

The Penrose and BSW effects in Vaidya spacetimes

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$$ds^2 = - \left(1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2} \right) + 2dvdr + r^2d\Omega^2. \quad (1)$$

Here $M(v)$ and $Q(v)$ are mass and charge function which depend upon the time v .

$$r < r_C = \frac{Q\dot{Q}}{\dot{M}}. \quad (2)$$

$$L = r^2 \frac{d\varphi}{d\lambda}, \quad (3)$$

$$E(v) = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) \frac{dv}{d\lambda} - \frac{dr}{d\lambda} + \frac{QQ^*}{r}. \quad (3)$$

Energy

$$E_{max} = \frac{QQ^*}{r} + \sqrt{\left(1 - \frac{2M}{r} + \frac{QQ^*}{r^2}\right) \left(\frac{L^2}{r^2} + 1\right) + \left(\frac{dr}{d\lambda}\right)^2}. \quad (4)$$

$$-QQ^* \geq r^2 \sqrt{1 - \frac{2M}{r} + \frac{QQ^*}{r^2}}. \quad (5)$$

$$\begin{aligned} M(v) &= \nu v, \nu > 0, \\ Q(v) &= \mu v, \mu \neq 0. \end{aligned} \quad (6)$$

Conformal coordinates

$$\begin{aligned} v &= r_0 e^{\frac{t}{r_0}}, \\ r &= R e^{\frac{t}{r_0}}. \end{aligned} \tag{7}$$

$$ds^2 = e^{\frac{2t}{r_0}} \left[-\left(1 - \frac{2\nu r_0}{R} + \frac{\mu^2 r_0^2}{R^2} - 2\frac{R}{r_0} \right) dt^2 + 2dtdR + R^2 d\Omega^2 \right]. \tag{8}$$

$$1 - \frac{2\nu r_0}{R} + \frac{\mu^2 r_0^2}{R^2} - 2\frac{R}{r_0} > 0. \tag{9}$$

New energy

$$E = e^{\frac{2t}{r_0}} \left[\left(1 - \frac{2\nu r_0}{R} + \frac{\mu^2 r_0^2}{R^2} - 2\frac{R}{r_0} \right) \frac{dt}{d\lambda} - \frac{dR}{d\lambda} \right] + \frac{\mu Q^* r_0}{R}, \quad (10)$$

$$L = e^{\frac{2t}{r_0}} R^2 \frac{d\varphi}{d\lambda}.$$

$$E = \frac{\mu Q^* r_0}{R} + \\ e^{\frac{t}{r_0}} \sqrt{\left(1 - \frac{2\nu r_0}{R} + \frac{\mu^2 r_0^2}{R^2} - 2\frac{R}{r_0} \right) \left(\frac{L^2}{R^2 e^{\frac{2t}{r_0}}} + 1 \right) + e^{\frac{2t}{r_0}} \left(\frac{dR}{d\lambda} \right)^2}. \quad (11)$$

$$-\mu Q^* r_0 \geq e^{\frac{t}{r_0}} R \sqrt{1 - \frac{2\nu r_0}{R} + \frac{\mu^2 r_0^2}{R^2} - 2\frac{R}{r_0}}. \quad (12)$$

Penrose process

$$E_0 m_0 = E_1 m_1 + E_2 m_2 , \quad (13)$$

$$m_0^2 \geq m_1^2 + m_2^2 . \quad (14)$$

$$\eta = \frac{m_2 E_2}{m_0 E_0} - 1 = -\frac{m_1 E_1}{m_0 E_0} . \quad (15)$$

Efficiency

$$\left(\frac{dR}{d\lambda} \right)_1 = \left(\frac{d\varphi}{d\lambda} \right)_1 = 0, \quad (16)$$

$$E = \frac{\mu Q^* r_0}{R} + e^{\frac{t}{r_0}} \sqrt{\left(1 - \frac{2\nu r_0}{R} + \frac{\mu^2 r_0^2}{R^2} - 2\frac{R}{r_0} \right)}. \quad (16)$$

$$m_2^2 = \left[\frac{m_0 \left(\frac{dR}{d\lambda} \right)_0}{\left(\frac{dR}{d\lambda} \right)_2} \right]^2. \quad (17)$$

$$\eta < - \left[\frac{\mu Q^* r_0}{R} + e^{\frac{t}{r_0}} \sqrt{\left(1 - \frac{2\nu r_0}{R} + \frac{\mu^2 r_0^2}{R^2} - 2\frac{R}{r_0} \right)} \right] \sqrt{1 - \frac{\left(\frac{dR}{d\lambda} \right)_0^2}{\left(\frac{dR}{d\lambda} \right)_2^2}}. \quad (18)$$

Conformal horizon

$$1 - \frac{2\nu r_0}{R_{max}} + \frac{\mu^2 r_0^2}{R_{max}^2} - 2\frac{R_{max}}{r_0} = 0. \quad (19)$$

$$\eta < -\frac{\mu Q_1^* r_0}{R_{max}} \sqrt{1 - \frac{\left(\frac{dR}{d\lambda}\right)_0^2}{\left(\frac{dR}{d\lambda}\right)_2^2}}. \quad (20)$$

Vaidya spacetime

$$ds^2 = e^{\frac{2t}{r_0}} \left[-\left(1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0}\right) dt^2 + 2dtdR + R^2 d\Omega^2 \right]. \quad (21)$$

$$1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0} > 0. \quad (22)$$

$$E = e^{\frac{2t}{r_0}} \left(1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0}\right) \frac{dt}{d\lambda} - e^{\frac{2t}{r_0}} \frac{dR}{d\lambda}. \quad (23)$$

Radial

$$e^{\frac{4t}{r_0}} \left(\frac{dR}{d\lambda} \right)^2 = E^2 - e^{\frac{2t}{r_0}} \left(1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0} \right) \left(\frac{L^2}{r^2 e^{\frac{2t}{r_0}}} + 1 \right) = e^{\frac{4t}{r_0}} P_R^2. \quad (24)$$

$$E_{c.m.} = m_0 \sqrt{2} \sqrt{1 - g_{ik} u_1^i u_2^k}. \quad (25)$$

BSW effect

$$\frac{E_{c.m.}^2}{2m_0} = 1 + \frac{E_1 \left(E_2 + e^{\frac{2t}{r_0}} P_{2R} \right)}{e^{\frac{2t}{r_0}} \left(1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0} \right)} - \frac{e^{\frac{2t}{r_0}} \left(E_1 + e^{\frac{2t}{r_0}} P_{1R} \right) P_{2R}}{1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0}} - \frac{L_1 L_2}{e^{\frac{2t}{r_0}} R^2}. \quad (26)$$

$$1 - \frac{2\mu r_0}{R_{kh}} - \frac{2R_{kh}}{r_0} = 0. \quad (27)$$

$$\lim_{R \rightarrow R_{kh}} e^{\frac{2t}{r_0}} P_{1R} = +E, \quad (28)$$

$$\lim_{R \rightarrow R_{kh}} e^{\frac{2t}{r_0}} P_{2R} = -E.$$

Unbound energy

$$\lim_{R \rightarrow R_{kh}} -\frac{e^{\frac{2t}{r_0}} \left(E_1 + e^{\frac{2t}{r_0}} P_{1R} \right) P_{2R}}{1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0}} = \frac{e^{2t} r_0 2E_1 E_2}{1 - \frac{2\mu r_0}{R} - \frac{2R}{r_0}} \rightarrow +\infty. \quad (29)$$