Magnon spectrum of skyrmion crystals in stereographic projection approach

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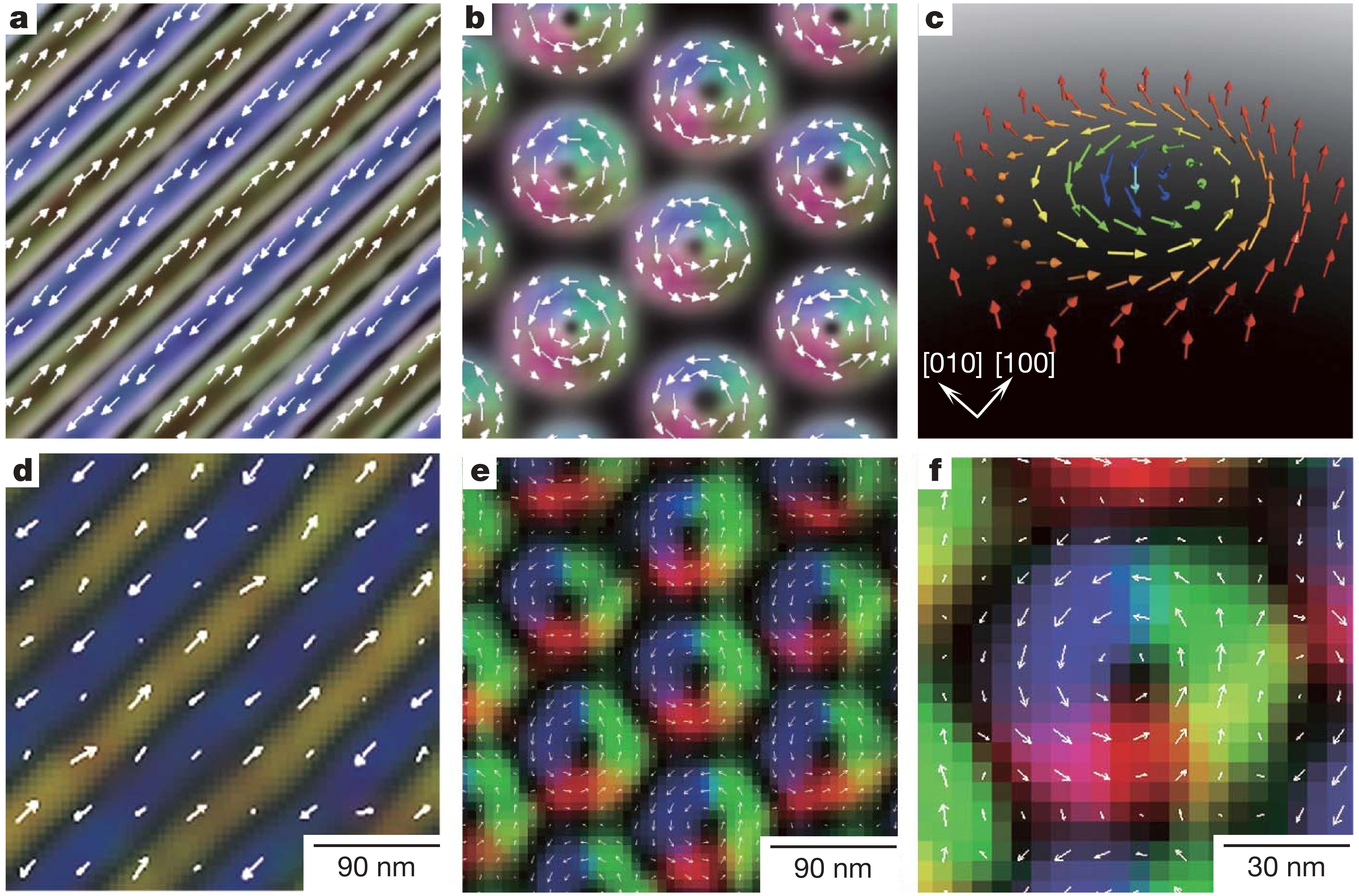
Plan of the talk

- Skyrmions, skyrmion crystal, minimal model, known results
- Stereographic projection, Ansatz for skyrmion crystal
- Energy and Lagrangian, equations of motion
- Spectrum, its evolution with magnetic field
- Magnon bands topology, Berry curvature, Chern numbers
- Conclusions and outlook

skyrmion crystals



theory

experiment
TEM

Yu et al., Nature (2010)

0 T 50 mT

helical magnet Fe0.5Co0.5Si

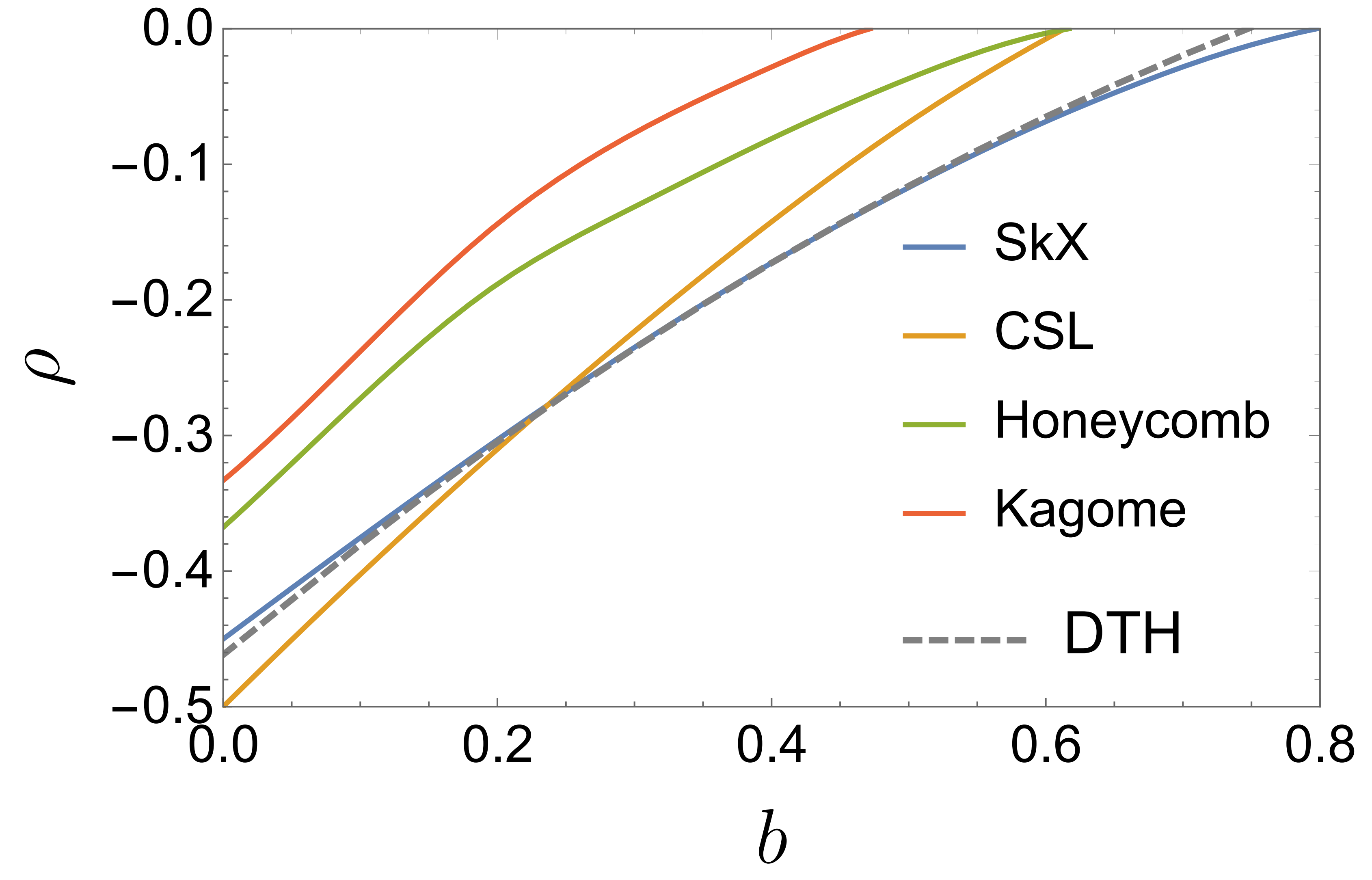
Minimal continuum model, 2D

$$\mathscr{E} = \frac{1}{2} C \partial_\mu S_i \partial_\mu S_i - D \epsilon_{\mu ij} S_i \partial_\mu S_j + B(1 - S_3)$$

Length in units of $l = C/D$ (helix pitch),
Magnetic field in units $D^2/C = B/b$

$$\mathscr{E} = \frac{1}{2} \partial_\mu S_i \partial_\mu S_i - \{\epsilon_{\mu ij} S_i \partial_\mu S_j\} + b(1 - S_3)$$

Phase diagram at $T = 0$:
Simple helix $0 < b < 0.25$
Skyrmion phase $0.25 < b < 0.8$
Uniform ferromagnet $b > 0.8$



Timofeev et al., PRB (2021)

Magnon bands in SkX and topology

Schütte, Garst, Phys.Rev. B (2014)

Roldán-Molina, Núñez, Fernández-Rossier, New J. Phys. (2016)

M.Garst in «The 2020 skyrmionics roadmap» J. Phys. D: Appl. Phys. (2020)

Dıaz, Hirosawa, Klinovaja, Loss, Phys. Rev. Research (2020)

Two-step procedure

1) Equilibrium local magnetization direction (Monte-Carlo?)
2) Boson representation of spins in local frame

Stereographic projection approach :

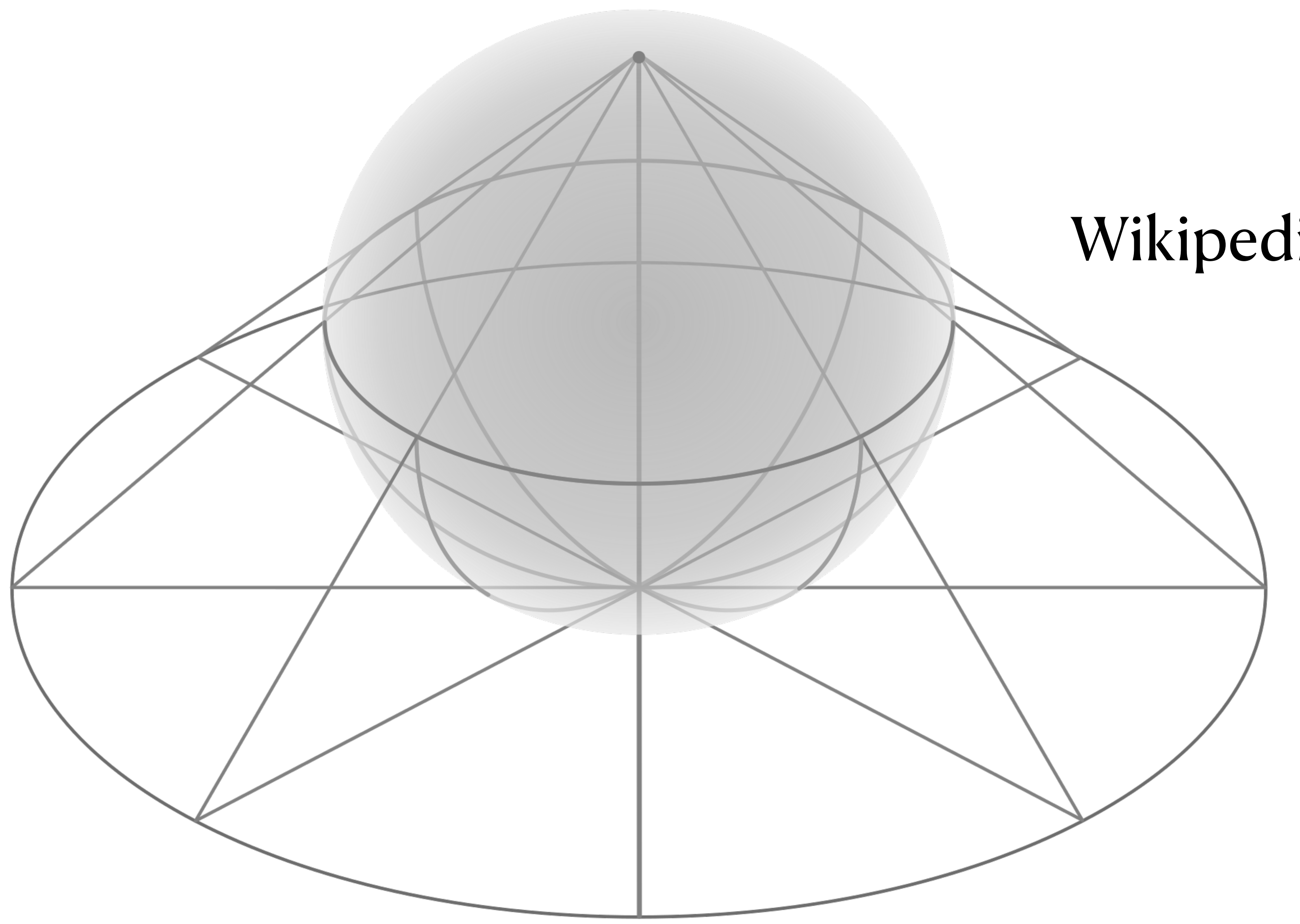
Timofeev, Aristov, Phys.Rev. B (2022)

Stereographic projection

$$S^1 + iS^2 = \frac{2f}{1+f\bar{f}}, \quad S^3 = \frac{1-f\bar{f}}{1+f\bar{f}}$$

$\mathbf{S} = (0,0,1) \leftrightarrow f = 0$

$\mathbf{S} = (0,0,-1) \leftrightarrow f = \infty$



Wikipedia

$(z = x + iy, \bar{z} = x - iy)$

Topological charge:

$$Q = \frac{1}{4\pi}\int d^2\mathbf{r}\ \frac{4(\partial_z f \partial_{\bar{z}} \bar{f} - \partial_z \bar{f} \partial_{\bar{z}} f)}{(1+f\bar{f})^2}$$

Energy and Lagrangian

$$L = \int d^2\mathbf{r}\,(\mathcal{T} - \mathcal{E}) \qquad \mathcal{T}[f] = \frac{i}{2}\frac{\bar{f}\partial_t f - f\partial_t \bar{f}}{1+f\bar{f}}$$

$$\mathcal{E} = \frac{4(\partial_z f \partial_{\bar{z}} \bar{f} + \partial_z \bar{f} \partial_{\bar{z}} f)}{(1+f\bar{f})^2} + \left\{ \frac{2i(\bar{f}^2 \partial_{\bar{z}} f + \partial_{\bar{z}} \bar{f} - \partial_z f - f^2 \partial_z \bar{f})}{(1+f\bar{f})^2} \right\} + \frac{2bf\bar{f}}{1+f\bar{f}}$$

Variation: $\delta L/\delta f = 0 \Rightarrow$

$$2f\partial_z \bar{f}\partial_{\bar{z}}\bar{f} - (1+f\bar{f})\partial_z\partial_{\bar{z}}\bar{f} - i\{\bar{f}\partial_{\bar{z}}\bar{f} + f\partial_z \bar{f}\} + \frac{1}{4}b\bar{f}(1+f\bar{f}) = 0$$

Any (anti)holomorphic f

Belavin,Polyakov (1975): $D = B = 0$

$$2f\partial_z \bar{f}\partial_{\bar{z}}\bar{f} - (1+f\bar{f})\partial_z\partial_{\bar{z}}\bar{f} = 0$$

See also: Metlov, PRB (2013)

Ansatz for skyrmion crystal

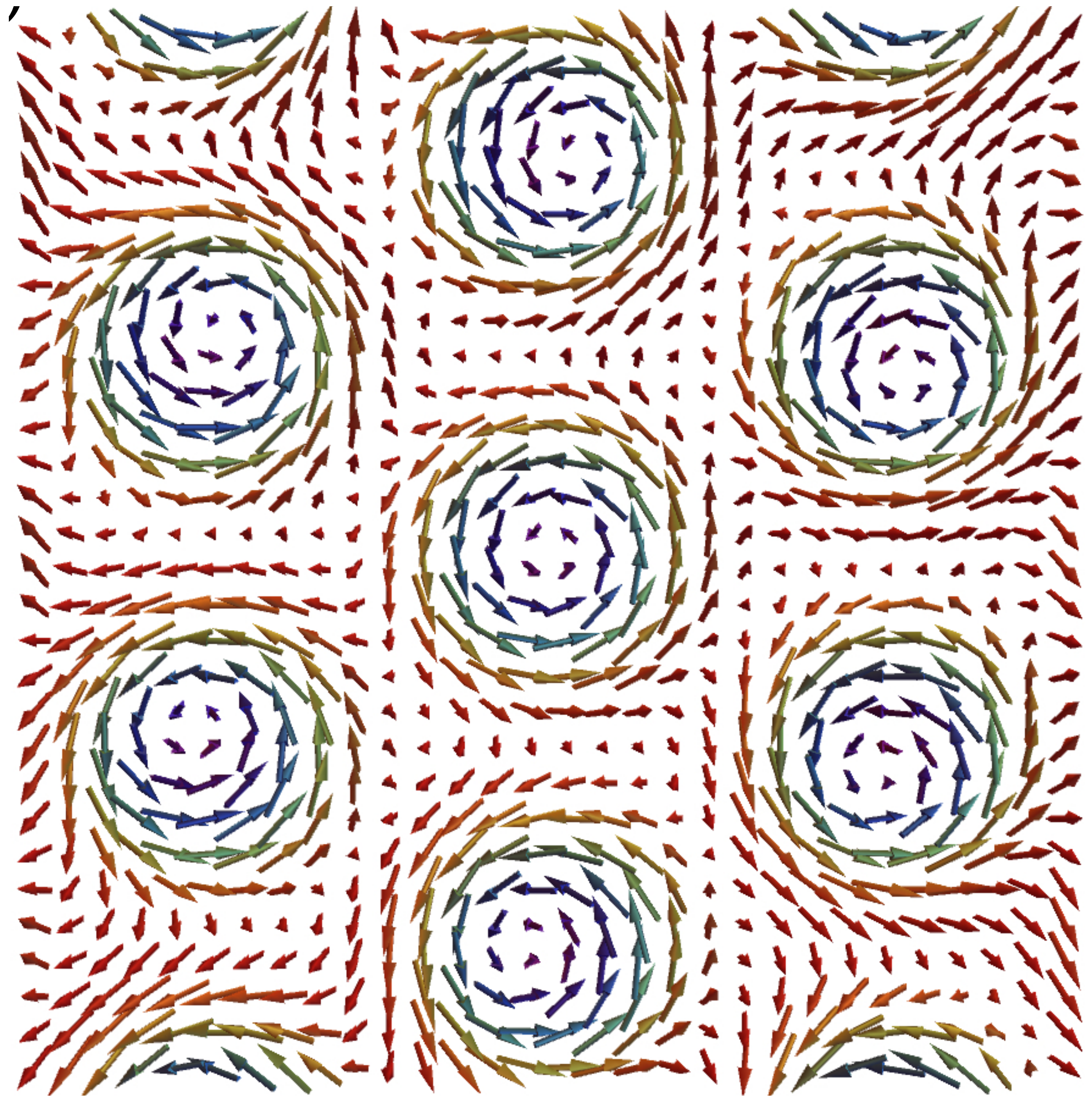
Belavin, Polyakov: $f = \sum z_j/(\bar{z} - Z_j)$

size z_j, position Z_j

Now $D \neq 0, B \neq 0$

$$f_{SkX}(a, z_0) = \sum_{n,m} f_1(\mathbf{r} - n\mathbf{a}_1 - m\mathbf{a}_2)$$

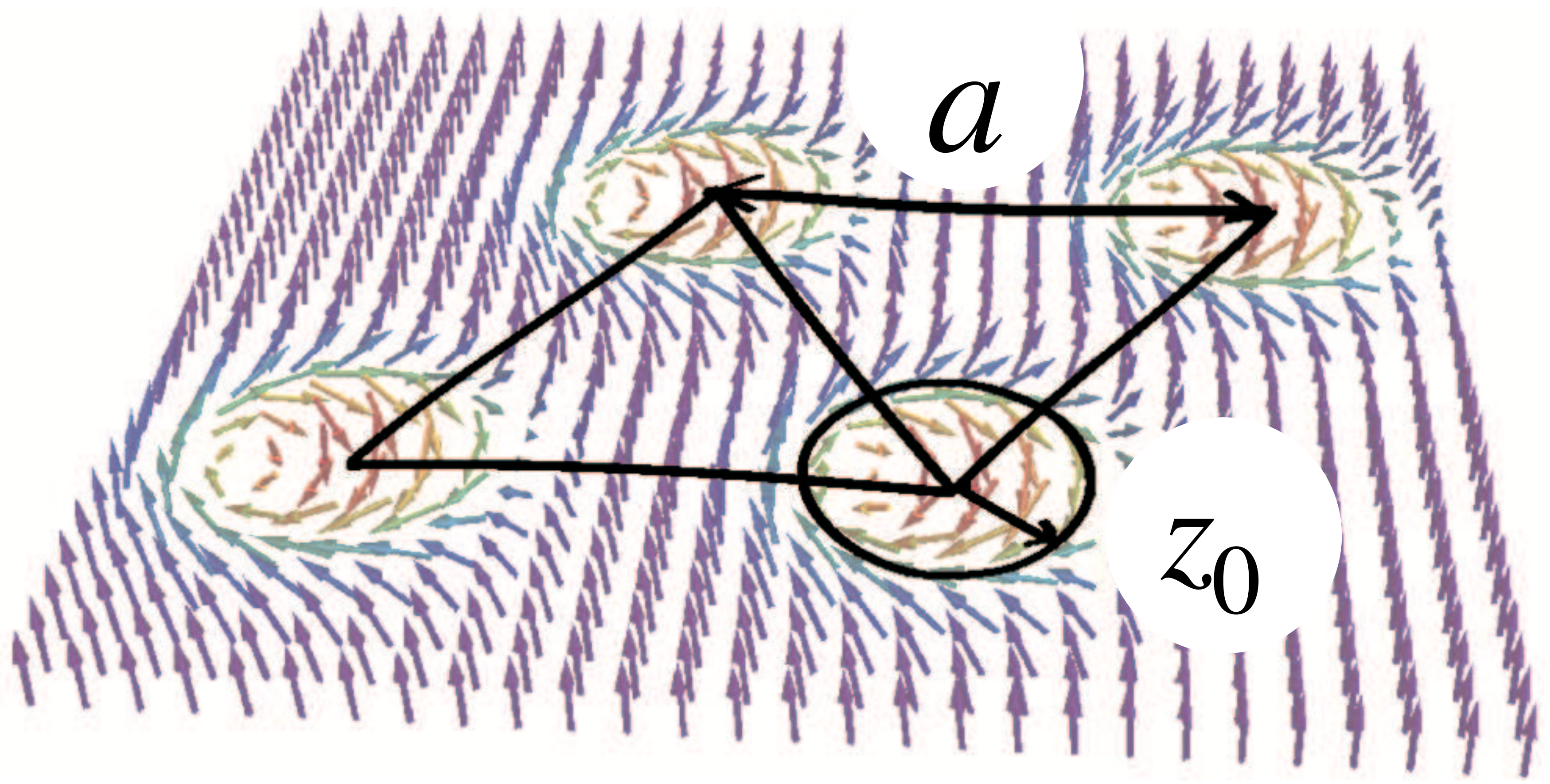
$$f_1 = \frac{i\, z_0\, \kappa(z\bar{z}/z_0^2)}{\bar{z}}$$

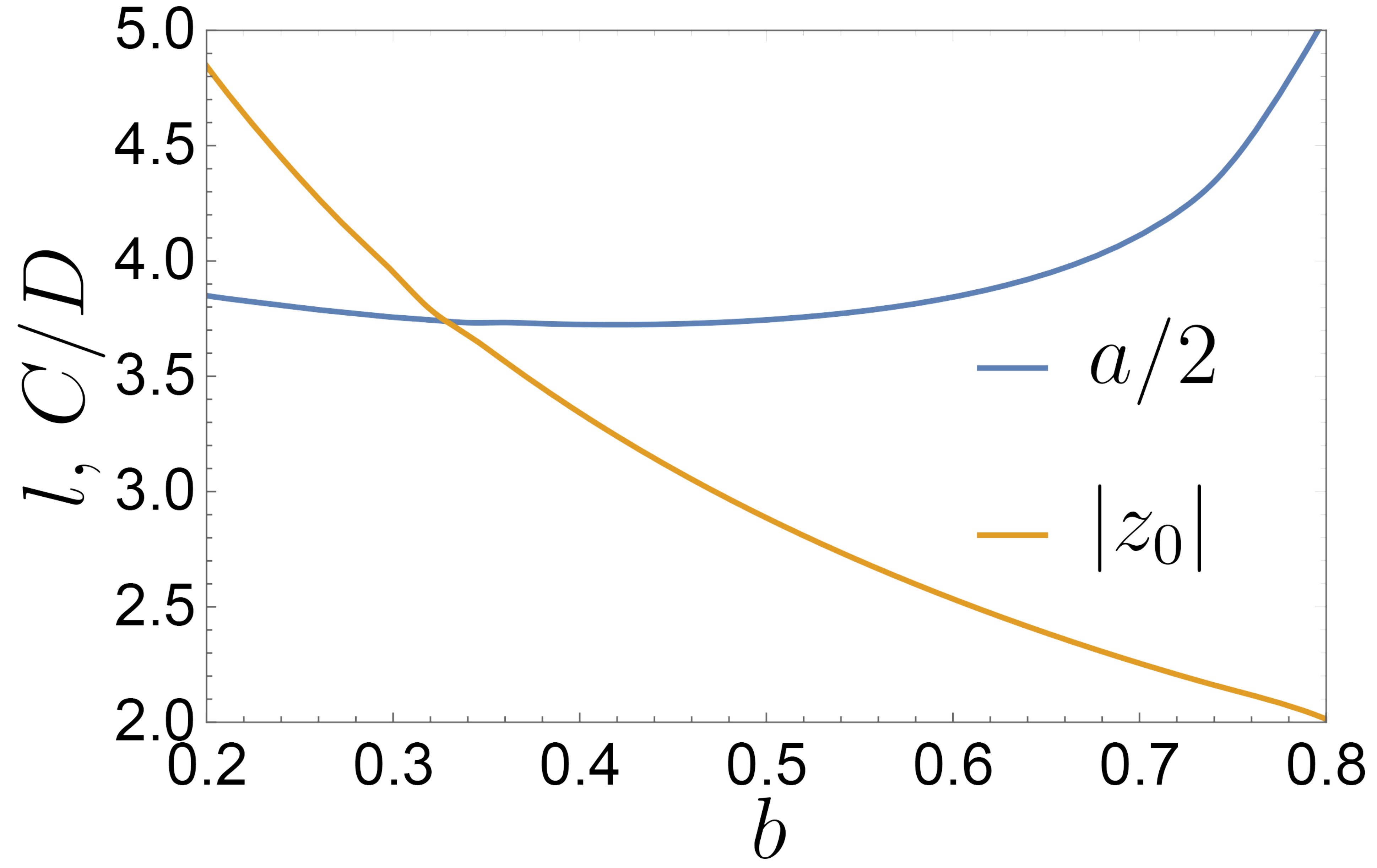


Timofeev, Sorokin, Aristov, *Towards an effective theory of skyrmion crystals*, JETP Letters (2019)

Timofeev, Sorokin, Aristov, *Triple helix versus skyrmion lattice …*, PRB (2021)

Ansatz for skyrmion crystal





$$f_1 = \frac{i\, z_0\, \kappa(z\bar{z}/z_0^2)}{\bar{z}}, \quad \kappa(r^2) \simeq \exp(-r^2)$$

Timofeev, Sorokin, Aristov, *Towards an effective theory of skyrmion crystals*, JETP Letters (2019)

Timofeev, Sorokin, Aristov, *Triple helix versus skyrmion lattice …*, PRB (2021)

Semiclassical method

$$f(t,z,\bar{z}) = f_0(z,\bar{z}) + \delta f(t,z,\bar{z})$$

$$\mathscr{L}[f_0+\delta f] = \mathscr{L}[f_0] + \delta f\,\mathscr{L}_1[f_0] + \frac{1}{2}\delta f\,\delta f\,\mathscr{L}_2[f_0] + \dots$$

Linear spin-wave theory

Overall translation $\mathbf{R}(t)$ = «Zero mode»

$$f(\mathbf{r}) = f_0 + (1 + f_0\bar{f}_0)\,\psi(\mathbf{r} - \mathbf{R}(t))$$

$$\mathscr{L} = \frac{1}{2}(\bar{\psi}, \quad \psi)\left(-i\begin{pmatrix}\partial_t & 0\\ 0 & -\partial_t\end{pmatrix} - \hat{\mathscr{H}}\right)\begin{pmatrix}\psi\\ \bar{\psi}\end{pmatrix}$$

Equations of motion

$$\hat{\mathscr{H}} = \begin{pmatrix} (-i\nabla + \mathbf{A})^2 + U & V \\ V^* & (i\nabla + \mathbf{A})^2 + U \end{pmatrix}$$

``Bogoliubov-de Gennes”

Any f providing extremum to the action

$$U = -4\frac{\partial_z f \partial_{\bar{z}} \bar{f} + \partial_z \bar{f} \partial_{\bar{z}} f}{(1+f\bar{f})^2} + b\frac{1-f\bar{f}}{1+f\bar{f}} + \left\{ \frac{2i(f^2\partial_z \bar{f} + \partial_z f - \partial_{\bar{z}} \bar{f} - \bar{f}^2 \partial_{\bar{z}} f + 2if\bar{f})}{(1+f\bar{f})^2} \right\}$$

$$V = 8\frac{\partial_z f \partial_{\bar{z}} f(1-2f\bar{f}) + f(1+f\bar{f})\partial_z \partial_{\bar{z}} f}{(1+f\bar{f})^2} - \left\{ \frac{4i(3f^2\partial_z f - \partial_{\bar{z}} f(1-2f\bar{f}))}{(1+f\bar{f})^2} \right\} - b\frac{2f^2}{1+f\bar{f}}$$

$$A_x = \frac{if\partial_x \bar{f} - i\bar{f}\partial_x f}{1+f\bar{f}} + \left\{ \frac{4\,\mathrm{Re} f}{1+f\bar{f}} \right\}$$

Gauge vector potential

Bogoliubov spinor

$$\boxed{\left(\epsilon_n \sigma_3 - \hat{\mathscr{H}}\right)\begin{pmatrix} u_n \\ v_n \end{pmatrix} = 0}$$

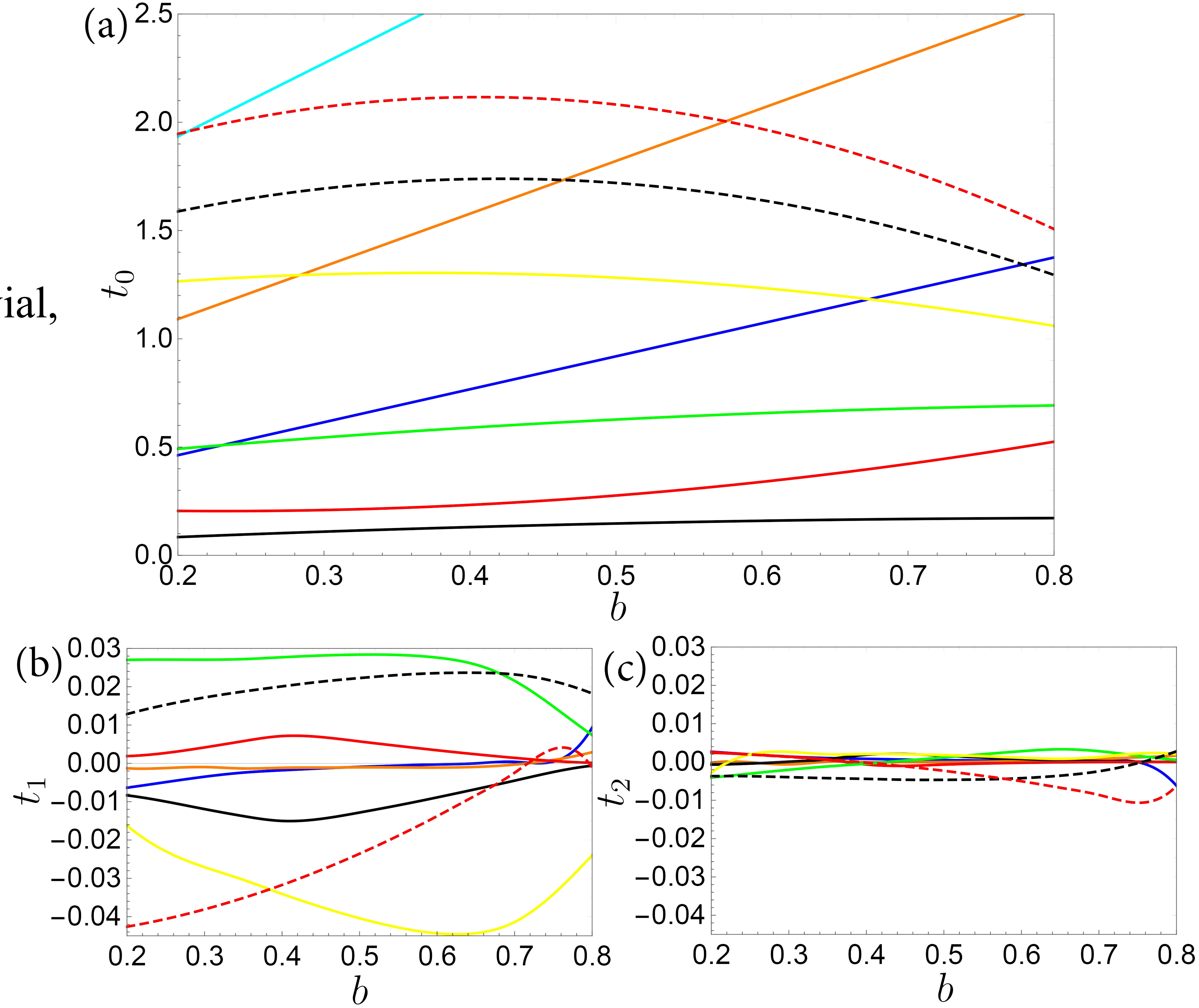
Spectrum: evolution with B

Spectrum: tight-binding fit

Two sets of bands:

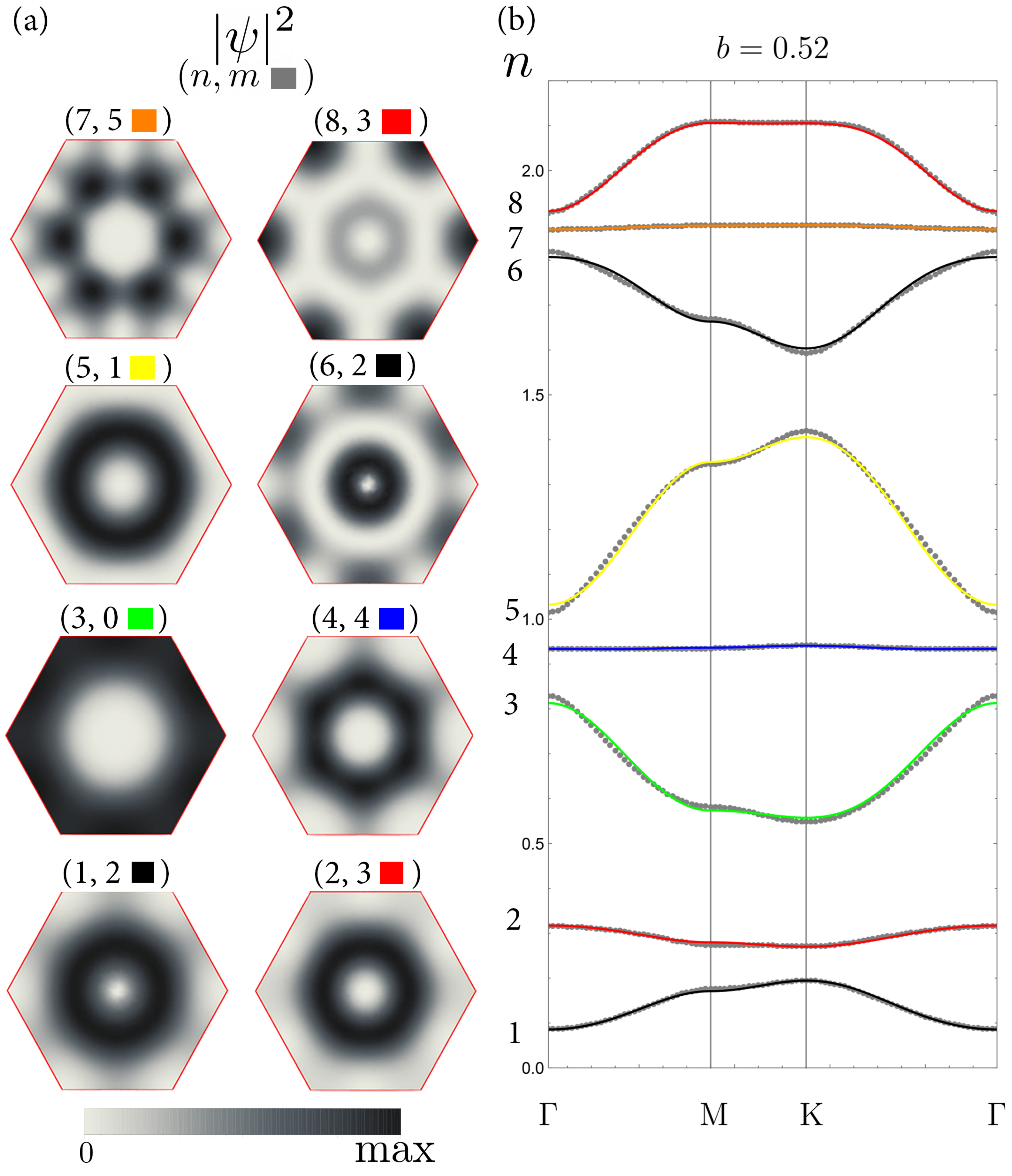
i) flat bands, topologically trivial,
fast evolution with B

ii) tight-binding form, topologically non-trivial,
steady with B



$$\hat{h} = \sum_i t_0\, c_i^\dagger c_i + \sum_{\langle i,j \rangle} t_1\, c_j^\dagger c_i + \sum_{\langle\langle i,j \rangle\rangle} t_2\, c_j^\dagger c_i$$

Spectrum: types of deformation



* Bogoliubov u-v spinors,
most weight in the upper (u) component

* Bloch function strongly varying in the unit cell

* behavior at centers of the skyrmions, $\psi \sim \exp im\phi$

Deformations of skyrmions:

m=0 counterclockwise rotation

m=1 breathing mode

m=2 clockwise rotation, «zero mode»

m=3 elliptical deformation

m=4 triangular deformation, etc.

Magnon bands topology

Berry curvature, Chern numbers

Bloch state of nth band. $\Psi_{n\mathbf{k}} = e^{i\mathbf{k}\mathbf{r}}\mathscr{V}_{n\mathbf{k}}(\mathbf{r})$

(assuming it is a smooth function of $\mathbf{k}$)

Berry connection $\mathscr{A}_{n,\mu}(\mathbf{k}) = -\langle \mathscr{V}_{n\mathbf{k}} | i\partial_\mu | \mathscr{V}_{n\mathbf{k}} \rangle$

Berry curvature $\Omega_{n,\mu\nu}(\mathbf{k}) = \partial_\mu \mathscr{A}_{n,\nu}(\mathbf{k}) - \partial_\nu \mathscr{A}_{n,\mu}(\mathbf{k})$

Chern number $C_n = \dfrac{1}{2\pi}\displaystyle\int_{\mathrm{BZ}} \Omega_{n,12}(\mathbf{k})\, d\mathbf{k}$

Link-variable method

Fukui, Hatsugai, Suzuki, JPSJ (2005)

Magnon bands topology

Clearly separated bands for $b = 0.52$

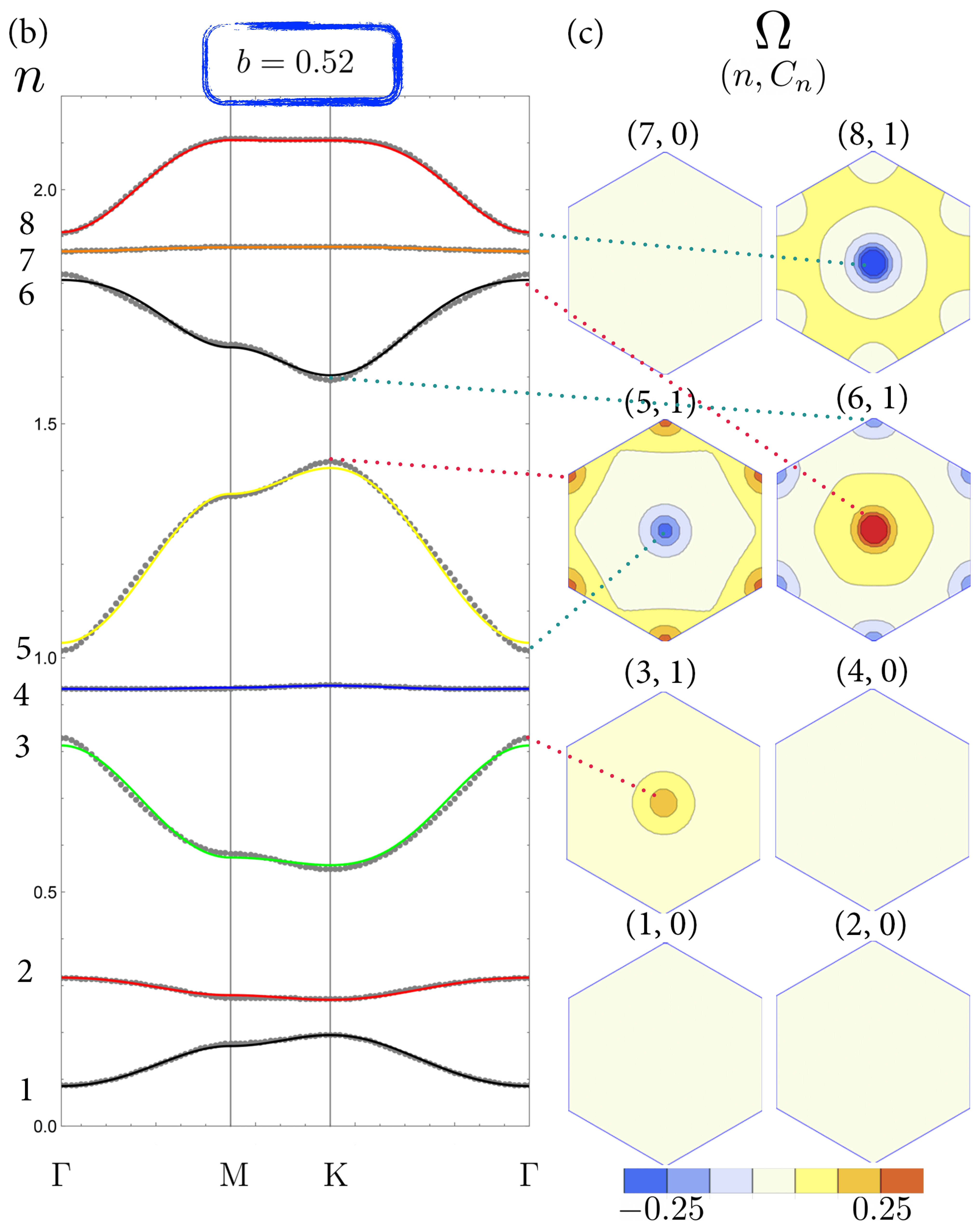
Flat bands => zero Berry curvature in B.z.

Non-flat bands => non-zero Berry curvature.
However Chern number may be zero,
always non-negative !!

Berry curvature:
smooth background + peaks around Γ, K

Influence of higher-energy bands

Neighboring bands



Conclusions and outlook

- Stereographic projection: unified approach to spin dynamics in SkX
- Other types of ordering (square lattice), interaction (uniaxial anisotropy)
- Spin susceptibilities
- Melting of Skyrmion crystal
- Thermal Hall effect
- Topological magnon edge states

Thank you !

Melting of Skyrmion crystal

$(b \geq 0.6)$

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nature nanotechnology

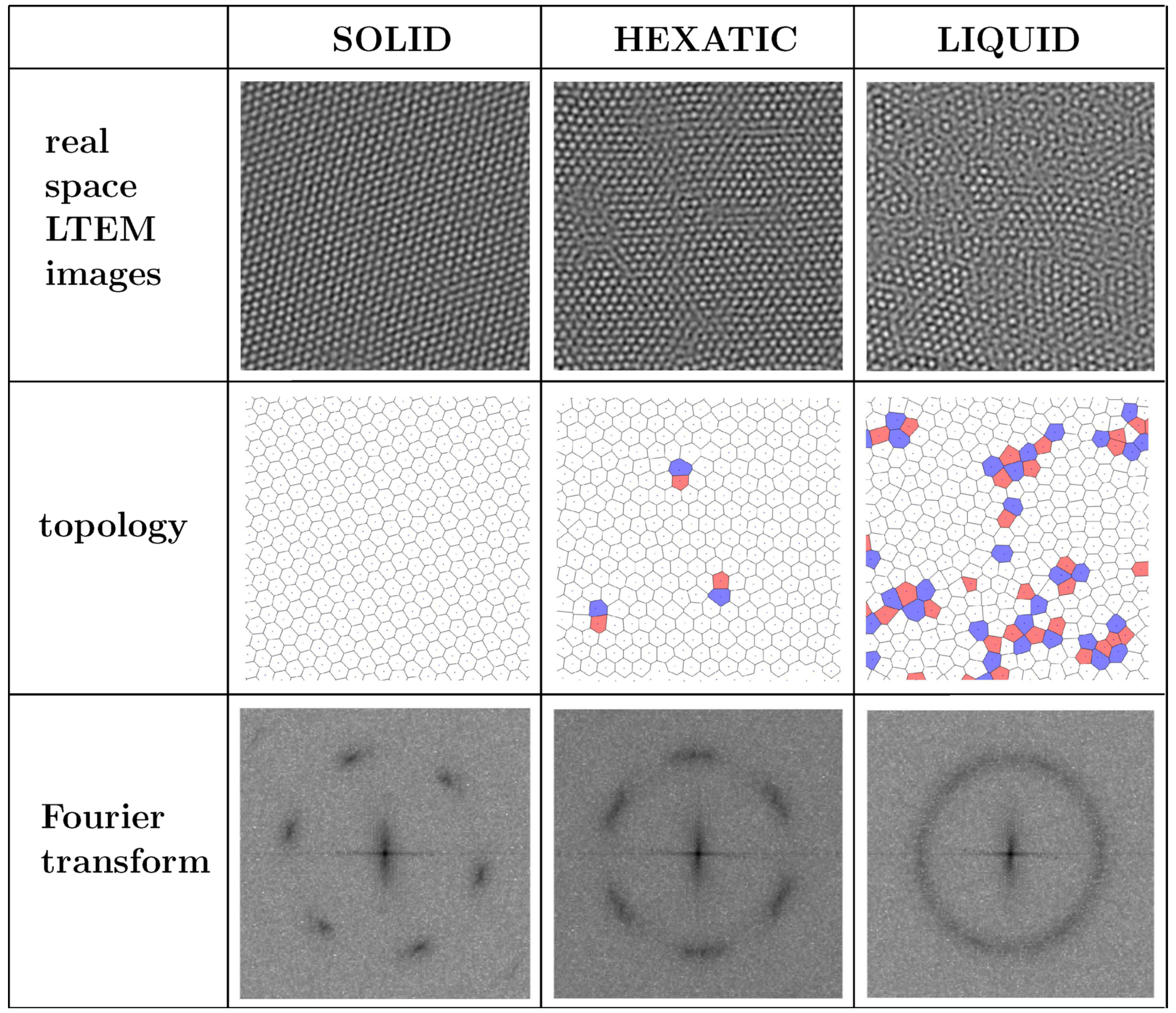
ARTICLES

https://doi.org/10.1038/s41565-020-0716-3

Check for updates

Melting of a skyrmion lattice to a skyrmion liquid via a hexatic phase

Ping Huang[1,2,3,7], Thomas Schönenberger[2,7], Marco Cantoni[4], Lukas Heinen[5], Arnaud Magrez[6], Achim Rosch[5], Fabrizio Carbone[3] and Henrik M. Rønnow[2]



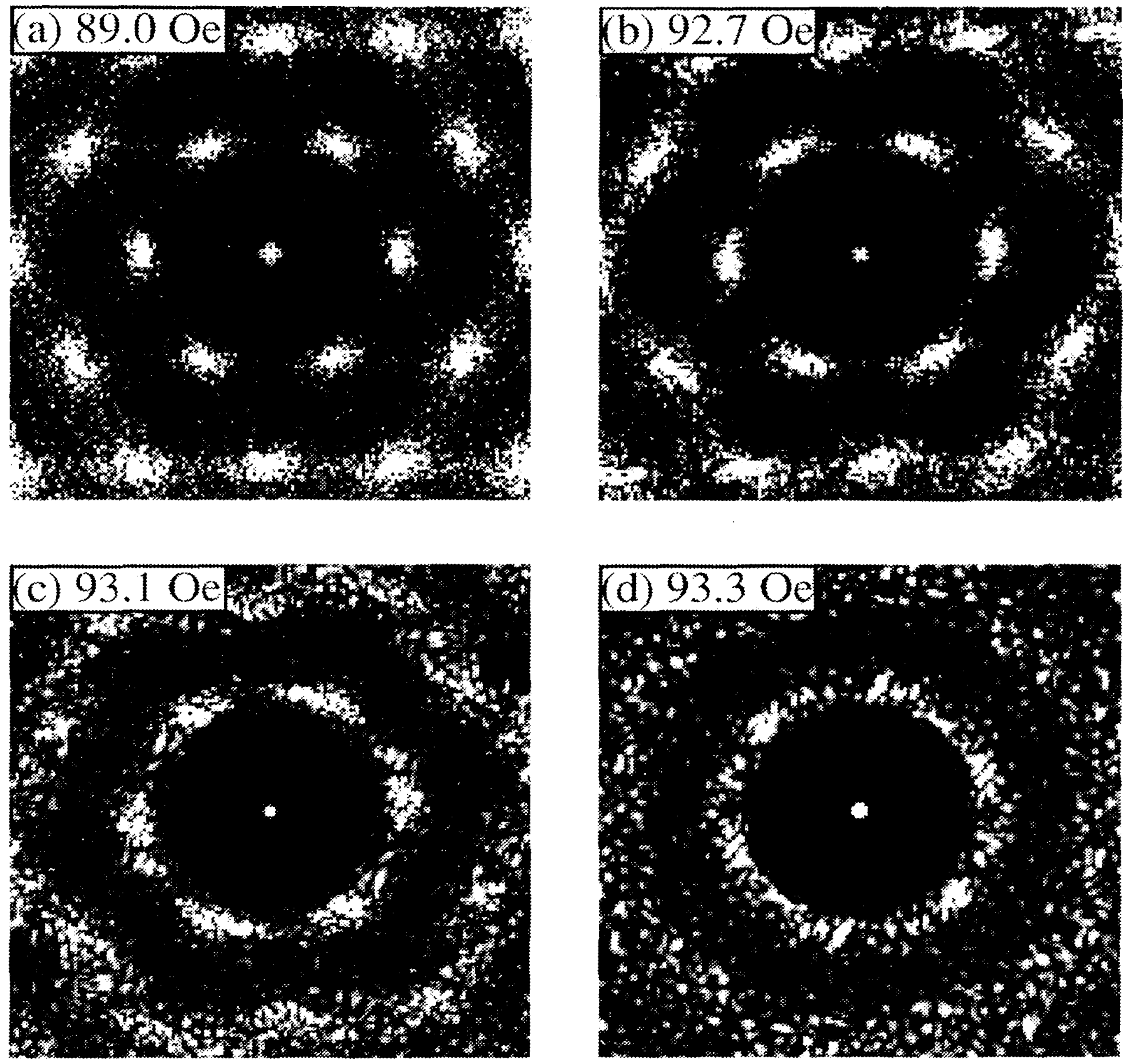


FIG. 2. Two-dimensional structure factor at the magnetic fields H_B indicated (see text).

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Hexatic-to-Liquid Melting Transition in Two-Dimensional Magnetic-Bubble Lattices

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