Beta functions of (3+1)-dimensional projectable Hořava gravity

Alexander Kurov

Lebedev Physics Institute

MQFT-2022

In collaboration with A. O. Barvinsky and S. M. Sibiryakov

Motivation for Hořava gravity

Einstein GR

$$S_{EH} = \frac{M_P^2}{2} \int dt\, d^d x \sqrt{-g} R \quad \Rightarrow \quad \frac{M_P^2}{2} \int dt\, d^d x \, \left(h_{ij} \Box h^{ij} + \dots \right) \tag{1}$$

Higher derivative gravity (Stelle 1977)

$$\int \left(R + R^2 + R_{\mu\nu} R^{\mu\nu} \right) \quad \Rightarrow \quad \int \left(h_{ij} \Box h^{ij} + h_{ij} \Box^2 h^{ij} + \dots \right) \tag{2}$$

The theory is renormalizable and asymptotically free. However the theory is not unitary due to presence of ghosts.

Hořava gravity (2009)

The key is the anisotropic scaling of time and space coordinates,

$$t \mapsto b^{-z} t \,, \quad x^i \mapsto b^{-1} x^i \,, \qquad i = 1, \ldots, d \tag{3}$$

The theory contains only second time derivatives

$$\int \underbrace{dt\, d^d x}_{\propto b^{-(z+d)}} \left(\dot{h}_{ij} \dot{h}_{ij} - h_{ij} (-\Delta)^z h_{ij} + \ldots \right) \tag{4}$$

And field scales as

$$h_{ij} \mapsto b^{(d-z)/2} h_{ij} \tag{5}$$

Critical theory

$$z = d \tag{6}$$

Foliation preserving diffeomorphisms

$$t \mapsto t'(t) \,, \qquad x^i \mapsto x'^i(t, \mathbf{x}) \tag{7}$$

Metric decomposition

The metric in the action of HG is expanded into the lapse N, the shift N^i and the spatial metric γ_{ij} like in the Arnowitt–Deser–Misner (ADM) decomposition,

$$\mathrm{d}s^2 = N^2 \mathrm{d}t^2 - \gamma_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t)\,. \tag{8}$$

Fields are assigned the following dimensions under the anisotropic scaling:

$$[N] = [\gamma_{ij}] = 0\ , \qquad [N^i] = d - 1\ . \tag{9}$$

The Lagrangian is then built out of all local FDiff-invariant operators that can be constructed from these fields and have dimension less or equal $2d$.

Projectable version

A.Barvinsky, D.Blas, M.Herrero-Valea, S.Sibiryakov, C.Steinwachs (2016)

We consider *projectable* version of Hořava gravity. The lapse N is restricted to be a function of time only, $N = N(t)$

$$S = \frac{1}{2G} \int \mathrm{d}t\, \mathrm{d}^d x \sqrt{\gamma} \big(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V} \big) \ , \tag{10}$$

where

$$K_{ij} = \frac{1}{2} \left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \ . \tag{11}$$

The potential part $\mathcal{V}$ in $d = 3$ reads,

$$\begin{aligned} \mathcal{V} =& 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij} R^{ij} \\ &+ \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} \ , \end{aligned} \tag{12}$$

This expression includes all relevant and marginal terms. It contains 9 couplings $\Lambda, \eta, \mu_1, \mu_2$ and $\nu_a, a = 1, \ldots, 5$.

Dispersion relations

The spectrum of perturbations contains a transverse-traceless graviton and a scalar mode. Both modes have positive kinetic terms when G is positive and

$$\lambda < 1/3 \quad \text{or} \quad \lambda > 1 \,. \tag{13}$$

Their dispersion relations around a flat background are

$$\omega_{tt}^2 = \eta k^2 + \mu_2 k^4 + \nu_5 k^6 \,, \tag{14a}$$

$$\omega_s^2 = \frac{1-\lambda}{1-3\lambda}\big(- \eta k^2 + (8\mu_1 + 3\mu_2)k^4\big) + \nu_s k^6 \,, \tag{14b}$$

where k is the spatial momentum and we have defined

$$\nu_s \equiv \frac{(1-\lambda)(8\nu_4 + 3\nu_5)}{1-3\lambda} \,. \tag{15}$$

These dispersion relations are problematic at low energies where they are dominated by the k^2-terms.

The choice of the background

A.Barvinsky, A.K., S.Sibiryakov (2022)

We focus on the part of the action consisting of the marginal operators with respect to the anisotropic scaling

$$S = \frac{1}{2G}\int d\tau\, d^3x\, \sqrt{\gamma}\,(K_{ij}K^{ij} - \lambda K^2 + \nu_1 R^3 + \nu_2 R R_{ij}R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}). \tag{16}$$

And choose static background metric and zero background shift

$$\gamma_{ij}(\tau, \mathbf{x}) = g_{ij}(\mathbf{x}) + h_{ij}(\tau, \mathbf{x}), \quad N^i(\tau, \mathbf{x}) = 0 + n^i(\tau, \mathbf{x})\,, \tag{17}$$

Background covariant gauge-fixing

The gauge-fixing action is chosen as

$$S_{\mathrm{gf}} = \frac{\sigma}{2G} \int d\tau \, d^3x \, \sqrt{g} \, F^i \mathcal{O}_{ij} F^j, \tag{18}$$

which is the quadratic form in the gauge-condition functions F^i with the kernel $\mathcal{O}_{ij}$

$$F^i = \dot{n}^i + \frac{1}{2\sigma} \mathcal{O}^{-1}_{ij} \big(\nabla_k h_{jk} - \lambda \nabla_j h \big), \tag{19}$$

$$\mathcal{O}_{ij} = \big(g^{ij} \Delta^2 + \xi \nabla^i \Delta \nabla^j \big)^{-1}. \tag{20}$$

The gauge-fixing matrix $\mathcal{O}_{ij}$ is the nonlocal Green's function of the covariant fourth-order differential operator.

Shift part of the action

From the sum of the kinetic action and the gauge-breaking terms we obtain the quadratic in n^i part of the gauge-fixed action,

$$\begin{aligned} S_n &= \frac{1}{2G}\int d\tau d^3x\sqrt{g}\, n^i \left[-\sigma\mathcal{O}_{ij}\partial_\tau^2 + \lambda\nabla_i\nabla_j - \frac{1}{2}\nabla_j\nabla_i - \frac{1}{2}g_{ij}\Delta\right] n^j \\ &= \frac{\sigma}{2G}\int d\tau d^3x\sqrt{g}\, n^i\mathcal{O}_{ij}\left[-\delta^j_k\partial_\tau^2 + \mathbb{B}^j{}_k(\nabla)\right] n^k, \end{aligned} \tag{21}$$

where the differential operator $\mathbb{B}^i{}_j(\nabla)$ in spatial derivatives reads

$$\begin{aligned} \mathbb{B}^i{}_j(\nabla) = -\frac{1}{2\sigma}\delta^i_j\Delta^3 - \frac{1}{2\sigma}\Delta^2\nabla_j\nabla^i - \frac{\xi}{2\sigma}\nabla^i\Delta\nabla^k\nabla_j\nabla_k - \frac{\xi}{2\sigma}\nabla^i\Delta\nabla_j\Delta \\ + \frac{\lambda}{\sigma}\Delta^2\nabla^i\nabla_j + \frac{\lambda\xi}{\sigma}\nabla^i\Delta^2\nabla_j. \end{aligned} \tag{22}$$

Ghost part of the action

The action of the ghost fields c^i and $\bar{c}_j$ reads

$$S_{\rm gh} = -\frac{1}{G}\int d\tau d^3x\sqrt{g}\,\bar{c}_i(\mathbf{s}F^i), \tag{23}$$

where $\mathbf{s}F^i$ is the BRST transform of the gauge conditions.

$$\mathbf{s}h_{ij} = \nabla_i c_j + \nabla_j c_i + h_{ik}\nabla_j c^k + h_{jk}\nabla_i c^k + c^k\nabla_k h_{ij}, \quad c_i = g_{ij}c^j, \tag{24a}$$

$$\mathbf{s}n^i = \dot{c}^i - n^j\nabla_j c^i + c^j\nabla_j n^i. \tag{24b}$$

After the substitution of (24) into (23), the ghost action in the quadratic order of all quantum fields takes the following form

$$S_{\rm gh} = \frac{1}{G}\int d\tau d^3x\sqrt{g}\,\bar{c}_i\left(-\delta^i_j\partial^2_\tau + \mathbb{B}^i{}_j(\nabla)\right)c^j, \tag{25}$$

where the operator $\mathbb{B}^i{}_j$ exactly coincides with that of shift part.

Metric part of the action

The kinetic part for the metric perturbations has the form

$$-\frac{\sqrt{g}}{2G}\, h^A \mathbb{G}_{AB} \partial_\tau^2 h^B \ , \quad \mathbb{G}^{ij,kl} = \frac{1}{8}(g^{ik}g^{jl} + g^{il}g^{jk}) - \frac{\lambda}{4} g^{ij} g^{kl} \ , \tag{26}$$

where $h^A \equiv h_{ij}$. The part of the quadratic action with space derivatives of the metric is too lengthy to be written explicitly. Schematically, it has the form,

$$\mathcal{L}_{\text{pot, hh}} + \mathcal{L}_{\text{gf}, hh} = \frac{\sqrt{g}}{2G}\, h^A \mathbb{D}_{AB} h^B, \tag{27}$$

where $\mathbb{D}_{AB}$ is a purely 3-dimensional differential operator of 6th order. In flat background it reduces to terms with exactly 6 derivatives,

$$h^A \mathbb{D}_{AB} h^B = \left(\frac{\nu_5}{2} - \frac{1}{4\sigma}\right) h^{ik} \Delta^2 \partial_i \partial_j h^{jk} + \left(2\nu_4 + \frac{\nu_5}{2} + \frac{\lambda(1+\xi)}{2\sigma}\right) h \Delta^2 \partial_k \partial_l h^{kl}$$
$$- \left(\nu_4 + \frac{\nu_5}{2} + \frac{\xi}{4\sigma}\right) h^{ij} \Delta \partial_i \partial_j \partial_k \partial_l h^{kl} + \left(- \nu_4 - \frac{\nu_5}{4} - \frac{\lambda^2(1+\xi)}{4\sigma}\right) h \Delta^3 h - \frac{\nu_5}{4} h^{ij} \Delta^3 h_{ij} \ .$$

Total one-loop action

The one-loop effective action is given by the Gaussian path integral

$$\exp\left(-\Gamma^{1-\text{loop}}\right) = \sqrt{\text{Det}\,\mathcal{O}_{ij}} \int \left[\, dh^A\, dn^i\, dc^i\, d\bar{c}_j \right] \exp\left(- S^{(2)}[\, h^A, n^i, c^i, c_j \,]\right),$$

where the quadratic part of the full action consists of three contributions — metric, shift vector and ghost ones,

$$\begin{aligned} S^{(2)}[\, h^A, n^i, c^i, \bar{c}_j \,] = \frac{1}{G} \int d\tau d^3x \sqrt{g} \Bigg[& \frac{1}{2} h^A \left(-\mathbb{G}_{AB} \partial_\tau^2 + \mathbb{D}_{AB} \right) h^B \\ & + \frac{1}{2} \sigma n^i \mathcal{O}_{ik} \left(-\delta^k_j \partial_\tau^2 + \mathbb{B}^k{}_j \right) n^j + \bar{c}_i \left(-\delta^i_j \partial_\tau^2 + \mathbb{B}^i{}_j \right) c^j \Bigg]. \end{aligned} \tag{28}$$

Total one-loop action

The result of the integration

$$\exp\left(-\Gamma^{1-\text{loop}}\right) = \sqrt{\text{Det}\,\mathcal{O}_{ij}}\,\frac{\text{Det}\left(-\delta^i_j\partial^2_\tau + \mathbb{B}^i{}_j\right)}{\sqrt{\text{Det}\left(-\mathbb{G}_{AB}\partial^2_\tau + \mathbb{D}_{AB}\right)}\sqrt{\text{Det}\left[\mathcal{O}_{ik}\left(-\delta^k_j\partial^2_\tau + \mathbb{B}^k{}_j\right)\right]}}.$$

The operator $\mathcal{O}_{ij}$ cancels out, while the shift and ghost parts reduce to the contribution of a single functional determinant.

$$\Gamma^{1-\text{loop}} = \frac{1}{2}\text{Tr}\ln(-\delta^A_B\partial^2_\tau + \mathbb{D}^A{}_B) - \frac{1}{2}\text{Tr}\ln\left(-\delta^i_j\partial^2_\tau + \mathbb{B}^i{}_j\right)\ , \tag{29}$$

where

$$\mathbb{D}^A{}_B = (\mathbb{G}^{-1})^{AC}\mathbb{D}_{CB}. \tag{30}$$

3D reduction

We begin by using the proper-time representation for the trace of the logarithm of an operator

$$\mathrm{Tr}\,\ln(-\partial_\tau^2+\mathbb{F}) = -\int_0^\infty \frac{ds}{s}\,\mathrm{Tr}\,\mathrm{e}^{-s(-\partial_\tau^2+\mathbb{F})}, \tag{31}$$

where $\mathbb{F}$ is either $\mathbb{D}^A{}_B$ or $\mathbb{B}^i{}_j$. Thus, we obtain the expression for the metric tensor part of the effective action,

$$\begin{aligned}\Gamma^{1-\mathrm{loop}}_{\mathrm{metric}} &= -\frac{1}{2}\int_0^\infty \frac{ds}{s}\,\mathrm{Tr}\,\mathrm{e}^{-s(-\delta^A_B\partial_\tau^2+\mathbb{D}^A{}_B)} \\ &= -\frac{1}{2}\int d\tau\, d^3x \int \frac{ds}{s}\,\mathrm{tr}\,\mathrm{e}^{-s(-\delta^A_B\partial_\tau^2+\mathbb{D}^A{}_B)}\delta(\tau-\tau')\,\delta(\mathbf{x}-\mathbf{x}')\Big|_{\tau=\tau',\,\mathbf{x}=\mathbf{x}'}.\end{aligned}$$

3D reduction

For a static background it can be transformed by the following chain of relations

$$
\begin{aligned}
\Gamma^{1-\text{loop}}_{\text{metric}} &= -\frac{1}{2}\int d\tau\, d^3x \int \frac{ds}{s}\, \mathrm{tr}\, \mathrm{e}^{-s(-\delta^A_B \partial_\tau^2 + \mathbb{D}^A{}_B)} \int \frac{d\omega}{2\pi} \mathrm{e}^{i\omega(\tau-\tau')} \delta(\mathbf{x}-\mathbf{x}')\Big|_{\tau-\tau',\, \mathbf{x}=\mathbf{x}'} \\
&= -\frac{1}{2}\int d\tau\, d^3x \int \frac{ds}{s} \int \frac{d\omega}{2\pi}\, \mathrm{e}^{-s\omega^2}\, \mathrm{tr}\, \mathrm{e}^{-s\mathbb{D}^A{}_B} \delta(\mathbf{x}-\mathbf{x}')\Big|_{\mathbf{x}=\mathbf{x}'} \\
&= -\frac{1}{4\sqrt{\pi}}\int d\tau\, d^3x \int \frac{ds}{s^{3/2}}\, \mathrm{tr}\, \mathrm{e}^{-s\mathbb{D}^A{}_B} \delta(\mathbf{x}-\mathbf{x}')\Big|_{\mathbf{x}=\mathbf{x}'} \\
&= -\frac{\Gamma(-1/2)}{4\sqrt{\pi}}\int d\tau\, d^3x\, \mathrm{tr}\, \sqrt{\mathbb{D}^A{}_B}\, \delta(\mathbf{x}-\mathbf{x}')\Big|_{\mathbf{x}=\mathbf{x}'} \\
&= \frac{1}{2}\int d\tau\, \mathrm{Tr}_3 \sqrt{\mathbb{D}^A{}_B}\,.
\end{aligned}
\tag{32}
$$

Introducing the notation $\mathbb{Q}_{\mathbb{D}}{}^A{}_B \equiv \sqrt{\mathbb{D}^A{}_B}$ and $\mathbb{Q}_{\mathbb{B}}{}^i{}_j \equiv \sqrt{\mathbb{B}^i{}_j}$ the full one-loop action can be expressed as

$$
\Gamma^{1-\text{loop}} = \frac{1}{2}\int d\tau \left[\mathrm{Tr}_3\, \mathbb{Q}_{\mathbb{D}}{}^A{}_B - \mathrm{Tr}_3\, \mathbb{Q}_{\mathbb{B}}{}^i{}_j\right]. \tag{33}
$$

The strategy for evaluation

The operators $\mathbb{F} = (\mathbb{D}, \mathbb{B})$ can be brought into the form:

$$\mathbb{F} = \sum_{a=0}^{6} \mathcal{R}_{(a)} \sum_{6 \geq 2k \geq a} \alpha_{a,k} \nabla_1 ... \nabla_{2k-a} (-\Delta)^{3-k} \ , \quad \mathcal{R}_{(a)} = O\Big(\frac{1}{l^a} \Big). \tag{34}$$

Their square roots are nonlocal pseudo-differential operators given by

$$\sqrt{\mathbb{F}} = \sum_{a=0}^{\infty} \mathcal{R}_{(a)} \sum_{k \geq a/2}^{K_a} \tilde{\alpha}_{a,k} \nabla_1 ... \nabla_{2k-a} \frac{1}{(-\Delta)^{k-3/2}} \ , \tag{35}$$

The UV divergent part of $\Gamma^{1-\text{loop}}$ follows from the calculation of UFTs

$$\int d^3x \, \mathcal{R}_{(a)}(\mathbf{x}) \nabla_1 ... \nabla_{2k-a} \frac{1}{(-\Delta)^{k-3/2}} \delta(\mathbf{x}, \mathbf{x}') \Big|_{\mathbf{x}=\mathbf{x}'}. \tag{36}$$

Since the divergences of HG have at maximum the dimensionality $a = 6$, only finite number of such traces will be needed. The problem is split in two steps — calculation of the operator square root and the evaluation of UFTs.

Square root of principal symbol

The principal symbol of the vector operator $\mathbb{B}$

$$\mathbb{B}^i{}_j(\mathbf{p}) = p^6 \left(\frac{1}{2\sigma}\delta^i_j + \frac{1-2\lambda+2\xi(1-\lambda)}{2\sigma}\hat{p}^i\hat{p}_j \right) \tag{37}$$

can be easily written in terms of the transverse and longitudinal projectors,

$$\mathbb{B}^i{}_j(\mathbf{p}) = \frac{p^6}{2\sigma}\mathbb{P}^{(VT)\,i}{}_j + \frac{(1-\lambda)(1+\xi)}{\sigma}p^6\,\mathbb{P}^{(VL)\,i}{}_j\ , \tag{38}$$

where

$$\mathbb{P}^{(VT)\,i}{}_j = \delta^i_j - \hat{p}^i\hat{p}_j, \qquad \mathbb{P}^{(VL)\,i}{}_j = \hat{p}^i\hat{p}_j\ . \tag{39}$$

Then the square root reads

$$\mathbb{Q}^{(0)\,i}_{\mathbb{B}}{}_j(\nabla) = \frac{1}{\sqrt{2\sigma}}\delta^i_j(-\Delta)^{3/2} + \left(\frac{1}{\sqrt{2\sigma}} + \sqrt{\frac{(1-\lambda)(1+\xi)}{\sigma}} \right) \nabla^i\nabla_j(-\Delta)^{1/2}. \tag{40}$$

An example of $\mathbb{Q}^{(0)}_{\mathbb{D}}$ in one of the gauges

$$\begin{aligned}\mathbb{Q}^{(0)\,kl}_{\mathbb{D}\,ij} =& \sqrt{\nu_5}\left[\left(\frac{1}{2}(\delta^k_i\delta^l_j + \delta^l_i\delta^k_j) + \frac{u_s-1}{2}g_{ij}g^{kl} \right)(-\Delta)^{3/2} + \frac{u_s-1}{2}g_{ij}\nabla^k\nabla^l(-\Delta)^{1/2} \right.\\ &\left. + \frac{u_s-1}{2}g^{kl}\nabla_i\nabla_j(-\Delta)^{1/2} + \frac{3(u_s-1)}{2}\nabla_i\nabla_j\nabla^k\nabla^l(-\Delta)^{-1/2} \right].\end{aligned}$$

Perturbative scheme

Square root contains all powers of curvature

$$\sqrt{\mathbb{F}} = \mathbb{Q}^{(0)} + \mathbb{Q}^{(2)} + \mathbb{Q}^{(3)} + \mathbb{Q}^{(4)} + \mathbb{Q}^{(5)} + \mathbb{Q}^{(6)} + \dots . \tag{41}$$

By denoting all curvature corrections in $\sqrt{\mathbb{F}}$ as $\mathbb{X}$,

$$\sqrt{\mathbb{F}} = \mathbb{Q}^{(0)} + \mathbb{X} \tag{42}$$

one obtains the equation for this correction term

$$\mathbb{Q}^{(0)}\mathbb{X} + \mathbb{X}\,\mathbb{Q}^{(0)} = \mathbb{F} - \left(\mathbb{Q}^{(0)}\right)^2 - \mathbb{X}^2, \quad \mathbb{F} - \left(\mathbb{Q}^{(0)}\right)^2 \propto R. \tag{43}$$

This nonlinear equation can be solved by iterations because its right hand side is at least linear in curvature.

Universal functional traces

I.Jack and H.Osborn (1984), A.Barvinsky and G.Vilkovisky (1985)

$$\nabla_{i_1}...\nabla_{i_p}\frac{\hat{1}}{(-\Delta)^{N+1/2}}\delta(x,y)\Big|_{y=x}=\frac{1}{\Gamma(N+1/2)}\nabla_{i_1}...\nabla_{i_p}\int_0^\infty ds\, s^{N-1/2}\,\mathrm{e}^{s\Delta}\,\hat{\delta}(x,y)\Big|_{y=x}^{\mathrm{div}}.$$

Heat-kernel (Schwinger-DeWitt) expansion

$$\mathrm{e}^{s\Delta}\,\hat{\delta}(x,y)=\frac{\mathcal{D}^{1/2}(x,y)}{(8\pi)^{d/2}}e^{-\frac{\sigma(x,y)}{2s}}\sum_{n=0}^{\infty}s^n\hat{a}_n(x,y).$$

Example of tensor UFTs

$$g^{ij}(-\Delta)^{1/2}\delta_{ij}{}^{kl}(x,y)|_{y=x}^{\mathrm{div}}=-\frac{\ln L^2}{16\pi^2}\sqrt{g}\,g^{kl}\frac{1}{30}\left(\frac{1}{2}R^{mn}R_{mn}+\frac{1}{4}R^2+\Delta R\right),$$

$$\int d^3x\,\delta^j_l\nabla_k\nabla^i(-\Delta)^{1/2}\delta_{ij}{}^{kl}(x,y)\Big|_{y=x}^{\mathrm{div}}$$

$$=-\frac{\ln L^2}{16\pi^2}\int d^3x\sqrt{g}\left(-\frac{23}{80}R^i_jR^j_kR^k_i+\frac{753}{1120}R_{ij}R^{ij}R-\frac{22}{105}R^3-\frac{1}{84}R\Delta R-\frac{61}{560}R_{ij}\Delta R^{ij}\right).$$

Beta functions

In the last step we obtain the divergent part of the one-loop effective action

$$\Gamma^{1-\text{loop}}\Big|^{\text{div}} = \ln L^2 \int d\tau\, d^3x\, \sqrt{g}\, \big(C_{\nu_1} R^3 + C_{\nu_2} R R_{ij} R^{ij} + C_{\nu_3} R^i_j R^j_k R^k_i + C_{\nu_4} \nabla_i R \nabla^i R + C_{\nu_5} \nabla_i R_{jk} \nabla^i R^{jk}\big). \tag{44}$$

The coefficients C_{ν_a}, which are functions of the couplings λ, $\nu_1, \ldots, \nu_5$, represent the key result of the calculation.
The UV divergent factor $\ln L^2$ is related to the integral over the proper-time parameter,

$$\ln L^2 = \int \frac{ds_2}{s_2} \simeq \ln\left(\frac{\Lambda^2_{\text{UV}}}{k_*^2}\right). \tag{45}$$

We are now ready to compute the β-functions of the couplings ν_a

$$\left(\frac{\nu_a}{2G}\right)_{\text{ren}} = \frac{\nu_a}{2G} + C_{\nu_a} \ln L^2 \quad \Rightarrow \quad \beta_{\nu_a} \equiv \frac{d\nu_{a,\,\text{ren}}}{d\ln k_*} = -4GC_{\nu_a} + \nu_a \frac{\beta_G}{G}. \tag{46}$$

Essential couplings

Background effective action Γ_{eff} depends on the choice of gauge fixing

$$\Gamma_{\text{eff}} \mapsto \Gamma_{\text{eff}} + \epsilon \mathcal{A}, \tag{47}$$

where $\mathcal{A}$ is a linear combination of equations of motion.
The UV behavior of the theory is parameterized by seven couplings G, λ, ν_a, $a = 1, \ldots, 5$. The essential couplings can be chosen as follows,

$$\mathcal{G} = \frac{G}{\sqrt{\nu_5}}, \qquad \lambda, \qquad u_s = \sqrt{\frac{\nu_s}{\nu_5}}, \qquad v_a = \frac{\nu_a}{\nu_5}, \quad a = 1, 2, 3. \tag{48}$$

The one-loop β-function of λ depends only on the first three of these couplings and reads,

$$\beta_\lambda = \mathcal{G} \frac{27(1-\lambda)^2 + 3u_s(11-3\lambda)(1-\lambda) - 2u_s^2(1-3\lambda)^2}{120\pi^2(1-\lambda)(1+u_s)u_s} \,. \tag{49}$$

The gauge-dependent β-function of G (not $\mathcal{G}$) was also computed.

A.Barvinsky, M.Herrero-Valea, S.Sibiryakov (2019)

Beta functions (main result)

Essential couplings

$$\mathcal{G} = \frac{G}{\sqrt{\nu_5}}, \qquad \lambda, \qquad u_s = \sqrt{\frac{(1-\lambda)(8\nu_4+3\nu_5)}{(1-3\lambda)\nu_5}}, \qquad v_a = \frac{\nu_a}{\nu_5}, \quad a = 1,2,3, \tag{50}$$

$$\beta_{\mathcal{G}} = \frac{\mathcal{G}^2}{26880\pi^2(1-\lambda)^2(1-3\lambda)^2(1+u_s)^3u_s^3}\sum_{n=0}^{7} u_s^n\,\mathcal{P}_n^{\mathcal{G}}[\lambda, v_1, v_2, v_3], \tag{51a}$$

$$\beta_{\chi} = A_{\chi}\frac{\mathcal{G}}{26880\pi^2(1-\lambda)^3(1-3\lambda)^3(1+u_s)^3u_s^5}\sum_{n=0}^{9} u_s^n\,\mathcal{P}_n^{\chi}[\lambda, v_1, v_2, v_3], \tag{51b}$$

where the prefactor coefficients $A_\chi = (A_{u_s}, A_{v_1}, A_{v_2}, A_{v_3})$ equal

$$A_{u_s} = u_s(1-\lambda), \quad A_{v_1} = 1, \quad A_{v_2} = A_{v_3} = 2. \tag{52}$$

Example of polynomial

$$\begin{aligned}\mathcal{P}_2^{u_s} &= -2(1-\lambda)^3\big[2419200v_1^2(1-\lambda)^2 + 8v_2^2(42645\lambda^2 - 86482\lambda + 43837)\\ &+ v_3^2(58698 - 106947\lambda + 48249\lambda^2) + 4032v_1\big(462v_2(1-\lambda)^2 + 201v_3(1-\lambda)^2\\ &+ 30\lambda^2 - 44\lambda - 10\big) + 8v_2(6252\lambda^2 - 9188\lambda - 1468) + 8v_2v_3(34335\lambda^2 - 71196\lambda\\ &+ 36861) + v_3(20556\lambda^2 - 30792\lambda - 3696) + 4533\lambda^2 - 3881\lambda + 1448\big].\end{aligned}$$

Fixed points of RG flow

$$\begin{cases} \beta_\lambda/\mathcal{G} = 0 \ , \\ \beta_\chi/\mathcal{G} = 0 \ , \qquad \chi = u_s, v_1, v_2, v_3 \ . \end{cases} \tag{53}$$

λ	u_s	v_1	v_2	v_3	$\beta_\mathcal{G}/\mathcal{G}^2$	AF?
0.1787	60.57	-928.4	-6.206	-1.711	-0.1416	yes
0.2773	390.6	-19.88	-12.45	2.341	-0.2180	yes
0.3288	54533	3.798×10^{8}	-48.66	4.736	-0.8484	yes
0.3289	57317	-4.125×10^{8}	-49.17	4.734	-0.8784	yes
0.333332	3.528×10^{11}	$-6.595\ \times10^{23}$	$-1.950\ \times10^{8}$	4.667	-3.989×10^{6}	yes

$\lambda \to \infty$ limit

A.Gumrukcuoglu and S.Mukohyama, Rev. D 83 (2011) 124033

$$\beta_\chi/\mathcal{G}\Big|_{\lambda=\infty} = 0\ , \qquad \chi = u_s, v_1, v_2, v_3\ . \tag{54}$$

u_s	v_1	v_2	v_3	$\beta_\mathcal{G}/\mathcal{G}^2$	AF?
0.01950	0.4994	-2.498	2.999	-0.2004	yes
0.04180	-0.01237	-0.4204	1.321	-1.144	yes
0.05530	-0.2266	0.4136	0.7177	-1.079	yes
12.28	-215.1	-6.007	-2.210	-0.1267	yes
21.60	-17.22	-11.43	1.855	-0.1936	yes
440.4	-13566	-2.467	2.967	0.05822	no
571.9	-9.401	13.50	-18.25	-0.07454	yes
950.6	-61.35	11.86	3.064	0.4237	no

Conclusions

- Beta functions for essential coupling in (3+1)-dimensional Hořava gravity were obtained.
- The results underwent a number of very powerful checks.
- Fixed points of RG flow were found. There are candidates for AF points of the theory.

Thank you!