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Loop corrections to a cosmological particle creation

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Plan

1. Setup for the problem. Geometry and modes
2. Loop corrections
3. Stress-energy tensor
4. Discussion and outlook

Setup for the problem. Geometry and modes

Consider the geometry given by the line element

$$ds^2 = C(\eta)(d\eta^2 - dx^2), \quad (1)$$

where the conformal factor $C(\eta)$ is given by:

$$C(\eta) = A + B \tanh(\rho\eta). \quad (2)$$

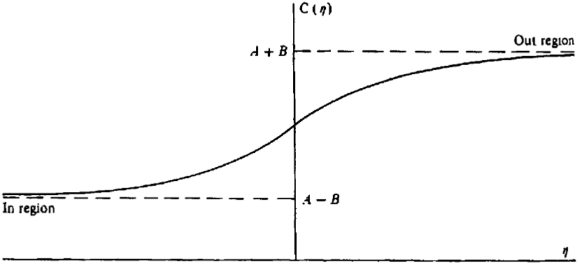

Figure: η variable dependence of the conformal factor.

In the past and future asymptotic regions the Ricci scalar is exponentially damped:

$$R(\eta) \to \pm \frac{8B\rho^2}{(A \pm B)^2} e^{\mp 2\rho\eta}. \quad (3)$$

Let us consider massive minimally coupled scalar field in the background (1):

$$S = \int d^2x\sqrt{-g}\left[\frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4\right]. \tag{4}$$

Among the solutions of the free field EOMs one distinguishes the in-modes:

$$u_k^{in}(\eta,x) = (4\pi\omega_{in})^{-\frac{1}{2}}e^{ikx - i\omega_+\eta - i\frac{\omega_-}{\rho}\log[2\cosh(\rho\eta)]}F\left(1+\frac{i\omega_-}{\rho}, \frac{i\omega_-}{\rho}; 1-\frac{i\omega_{in}}{\rho}; \frac{1+\tanh(\rho\eta)}{2}\right), \tag{5}$$

where $F(a, b\,; c\,; z)$ is the gaussian hypergeometric function and

$$\begin{aligned} \omega_{in}(k) &= \sqrt{k^2 + m^2(A-B)}, \\ \omega_{out}(k) &= \sqrt{k^2 + m^2(A+B)}, \\ \omega_\pm &= \frac{1}{2}(\omega_{out} \pm \omega_{in}). \end{aligned} \tag{6}$$

In the asymptotic regions the in-modes have the following behavior:

$$u_k^{in}(\eta, x) = e^{ikx} g_k^{in}(\eta) \sim \begin{cases} \frac{1}{\sqrt{4\pi\omega_{in}}} e^{ikx - i\omega_{in}\eta}, & \text{as} \quad \eta \to -\infty \\ \frac{1}{\sqrt{4\pi\omega_{in}}} e^{ikx} \left[\alpha_k e^{-i\omega_{out}\eta} + \beta_k e^{i\omega_{out}\eta}\right], & \text{as} \quad \eta \to +\infty, \end{cases} \tag{7}$$

where

$$\alpha_k = \frac{\Gamma(1 - i\omega_{in}/\rho)\Gamma(-i\omega_{out}/\rho)}{\Gamma(-i\omega_+/\rho)\Gamma(1 - i\omega_+/\rho)}, \qquad \beta_k = \frac{\Gamma(1 - i\omega_{in}/\rho)\Gamma(i\omega_{out}/\rho)}{\Gamma(1 + i\omega_-/\rho)\Gamma(i\omega_-/\rho)}. \tag{8}$$

Mode decomposition of the field operator is as follows:

$$\phi(\eta, x) = \int_{-\infty}^{\infty} dk \left[a_k u_k^{in}(\eta, x) + a_k^\dagger u_k^{in*}(\eta, x)\right]. \tag{9}$$

Alternatively, one may define out-modes as

$$u_k^{out}(\eta, x) = (4\pi\omega_{in})^{-\frac{1}{2}} e^{ikx - i\omega_+\eta - i\frac{\omega_-}{\rho}\log[2\cosh(\rho\eta)]} F\left(1 + \frac{i\omega_-}{\rho}, \frac{i\omega_-}{\rho}; 1 + \frac{i\omega_{out}}{\rho}; \frac{1 - \tanh(\rho\eta)}{2}\right), \tag{10}$$

which have the following asymptotic behavior:

$$u_k^{out}(\eta, x) = e^{ikx} g_k^{out}(\eta) \sim \begin{cases} \frac{1}{\sqrt{4\pi\omega_{out}}} e^{ikx} \left[\gamma_k e^{-i\omega_{in}\eta} + \delta_k e^{i\omega_{in}\eta}\right], & \text{as} \quad \eta \to -\infty \\ \frac{1}{\sqrt{4\pi\omega_{out}}} e^{ikx - i\omega_{out}\eta}, & \text{as} \quad \eta \to +\infty, \end{cases} \tag{11}$$

where

$$\gamma_k = \frac{\Gamma(1 + i\omega_{out}/\rho)\Gamma(i\omega_{in}/\rho)}{\Gamma(i\omega_+/\rho)\Gamma(1 + i\omega_+/\rho)}, \qquad \delta_k = \frac{\Gamma(1 + i\omega_{out}/\rho)\Gamma(-i\omega_{in}/\rho)}{\Gamma(1 + i\omega_-/\rho)\Gamma(i\omega_-/\rho)}. \tag{12}$$

Mode decomposition of the field operator in this case reads:

$$\phi(\eta, x) = \int_{-\infty}^{\infty} dk \left[b_k u_k^{out}(\eta, x) + b_k^\dagger u_k^{out*}(\eta, x)\right]. \tag{13}$$

Equating the two mode decompositions over in- and out-modes

$$\phi(\eta, x) = \int_{-\infty}^{\infty} dk \left[a_k u_k^{in}(\eta, x) + a_k^\dagger u_k^{in*}(\eta, x)\right] = \int_{-\infty}^{\infty} dk \left[b_k u_k^{out}(\eta, x) + b_k^\dagger u_k^{out*}(\eta, x)\right]. \quad (14)$$

one obtains the following relation between creation and annihilation operator for the in- and out-modes:

$$b_k = \alpha_k a_k + \beta_k^* a_{-k}^\dagger, \quad (15)$$

$$b_k^\dagger = \alpha_k^* a_k^\dagger + \beta_k a_{-k}, \quad (16)$$

with the Bogoliubov coefficients

$$|\alpha_k|^2 = \frac{\omega_{in}}{\omega_{out}} \frac{\sinh^2(\pi\omega_+/\rho)}{\sinh(\pi\omega_{in}/\rho)\sinh(\pi\omega_{out}/\rho)}, \qquad |\beta_k|^2 = \frac{\omega_{in}}{\omega_{out}} \frac{\sinh^2(\pi\omega_-/\rho)}{\sinh(\pi\omega_{in}/\rho)\sinh(\pi\omega_{out}/\rho)}. \quad (17)$$

One may define the Fock vacuum for the in-modes as:

$$|0, in\rangle : \qquad a_k|0, in\rangle = 0, \ \forall k. \tag{18}$$

Similarly, the vacuum for the out-modes is:

$$|0, out\rangle : \qquad b_k|0, out\rangle = 0, \ \forall k. \tag{19}$$

Then, one can obtain that

$$\langle 0, out|a_k^\dagger a_k|0, out\rangle = \int dk\, |\beta_k|^2 = \int dk\, \frac{\omega_{in}}{\omega_{out}} \frac{\sinh^2(\pi\omega_-/\rho)}{\sinh(\pi\omega_{in}/\rho)\sinh(\pi\omega_{out}/\rho)}, \tag{20}$$

i.e. the occupation number of the in-modes in the distant future is a finite non-zero number.

Loop corrections

Example. Scalar field in the flat spacetime. Let us consider the scalar field with the quartic self-interaction in the Minkowski spacetime, where the modes are

$$u_k(t, x) = e^{i\omega_k t - ikx}. \tag{21}$$

From the Dyson-Schwinger equation for the propagators one may obtain the kinetic equation for the occupation number:

$$\frac{\partial n_p}{\partial t} + \vec{v}\frac{\partial n_p}{\partial \vec{r}} \sim \int \dots \Big[(1+n_p)(1+n_{q_1})n_{q_2}n_{q_3} - n_p n_{q_1}(1+n_{q_2})(1+n_{q_3})\Big] \times \\ \times \delta(\omega_p + \omega_{q_1} - \omega_{q_2} - \omega_{q_3}). \tag{22}$$

One can see that the thermal distribution

$$n_p = \frac{1}{e^{\beta\omega_p} - 1} \tag{23}$$

solves the kinetic equation.

Loop corrections

Consider now the theory with $\lambda\phi^4$ self-interaction:

$$S_{int} = -\frac{\lambda}{4!}\int d^2x\sqrt{-g}\left(\phi_+^4 - \phi_-^4\right) = -\frac{\lambda}{4!}\int d^2x\sqrt{-g}\left(4(\phi_{cl})^3\phi_q + \phi_{cl}(\phi_q)^3\right). \tag{24}$$

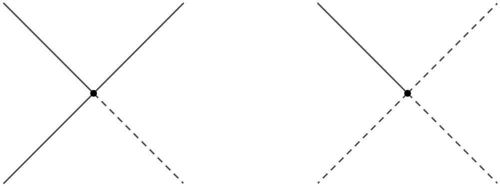

Figure: Vertices for the quartic self-interaction.

Tadpole diagrams

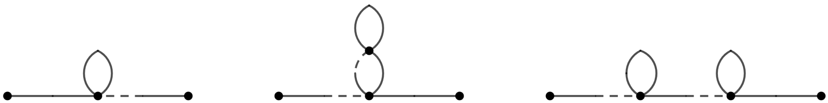

Figure: Different types of tadpole diagrams

In the IR limit $\frac{\eta_1+\eta_2}{2} \equiv \eta \gg \eta_1 - \eta_2$ the Dyson-Schwinger equation for the "exact" Keldysh function:

$$\tilde{G}^K_{12} = G^K_{12} - \frac{i\lambda}{2} \int_{\eta_0}^{\infty} d\eta_3\, dx_3\, C(\eta_3) \left(G^K_{13} G^K_{33} \tilde{G}^A_{32} + G^R_{13} G^K_{33} \tilde{G}^K_{32} \right). \tag{25}$$

Applying the differential operator $(\Box + m^2)$ to the both sides of this equation, one obtains

$$\left(\Box + m^2 + \frac{\lambda}{2} G^K_{11} \right) \tilde{G}^K_{12} = 0, \tag{26}$$

i.e. sum of the tadpole diagrams leads to the mass renormalization

$$m^2_{ren} = m^2 + \frac{\lambda}{2} G^K_{11}. \tag{27}$$

Tadpole diagrams

Due to the mass renormalization the modes change their behavior. For the in-modes asymptotics one has

$$\tilde{g}_k^{in}(\eta) \approx \begin{cases} \frac{1}{\sqrt{4\pi\tilde{\omega}_{in}}} e^{-i\tilde{\omega}_{in}\eta}, & \text{as} \quad \eta \to -\infty \\ \frac{1}{\sqrt{4\pi\tilde{\omega}_{in}}} \left[C_1(k)\, e^{-i\tilde{\omega}_{out}\eta} + C_2(k)\, e^{i\tilde{\omega}_{out}\eta} \right], & \text{as} \quad \eta \to +\infty, \end{cases} \tag{28}$$

where

$$\begin{aligned} \tilde{\omega}_{in}^2(k) &\approx k^2 + (A - B)\left(m^2 + \int \frac{dk}{4\pi\omega_{in}} \right), \\ \tilde{\omega}_{out}^2(k) &\approx k^2 + (A + B)\left(m^2 + \int \frac{dk}{4\pi\omega_{out}} \frac{\sinh^2(\pi\omega_+/\rho) + \sinh^2(\pi\omega_-/\rho)}{\sinh(\pi\omega_{in}/\rho)\sinh(\pi\omega_{out}/\rho)} \right). \end{aligned} \tag{29}$$

Similarly, for the out-modes one has

$$\tilde{g}_k^{out}(\eta) \approx \begin{cases} \frac{1}{\sqrt{4\pi\tilde{\omega}_{in}}} \left[C_3(k)\, e^{-i\tilde{\omega}_{in}\eta} + C_4(k)\, e^{i\tilde{\omega}_{in}\eta} \right], & \text{as} \quad \eta \to -\infty \\ \frac{1}{\sqrt{4\pi\tilde{\omega}_{in}}} e^{-i\tilde{\omega}_{out}\eta}, & \text{as} \quad \eta \to +\infty. \end{cases} \tag{30}$$

Sunset diagrams

Two-loop sunset diagram contributions to the Keldysh propagator are:

$$G^K_{(2)}(\eta_1, x_1|\eta_2, x_2) = -\frac{\lambda^2}{6}\int_{\eta_0}^{\infty} d\eta_3\, dx_3\, C(\eta_3)\int_{\eta_0}^{\infty} d\eta_4\, dx_4\, C(\eta_4)\Big[3G^K_{13}(G^K_{34})^2 G^A_{34} G^A_{42} + \\ + G^R_{13}(G^K_{34})^3 G^A_{42} + 3G^R_{13}G^R_{34}(G^K_{34})^2 G^K_{42} + \frac{3}{4}G^R_{13}(G^R_{34})^2 G^K_{34} G^A_{42} + \frac{1}{4}G^R_{13}(G^R_{34})^3 G^K_{42} + \\ + \frac{1}{4}G^K_{13}(G^A_{34})^3 G^A_{42} + \frac{3}{4}G^R_{13}(G^A_{34})^2 G^K_{34} G^A_{42}\Big]. \quad (31)$$

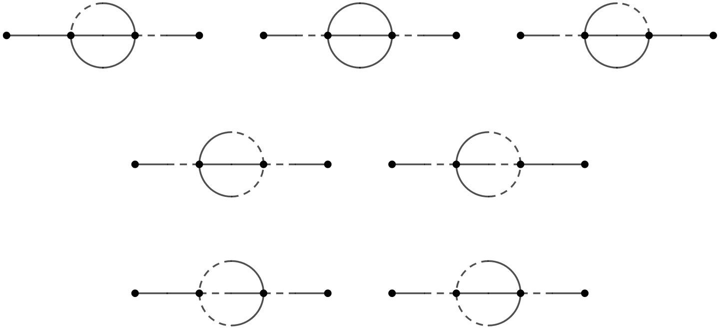

Figure: Sunset diagram corrections to the Keldysh propagator

Sunset diagrams

$$G^K(\eta, x|\eta', x') = \int dk \left\{ \left(\frac{1}{2} + \left\langle a_k^\dagger a_k \right\rangle \right) \left[u_k^{in}(\eta, x)\, u_k^{in*}(\eta', x') + u_k^{in}(\eta', x')\, u_k^{in*}(\eta, x) \right] + \right.$$

$$\left. + \left\langle a_k a_{-k} \right\rangle u_k^{in}(\eta, x)\, u_{-k}^{in}(\eta', x') + \left\langle a_k^\dagger a_{-k}^\dagger \right\rangle u_k^{in*}(\eta, x)\, u_{-k}^{in*}(\eta', x') \right\}, \qquad (32)$$

In the IR limit

$$\frac{\eta_1 + \eta_2}{2} \equiv \eta \gg \eta_1 - \eta_2, \qquad (33)$$

After evaluation of the integrals over η_3 и η_4 one obtains

$$n_p^{(2)} \approx \frac{\lambda^2 (A+B)^2\, \eta}{64\pi} \int \frac{dq\, dr\, ds\, \delta\Big(p - q - r - s\Big)}{\omega_{in}(p)\omega_{in}(q)\omega_{in}(r)\omega_{in}(s)} \delta\Big(\omega_{out}(p) + \omega_{out}(q) - \omega_{out}(r) - \omega_{out}(s)\Big) \times$$

$$\times \Big\{ \mathcal{N}_1(p,q,r,s) \Big[(1+n_p)(1+n_q)(1+n_r)(1+n_s) - n_p n_q n_r n_s \Big] +$$

$$+ \mathcal{N}_2(p,q,r,s) \Big[(1+n_p)(1+n_q)(1+n_r) n_s - n_p n_q n_r (1+n_s) \Big] +$$

$$+ \mathcal{N}_3(p,q,r,s) \Big[(1+n_p) n_q (1+n_r)(1+n_s) - n_p (1+n_q) n_r n_s \Big] +$$

$$+ \mathcal{N}_4(p,q,r,s) \Big[(1+n_p) n_q n_r (1+n_s) - n_p (1+n_q)(1+n_r) n_s \Big] +$$

$$+ \mathcal{N}_5(p,q,r,s) \Big[(1+n_p) n_q n_r n_s - n_p (1+n_q)(1+n_r)(1+n_s) \Big] \Big\}, \qquad (34)$$

Sunset diagrams

Analogously, one obtains for the anomalous quantum averages

$$\kappa_p^{(2)} \approx -\frac{\lambda^2(A+B)^2\,\eta}{64\pi}\int \frac{dq\,dr\,ds\,\delta\Big(p-q-r-s\Big)}{\omega_{in}(p)\omega_{in}(q)\omega_{in}(r)\omega_{in}(s)}\times$$
$$\times\Big\{\mathcal{K}_1(p,q,r,s)\Big[(1+n_q)(1+n_r)(1+n_s)+n_q n_r n_s\Big]+$$
$$+\mathcal{K}_2(p,q,r,s)\Big[(1+n_q)(1+n_r)n_s+n_q n_r(1+n_s)\Big]+$$
$$+\mathcal{K}_3(p,q,r,s)\Big[n_q(1+n_r)(1+n_s)+(1+n_q)n_r n_s\Big]+$$
$$+\mathcal{K}_4(p,q,r,s)(1+2n_p)\Big[(1+n_q)(1+n_r)(1+n_s)-n_q n_r n_s\Big]+$$
$$+\mathcal{K}_5(p,q,r,s)(1+2n_p)\Big[(1+n_q)(1+n_r)n_s-n_q n_r(1+n_s)\Big]\Big\}, \qquad (35)$$

Out-modes

As the out-modes correspond to the usual notion of a particle in the distant future, in the IR limit (33) one obtains the same contribution as in the flat spacetime:

$$\tilde{n}_p^{(2)} \approx \frac{\lambda^2 (A+B)^2 (\eta - \bar{\eta}')}{64\,\pi} \int \frac{dq\,dr\,ds\,\delta(p-q-r-s)}{\omega_{out}(p)\omega_{out}(q)\omega_{out}(r)\omega_{out}(s)} \delta(\omega_{out}(p)+\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)) \times \left[(1+\tilde{n}_p)(1+\tilde{n}_q)\tilde{n}_r\tilde{n}_s - \tilde{n}_p\tilde{n}_q(1+\tilde{n}_r)(1+\tilde{n}_s)\right], \quad (36)$$

with the anomalous quantum average not growing secularly.

Stress-energy tensor

Regularizing the stress-energy tensor using Pauli-Villars method, one obtains in the distant past

$$\langle T^{\mu\nu} \rangle \xrightarrow[\eta \to -\infty]{} 0, \tag{37}$$

while in the distant future

$$\begin{aligned} &\langle \text{in} | T^{\mu\nu} | \text{in} \rangle = \langle \text{in} | T_0^{\mu\nu} | \text{in} \rangle + \langle \text{in} | T_{\text{loop}}^{\mu\nu} | \text{in} \rangle, \\ &\langle \text{in} | T_0^{\mu\nu} | \text{in} \rangle \xrightarrow[\eta \to +\infty]{} \int_{-\infty}^{\infty} \frac{dk}{4\pi \tilde{\omega}_{out}} k^\mu k^\nu \left(\frac{\tilde{\omega}_{out}}{\tilde{\omega}_{in}} \left(|C_1|^2 + |C_2|^2 \right) - 1 \right), \\ &\langle \text{in} | T_{\text{loop}}^{\mu\nu} | \text{in} \rangle \xrightarrow[\eta \to +\infty]{} \int_{-\infty}^{\infty} \frac{dk}{2\pi \tilde{\omega}_{in}} k^\mu k^\nu \left[n_k^{(2)} \left(|C_1|^2 + |C_2|^2 \right) + \kappa_k^{(2)} C_1 C_2 + \kappa_k^{(2)*} C_1^* C_2^* \right], \end{aligned} \tag{38}$$

where

$$k^\mu = \left(\frac{\tilde{\omega}_{out}}{A+B}, \frac{k}{A+B} \right). \tag{39}$$

Discussion and outlook

- Therefore, it has been shown that in the interacting theory initial occupation numbers and anomalous quantum averages acquire secularly growing contributions. Moreover, it has been shown that these secularly growing contribution are non-zero even if the initial state is the Fock vacuum;
- It is shown that the stress-energy tensor also acquires secularly growing contributions;
- The analysis can be generalised to the arbitrary number of dimensions of spacetime. Ultimately, all the results stay the same;
- The next step is to sum all the secularly growing contributions in the leading order and to find the "exact" propagators in this limit. With the "exact" propagator it is interesting to calculate regularized stress-energy tensor and with the use of the semiclassical Einstein equations estimate whether the quantum corrections should change the background geometry.

Thank you for your attention!

$$
\begin{aligned}
\mathcal{N}_1(p,q,r,s) &= |C_1(p)C_1(q)C_2(r)C_2(s)|^2 + |C_2(p)C_2(q)C_1(r)C_1(s)|^2, \\
\mathcal{N}_2(p,q,r,s) &= 2\left(|C_1(p)C_1(q)C_2(r)C_1(s)|^2 + |C_2(p)C_2(q)C_1(r)C_2(s)|^2\right), \\
\mathcal{N}_3(p,q,r,s) &= |C_1(p)C_2(q)C_2(r)C_2(s)|^2 + |C_2(p)C_1(q)C_1(r)C_1(s)|^2, \\
\mathcal{N}_4(p,q,r,s) &= 2\left(|C_1(p)C_2(q)C_1(r)C_2(s)|^2 + |C_2(p)C_1(q)C_2(r)C_1(s)|^2\right), \\
\mathcal{N}_5(p,q,r,s) &= |C_1(p)C_2(q)C_1(r)C_1(s)|^2 + |C_2(p)C_1(q)C_2(r)C_2(s)|^2.
\end{aligned} \tag{40}
$$

$$\mathcal{K}_1(p,q,r,s) = \Big(C_1^*(p)C_1^*(q)C_2^*(r)C_2^*(s)C_2^*(p)C_1(q)C_2(r)C_2(s)+$$

$$+C_2^*(p)C_2^*(q)C_1^*(r)C_1^*(s)C_1^*(p)C_2(q)C_1(r)C_1(s)\Big)\delta\Big(\omega_{out}(p)+\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)\Big),$$

$$\mathcal{K}_2(p,q,r,s) = 2\Big(C_1^*(p)C_1^*(q)C_2^*(r)C_1(s)C_2^*(p)C_1(q)C_2(r)C_1^*(s)+$$

$$+C_2^*(p)C_2^*(q)C_1^*(r)C_2(s)C_1^*(p)C_2(q)C_1(r)C_2^*(s)\Big)\delta\Big(\omega_{out}(p)+\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)\Big),$$

$$\mathcal{K}_3(p,q,r,s) = \Big(C_1^*(p)C_2(q)C_2^*(r)C_2^*(s)C_2^*(p)C_2^*(q)C_2(r)C_2(s)+$$

$$+C_2^*(p)C_1(q)C_1^*(r)C_1^*(s)C_1^*(p)C_1^*(q)C_1(r)C_1(s)\Big)\Big)\delta\Big(\omega_{out}(p)+\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)\Big),$$

$$\mathcal{K}_4(p,q,r,s) =$$

$$= -\frac{i}{3\pi}\left(\frac{C_{\mathbf{1}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{1}}^*(s)C_{\mathbf{2}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{1}}(r)C_{\mathbf{1}}(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{2}}^*(s)C_{\mathbf{1}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{2}}(r)C_{\mathbf{2}}(s)}{\omega_{out}(p)+\omega_{out}(q)+\omega_{out}(r)+\omega_{out}(s)} + \right.$$

$$+3\,\frac{C_{\mathbf{1}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{2}}^*(s)C_{\mathbf{2}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{1}}(r)C_{\mathbf{2}}(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{1}}^*(s)C_{\mathbf{1}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{2}}(r)C_{\mathbf{1}}(s)}{\omega_{out}(p)+\omega_{out}(q)+\omega_{out}(r)-\omega_{out}(s)} +$$

$$+3\,\frac{C_{\mathbf{1}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{2}}^*(r)C_{\mathbf{2}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{2}}(r)C_{\mathbf{2}}(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{1}}^*(s)C_{\mathbf{1}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{1}}(r)C_{\mathbf{1}}(s)}{\omega_{out}(p)+\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)} +$$

$$\left. + \frac{C_{\mathbf{1}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{2}}^*(s)C_{\mathbf{2}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{2}}(r)C_{\mathbf{2}}(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{1}}^*(s)C_{\mathbf{1}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{1}}(r)C_{\mathbf{1}}(s)}{\omega_{out}(p)-\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)} \right),$$

$$\mathcal{K}_5(p,q,r,s) =$$

$$= -\frac{i}{\pi}\left(\frac{C_{\mathbf{1}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{2}}(s)C_{\mathbf{2}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{1}}(r)C_{\mathbf{2}}^*(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{1}}(s)C_{\mathbf{1}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{2}}(r)C_{\mathbf{1}}^*(s)}{\omega_{out}(p)+\omega_{out}(q)+\omega_{out}(r)+\omega_{out}(s)} + \right.$$

$$+ \frac{C_{\mathbf{1}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{1}}(s)C_{\mathbf{2}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{1}}(r)C_{\mathbf{1}}^*(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{2}}(s)C_{\mathbf{1}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{2}}(r)C_{\mathbf{2}}^*(s)}{\omega_{out}(p)+\omega_{out}(q)+\omega_{out}(r)-\omega_{out}(s)} +$$

$$+2\,\frac{C_{\mathbf{1}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{2}}(s)C_{\mathbf{2}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{1}}(r)C_{\mathbf{2}}^*(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{1}}(s)C_{\mathbf{1}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{2}}(r)C_{\mathbf{1}}^*(s)}{\omega_{out}(p)-\omega_{out}(q)+\omega_{out}(r)+\omega_{out}(s)} +$$

$$+2\,\frac{C_{\mathbf{1}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{1}}(s)C_{\mathbf{2}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{2}}(r)C_{\mathbf{1}}^*(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{2}}(s)C_{\mathbf{1}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{1}}(r)C_{\mathbf{2}}^*(s)}{\omega_{out}(p)+\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)} +$$

$$+ \frac{C_{\mathbf{1}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{2}}(s)C_{\mathbf{2}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{2}}(r)C_{\mathbf{2}}^*(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{1}}(s)C_{\mathbf{1}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{1}}(r)C_{\mathbf{1}}^*(s)}{\omega_{out}(p)-\omega_{out}(q)-\omega_{out}(r)+\omega_{out}(s)} +$$

$$\left. + \frac{C_{\mathbf{1}}^*(p)C_{\mathbf{2}}^*(q)C_{\mathbf{2}}^*(r)C_{\mathbf{1}}(s)C_{\mathbf{2}}^*(p)C_{\mathbf{2}}(q)C_{\mathbf{2}}(r)C_{\mathbf{1}}^*(s) - C_{\mathbf{2}}^*(p)C_{\mathbf{1}}^*(q)C_{\mathbf{1}}^*(r)C_{\mathbf{2}}(s)C_{\mathbf{1}}^*(p)C_{\mathbf{1}}(q)C_{\mathbf{1}}(r)C_{\mathbf{2}}^*(s)}{\omega_{out}(p)-\omega_{out}(q)-\omega_{out}(r)-\omega_{out}(s)} \right).$$