Can we avoid explosions of black holes?

Irina Aref'eva

Steklov Mathematical Institute, RAS

VII International Conference
"Models in Quantum Field Theory" (MQFT-2022)

October 10-14, 2022
Saint Petersburg

Overview

1. Black hole explosion
2. Proposal of this talk — constraints on equations of state
   - Reissner-Nordstrom, Kerr, Schwarzschild de Sitter,...
   - Does not work for Schwarzschild BH
3. Schwarzschild BH: new coordinates instead of Kruskal
4. Explosion and information loss problem
5. Conclusions

Based on joint works with I. Volovich

"Complete evaporation of black holes and Page curves ", **2202.00548**

"Quantum explosions of black holes and thermal coordinates", **2104.12724**

Black hole explosion

- **Hawking temperature of Schwarzschild BH**

$$\mathbf{T = \frac{1}{8\pi M},}$$

M is the mass of BH

- **If M $\rightarrow$ 0 (evaporation) then T $\rightarrow \infty$**

- **Energy density of radiation**

$$\mathbf{E \sim \frac{1}{M^4}}$$

S. W. Hawking, "Black hole explosions?"Nature, v.248 (1974), 30.

Complete evaporation of BH

- **Quantum gravity, Planck scale, Modified gravity,...**

- **Complete evaporation:**

  Hawking, Page, Susskind, Maldacena,...

  This talk based on 2202.00548 & 2104.12724

- **Semiclassical effective theory**

Proposal of this talk - constraints

- **Reissner-Nordstrom, Schwarzschild de Sitter, ...**
  - **Special form of the state equations:**

    Special charge dependence on BH mass
- **Schwarzschild BH**
  - **open problem**
  - **new thermal coordinates**

Hawking temperature of Black Hole

$$\mathbf{ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2}$$

$$\mathbf{f(r_h) = 0, \qquad f'(r_h) \neq 0}$$

Hawking temperature

$$\mathbf{T = \frac{1}{4\pi} f'(r_h)}$$

$$\mathbf{f = 1 - \frac{2M}{r}, \qquad r_h = 2M, \qquad T = \frac{1}{8\pi M}}$$

- **There is the Big Explosion problem with the completely evaporated Schwarzschild black holes.**

Reissner-Nordstrom BH

- Reissner-Nordstrom black hole

$$\mathbf{ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2}$$

$$\mathbf{f(r) = (1 - \frac{2M}{r} + \frac{Q^2}{r^2}) = (r - r_+)(r - r_-)/r^2}$$

  - horizons $\mathbf{r_\pm = M \pm \sqrt{M^2 - Q^2}}$
  - temperature

$$\mathbf{T = \frac{1}{4\pi} f'(r_+) = \frac{1}{2\pi} \frac{\sqrt{M^2 - Q^2}}{(M + \sqrt{M^2 - Q^2})^2}}$$

  - Kretschmann scalar

$$K_{RN} = \frac{48M^2}{r^6}(1 + \frac{2Q^2}{rM} + \frac{7Q^4}{48r^2M^2})$$

Main result

- **A scenario to avoid the BH explosion is proposed.**
- **We propose to use special EOS (Equations Of State), i.e. special relations of M with other parameters.**

RN black hole under constraint

- **We take the charge in the form**

$$\mathbf{Q^2 = M^2 - \lambda(M)^2}$$

where $\lambda(M)$ is some given function, $0 < \lambda(M) < M$.

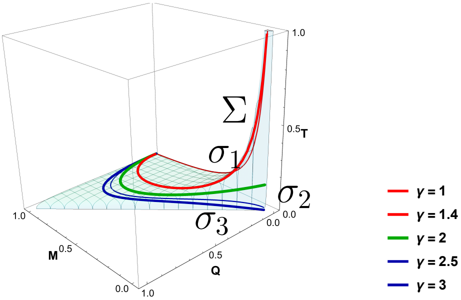
- **The Hawking temperature T becomes**

$$\mathbf{T = \frac{\lambda(M)}{2\pi(M + \lambda(M))^2}}$$

- **If we take $\lambda(M)$ such that for small M it obeys $\boldsymbol{\lambda(M) = o(M^2)}$, then T tends to 0 as $\mathbf{M \to 0}$ and we get the complete evaporation of BH without $T \to \infty$ at the end of evaporation.**
- **Hence, for small M the evaporating black hole should be in the near extremal state.**

RN under constraint. Curves σ_γ on surface Σ

- $\mathbf{T} = \frac{1}{2\pi} \frac{\sqrt{\mathbf{M}^2 - \mathbf{Q}^2}}{(\mathbf{M} + \sqrt{\mathbf{M}^2 - \mathbf{Q}^2})^2}$ **defines surface Σ in (M, Q, T)-space.**
- **Constraint** $\mathbf{Q}^2 = \mathbf{M}^2 - \lambda(\mathbf{M})^2$. **We take** $\lambda(\mathbf{M}) = \mathbf{C}\mathbf{M}^\gamma \quad (*)$



- $T \to \infty$ **as** $M \to 0$ **for** $\gamma < 2$,
- $T \to const \neq 0$ **as** $M \to 0$ **for** $\gamma = 2$
- $T \to 0$ **as** $M \to 0$ **for** $\gamma > 2$

Thermodynamics of RN BHs

- **Temperature**

$$T = \frac{\lambda(M)}{2\pi(M + \lambda(M))^2}.$$

- **Entropy**

$$S_{RN} = \pi r_+^2 = \pi(M + \lambda(M))^2$$

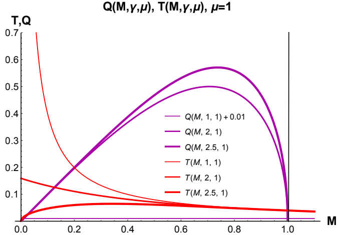
- **Free energy**

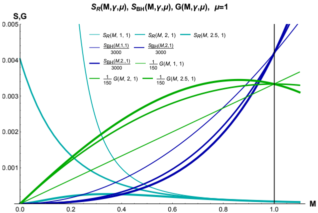
$$G_{RN} = M - TS = M - \frac{1}{2}\lambda(M)$$

- **entropy S_{RN} and free energy G_{RN} go to 0 as $M \to 0$ for λ satisfying**

$$\mathbf{0 < \lambda(M) \leq CM^{\gamma}, \quad C > 0, \; \gamma > 2}$$

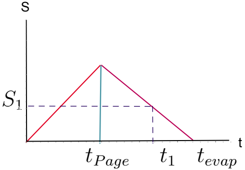
Mass dependence of Q, T, S_R, S_{BH} and F at $\lambda(M) = (M/\mu)^\gamma$ with different γ





- **for all $\gamma > 1$ there is a restriction on M, $M \leq 1$**
- **T and S_R at $\gamma > 2$ starting from $M = 1$, increase to a certain maximum value, then decrease to zero, i.e. the mass dependencies – deformed bell shapes**
- **for $\gamma = 2$ $T \to T_0 \neq 0$ as $M \to 0$**
- **for $\gamma < 2$ $T, S_R \to \infty$**
- **S_{BH} and F tend to zero at $M \to 0$ for all values of γ**
- **F first increases as the black hole mass decreases, but after some M_0 starts to decrease**

- **- Page form of the time evolution of S**



Explosion and information loss problem

- **The information loss problem can be formulated as follows:**
  - **If the Schwarzschild black hole is formed from a pure quantum state and then evaporates completely, this means**

    $|$**pure state** $> \to |$**mixed temperature state of radiation**$>$

    Information paradox appears when BH complete evaporates

- **The entropy of the initial pure quantum state is zero while entropy of the final radiation state is infinity.**

Black Hole Explosions Problem and Information Paradox

- **The information loss problem (Hawking, 1976) is closely related to the Black Hole Explosions Problem, since the radiation entropy S_R diverges for small M as M^{-3}.**

- **Alternative: the evaporation is incomplete and it stops when the Schwarzschild radius is close to the Planck length.**

  However, Hawking and Page considered complete evaporation process.

Time evolution of RN BH under constraint

- **The loss of the mass and charge during evaporation of RN black hole is a subject of numerous consideration**
  Gibbons, Zaumen, Carter, Damour, Page'76, Hiscock'90, Gabriel'00, Sorkin'01,...

$$\frac{\mathbf{dM}}{\mathbf{dt}} = -\mathbf{A}\sigma\mathbf{T^4} + \frac{\mathbf{Q}}{\mathbf{r_+}}\frac{\mathbf{dQ}}{\mathbf{dt}},$$

- **A is a positive constant and the cross-section σ is proportional to M^2 for small M**
- **in the second term**

$$\frac{\mathbf{dQ}}{\mathbf{dt}} = \frac{\mathbf{d}}{\mathbf{dt}}\sqrt{\mathbf{M^2} - \lambda^2} = \frac{\mathbf{M} - \lambda\lambda'}{\sqrt{\mathbf{M^2} - \lambda^2}}\frac{\mathbf{dM}}{\mathbf{dt}}$$

Time evolution of RN BH under constraint

- **For** $\lambda(M) = CM^\gamma$ **and small** M **one gets** $\frac{dM}{dt} = -C_1 M^{3\gamma-5} (*)$
- **(*) for** $\gamma > 2$ **and** $M(0) = M_0$ **has a solution**

$$\mathbf{M(t) = \frac{M_0}{(1 + B\,t)^{\frac{1}{3(\gamma-2)}}}, \quad B = \frac{3(\gamma-2)C_1}{M_0^{6-3\gamma}}, \quad \gamma > 2,}$$

M_0 **and** B **are positive constants**

- **(*) for** $\gamma = 2$ **has solution** $\mathbf{M(t) = M_0 e^{-C_1 t}}$
  - **i.e. one gets an infinite large time of the complete evaporation of charged black hole under our constraint.**

Schwarzschild Black Hole
Problems with Kruskal coordinates for $M \to 0$

$$ds^2 \;=\; -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

Schwarzschild $f(r) = 1 - \frac{2M}{r}$, $M \to 0$ **- OK**

Surface gravity $\tau_g = \frac{1}{2}|f'(r_h)| = \frac{1}{4M}$ **- diverges as** $M \to 0$

Tortoise coordinate r_*: $dr_* = \frac{dr}{f(r)} \quad u = t - r_*, \quad v = t + r_*$

Kruskal coordinates: $\mathbf{U = -e^{-\tau_g u} = e^{-\frac{u}{4M}},\ V = e^{v/4M}}$.

- **Advantage: Kruskal coordinates describe Kruskal (Szekeres) extension that is a maximal, analytic, simply connected vacuum solution**
- **Disadvantage: Kruskal coordinates are singular in the limit** $M \to 0$**.**

New coordinates

- **To improve the situation, new coordinates $\mathscr{U}$ and $\mathscr{V}$ for the Schwarzschild metric are introduced,**

$$\mathscr{U} = -e^{-\frac{u}{4M+b}}, \quad \mathscr{V} = e^{\frac{v}{4M+b}},$$

  which depend on the black hole mass M and a parameter $b > 0$.
  This parameter b sets the observer's motion along a special trajectory - for explanation see next pages
- **Temperature**

$$T = \frac{1}{2\pi(4M+b)}$$

- **Limit $M \to 0$**

$$T_0 = \frac{1}{2\pi b}$$

Schwarzschild metric in new coordinates

- **2-dim part of Schwarzschild metric**

$$ds_2^2 \;=\; -(1-\frac{2M}{r})dudv = (4M+b)^2(1-\frac{2M}{r})\frac{d\mathscr{U}\,d\mathscr{V}}{\mathscr{U}\mathscr{V}}, \quad r>2M.$$

$$(r-2M)^{\frac{M}{M+b/4}} = (2M)^{M/(M+b/4)}(-\mathscr{U}\mathscr{V})e^{-\frac{r}{2(M+b/4)}}$$

- **in the limit $M\to 0$ the 2-dimensional part becomes**

$$ds_2^2 = b^2\frac{d\mathcal{U}\,d\mathcal{V}}{\mathcal{U}\mathcal{V}}, \qquad r = b/2\log(-\mathcal{U}\mathcal{V})$$

$r>0$ **corresponds to** $-\mathcal{U}\mathcal{V}>1$**.**

New (thermal/exponential/quasi Kruskal) coordinates in Minkowski

- **New coordinates**

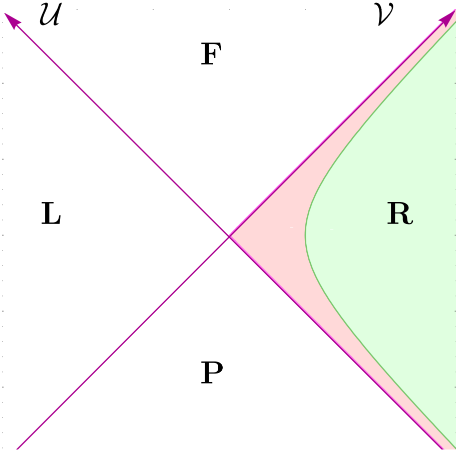
$$\begin{aligned}
\mathcal{U} &= \mathcal{U}^{(R)}(t,r) = -\exp\{-\frac{u}{b}\}, \qquad u = t-r,\ b>0,\\
\mathcal{V} &= \mathcal{V}^{(R)}(t,r) = \exp\{\frac{v}{b}\}, \qquad v = t+r\\
r &= b/2\log(-\mathcal{U}\mathcal{V}) \qquad \textbf{R-region}: \mathcal{U}\mathcal{V}<0
\end{aligned}$$

- **New coordinate** $\mathcal{T} = \frac{\mathcal{U}+\mathcal{V}}{2}$, $\mathcal{X} = \frac{\mathcal{V}-\mathcal{U}}{2}$

$$\textbf{R-region} \;=\; \{(\mathcal{T},\mathcal{X}) \in \mathbb{R}^2 \,|\, \mathcal{X}^2 - \mathcal{T}^2 > 0, \mathcal{X} > 0\}$$

- **Similarity with Rindler coordinates (see below)**

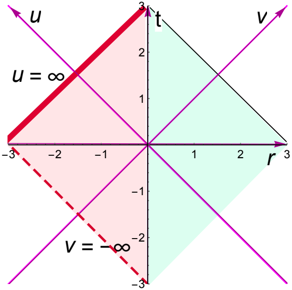
New coordinates in Minkowski



$\leftarrow$

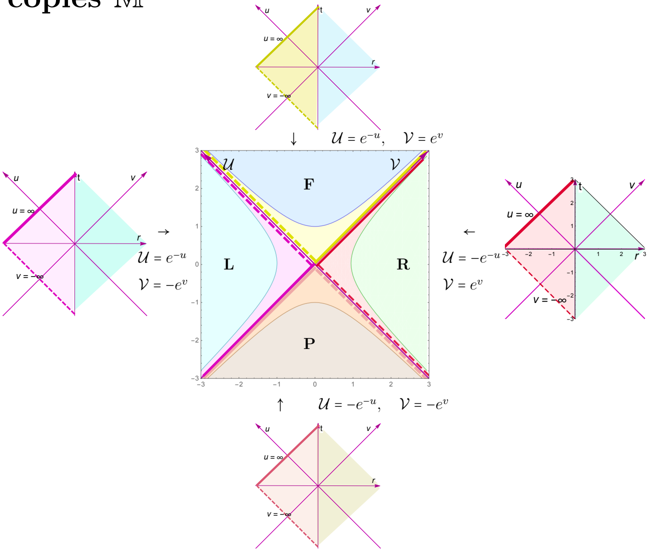
$$\mathcal{U} = -e^{-u}$$

$$\mathcal{V} = e^{v}$$



New coordinates in Minkowski.

Map of 4 copies $\mathbb{M}^{1,1}$



New coordinates in Minkowski. Geodesics

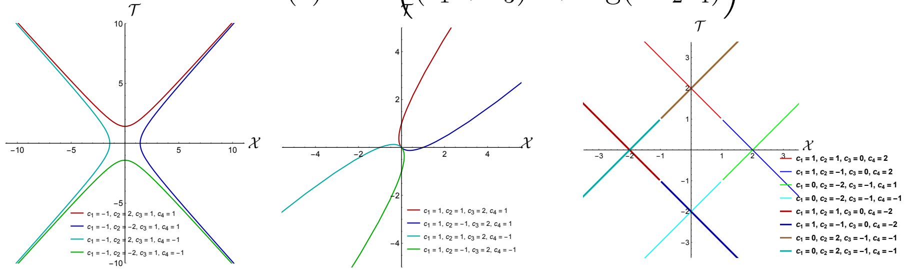
Metric: $ds_2^2 = b^2 \frac{d\mathcal{U}\, d\mathcal{V}}{\mathcal{U}\mathcal{V}}$

Geodesics eqs.: $\mathcal{V}'(\mathfrak{s})^2 - \mathcal{V}(\mathfrak{s})\mathcal{V}''(\mathfrak{s}) = 0 = \mathcal{U}'(\mathfrak{s})^2 - \mathcal{U}(\mathfrak{s})\mathcal{U}''(\mathfrak{s})$

Geodesics: $\mathcal{V}(\mathfrak{s}) = c_2 e^{c_1 \mathfrak{s}}, \qquad \mathcal{U}(\mathfrak{s}) = c_4 e^{c_3 \mathfrak{s}}$

in Mink.coord.: $t(\mathfrak{s}) = \frac{1}{2}\left(\mathfrak{s}(-c_1 - c_3)) - \log\left(-\frac{c_2}{c_4}\right)\right)$

$$r(\mathfrak{s}) = 2b\Big((c_1 + c_3)\mathfrak{s} + \log(-c_2 c_4)\Big)$$

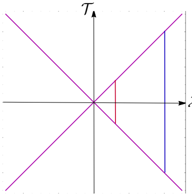


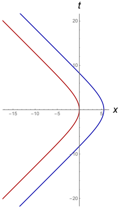
Geodesics cannot cross the light-cone with vertex at O

New coordinates in Minkowski. Acceleration

The "lift" $\mathcal{X} = \mathcal{X}_0$

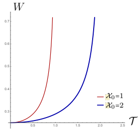
The proper time $d\tau = \frac{4b}{\sqrt{\mathcal{X}_0^2 - \mathcal{T}^2}}\, d\mathcal{T} \qquad \mathcal{T} = \pm \mathcal{X}_0 \sin\left(\frac{\tau - \tau_0}{b}\right)$





$$
\begin{aligned}
t &= b\,\mathbf{arctanh}\left(\sin\left(\frac{\tau - \tau_0}{b}\right)\right)\\
r &= \frac{b}{2}\log\left(\mathcal{X}_0^2\cos^2\left(\frac{\tau - \tau_0}{b}\right)\right)\\
V^0 &= \frac{dt}{d\tau} = \frac{\mathcal{X}_0}{\sqrt{\mathcal{X}_0^2 - \mathcal{T}^2}}\\
V^1 &= \frac{dx}{d\tau} = \frac{\mathcal{T}}{\sqrt{\mathcal{X}_0^2 - \mathcal{T}^2}}
\end{aligned}
$$

$$W^0 = \frac{dV^0}{d\tau} = \frac{\mathcal{X}_0 \mathcal{T}}{b\left(\mathcal{X}_0^2 - \mathcal{T}^2\right)} \qquad W^1 = \frac{dV^1}{d\tau} = \frac{\mathcal{X}_0^2}{b\left(\mathcal{X}_0^2 - \mathcal{T}^2\right)}$$



Acceleration $W \neq$ **const**

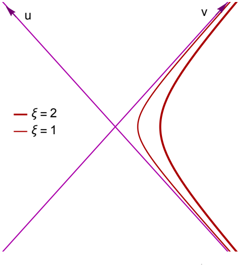
$$
\begin{aligned}
W^2 &\equiv -(W^0)^2 + (W^1)^2\\
&= \frac{\mathcal{X}_0^2}{b^2\left(\mathcal{X}_0^2 - \mathcal{T}^2\right)}\\
&= \frac{1}{b^2}\cosh^2\frac{t}{b}
\end{aligned}
$$

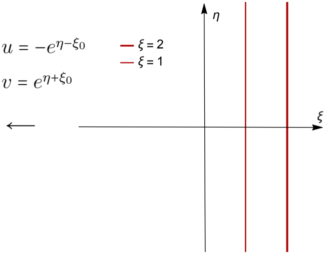
Comparison with Rindler coordinates

An observer traveling in Minkowski with constant acceleration a is described by the Rindler coordinates (ν, ϑ)

$$u = -\frac{1}{a}e^{-a\nu}, \quad u = t - x, \qquad v = t + x,$$
$$v = \frac{1}{a}e^{a\vartheta}, \qquad \nu = \eta - \xi, \qquad \vartheta = \eta + \xi \quad a > 0$$

The Rindler observer is "at rest"in Rindler coordinates and travels along the hyperbola in $uv = e^{\xi_0}$ in the inertial coordinates (t, x)





New coordinates for general $f(r)$

- **Metric** $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$
  $f(r)$**-general blackening function,** $\quad r_*(r) = \int \frac{dr}{f(r)}$
- $\mathscr{U} = -e^{-\frac{u}{B}}, \quad \mathscr{V} = e^{\frac{v}{B}}, \quad B > 0,$
- **Metric** $ds_2^2 = -f(r)dudv = f(r)B^2\frac{d\mathscr{U}d\mathscr{V}}{\mathscr{U}\mathscr{V}} = f(r)B^2\frac{-d\mathscr{T}^2+d\mathscr{X}^2}{-\mathscr{T}^2+\mathscr{X}^2}$
- r **is implicitly defined by** $e^{2r_*/B} = -\mathscr{U}\mathscr{V}$
- **temperature** $T = \frac{1}{2\pi B}$
- **acceleration along trajectory** $\mathscr{X} = \mathscr{X}_0$

$$w^2 \;=\; -f(w^0)^2 + f^{-1}(w^1)^2 = \frac{1}{f}\left(\frac{f'}{2} - \frac{1}{B}\right)^2 \cdot \frac{\mathscr{X}_0^2}{\mathscr{X}_0^2 - \mathscr{T}^2}$$

Acceleration along trajectory $\mathscr{X} = \mathscr{X}_0$ in Schwarzschild metric

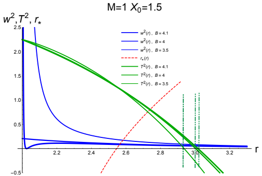
- $\mathscr{T}^2 = \mathscr{X}_0^2 - e^{2r/B}\left(\frac{r}{2M} - 1\right)^{4M/B}$
- **Acceleration**

$$w^2 = \frac{\mathscr{X}_0^2\left(\frac{1}{B} - \frac{M}{r^2}\right)^2}{1 - \frac{2M}{r}} e^{-2r/B}\left(\frac{r}{2M} - 1\right)^{-4M/B}$$

- **We see that for $B < 4M$ the acceleration is ∞ at $z_h = 2M$**
- **For $B = 4M$ the acceleration at $z_h = 2M$ is related with the surface gravity $\kappa = 1/4M$,**

$$w^2|_{r=r_h} = \frac{4\mathscr{X}_0^2}{e}\kappa^2$$

- **For $B > 4M$ the acceleration near $z_h = 2M$ is ∞ and $\to 0$ at $r = r_0 = \sqrt{BM}$**



Conclusions

- **The problems of the BH explosion of completely evaporating black holes is considered.**
- **It is shown that the constraint permits to avoid the RN BH explosion.**
- **Similar results are obtained for Kerr and Schwarzschild-de Sitter black holes.**
- **Problem with Kruskal coordinates for small mass are indicated and new thermal coordinates are suggested**

Thank you !