# Tsallis holographic dark energy model with event horizon cutoff in modified gravity

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# Outline

1. Introduction
2. Previous articles
3. Present study

# Introduction

# Accelerating expansion of the universe

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Problems with Standard Model:

- Smallness of the cosmological constant
- The coincidence problem

# Modified gravity

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then in the FLRW metric the Friedmann equation has the form

$$H^2 = \frac{1}{3f'(R)}\left(\rho + \frac{Rf'(R) - f(R)}{2} - 3H\dot{R}f''(R)\right)$$

# Holographic dark energy model

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A model of holographic dark energy was proposed:

$$\rho_{de} = \frac{3C^2}{L^2},$$

where C is a constant and L can be chosen as:

$$L = \frac{1}{H}, \qquad L = a\int_0^t \frac{dt}{a}, \qquad L = a\int_t^\infty \frac{dt}{a}, \qquad L = R = 6\left(\dot{H} + 2H^2\right)$$

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# Previous articles

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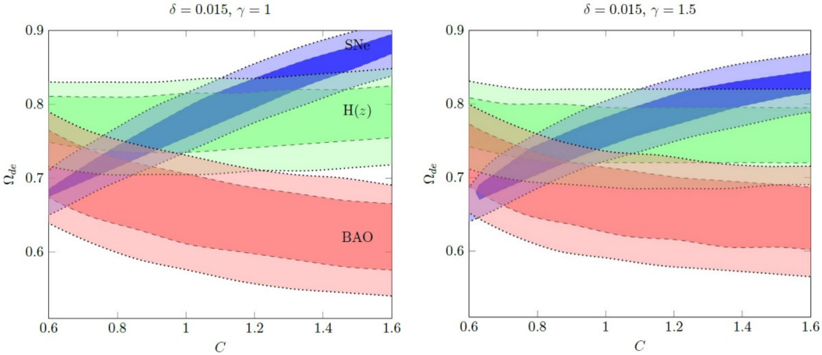


$1\sigma$ (dashed lines) and $2\sigma$ (dotted lines) allowed areas on plane $C - \Omega_{de}$

- Tsalis models of holographic dark energy in Friedman cosmology ($\delta = 0$) are in good agreement with observational data
- As $\delta$ increases, the range of acceptable parameter values decreases, but still, for some values, the models will fit the observables almost as well as the standard model
- In all models, a Big Rip singularity occurs

# Crossing of phantom divide line in model of interacting Tsallis holographic dark energy

Model with interaction:

$$H^2 = \frac{1}{3}(\rho_m + \rho_{de})$$

$$\dot{\rho_m} + 3H\rho_m = Q$$

$$\dot{\rho_{de}} + 3H(\rho_{de} + p_{de}) = -Q$$

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*Universe 2022, 8, 265.*

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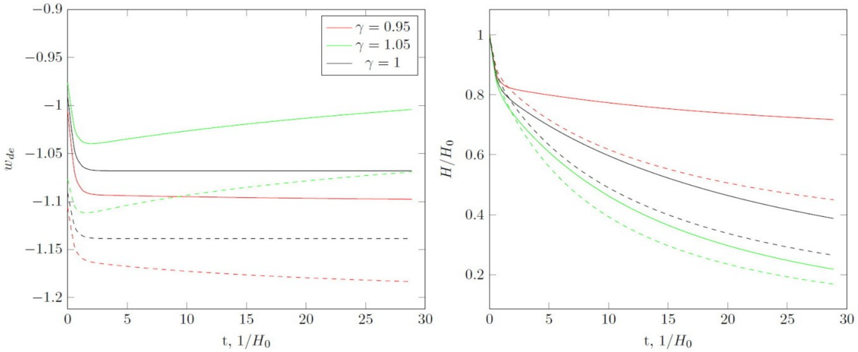


The effective state parameter for holographic dark energy and the Hubble parameter

# Results

- The interaction can lead to phantomization for dark energy but without a singularity in the future
- For $C = 1$, $\gamma > 1$ and $d^2 > 0$, the value of the state parameter $w_{de}$ can intersect the phantom line $w_{ph} = -1$
- For $C > 1$ and $\gamma \geq 1$, the "quintessence" with $w > -1$ due to the interaction is possible: the Hubble parameter decreases and the universe expansion decelerates

# Present study

Modified gravity model of Carroll-Duvvuri-Trodden-Turner (CDTT):

$$H^2 = \frac{1}{3f'(R)}\left(\rho + \frac{Rf'(R) - f(R)}{2} - 3H\dot{R}f''(R)\right)$$

$$f(R) = R + \lambda R^2 - \sigma\frac{\mu}{R}$$

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*Int. J. Geom. Meth. Mod. Phys., 2022. arXiv:2208.13320v1*

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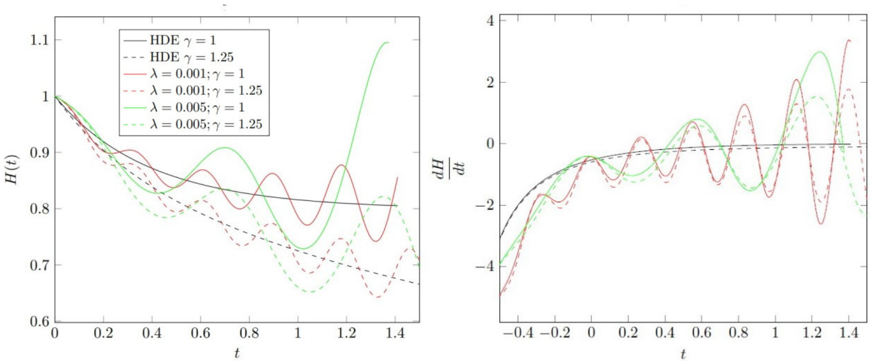
We got a system of equations:

$$\frac{d^2H}{dz^2} = -\frac{f'(R)}{6H(z+1)^2 f''(R)} + \frac{1}{18H^3(z+1)^2 f''(R)}\left(\rho + \frac{Rf'(R) - f(R)}{2}\right) + \\ + \frac{1}{H(z+1)}\left(\left(\frac{dH}{dz}\right)^2 (z+1) - 3H\frac{dH}{dz}\right),$$

$$\frac{dL'}{dz} = \frac{1}{H}$$

# Oscillations




Dependence of the Hubble parameter $H$ and the derivative of the Hubble parameter $\dot{H}$ with time in the past and future (time is given in units of $\frac{1}{H_0}$ and Hubble parameter in units of $H_0$ )

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  $\chi^2$ = 17.097 ($H_0 = 67.89$ km/s/Mpc) for $\Omega = 0.717$, $\dot{H}(0) = -0.21H_0^2$;
  $\chi^2$ = 16.955 ($H_0 = 67.64$ km/s/Mpc) for $\Omega = 0.713$, $\dot{H}(0) = -0.42H_0^2$;
  $\chi^2$ = 16.799 ($H_0 = 67.13$ km/s/Mpc) for $\Omega = 0.705$, $\dot{H}(0) = -0.84H_0^2$.
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  For ΛCDM model $\bar{\chi}_H^2$ is minimal for $\Omega_\Lambda = 0.737$ and equal to 19.262
- But for these parameters SNe data are described worse: for given $\dot{H}(0)$ there is a significant discrepancy between optimal value of $\Omega$ from two data sets.

| $\lambda$ \ $\sigma\mu$ | -0.003 | -0.001 | 0 | 0.001 | 0.003 |
|---|---|---|---|---|---|
| $\gamma = 1$ | | | | | |
| 0.001 | 1.5005 | 1.4696 | 2.3678 | 0.9069 | 0.6107 |
| 0.005 | 2.0362 | 1.6923 | 1.9583 | 1.4120 | 0.8412 |
| $\gamma = 1.25$ | | | | | |
| 0.001 | 1.5112 | 1.4551 | 3.2991 | 0.6302 | 0.3409 |
| 0.005 | 1.5234 | 1.7937 | 2.6955 | 0.8382 | 0.7934 |

Time to the final singularity for different $\mu$ and $\lambda$ ($C = 1$, $\Omega = 0.72$).

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- The amplitude of this oscillations grows with time in future. For $\mu \neq 0$ a future singularity arises corresponding to zero of first derivative of $f(R)$ for some $R$.
- The time before singularity, as determined by the value of $\dot{H}$ for the initial moment in time, can vary in wide limits.
- Such models for some parameters can describe observational data for SN Ia and dependence $H(z)$ with sufficient accuracy especially for $\gamma = 1$ and larger values of $\dot{H}$ in comparison with $\Lambda$CDM model.

# Thanks for attention