

Anomalous kinetics of multi-species reaction-diffusion system: Effect of thermal fluctuations

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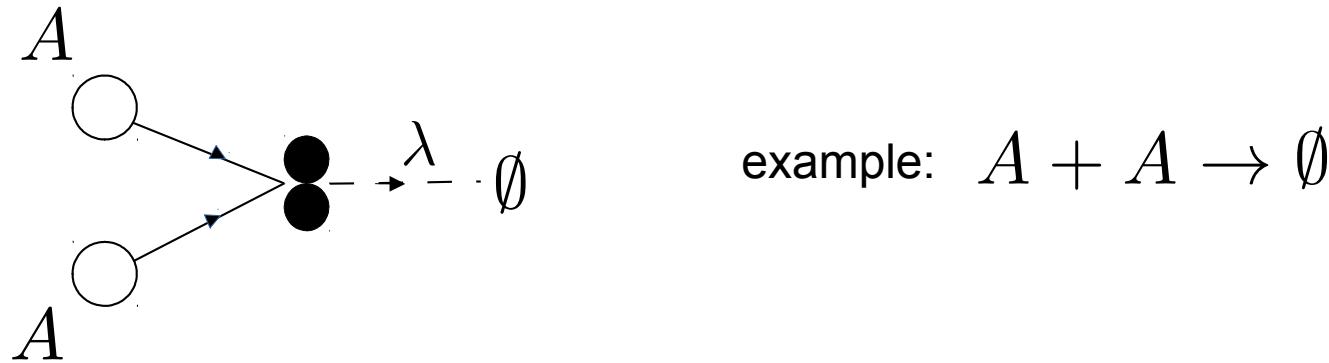
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Introduction

Particle diffuse and can react on mutual contact



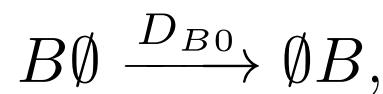
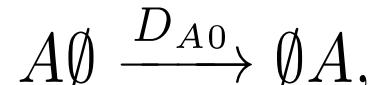
Irreversible unidirectional reactions

Non-equilibrium phenomenon

Tauber 2014; Krapivsky et. al 2010; Ovchinnikov & Timashev 1989;

Introduction

Movement:



Reaction scheme: $A + A \rightarrow \begin{cases} A, & \text{with prob. } p, \\ \emptyset, & \text{with prob. } 1 - p, \end{cases}$

$$A + B \rightarrow A.$$

Rajesh & Zaboronski 2004; Vollmayr-Lee et. al 2017; Hellerick et. al 2020;
Birnštejnová & Hnatič & Lučivjanský 2020;

Introduction

Reaction-limited regime: $\tau_{diff} \ll \tau_{reac}$

No spatial fluctuations

Introduction

Reaction-limited regime: $\tau_{dif} \ll \tau_{reac}$

No spatial fluctuations

Kinetic rate equations, $\frac{da}{dt} = -\lambda a^2, \frac{db}{dt} = -2\lambda' Qab,$

where

$$Q = \frac{1}{2-p}$$

Long-time limit: $a \sim (\lambda t)^{-1}, b \sim t^{-2\lambda' Q/\lambda}, t \rightarrow \infty.$

Ovchinnikov & Timashev 1989; Tauber & Howard & Lee 2005; Rajesh & Zaboronski 2004;
Vollmayr-Lee et. al 2017; Hellerick et. al 2020; Birnštejnová & Hnatič & Lučivjanský 2020;

Introduction

Diffusion-limited regime: $\tau_{dif} \gg \tau_{reac}$

Strong-density fluctuations in $d \leq d_c$, in this case $d_c = 2$

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Strong-density fluctuations in $d \leq d_c$, in this case $d_c = 2$

Methods of solution:

Simulations: Hellerick et. al 2020;

Perturbative RG: Rajesh & Zaboronski 2004; Vollmayr-Lee et. al 2017;

Generalization to Lévy flight spreading: Birnšteinová & Hnatič & Lučivjanský 2020;

Tauber & Howard & Lee 2005; Tauber 2014;

Inclusion of velocity fluctuations

Velocity field

Additional drift of particles

Origin: thermal fluctuations, external stirring

Introduction of $\mathbf{v} = \mathbf{v}(t, \mathbf{x})$ to model this phenomenon

In this work: passive advection, incompressible fluid, thermal fluctuations

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What is the effect of the introduction of velocity field?

How does velocity field affect the stability of large scale behavior?

Velocity field

Let $\mathbf{v}(t, \mathbf{x})$ have the following properties:

$$\langle \mathbf{v} \rangle = 0,$$

$$\partial_i v_i = 0,$$

Dynamics described by stochastic Navier-Stokes equation:

$$\partial_t v_i + (\mathbf{v} \cdot \partial) v_i = \nu_0 \partial^2 v_i - \partial_i P + F_i,$$

Velocity field

Let random force \mathbf{F} obey Gaussian statistics, have zero mean.

And let the correlation function given by

$$\langle F_i(t, \mathbf{x}) F_j(t', \mathbf{x}') \rangle = \delta(t - t') D_0 \int_{k > m} \frac{d^d k}{(2\pi)^d} P_{ij}(\mathbf{k}) k^2 \exp(i(\mathbf{k} \cdot \mathbf{x}))$$
$$P_{ij}(\mathbf{k}) = \delta_{ij} - \frac{k_i k_j}{k^2},$$

And a cutoff m providing an IR regularization.

Field theory

Field theory

Action functional:

Reaction-diffusion part: Doi-Peliti approach

Velocity field: De Dominicis-Janssen action functional

Additional terms: $-\psi_A^\dagger(\mathbf{v} \cdot \partial)\psi_A - \psi_B^\dagger(\mathbf{v} \cdot \partial)\psi_B$

Doi 1976; Peliti 1985; Zinn-Justin 2002; Vasiliev 2004;

Field theory

$$\begin{aligned} S = & \psi_A^\dagger (-\partial_t + D_{A0}\partial^2) \psi_A + \psi_B^\dagger (-\partial_t + D_{B0}\partial^2) \psi_B - D_{A0}\lambda_0 \psi_A^\dagger \psi_A^2 \\ & - D_{A0}\lambda_0 \psi_A^{\dagger 2} \psi_A^2 - \lambda'_0 Q D_{A0} \psi_B^\dagger \psi_A \psi_B - D_{A0}\lambda'_0 \psi_A^\dagger \psi_B^\dagger \psi_A \psi_B \\ & + \frac{1}{2} v_i' D_{ij} v_j' + v_i' [-\partial_t v_i - (\mathbf{v} \cdot \partial) v_i + \nu_0 \partial^2 v_i] - \psi_A^\dagger (\mathbf{v} \cdot \partial) \psi_A \\ & - \psi_B^\dagger (\mathbf{v} \cdot \partial) \psi_B + \psi_A^\dagger a_0 + \psi_B^\dagger b_0 \end{aligned}$$

Field theory

$$\langle \psi_A \psi_A^\dagger \rangle_0 = \text{---} +$$

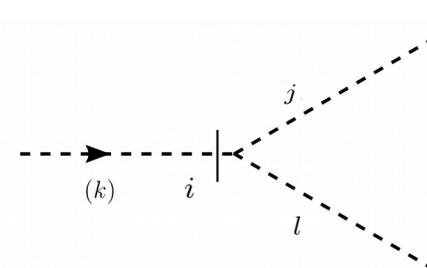
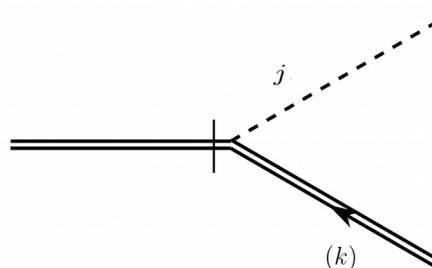
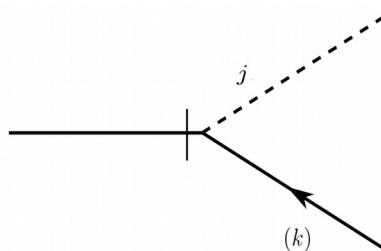
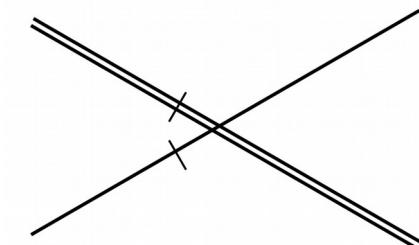
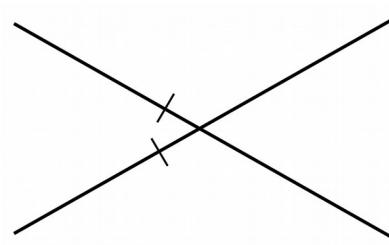
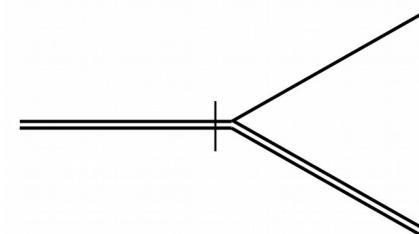
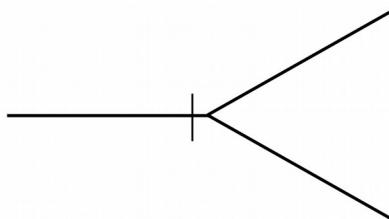
$$\langle \psi_B \psi_B^\dagger \rangle_0 = \text{---} +$$

$$\langle v_i v_j' \rangle_0 = -\cdots +$$

i *j*

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i *j*



Field theory

We employ: dimensional regularization in minimal subtraction scheme

Calculation of Z

Zero points of beta functions

We determine stability of fixed points

On general approach: e.g. Vasiliev 2004;

Results

Field theory

$$\langle v'v' \rangle_{1-ir}$$

$$\langle v'v \rangle_{1-ir}$$

$$\langle \psi_A^\dagger \psi_A \rangle_{1-ir}$$

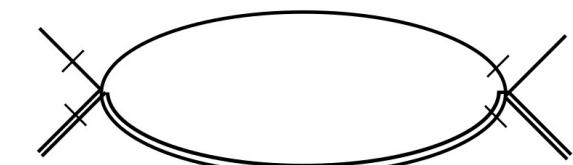
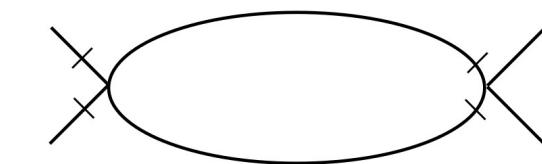
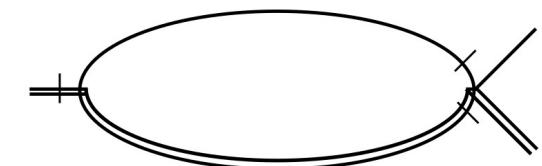
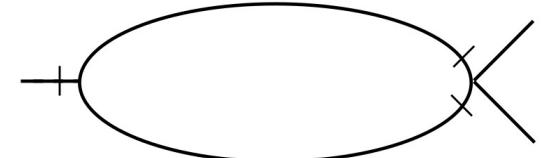
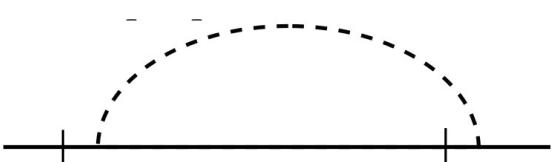
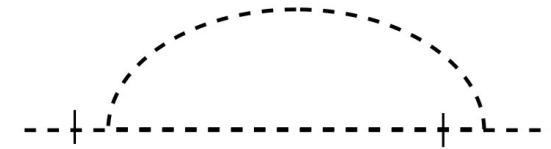
$$\langle \psi_B^\dagger \psi_B \rangle_{1-ir}$$

$$\langle \psi_A^\dagger \psi_A \psi_A \rangle_{1-ir}$$

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$$\langle \psi_B^\dagger \psi_A^\dagger \psi_B \psi_A \rangle_{1-ir}$$



Results

β Functions: $\beta_e = \mu \partial_\mu e|_0$, where $e = \{g, u_A, u_B, \lambda, \lambda'\}$, and $g = D/\nu^3$

$$\beta_g = -g \left(\epsilon - \frac{\hat{g}}{8} \right),$$

$$\beta_{u_A} = -u_A \hat{g} \left(\frac{1}{4u_A(1+u_A)} - \frac{1}{16} \right), \quad \beta_{u_B} = -u_B \hat{g} \left(\frac{1}{4u_B(1+u_B)} - \frac{1}{16} \right),$$

$$\beta_\lambda = -\lambda \left(\epsilon - \hat{\lambda} - \frac{\hat{g}}{4u_A(1+u_A)} \right), \quad \beta'_\lambda = -\lambda \left(\epsilon - \hat{\lambda}' - \frac{\hat{g}}{4u_A(1+u_A)} \right).$$

Where $\hat{g} = g S_d / (2\pi)^d$, etc.

Results

FP	\hat{g}^*	u_A^*, u_B^*	$\hat{\lambda}^*$	$\hat{\lambda}'^*$	Stability
FP1	0	arbitrary	0	0	$\epsilon < 0$
FP2	0	arbitrary	0	$\epsilon \frac{u_A + u_B}{u_A}$	never IR stable
FP3	0	arbitrary	ϵ	0	never IR stable
FP4	0	arbitrary	ϵ	$\epsilon \frac{u_A + u_B}{u_A}$	never IR stable
FP5	8ϵ	$\frac{-1 + \sqrt{17}}{2}$	0	0	never IR stable
FP6	8ϵ	$\frac{-1 + \sqrt{17}}{2}$	0	ϵ	never IR stable
FP7	8ϵ	$\frac{-1 + \sqrt{17}}{2}$	$\epsilon/2$	0	never IR stable
FP8	8ϵ	$\frac{-1 + \sqrt{17}}{2}$	$\epsilon/2$	ϵ	$\epsilon > 0$

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FP6	8ϵ	$\frac{-1 + \sqrt{17}}{2}$	0	ϵ	never IR stable
FP7	8ϵ	$\frac{-1 + \sqrt{17}}{2}$	$\epsilon/2$	0	never IR stable
FP8	8ϵ	$\frac{-1 + \sqrt{17}}{2}$	$\epsilon/2$	ϵ	$\epsilon > 0$

Summary

We constructed field-theoretic model for multi-species reaction diffusion system in the presence of incompressible velocity field.

The model was studied in one-loop order.

Fixed points and their stability was determined.

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Thank you!