Lambda}{\mu} + b_{2,3U}\right) + \frac{1}{4\pi^2}tr(Y_D Y_D^\dagger)\left(\ln\frac{\Lambda}{\mu} + b_{2,3D}\right)\right]$$
$$+ O(\alpha^2, \alpha Y^2, Y^4);$$

$$(Z_Q)^T = 1 + \frac{\alpha_1}{60\pi}\left(\ln\frac{\Lambda}{\mu} + g_{Q1}\right) + \frac{3\alpha_2}{4\pi}\left(\ln\frac{\Lambda}{\mu} + g_{Q2}\right) + \frac{4\alpha_3}{3\pi}\left(\ln\frac{\Lambda}{\mu} + g_{Q3}\right)$$
$$- \frac{1}{8\pi^2}tr(Y_U Y_U^\dagger)\left(\ln\frac{\Lambda}{\mu} + g_{QU}\right) - \frac{1}{8\pi^2}tr(Y_D Y_D^\dagger)\left(\ln\frac{\Lambda}{\mu} + g_{QD}\right) + O(\alpha^2, \alpha Y^2, Y^4);$$

$$Y_{0U} = \left[1 - \frac{13\alpha_1}{60\pi}\left(\ln\frac{\Lambda}{\mu} + j_{U1}\right) - \frac{3\alpha_2}{4\pi}\left(\ln\frac{\Lambda}{\mu} + j_{U2}\right) - \frac{4\alpha_3}{3\pi}\left(\ln\frac{\Lambda}{\mu} + j_{U3}\right) + \frac{3}{16\pi^2}tr(Y_U Y_U^\dagger)\right.$$
$$\left. \times\left(\ln\frac{\Lambda}{\mu} + j_{UtU}\right) + \frac{1}{16\pi^2}(Y_D Y_D^\dagger)\left(\ln\frac{\Lambda}{\mu} + j_{UD}\right) + \frac{3}{16\pi^2}(Y_U Y_U^\dagger)\left(\ln\frac{\Lambda}{\mu} + j_{UU}\right)\right] Y_U$$
$$+ O(\alpha^2, \alpha Y^2, Y^4).$$

The three-loop MSSM β-functions for MSSM

The result for the three-loop MSSM β-functions defined in terms of the bare couplings was obtained by using the NSVZ equations for MSSM and is presented here only for α_{03}. Other three-loop β-functions (defined both in terms of the bare and renormalized couplings) can be found in

JHEP **09** (2022) 189.

$$
\begin{aligned}
\frac{\beta_3(\alpha_0,Y_0)}{\alpha_{03}^2} &= -\frac{1}{2\pi}\left[3-\frac{11}{20}\frac{\alpha_{01}}{\pi}-\frac{9}{4}\frac{\alpha_{02}}{\pi}-\frac{7}{2}\frac{\alpha_{03}}{\pi}+\frac{1}{4\pi^2}tr(Y_{0U}Y_{0U}^\dagger+Y_{0D}Y_{0D}^\dagger)+\right.\\
&+\frac{1}{2\pi^2}\left[\frac{137}{1200}\alpha_{01}^2+\frac{27}{16}\alpha_{02}^2+\frac{1}{6}\alpha_{03}^2+\frac{3}{40}\alpha_{01}\alpha_{02}-\frac{11}{60}\alpha_{01}\alpha_{03}-\frac{3}{4}\alpha_{02}\alpha_{03}-\right.\\
&-\frac{27}{2}\alpha_{02}^2(\ln a_{\varphi,SU(2)}+1+\frac{A}{2})-72\alpha_{03}^2(\ln a_{\varphi,SU(3)}+1+\frac{A}{2})+\\
&+\frac{363}{100}\alpha_{01}^2(\ln a_{U(1)}+1+\frac{A}{2})+\frac{63}{4}\alpha_{02}^2(\ln a_{SU(2)}+1+\frac{A}{2})+\\
&+48\alpha_{03}^2(\ln a_{SU(3)}+1+\frac{A}{2})\Big]+\frac{1}{8\pi^3}\Big(tr(Y_{0U}Y_{0U}^\dagger)\Big[\alpha_{01}\left(\frac{3}{20}+\frac{13}{30}(B-A)\right)+\\
&+\alpha_{02}\left(\frac{3}{4}+\frac{3}{2}(B-A)\right)+\alpha_{03}\left(3+\frac{8}{3}(B-A)\right)\Big]+\\
&+tr(Y_{0D}Y_{0D}^\dagger)\Big[\alpha_{01}\left(\frac{3}{20}+\frac{7}{30}(B-A)\right)+\alpha_{02}\left(\frac{3}{4}+\frac{3}{2}(B-A)\right)+\\
&+\alpha_{03}\left(3+\frac{8}{3}(B-A)\right)\Big]\Big)-\frac{1}{(8\pi^2)^2}\Big(\frac{3}{2}tr\left((Y_{0U}Y_{0U}^\dagger)^2\right)+\frac{3}{2}tr\left((Y_{0D}Y_{0D}^\dagger)^2\right)+\\
&+3tr^2(Y_{0U}Y_{0U}^\dagger)+3tr^2(Y_{0D}Y_{0D}^\dagger)+tr(Y_{0E}Y_{0E}^\dagger)tr(Y_{0D}Y_{0D}^\dagger)+tr(Y_{0D}Y_{0D}^\dagger Y_{0U}Y_{0U}^\dagger)\Big)\Big]\\
&+O(\alpha_0^3,\alpha_0^2Y_0^2,\alpha_0Y_0^4,Y_0^6).
\end{aligned}
$$

The three-loop MSSM β-functions for MSSM

Next, we substitute this expression into the corresponding renormalization group equation and find the relation between the bare couplings and the renormalized ones, which allows us to obtain the three-loop MSSM β-functions defined in terms of the renormalized couplings.

$$
\begin{aligned}
\frac{\tilde{\beta}_3(\alpha,Y)}{\alpha_3^2} &= -\frac{1}{2\pi}\Bigg[3-\frac{11}{20}\frac{\alpha_1}{\pi}-\frac{9}{4}\frac{\alpha_2}{\pi}-\frac{7}{2}\frac{\alpha_3}{\pi}+\frac{1}{4\pi^2}tr(Y_UY_U^\dagger+Y_DY_D^\dagger)\\
&+\frac{1}{2\pi^2}\Bigg[\frac{137}{1200}\alpha_1^2+\frac{27}{16}\alpha_2^2+\frac{1}{6}\alpha_3^2+\frac{3}{40}\alpha_1\alpha_2-\frac{11}{60}\alpha_1\alpha_3-\frac{3}{4}\alpha_2\alpha_3+\frac{363}{100}\alpha_1^2\\
&\times\left(\ln a_{U(1)}+1+\frac{A}{2}+b_{2,31}-b_{1,1}\right)+\frac{9}{4}\alpha_2^2\left(7\ln a_{SU(2)}-6\ln a_{\varphi,SU(2)}+1+\frac{A}{2}+b_{2,32}-b_{1,2}\right)\\
&+24\alpha_3^2\left(2\ln a_{SU(3)}-3\ln a_{\varphi,SU(3)}+1+\frac{A}{2}+\frac{7}{16}b_{2,33}-\frac{7}{16}b_{1,3}\right)\Bigg]+\frac{1}{8\pi^3}\Bigg(tr(Y_UY_U^\dagger)\\
&\times\Bigg[\alpha_1\left(\frac{3}{20}+\frac{13}{30}(B-A+2b_{2,3U}-2j_{U1})\right)+\alpha_2\left(\frac{3}{4}+\frac{3}{2}(B-A+2b_{2,3U}-2j_{U2})\right)+\alpha_3\\
&\times\left(3+\frac{8}{3}(B-A+2b_{2,3U}-2j_{U3})\right)\Bigg]+tr(Y_DY_D^\dagger)\Bigg[\alpha_1\left(\frac{3}{20}+\frac{7}{30}(B-A+2b_{2,3D}-2j_{D1})\right)\\
&+\alpha_2\left(\frac{3}{4}+\frac{3}{2}(B-A+2b_{2,3D}-2j_{D2})\right)+\alpha_3\left(3+\frac{8}{3}(B-A+2b_{2,3D}-2j_{D3})\right)\Bigg]\Bigg)\\
&-\frac{1}{(8\pi^2)^2}\left(\frac{3}{2}tr\left((Y_UY_U^\dagger)^2\right)(1+4b_{2,3U}-4j_{UU})+\frac{3}{2}tr\left((Y_DY_D^\dagger)^2\right)(1+4b_{2,3D}-4j_{DD})\right.\\
&+3tr^2(Y_UY_U^\dagger)(1+2b_{2,3U}-2j_{UtU})+3tr^2(Y_DY_D^\dagger)(1+2b_{2,3D}-2j_{DtD})\\
&+tr(Y_EY_E^\dagger)tr(Y_DY_D^\dagger)(1+2b_{2,3D}-2j_{DtE})+tr(Y_DY_D^\dagger Y_UY_U^\dagger)\\
&\times(1+2b_{2,3U}+2b_{2,3D}-2j_{UD}-2j_{UD}))\Bigg]+O(\alpha^3,\alpha^2Y^2,\alpha Y^4,Y^6).
\end{aligned}
$$

RGFs for MSSM in the $\overline{\text{DR}}$-scheme

Under certain renormalization prescription both three-loop β-functions and two-loop anomalous dimensions exactly reproduce the $\overline{\text{DR}}$ expressions obtained earlier, for example,

$$\tilde{\gamma}_Q(\alpha,Y)^T = -\frac{\alpha_1}{60\pi} - \frac{3\alpha_2}{4\pi} - \frac{4\alpha_3}{3\pi} + \frac{1}{8\pi^2}\left(Y_U Y_U^\dagger + Y_D Y_D^\dagger\right) + \frac{1}{2\pi^2}\left[\frac{199}{3600}\alpha_1^2 + \frac{15}{16}\alpha_2^2 - \frac{2}{9}\alpha_3^2 + \frac{1}{40}\alpha_1\alpha_2 + \frac{2}{45}\alpha_1\alpha_3 + 2\alpha_2\alpha_3\right] + \frac{1}{8\pi^3}Y_U Y_U^\dagger \cdot \frac{\alpha_1}{5} + \frac{1}{8\pi^3}Y_D Y_D^\dagger \cdot \frac{\alpha_1}{10} - \frac{1}{(8\pi^2)^2}\left[(Y_U Y_U^\dagger)^2 + (Y_D Y_D^\dagger)^2 + \frac{3}{2}tr(Y_U Y_U^\dagger)Y_U Y_U^\dagger + \frac{3}{2}tr(Y_D Y_D^\dagger)Y_D Y_D^\dagger + \frac{1}{2}tr(Y_E Y_E^\dagger)Y_D Y_D^\dagger\right] + O(\alpha^3,\alpha^2 Y^2,\alpha Y^4,Y^6)$$

$$\frac{\tilde{\beta}_3(\alpha,Y)}{\alpha_3^2} = -\frac{1}{2\pi}\left[3 - \frac{11}{20}\frac{\alpha_1}{\pi} - \frac{9}{4}\frac{\alpha_2}{\pi} - \frac{7}{2}\frac{\alpha_3}{\pi} + \frac{1}{4\pi^2}tr(Y_U Y_U^\dagger + Y_D Y_D^\dagger)\right. + \frac{1}{2\pi^2}\left[\frac{851}{300}\alpha_1^2 + \frac{27}{8}\alpha_2^2 - \frac{347}{24}\alpha_3^2 + \frac{3}{40}\alpha_1\alpha_2 - \frac{11}{60}\alpha_1\alpha_3 - \frac{3}{4}\alpha_2\alpha_3\right] + \frac{1}{8\pi^3}\left(tr(Y_U Y_U^\dagger)\right.$$
$$\times\left[\frac{11}{30}\alpha_1 + \frac{3}{2}\alpha_2 + \frac{13}{3}\alpha_3\right] + tr(Y_D Y_D^\dagger)\left[\frac{4}{15}\alpha_1 + \frac{3}{2}\alpha_2 + \frac{13}{3}\alpha_3\right]\Big) - \frac{1}{(8\pi^2)^2}\left(3tr\left((Y_U Y_U^\dagger)^2\right)\right.$$
$$\left.\left. + 3tr\left((Y_D Y_D^\dagger)^2\right) + \frac{9}{2}tr^2(Y_U Y_U^\dagger) + \frac{9}{2}tr^2(Y_D Y_D^\dagger) + \frac{3}{2}tr(Y_E Y_E^\dagger)tr(Y_D Y_D^\dagger) + 2tr(Y_D Y_D^\dagger Y_U Y_U^\dagger)\right)\right]$$
$$+ O(\alpha^3,\alpha^2 Y^2,\alpha Y^4,Y^6).$$

A class of the NSVZ schemes for MSSM

The NSVZ equations is scheme-dependent in the considered approximation, but it is possible to find restrictions to the finite constants contained in the RGFs defined in terms of the renormalized couplings under which the NSVZ equations remain valid:

$$b_{2,11} = \frac{1}{398}(g_{Q1} + 128g_{U1} + 8g_{D1} + 27g_{L1} + 216g_{E1} + 9g_{H_u1} + 9g_{H_d1});$$

$$b_{2,12} = \frac{1}{6}(g_{Q2} + 3g_{L2} + g_{H_u2} + g_{H_d2}); \quad b_{2,13} = \frac{1}{11}(g_{Q3} + 8g_{U3} + 2g_{D3});$$

$$b_{2,21} = \frac{1}{6}(g_{Q1} + 3g_{L1} + g_{H_u1} + g_{H_d1}); \quad b_{2,22} = \frac{1}{50}(27g_{Q2} + 9g_{L2} + 3g_{H_u2} + 3g_{H_d2} + 8b_{1,2});$$

$$b_{2,23} = g_{Q3}; \quad b_{2,31} = \frac{1}{11}(g_{Q1} + 8g_{U1} + 2g_{D1}); \quad b_{2,32} = g_{Q2};$$

$$b_{2,33} = \frac{1}{7}(8g_{Q3} + 4g_{U3} + 4g_{D3} - 9b_{1,3}); \quad b_{2,1U} = \frac{1}{26}(g_{QU} + 16g_{UU} + 9g_{H_uU});$$

$$b_{2,1D} = \frac{1}{14}(g_{QD} + 4g_{DD} + 9g_{H_dD}); \quad b_{2,1E} = \frac{1}{6}(g_{LE} + 4g_{EE} + g_{H_dE});$$

$$b_{2,2U} = \frac{1}{2}(g_{QU} + g_{H_uU}); \quad b_{2,2D} = \frac{1}{2}(g_{QD} + g_{H_dD}); \quad b_{2,2E} = \frac{1}{2}(g_{LE} + g_{H_dE});$$

$$b_{2,3U} = \frac{1}{2}(g_{QU} + g_{UU}); \quad b_{2,3D} = \frac{1}{2}(g_{QD} + g_{DD}).$$

A class of the NSVZ schemes for MSSM

The HD+MSL scheme, in which all finite constants are equal to zero, is obviously NSVZ (in this scheme RGFs defined in terms of the bare and renormalized couplings differ from each other only by a formal renaming). However, there are others schemes in which the NSVZ equations are valid. Different NSVZ schemes are related to each other by the finite renormalization

$$\alpha'_K = \alpha'_K(\alpha,\lambda); \quad \lambda'_K = \lambda'_K(\alpha,\lambda); \quad Z'_a(\alpha',\lambda',\ln\frac{\Lambda}{\mu}) = z_a(\alpha,\lambda)Z_a(\alpha,\lambda,\ln\frac{\Lambda}{\mu})$$

which satisfies the equation

$$\frac{1}{\alpha'_K} - \frac{1}{\alpha_K} + \frac{C_2(G_K)}{2\pi}\ln\frac{\alpha'_K}{\alpha_K} - \frac{1}{2\pi}\sum_a T_{aK}\ln z_a = B_K,$$

where B_K are some constants.

I. O. Goriachuk, A. L. Kataev and K. V. Stepanyantz, Phys. Lett. B **785** (2018) 561; D. Korneev, D. Plotnikov, K. Stepanyantz and N. Tereshina, The NSVZ relations for $\mathcal{N}=1$ supersymmetric theories with multiple gauge couplings, JHEP **10** (2021) 046.

A class of the NSVZ schemes for MSSM

The equations that define a finite renormalization alowing to change one NSVZ scheme to another for MSSM are as follows:

$$\frac{1}{\alpha_3'} - \frac{1}{\alpha_3} + \frac{3}{2\pi}\ln\frac{\alpha_3'}{\alpha_3} - \frac{1}{2\pi}tr\Big(\ln(z_Q)^T + \frac{1}{2}\ln z_U + \frac{1}{2}\ln z_D\Big) = B_3;$$

$$\frac{1}{\alpha_2'} - \frac{1}{\alpha_2} + \frac{1}{\pi}\ln\frac{\alpha_2'}{\alpha_2} - \frac{1}{2\pi}tr\Big(\frac{3}{2}\ln(z_Q)^T + \frac{1}{2}\ln(z_L)^T\Big)$$

$$-\frac{1}{2\pi}\Big(\frac{1}{2}\ln(z_{H_u})^T + \frac{1}{2}\ln z_{H_d}\Big) = B_2;$$

$$\frac{1}{\alpha_1'} - \frac{1}{\alpha_1} - \frac{1}{2\pi}\cdot\frac{3}{5}\Big[tr\Big(\frac{1}{6}\ln(z_Q)^T + \frac{4}{3}\ln z_U + \frac{1}{3}\ln z_D + \frac{1}{2}\ln(z_L)^T + \ln z_E\Big)$$

$$+\frac{1}{2}\ln(z_{H_u})^T + \frac{1}{2}\ln z_{H_d}\Big] = B_1.$$

Solving these equations for the case when α_K correspond to the HD+MSL scheme and α_K' correspond to the arbitrary renormalization prescription used earlier, we find the relations presented at slide 22 together with the equations

$$B_1 = \frac{33}{10\pi}b_{1,1}; \qquad B_2 = \frac{1}{2\pi}b_{1,2}; \qquad B_3 = -\frac{3}{2\pi}b_{1,3}.$$

Conclusion

Using the Slavnov higher derivative regularization, we found the two-loop anomalous dimensions for all MSSM chiral matter superfields and the three-loop MSSM gauge β-functions defined both in terms of the bare and renormalized couplings. We used an arbitrary supersymmetric renormalization prescription, that is a prescription to renormalize all superfields as a whole.

Firstly we obtained the two-loop anomalous dimensions defined in terms of the bare couplings and calculated the three-loop MSSM β-functions with the help of the NSVZ equations which are valid in all orders with this regularization. Next we obtained RGFs standardly defined in terms of the renormalized couplings and constracted a class of the renormalization prescriptions under which renormalized RGFs continue to satisfy the NSVZ relations.

As a test of the calculation correctness, we checked that for a certain choice of a subtraction scheme the results (for both the two-loop anomalous dimensions and the three-loop β-functions) coincide with the ones obtained earlier in the $\overline{\text{DR}}$-scheme.

Thank you for the attention!