

Temperature and Time dependent at finite temperature Green functions. Features and results of applications.

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Content:

Temperature and Time dependent at finite temperature Green functions.

New model of superfluid phase transition dynamics.

New model of high temperature superconducting phase transition.

About the UV divergences, Lorentz invariance and Wick rotation in the content of the statistical physics.

Time dependent at finite temperature Green functions were used to describe the dynamics of the phase transition of quantum Bose systems to the superfluid state. The obtained renormalization group results contradict the currently accepted dynamic stochastic model E for this phenomenon. General issues of introduction of temperature Green's functions are also discussed.

The quantum statistical physics can be constructed using the field operators

$$\hat{\psi}^+(x) = \int d\mathbf{p} \hat{a}_p^+ \phi_p(x)^*, \quad \hat{\psi}(x) = \int d\mathbf{p} \hat{a}_p \phi_p(x),$$

$$\hat{H}_0 \phi_p(x) = \varepsilon_p \phi_p(x), \quad \hat{\rho}(x) = \hat{\psi}^+(x) \hat{\psi}(x),$$

$$\hat{H} = \int d\mathbf{x} [\hat{\psi}^+(x) \hat{H}_0 \hat{\psi}(x) + \int d\mathbf{y} \hat{\psi}^+(x) \hat{\psi}(x) \hat{V}(\mathbf{x} - \mathbf{y}) \hat{\psi}^+(y) \hat{\psi}(y)]$$

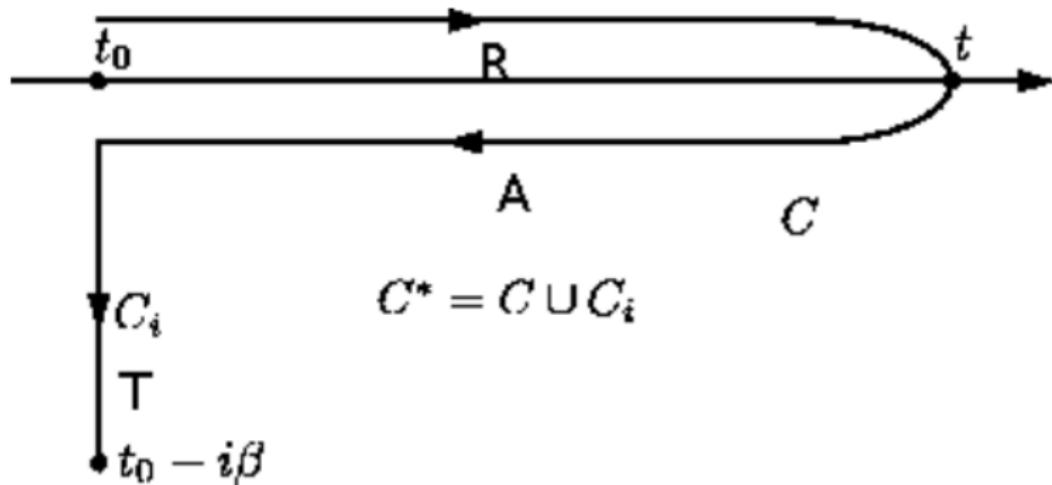
Time dependent at finite temperature Green functions

$$\begin{aligned}
 \text{Sp}(T(\hat{\psi}^+(\mathbf{x}_1, t_1) \dots \hat{\psi}(\mathbf{x}_n, t_n)) e^{-\beta \hat{H}}) &= \text{Sp}((\hat{\psi}^+(\mathbf{x}_1) e^{-i\hat{H}(t_2-t_1)} \dots \\
 &\dots e^{-i\hat{H}(t_n-t_{n-1})} \hat{\psi}(\mathbf{x}_n, t_n)) e^{-\beta \hat{H}} e^{-i(t_1-t_n)\hat{H}}) = \\
 &= \int D\psi D\psi^+ (\psi^+(\mathbf{x}_1, t_1) \dots \psi(\mathbf{x}_n, t_n)) e^{iS} \\
 S(\psi, \psi^+) &= \int d\mathbf{x} \int_C dt [\psi^+(\partial_t + i\frac{\Delta}{2m} + i\mu)\psi - i\frac{g}{4}(\psi^+\psi)^2],
 \end{aligned}$$

t is at the contour C^* which depends from the time and temperature.
 $\text{Re}(t) \in (t_0, t_f)$, where t_0 and t are initial and final times,

$$\text{Im}t \in (0, -\beta), \quad \beta = \frac{1}{kT}.$$

Keldysh-Schwinger contour



We use this formalism to the dynamics of superfluid phase transition description

P. Hohenberg and B. Halperin, Rev. Mod. Phys. 49, 435 (1977):

The critical dynamics is described by the phenomenological stochastic equations with the random forces f

$$\begin{aligned}\partial_t \psi = & f_\psi + \lambda(1 + ib)[\partial^2 \psi - g_1(\psi^+ \psi) \psi / 3 + g_2 m \psi] \\ & + i \lambda g_3 \psi [g_2 \psi^+ \psi - m].\end{aligned}$$

The field $m(x, t)$ characterizes *temperature and density fluctuations*. For m field the equation is

$$\partial_t m = f_m - \lambda u \partial^2 [g_2 \psi^+ \psi - m] + i \lambda g_3 [\psi^+ \partial^2 \psi - \psi \partial^2 \psi^+]$$

Propagators of the theory

R, A, T indicate the branches of C contour for times of fields

$$G_{RR} = e^{-i\epsilon(t-t')}(\Theta(t-t') + n(\epsilon)), \quad G_{AA} = e^{-i\epsilon(t-t')}(\Theta(t'-t) + n),$$

$$G_{TT} = e^{-i\epsilon(\tau-\tau')}(\Theta(\tau-\tau') + n), \quad G_{RT} = e^{-i\epsilon(t-t_0+i\tau')}n,$$

$$G_{RA} = e^{-i\epsilon(t-t')}n, \quad G_{AT} = e^{-i\epsilon(t-t_0+i\tau')}n, \quad G_{AR} = e^{-i\epsilon(t-t')}(n+1),$$

$$G_{TR} = e^{-i\epsilon(t_0-i\tau-t')}(n+1), \quad G_{TA} = e^{-i\epsilon(t_0-i\tau-t')}(n+1).$$

$$\langle \psi_i(t)\psi_j^+(t') \rangle \equiv G_{ij}, \quad n(\epsilon) = 1/(e^{\beta\epsilon} - 1), \quad \epsilon \equiv p^2/(2m) - \mu.$$

p is a momentum, $\tau \equiv -\text{Im}t$.

Unusual for QFT singularities of Tt diagram expansion at large t were found.

α regularization was introduced. $n \rightarrow t_0 \rightarrow -\infty$ $\xi = \psi_R + \psi_A$,
 $\eta = \psi_r - \psi_A$
Renormalizable model

$$\begin{aligned} S' = & 4\eta\alpha\eta^+ + \eta^+(\partial_t - iu\Delta - \alpha\Delta)\xi + \xi^+(\partial_t - iu\Delta + \alpha\Delta)\eta \\ & + \frac{i\lambda\alpha}{2}\eta^+\xi^+\xi\xi + \frac{i\bar{\lambda}\alpha}{2}\eta\xi\xi^+\xi^+, \end{aligned}$$

Nonzero α is generated due to the renormalization with the initial
 $\alpha = 0$

Superfluid phase transition

Effective large-scale model of boson gas from microscopic theory

Honkonen, J;Komarova, M., V; Molotkov, Yu G.;Nalimov, M. Yu, N.Phys B, 939 105-129, (2019) CRITICAL DYNAMICS OF THE PHASE TRANSITION TO THE SUPERFLUID STATE Yu. A. Zhavoronkov, M. V.

Komarova, Yu. G. Molotkov, M. Yu. Nalimov, and J. Honkonen TMP, 200(2): 1237–1251 (2019) Kinetic Theory of Boson Gas J. Honkonen, M. V. Komarova, Yu. G. Molotkov : M. Yu. Nalimov TMP 200, 1360–1373(2019) Renormalization Group in Non-Relativistic Quantum Statistics Honkonen J , Komarova M.V., Molotkov Yu.G. , Nalimov M.Yu., Zhavoronkov Yu. A. (MMCP 2019), 226, 01005, (2020)

Critical dynamics of the superfluid phase transition: Multiloop calculation of the microscopic model *J. Honkonen , M. Komarova , Yu. Molotkov , M. Nalimov , A. Trenogin, Phys.Rev. E 106, 014126 (2022)*

The results obtained: the real stochastic equation is

$$\partial_t \psi = f_\psi + \lambda [\partial^2 \psi - g_1(\psi^+ \psi) \psi / 3].$$

Lets compare with initial

$$\partial_t \psi = f_\psi + \lambda(1 + ib)[\partial^2 \psi - g_1(\psi^+ \psi) \psi / 3 + g_2 m \psi]$$

$$+ i \lambda g_3 \psi [g_2 \psi^+ \psi - m].$$

$$\partial_t m = f_m - \lambda u \partial^2 [g_2 \psi^+ \psi - m] + i \lambda g_3 [\psi^+ \partial^2 \psi - \psi \partial^2 \psi^+]$$

Not every phenomenological theory is true

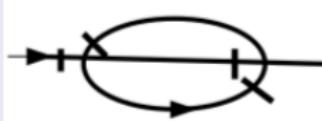
Using the Dyson equation in two-loops approximations the dissipation was found

The equilibrium ensemble with fixed temperature is sufficient to find the dissipation.

$$G^{-1} = G_0^{-1} - \Sigma,$$

G is the matrix of dressed propagators, G_0 – initial propagators, Σ – self energy.

It was calculated the contribution $\sim g^2$ to the real part of Σ



– dissipation

No thermal basin to obtain the equilibrium.

Temperature Green functions

$$\hbar = c = 1, \beta = 1/(kT)$$

Functional integral e.g. $\int D\phi e^{-\frac{1}{2}\phi K\phi - g\phi^4}$, can describes S-matrix or partition function, the operator of evolution $\langle q_{out} | e^{-i\hat{H}t} | q_{in} \rangle$ or $Sp(e^{-\beta\hat{H}})$. The difference is in $it \rightarrow \beta$.

The quantum statistical physics can be constructed using the field operators

$$\psi^+(x) = \int d\mathbf{p} \hat{a}_p^+ \phi_p(x)^* \quad \psi(x) = \int d\mathbf{p} \hat{a}_p \phi_p(x)$$

$$\Sigma = C \int D\psi^+ D\psi e^{-S_\beta(\psi^+, \psi)},$$

$$S_\beta = \int_0^\beta dt \int d\mathbf{x} \psi^+(\mathbf{x}, t) \left(\partial_t + \hat{H}_0 \right) \psi(\mathbf{x}, t) + \frac{g}{4} \psi^{+2}(\mathbf{x}, t) \psi^2(\mathbf{x}, t),$$

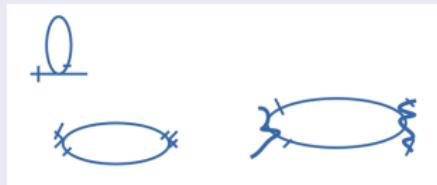
Where $\psi^+(\mathbf{x}, \mathbf{t})$, $\psi(\mathbf{x}, \mathbf{t})$ are complex conjugated fields with the periodic boundary conditions $t \in [0, \beta]$

$$\hat{H}_0 = -\frac{\Delta}{2m} - \mu \rightarrow \frac{\mathbf{p}^2}{2m} - \mu$$

UV divergences

$$G(t, t', \mathbf{p}) = e^{-\varepsilon_p(t-t')} [\Theta(t - t') + n(\varepsilon_p)] \rightarrow \frac{1}{-i\omega_n + \varepsilon(p)},$$

$$n(\varepsilon_p) = \frac{1}{e^{\beta\varepsilon(p)} - 1}, \quad \varepsilon(p) = \frac{p^2}{2m} - \mu.$$



$$= \int \frac{d\mathbf{p}}{(2\pi)^d} \frac{1}{e^{\beta(p^2/(2m)-\mu)} - 1}$$

$$\Theta(0) = 0, V(p)$$

Critical behavior, universality

One can construct the IR effective theory in the limit $p \rightarrow 0$, $\mu \rightarrow 0$, $t, t' \in [0, \beta]$. Then we lost the quantum properties of the theory

$$\begin{aligned} G(t, t', \mathbf{p}) &= e^{-(\frac{p^2}{2m} - \mu)(t-t')} [\Theta(t - t') + n(\varepsilon_p)] \\ &\rightarrow \frac{1}{\beta(p^2/(2m) - \mu)} \end{aligned}$$

$$S_{\beta,IR} = \beta \int d\mathbf{x} \psi^+(\mathbf{x}) \left(p^2/(2m) - \mu \right) \psi(\mathbf{x}) + \\ + \frac{g}{4} \psi^{+2}(\mathbf{x}) \psi^2(\mathbf{x}) \right),$$

Similar the relativistic properties are lost in the critical region due to

$$p^2/(2m) \rightarrow \sqrt{m_0^2 c^4 + p^2 c^2} \rightarrow m_0 c^2 + p^2/(2m_0)$$

This formalism was used in our works to construct the hight temperature super-conductivity theory.

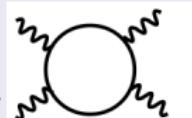
With the help of Temperature Green functions, a new model of the phase transition of Fermi systems to the superconducting state was created. The renormalization group results testify in favor of a high phase transition temperature in layered or heterogeneous conductors, which is confirmed experimentally.

$$S_\beta = \int_0^\beta dt \int d\mathbf{x} \psi^+(\mathbf{x}, t) \left(\partial_t + \frac{-\Delta}{2m} - \mu \right) \psi(\mathbf{x}, t) - \frac{\lambda}{4} \psi^{+2}(\mathbf{x}, t) \psi^2(\mathbf{x}, t),$$

$$S = \psi_i^+ \left(\partial_t - \frac{\Delta}{2m} - \mu \right) \psi_i + \chi_{ji}^+ \chi_{ij} + \sqrt{\frac{\lambda}{2}} \chi_{ij} (\psi_i^+ \psi_j^+) + \sqrt{\frac{\lambda}{2}} \chi_{ij}^+ (\psi_i \psi_j).$$

The critical modes of the superconducting phase transition

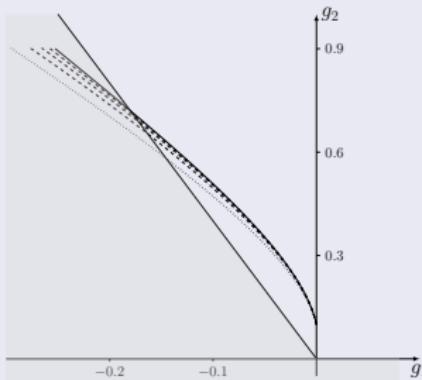
$$S = \chi_{ji}^+ \chi_{ij} + \frac{1}{2} \text{Diagram A} + \frac{1}{4} \text{Diagram B} + \dots$$

$$S = \partial_\alpha \chi_{ji}^+ \partial_\alpha \chi_{ij} + \tau \chi_{ji}^+ \chi_{ij} + \frac{g_1}{2} (tr(\chi^+ \chi))^2 + \frac{g_2}{2} tr(\chi^+ \chi \chi^+ \chi) + \dots + A$$

RG

$$\partial_\zeta \bar{g}_1 = \beta_1(g_1, g_2) \quad \partial_\zeta \bar{g}_2 = \beta_2(g_1, g_2)$$



Quantum mechanics is not applied for some phenomena

About the rotation into the Euclidean space

$$S_\beta = \int_0^\beta dt \int d\mathbf{x} \psi^+(\mathbf{x}, t) \left(\partial_t + \left(-\frac{\Delta}{2m} - \mu \right) \right) \psi(\mathbf{x}, t) + \frac{g}{4} \psi^{+2}(\mathbf{x}, t) \psi^2(\mathbf{x}, t),$$

$$S_\beta = \int_0^\beta dt \int d\mathbf{x} \left(\partial_t \phi(\mathbf{x}, t) \partial_t \phi(\mathbf{x}, t) + \partial_i \phi(\mathbf{x}, t) \partial_i \phi(\mathbf{x}, t) + m^2 \phi(\mathbf{x}, t)^2 \right. \\ \left. + \frac{g}{4!} \phi^4(\mathbf{x}, t) \right)$$

$$G(t, t', \mathbf{p}) = e^{-(\frac{p^2}{2m} - \mu)(t-t')} [\Theta(t - t') - n(\varepsilon_p)]$$

$$n(\varepsilon_p) = \frac{1}{e^{\beta\varepsilon_p} - 1}, \quad \varepsilon_p = \frac{p^2}{2m} - \mu$$

$$G(t, t', \mathbf{p}) = \frac{1}{2\varepsilon_p} n(\varepsilon_p) \left(e^{\beta - \varepsilon_p |t-t'|} + e^{\varepsilon_p |t-t'|} \right)$$

It is common opinion that the rotation into the Euclidean space turns quantum field theory into an equilibrium statistical physics. But each classical science corresponds to a set of quantum ones. The statistical equilibrium theory corresponds to the single correct only. The report will show that equilibrium statistical physics not only loses the property of Lorentz invariance (due to the use of a co-moving coordinate system), but also fundamentally distinguishes between "partitital" and "wave" descriptions of systems (in the sense of formalisms of the first and second order in time). It is argued that preference should be given to the first-order formalism, which rejects oscillatory types models.