What is the appropriate field theory?

Kay Wiese

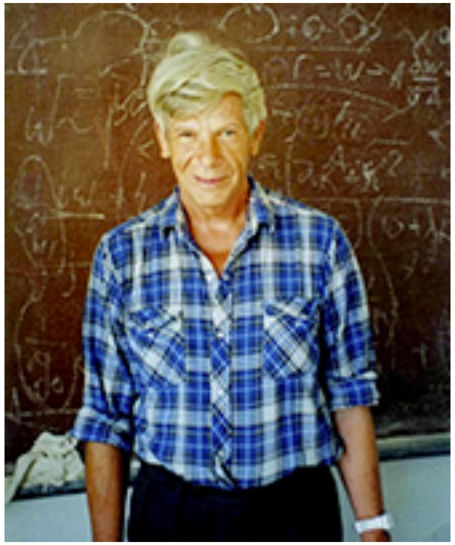
Ecole Normale Supérieur (ENS), Paris
with Cathelijne ter Burg, Gauthier Mukerjee

Sankt Petersburg, October 2022

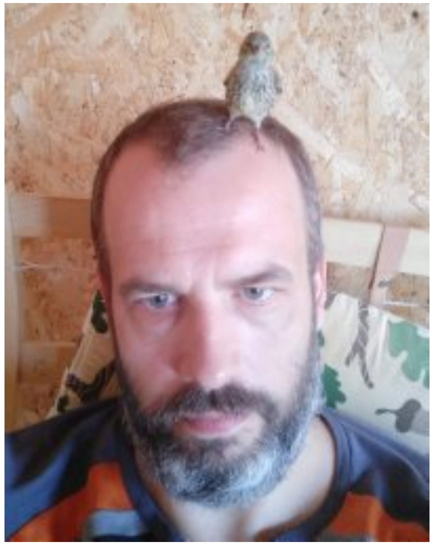
http://www.phys.ens.fr/~wiese/

Review: arXiv:2102.01215

…dedicated to my Russian and Ukrainian friends and colleagues…

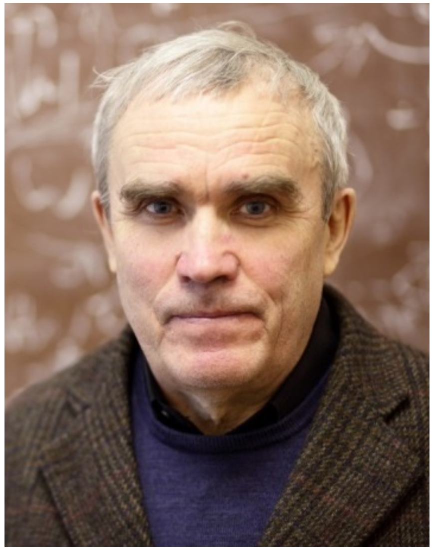


Sasha



Misha

Andrei



Juri

Nicolai

Kolya

Boris

Mykola

What is the appropriate field theory?

- Ising model: ϕ^4-theory ✔
- quantum gravity ?
- turbulence ?
- KPZ equation ?

we believe that for KPZ

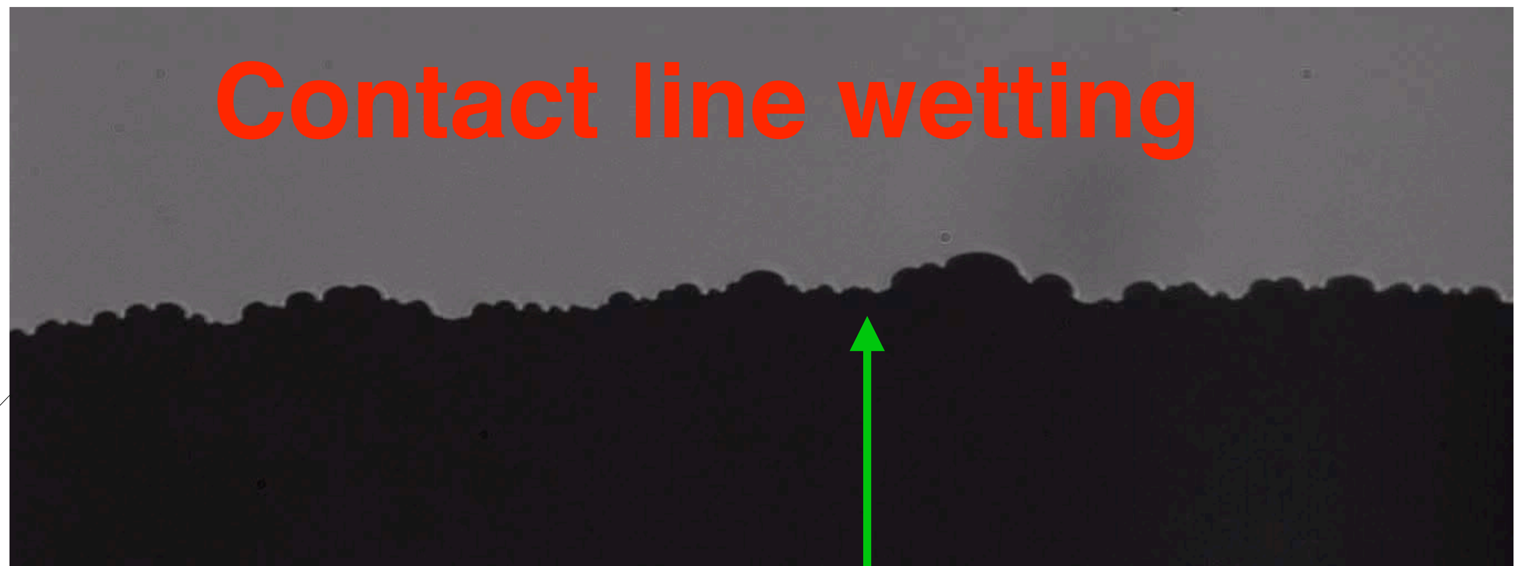
$$\mathcal{S} = \int_{x,t} \tilde{h}(x,t)\Big[\partial_t h(x,t) + \nabla^2 h(x,t) + \lambda \left[\nabla h(x,t)\right]^2 + 2D\tilde{h}(x,t)\Big]$$

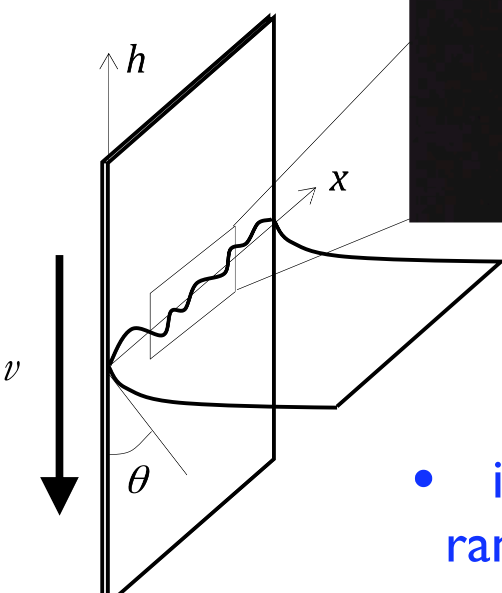
- but the perturbative expansion describes weak-strong coupling crossover in $d = 2 + \epsilon$ dimensions. Something is missing here…
- strong-coupling regime accessible via the directed polymer, but in completely different variables!

How to find the appropriate field theory?

- how to get out of this dilemma?
- try to measure the effective theory!
- here: disordered elastic systems

(C) E. Rolley

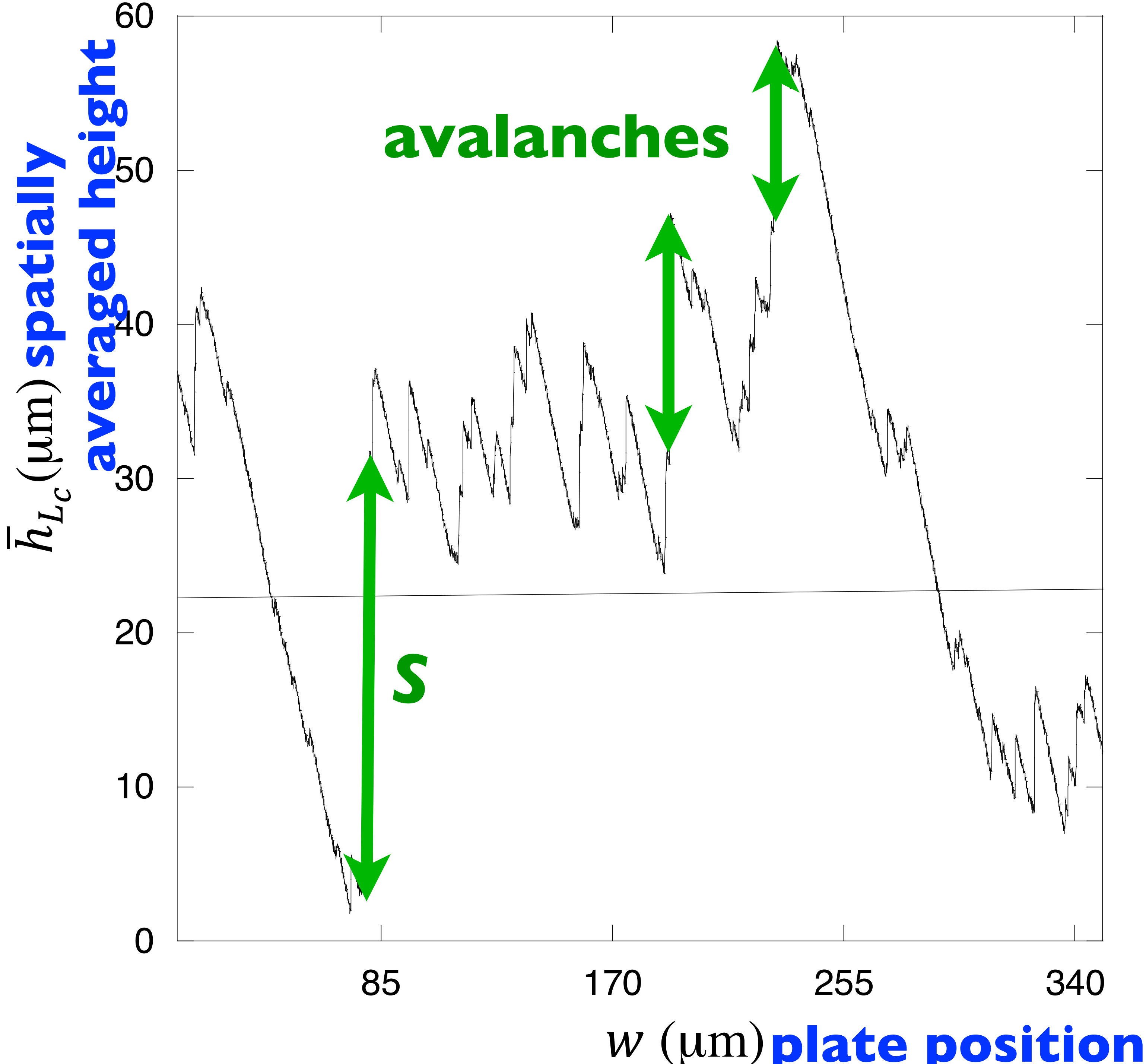




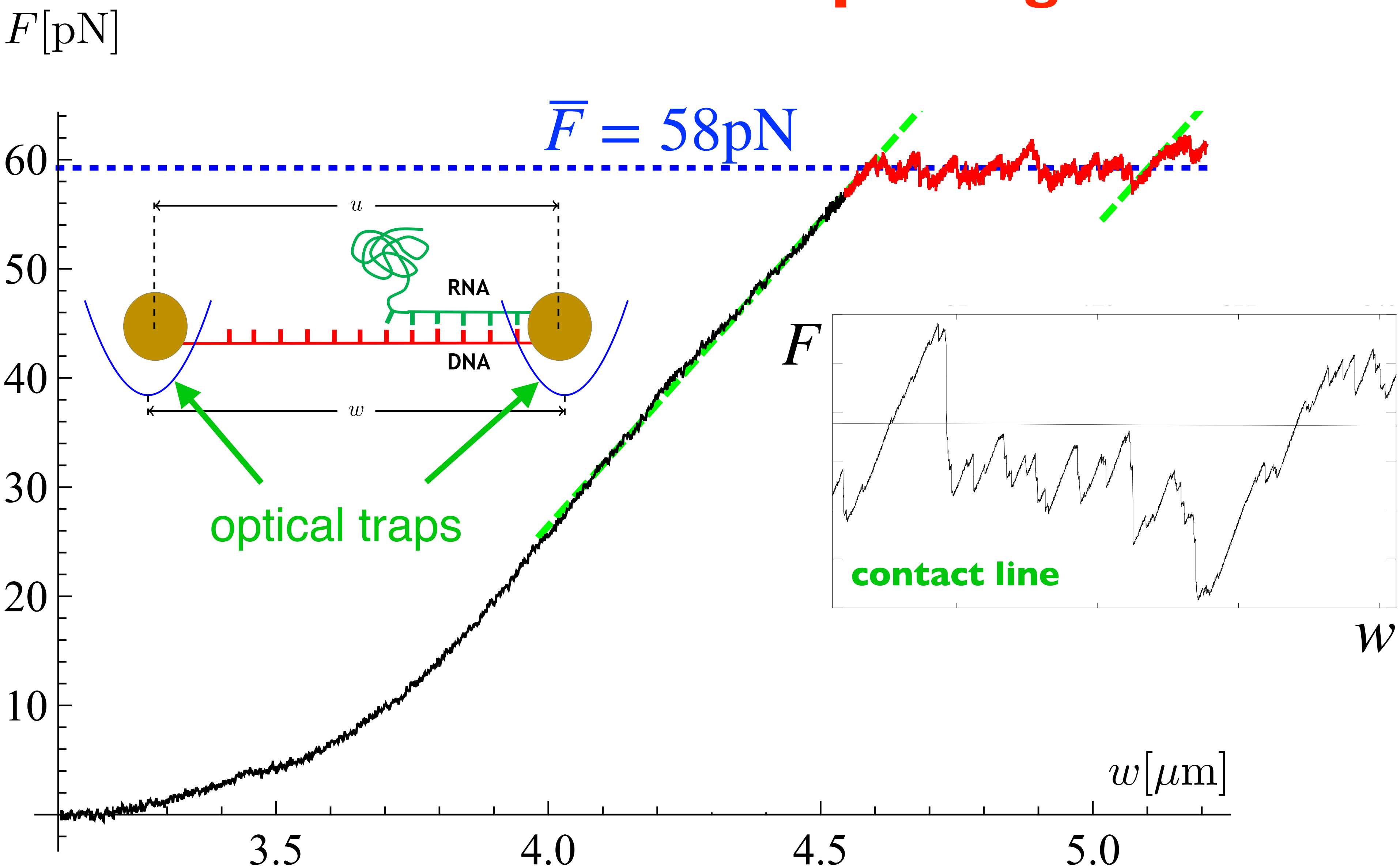
- isobutanol on a randomly silanized

avalanche

height jumps = avalanches

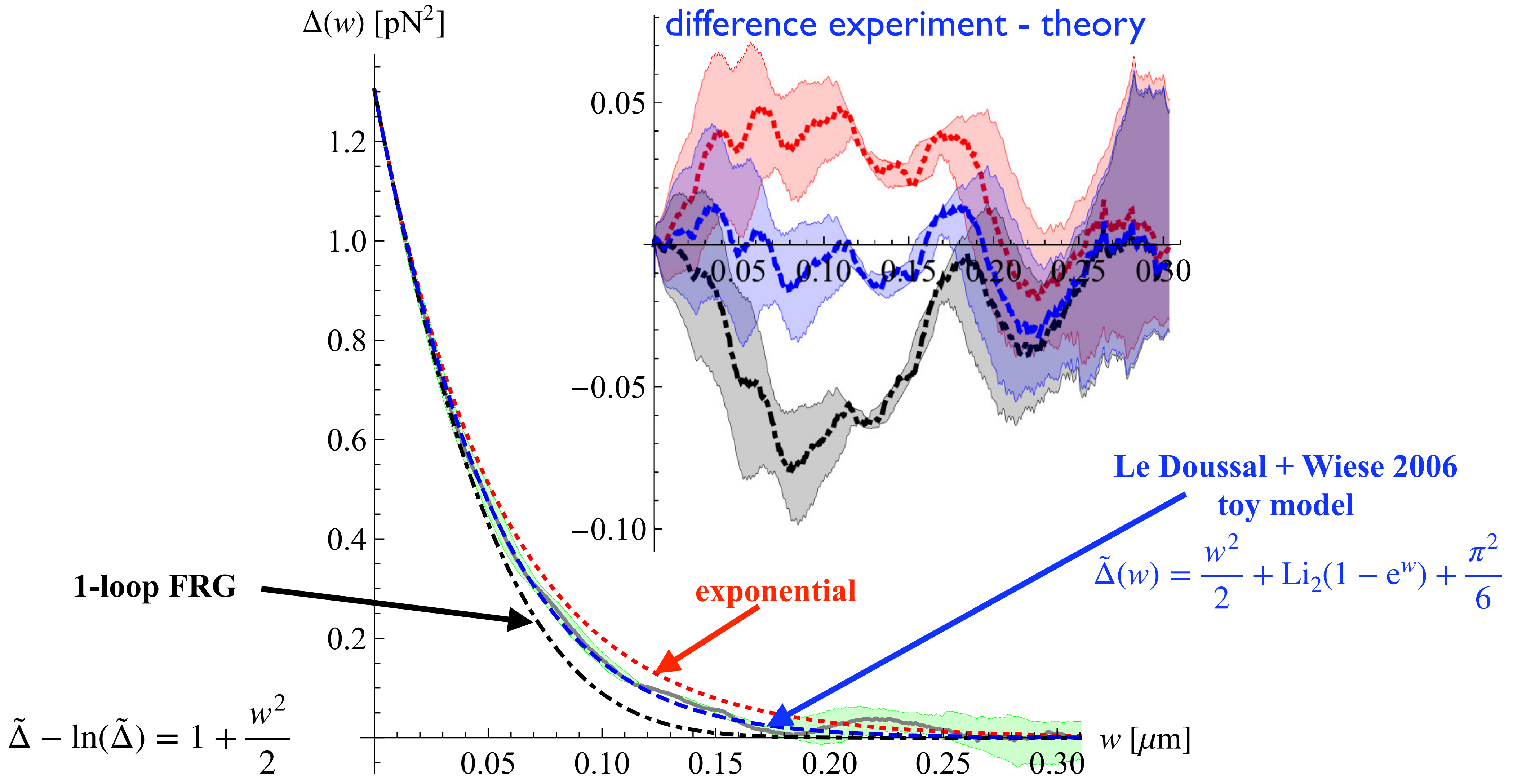


Force as a function of distance for RNA/DNA peeling



Force-force correlations

$$\Delta(w-w') := \overline{F_w F_{w'}}^{\,c} \equiv \overline{F_w F_{w'}} - \overline{F_w}\ \overline{F_{w'}}$$



Field theory

Equation of motion (for SR elasticity for simplicity)

height of the interface

$w = vt$

$$\partial_t u(x,t) = \nabla^2 u(x,t) + m^2[w - u(x,t)] + F(x, u(x,t))$$

Forces are drawn from a **Gaussian**, and have correlations

$$\overline{F(x,u)F(x',u')}^{\mathrm{c}} = \delta^d(x-x')\Delta(u-u')$$

Field theory (MSR=classical limit $\hbar \to 0$ of Keldysh)

$$\mathcal{S}[\tilde{u}, u] = \int_{x,t} \tilde{u}(x,t)\left[\partial_t u(x,t) - \nabla^2 u(x,t) + m^2\Big(u(x,t) - w\Big)\right]$$

$$-\frac{1}{2}\int_{x,t,t'} \tilde{u}(x,t)\tilde{u}(x,t')\Delta\Big(u(x,t) - u(x,t')\Big)$$

was measured

Why did we measure Δ?

action

unrenormalized

$$\mathcal{S}[\tilde u, u] = \int_{x,t} \tilde u(x,t)\Big[\partial_t u(x,t) - \boxed{\nabla^2} u(x,t) + \boxed{m^2}\Big(u(x,t) - \boxed{w}\Big)\Big]$$

IR scale

$$-\frac{1}{2}\int_{x,t,t'} \tilde u(x,t)\tilde u(x,t')\boxed{\Delta}\Big(u(x,t) - u(x,t')\Big)$$

want to measure

external field

$$u_w := \lim_{t\to\infty} \frac{1}{L^d}\int_x u(x,t)\Big|_w$$

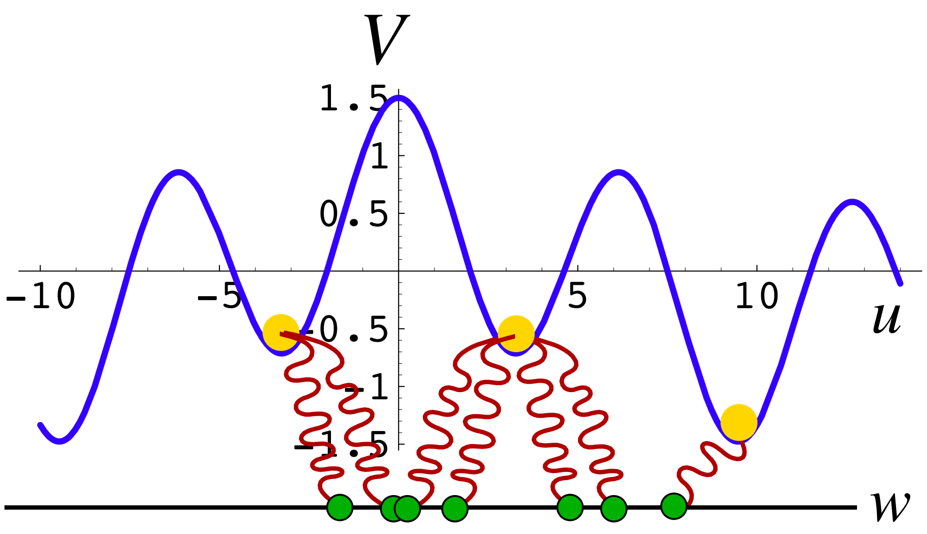
center of mass at large t, i.e. $\omega \to 0$

$$\Delta(w-w') \equiv \Gamma^{(2)} = \mathscr{L} \circ \overline{u_w u_{w'}} = \left[\mathscr{R}^{-1}\right]^2 \overline{u_w u_{w'}} = (m^2)^2 \overline{u_w u_{w'}}$$

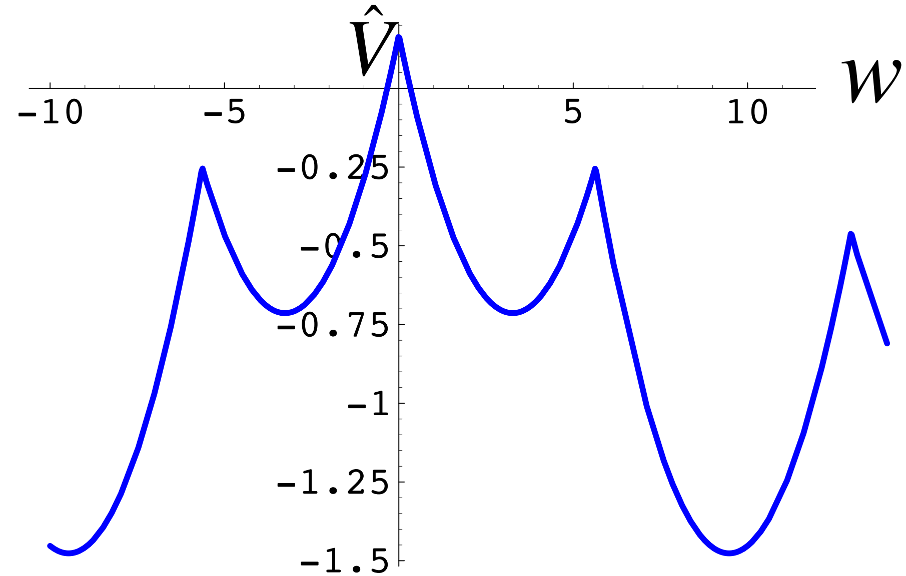
Legendre transform

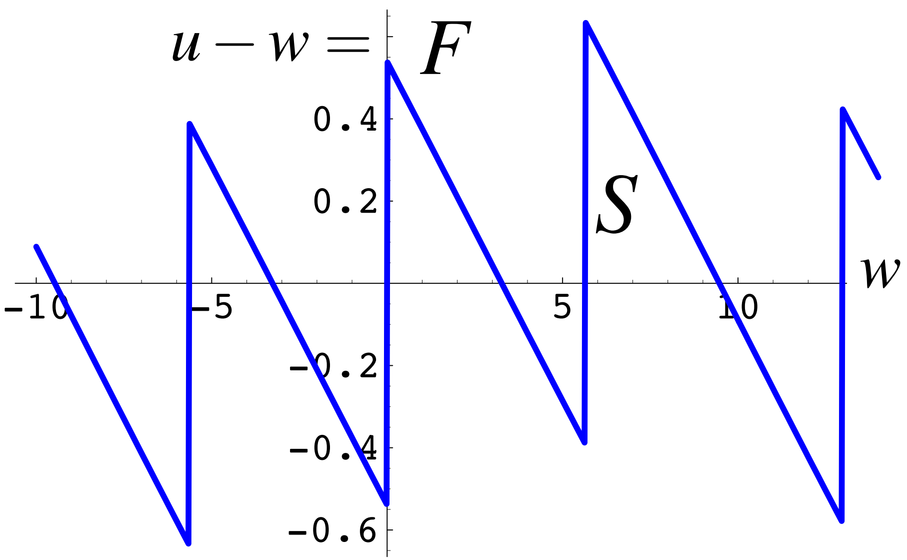
amputate 2-point function (response)

Why a cusp in the effective action?



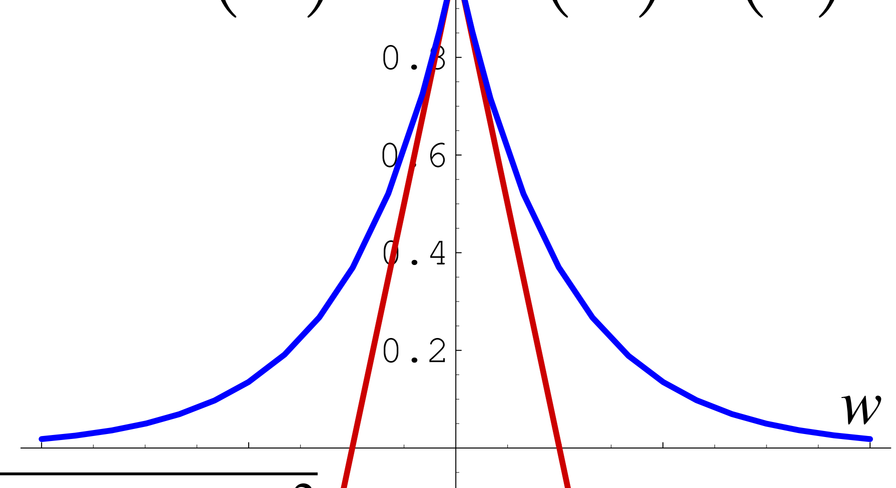
$$\xrightarrow{\text{Min}}$$





$$\xrightarrow{\text{average}}$$

$$\Delta(w) = \overline{F(w)F(0)}^{c}$$



$$\overline{[F(w) - F(0)]^2} = \underbrace{\rho_{\text{shock}}|w|}_{p_{\text{shock}}} \langle S^2 \rangle$$

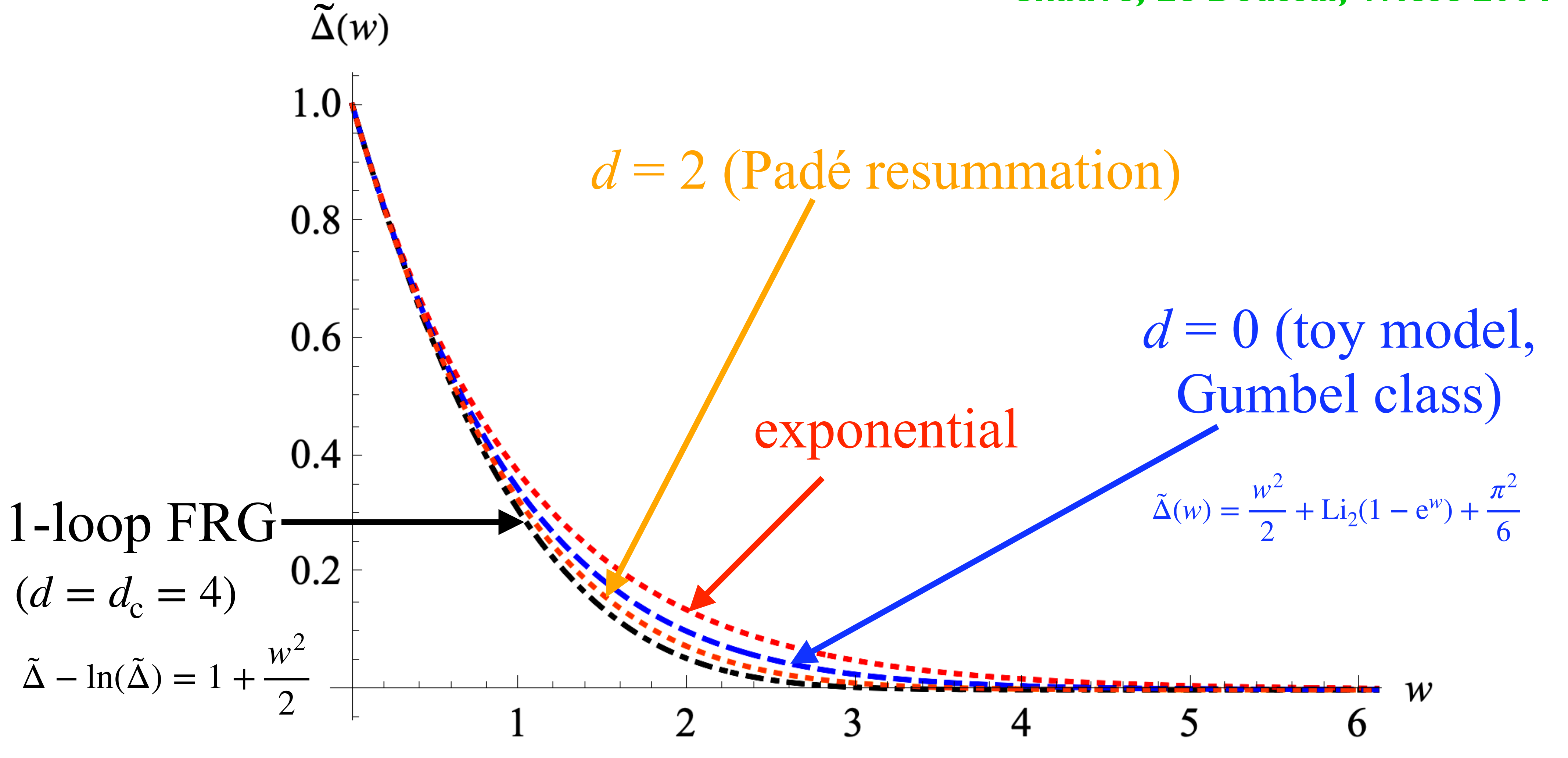
$$\boxed{-\Delta'(0^+) = \frac{\langle S^2 \rangle}{2\langle S \rangle}} \quad \Longleftarrow \quad \rho_{\text{shock}} = \langle S \rangle^{-1}$$

Renormalization of disorder

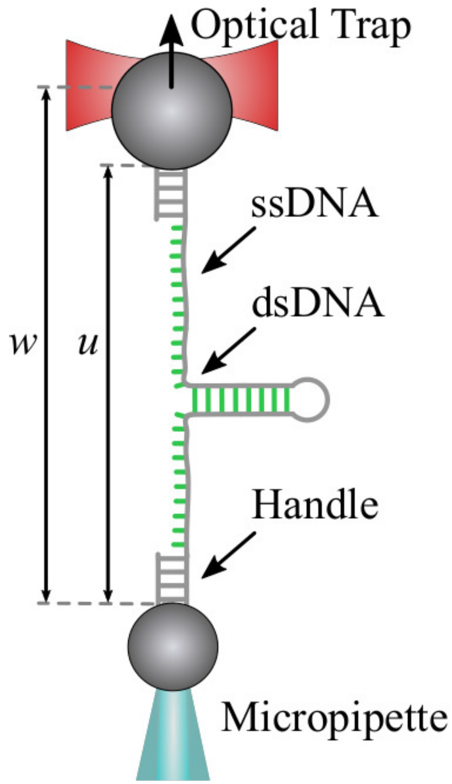
FRG

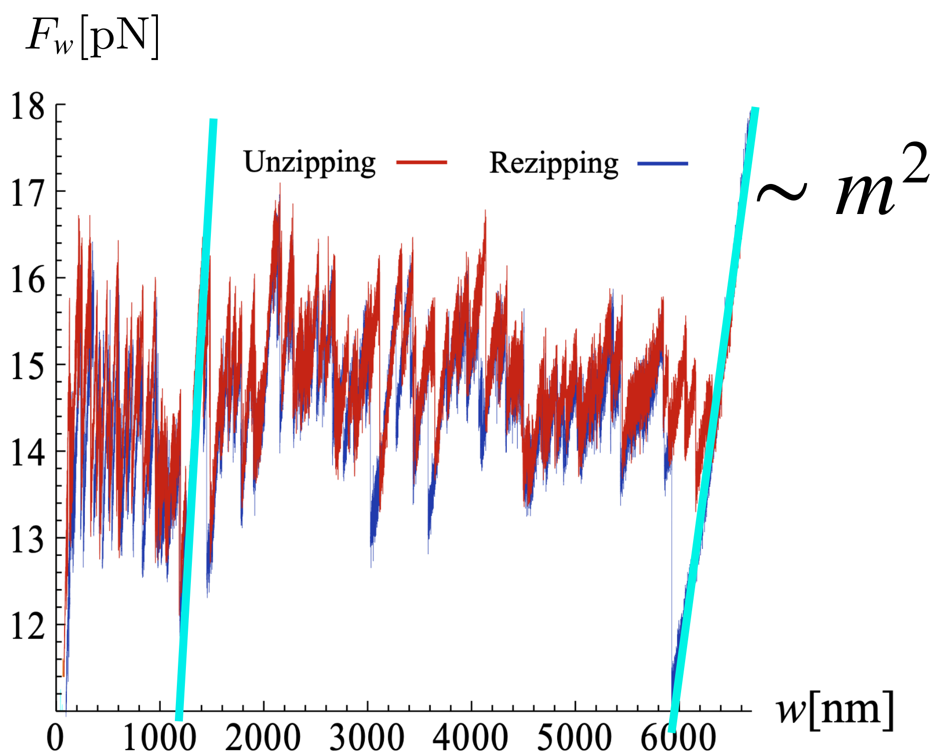
$$-\frac{m\mathrm{d}}{\mathrm{d}m}\tilde{\Delta}(w) = (\epsilon - 2\zeta)\tilde{\Delta}(w) + \zeta w \tilde{\Delta}'(w) - \frac{1}{2}\partial_w^2\left[\tilde{\Delta}(w) - \tilde{\Delta}(0)\right]^2 + \frac{1}{2}\partial_w^2\left\{\left[\tilde{\Delta}(w) - \tilde{\Delta}(0)\right]\tilde{\Delta}'(w)^2 + \tilde{\Delta}'(0^+)^2\tilde{\Delta}(u)\right\}$$

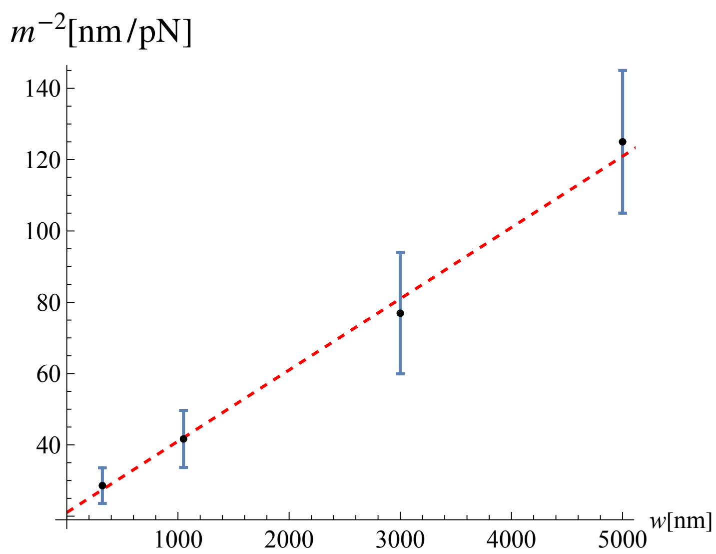
Chauve, Le Doussal, Wiese 2004

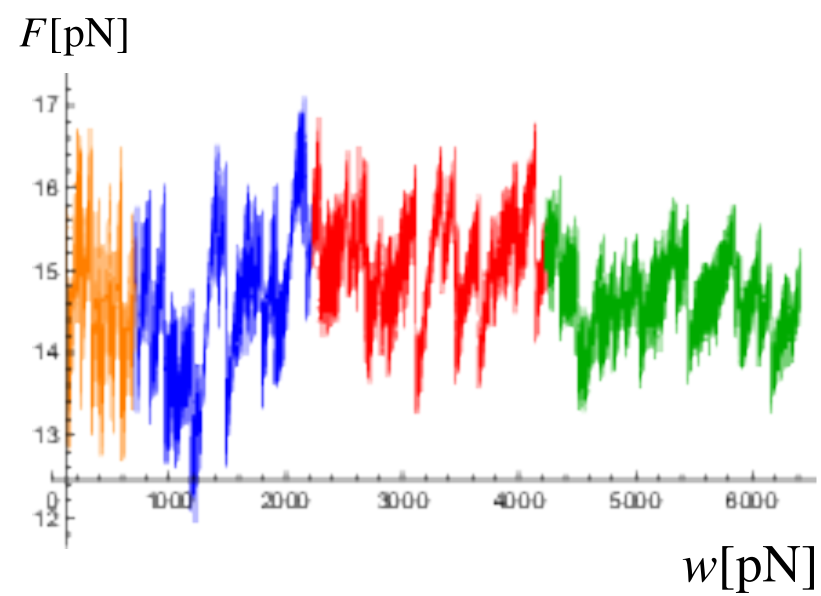


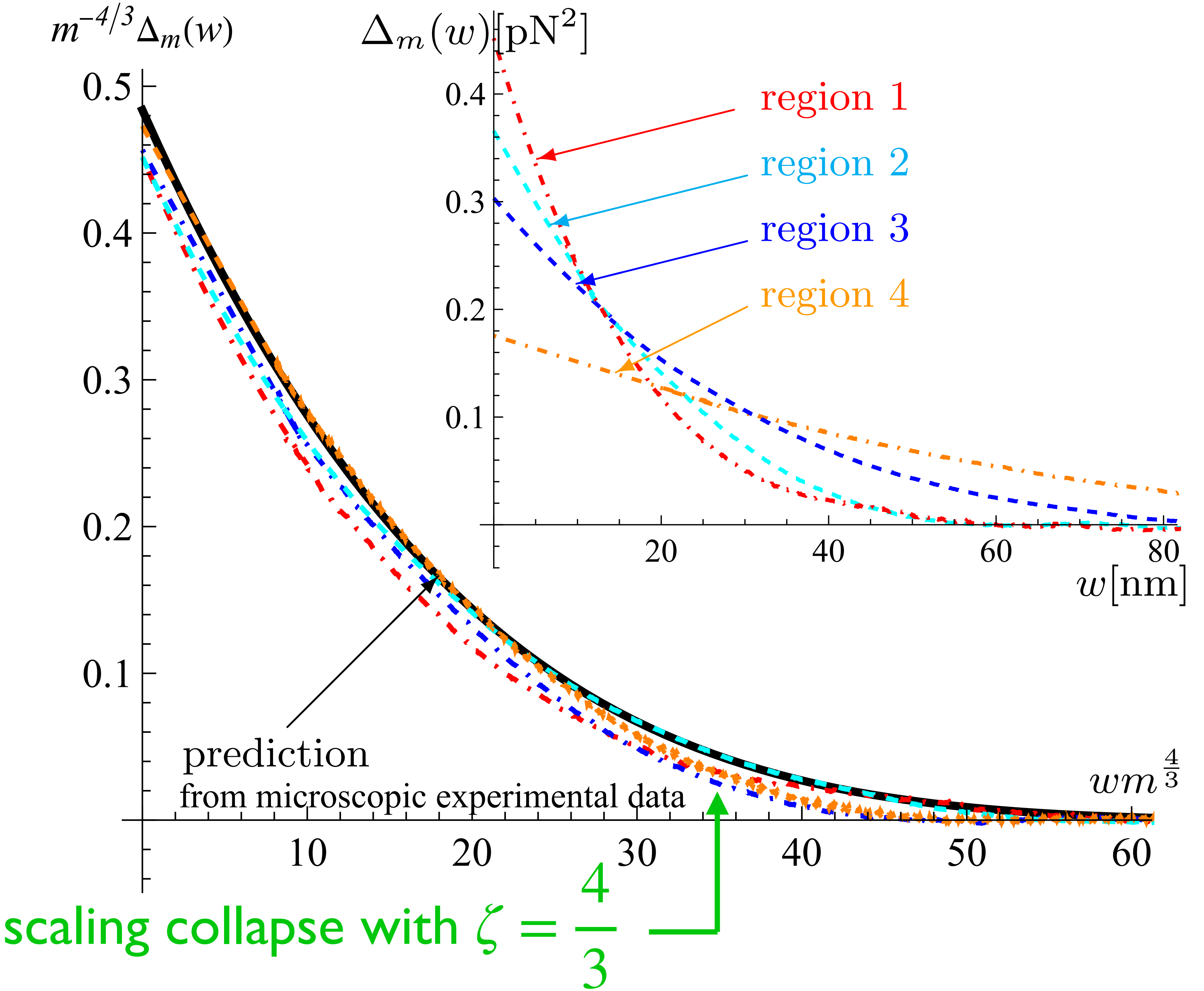
Renormalization in DNA-unzipping









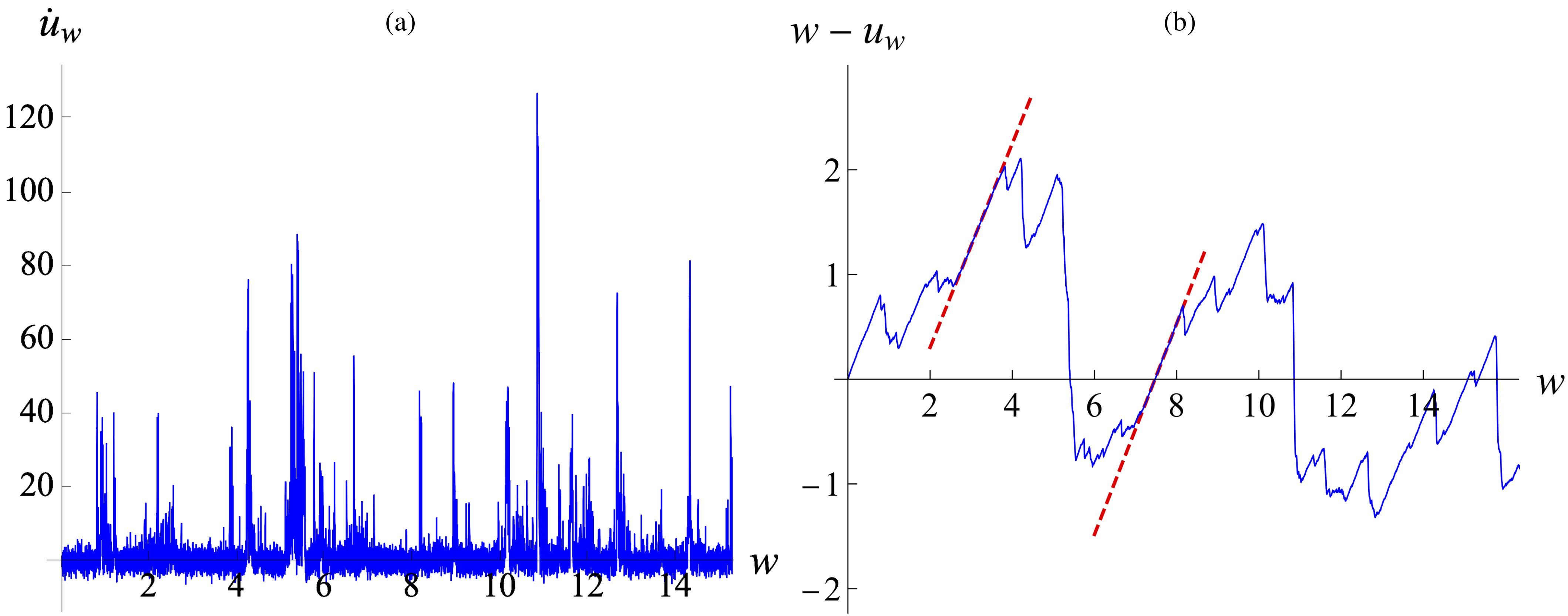


$m^{-4/3}\Delta_m(w)$
0.5
0.4
0.3
0.2
0.1
$\Delta_m(w)[\mathrm{pN}^2]$
0.4
0.3
0.2
0.1
region 1
region 2
region 3
region 4
20
40
60
80
$w[\mathrm{nm}]$
prediction
from microscopic experimental data
$wm^{\frac{4}{3}}$
10
20
30
40
50
60
scaling collapse with $\zeta = \frac{4}{3}$

Magnetic domain walls $(d = 2)$

(data by F. Bohn, G. Durin, R.L. Sommer)

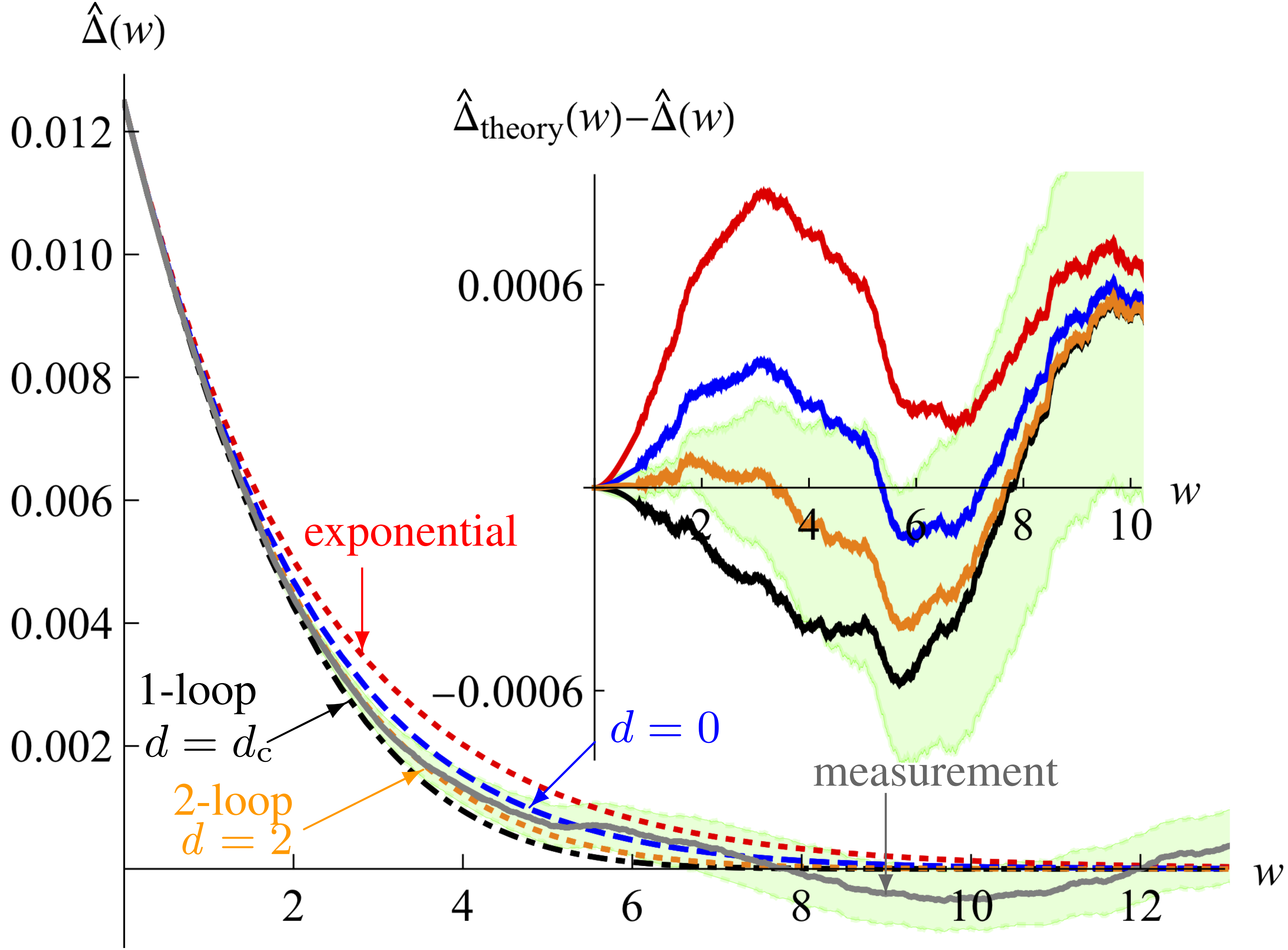
current in a pickup coil .... allows to construct :



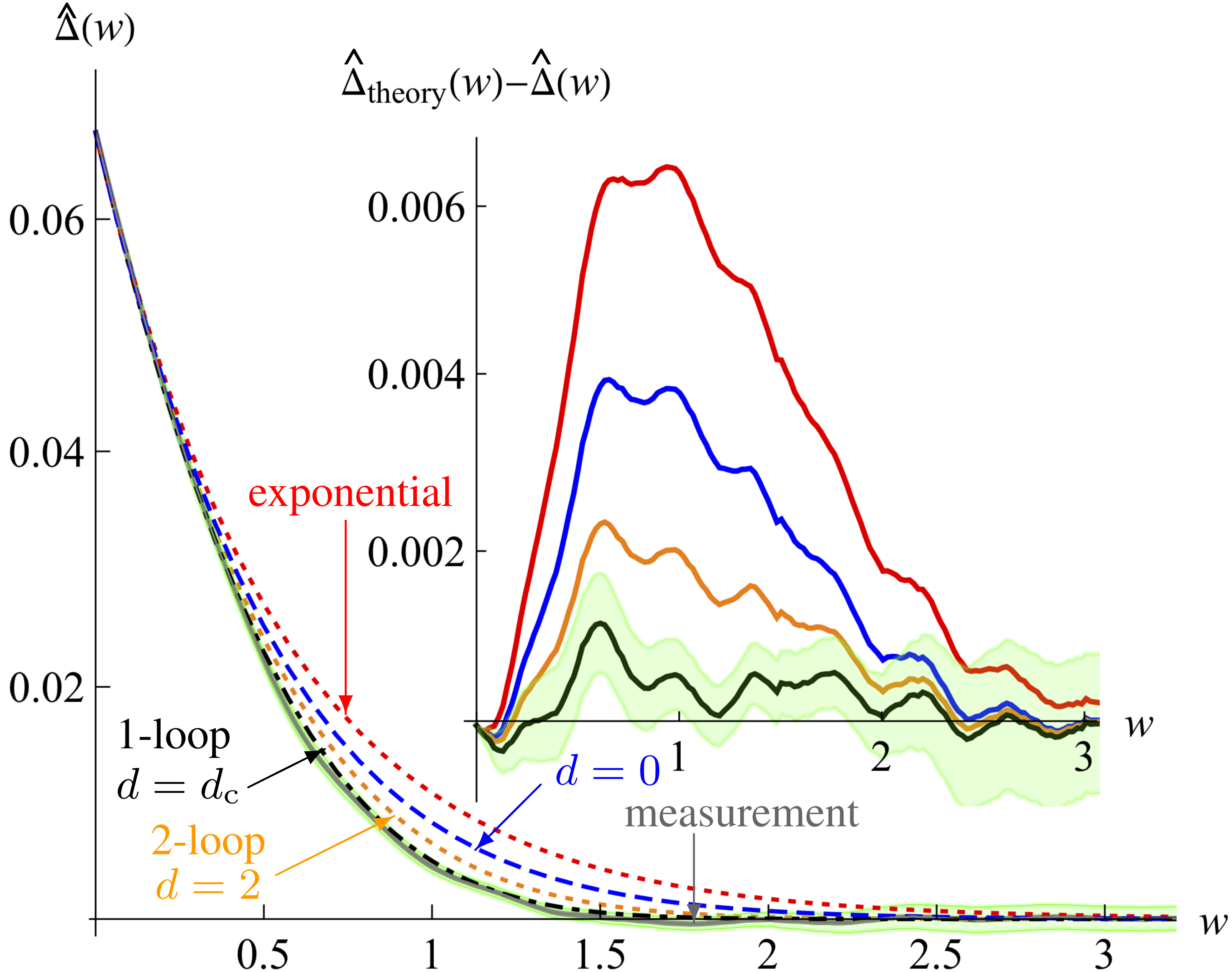
eliminate one unknown scale by the definition

$$\hat{\Delta}_v(w - w') := \overline{\left[w - u_w\right]\left[w' - u_{w'}\right]}^{\mathrm{c}} = \frac{1}{m^4}\overline{F_w F_{w'}}^{\mathrm{c}}$$

Magnetic domain walls SR elasticity ($d = 2$)

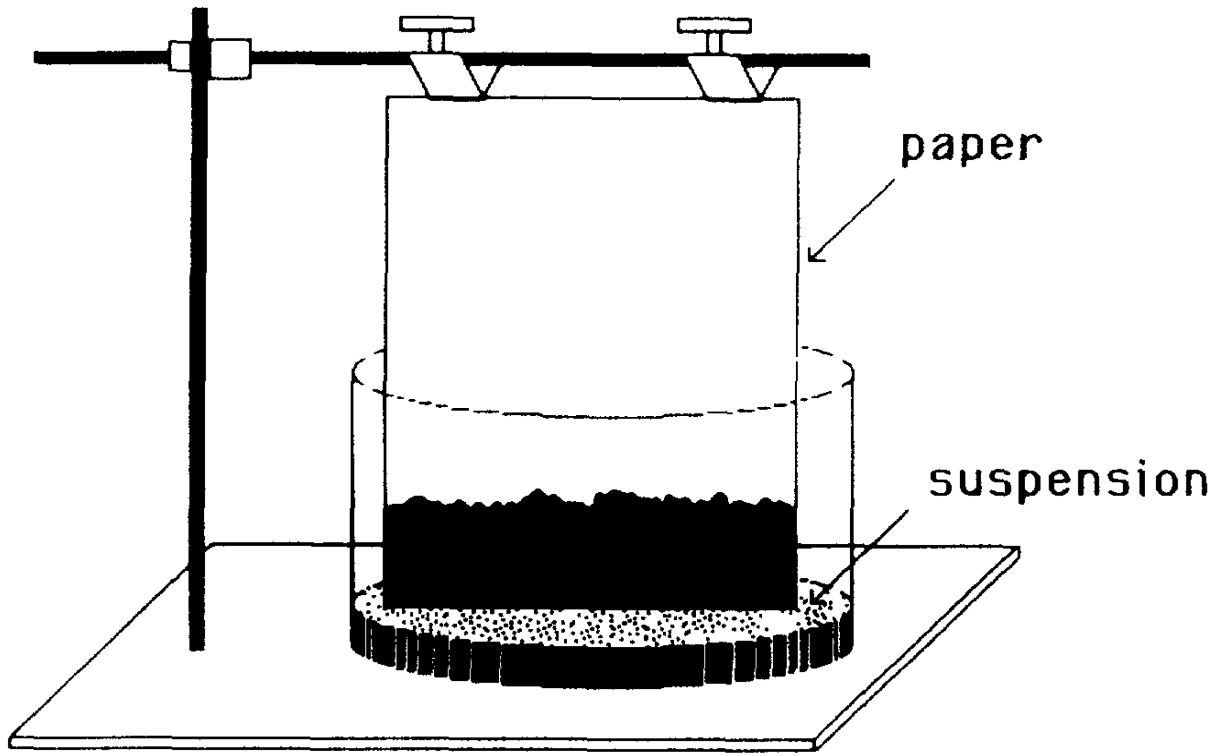


Domain walls ($d = 2$) with long-range elasticity



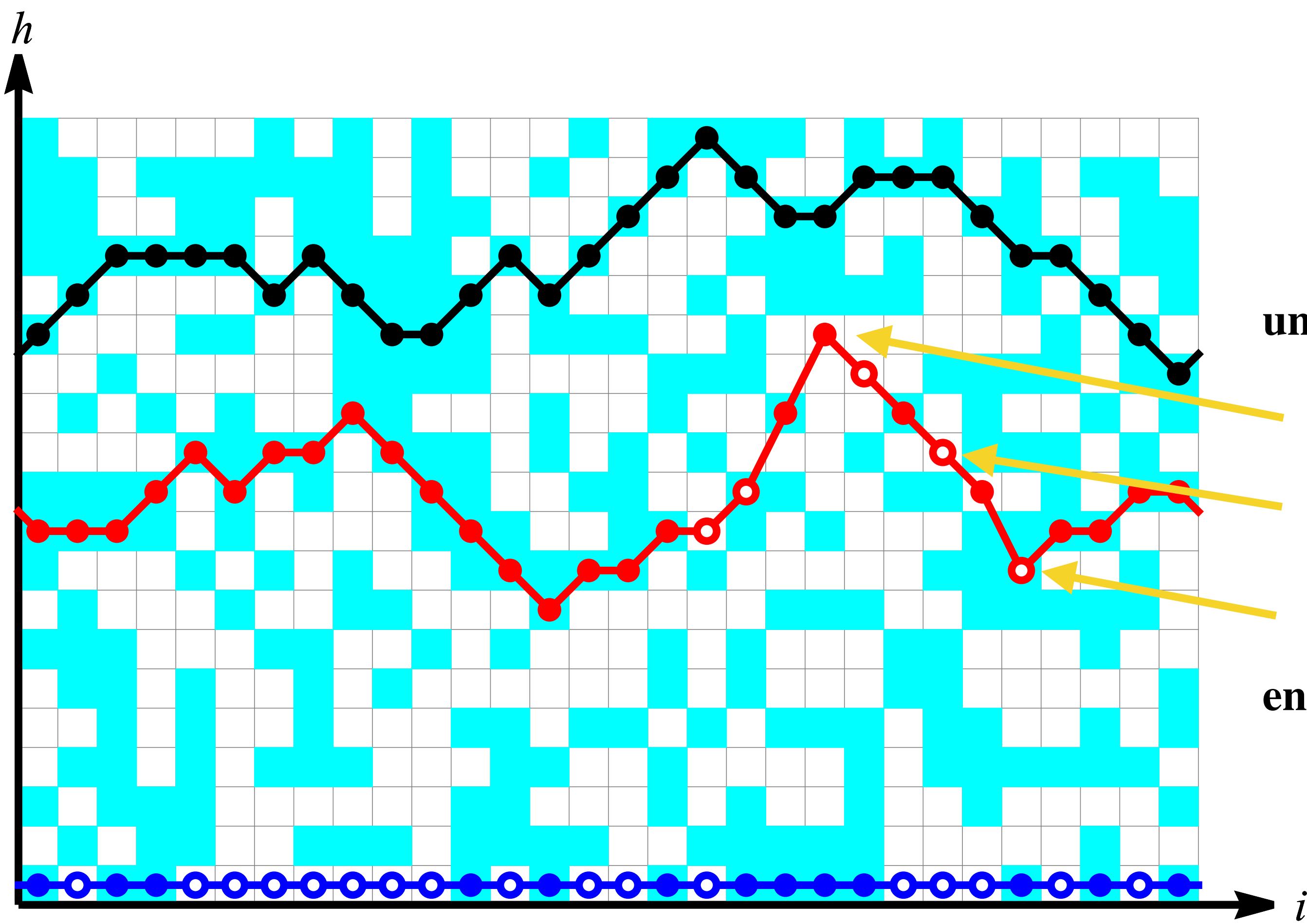
- **1-loop FRG gives fixed point.**
- **this is not ABBM disorder:** $\Delta(0) - \Delta(w) \neq \sigma|w|$
- **ABBM only gives short-scale behavior correctly**

Imbibition



S.V. Buldyrev, et al., Phys. Rev. A 45 (1992) R8313–16.

The Tang-Leschhorn cellular automaton of 1992 TL92

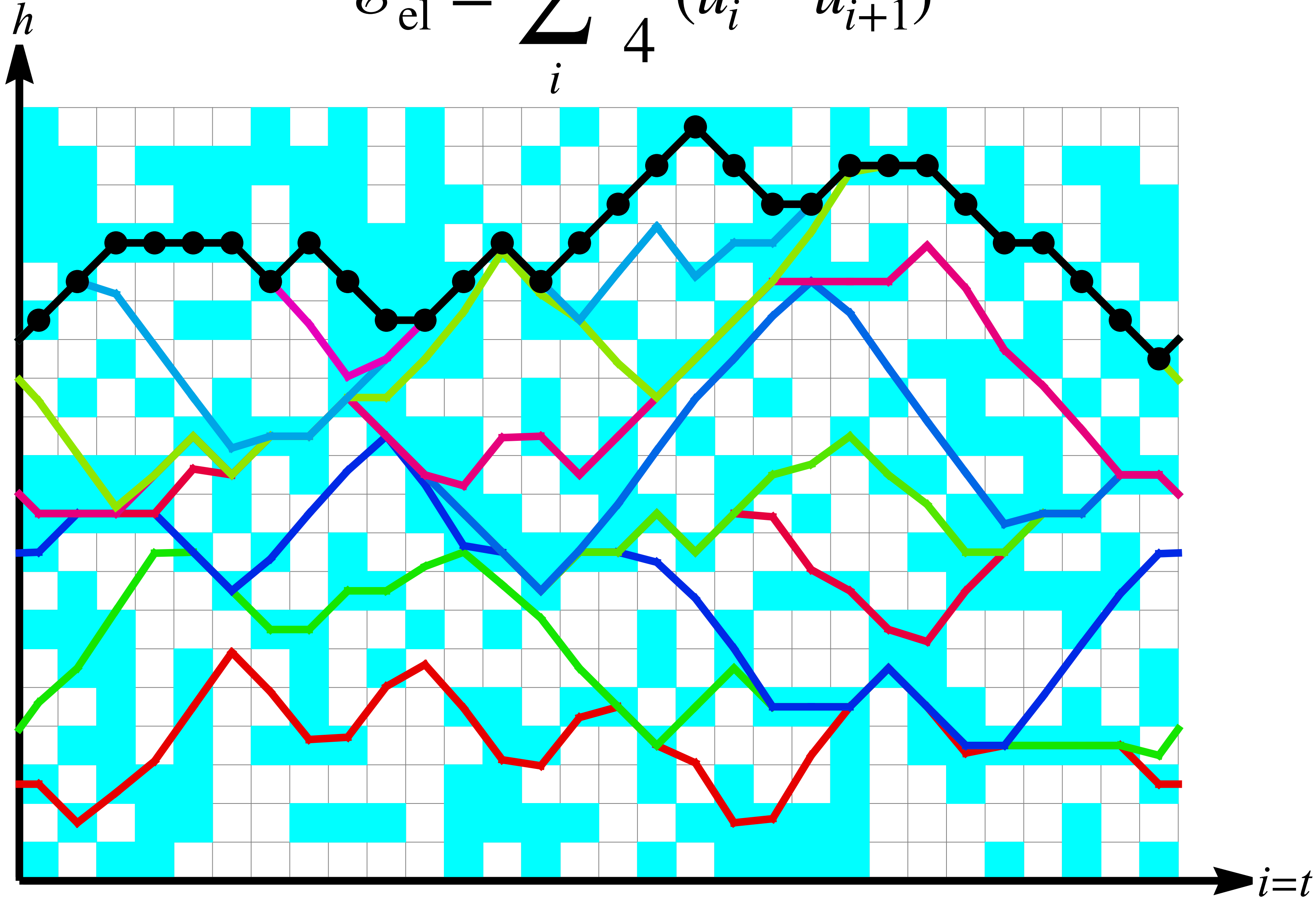


unstable(i)
 # links cannot be longer than 2
if $h(i) - h(\text{neighbor}) \geq 2$ **return** false
 # move forward if open
if $f\big(i, h(i)\big) > f_c$ **return** true
 # move forward if a neighbor is 2 ahead
if $h(\text{neighbor}) - h(i) \geq 2$ **return** true
end

variants: Buldyrev, S. Havlin and H.E. Stanley1992

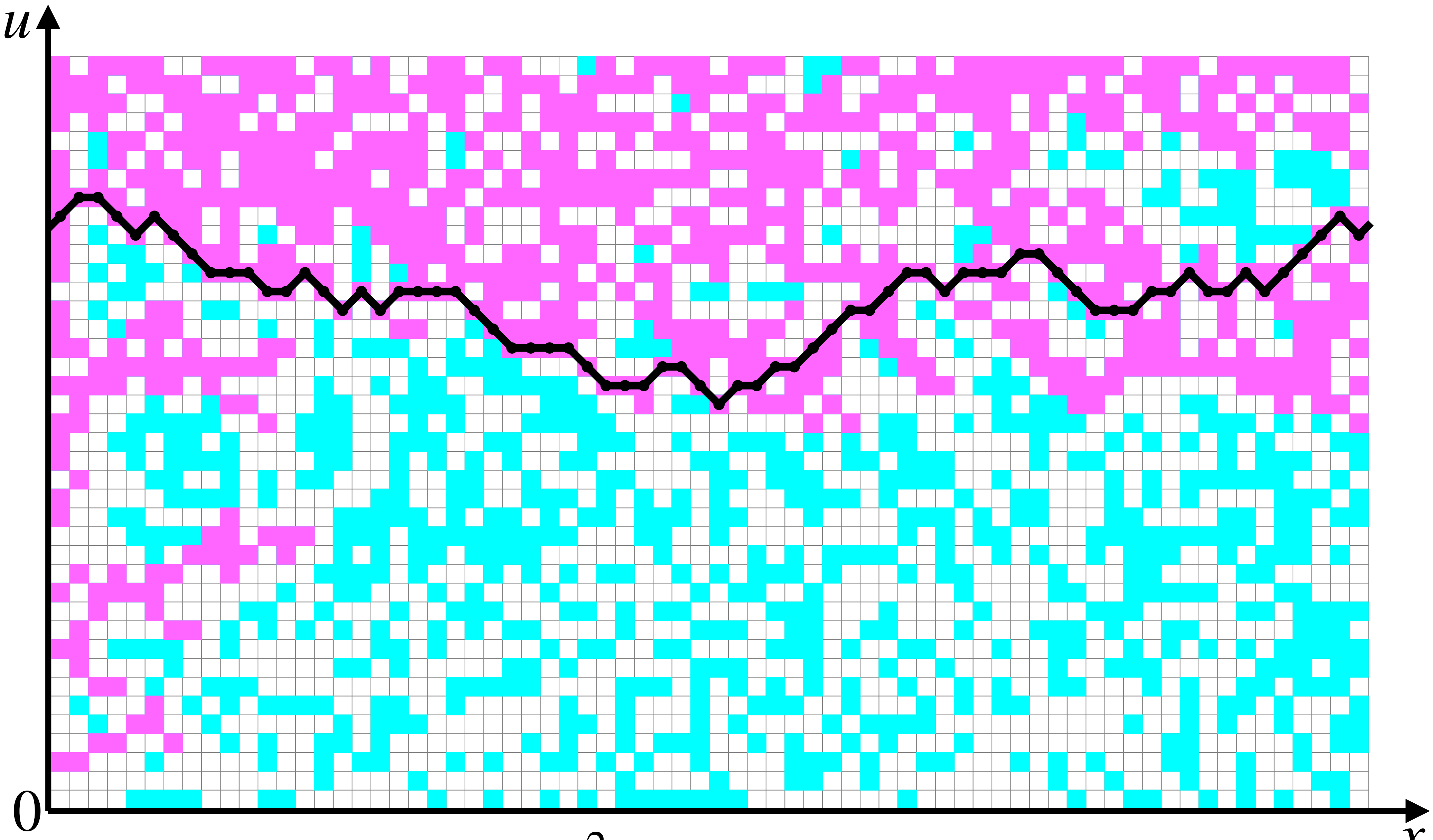
Anharmonic depinning = TL92

$$\mathcal{E}_{\text{el}} = \sum_i \frac{c_4}{4}(u_i - u_{i+1})^4$$



anharmonic depinning respects the Middleton theorem = return point memory (not guaranteed for qKPZ)

TL92 and directed percolation $(d = 1)$

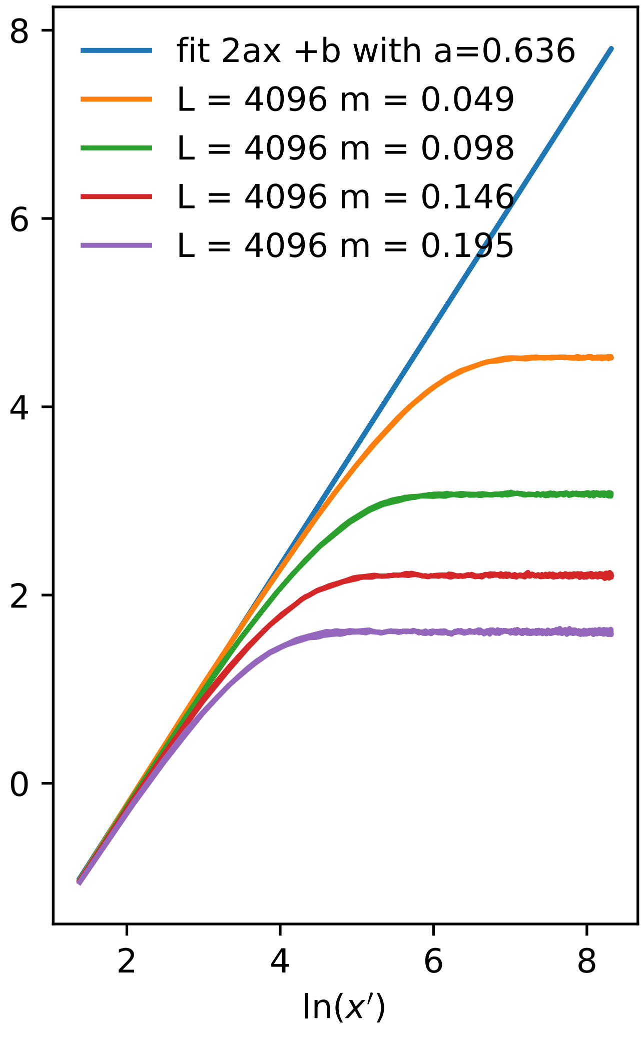


$$f_c := m^2(u - w)$$

used for driving

2-point function

$$\frac{1}{2}\overline{[u(x)-u(y)]^2} \sim \begin{cases} A\,|x-y|^{2\zeta}, & |x-y|<\xi \\ B\,m^{-2\zeta_m}, & |x-y|>\xi \end{cases}$$



from directed percolation

$$\zeta^{d=1} = \frac{\nu_\perp}{\nu_\parallel} = 0.632613(3)$$

$$\zeta_m^{d=1} = \frac{2\nu_\perp}{1+\nu_\perp} = 1.046190(4)$$

two distinct exponents in all d

$$\boxed{\zeta_m > \zeta}$$

What is the appropriate long-distance theory?

Can we measure it?

standard elasticity

$c \to 0$

non-linear elasticity

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + c_4 \nabla[\nabla u(x,t)]^3 - m^2[u(x,t) - w]$$
$$+F\big(x, u(x,t)\big)$$

disorder force

confining potential

background field

What is the appropriate long-distance theory?

Can we measure it?

standard elasticity $c > 0$

non-linear elasticity

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + c_4 \nabla[\nabla u(x,t)]^3 - m^2[u(x,t) - w(x,t)]$$
$$+\lambda[\nabla u(x,t)]^2 + F\big(x, u(x,t)\big)$$

KPZ term

disorder force

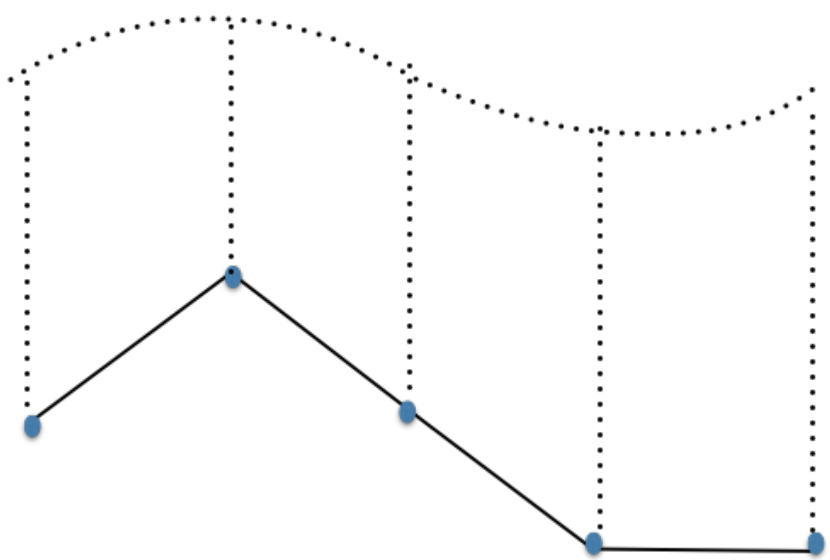
confining potential (unrenormalized)

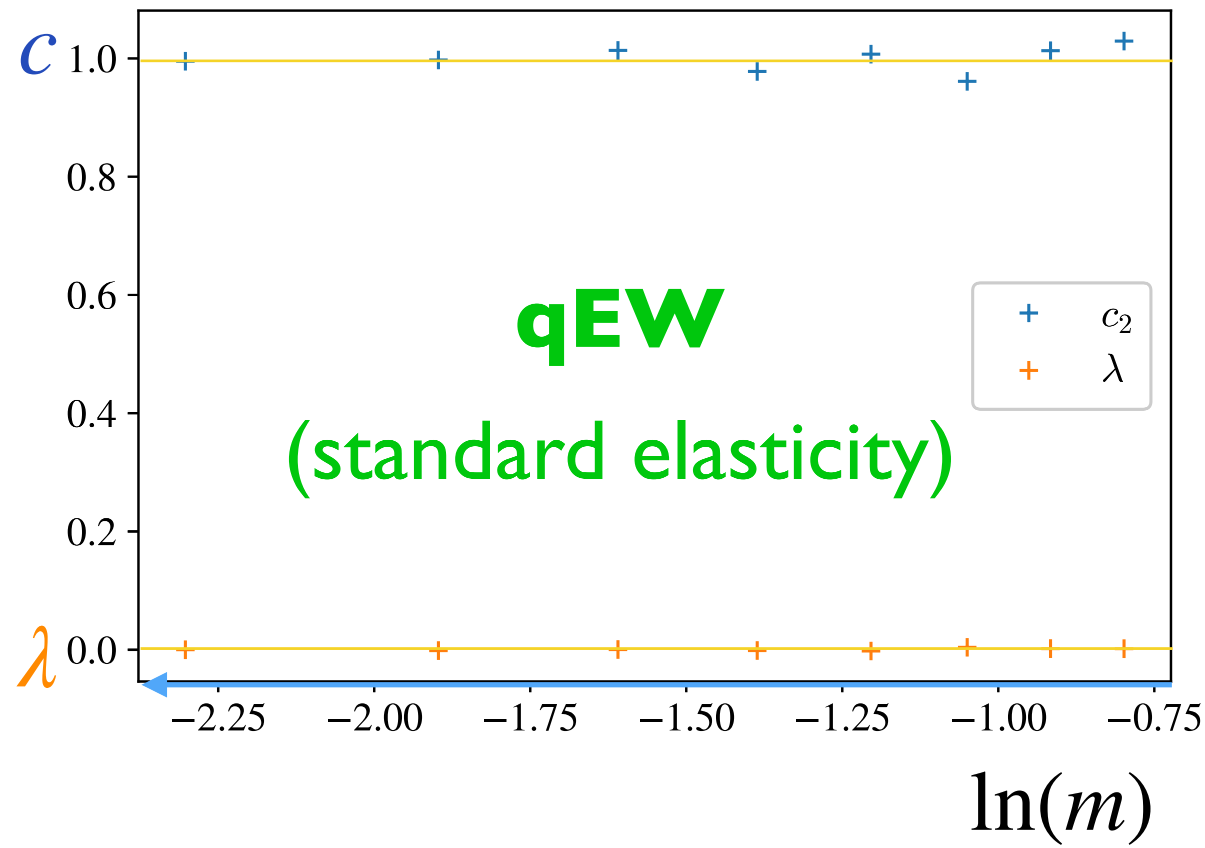
background field (modulated)

Measuring the elastic constants for harmonic depinning (qEW)

$$\eta\partial_t u(x,t) = c\nabla^2 u(x,t) - m^2[u(x,t) - w(x,t)] + F(x,u(x,t))$$

$$w(x) = w_0 + A sin(\frac{\pi x}{L})$$

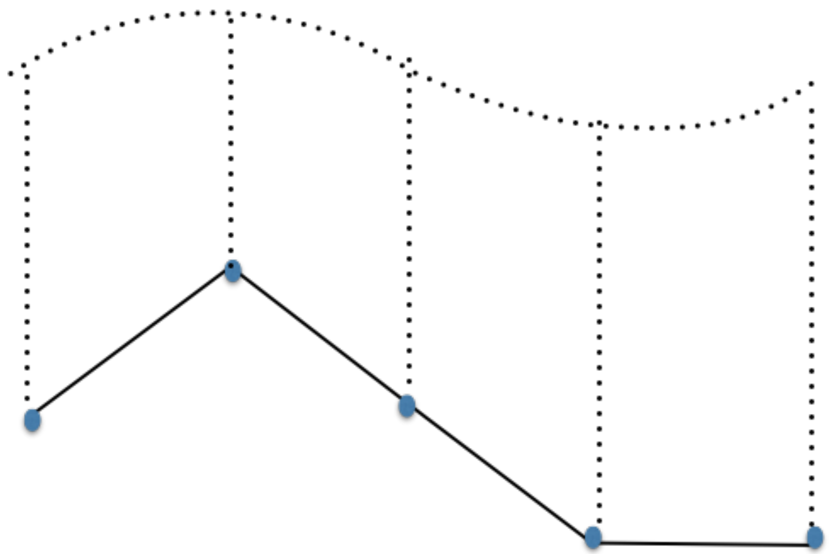


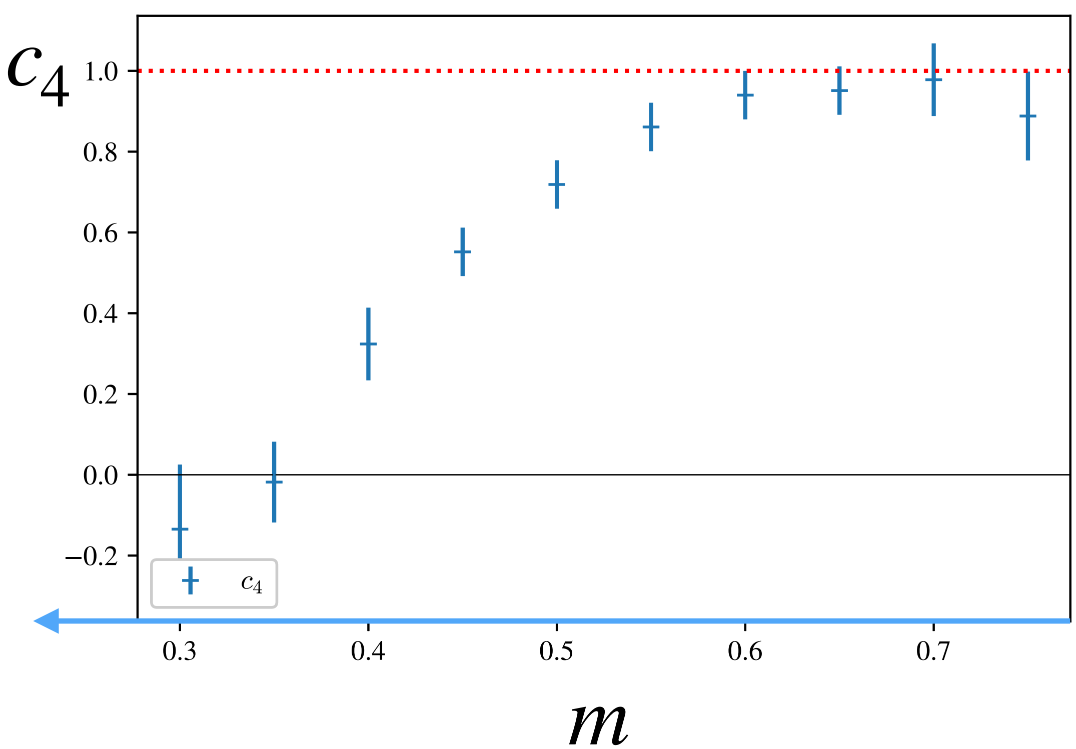


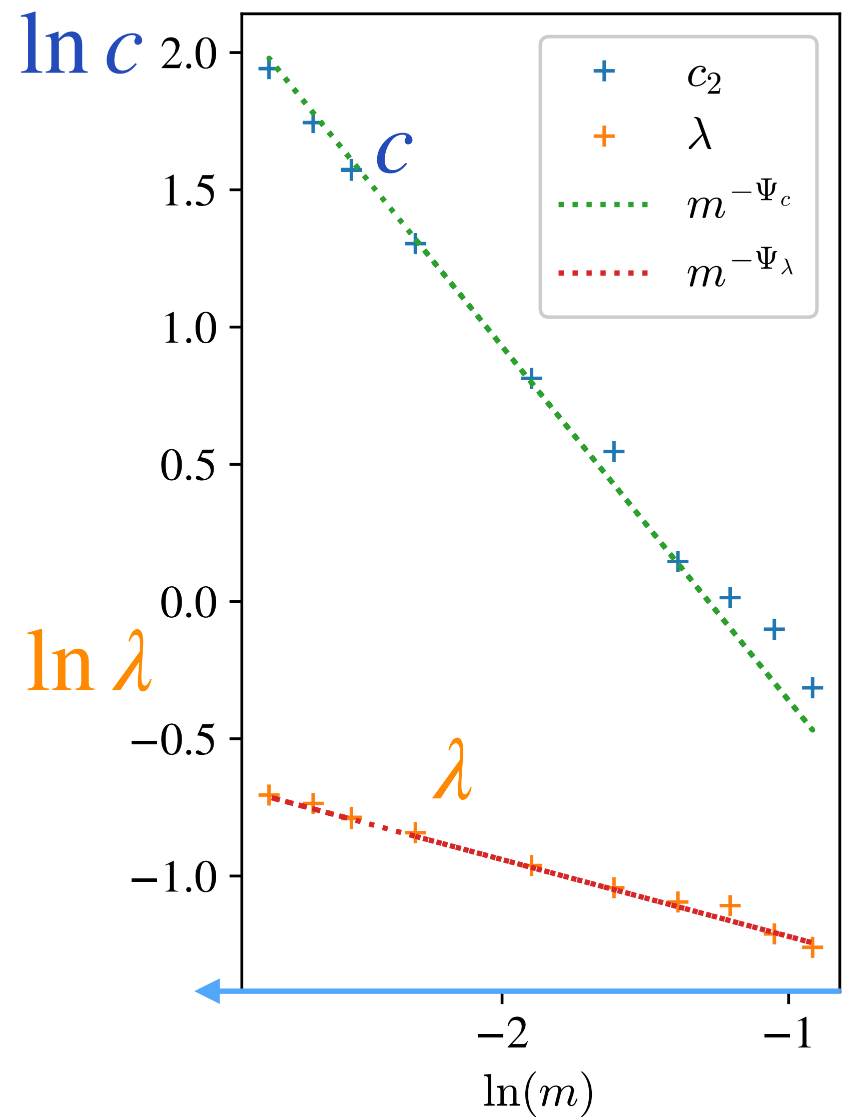
Measuring the elastic constants

anharmonic depinning ($c_4 > 0$)

$$w(x) = w_0 + A sin(\frac{\pi x}{L})$$





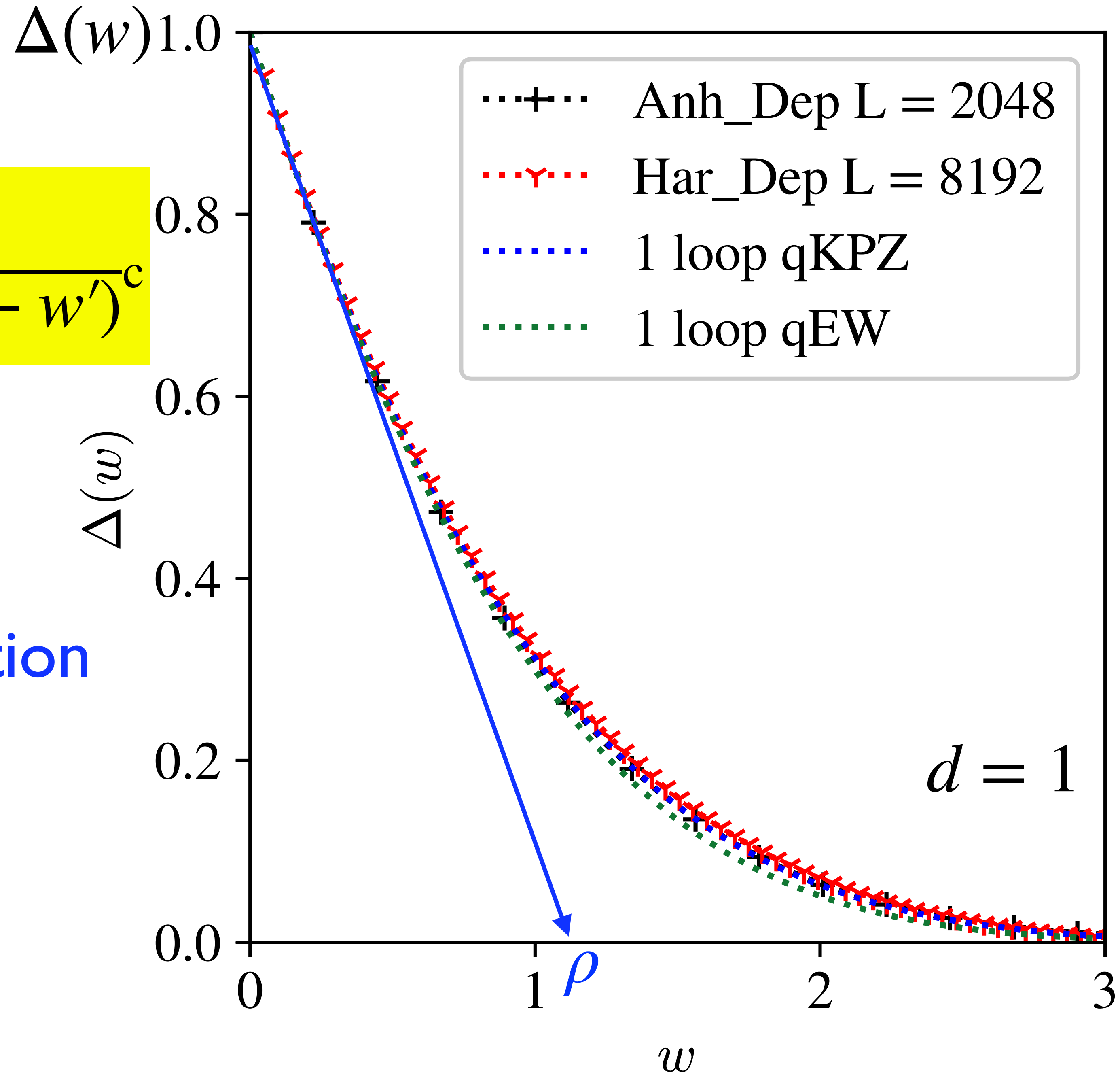


Measuring the effective force correlator

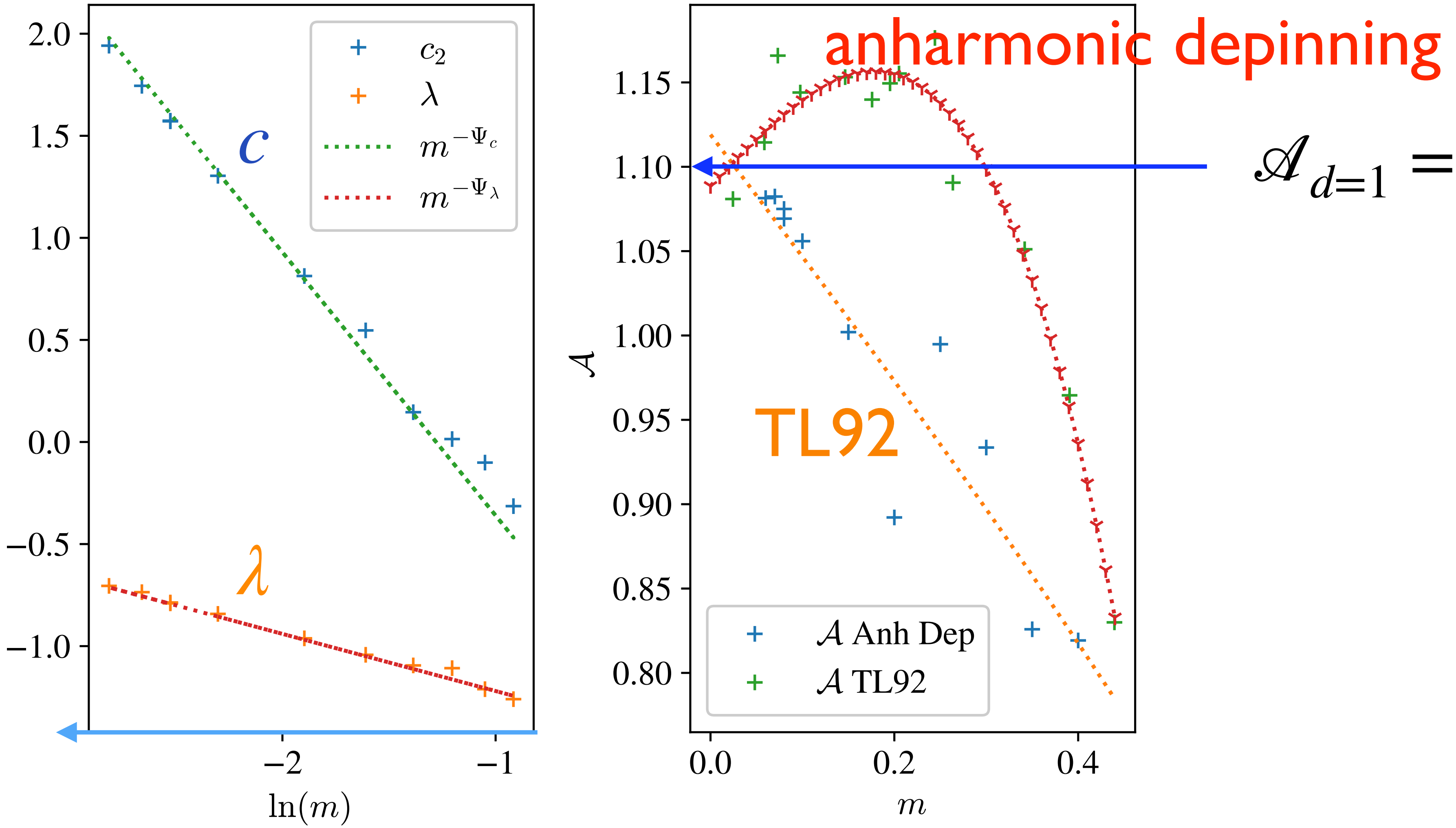
$$\Delta(w-w') = m^4 L^d \overline{(u_w - w)(u_{w'} - w')}^{\mathrm{c}}$$

$$u_w = \frac{1}{L^d}\int_x u_w(x)$$

centre-of-mass position given w



Coupling constant for qKPZ



$$\mathscr{A}_{d=1} = 1.10(2)$$

scale-free universal
KPZ amplitude

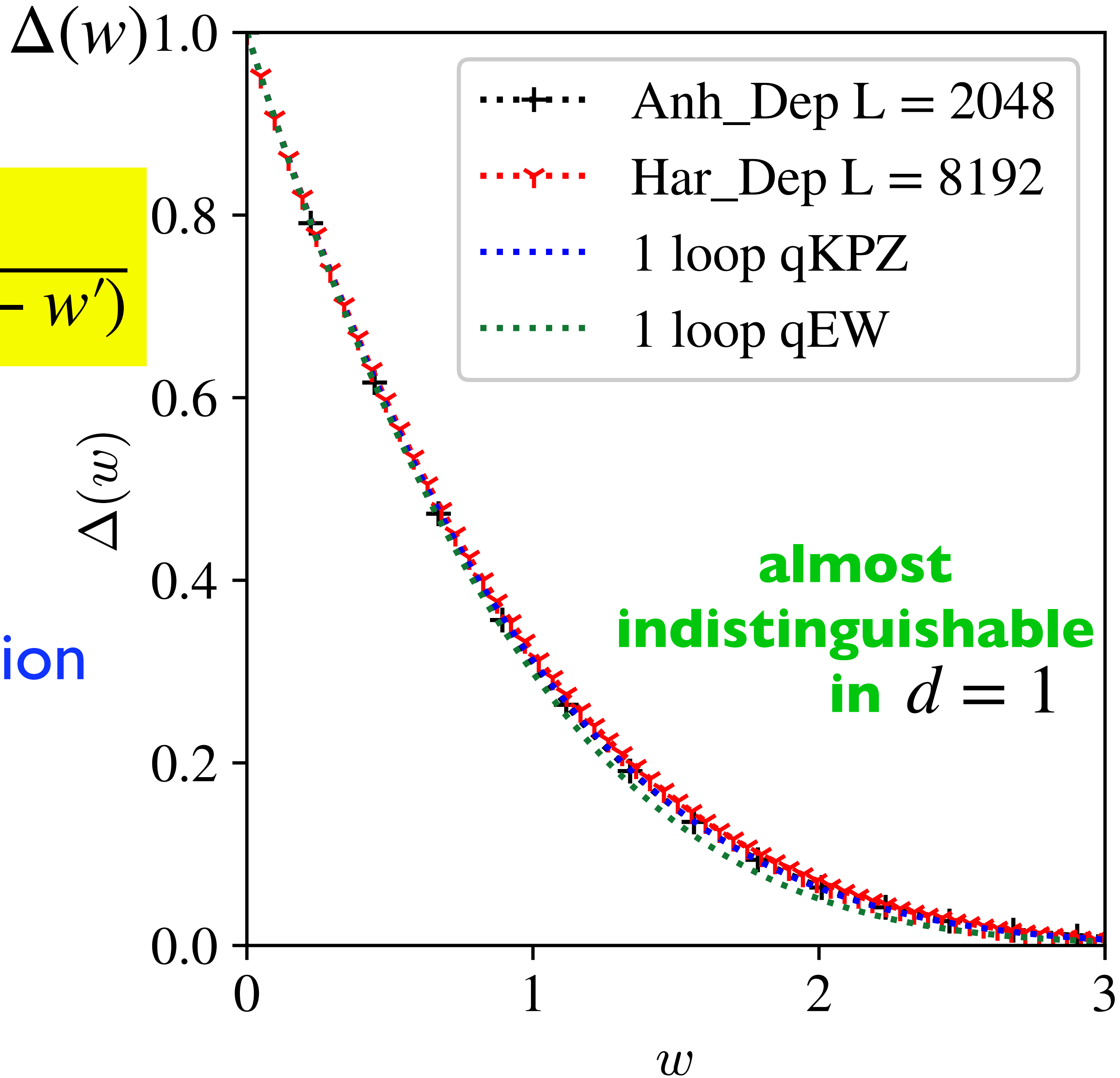
$$\mathscr{A} := \rho \frac{\lambda}{c} \equiv \frac{\Delta(0)}{|\Delta'(0^+)|} \frac{\lambda}{c}$$

Measuring the effective force correlator

$$\Delta(w - w') = m^4 L^d \overline{(u_w - w)(u_{w'} - w')}$$

$$u_w = \frac{1}{L^d} \int_x u_w(x)$$

centre-of-mass position given w



Universality classes for depinning

qKPZ SR-elasticity

$d = 4$ → $d = 3$ → $d = 2$ **magnetic domain wall** → $d = 1$ **imbibition** → $d = 0$

qEW SR-elasticity

$d = 4$ → $d = 3$ **vortex lattice/CDW** → $d = 2$ **magnetic domain wall** → $d = 1$ **magnetic domain wall** → $d = 0$

qEW LR-elasticity

$d = 2$ **magnetic domain walls, earthquakes, knitting** → $d = 1$ **contact line, fracture** → $d = 0$

$d = 0$

analytically solvable: dragged particle (RNA/DNA peeling)

FRG flow equations

shooting parameter

Flow of the disorder for qKPZ

$$\partial_\ell \tilde{\Delta}(u) = \left(4 - d\frac{\zeta_m}{\zeta} - 2\zeta_m\right)\tilde{\Delta}(u) + u\zeta_m \tilde{\Delta}'(u)$$
$$+\frac{d(d+2)}{12}\tilde{\lambda}^2\tilde{\Delta}(u)^2 - \tilde{\Delta}'(u)^2 - \tilde{\Delta}''(u)\left[\tilde{\Delta}(u) - \tilde{\Delta}(0)\right]$$

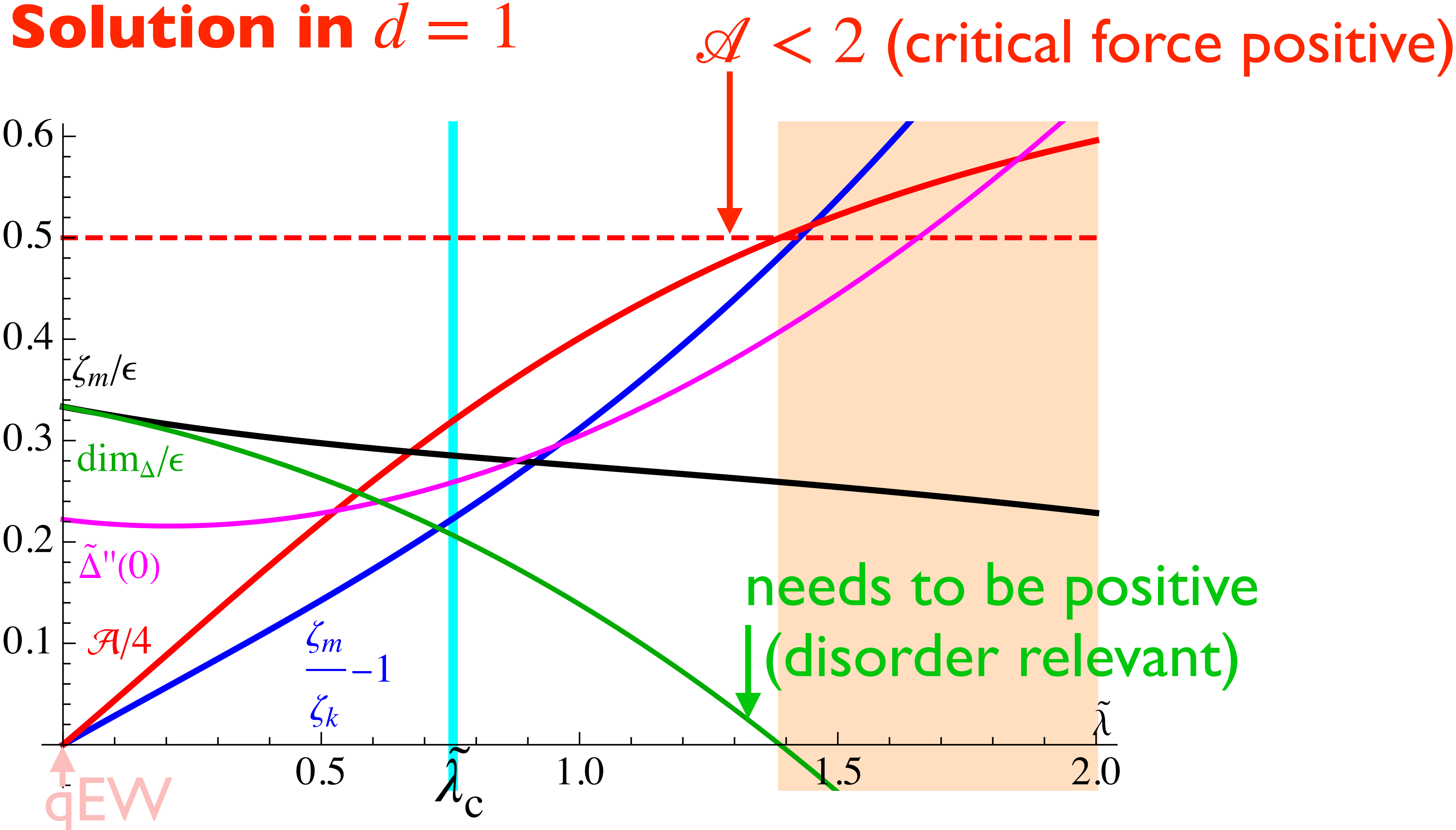
replace ζ_m/ζ

$$\frac{\zeta_m}{\zeta} = 1 + \frac{1}{2}\left[-\tilde{\lambda}\tilde{\Delta}'\left(0^+\right) - \frac{d-1}{3}\tilde{\lambda}^2\tilde{\Delta}(0)\right].$$

flow for λ (with confining potential, i.e. massive theory)

$$-m\partial_m\tilde{\lambda} = \zeta_m\tilde{\lambda} - \frac{4-d}{6}\tilde{\lambda}^3\tilde{\Delta}(0) \implies \tilde{\lambda}_c = \sqrt{\frac{6\zeta_m}{(4-d)\tilde{\Delta}(0)}}$$

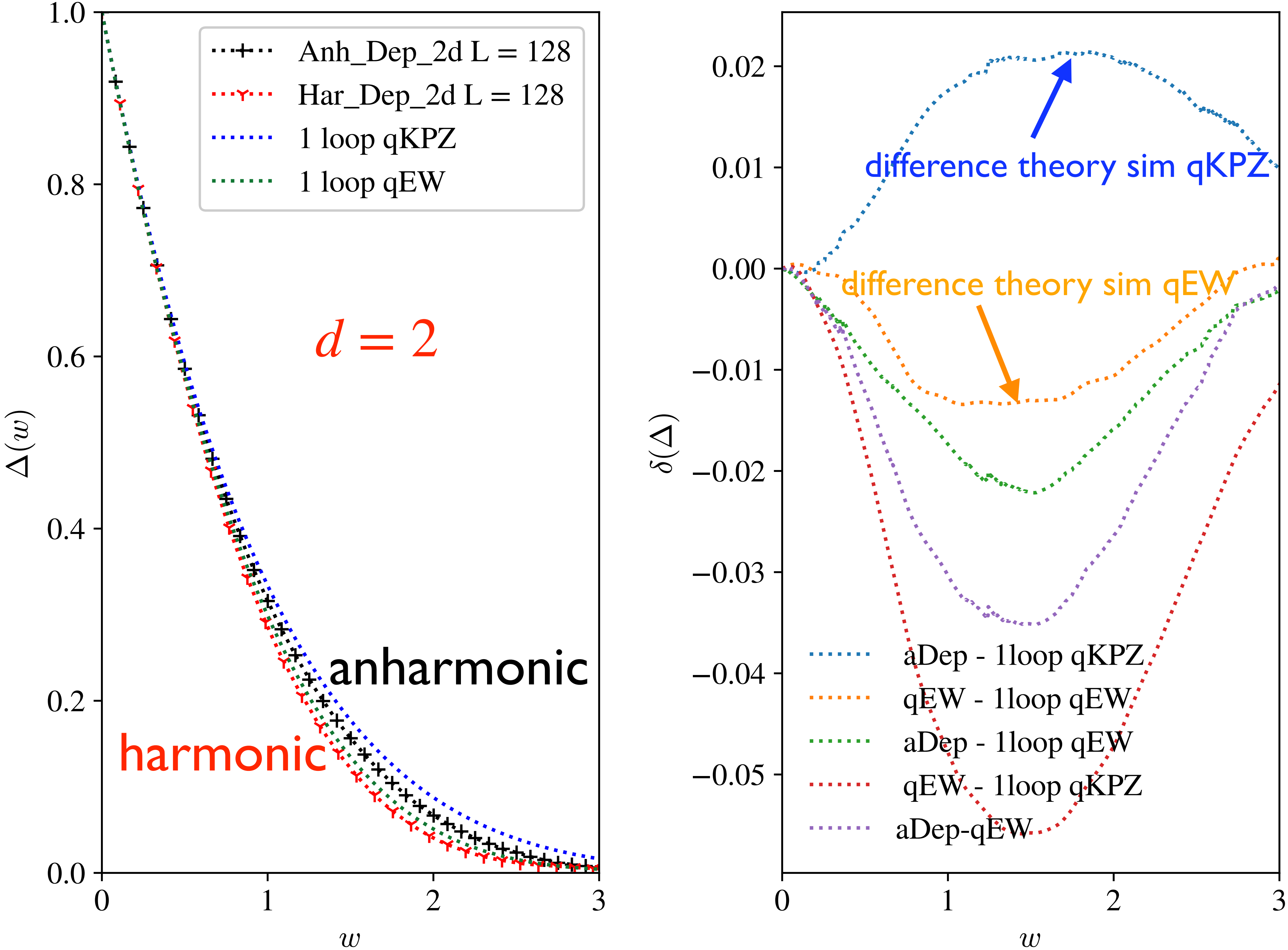
Solution in $d=1$



RG: $\zeta_m^{d=1} = 0.86$, $\zeta^{d=1} = 0.69$, $z^{d=1} = 1.27$, $\mathscr{A}^{d=1} = 1.27$

numerics: $\zeta_m^{d=1} = 1.05$, $\zeta^{d=1} = 0.63$, $z^{d=1} = 1.10(2)$, $\mathscr{A}^{d=1} = 1.10(2)$

Shape of $\Delta(w)$ different in $d = 2$



Conclusions

- when in doubt: measure effective long-distance action (= theory/description)
- standard elastic depinning (**qEW**) has non-trivial disorder correlator given by FRG
- imbibition (e.g. TL92), anharmonic depinning and qKPZ all belong to the same universality class: the effective long-wavelength theory is **qKPZ**
- you need to introduce a confining potential $m^2[w - u(x,t)]$ to measure disorder correlations
  $\Rightarrow$ give up the Cole-Hopf transform
  $\Rightarrow$ yields an RG fixed point
- a field theory can be build